CESifo Working Papers

www.cesifo.org/wp

A Game-Theoretic Foundation for the Wilson Equilibrium in Competitive Insurance Markets with Adverse Selection

Wanda Mimra
Achim Wambach

CESIFO WORKING PAPER NO. 3412
CATEGORY 11: INDUSTRIAL ORGANISATION
APRIL 2011

CESifo *Working Paper No. 3412*

A Game-Theoretic Foundation for the Wilson Equilibrium in Competitive Insurance Markets with Adverse Selection

Abstract

We extend the seminal Rothschild and Stiglitz (1976) model on competitive insurance markets with asymmetric information in the spirit of Wilson (1977)'s 'anticipatory equilibrium' by introducing an additional stage in which initial contracts can be withdrawn after observation of competitors' contract offers. We show that an equilibrium always exists where consumers obtain their respective Wilson-Miyazaki-Spence (WMS) contract. Jointly profit-making contracts can also be sustained as equilibrium contracts. However, the second-best efficient WMS allocation is the unique equilibrium allocation under entry.

JEL-Code: C720, D820, G220, L100.

Keywords: asymmetric information, competitive insurance market, contract withdrawal.

Wanda Mimra
University of Cologne
Department of Economics
Gyrhofstrasse 8c
50931 Cologne
Germany
mimra@wiso.uni-koeln.de

Achim Wambach
University of Cologne
Department of Economics
Gyrhofstrasse 8c
50931 Cologne
Germany
wambach@wiso.uni-koeln.de

March 2011
We would like to thank Keith Crocker, Vitali Gretschko, Tore Nilssen, Andreas Richter, Klaus Ritzberger and seminar participants at Bonn, Frankfurt and Toulouse for helpful comments and discussions.

1 Introduction

The Rothschild and Stiglitz (1976) model on competitive insurance markets with adverse selection is widely considered as one of the seminal works on asymmetric information common value markets besides Akerlof (1970). Yet, it entails a puzzle: an equilibrium in pure strategies may fail to exist altogether. To be precise, Rothschild and Stiglitz (1976) show in a simple screening game in which insurers offer contracts first and then consumers choose, that if the share of high risk types is low, the candidate separating, separately zero-profit making Rothschild-Stiglitz (RS) contracts cannot be tendered in equilibrium as they might be overturned by a pooling contract. However, pooling cannot be an equilibrium as insurers would try to cream skim low risks. This potential non-existence of equilibrium has received much attention ever since; subsequent research has addressed the non-existence problem by considering mixed strategies (Dasgupta and Maskin, 1986), introducing equilibrium concepts that differ from Nash-equilibrium (Wilson 1977; Riley 1979), extending the dynamic structure of the game (Jaynes 1978; Hellwig 1987; Engers and Fernandez 1987; Asheim and Nilssen 1996) or modifying assumptions about insurer or contract characteristics (Inderst and Wambach 2001; Faynzilberg 2006; Picard 2009; Mimra and Wambach 2010).[1]

To date, due to its intuitive appeal, one of the most referred to solutions is still the Wilson (1977) equilibrium. Wilson (1977) considers the RS game structure, but introduces the 'anticipatory equilibrium' concept: In this concept, an expectation rule is imposed such that "each firm assumes that any policy will be immediately withdrawn which becomes unprofitable after that firm makes its own policy offer". Wilson shows that the anticipatory equilibrium concept leads to a pooling equilibrium in which low risk utility is maximized subject to a zero-profit condition. Extending the analysis to contract menus, Miyazaki (1977) and Spence (1978) show that the anticipatory equilibrium concept results in an allocation with separating, cross-subsidizing, jointly zero-profit making contracts that are second-best efficient, the famous Wilson-Miyazaki-Spence (WMS) contracts. However, surprisingly, despite its appeal the logic behind the WMS equilibrium lacks a sound game-theoretic

[1] There are yet methodically different strands in the literature. Ania, Tröger, and Wambach (2002) take an evolutionary game theory approach, Guerrieri, Shimer, and Wright (2010) consider a competitive search model; other directions use cooperative concepts (Lacker and Weinberg, 1999) or a general-equilibrium framework (see e.g. Dubey and Geanakoplos (2002) or Bisin and Gottardi (2006)).

foundation to date.[2]

The present paper spells out the idea by introducing an additional stage into the RS model in which firms can withdraw contracts (repeatedly) before consumers make their choice but after observing the contract offers of competitors. We show that an equilibrium always exists where every consumer obtains her respective WMS contract. Intuitively, the possibility of contract withdrawal prevents cream-skimming deviations that upset the WMS contracts in the original RS set-up. However, to sustain the WMS allocation, not only the WMS contracts, but also a continuum of low risk contracts as well as the RS contracts have to be on offer as latent contracts. The reason is that the explicit possibility to withdraw contracts allows for sophisticated deviating strategies that are prevented by latent contracts that, off the equilibrium path, attract low risks away from such possible deviations.[3]

We show moreover that, besides the WMS contracts, profit-making contracts can also be enforced as equilibrium contracts as the possibility to retract contracts provides firms with adequate threat points. More generally, contract withdrawal leads to a multiplicity of equilibrium allocations. This multiplicity remains if, instead of only considering contract withdrawal, we allow for the addition of contracts in the second stage.[4] We then extend the game to allow for entry as would be expected in a model of a competitive market. Then, positive profits cannot be sustained in equilibrium. More strongly, the WMS allocation is generically unique under entry.

There is a small literature where contract withdrawal is added to a market with adverse selection. This literature differs from the present work in that while we allow the withdrawal of individual contracts to model the logic behind the Wilson equilibrium, contract withdrawal in the literature so far implies exit from the market, i.e. only complete contracts withdrawal. One contribution is Hellwig (1987), which furthermore differs in the timing of the game: firms make single contract offers and may decline to fulfill a contract and thus exit the market after consumers have already cho-

[2]Some, in particular more recent research on the equilibrium inexistence problem yields the WMS allocation (Asheim and Nilssen 1996, Picard 2009 and Mimra and Wambach 2010), however, the economics in these models is quite different.

[3]Latent contracts are not new in adverse selection environments. Attar, Mariotti, and Salanie (2009) model nonexclusive competition in an adverse selection market. In their model, infinitely many contracts need to be issued as latent contracts to sustain the equilibrium allocation.

[4]The addition of contracts, instead of withdrawal, is at the heart of Riley (1979)'s 'reactive equilibrium' concept. In their discussion on markets and contracts, Bolton and Dewatripont (2005) state that "it remains an open question whether either the Wilson or Riley equilibrium would continue to exist in the larger, more natural, game where in stage 2 offers can be added or withdrawn." We show that both allocations can be sustained in equilibrium.

sen their insurance contract. Hellwig (1987) shows that the Wilson pooling contract corresponds to a stable equilibrium of this three-stage game. However, when allowing for contract menus and individual contract withdrawal, the WMS contracts do not constitute equilibrium contracts in Hellwig's game as any firm would have an incentive to withdraw the loss-making high risk contract. A study with a similar timing structure as ours is Netzer and Scheuer (2008). In a model of moral hazard without commitment, Netzer and Scheuer (2008) model competition after unobservable effort choice such that firms offer contract menus and can, after observation of competitor's offers, decide to exit the market before consumers choose contracts. Again, contrary to our model, the restriction to exit does not allow to withdraw individual loss-making contracts. The consideration of individual contract rather than complete contracts withdrawal relates to a more general problem: why should a firm in a competitive market offer a loss-making contract? We show that, even when individual contracts can be withdrawn, firms may offer loss-making contracts in a competitive market.

The rest of the paper is organized as follows: In the next section, the model is introduced. Section 3 provides an existence result for the WMS equilibrium, and section 4 derives an equilibrium with positive profits. In Section 5, the analysis is extended to allow for the addition of contracts. Section 6 introduces entry and establishes that the unique equilibrium allocation with entry are the WMS contracts.

2 The model

The set-up closely follows Rothschild and Stiglitz (1976) and Wilson (1977): There is a continuum of individuals with mass 1. Each individual faces two possible states of nature: In state 1, no loss occurs and the endowment is w_{01}, in state 2 a loss occurs and the endowment is w_{02} with $w_{01} > w_{02} > 0$. There are two types of individuals, an individual may be a high risk type (H) with loss probability p^H, or a low-risk type (L) with loss probability p^L, with $0 < p^L < p^H < 1$. Insurance is provided by firms in the set $F := \{1, ..., f, ...n\}$. Firms do not know, ex ante, any individual's type. If an individual buys insurance, then the initial endowment $\omega_0 = (w_{01}, w_{02})$ is traded for another state-contingent endowment $\omega = (w_1, w_2)$; we say the individual buys insurance contract ω. The set of feasible contracts, Ω, is given by $\Omega := \{(w_1, w_2) \,|w_1 \geq w_2 > 0\}$ where $w_1 < w_2$ is ruled out for moral hazard considerations. The expected utility of a J-type individual, $J \in \{H, L\}$ from chosing a contract $\omega \in \Omega$ is abbreviated by $u^J(\omega) := (1 - p^J)v(w_1) + p^J v(w_2)$ where v is a strictly increasing, twice continuously

differentiable and strictly concave von Neumann-Morgenstern utility function.
The timing of the game is as follows: First, firms set contracts simultaneously and observe their competitors' contract offers. Then, firms can withdraw contracts potentially repeatedly for several rounds whereby firms observe their competitors remaining contract offers after each round. Contract withdrawal is possible as long as at least one contract was withdrawn by any firm in the previous round. After contract withdrawal ends, consumers make their contract choice. Formally, the game proceeds as follows:

Stage 0: The risk type of each individual is chosen by nature. Each individual has a chance of γ, $0 < \gamma < 1$ to be a H-type, and of $(1-\gamma)$ to be a L-type.

Stage 1: Each firm $f \in F$ offers an initial set of contracts $\Omega_0^f \subset \Omega$. The offered sets are observed by all firms before the beginning of the next stage.

Stage 2: Stage 2 consists of $t = 1, 2, ...$ rounds. In each round t, each firm $f \in F$ can withdraw a set from its remaining contracts. After each round, firms observe the remaining contract offers of all firms. Denote by Ω_t^f firm f's contract set on offer at the end of t; $\Omega_t^f \subseteq \Omega_{t-1}^f$. If, for any t, $\Omega_t^f = \Omega_{t-1}^f$ for all $f \in F$, this stage ends. Denote the final round in stage 2 by $\hat{t}$.

Stage 3: Individuals choose among the remaining contracts $\bigcup\limits_F \Omega_{\hat{t}}^f$ or remain uninsured.

Before proceeding, let us discuss the difference of our setup to Rothschild-Stiglitz and how this implements the Wilson concept: The Rothschild-Stiglitz game corresponds to stages $0, 1$ and 3. In this reduced game, a pooling contract or more generally cross-subsidizing contracts cannot be sustained as equilibrium contracts as insurers would always try to cream skim low risks. In Wilson's 'anticipatory equilibrium' concept, such cream skimming deviations are not profitable because the expectation rule is that cross-subsidized contracts at non-deviating insurers would be withdrawn since they become unprofitable after introduction of the cream-skimming contract. We implement this concept by adding stage 2. However, when instead of imposing an expectation rule, firms are explicitly allowed to withdraw contracts after observation of competitor's contract offers, for the Wilson reasoning to hold in a game with contract menus contract withdrawal has to end endogenously as in our model specification: with a fixed number of withdrawal rounds, a single firm would always be able to profitably deviate by withdrawing a cross-subsidized contract in the last round.

When stage 2 ends after round $\hat{t}$ and contract $\omega_j^f \in \Omega_{\hat{t}}^f$ is taken out by a mass of individuals λ_j^f among which the share of H-types is σ_j^f, then the expected profit of firm $f \in F$ is:

$$\pi^f = \int_{\Omega_{\hat{t}}^f} \lambda_j^f \left[(w_{01} - w_{j,1}^f) - (p^H \sigma_j^f + p^L(1 - \sigma_j^f))(w_{j,2}^f - w_{02} + w_{01} - w_{j,1}^f) \right] d\omega$$

As we did not restrict the sets of contract offers in stage 1 to be finite, stage 2 does not necessarily end. For $t \to +\infty$, we specify that firms make zero (expected) profits. Let us stress that it is solely out of simplicity that we do not restrict the set of feasible contracts Ω and hence do not assume contract offers to be finite such that stage 2 does not necessarily end. As will become clear below, all our results remain to hold if we would consider a discrete contract grid and thus a finite number of stage 1 contract offers and stage 2 contract withdrawals.

3 Equilibrium with WMS allocation

Let us first recall the Wilson-Miyazaki-Spence (WMS) contracts, which of all separating contracts that jointly break-even are the pair that is most preferred by the L-type. Formally, consider the following maximization problem:

$$\max_{\omega^L, \omega^H} \; u^L(\omega^L) \tag{1}$$

s.t.

$$u^H(\omega^H) \geq u^H(\omega^L) \tag{2}$$

$$u^H(\omega^H) \geq u^H(\omega_{RS}^H) \tag{3}$$

$$\gamma_H[(1 - p_H)(w_{01} - w_1^H) + p_H(w_{02} - w_2^H)] + (1 - \gamma_H)[(1 - p_L)(w_{01} - w_1^L) + p_L(w_{02} - w_2^L)] \geq 0 \tag{4}$$

where $\omega_{RS}^H = (w_{01} - p_H(w_{01} - w_{02}), w_{01} - p_H(w_{01} - w_{02}))$ is the fair full insurance contract for the high risk type. The above maximization problem has a unique solution.[5]

[5] See e.g. Asheim and Nilssen (1996).

Definition 1. The unique solution to the above maximization problem are the Wilson-Miyazaki-Spence contracts, denoted by ω^H_{WMS} and ω^L_{WMS}.

Note that the WMS contracts are second-best efficient.[6] Denote by ω^H_{RS} and ω^L_{RS} the H-type and L-type RS contracts; ω^H_{RS} specifies full coverage while the expected zero-profit condition for insurers on this contract holds while ω^L_{RS} is pinned down by maximizing $u^L(\omega)$ subject to the expected zero-profit condition for the insurer on this contract and the H-type incentive compatibility constraint, which both become binding. Note that the WMS contracts correspond to the RS contracts when (3) is binding. When (3) is not binding, WMS contracts are such that the fully-insured H-types are subsidized by the partially insured L-types. We will focus on this more interesting case for the remainder of this paper.[7] The WMS and RS contracts are shown below in Figure 1. The axes in Figure 1 depict wealth in state 1 and 2 respectively, and point (w_{01}, w_{02}) shows the initial endowment. The solid lines are the fair insurance, i.e. zero profit lines for each type, where the flatter one corresponds to the fair insurance line for H-types. The dashed line is the pooling zero profit line. The dotted curve gives all L-type contracts that jointly with a corresponding incentive compatible full insurance H-type contract yield zero profits overall.

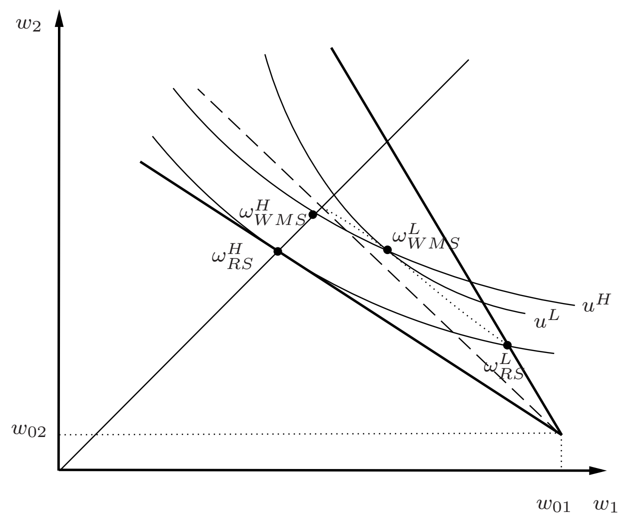

Figure 1: WMS and RS contracts

We will show that the WMS allocation can be sustained as equilibrium allocation. As suggested by the Wilson logic, contract withdrawal prevents simple cream-skimming

[6] This was shown by Crocker and Snow (1985).

[7] This is precisely when equilibrium fails to exist in the RS set-up when firms are allowed to offer contract menues. Our results hold trivially for the case that the WMS contracts correspond to the RS contracts.

deviations. However, since a deviator might as well withdraw some of his contracts in subsequent rounds, more complex deviating strategies than simple cream-skimming deviations emerge. To prevent such deviations, latent contracts have to be offered alongside the WMS contracts.

Proposition 1. *There exists a symmetric equilibrium where every individual obtains her respective WMS contract in stage 3.*

Proof. See Appendix. □

The intuition of the proof is as follows. Consider the following firm strategy: In stage 1, firms offer the WMS contracts and additionally the RS contracts as well as a continuum of contracts that lie on the L-type fair insurance line and give the L-type a lower expected utility than her WMS contract but higher expected utility than her RS contract. We name this continuum of contracts 'LR contracts'. These contracts are shown in Figure 2.

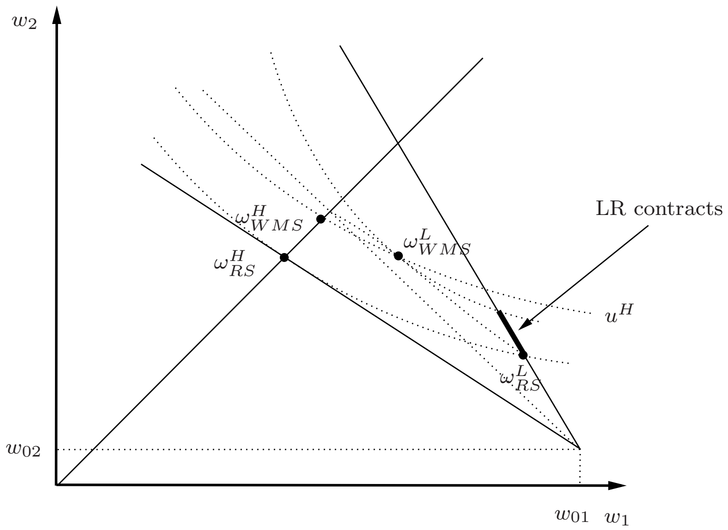

Figure 2: Contracts on offer in equilibrium

Then, in stage 2, in each round t, each firm computes the hypothetical profit it would make if stage 2 ended after round $t-1$, and, if it makes a loss, withdraws the loss-making contract(s), but does not withdraw any contracts if it makes zero expected profits.

This strategy supports the WMS allocation for the following reasoning: A simple cream-skimming deviation is prevented by withdrawing WMS contracts as they become unprofitable. Thus the firm trying to cream skim would also attract the high risks which makes cream skimming unprofitable. This case is the direct application of the reasoning by Wilson. Similarly, as stage 2 ends endogenously, a deviation that involves the withdrawal of the H-type WMS contract in some round is prevented as all other firms would withdraw their loss-making H-type WMS contract subsequently. Therefore a strategy of only serving L-types is prevented. A more subtle deviation strategy is the following: a deviatior could force firms to withdraw WMS contracts by offering a cream-skimming contract, then withdraw this cream-skimming contract and subsequently make a positive profit on e.g. a pooling contract. This is prevented by the LR contracts, as firms will withdraw those contracts from the LR contracts that would be taken up by H-types and hence be loss-making, but will leave exactly those LR contracts that would not be taken up by H-types but only by L-types and hence cream-skim the L-types from any deviating contract or contract menu. This type of deviation and the reaction according to the equilibrium strategy is shown in Figure 3, where a potential deviating contract menu is given by the cream-skimming contract ω_A and an intended pooling contract ω_B.[8] Finally, the RS contracts always remain on offer since they are separately zero profit making.

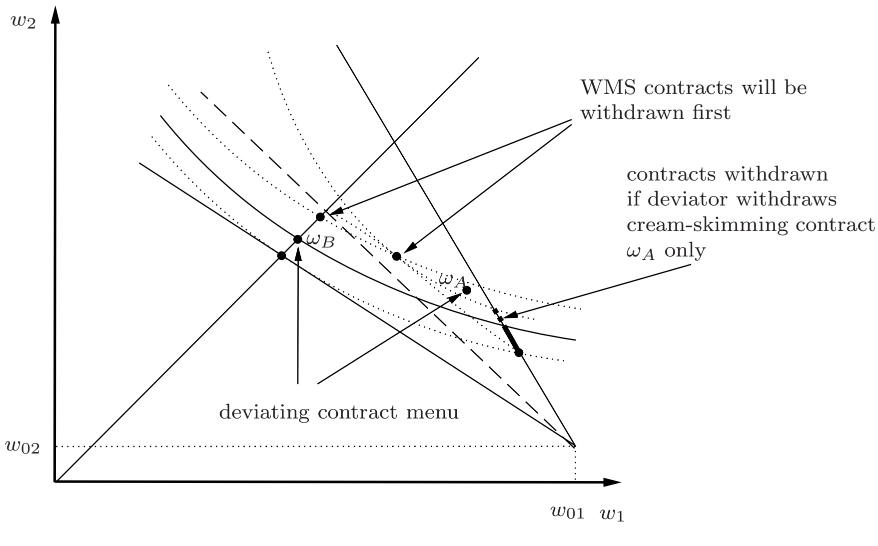

Figure 3: Reaction to a deviation

In this equilibrium, latent contracts are offered: the RS contracts and the LR contracts. A standard criticism of latent contracts is that they are loss-making off the equilibrium path.[9] Note that this is not the case here: If, off the equilibrium path, latent contracts would be the best available contracts on offer for some type, they

[8]Instead of a pooling contract ω_B, this of course also works for any profitable contract menu.
[9]Criticism of latent contracts is e.g. reviewed in Attar et al. (2009).

would either not be loss-making, or they would be withdrawn such that they cannot be chosen in stage 3. In particular, either the LR contracts are taken up only by low risks, or, potentially in more than one round, they are withdrawn. The other latent contracts, the RS contracts, will never be withdrawn, however, they are zero-profit making anyway.

The above Proposition provides an existence result for an equilibrium with an allocation that yields zero expected profits and is second-best efficient. It has a particular property, namely that on the equilibrium path no contract is withdrawn. If one allows for contract withdrawal on the equilibrium path, further outcomes are possible, in particular there exist equilibria where firms share positive profits.

4 Equilibrium with positive profits

To show that equilibria exist in which firms share positive profits, we concentrate on a simple case: Consider the full insurance contract that extracts all consumer surplus from H-types. We denote this contract by ω_P. Now as ω_P just leaves H-types indifferent between purchasing insurance and remaining uninsured, ω_P will not be taken up by L-types, but it yields a per (H-type) customer profit equal to the H-type risk premium and hence, as $0 < \gamma$, strictly positive profits overall. Note that, if the share of H-types is sufficiently high, ω_P corresponds to the monopoly allocation.

Proposition 2. *The profit-making full insurance contract ω_P can be sustained as equilibrium contract in a symmetric equilibrium for any number of firms in the market.*

Proof. See Appendix. □

The possibility to withdraw contracts allows firms to coordinate on a profit-making allocation: Consider offering ω_P and the set of contracts from the equilibrium strategy in Proposition 1, i.e. the WMS and RS contracts and LR contracts. If only those contracts are observed, all contracts different from ω_P are withdrawn sequentially in stage 2. In particular, firms withdraw the H-type WMS and RS contracts first such that there is no pooling deviation on any of those contracts. After that, all remaining contracts different from ω_P are withdrawn since they would be loss-making if taken out by both risk types. Initial contract offers and the equilibrium contract are shown in Figure 4.

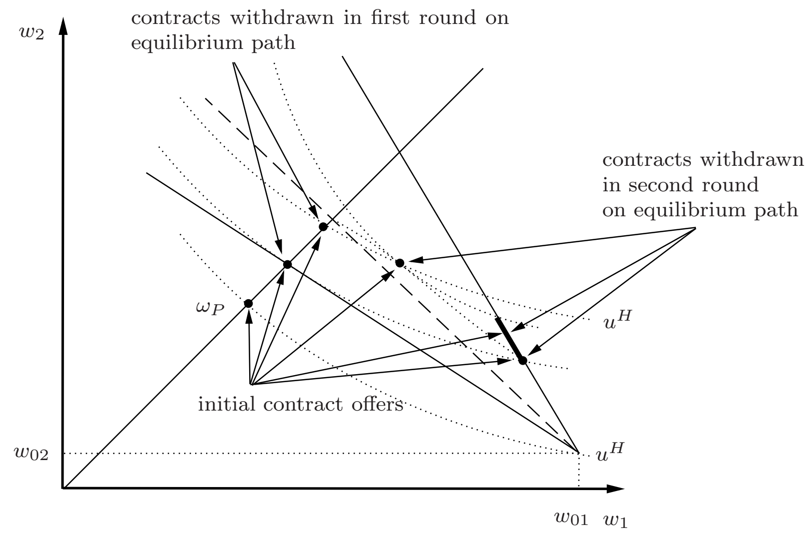

Figure 4: profit-making allocation

Then, if any deviating, stand-alone profit-making contracts are observed, the WMS (and all other initial contracts) are not withdrawn. This intuition works as it is credible for firms not to withdraw the WMS contracts and make zero profits on WMS contracts when they observe deviation. It is credible because any profitable deviation from ω_P implies that if WMS, RS and LR contracts are withdrawn, insurers make zero expected profits. Hence, it is sequentially rational not to withdraw the WMS, RS and LR contracts and make zero expected profits on WMS contracts. Again, as was the logic in the Proof of Proposition 1, attempting a deviation by initially offering a cream-skimming deviation such that WMS contracts have to been withdrawn is prevented by the offer of RS and LR contracts.[10]

The logic of Proposition 2 applies to any profit-making contract taken out by high risks only or more generally, to any contract menu or pooling contract that lies below the H-type indifference curve through the H-type WMS contract in which a positive profit is at most made on one risk type. The intuition is clear: Any deviation on these contracts or contract menus leads to nonpositive expected profits for the remaining firms such that not withdrawing the WMS contracts is a credible threat. In particular, it follows that the RS contracts can also be supported as equilibrium contracts.

The discussion also illustrates that the above logic does not necessarily work for any profitable contract menu, as each contract might be separately profit-making or similarly for a pooling contract where high risks are not cross-subsidized. In these

[10]Note that, as discussed in section 2, the possibility to sustain positive profits in equilibrium does not stem from the fact that stage 2 is potentially infinite.

cases, a deviator might only deviate on one contract/risk type, leaving firms with a positive profit on the other contract/risk type such that the threat of not withdrawing the WMS contracts is not credible. However, even in the case of a contract menu with separately profit-making contracts, this contract menu might be supported as the equilibrium allocation in an asymmetric equilibrium of the following form: In stage 1, one firm offers intermediate contracts that yield positive and in particular more than $1/n$th of the profit of each contract from the menu separately, but less than $1/n$th of total profits from the menu. All other firms follow the strategy described previously. If there is no deviation, then all contracts except the profit-making contract menu will be withdrawn, and this is sequentially rational for all firms, even for the firm offering the intermediate contracts, as the intermediate contracts yield less than $1/n$th of total profits from the contract menu. If a deviation 'below' intermediate contracts is observed, the intermediate contracts will not be withdrawn, and this is again sequentially rational, as intermediate profits yield more than $1/n$th of the profit of each contract from the menu separately.[11]

5 Riley extension

So far, we have only considered contract withdrawal in stage 2 in the spirit of Wilson. However, if contracts can be withdrawn, it seems plausible to enlarge the action space and allow for also offering new contracts in stage 2. This is the dynamic proposed in Riley (1979)'s 'reactive equilibrium' concept, in which instead of contract withdrawal, firms anticipate that contracts will be added in response to a deviation, which results in the RS allocation.[12] In a survey, Riley (2001) conjectures that in a game where firms are allowed to either add or drop offers in the second stage "both the Wilson and reactive equilibria are a Nash equilibrium of this new game". As Proposition 3 below shows this is true. Consider the game with the following modification:

Stage 2': Stage 2 consists of $t = 1, 2, ...$ rounds. In each round t, each firm $f \in F$ can withdraw a set from its remaining contracts and add any set of contracts to the remaining contract. After each round, firms observe the contract offers of all

[11] Note that, with an analoguous contract set to the LR contracts, a deviation on the intermediate contracts can easily be prevented as well.

[12] Engers and Fernandez (1987) generalize the reactive equilibrium concept and give the game-theoretic interpretation of repeated addition of contracts where offers once made cannot be withdrawn but always be reacted to by another addition of contracts. They show that this game has a multiplicity of perfect Nash equilibrium outcomes.

firms. Again, denote by Ω_t^f firm f's contract set on offer at the end of t. If, for any t, $\Omega_t^f = \Omega_{t-1}^f$ for all $f \in F$, this stage ends.

This extension of the action space does not eliminate equilibrium allocations, in particular profit-making equilibria can still be sustained.

Proposition 3. *Any equilibrium allocation of the original game can be supported as an equilibrium allocation in the extended game that allows for additional contract offers in stage 2.*

Proof. See Appendix. □

To see why, pick an equilibrium in the original game with the corresponding equilibrium strategy of firms. Then, consider that firms have the same strategy, with the addition that whenever they observe any new contract offer by any other firm in stage 2, round t, then in round $t+1$ they add the complete set of contracts offered in stage 1 in the equilibrium strategy. That way, if, e.g. in a profit-making equilibrium, after WMS, RS and LR contracts have been withdrawn, a firm attempts to make a profit by offering a contract that profitably attracts the whole population, this strategy replicates, in round $t+1$, any possible configuration of contract offers at the end of stage 1 in the original game. However, then there is no profitable deviation since it was an equilibrium in the original game. This result allows us to formally confirm Riley (2001)'s conjecture:

Corollary 1. *In the game in which contracts can be withdrawn and added in stage 2, both the WMS and RS allocation can be sustained as equilibrium allocations.*

6 Entry

We now return to the Wilson setup, but allow for entry in any round in stage 2. In particular, entry takes the following form: There are $m \geq 2$ potential entrants. A potential entrant can decide to enter in any round t in stage 2 as long as stage 2 has not ended. If an entrant enters in some round t, firms in the market can withdraw contracts in the subsequent round. Once an entrant has offered a nonempty set of contracts in some round t, he can, as incumbents, withdraw contracts from the offered contracts in subsequent rounds. Formally, the game proceeds as follows:

Stage 2″: Stage 2″ consists of $t = 1, 2, ...$ rounds. There is a set of entrants $E := \{1, ..., f, ...m\}$ with $m \geq 2$. As long as firm $f \in E$ does not enter, we say that f offers $\Omega_t^f = \emptyset$ in round t. In any round t for which $\Omega_t^f \neq \Omega_{t-1}^f$ for some $f \in F \cup E$, any $f \in E$ with $\Omega_j^f = \emptyset$ for all $j = 1, .., t-1$ can decide on entering the market and offer a set of contracts $\Omega_t^f \in \mathcal{P}(\Omega) \setminus \emptyset$ in t. We denote the round in which $f \in E$ enters by $\bar{t}^f$. In each round t, each firm $f \in F \cup E$ can withdraw a set from its remaining contracts. After each round, firms observe the contract offers of all firms. Denote by Ω_t^f firm f's contract set on offer at the end of t. If, for any t, $\Omega_t^f = \Omega_{t-1}^f$ for all $f \in F \cup E$, this stage ends. Define $\hat{t}$ by $\Omega_{\hat{t}}^f = \Omega_{\hat{t}-1}^f$ for all $f \in F \cup E$.

Stage 3": Individuals choose among the contracts $\bigcup\limits_{F \cup E} \Omega_{\hat{t}}^f$.

We further specify that if an entrant is indifferent between entering the market or not, the entrant enters.

Proposition 4. *In the game with entry, an equilibrium with the WMS allocation exists and is generically unique.*

Proof. See Appendix. □

The proof proceeds in two steps. First, it is shown that an equilibrium with the WMS allocation always exists. In the second step, it is shown that any equilibrium yields the WMS allocation.
The reasoning why an equilibrium with the WMS allocation always exists is similar to the one for existence of WMS equilibrium without entry: Assume that firms initially on the market follow the strategy specified in proof of Proposition 1, i.e. they offer the WMS, RS and LR contracts in stage 1 and, in case they would make a loss if stage 2 were to end after round $t-1$, they withdraw the loss-making contracts in round t and do not withdraw any contract if they make zero expected profits. The strategy of any entrant is the following: If, after any round $t-1$, incumbent firms (firms initially on the market and previous entrants) would either make zero or positive profits if stage 2 ended after round $t-1$, then the entrant enters the market in t. If incumbent firms make zero expected profits on WMS contracts, then the entrant offers WMS, RS and LR contracts in t, otherwise, the entrant offers the largest contract set that maximizes her expected profit given the contract offers of incumbents at the end of $t-1$. The entrant's strategy in all subsequent rounds is the same as that of initial

firms. This constitutes an equilibrium with the WMS allocation since any entrant cannot profitably deviate from the WMS contracts: Firstly, some entrant will have to enter in $t = 1$ as otherwise stage 2 ends. Secondly, as incumbent firms offer WMS, RS and LR contracts, there is no profitable deviation as shown in proof of Proposition 1.

For the second step, assume on the contrary that an equilibrium exists that yields an allocation that differs from WMS. Since it is an equilibrium, it yields nonnegative profits to all firms. Then, independent of whether it is (an) initial firm(s) or (an) entrant(s) that serve customers, since the allocation is not WMS, at least one entrant can profitably deviate by waiting to enter until the last round and, in the last round, offering a slightly better contract menu attracting all customers. Note that, as the deviating contract menu attracts all types, i.e. yields a utility for both types at least as high as that on the contracts that would have been the best on offer without the deviation, then there are no latent contracts by incumbents (firms active in stage 1 or previous entrants) that can prevent this deviation.

Note that, although entry implies additional contract offers in stage 2, there is a subtle difference to allowing additional contract offers by incumbent firms: The situation under entry is asymmetric in the sense that, if there are positive profits to be made, a firm can enter without the possibility of incumbent firms to punish additional contract offers by own new contract offers.

Again, latent contracts need to be issued to sustain the WMS equilibrium. Note that under entry another criticsm of latent contracts, namely that they lead to a multiplicity of equilibrium allocations, does not apply here either as the equilibrium is generically unique.

7 Conclusion

We modify the seminal Rothschild and Stiglitz (1976) model in the spirit of Wilson (1977)'s "anticipatory equilibrium" concept by introducing an additional stage in which firms can withdraw contracts (repeatedly) after observation of competitor's contract offers. It is shown that an equilibrium always exists where consumers obtain their respective Wilson- Miyazaki-Spence (WMS) contract, i.e. second-best efficiency can be achieved for any share of high-risk types in the population. However, contrary to intuition the game-theoretic analysis of the Wilson concept is not that straightforward: Firstly, latent contracts have to be offered alongside the WMS contracts for the Wilson logic to work. Furthermore, contract withdrawal also gives rise to more subtle strategies which allow for allocations in which firms make positive profits. This remains valid if, besides contract withdrawal, additional contracts can be offered in the second stage. However, if entry is allowed, then the WMS allocation is the unique equilibrium allocation. Interestingly, cross-subsidization can prevail in equilibrium.

Appendix

Proof of Proposition 1.
Let $\Omega_{WMS} := \left\{\omega^H_{WMS}, \omega^L_{WMS}\right\}$ denote the set of WMS contracts, $\Omega_{RS} := \left\{\omega^H_{RS}, \omega^L_{RS}\right\}$ denote the set of RS contracts and

$$\Omega_{LR} := \Big\{\omega \in \Omega \,\Big|\, u^L(\omega) < u^L(\omega^L_{WMS}), u^L(\omega) > u^L(\omega^L_{RS}) \\ \text{and } (1-p_L)(w_{01}-w_1) + p_L(w_{02}-w_2) = 0\Big\}$$

the continuum of contracts that lie on the L-type fair insurance line and yield an L-type a higher expected utility than her RS contract but lower expected utility than her WMS contract. Furthermore, let

$$\Omega_{CS} := \Big\{\omega \in \Omega \,\Big|\, u^L(\omega) \geq u^L(\omega^L_{WMS}) \text{ and } u^H(\omega) \leq u^H(\omega^L_{WMS}), \omega \neq \omega^L_{WMS}\Big\}$$

denote the cream-skimming region with respect to the WMS contracts.
Let $\Omega_t := (\Omega^1_t, ...\Omega^n_t)$,and let $h_t = (\Omega_0, \Omega_1, ..., \Omega_{t-1})$ denote the history in the beginning of round t. Furthermore, $\Delta_t := \bigcup\limits_F \Omega^f_t$. We denote by $\bar{\omega}^J_t$ the contract such that

$$\bar{\omega}^J_t \in \arg\max_{\omega\in\Delta_t} \; u^J(\omega)$$

and

$$\bar{w}^J_2 \geq \tilde{w}^J_2 \;\forall\; \tilde{\omega}^J \in \arg\max_{\omega\in\Delta_t} \; u^J(\omega)$$

Let $\bar{k}^J(\Omega_t)$ denote the number of firms offering $\bar{\omega}^J_t$ at the end of t, i.e. $\bar{K}^J(\Omega_t) := \left\{f \in F \middle|\, \bar{\omega}^J_t \in \Omega^f_t\right\}$ and $\bar{k}^J(\Omega_t) := |\bar{K}^J(\Omega_t)|$.
The strategy of a consumer of type J is to choose $\bar{\omega}^J_{\hat{t}}$ at firm $f \in \bar{K}^J(\Omega_{\hat{t}})$ with probability $1/\bar{k}^J(\Omega_{\hat{t}})$.
A strategy of a firm f specifies a set of contracts in stage 1, and in stage 2, round t, a map from the history to a set of remaining contracts of firm f at the end of t in stage 2, i.e. $\alpha^f_t :\; h_t \longmapsto \Omega^f_t$ with $\Omega^f_t \subseteq \Omega^f_{t-1}$.

We denote the hypothetical profit of firm f if stage 2 would end after $t-1$ by

$$\begin{aligned}\pi^f(\Omega_{t-1}) \;&:=\; \gamma_H[(1-p_H)(w_{01}-\bar{w}_1^H)+p_H(w_{02}-\bar{w}_2^H)](1/\bar{k}^H(\Omega_{t-1}))\mathbb{1}_{\Omega^f_{t-1}}(\bar{\omega}^H_{t-1})+\\ &\quad(1-\gamma_H)[(1-p_L)(w_{01}-\bar{w}_1^L)+p_L(w_{02}-\bar{w}_2^L)](1/\bar{k}^L(\Omega_{t-1}))\mathbb{1}_{\Omega^f_{t-1}}(\bar{\omega}^L_{t-1})\end{aligned}$$

where $\mathbb{1}$ is an indicator function. Similarly, we denote by

$$\pi^{f,J}(\Omega_{t-1}) \;=\; \gamma_H[(1-p_H)(w_{01}-\bar{w}_1^J)+p_H(w_{02}-\bar{w}_2^J)](1/\bar{k}^J(\Omega_{t-1}))\mathbb{1}_{\Omega^f_{t-1}}(\bar{\omega}^J_{t-1})$$

the hypothetical profits on J-types respectively. Finally, let

$$A := \left\{f\in F \,\middle|\, \Omega^f_0 \subseteq \Omega_{WMS}\cup\Omega_{RS}\cup\Omega_{LR}\right\}.$$

We propose that a possible equilibrium strategy of firms is the following: In stage 1, firm $f\in F$ offers $\Omega^f_0 = \Omega_{WMS}\cup\Omega_{RS}\cup\Omega_{LR}$. In stage 2, round t the strategy of firm f specifies

$$\alpha^f_t(h_t) = \begin{cases} \Omega^f_{t-1} & \text{if } \pi^f(\Omega_{t-1}) = 0;\\ \Omega^f_{t-1}\setminus\left\{\hat{\Omega}^H_{t-1}\right\} & \text{if } \pi^f(\Omega_{t-1})<0 \text{ and } \pi^{f,L}(\Omega_{t-1})\geq 0;\\ \Omega^f_{t-1}\setminus\{\bar{\omega}^L_{t-1}\} & \text{if } \pi^f(\Omega_{t-1})<0 \text{ and } \pi^{f,H}(\Omega_{t-1})\geq 0;\\ \Omega^f_{t-1}\setminus\left\{\bar{\omega}^H_{t-1},\bar{\omega}^L_{t-1}\right\} & \text{if } \pi^f(\Omega_{t-1})<0 \text{ and } \pi^{f,L}(\Omega_{t-1}),\pi^{f,H}(\Omega_{t-1})<0;\\ \bar{\Omega}^f_t & \text{if } \pi^f(\Omega_{t-1})>0. \end{cases}$$

where

$$\begin{aligned}\hat{\Omega}^H_{t-1} := \Big\{\omega\in\Omega^f_{t-1}\setminus\{\Omega_{RS}\} \text{ such that if } \Omega^j_{t-1}=\{\omega\}\ \forall j\in A,&\\ \text{then } \omega=\bar{\omega}^H_{t-1} \text{ and } \pi^{f,H}(\Omega_{t-1})<0\Big\}&\end{aligned}$$

i.e. if firm f makes losses on H-types, it withdraws any contract that, if all firms that in stage 1 offered the contracts according to equilibrium strategy (or less contracts) only offered this one contract, would attract the H-types and be loss-making. Furthermore, $\bar{\Omega}^f_t$ denotes the *largest* set of contracts such that for $\Omega^j_t=\Omega^j_{t-1}\ \forall j\in F\setminus\{f\}$, then $\pi^f(\Omega_t)$ is maximal.

If all firms follow the above strategy, no firm withdraws any contract in $t=1$ and stage 2 ends after $t=1$, firms make zero expected profit and a customer of type J receives her J-type WMS contract.

It remains to show that there is no profitable deviation. We will proceed in two steps:

First, we show that a deviator serves some H-types. In a second step, we show that if the deviator serves some H-types, she cannot be making a strictly positive profit. Consider firm $\bar{f}$ that offers $\Omega_0^{\bar{f}}$ in stage 1 and has a strategy $\hat{\alpha}^{\bar{f}} : \ h_t \longmapsto \Omega_t^{\bar{f}}$ in stage 2. Let $\Omega_{\hat{t}}^f$ be the final set of contract offers of firm f, i.e. $\Omega_{\hat{t}}^f = \Omega_{\hat{t}-1}^f \ \forall \ f \in F$. Then it must be that $\pi^f(\Omega_{\hat{t}-1}) \geq 0 \ \forall \ f \in F \setminus \{\bar{f}\}$ as otherwise a firm $f \in F \setminus \{\bar{f}\}$ would withdraw a nonempty set of contracts in $\hat{t}$ and $\hat{t}$ would not be the last round in stage 2.

Now assume $\pi^{\bar{f}}(\Omega_{\hat{t}}) > 0$. As $\pi^f(\Omega_{\hat{t}-1}) \geq 0 \ \forall \ f \in F \setminus \{\bar{f}\}$, we will show that $\bar{w}_{\hat{t}}^H \in \Omega_{\hat{t}}^{\bar{f}}$: Since $\pi^{\bar{f}}(\Omega_{\hat{t}}) > 0$, $\bar{f}$ serves some customers. To show that it cannot be possible that $\bar{f}$ serves only L-types, assume on the contrary that $\bar{f}$ only serves L-types. If L-types prefer an insurance contract to remaining uninsured, than H-types prefer to be insured as well. As $\bar{f}$ only serves L-types, then at least one firm $f \in F \setminus \{\bar{f}\}$ serves H-types and the share of L-types among customers at f is less than $1-\gamma$. There are three possible cases:

Case 1: $\bar{\omega}_{\hat{t}}^H = \omega_{WMS}^H$. Now any firm $f \in F \setminus \{\bar{f}\}$ that serves H-types with ω_{WMS}^H and has a share of L-types among customers that is less than $1-\gamma$ does not make a nonnegative profit.[13] This contradicts $\pi^f(\Omega_{\hat{t}-1}) \geq 0 \ \forall \ f \in F \setminus \{\bar{f}\}$.

Case 2: $\bar{\omega}_{\hat{t}}^H \in \Omega_{LR}$. Any contract $\omega \in \Omega_{LR}$ if taken up by some H-types is loss-making, independent of whether it is also taken up by some L-types. This contradicts $\pi_f(\Omega_{\hat{t}-1}) \geq 0 \ \forall \ f \in F \setminus \{\bar{f}\}$.

Case 3: $\bar{\omega}_{\hat{t}}^H = \omega_{RS}^H$. From the strategy of all $f \in F \setminus \{\bar{f}\}$, both RS contracts will never be withdrawn, i.e. the L-type contract is still on offer when $\bar{\omega}_{\hat{t}}^H = \omega_{RS}^H$. Then, there is no contract that $\bar{f}$ can offer attracting L-types and making a positive profit, which is a contradiction.

Hence, $\bar{\omega}_{\hat{t}}^H \in \Omega_{\hat{t}}^{\bar{f}}$. We will now show that if $\bar{\omega}_{\hat{t}}^H \in \Omega_{\hat{t}}^{\bar{f}}$, $\bar{f}$ cannot be making a positive profit. First, note that, the RS contracts are always, i.e. in any t, offered by each firm $f \in F \setminus \{\bar{f}\}$. Then, it follows that $u^H(\bar{\omega}_{\hat{t}}^H) \geq u^H(\omega_{RS}^H)$. There are again three possible cases:

Case 1: $u^H(\bar{\omega}_{\hat{t}}^H) \geq u^H(\omega_{WMS}^H)$. Then, ω_{WMS}^L will not have been withdrawn by any firm $f \in F \setminus \{\bar{f}\}$. As ω_{WMS}^L is on offer from firms $f \in F \setminus \{\bar{f}\}$, by construction of the WMS contracts, $\pi^{\bar{f}}(\Omega_{\hat{t}}) \leq 0$ for the cases that $\bar{f}$ only serves H-types or both types.

Case 2: $u^H(\bar{\omega}_{\hat{t}}^H) < u^H(\omega_{WMS}^H)$ and $\bar{\omega}_{\hat{t}}^L \in \Omega_{CS}$. Hence, both WMS contracts are not on offer at any firm $f \in F \setminus \{\bar{f}\}$ and any firm $f \in F \setminus \{\bar{f}\}$ does not serve L-types since it does not offer any contract $w \in \Omega_{CS}$. However, by construction of the WMS

[13] This is because firm $f \in F \setminus \{\bar{f}\}$ at best serves some L-types with ω_{WMS}^L, however, since the share of L-types is less than $1-\gamma$, this is loss-making.

contracts, there is no incentive compatible menu of contracts with $\bar{\omega}_{\hat{t}}^{L} \in W_{CS}$ that is profit-making, hence $\pi_{\bar{f}}(\Omega^{\hat{t}}) < 0$.
Case 3: $u^H(\bar{\omega}_{\hat{t}}^{H}) < u^H(\omega_{WMS}^{H})$ and $\bar{\omega}_{\hat{t}}^{L} \notin \Omega_{CS}$. If $\bar{\omega}_{\hat{t}}^{H} \in \Omega_{LR} \cup \Omega_{RLR}$ with

$$\begin{aligned}\Omega_{RLR} := \big\{\omega \in \Omega \,\big|\, & u^L(\omega) \leq u^L(\omega_{WMS}^{L}) \text{ and } u^H(\omega) \geq u^H(\omega_{RS}^{H}); \\ & (1-p_L)(w_{01}-w_1) + p_L(w_{02}-w_2)] < 0\big\}\end{aligned}$$

then $\pi^{\bar{f}}(\Omega_{\hat{t}}) < 0$. If $\bar{\omega}_{\hat{t}}^{H} \notin \Omega_{LR} \cup \Omega_{RLR}$, then from the strategy of any firm $f \in F \setminus \{\bar{f}\}$, $\bar{\omega}_{\hat{t}}^{L} \in \Omega_{\hat{t}}^{f}\ \forall\ f \in F \setminus \{\bar{f}\}$. Then, $\pi^{\bar{f}}(\Omega_{\hat{t}}) \leq 0$.

Hence, $\pi^{\bar{f}}(\Omega_{\hat{t}}) \leq 0$ which is a contradiction.

Proof of Proposition 2.
Let $\Omega_P := \{\omega_P\}$.
Again, the strategy of a consumer of type J is to choose $\bar{\omega}_{\hat{t}}^{J}$ at firm $f \in \bar{K}^J(\Omega_{\hat{t}})$ with probability $1/\bar{k}^J(\Omega_{\hat{t}})$. Let $\Omega_{WH} := \big\{\omega \in \Omega \,\big|\, u^H(\omega) < u^H(\omega_P)\big\}$ and let

$$B := \left\{ f \in F \,\middle|\, \Omega_0^f \subseteq \Omega_P \cup \Omega_{WMS} \cup \Omega_{RS} \cup \Omega_{LR} \cup \Omega_{WH} \right\}.$$

We claim that a possible equilibrium strategy of firms is the following: In stage 1, firm $f \in F$ sets $\Omega_0^f = \Omega_P \cup \Omega_{WMS} \cup \Omega_{RS} \cup \Omega_{LR}$. In stage 2, round t the strategy of firm f specifies

$$\beta_t^f(h_t) = \begin{cases} \Omega_{t-1}^f \setminus \{\omega_{RS}^H, \omega_{WMS}^H\} & \text{if } \pi^f(\Omega_{t-1}) = 0 \text{ and } B = F; \\ \Omega_{t-1}^f & \text{if } \pi^f(\Omega_{t-1}) = 0 \text{ and } B \neq F; \\ \Omega_{t-1}^f \setminus \left\{\hat{\Omega}_{t-1}^H\right\} & \text{if } \pi^f(\Omega_t) < 0 \text{ and } \pi^{f,L}(\Omega_{t-1}) \geq 0; \\ \Omega_{t-1}^f \setminus \{\bar{\omega}_{t-1}^L\} & \text{if } \pi^f(\Omega_{t-1}) < 0 \text{ and } \pi^{f,H}(\Omega_{t-1}) \geq 0; \\ \Omega_{t-1}^f \setminus \{\bar{\omega}_{t-1}^H, \bar{\omega}_{t-1}^L\} & \text{if } \pi^f(\Omega_{t-1}) < 0 \text{ and } \pi^{f,L}(\Omega_{t-1}), \pi^{f,H}(\Omega_{t-1}) < 0; \\ \bar{\Omega}_t^f & \text{if } \pi^f(\Omega_{t-1}) > 0. \end{cases}$$

$$\begin{aligned}\hat{\Omega}_{t-1}^H := \Big\{\omega \in \Omega_{t-1}^f \setminus \{\Omega_{RS}\} \text{ such that if } \Omega_{t-1}^j = \{\omega\}\ \forall j \in B, & \\ \text{then } \omega = \bar{\omega}_{t-1}^H \text{ and } \pi^{f,H}(\Omega_{t-1}) < 0\Big\}, &\end{aligned}$$

and $\bar{\Omega}_t^f$ denotes the *largest* set of contracts such that for $\Omega_t^j = \Omega_{t-1}^j\ \forall j \in F \setminus \{f\}$, then $\pi^f(\Omega_t)$ is maximal.
The strategy specifies that if, after stage 1, there is no contract ω with

$\omega \notin \{\Omega_P \cup \Omega_{WMS} \cup \Omega_{RS} \cup \Omega_{LR} \cup \Omega_{WH}\}$ on offer, then the H-type WMS and RS contracts are withdrawn first. Furthermore, the strategy specifies that if firm f makes losses on H-types, it withdraws any contract that, if all firms that in stage 1 offered the contracts according to equilibrium strategy (or less contracts) only offered this one contract, would attract the H-types and be loss-making. This covers the case that after withdrawal of the H-type WMS and RS contracts, firms subsequently withdraw the L-type WMS contract as well as the LR contracts and L-type RS contract. The strategy thus specifies that if, after stage 1, there is no contract ω with $\omega \notin \{\Omega_P \cup \Omega_{WMS} \cup \Omega_{RS} \cup \Omega_{LR} \cup \Omega_{WH}\}$ on offer, then the WMS, RS and LR contracts will be sequentially withdrawn, however, if a contract ω with $\omega \notin \{\Omega_P \cup \Omega_{WMS} \cup \Omega_{RS} \cup \Omega_{LR} \cup \Omega_{WH}\}$ is observed after stage 1, the WMS contracts will not be withdrawn (if the firm's hypothetical expected profit is zero) and the strategy is the same as the strategy in proof of Proposition 1.

If all firms follow the above strategy, then all firms withdraw the H-type WMS contract and the H-type RS contracts in $t = 1$ and the L-type WMS and RS contracts as well as LR contracts in $t = 2$. Stage 2 ends after $t = 3$, firms share the profit on ω_P, H-type customers buy ω_P and L-types remain uninsured.
It remains to show that there is no profitable deviation. First, note that, since firms share the profit from the single contract ω_P, any deviation yielding a higher profit than $1/n$th of the profit from ω_P necessarily yields zero expected profits to nondeviating firms as they do not serve any customer. Then, it is sequentially rational not to withdraw the WMS contracts as prescribed by the equilibrium strategy.
The rest of the proof proceeds along the same lines as the proof of Proposition 1 and is therefore omitted.[14]

Proof of Proposition 3.
Fix an equilibrium in the original game. In this equilibrium, the equilibrium strategy of firm $f \in F$ specifies a contract offer Ω_0^f in stage 1 and in stage 2, round t a withdrawal strategy $\chi_t^f(h_t)$ specifying remaining contract offers in the end of t. Note that, we neither assume that equilibrium strategies are symmetric nor put any restrictions on stage 2 strategies.
In the extended game, a strategy specifies a contract offer in stage 1, and map from the history h_t to a contract offer in stage 2, round t, $\bar{\chi}_t^f : \; h_t \longmapsto \Omega_t^f$.

[14]Note that, in particular a deviation aiming at offering the H-type WMS or RS contract as a pooling contract is covered by proof of Proposition 1.

Then consider the following strategy in the extended game: Firm $f \in F$ offers Ω_0^f in stage 1. In stage 2, round $t = 1$, the strategy specifies $\bar{\chi}_t^f(h_t) = \chi_t^f(h_t)$ and for $t \geq 2$, the strategy specifies

$$\bar{\chi}_t^f(h_t) = \begin{cases} \chi_t^f(h_t) & \text{if } \Omega_{t-1}^j \subseteq \Omega_{t-2}^j \ \forall j \in F; \\ \widetilde{\Omega}_t^f & \text{if } \Omega_{t-1}^j \subseteq \Omega_{t-2}^j \ \forall j \in F \setminus f \text{ and there exists a contract} \\ & \omega \in \Omega_{t-1}^f \ \text{ with } \omega \notin \Omega_{t-2}^f; \\ \Omega_0^f & \text{otherwise.} \end{cases}$$

where $\widetilde{\Omega}^f$ is a set of contracts such that for $\Omega_t^j = \Omega_{t-1}^j \ \forall j \in F \setminus \{f\}$, then $\pi^f(\Omega_t)$ is maximal. Note that, in this case $\widetilde{\Omega}$ does not need to be the largest set such that the profit is maximal as if there are some contracts withdrawn or added in t, f can add contracts in $t + 1$.

This strategy implies that firms have the same strategy as in the original game, however, whenever a firm f observes another firm j adding contracts in the previous round, then f replicates contract offers after stage 1 as it throws all stage 1 contracts on the market.

Now firstly, this strategy yields the same equilibrium allocation as in the original game as on the equilibrium path, each firm $f \in F$ takes the same action in stage 1 and in all rounds of stage 2 as on the corresponding equilibrium path in the original game.

It remains to show that there is no profitable deviation. A profitable deviation here means a deviation such that profits are higher than in equilibrium in the original game. Assume a firm $\bar{f}$ offers $\widehat{\Omega}_0^{\bar{f}}$ in stage 1 and has a strategy that specifies some $\widehat{\chi}_t^{\bar{f}} : h_t \longmapsto \Omega_t^{\bar{f}}$ in stage 2 and makes a profit that is strictly higher than in the equilibrium in the original game. Firstly, note that this implies that stage 2 ends in some t. We need to distinguish 4 cases:

Case 1: $\widehat{\Omega}_0^{\bar{f}} \neq \Omega_0^{\bar{f}}$ and $\widehat{\chi}_t^{\bar{f}}(h_t) = \bar{\chi}_t^{\bar{f}}(h_t)$. This implies that no contract will be added by any firm $f \in F$ in any round in stage 2. However, then either it involves the same allocation and same profits for all firms as in the equilibrium in the original game, or $\chi^f(h_t)$ cannot have been part of an equilibrium strategy in the original game for some $f \in F$.

Case 2: $\widehat{\Omega}_0^{\bar{f}} = \Omega_0^{\bar{f}}$, $\widehat{\chi}_t^{\bar{f}}(h_t) \neq \bar{\chi}_t^{\bar{f}}(h_t)$ and $\bar{f}$ does not add any contract in any t. Again, this implies that no contract will be added by any firm $f \in F$ in any round in stage 2. As in Case 1, then either it involves the same allocation and same profits for all firms as in the equilibrium in the original game, or $\chi^f(h_t)$ cannot have been part of an equilibrium strategy in the original game for some $f \in F$.

Case 3: $\widehat{\Omega}_0^{\bar{f}} = \Omega_0^{\bar{f}}$, $\widehat{\chi}_t^{\bar{f}}(h_t) \neq \bar{\chi}_t^{\bar{f}}(h_t)$ and $\bar{f}$ adds at least one contract in some t. Assume first that a contract will only be added by $\bar{f}$ in at most one round t and let $\tilde{t}$ denote this round. Then, the strategy of firms $f \in F \setminus \bar{f}$ specifies that $\Omega_{\tilde{t}+1}^f = \Omega_0^f$ $\forall f \in F \setminus \bar{f}$. However, then, for $t \geq \tilde{t}+1$ this replicates either Case 1 or 2 above. Now assume that $\bar{f}$ adds contracts in more than one round t. Let $\check{t}$ denote the last round in which a contract will be added by $\bar{f}$. Again, the strategy of firms $f \in F \setminus \bar{f}$ specifies that $\Omega_{\check{t}+1}^f = \Omega_0^f$ $\forall f \in F \setminus \bar{f}$. However, then, this again replicates either Case 1 or 2 above.

Case 4: $\widehat{\Omega}_0^{\bar{f}} \neq \Omega_0^{\bar{f}}$ and $\widehat{\chi}_t^{\bar{f}}(h_t) \neq \bar{\chi}_t^{\bar{f}}(h_t)$. We can transform this case in the following way: Instead of $\widehat{\Omega}_0^{\bar{f}} \neq \Omega_0^{\bar{f}}$, let $\widehat{\Omega}_0^{\bar{f}} = \Omega_0^{\bar{f}}$ and $\bar{f}$ either adds or withdraws some contract in stage 2, round 1 and plays $\widehat{\chi}_t^{\bar{f}}(h_t)$ thereafter. However, then, this falls under one of the above cases.

Proof of Proposition 4.

We will proceed in two steps: First, we show that an equilibrium with the WMS allocation always exists. In the second step, we show that any equilibrium yields the WMS allocation.

For the first part, again, the strategy of a consumer of type J is to choose $\bar{\omega}_{\hat{t}}^J$ at firm $f \in \bar{K}^J(\Omega_{\hat{t}})$ with probability $1/\bar{k}^J(\Omega_{\hat{t}})$.

For any t, let $\Omega_t^{FE} := (\Omega_t^1, ..., \Omega_t^n ..., \Omega_t^{n+m})$ denote the observed contract offers of *all* firms, that is initial firms and (potential) entrants and denote by M_t the set of *active* firms on the market in t. We can then denote the history in t by $h_t^{FE} = (\Omega_0^{FE}, \Omega_1^{FE}, ..., \Omega_{t-1}^{FE})$.

A strategy of a firm $f \in F$ specifies a set of contracts in stage 1, and in stage 2, round t, a map from the history to a set of remaining contracts of firm f at the end of t in stage 2, i.e. $\alpha_t^f : h_t^{FE} \longmapsto \Omega_t^f$ with $\Omega_t^f \subseteq \Omega_{t-1}^f$.

We propose that the equilibrium strategy of any firm $f \in F$ specifies the following: In stage 1, firm $f \in F$ offers $\Omega_0^f = \Omega_{WMS} \cup \Omega_{RS} \cup \Omega_{LR}$. In stage 2, round t the strategy of firm f is

$$\alpha_t^f(h_t^{FE}) = \begin{cases} \Omega_{t-1}^f & \text{if } \pi^f(\Omega_{t-1}^{FE}) = 0; \\ \Omega_{t-1}^f \setminus \left\{\hat{\Omega}_{t-1}^H\right\} & \text{if } \pi^f(\Omega_{t-1}^{FE}) < 0 \text{ and } \pi^{f,L}(\Omega_{t-1}) \geq 0; \\ \Omega_{t-1}^f \setminus \left\{\bar{\omega}_{t-1}^L\right\} & \text{if } \pi^f(\Omega_{t-1}^{FE}) < 0 \text{ and } \pi^{f,H}(\Omega_{t-1}) \geq 0; \\ \Omega_{t-1}^f \setminus \left\{\bar{\omega}_{t-1}^H, \bar{\omega}_{t-1}^L\right\} & \text{if } \pi^f(\Omega_{t-1}^{FE}) < 0 \text{ and } \pi^{f,L}(\Omega_{t-1}^{FE}), \pi^{f,H}(\Omega_{t-1}^{FE}) < 0; \\ \bar{\Omega}_t^f & \text{if } \pi^f(\Omega_{t-1}^{FE}) > 0. \end{cases}$$

where

$$\hat{\Omega}^H_{t-1} := \Big\{\omega \in \Omega^f_{t-1} \setminus \{\Omega_{RS}\} \text{ such that if } \Omega^j_{t-1} = \{\omega\} \ \forall j \in C,$$
$$\text{then } \omega = \bar{\omega}^H_{t-1} \text{ and } \pi^{f,H}(\Omega_{t-1}) < 0\Big\}$$

with
$$C := \Big\{ f \in M_t \Big| \Omega^f_0, \Omega^f_{\bar{t}f} \subseteq \Omega_{WMS} \cup \Omega_{RS} \cup \Omega_{LR} \Big\}.$$

and $\bar{\Omega}^f_t$ denotes the *largest* set of contracts such that for $\Omega^j_t = \Omega^j_{t-1}$ for all $j \in F \cup E \setminus \{f\}$, then $\pi^f(\Omega_t)$ is maximal.

The strategy of an entrant $f \in E$ in stage 2, round t specifies the following: As long as f has not entered, the strategy consists of a map from the history in t to a decision to enter the market in round t, i.e. $\theta^f_t : h^{FE}_t \longmapsto \eta \in \{entry, noentry\}$. When f enters in t, i.e. $\theta^f_t(h^{FE}_t) = entry$, the strategy specifies a map from the history to a set of contract offers Ω^f_t, $\gamma^f_t : h^{FE}_t \longmapsto \Omega^f_t$. For all subsequent rounds t, the strategy specifies a map from the history to a set of remaining contracts of firm f at the end of t i.e. $\phi^f_t : h^{FE}_t \longmapsto \Omega^f_t$ with $\Omega^f_t \subseteq \Omega^f_{t-1}$.

We propose that a possible equilibrium strategy of an entrant $f \in E$ specifies the following:

$$\theta^f_t(h^{FE}_t) = \begin{cases} entry & \text{if } \pi^j(\Omega^{FE}_{t-1}) \geq 0 \ \forall j \in F \cup E \setminus f; \\ noentry & \text{otherwise.} \end{cases}$$

$$\gamma^f_t(h^{FE}_t) = \begin{cases} \Omega_{WMS} \cup \Omega_{RS} \cup \Omega_{LR} & \text{if } \pi^j(\Omega^{FE}_{t-1}) = 0 \text{ for all } j \in F \cup E \setminus f \text{ and } C = M_t; \\ \bar{\Omega}^f & \text{otherwise.} \end{cases}$$

where $\bar{\Omega}^f$ denotes the *largest* set of contracts such that for $\Omega^j_t = \Omega^j_{t-1} \ \forall j \in F \cup E \setminus f$, then $\pi^f(\Omega_t)$ is maximal and lastly,

$$\phi^f_t(h^{FE}_t) = \alpha^f_t(h^{FE}_t).$$

If all initial firms and entrants follow the above respective strategies, no initial firm withdraws any contract in $t = 1$ and all entrants enter in $t = 1$, no firm withdraws any contract in $t = 2$ and stage 2 ends after $t = 2$, firms make zero expected profit and a customer of type J receives her J-type WMS contract.

It remains to show that there is no profitable deviation. Firstly, note that, for the same reasoning as in proof of Proposition 1, no initial firm can deviate profitably.

Then, assume that an entrant $\bar{f}$ in $\bar{t}^{\bar{f}}$ offers $\Omega^{\bar{f}}_{\bar{t}^{\bar{f}}}$ and has a withdrawal strategy $\hat{\phi}^{\bar{f}}_{t}$ and assume that $\bar{f}$ makes a strictly positive profit. By the strategy of all firms $f \in F \cup E \setminus \bar{f}$, it follows that $\bar{t}^{\bar{f}} \leq 2$. Then, however, independent of whether $\bar{t}^{\bar{f}} = 1$ or $\bar{t}^{\bar{f}} = 2$, from the strategy of all firms $f \in F \cup E \setminus \bar{f}$ it follows that round $\bar{t}^{\bar{f}}$ is equivalent to stage 1 in the game without entry. The rest of the proof corresponds to the proof of Proposition 1.

For the second part of the proof, assume that an equilibrium exists that yields an allocation different from WMS, i.e. $(\bar{w}^H_{\hat{t}}, \bar{w}^L_{\hat{t}}) \neq (\omega^H_{WMS}, \omega^L_{WMS},)$. Since it is an equilibrium, each firm $f \in F \cup E$ makes a nonnegative expected profit. Since we specified that if an entrant is indifferent between entering the market or not, the entrant enters, nonnegative expected profits imply that all firms $f \in E$ enter in some round $t < \hat{t}$. Now since $(\bar{w}^H_{\hat{t}}, \bar{w}^L_{\hat{t}}) \neq (\omega^H_{WMS}, \omega^L_{WMS},)$ there exist contracts $\hat{\omega}^H$, $\hat{\omega}^L$ such that $u^H(\hat{\omega}^H) \geq u^H(\bar{w}^H_{\hat{t}})$, $u^H(\hat{\omega}^H) \leq u^H(\hat{\omega}^L)$, $u^L(\hat{\omega}^L) > u^L(\bar{w}^H_{\hat{t}})$ and for firm $\hat{f}$ offering $\Omega^{\hat{f}} = \left\{\hat{\omega}^H, \hat{\omega}^L\right\}$ and attracting the whole population, $\pi^{\hat{f}} > 0$ and $\pi^{\hat{f}} \geq \sum_{F \cup E} \pi^j$. As $\pi^{\hat{f}} \geq \sum_{F \cup E} \pi^j$, an entrant can profitably deviate by waiting to enter the market until $\hat{t}$ and offer $\hat{\omega}^H$, $\hat{\omega}^L$ in $\hat{t}$. Note that this entry cannot be prevented by firms as $u^L(\hat{\omega}^L) > u^L(\bar{w}^H_{\hat{t}})$, i.e. there exists no contract that cream skims low risks from $u^L(\hat{\omega}^L)$.

References

Akerlof, G. A., 1970. The market for lemons: Quality uncertainty and the market mechanism. The Quarterly Journal of Economics 84 (3), 488–500.

Ania, A. B., Tröger, T., Wambach, A., 2002. An evolutionary analysis of insurance markets with adverse selection. Games and Economic Behavior 40 (2), 153–184.

Asheim, G. B., Nilssen, T., 1996. Non-discriminating renegotiation in a competitive insurance market. European Economic Review 40 (9), 1717–1736.

Attar, A., Mariotti, T., Salanie, F., 2009. Non-exclusive competition in the market for lemons. IDEI Working Paper no. 558.

Bisin, A., Gottardi, P., 2006. Efficient competitive equilibria with adverse selection. Journal of Political Economy 114 (3), 485–516.

Bolton, P., Dewatripont, M., 2005. Contract theory. The MIT Press.

Crocker, K. J., Snow, A., 1985. The efficiency of competitive equilibria in insurance markets with asymmetric information. Journal of Public Economics 26 (2), 207–219.

Dasgupta, P., Maskin, E., 1986. The existence of equilibrium in discontinuous economic games, ii: Applications. The Review of Economic Studies 53 (1), 27–41.

Dubey, P., Geanakoplos, J., Nov. 2002. Competitive pooling: Rothschild-Stiglitz reconsidered. Quarterly Journal of Economics 117 (4), 1529–1570.

Engers, M., Fernandez, L., 1987. Market equilibrium with hidden knowledge and self-selection. Econometrica 55 (2), 425–439.

Faynzilberg, P. S., 2006. Credible forward commitments and risk-sharing equilibria. mimeo.

Guerrieri, V., Shimer, R., Wright, R., 2010. Adverse selection in competitive search equilibrium. Econometrica, forthcoming.

Hellwig, M., 1987. Some recent developments in the theory of competition in markets with adverse selection. European Economic Review 31 (1-2), 319–325.

Inderst, R., Wambach, A., 2001. Competitive insurance markets under adverse selection and capacity constraints. European Economic Review 45 (10), 1981–1992.

Jaynes, G. D., 1978. Equilibria in monopolistically competitive insurance markets. Journal of Economic Theory 19 (2), 394–422.

Lacker, J. M., Weinberg, J. A., 1999. Coalition-proof allocations in adverse-selection economies. The GENEVA Papers on Risk and Insurance - Theory 24 (1), 5–17.

Mimra, W., Wambach, A., 2010. Endogenous capital in the Rothschild-Stiglitz model.

Miyazaki, H., 1977. The rat race and internal labor markets. The Bell Journal of Economics 8 (2), 394–418.

Netzer, N., Scheuer, F., 2008. Competitive markets without commitment. University of Zurich, SOI Working Paper No. 0814.

Picard, P., 2009. Participating insurance contracts and the Rothschild-Stiglitz equilibrium puzzle, mimeo.

Riley, J. G., 1979. Informational equilibrium. Econometrica 47 (2), 331–359.

Riley, J. G., 2001. Silver signals: Twenty-five years of screening and signaling. Journal of Economic Literature 39 (2), 432–478.

Rothschild, M., Stiglitz, J., 1976. Equilibrium in competitive insurance markets: An essay on the economics of imperfect information. The Quarterly Journal of Economics 90 (4), 629–649.

Spence, M., Dec. 1978. Product differentiation and performance in insurance markets. Journal of Public Economics 10 (3), 427–447.

Wilson, C., Dec. 1977. A model of insurance markets with incomplete information. Journal of Economic Theory 16 (2), 167–207.