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Jay Pil Choi
Heiko Gerlach

CESifo Working Paper No. 3425
Category 11: Industrial Organisation
April 2011

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Abstract

This paper analyzes selection biases in the project choice of complementary technologies that are used in combination to produce a final product. In the presence of complementary technologies, patents allow innovating firms to hold up rivals who succeed in developing other system components. This hold-up potential induces firms to preemptively claim stakes on component property rights and excessively cluster their R&D efforts on a relatively easier technology. This selection bias is persistent and robust to several model extensions. Implications for the optimal design of intellectual property rights are discussed. We also analyze selection biases that arise when firms differ in research capabilities.

JEL-Code: D430, L130, O300.

Keywords: R&D project choice, preemptive duplication, complementary innovations.

Jay Pil Choi
Department of Economics
Michigan State University
110 Marshall-Adams Hall
USA – East Lansing, MI 48824-1038
choijay@msu.edu

Heiko Gerlach
School of Economics
University of Queensland
St. Lucia, QLD 4072
Australia
h.gerlach@uq.edu.au

March 2011
We thank Eric Maskin and Jean Tirole for useful discussions. Sanghyun Kim provided excellent research assistance. All remaining errors are ours.

1 Introduction

This paper analyzes selection biases in the project choice of complementary innovations. In the presence of complementarity, patents allow innovating firms to hold up rivals who succeed in developing other system components. This hold-up potential induces firms to preemptively stake claims on component property rights and excessively cluster their R&D effort on a relatively easier technology. With the convergence of digital technologies, a typical product in today's high-tech industries encompasses multiple complementary innovations. For instance, a cell phone can employ a variety of technologies covered by different patents in the areas of wireless communication, digital technology, high speed broadband, and so on. The importance of complementary technologies can also be inferred by numerous patent suits and cross-licensing agreements among major players in the industry. The analysis thus has important implications for the optimal design of intellectual property rights in this new technological environment of digital convergence. The recognition of the complementary nature of innovations demands for a new way to reward innovators in order to eliminate wasteful R&D duplications and align private incentives with the socially optimal one.

To analyze the nature of selection biases in complementary R&D projects and the role of R&D competition as the main driver of these selection biases, we consider the following scenario as our basic set-up. There are two complementary innovations, A and B, both of which are needed to produce a final product. Thus, each innovation has no stand-alone value and can generate value only when used in conjunction with the other. Further assume that a firm can engage in only one project in each period. In such a case, if there is only one firm that can engage in the R&D projects, the sequence of project choice is irrelevant since the firm needs to make both innovations.[2] However, sequence choice becomes important when there is competition and the first firm that innovates receives a patent on the innovation made. We allow asymmetry in the research projects in terms of the difficulty of success and show that there is a tendency for preemptive duplication in the easier project compared to the socially optimal allocation of research project choices.

To understand the selection bias towards the easier project, consider the following nu-

[2] This holds true as long as there is no learning effect from successfully completing one project before another.

merical example. Let p and q be the success probabilities for project A and B, respectively, where $p > q$, i.e., project A has a relatively higher chance of success compared to B. Each firm can engage in only one project in each period. There are two periods and there is no discounting. Let the value of the final product comprised of the two innovations be normalized to 1. If one firm has patents for both innovations, the firm's profit is 1. If one firm has a patent for one innovation and the other firm has a patent for the other, we assume that they split the value and each firm receives 1/2. If both firms succeed in the same innovation, we assume that each firm has an equal chance of receiving the patent. To make the intuition clear, suppose that $p = 1$ and $q = 1/2$, that is, innovation in project A is certain. Then, it is clear that there should never be duplication in project A for the social optimum to maximize the probability of completing both projects by the end of the second period. However, it can be easily shown that in equilibrium both firms engage in project A in the first period.[3] The intuition for this discrepancy between the private and social incentives is that firms care about the division of market value of the final product as well as the overall probability of completing both projects by the second period. The privately optimal strategy to maximize their share of market value is to stake an early claim on the patent that is easily achievable because it allows them to hold up against anyone who succeeds in the other project.

We demonstrate that the biases identified above in complementary project choices are persistent and robust. We extend the basic set-up to allow for more than 2 firms, free entry to the R&D race, and an infinite horizon. In all variations of the basic model, we find that the biases towards preemptive duplication in the easier project persist. We also analyze selection biases in complementary R&D projects that arise from asymmetry in firm capabilities. To illustrate this, consider the same example above except that firm 1 is a "specialist" that has the capability to innovate in only project A. More specifically, assume that $p_1 = 0$, $q_1 = p_2 = q_2 = 2/3$,where subscript $i = 1, 2$, denotes firm identity. Since firm 1 can engage in only project A, the socially optimal outcome is that firm 1 engages in project A and firm 2 engages in project B in the first period. In the market equilibrium, firm 1

[3] Under the parameter assumption in this example, the equilibrium expected payoff from both choosing project A is given by 12/32. However, when one firm chooses project B while the other firm chooses A, which is the socially optimal outcome, its expected payoff is only 11/32(<12/32).

engages in project A. We can show that firm 2 also selects project A in the first period.[4] The intuition for this result is the same. Firm 2 does not face any threat in project B and thus engages in preemptive duplication in project A to secure the maximal market share.

Our paper departs from most of the R&D literature in two respects. First, the main focus of the literature has been on the aggregate amount of R&D and the comparison of equilibrium R&D spending to the socially optimal level.[5] However, as pointed out by Mansfield (1981), "the *composition* of R&D expenditures may be as important as their total size (italics added, p. 614)." The R&D management literature also emphasizes the importance of R&D project selection decisions. For instance, Ofek (2008) states that "one of the most important dilemmas confronting firms" in the product development process is "where should these efforts be directed?". Second, most of the R&D literature analyzes R&D competition for a single isolated innovation. Even though this may be a good description of many innovations in the past, it is increasingly at odds in today's high-tech industries. In our paper, we consider complementary innovations and focus on the allocation of R&D resources across projects to reflect this reality.

Early contributions that analyzed the issue of R&D resource allocations across projects include Dasgupta and Maskin (1986), Bhattacharya and Mookherjee (1986), and Klette and de Meza (1986). They consider an isolated innovation case and show that the market is biased towards the selection of more risky projects compared to the social optimum. The main intuition comes from the "winner-takes-all" nature of patent races that gives rise to a convex payoff function for potential innovators.[6] Our paper, in contrast, considers an environment of complementary innovations and analyzes the allocation of resources between the two projects.

In terms of the preemptive nature of R&D, our paper is closest to Cardon and Sasaki (1998). They show that firms have incentives to cluster on the same project even if potential technologies are ex ante equally promising and there are no informational spillovers.

[4] Firm 2's expected payoff from engaging in project A in the first period is given by $38/81$ whereas its expected payoff from engaging in project B, which is the socially optimal outcome, is only $35/81$.

[5] The classical contributions to this question include Loury (1979), Lee and Wilde (1980), Dasgupta and Stiglitz (1980), and Tandon (1983), among others. See Reinganum (1988) for an excellent survey of the literature on R&D.

[6] Cabral (1994), in contrast, derives conditions under which the market is biased against risky R&D portfolios in a setup where there can be more than one winner in the R&D competition. He analyzes firms' decisions to allocate their fixed R&D budgets across two *independent* markets, one in which innovation is easier and a larger market in which innovation is more difficult. He shows that there is a bias against risky R&D (i.e., innovation in the larger market with a lower probability of success).

However, the mechanism for clustering in their paper is very different from ours. In their paper, clustering takes place in the R&D stage to delay product market competition. This effect arises because they assume that the two potential technologies are substitutes rather than complements as in our case. If the two firms engage in different projects, there is a possibility that both firms succeed and compete in the product market. By clustering on the same project, there will only be one firm that receives a patent and the market structure is guaranteed to be monopolistic until the firm that loses out pursues and succeeds in the other competing project. In a related paper, Gerlach et al. (2005) consider firms' decisions of which products to target for innovation in a Hotelling model with stochastic R&D outcomes. They analyze how project choice depends on the degree of technical risk. As the degree of technical risk increases, i.e., innovations become harder to succeed in, there is more clustering at the mid point of the market because the risk of facing competition decreases. The main difference is that they consider substitute technologies as in most of the literature whereas we are concerned with complementary innovations.[7]

As in our paper, Gilbert and Katz (forthcoming) consider a situation in which the introduction of a new product requires complementary technologies. They derive the optimal mechanism to split profits form complementary innovations to support efficient investment in R&D. Their model assumes a pre-determined sequence of innovation races for complementary technologies. Therefore, the issue of project choice does not arise in their model. The main focus of their paper is on the aggregate investment level as in the traditional R&D literature. Fershtman and Kamien (1992) also consider complementary technologies. However, their setup and focus are quite different from those in our paper. In their model, only the final product that incorporates all complementary technologies is patentable and patents for intermediate (component) technologies are not granted. Therefore, it is possible that firms continue to develop component technologies already developed by other firms as long as all component technologies necessary for the new final product have not been developed by the same firm. Nevertheless, cross-licensing can take place when different firms have developed different component technologies. The basic trade-offs in the cross-licensing

[7] See also Lin (2009) who considers allocation of R&D resources for multiproduct firms and analyzes how competition affects the optimal R&D portfolios. He shows that, relative to a monopoly, multiproduct duopolists choose more specialized R&D portfolios with regard to their core products in order to avoid head-to-head competition. Once again, he considers substitute products and the equilibrium configuration is characterized by diversification, rather than duplication.

decision are the benefits of avoiding unnecessary duplication and speeding up the introduction of the new product against the costs of intensified competition in the final product market. The focus of their paper is how the possibility of cross-licensing affects the firms' R&D investment levels, and hence the pace of the innovation race in a model of stochastic R&D process.[8] In contrast, we consider a situation in which component technologies themselves are patentable and analyze how the patentability of component technologies drives inefficiency in the choice of R&D projects.

The remainder of the paper is organized in the following way. In section 2, we set up a very basic model of project selection for complementary innovations to illustrate the incentive to preempt on the easier project, which leads to clustering and wasteful duplication of efforts from the social planner's viewpoint. Section 3 considers prior user rights as a solution to mitigate preemptive duplication. Section 4 expands on the basic model and considers various extensions of the model to check the robustness of the main result. Section 5 analyzes selection biases in complementary R&D projects that are driven by asymmetric firm capabilities rather than asymmetric projects. Section 6 closes the paper with concluding remarks. The proofs for lemmas and propositions are relegated to Appendices A and B.

2 Model of Project Choice in Complementary Innovations

2.1 Basic Set-up

Consider two firms and two complementary innovations, A and B. Both technologies are needed for a final product.[9] Alternatively, we can consider A and B as complementary components that form a system. Let m be the monopoly profit from the final product/system good that incorporates these two innovations. Thus, if one firm has patents for both innovations A and B, the firm's profit is given by m. With only one innovation available, there is no intermediate payoff. When the patents are owned by two different firms, each patent holder's profit is denoted by d. In this situation, there is typically a pricing externality due to the complementary nature of the two innovations. We assume that the two firms can

[8] Fershtman and Kamien (1992) also consider a scenario in which component technologies are patentable. In that case, the race, in the absence of cross-licensing, is essentially over when one component technology is developed because the innovator can hold-up. The analysis in this case thus is more or less trivial since their focus is on the role of cross-licensing in the R&D race.

[9] In section 4, we extend our analysis to N firms and show that the main result is robust to this change.

internalize the externality and coordinate on pricing. This assumption is justified because price coordination reduces the overall price of the two innovations and thus benefits consumers; there is no reason for an antitrust authority to intervene. We further suppose that the two firms split the industry profit through Nash bargaining, which implies that $d = m/2$. This is a natural assumption to make in the context of complementary innovations where each innovator has the ability to hold up the marketing of the final product.

We consider a two-period model in which each firm independently decides which project to carry out in each period.[10] There is no discounting. The industry profit m can be realized only when both innovations have been made by the end of the second period. Even though the two innovations are both necessary to generate any surplus, the two projects are *asymmetric* in terms of success probability.[11] More specifically, we assume that the probability of success for projects A and B in each period are respectively given by p and q, where $0 < p, q < 1$ and $p > q$, that is, technology A is relatively easier to achieve. When a firm succeeds in an innovation, the firm gets a patent on the innovation. If both firms are successful in the same innovation in the same period, we assume that each firm gets the patent with a probability of $\frac{1}{2}$.[12] [13]

The main focus of the paper is the composition of the R&D project portfolios rather than the level of R&D investments. We thus assume away the cost of R&D, and instead suppose that each firm can engage in only one R&D project in each period. One way to justify this assumption is that each firm has a fixed R&D budget or limited R&D personnel, which does not allow implementation of simultaneous R&D projects in the same period.[14]

When both projects are successfully carried out and the final product or system good is introduced in the market, social welfare is given by w.[15] In this framework, it is easy to verify that under monopoly there is no discrepancy between the private choice and the

[10] In section 4, we extend our analysis to an infinite horizon model and show that the main result is robust to changes in the time horizon.

[11] In section 5, we extend our analysis to the case where markets are symmetric but firms differ in their research capabilities.

[12] The event of simultaneous discovery arises due to the discrete time framework we use. In a continuous time framework with a Poisson distribution of a successful innovation, the probability of simultaneous innovation is a measure zero event and the probability of each firm winning is 1/2.

[13] In the next section we consider the prior user rights policy which grants patents to all innovators when firms innovate at the same time.

[14] In section 4, we consider the possibility of costly entry into the R&D stage and derive the same qualitative results.

[15] Note that the social surplus is the same regardless of the number of firms that hold patents because we assume that price coordination takes place if patents are held by two different firms.

socially optimal choice. In fact, in both cases the sequence of project choices does not matter. With two periods, private benefits and social surplus can be realized only when an innovation is made in each period, and the probability of such an event is given by pq regardless of the sequence. This implies that if there is any discrepancy between the market equilibrium and the socially optimal outcome in our setup, it can be attributed only to the rivalry in R&D competition.

Finally, the following notation is helpful for exposition throughout the paper. Suppose in period 1 exactly one patent is awarded. This occurs if *(i)* both firms pursue the same technology and at least one is successful, or if *(ii)* they choose different projects and exactly one firm is successful. Further assume that the success probability for the remaining technology is x for the patentholder and y for the unsuccessful firm.[16] Let $\pi_1(x,y)$ denote the overall expected market share for the firm that obtains the patent in period 1,

$$\pi_1(x,y) \equiv xy\frac{3}{4} + x(1-y) + (1-x)y\frac{1}{2}.$$

This market share takes values between 1/2 and 1. It strictly increases in the patentholder's capability x. The effect of an increase in the rival's capability is ambiguous due to the complementarity of the technologies. A higher y reduces the patentholder's expected market share when this firm is successful in period 2. By contrast, a more capable rival is beneficial in the event that the patentholder is not successful in period 2. In this case a higher y increases the probability that both technologies are available at the end of period 2 (and that the firm's patent from period 1 becomes valuable). The negative effect of a strong rival increases in x, whereas the positive effect decreases in x. Verify that if the patentholder's capability is less than 2/3, then the firm benefits from a stronger rival with $\partial\pi_1(x,y)/\partial > 0$. Otherwise, we have $\partial\pi_1(x,y)/\partial y < 0$. Similarly, let $\pi_0(x,y)$ denote the overall expected market share of the unsuccessful firm in the first period,

$$\pi_0(x,y) \equiv xy\frac{1}{4} + x(1-y)\frac{1}{2}.$$

This value increases in the firm's own capability x, decreases in its rival's success rate and is between 0 and 1/2. Note that these market shares are defined conditional on both

[16] We allow different success probabilities for the same project because we consider asymmetric firm capabilities in section 5.

technologies being available by the end of the second period. Hence, the sum of the market shares $\pi_0(x, y) + \pi_1(y, x)$ is the probability that at least one of the firms is successful in the second period.

2.2 Social Optimum

As a benchmark we first derive the second best social optimum for project choice, given the pricing decisions and the patent system. We show that the socially optimal project choice is for each firm to diversify, i.e., for one firm to pursue project A and the other to pursue project B to eliminate duplication of R&D output in the first period.

We proceed by backward induction. Consider the efficient project choices in the second period. If there is no innovation in the first period, it is clear that the social optimum is for each firm to diversify. If both firms engage in the same project, the expected social surplus is zero because both innovations are needed due to the complementary nature of the innovations. If there is only one innovation in the first period, the optimum is for both firms to engage in the remaining innovation project.

Now consider the first period project choices. Let the expected social surplus when both firms engage in project $i \in \{A, B\}$ in the first period be denoted by $W(i, i)$. Assume x is the success probability of technology i whereas y is the success rate of technology j. We get

$$W(i, i) = (1 - (1 - x)^2)[\pi_1(y, y) + \pi_0(y, y)]w + (1 - x)^2 xyw.$$

If at least one firm is successful in i, both firms dedicate themselves to technology j where the joint probability of success is $\pi_1(y, y) + \pi_0(y, y) = 1 - (1 - y)^2$. Otherwise, they diversify in period 2. Similarly, let $W(i, j)$ denote the expected social surplus when firms choose different projects $j \neq i$. We get

$$\begin{aligned} W(i, j) = pqw + (1 - p)q[\pi_1(p, p) + \pi_0(p, p)]w + \\ (1 - q)p[\pi_1(q, q) + \pi_0(q, q)]w + (1 - p)(1 - q)pqw \end{aligned}$$

A social planner chooses the project allocation that maximizes the probability of completing both projects by the end of period 2. Comparing diversification and clustering in period one yields

$$W(i,j) - W(i,i) = pq[1 - \pi_1(y,y) - \pi_0(y,y)]w > 0$$

and we get the following welfare benchmark.

Lemma 1 *The socially optimal project choice in the first period is to have diversification in the project choices across firms.*

When firms engage in different projects in period 1, both innovations are completed in the event that each firm's research project is successful. By contrast, when firm pursue the same technology, only one innovation is completed in period 1 and there is a technical risk that the remaining technology is not achieved by the end of period 2. In other words, diversification avoids R&D duplication which is purely wasteful from the social planner's viewpoint. Note that this holds for any $0 < p, q < 1$.

2.3 Market Equilibrium

We can easily verify that in the second period, the social optimum and the market equilibrium coincide. However, in the following we show that in the first period situations arise where both firms cluster on the same project whereas the social optimum dictates that each firm engages in different projects.

As a first step, suppose firm 1 is research active in both periods while firm 2 is only active in the second period. If firm 1 is successful with its research in the first period, then both firms engage in the remaining project. If firm 1 is not successful, then each firm tackles a different technology and expects profits of $pqm/2$. Firm 1 as the only research active firm in period 1 prefers project A over B if and only if

$$p\pi_1(q,q) + (1-p)pq\frac{1}{2} \geq q\pi_1(p,p) + (1-q)pq\frac{1}{2} \quad (1)$$
$$\text{or} \quad p\pi_1(q,q) - q\pi_1(p,p) - (p-q)pq\frac{1}{2} \geq 0.$$

Obtaining a technology B patent in period 1 is more valuable since it is more likely that at least one of the two firms develops technology A in the second period. At the same time technology B is harder to develop. However, we can easily verify that this trade-off is umbiguously resolved towards firm 1 choosing project A. To see the economic intuition behind this result, note that both options yield the same probability of completing both projects by the end of period 2.[17] Hence, firm 1 picks the one that maximizes its total expected market share in the system. Since each firm expects to secure half of the patents obtained in the second period, the profit maximizing choice for firm 1 in the first period is to engage in the technology that is easier to develop. In other words, firm 1 uses its first mover advantage to preempt its rival by securing the patent that is easier to obtain.

Now consider the case where both firms are active in both periods. Let us denote $\Pi(i,j)$ to represent a firm's expected profits when the firm engages in project i and the other firm engages in project j in the first period, where $i, j \in \{A, B\}$. First, suppose firms choose the same project i which has a success probability x whereas the other project succeeds with probability y. Then, we have

$$\Pi(i,i) = x^2 \frac{1}{2}[\pi_0(y,y) + \pi_1(y,y)]m + x(1-x)[\pi_0(y,y) + \pi_1(y,y)]m + (1-x)^2 xy \frac{1}{2} m.$$

With probability x^2 both firms succeed in the first period, in which case each firm gets a patent on the first innovation with probability 1/2. In the second period, both firms engage in R&D for the second innovation. With probability $x(1-x)$ one firm is successful in period 1 while the other one is not. As patentholder the firm gets $\pi_1(y,y)$ in the second innovation; if unsuccessful, it gets $\pi_0(y,y)$. If both firms are unsuccessful in the first period, they diversify in the second period and with probability xy each receives half of the market value.

Similarly, suppose firms choose different projects. One firm chooses project i with success rate x whereas its rival engages in technology j with success rate y. Then the expected profits of the firm with project i is given by

$$\Pi(i,j) = xy\frac{1}{2}m + x(1-y)\pi_1(y,y) + (1-x)y\pi_0(x,x)m + (1-x)(1-y)xy\frac{1}{2}m.$$

With probability xy both firms are successful and each holds a patent for one of the two technologies. With probability $x(1-y)$ only the firm in project i was successful and receives a patent. Both firms then engage in the research of project j. Conversely, with probability $y(1-x)$ only the rival succeeds and firms continue with technology i.

[17] If firm 1 chooses project A in the first period, the probability of completing both projects by the end of period is given by $p[1-(1-q)^2] + (1-p)pq$. The corresponding probability when firm 1 chooses project B in the first period is given by $q[1-(1-p)^2] + (1-q)pq$. A simple manipulation shows that both expressions are equal to $pq(3-p-q)$.

Let us turn to the equilibrium analysis. First, suppose firm 2 chooses project A. Firm 1 prefers to cluster and choose the same project if and only if $\Pi(A,A) \geq \Pi(B,A)$ or

$$(1-p)[p\pi_1(q,q) - q\pi_1(p,p) - (p-q)pq\frac{1}{2}] + p(p-q)\frac{1}{2}[\pi_1(q,q) - \pi_0(q,q)] \geq \quad (2)$$
$$pq\frac{1}{2}[1 - \pi_1(q,q) - \pi_0(q,q)].$$

This incentive constraint describes the basic trade-off for firms between preemptive clustering and gains from diversification. The LHS is the net benefit from clustering in the easier technology in order to preemptively secure a patent in the first period. The first term is the expected gain from preemption when the rival firm in A is unsuccessful. In particular, the expression in the squared bracket is identical to the net benefit from clustering when the rival is not active in period one. As discussed in (1), this term is always positive. The second term on the LHS is the gain from clustering when the rival is successful. With probability mass *p*-*q* firm 1 is successful when joining market A but not when pursuing B. In market A each firm obtains the patent with probability 1/2. In the latter case rival firm 2 always obtains the patent. The difference is half of the value of the second squared bracket on the LHS. The RHS is the expected gain from diversification when both firms' projects are successful. Choosing different projects avoids duplication of R&D and enables the firms to complete both innovations at the end of period 1. By contrast, clustering yields one successful innovation in period 1 and the technical risk as to whether both projects are accomplished by the end of period 2. To quantitify the trade-off between preemption and diversification, we simplify and rewrite (2) as

$$(p-q)\frac{1}{4}(3-p-q) \geq \frac{1}{2}(1-q)^2. \quad (2')$$

For p=q the LHS of (2) is equal to zero whereas the gains from avoiding duplication are positive. By contrast, if p=1, then the preemption incentive dominates diversification and firm 1 always clusters.[18] Finally, it can be verified that (2) holds if and only if $p \geq p^*$ where p^* is the smaller root of the solution to (2') as an equality.[19]

[18] In this case the incentive constraint reduces to $1/2 - q/4 \geq 1/2 - q/2$.

[19] Rewrite (2') as equality, i.e. $(1-p^*)p^* + (1-q)q - 2(1-p^*) = 0$, and check that the difference between LHS and RHS is concave in q and has a strictly positive derivative $3-2p$ for all p in $[0,1]$. The threshold is thus given by

$$p^* = 3/2 - \sqrt{1/4 + q(1-q)}.$$

Alternatively, suppose one firm chooses project B. The rival firm prefers project A if and only if $\Pi(A,B) \geq \Pi(B,B)$ or

$$(1-q)[p\pi_1(q,q) - q\pi_1(p,p) - (p-q)pq\frac{1}{2}] + q(p-q)[\frac{1}{2} - \pi_0(p,p)] \qquad (3)$$
$$+q^2\frac{1}{2}[1 - \pi_1(p,p) - \pi_0(p,p)] \geq 0.$$

Verify that this condition always holds. The first term is the expected gain from preemption when the rival in B is not innovating. By condition (1) the first squared bracket is strictly positive. The second term is the expected preemption benefit when the rival is successful. With probability mass *p*-*q* firm 1 is successful in market A but not in B. In this case, the relative market share advantage from being in market A is the term in the second squared bracket. The last term is the benefit from avoiding R&D duplication when both innovate. Thus, when one firm opts for market B, its rival is always better off choosing technology A.

From this analysis two equilibrium regimes follow. If (2) is satisfied, then the unique equilibrium is (A,A), i.e. both firms engage in project A in the first period, causing inefficiency in the project choice. Otherwise, the equilibrium involves firms choosing different projects, either firm 1 in A and firm 2 in B or vice versa, i.e. (A,B) or (B,A).[20] This is illustrated in Figure 1 below. It is instructive to pinpoint circumstances under which there is inefficient clustering on the technology that is easier to implement. When both projects are relatively difficult, that is, both p and q are small (with the maintained assumption of $p > q$), there is diversification of project selection. As discussed above, preemption is only profitable if project A has a sufficiently high success rate to overcome the gains from R&D diversification. By contrast, preemption incentives are non-monotonic in the success rate of project B. Preemption is most likely to occur if q is intermediate. First, if q is high and sufficiently close to p, then the relative gain from preemption is small. Second, preemptive patenting in technology A is only valuable if there is an innovation in technology B in period 2. Hence, for low values of q, there is a countervailing incentive against clustering to maximize the overall probability that both innovations are made. As a consequence,

[20] We assume that the two firms can coordinate on one of the two pure strategy equilibria. There is also a symmetric mixed strategy equilibrium in which both firms choose project A with probability $\frac{\Pi(A,B)-\Pi(B,B)}{[\Pi(A,B)+\Pi(B,A)]-[\Pi(A,A)+\Pi(B,B)]}$. In the mixed strategy equilibrium, there can also be inefficient duplication when both firms choose the same project A or B due to a coordination failure.

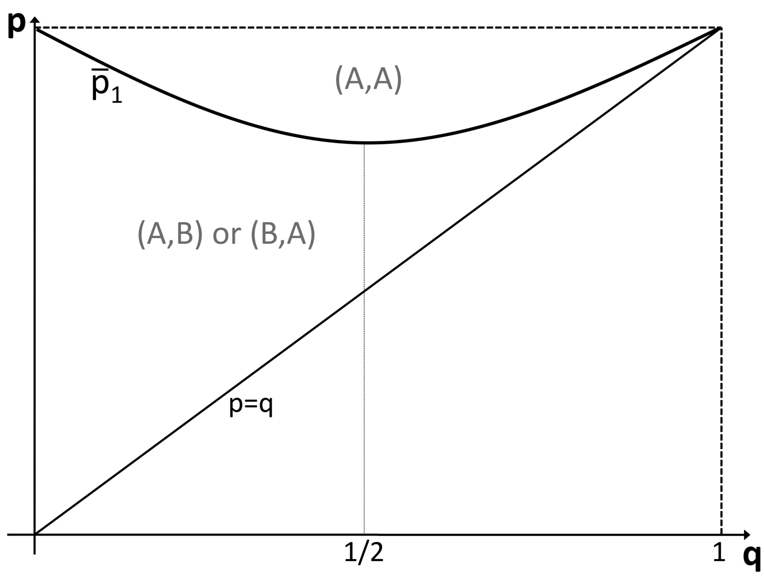

Figure 1: *Project choice equilibrium with asymmetric markets*

preemption is most profitable for sufficiently high values of p and intermediate values of q. We summarize as follows.

Proposition 1 *For complementary innovations with asymmetric success probabilities, the equilibrium project choices may be biased towards the easier technology.*

Note that the market equilibrium and the socially optimal outcome coincide if we consider a static model in which the firms have only one opportunity to engage in R&D, because the only way they can collectively bring the final product to the market is to be successful in both innovations due to the complementary nature of the innovations. This indicates that the source of inefficiency in the simple model arises solely due to the *dynamics of competition*.[21]

3 Prior User Rights as a Solution to Preemptive Duplication

In a recent paper, Shapiro (2006) asks the question of how property rights should be defined and allocated when innovations are made independently and at roughly the same time by more than one individual or firm. This is a pertinent question in our setup where duplicative

[21] In section 4 we analyze the case where the number of firms is larger than the number of complementary projects and show that this can lead to inefficiencies in the static case, too.

research efforts lead to a possibility of simultaneous discovery. In consistency with the current patent system, we have assumed that each firm has an equal chance of receiving the patent and obtaining exclusive property rights on the innovation. Shapiro assumes that both firms have the right to use the invention with prior user rights. In other words, he abstracts from the details of which party discovered the invention first, and treats prior user rights as an independent invention defense. Slight difference in invention timing are considered random. With such an abstraction, Shapiro analyzes the effects of prior user rights on R&D resource allocations and explores their welfare implications in the context of a single isolated innovation. In the following we consider the effects of prior user rights in the context of complementary innovations and show that prior user rights have a salutatory effect of mitigating duplicative research project choice.[22] Thus, the positive effects of prior user rights extend beyond the ones identified in Shapiro (2006).

To analyze the prior user rights policy, we need to consider two additional market structures at the end of period 2, which could not arise in the set-up of section 2. First, suppose one firm holds patents for both technologies whereas its rival holds exactly one patent. In this case, we suppose that the dominant firm with two patents is able to extract the monopoly rent from the system and earn profits of m. Second, when both firms end up holding patents for both technologies, the monopolistic rent is entirely competed away. From this we can construct the expected conditional market shares when exactly one firm receives a patent in period 1. The expected market share of a firm holding one patent while its rival was not successful is given by

$$\widehat{\pi}_1(x,y) \equiv x + (1-x)y\frac{1}{2}.$$

Conversely, a firm without patent after period 1 who faces a rival with exactly one patent expects a total market share of

$$\widehat{\pi}_0(x,y) \equiv x(1-y)\frac{1}{2}.$$

[22] Maurer and Scotchmer (2003) make a similar point in a static model with determistic R&D outcomes.

Notice that

$$\widehat{\pi}_1(x,y) > \pi_1(x,y), \widehat{\pi}_0(x,y) < \pi_0(x,y), \text{ and } \widehat{\pi}_1(x,y) + \widehat{\pi}_0(x,y) = \pi_1(x,y) + \pi_0(x,y)$$

Compared to the standard patent system, a patent holder from period 1 has a higher expected overall market share whereas the unsuccessful firm's expected market share is lower. This is due to the fact that with prior user rights the patentholder obtains a market share of one if he innovates in the second period independent of the success of its rival.[23]

Next consider ex ante profits as function of project choices. Suppose both firms choose project i with success probability x in the first period. The expected profits from clustering with *prior user rights* are

$$\widehat{\Pi}(i,i) \equiv \Pi(i,i) - x^2 y^2 \frac{1}{2} m.$$

The only difference to the standard patent system arises if both firms succeed and receive user rights in the first period. In this case a firm only makes positive profits at the end of period 2 if it is the sole innovator of the remaining technology. Hence, relative to the standard patent system, prior user rights reduce the value of clustering. Similarly, consider the expected profits of a firm pursuing project i (with success rate x) whereas the other firm engages in project j in the first period,

$$\widehat{\Pi}(i,j) \equiv \Pi(i,j) + x(1-y)[\widehat{\pi}_1(y,y) - \pi_1(y,y)]m + (1-x)y[\widehat{\pi}_0(x,x) - \pi_0(x,x)]m.$$

Here the difference to the standard patent system arises when one firm is successful in period 1 while the other one is not. As mentioned above, the successful firm gains whereas the unsuccessful firm loses relative to a standard patent system. The overall effect is thus ambiguous.

Before deriving the incentive constraint for clustering, consider the project choice in the case where firm 1 is active in both periods whereas firm 2 is only active in period 2. In this setting prior user rights remove the preemption incentive of the standard patent system and

[23] Note that at the same time, the probability that at least one firm is successful, $\widehat{\pi}_0(x,y) + \widehat{\pi}_1(y,x)$, is the same as in the previous section.

firm 1 is indifferent as to which project to choose in period 1, i.e.

$$p\widehat{\pi}_1(q,q) + (1-p)pq\frac{1}{2} = q\widehat{\pi}_1(p,p) + (1-q)pq\frac{1}{2}. \tag{4}$$

To see this, consider the two pathways that lead to the completion of both innovations and verify that firm 1 is indifferent between the two projects in both cases. If firm 1 is successful in both periods, it will have a market share of one, independent of firm 2's R&D outcome. Hence, the initial project choice is irrelevant. If firm 1 succeeds exactly once and firm 2 succeeds in period 2, then firm 1's market share is 1/2 and both projects have the same overall probability of completing the two technologies.

Given that there is no preemption incentive when the rival is not successful, we can write $\widehat{\Pi}(B,A) \geq \widehat{\Pi}(A,A)$ as

$$pq[\frac{1}{2} - q(1-q)] \geq (p-q)p[q(1-q) - \widehat{\pi}_0(q,q)] \tag{5}$$

The LHS is the gain from diversification when both firms are successful. The RHS is the preemption gain from raising the success probability by pursuing A instead of B given that the rival is successful. With probability mass $p-q$ the firm is successful in A and jointly awarded a patent whereas it would not innovate in B. Note that the RHS is strictly smaller than the second term in (2) and the LHS is strictly larger than the RHS in (2). Thus, prior user rights reduce preemption incentives and increase the loss from R&D duplication. Simplifying (5) further yields

$$1 - 2q(1-q) \geq (p-q)(1-q) \tag{5'}$$

which, by inspection, is always satisfied.[24] Hence, an equilibrium, in which both firms choose technology A in the first period, does not exist. Furthermore, consider a situation where firm 1 chooses technology A and firm 2 technology B. Firm 2 always gains more from preemption by deviating to A than firm 1 from deviating to B. From this and (5) follows that, with prior user rights, the unique (modulo symmetry) pure strategy equilibrium involves

[24] The RHS is maximized at p=1. For this value the condition simplifies to

$$1 - 2q + 2q^2 \geq 1 - 2q + q^2.$$

the two firms choosing different research projects. Prior user rights increase the gains from diversification and reduce the benefits from preemption. As a result, the bias towards joint discovery due to preemptive duplication incentives can be mitigated.

Proposition 2 *In the only pure strategy equilibrium with prior user rights the two firms choose different research projects. Hence, the market equilibrium is efficient with prior user rights.*

4 Extensions and the Robustness of the Basic Model

In this section, we extend the basic model in several dimensions and illustrate that the main result of the paper – preemptive duplication in the easier project for complementary innovations – is robust to changes in the basic model.

4.1 N Firm Model

We first extend our model to consider a more general N firm case. Suppose that there are N (>2) firms that can engage in the two R&D projects. We implicitly assume barriers to entry, and the number of firms is fixed at N.[25] Apart from this we follow the assumptions of the model in section 2. In particular, each firm can engage in only one project and R&D outcomes are independent across firms and projects. Let n_A and n_B denote the number of firms that engage in R&D projects A and B, respectively, with $n_A + n_B = N$. The probability that all firms fail in project A is given by $(1-p)^{n_A}$. Let $P(n_A)$ denote the probability that at least one firm is successful in project A when n_A firms invest in project A. Then, we have $P(n_A) = 1 - (1-p)^{n_A}$. Similarly, we can denote the probability that at least one firm succeeds in project B as $Q(n_B) = 1 - (1-q)^{n_B}$.

4.1.1 Static Model as a Building Block

In this set-up, the main efficiency question is how many firms will engage in each project and how the market equilibrium configuration compares to the socially optimal outcome. To address this issue, we first analyze the static case in which each firm has only one chance to engage in R&D. In the simple model we analyze in section 2, there is no inefficiency in

[25] We consider the free entry case below and show that the conclusions are robust.

the static framework since two firms are always better off diversifying. We now show that this result does not carry over to situations with more than two firms.

To see this, note that the social planner maximizes the probability that both innovations are made, given the total number of firms N. In other words, the social planner solves

$$\begin{aligned} \underset{n_A, n_B}{Max} \qquad & W \equiv P(n_A)Q(n_B) = [1-(1-p)^{n_A}][1-(1-q)^{n_B}] \\ & \text{subject to } n_A + n_B = N \end{aligned}$$

For the simplicity of the analysis, we treat n as a continuous variable. The first order conditions imply the following relationship between n_A and n_B,

$$\frac{\partial P(n_A)}{\partial n_A} Q(n_B) = \frac{\partial Q(n_B)}{\partial n_B} P(n_A).$$

The optimal allocation of firms equates the *marginal* gain in the system success rate across the two projects. The next lemma characterizes the socially optimal allocation of firms.

Lemma 2 *Let n_A^o and n_B^o be the socially optimal division of projects among N firms in the static problem, where $n_A^o + n_B^o = N$. Then, it holds that $n_A^o < n_B^o$. Furthermore, the socially optimal number of firms in project A decreases in p and increases in q.*

Lemma 2 states that the socially optimal division of firms across projects requires more resources to be devoted to the more difficult project. This result can also be interpreted as the optimal resource allocation for a monopolist with a fixed research budget. For instance, consider a monopolistic firm with N research teams that can be assigned to one of the two projects. The optimal solution to this problem is isomorphic to the social planner's problem in that both problems maximize the probability that each one of both innovations is made by at least one team. The monopolist will devote more resources to the more difficult project.

In the market equilibrium, the expected benefits of participating in each project should be equalized. Otherwise, a firm in the less profitable project has an incentive to switch to the other project. The equilibrium numbers of participants, n_A^* and n_B^*, in each project are thus characterized by

$$\frac{P(n_A)}{n_A} Q(n_B) = \frac{Q(n_B)}{n_B} P(n_A).$$

In other words, a firm's *average* contribution to system success has to be equal in both projects. This, of course, requires that the number of firms in each project is identical, i.e. $n_A^* = n_B^*$. Since the total number of participating firms is fixed and the social planner prefers to devote more firms to the more difficulty project, we get the following result.

Proposition 3 *In the static problem with N>2 firms participating in the R&D projects, too many firms engage in the easy project compared to the socially optimal configuration.*

Note that the market equilibrium configuration is independent of p and q whereas the socially optimal configuration is sensitive to the relative magnitude of the success probabilities. Suppose we are in a symmetric situation in which $p = q$. Then, the socially optimal outcome and the market equilibrium coincide with $n_A^* = n_B^* = n_A^o = n_B^o = N/2$. From Lemma 2 follows that as we move further away from this situation, the discrepancy between the two outcomes becomes magnified since the market equilibrium is invariant to changes in the success probabilities. In other words, as the more difficult project becomes harder, or the less difficult project becomes easier, there is more excessive clustering in the easier project.

4.1.2 Dynamic Model with N firms

Now consider a two period model of innovations like in Section 2. Each firm can engage in only one project in a given period, but each firm has two chances. We show that in this dynamic setting, the excessive clustering problem worsens relative to the static model analyzed above.

In the second period, there are four possible scenarios. If both innovations were made in the first period, the system can be marketed and then the game ends. If only one innovation A (B) was made in the first period, then the market equilibrium and the socially optimal outcome are the same; all firms engage in the remaining project B (A). If neither innovation was made in the first period, the second period problem is the same as the static problem from section 4.1.1.

Our focus is therefore the first period decision problem for each firm and the social planner. First, consider the social planner's problem, which is to maximize the probability of making both innovations by the end of the second period. Let Λ^o be the probability that both innovations will be made when N firms are allocated optimally in the static problem, i.e. $\Lambda^o \equiv P(n_A^o)Q(n_B^o)$. Then the social planner's problem is

$$\begin{aligned} \underset{n_A,n_B}{Max} \quad & \widehat{W} \equiv P(n_A)Q(n_B) + P(n_A)[1-Q(n_B)]Q(N) \\ & +[1-P(n_A)]Q(n_B)P(N) + [1-P(n_A)][1-Q(n_B)]\Lambda^o \\ & \text{subject to } n_A + n_B = N \end{aligned}$$

The solution to this problem implies that the marginal value of increasing the number of firms in each project is equal, i.e.

$$\begin{aligned} & \frac{\partial P(n_A)}{\partial n_A}[Q(n_B) + [1-Q(n_B)]Q(N) - Q(n_B)P(N) - [1-Q(n_B)]\Lambda^o] = \\ & \qquad \frac{\partial Q(n_B)}{\partial n_B}[P(n_A) + [1-P(n_A)]P(N) - P(n_A)Q(N) - [1-P(n_A)]\Lambda^o]. \end{aligned}$$

From this the socially optimal allocation can be characterized as follows.

Lemma 3 *Let $\widehat{n}_A^o$ and $\widehat{n}_B^o$ be the socially optimal division of projects among N firms in the first period of the dynamic model where $\widehat{n}_A^o + \widehat{n}_B^o = N$. Then, it holds that $\widehat{n}_A^o < n_A^o < N/2$.*

Compared to the efficient solution in the static setting, the social planner allocates relatively more firms to pursue the more difficult project B in the first period of the dynamic problem. The reason for this is that the social opportunity cost of adding a firm to the harder project is higher in the static setting relative to the first period of the dynamic problem. Thus, when firms are unsuccessful in the first period of the dynamic model, the social planner optimally re-allocates some firms from the harder to the easier project.

Let us turn to the market equilibrium in the first period. Given the market equilibrium configuration in the second period, define $\Pi_A(n_A, n_B)$ and $\Pi_B(n_A, n_B)$ to be each firm's expected profits of entering into project A and B, respectively, when there are n_A and n_B firms for each project. Let Λ^* be the probability that both innovations will be made when N firms are allocated according to the market equilibrium in the static problem, i.e. $\Lambda^* \equiv P(n_A^*)Q(n_B^*)$. Then, we have

$$\begin{aligned} \Pi_A(n_A, n_B) \quad = \quad & \Big\{ \frac{P(n_A)}{n_A}Q(n_B) + P(n_A)[1-Q(n_B)]Q(N)(\frac{1}{n_A} + \frac{1}{N}) \\ & +[1-P(n_A)]Q(n_B)\frac{P(N)}{N} + [1-P(n_A)][1-Q(n_B)]\Lambda^* \frac{2}{N} \Big\} \frac{m}{2} \end{aligned}$$

and

$$\Pi_B(n_A, n_B) = \left\{P(n_A)\frac{Q(n_B)}{n_B} + [1-P(n_A)]Q(n_B)P(N)(\frac{1}{n_B}+\frac{1}{N}) \right.$$
$$\left. +P(n_A)[1-Q(n_B)]\frac{Q(N)}{N} + [1-P(n_A)][1-Q(n_B)]\Lambda^*\frac{2}{N}\right\}\frac{m}{2}$$

In a market equilibrium no firm should have an incentive to switch to the other project, i.e. it has to hold that $\Pi_A(n_A, n_B) = \Pi_B(n_A, n_B)$. The next lemma characterizes the project choice equilibrium of the dynamic game.

Lemma 4 *Let $\widehat{n}_A^*$ and $\widehat{n}_B^*$ be the allocation of firms to projects in the equilibrium of the first period of the dynamic model where $\widehat{n}_A^* + \widehat{n}_B^* = N$. Then, it holds that $\widehat{n}_A^* > N/2$.*

In the equilibrium of the dynamic model with N firms, preemption incentives induce more firms to engage in the easier project A than in project B. By contrast, we know from Lemma 3 that a social planner would allocate a majority of firms to the harder project B. Hence, we obtain the same selection bias towards the easier project as in section 2. Our results actually allow us to characterize this bias further. Notice that in the static problem of section 4.1.1 the selection bias is given by $n_A^* - n_A^o > 0$. Since $\widehat{n}_A^o < n_A^o$ and $\widehat{n}_A^* > n_A^* = N/2$, the outcomes of the static and dynamic model can be ranked as follows,

$$\widehat{n}_A^* - \widehat{n}_A^o > n_A^* - n_A^o > 0.$$

Thus, compared to the static selection model, there will be an *additional* preemptive incentive to concentrate on the easier project in a dynamic model. Or, put differently, a selection bias occurs in each stage of the dynamic R&D selection model.

Proposition 4 *In the dynamic R&D project selection model with N>2 firms, too many firms engage in the easier project. A selection bias towards the easier project occurs in each stage of the dynamic model.*

4.2 R&D Costs and Free Entry

4.2.1 R&D Selection Model with Endogenous Entry

So far, we have assumed that there is no cost of R&D with a fixed number of firms who are capable of engaging in R&D. In this subsection, we demonstrate that the main intuition holds even as we introduce free entry with R&D costs. To this end, consider a simple extension of the model with $N>2$ firms, in which each firm can engage in one of the two projects at a cost of c. We further make the assumption that the private value and the social value of completing both innovations are the same ($m = w$). This allows us to eliminate the source of inefficiency that arises from the discrepancy between market and social value; any inefficiency in this setup is due to the *competition effect*.

In the market equilibrium of the static model, there will be entry until the expected benefits of participating will be equal to the cost of entry c for both projects. This implies that the equilibrium numbers of participants in each project, n_A^* and n_B^*, are characterized by

$$\frac{P(n_A)}{n_A}Q(n_B)\frac{m}{2} = P(n_A)\frac{Q(n_B)}{n_B}\frac{m}{2} = c. \tag{6}$$

The socially optimal number of entrants in each project solves

$$\underset{n_A,n_B}{Max} \quad P(n_A)Q(n_B)m - cn_A - cn_B$$

and the socially optimal number of entrants (n_A^o, n_B^o) satisfies the first order conditions

$$\frac{\partial P(n_A)}{\partial n_A}Q(n_B)m = P(n_A)\frac{\partial Q(n_B)}{\partial n_B}m = c. \tag{7}$$

By comparing the market equilibrium condition (6) to the social optimality condition (7), we can identify two sources for inefficiencies. First, as is the case with the commons problem, the free entry equilibrium associates the number of entrants with the average benefit whereas the socially optimal outcome is concerned with the marginal benefit of an additional entrant. Since the probability of at least one success for each project is a concave function of the number of participants, the average profit exceeds the marginal profit, leading to excessive entry. This is consistent to Tandon (1983) who shows that there is excessive entry for the case of *isolated* innovations because the average benefit exceeds the marginal benefit of the marginal entrant. However, with complementary innovations, there is a countervailing effect. More precisely, with complementary innovations, an additional entrant in one project confers positive externalities on the other innovation, which is ignored in the private entry decision. This non-internalization of the positive externality is reflected by the factor $m/2$ (rather than m) in the condition for the market equilibrium. This positive externality effect induces the extent of market entry to be insufficient compared to the social optimum. Due to the coexistence of these two conflicting effects, we do not have an unambiguous result on the extent of market entry compared to the socially optimal entry level. The extent of entry in the market equilibrium can be either excessive or insufficient.

In addition, we derive another source of inefficiency with complementary R&D projects. Given the total number of firms that engage in R&D, too many firms choose project A compared to the socially optimal division between projects A and B. As shown above, the number of entrants in each project will be the same in the market equilibrium. The social optimum, however, requires that more firms to engage in the more difficult project B.

Proposition 5 *With free entry into R&D project participation, there are two sources of inefficiency. First, there is excessive or insufficient entry in that the total number of firms participating in the project exceeds or falls short of the socially optimal number of participants. Second, given the total number of firms participating in the R&D projects, too many firms engage in the easy project compared to the socially optimal configuration.*

Conceptually, the analysis of the overall extent of entry in the static model with complementary innovations can be easily extended to a dynamic setting. However, the full characterizations of the extent of entry in the market equilibrium and the socially optimal outcome are tedious, and do not yield sharp predictions about the relationship between them due to the two countervailing effects identified above. Nevertheless, *given* the aggregate number of entrants in the first period, we can apply the same logic developed earlier and conclude that there are too many firms engaged in the easy project.

4.2.2 Intermediate Licensing

With free entry, we briefly remark on the role of the *intermediate* stage licensing as a mechanism to encourage R&D for complementary innovations. More precisely, we consider an intermediate stage in which only one innovation has been made and analyze the patent holder's incentive to offer its innovation at a fixed price before investments for complementary innovations are made. Such intermediate licensing can serve as a commitment mechanism not to hold up against complementary innovations. To understand this, let us consider a subgame in which a firm has a patent on one innovation. Let x be the probability of success for the remaining innovation. Let $P(n; x)$ be the probability of success that at least one firm is successful in the remaining project, that is, $P(n;x) = 1-(1-x)^n$. Suppose that the patent holder can offer its innovation at any price αm, where $\alpha \in [0, 1/2]$. Then, the profit to any firm who receives the remaining patent is given by $(1-\alpha)m$. Thus, given the intermediate licensing price of αm, the number of firms that participate in the remaining R&D project is implicitly defined by

$$\frac{P(n;x)}{n}(1-\alpha)m = c \tag{8}$$

Let $n^*(\alpha; x)$ be the number of entrants that satisfies the relationship above. It can be easily verified that $n^*(\alpha; x)$ is deceasing in α and increasing in x. For the moment, let us ignore the constraint $\alpha \in [0, 1/2]$ and assume that the patent holder can choose any α. Then, the patent holder's optimal licensing contract can be derived by solving

$$\underset{\alpha}{Max} \quad \alpha P(n^*(\alpha, x); x)$$

The first order condition for this problem is given by

$$P(n^*(\alpha, x); x) + \alpha \frac{\partial P}{\partial n} \frac{\partial n^*}{\partial \alpha} = 0$$

which implicitly defines the optimal choice $\alpha^*(x)$. By totally differentiating the first order condition, we can easily verify that α^* is an increasing function of x, that is, the patent holder would like to charge a higher intermediate licensing fee when the complementary innovation is easier. This implies that there is a critical x^* where $\alpha^*(x^*) = 1/2$. Note that the patent holder cannot charge more than 1/2 since the innovator of the complementary innovation would also have an ability to hold up. We can conclude that the optimal licensing contract is given by $min[\alpha^*, 1/2]$. In other words, when $x \geq x^*$, there is no role for intermediate licensing. However, if the complementary innovation is sufficiently hard, that is, $x < x^*$, the patent holder has an incentive to commit to receive less than half of the monopoly profit to encourage entry into the remaining R&D project. For instance, the need to encourage complementary innovations may explain IBM's recent pledge to grant free access to its patents.[26] It is straightforward to show that such intermediate licensing contracts maximize

[26] See Lohr (2005). See also Green and Scotchmer (2005) and Bessen and Maskin (2009) who analyze a similar issue in the context of *sequential* innovations.

ex post social welfare and thus are *ex post* optimal.[27] However, more importantly, the possibility of intermediate licensing can increase the *ex ante* expected profits from choosing the easier project in the first place and exacerbate the incentive problem of clustering towards the easier project.

4.3 Infinite Horizon Model

We now return to the simple model of two firms and two complementary projects, but we extend the model to an infinite horizon to eliminate the end game effect of a finite horizon, which forces the two firms to pursue different projects in the final period if neither firm has made any innovation by that time. In an infinite horizon model both innovations are eventually made. Since the issue is the timing of innovations with earlier innovations being preferred, we introduce a discount factor, which is denoted by $\delta(< 1)$.

As in the case of the two-period model, we first show that the socially optimal project choice is for each firm to diversify. To see this, suppose one innovation has been made by any firm and both firms subsequently pursue the other project until one of the firms is successful. The game ends when both innovations are made, at which point the final product can be brought to the market and the social value m obtains. The social welfare value function in this case is simply $w(x)m$ where x is the innovation probability of the remaining project and

$$w(x) = \frac{\pi_0(x,x) + \pi_1(x,x)}{1 - \delta(1-x)^2}.$$

Let $W(i,j)$, denote the expected social surplus when no innovation has been made with firm 1 choosing project i and firm 2 choosing project j. Suppose project i has success probability x whereas project j succeeds with probability y. If both firms pursue project i, then welfare $W(i,i)$ is recursively given by

$$W(i,i) = \delta[1-(1-x)^2]w(y)m + \delta(1-x)^2 W(i,i)$$

[27] To see this, notice that the patent holder's problem can be restated as follows. Using (8) and treating n as an indirect control variable, we get

$$\begin{aligned} Max\ \alpha P(n^*(\alpha,x);x)m &= \underset{\alpha}{Max}[1 - \frac{cn^*(\alpha;x)}{P(n^*(\alpha,x);x)m}]P(n^*(\alpha,x);x)m \\ &= \underset{\alpha}{Max} P(n^*(\alpha,x);x)m - cn^*(\alpha;x). \end{aligned}$$

which simplifies to

$$W(A,A) = W(B,B) = w(p)w(q)m.$$

If firms pursue different projects, expected welfare is defined by

$$W(i,j) = xym + \delta x(1-y)w(y)m + \delta(1-x)yw(x) + \delta(1-x)(1-y)W(i,j),$$

which yields

$$W(A,B) = W(B,A) = w(p)w(q)m + \frac{pq(1-\delta)m}{[1-\delta(1-p)^2][1-\delta(1-q)^2]}.$$

It follows straightforward that a social planner always prefers diversification over project clustering. As in the two period model, diversification eliminates the possibility of excessive duplication and reduces the time until both innovations are made.

We now analyze the market equilibrium and show that there is a tendency for firms to cluster and choose the easier project first in contrast to the social optimum of diversification. When exactly one innovation is made, both firms start pursuing the remaining R&D project as in the social optimum. Let x denote the success probability of the remaining technology. The expected profit stream of the unsuccessful firm and the patentholder are $v_0(x)m$ and $v_1(x)m$, respectively, where

$$v_0(x) = \frac{\pi_0(x,x)}{1-\delta(1-x)^2}, v_1(x) = \frac{\pi_1(x,x)}{1-\delta(1-x)^2}.$$

Denote $V(i,j)$ to represent a firm's value function when the firm engages in project i and the other firm engages in project j when no innovation has been made. When firms choose the same project, the value function is recursively defined as

$$V(i,i) = x(1-x)\delta v_1(y)m + (1-x)x\delta v_0(y)m + x^2\delta\frac{1}{2}[v_0(y)+v_1(y)]m + (1-x)^2\delta V(i,i)$$

which yields

$$V(A,A) = V(B,B) = \frac{1}{2}\delta m[v_0(p)+v_1(p)][v_0(q)+v_1(q)].$$

When firms pursue different projects i and j with success probabilities x and y, respectively, we get

$$V(i,j) = x(1-y)\delta v_1(y)m + (1-x)y\delta v_0(x)m + xy\delta\frac{1}{2}m + (1-x)(1-y)\delta V(i,j).$$

As long as no innovation has been made, firms simultaneously choose which project to pursue in the current period. The following proposition compares the above value functions and gives the project choice equilibrium as a function of the discount factor.

Proposition 6 *In the infinite horizon model, there exists a δ^* such that for all $\delta \geq \delta^*$, both firms pursue the easier project A, while for $\delta < \delta^*$, firms diversify their choices over the projects, i.e. one firm chooses A and its rival B.*

If the discount factor is sufficiently large, choosing project A is a dominant strategy and (A, A) is the only project selection equilibrium. The reason is that for sufficiently large δ, what matters for each firm is how many patents it has. The timing as to when the overall innovations are made is of secondary importance. As a result, choosing the easier project A regardless of the other firm's choice is optimal. In contrast, when δ is small, firms have incentives to diversify to hasten the innovation speed of the overall system. For instance, as $\delta \to 0$, it is easy to check that $V(A, A) - V(B, A) \to -V(B, A) < 0$. The stategic situation becomes a one-shot game, and the two firms diversify on their choices to maximize the probability of both innovations in the current period.

5 Project Choice with Asymmetric Firms

In this section we extend the basic model to situations where firms differ in their capabilities to innovate. In particular, we are interested to investigate the project choice equilibrium when one firm is dominant and has higher success rates in both technologies and the other firm is more (or less) specialized in one project. We show that preemption incentives by the dominant firm can induce inefficient clustering, inefficient diversification and lead to non-existence of pure-strategy Nash equilibria.

Suppose firm 1 develops product A with probability $q_A = \alpha q$ and product B with probability $q_B = (1-\alpha)q$, where $\alpha \in [\frac{1}{2}, 1]$ represents the degree of specialization in technology A. For $\alpha = \frac{1}{2}$, firm 1 is equally strong in both technologies; for $\alpha = 1$, firm 1 is fully specialized in technology A and inactive in B. Firm 2 is the dominant firm and has a per period success rate of $p\_(> q)$ with both technologies.[28] Apart from this, the set-up follows the benchmark model of section 2. Let $\Pi_1(i,j)$ and $\Pi_2(i,j)$ denote the expected profits of firm 1 and 2, respectively, when firm 1 engages in project i and firm 2 in project j in the first period. These expressions are derived in the same way as the profit functions in the model of Section 2.[29] We further assume that when both firms are unsuccessful in the first period, they can coordinate on the efficient project choice in the second period, i.e. firm 1 in market A and firm 2 in market B. Lastly, it will be useful to note the following property of the expected market shares $\pi_0(x,y)$ and $\pi_1(x,y)$.

Remark 1 *There exists a x' with $y < x' < 1$ and $x' \geq 2/3$ such that $\frac{1}{2}\pi_0(x,y)+\frac{1}{2}\pi_1(x,y) \geq \frac{1}{2}$ if and only if $x \geq x'$.*

To understand the economic intuition, suppose for a moment that both firms are successful with probability one in both markets in period 1. If they choose the same market, the firms have a success rate in the remaining market of x and y, respectively. If they choose different projects, they have a certain market share of $1/2$. The above remark implies that if the firm with success rate x has a sufficiently high innovation advantage in the remaining market, then the firm prefers clustering over diversifying. Vice versa, if the advantage is small or the rival is more capable in the remaining market, then the firm prefers to diversify.

Before proceeding with the equilibrium analysis, consider the socially efficient outcome. In the second period, the equilibrium and the socially efficient choice coincide for any possible outcome for the first period. In period 1, the social planner maximizes the probability that the two technologies are available at the end of period 2. While the dominant firm is equally good in both projects, firm 1 is specialized in technology A. Hence, allocating firms according to their relative innovation advantage is efficient. Firm 1 engages in its specialized technology A whereas firm 2 pursues technology B in order to avoid duplication in period 1.[30]

Lemma 5 *The socially efficient project allocation in the first period is to assign firm 1 to technology A and firm 2 to technology B.*

[28] We can say that firm 2 has absolute advantages in both projects, but firm 1 has a comparative advantage in project A.

[29] See Appendix B for the definition of the expected profit functions.

[30] This result is formally derived in Appendix B.

Now consider the market equilibrium in project choice. First consider the incentives of firms to engage in the socially efficient outcome (A, B). Suppose firm 2 engages in project B and verify that $\Pi_1(A, B) > \Pi_1(B, B)$ if and only if

$$(1-p)\left[q_A\pi_1(q_B,p) - q_B\pi_1(q_A,p) - (q_A - q_B)q_A p\frac{1}{2}\right] + \tag{9}$$
$$p(q_A - q_B)[\frac{1}{2} - \pi_0(q_A,p)] + pq_B\frac{1}{2}[1 - \pi_1(q_A,p) - \pi_0(q_A,p)] > 0.$$

Diversifying and engaging in technology A has three benefits for firm 1. The first term is the relative advantage of pursuing A rather than B when the rival is not successful in the first period. As above, when its rival is inactive in period 1, firm 1 maximizes its overall expected market share by pursuing the technology in which it has the highest success rate. Since firm 1 is specialized in technology A, this term is strictly positive. The second term is the difference in expected profits of a presence in market A relative to B in the event that rival firm 2 is successful. Pursuing A increases the success chances by $q_A - q_B$ and leads to a higher expected market share. Finally, the last term is the gain from avoiding R&D duplication when clustering in B which is by Remark 1 strictly positive. As a result, (9) is always satisfied. Given the dominant firm chooses project B, firm 1 strictly prefers to diversify and engage in market A. From this follows immediately that an equilibrium (B, B) fails to exist.

Next consider the R&D project choice of the dominant firm. Firm 2 chooses to diversify if and only if $\Pi_2(A, B) \geq \Pi_2(A, A)$ or

$$q_A\frac{1}{2}[1 - \pi_1(p,q_B) - \pi_0(p,q_B)] \geq (1-q_A)\left[\pi_1(p,q_B) - \pi_1(p,q_A)\right]. \tag{10}$$

This condition reflects that the dominant firm's project choice is only concerned with outcomes in which it is successful. If firm 2 is not successful (which occurs with probability $1-p$ in both markets), it receives the same expected profits in both markets.[31] Thus, the RHS is the expected relative market share gain from clustering in A when firm 2 succeeds while firm *1* does not. In this case pursuing technology A in period 1 implies that the dominant firm 2 faces a rival with a lower innovation capability $q_B = (1-\alpha)q$ in the second period.

[31] In fact, with probability $q_A = \alpha q$ firm 1 is successful and firm 2 receives a continuation profit of $\pi_0(p, q_B = (1-\alpha)q)$ independent of the stage 1 project choice. Likewise, if firm 1 is not successful firm 2 receives $pq_A/2$ no matter which technology it chose in stage 1.

As discussed above, this is profitable if and only firm 2's own capability is larger than 2/3. Hence, the RHS is positive if and only if $p \geq 2/3$. The LHS is the expected relative gain from diversifying when both firms are successful in the first period. If firm 2 pursues project B, each firm can secure itself half of the market. However, if the dominant firm clusters, it has a 50% chance of getting a patent on technology A plus it is more likely to succeed in the other market in the second period. From Remark 1, it follows that if p is less than 2/3, then diversifying is more profitable and the LHS is positive. By contrast, if p is sufficiently high, then clustering dominates and the LHS is negative. Accordingly, it can be shown that there exists a $\overline{p}_2^A(\alpha, q) \geq 2/3$ such that diversifying is profitable for the dominant firm if and only if $p \leq \overline{p}_2^A(\alpha, q)$. Since firm 1 has no incentive to deviate, (A, B) is an equilibrium if $p \leq \overline{p}_2^A(\alpha, q)$. Figure 2 depicts this threshold in an $\alpha - q$ space.

Second, consider firms' incentives to cluster in technology A. From (10) follows that the dominant firm does not deviate from (A,A) if and only if it innovation capability is sufficiently high, $p \geq \overline{p}_2^A(\alpha, q)$. Firm 1 prefers project A if and only if $\Pi_1(A, A) \geq \Pi_1(B, A)$ or

$$(1-p)\left[q_A\pi_1(q_B, p) - q_B\pi_1(q_A, p) - (q_A - q_B)q_A p\frac{1}{2}\right] + \qquad (11)$$
$$p(q_A - q_B)\frac{1}{2}[\pi_1(q_B, p) - \pi_0(q_B, p)] > pq_B\frac{1}{2}[1 - \pi_1(q_B, p) - \pi_0(q_B, p)].$$

The terms on the LHS correspond to the first two terms in (9). Choosing project A is beneficial if the rival is not successful and yields a higher expected market share if both firms are successful. However, when firm 2 is engaged in A, then these preemption benefits have to be weighed against the gain from diversification on the RHS. The relative advantage of preemption in market A is higher, the more specialized firm 1 is in project A. In particular, it is shown in Appendix B that there exists a threshold value $\overline{p}_1^A(\alpha, q)$ such that if $p < \overline{p}_1^A(\alpha, q)$, then firm 1 diversifies whereas if $p \geq \overline{p}_1^A(\alpha, q)$ it clusters in A. Accordingly, an equilibrium (A,A) exists if $p \geq max\{\overline{p}_1^A(\alpha, q), \overline{p}_2^A(\alpha, q)\}$, i.e. when firm 1 is sufficiently specialized and firm 2's innovation capability is high (see Figure 2 below for an illustration).

Finally, consider the incentives to form a (B,A) equilibrium. Clearly, firm 1 does not deviate if (11) is not satisfied. The dominant firm 2 prefers project A while its rival engages in B if and only if $\Pi_2(B,A) \geq \Pi_2(B,B)$ or

$$q_B \frac{1}{2}[1 - \pi_1(p, q_A) - \pi_0(p, q_A)] \geq (1 - q_B)\,[\pi_1(p, q_A) - \pi_1(p, q_B)]\,. \quad (12)$$

A similar trade-off as in (10) arises. The LHS are the gains from diversifying when both firms innovate. The RHS are the expected gains from clustering in B when firm 2 is the sole innovator. The only qualitative difference is that when firm 2 is the sole innovator and clusters, it faces a rival with a higher innovation capability in the second period. This is profitable if and only if its own capability is less than 2/3. Two cases arises. Clustering dominates diversifying if either $p<2/3$ and $p \leq \overline{p}_2^B$, or, if $p>2/3$ and $p \geq \overline{p}_2^B$. In the former case, the gains from clustering when firm 2 is the sole innovator outweigh the benefit from clustering. In the latter case, the LHS and the RHS are negative but if p is large and α small, the relative loss from clustering when only firm 2 innovates is smaller. We can characterize the project choice equilibrium as follows.

Proposition 7 *Consider the project choice equilibrium with a specialized and a dominant firm. (i) The socially efficient project choice obtains in equilibrium if and only if firm 1 is not too specialized and firm 2's innovation advantage is small. (ii) There is excessive clustering in A if and only if firm 1 is sufficiently specialized and firm 2 sufficiently dominant. (iii) There exist parameter values such that (B,A) is the unique project choice equilibrium. (iv) There exist parameter values such that no pure strategy project choice equilibrium exists.*

The project choice equilibrium may differ from the socially efficient outcome because the dominant firm 2 has an incentive to preempt by first developing the technology its rival specializes in. This allows firm 2 to secure itself a higher expected market share in technology A. Such a preemption strategy is profitable if the dominant firm has a high success rate across both technologies and its rival is sufficiently specialized. A high innovation capability makes preemption more profitable and reduces the cost of duplicating its rival's R&D effort. A high degree of specialization implies that firm 1 has no incentive to avoid competition for a patent in A by switching to project B.

Interestingly, for lower degrees of specialization of firm 1, firm 2's preemption strategy leads to a (B,A) equilibrium in which both firms are inefficiently allocated relative to the social optimum. In this equilibrium, the specialist firm 1 has sufficient expertise in its less efficient project B in order to make a switch profitable. While firm 1 avoids competition from

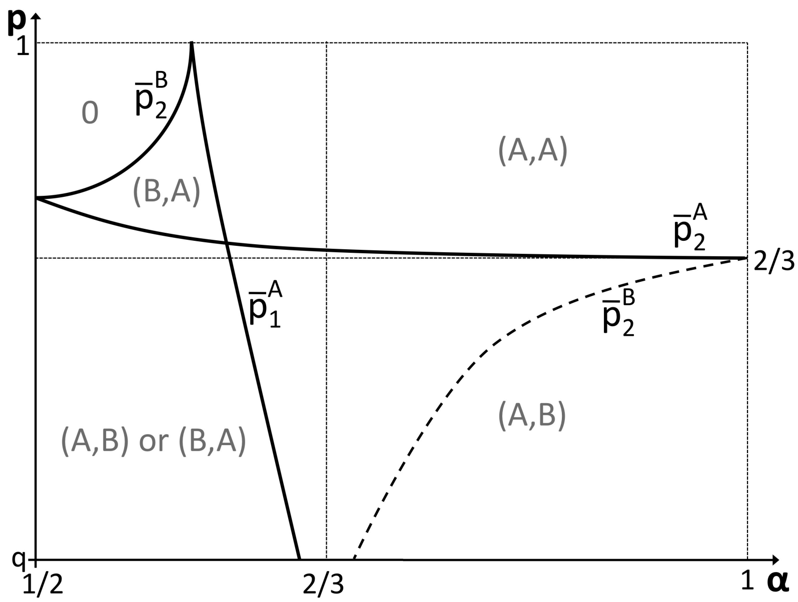

Figure 2: *Project choice equilibrium with asymmetric firm capabilities*

the dominant firm, firm 2 maintains its preemption stronghold on firm 1's specialization technology. This equilibrium is unique if firm 1 has an intermediate degree of specialization and firm 2 is moderately dominant.

Finally, as firm 2 becomes more dominant and firm 1's degree of specialization decreases, the preemption strategy changes. Instead of preempting the project that firm 1 is best at, firm 2 tries to engage in the same technology as its rival. However, firm 1 has the exact opposite strategy and prefers to avoid head-to-head competition by choosing a different R&D project than the dominant firm. As a result, no equilibrium in pure strategies exists.[32]

6 Concluding Remarks

This paper analyzes biases in the project choice of complementary innovations that are used in combination to produce a final product. In order to highlight the effects of rivalry on project selections, we set up a model in which there is no bias under monopoly. We show that, relative to the socially optimal allocation, R&D competition induces selection biases in

[32] In the resulting mixed strategy equilibrium firm $k \in \{1, 2\}$ chooses project A with probability

$$\mu_k = \frac{\Pi_{-k}(A,B) + \Pi_{-k}(B,A) - \Pi_{-k}(A,A) - \Pi_{-k}(B,B)}{\Pi_{-k}(A,B) - \Pi_{-k}(B,B)}$$

where $-k$ is the rival's subscript.

the choice of R&D projects. In the presence of complementary technologies, patents allow innovating firms to hold up rivals who succeed in developing other system components. This hold-up problem induces firms to preemptively claim stakes on the property rights of the complementary technologies. When there is asymmetry across project with respect to innovation success rates, firms have an incentive to excessively cluster their R&D efforts on a relatively easier technology. This selection bias is persistent and robust to several extensions including number of firms, number of R&D stages, free entry, intermediate licensing and an infinite horizon. We also analyze selection biases in complementary R&D projects that arise when firms differ in research capabilities. In particular, we demonstrate that a dominant firm has incentives to excessively engage in either the same technology as its weaker rival or in the technology in which its rival has relatively less research capabilities.

Recently, the FTC and several authors have expressed concerns regarding patents issued for obvious or nearly obvious inventions, which can be used as blocking patents and thus hinder the developments of new products.[33] Our analysis points out that excessive resource allocation towards such obvious inventions can further exacerbate the problem. The inefficient clustering in R&D project choice identified in our paper calls for policy interventions and patent reform. Shapiro (2006) highlighted the attractiveness of prior user rights as a way to partially correct for various inefficiencies that arise for isolated innovations. Interestingly, the feature of prior user rights that favors sole innovations over joint innovations has a beneficial effect in our case of complementary innovations. Prior user rights can mitigate the problem of preemptive duplication and promote diversification of project choice, which can hasten the expected time of arrival for all innovations required for the final product.

[33] See Federal Trade Commission (2011) and Jaffe and Lerner (2004).

Appendix A

Proof of Lemma 2. The first order condition, $P'Q(N-n_A)-P(n_A)Q'=0$ can be written as

$$-(1-p)^{n_A}(1-(1-q)^{N-n_A})\ln(1-p)+(1-q)^{N-n_A}(1-(1-p)^{n_A})\ln(1-q)=0$$

Let us define the LHS as $\Psi(n_A)$. The necessary condition for an optimum $(n_A^o, n_B^o = N-n_A^o)$ is $\Psi(n_A^o)=0$. We show that the unique solution implies $n_A^o < n_B^o$, or equivalently $n_A^o < N/2$. First, check that $\Psi(n_A)$ is strictly decreasing since

$$\begin{aligned}\frac{\partial\Psi(n_A)}{\partial n_A} &= \frac{(1-p)^{n_A}}{(1-q)^{n_A}}[(1-q)^N-(1-q)^{n_A}][\ln(1-p)]^2 \\ &\quad -(1-q)^{N-n_A}\ln(1-q)\,[2(1-p)^{n_A}\ln(1-p)+(1-(1-p)^{n_A})\ln(1-q)]<0\end{aligned}$$

for all $n_A \in [0,N]$. Next, check that $\Psi(0)=-[1-(1-q)^N]\ln(1-p)>0$. It remains to show that $\Psi(N/2)<0$. For this, note that $\Psi(N/2)|_{p=q}=0$ and

$$\frac{\partial\Psi(N/2)}{\partial p}=2-2(1-q)^{N/2}+N\ln(1-p)-N(1-q)^{N/2}[\ln(1-p)-\ln(1-q)]<0.$$

To verify the negative sign of this derivative, check that

$$\frac{\partial^2\Psi(N/2)}{\partial p\partial q}=\frac{N^2}{4}(1-p)^{N/2-1}(1-q)^{N/2-1}[\ln(1-p)-\ln(1-q)]<0.$$

Hence, the derivative takes its highest value at $q=0$ where

$$\frac{\partial\Psi(N/2)}{\partial p}|_{q=0}=0.$$

It follows that $\partial\Psi(N/2)/\partial p<0$ for all $q>0$. It is then immediate that $\Psi(N/2)<0$ for all $p>q$. It follows that $n_A^o<N/2$. Finally, consider the comparative statics of n_A^o with respect to p and q. Since $\partial\Psi(n_A)/\partial n_A<0$, it holds that $sign\{dn_A^o/dp\}$=$sign\{\partial\Psi/\partial p\}|_{n_A=n_A^o}$. Using the fact that in the optimum,

$$\ln(1-q)=\ln(1-p)\frac{(1-p)^{n_A}(1-(1-q)^{N-n_A})}{(1-q)^{N-n_A}(1-(1-p)^{n_A})},$$

we obtain

$$\frac{\partial \Psi}{\partial p}|_{n_A=n_A^o} = \frac{(1-p)^{n_A-1}}{(1-q)^{n_A}}[(1-q)^{n_A}(1+n_A\ln(1-p)) - (1-q)^N(1+n_A\ln(1-p)-\ln(1-q))]$$
$$= \frac{(1-p)^{n_A-1}}{(1-q)^{n_A}}\frac{(1-q)^{n_A}-(1-q)^N}{1-(1-p)^{n_A}}[1-(1-p)^{n_A}+n_A\ln(1-p)] < 0$$

since the value in the squared bracket is zero at $p = 0$ and strictly decreasing in p. Similarly, we can establish that $sign\{dn_A^o/dq\}{=}sign\{\partial\Psi/\partial q\}|_{n_A=n_A^o}$ since

$$\frac{\partial \Psi}{\partial q}|_{n_A=n_A^o} = (1-q)^{N-n_A-1}[(1-(1-p)^{n_A})(1+(N-n_A)\ln(1-q)) - (N-n_A)(1-p)^{n_A}\ln(1-p)]$$
$$= \frac{(1-(1-p)^{n_A})(1-q)^{N-1}}{(1-q)^{n_A}-(1-q)^N}[(1-q)^{N-n_A}-1-(N-n_A)\ln(1-q)] > 0$$

since the value in the squared bracket is zero at $q = 0$ and strictly increasing in q. The lemma follows. ■

Proof of Lemma 3. In order to show that $\widehat{n}_A^o < n_A^o$ it is sufficient to show that (i) $\widehat{W}(n_A)$ is strictly concave and (ii) $\widehat{W}'(n_A^o) < 0$. Let us start with the second part. Differentiating $\widehat{W}(n_A)$ with respect to n_A yields

$$\frac{d\widehat{W}(n_A)}{dn_A} = P'(n_A)Q(N-n_A) - P(n_A)Q'(N-n_A) + P(n_A)Q(N)Q'(N-n_A)$$
$$+ P'(n_A)(1-Q(N-n_A))Q(N) - P(N)Q'(N-n_A)(1-P(n_A))$$
$$+ P'(n_A)Q(N-n_A)P(N) + \Lambda^o(1-P(n_A))Q'(N-n_A) + \Lambda^o(1-Q(N-n_A))P'(n_A).$$

At $n_A = n_A^o$ and using the first order condition for $n_A^o, Q'(N-n_A^o) = P'(n_A^o)Q(N-n_A^o)/P(n_A^o)$ we get

$$\frac{d\widehat{W}(n_A)}{dn_A}|_{n_A=n_A^o} = -\frac{P'(n_A^o)}{P(n_A^o)}\left\{Q(N-n_A^o)[P(N)+P(n_A^o)^2] - P(n_A^o)[Q(N)+Q(N-n_A^o)^2]\right\}$$
$$= \frac{(1-p)^N\ln(1-p)}{1-(1-p)^N}(\Upsilon_1+\Upsilon_2) \leq 0$$

for all $n_A^o \leq N/2$ since

$$\Upsilon_1 \equiv -(1-q)^N[1-(1-p)^{n_A^o}][(1-q)^N-(1-q)^{2n_A^o}] \geq 0 \text{ and}$$
$$\Upsilon_2 \equiv (1-q)^{n_A^o}[(1-p)^N-(1-p)^{2n_A^o}][(1-q)^N-(1-q)^{n_A^o}] \geq 0.$$

Next check concavity by taking the second derivative which yields

$$\frac{d\widehat{W}^2(n_A)}{(dn_A)^2} = [\ln(1-p) - \ln(1-q)]^2\Delta + \ln(1-q)(1-p)^{N+n_A}[\ln(1-q) - 2\ln(1-p)]$$
$$- [\ln(1-q)]^2(1-q)^{2N-n_A} < 0$$

for all $n_A \leq N$ since

$$\Delta \equiv \frac{(1-p)^{n_A}}{(1-q)^{n_A+n_A^o}}\{-(1-q)^{2N}[1-(1-q)^{n_A^o}] - (1-p)^N(1-q)^{n_A^o}[(1-q)^{n_A} - (1-q)^N]$$
$$- (1-p)^{n_A^o}(1-q)^{N+n_A^o}[1-(1-q)^{N-n_A^o}]\} < 0.$$

The lemma follows. ■

Proof of Lemma 4. To prove the lemma we show that (i) $\partial[\Pi_A(n_A, N-n_A) - \Pi_B(n_A, N-n_A)]/\partial n_A < 0$ and (ii) $\Pi_A(N/2, N/2) > \Pi_B(N/2, N/2)$. First, observe that

$$\Pi_A(n_A, N-n_A) - \Pi_B(n_A, N-n_A) =$$
$$\frac{m}{2}\{P(n_A)Q(N-n_A)\frac{1}{n_A} + P(n_A)[1-Q(N-n_A)]Q(N)\frac{1}{n_A}$$
$$- P(n_A)Q(N-n_A)\frac{1}{N-n_A} - (1-P(n_A))Q(N-n_A)P(N)\frac{1}{N-n_A}\}$$

Let Φ_1 and Φ_2 denote the sum of the first two and last two terms in the curly bracket, respectively. Check that

$$\frac{\partial\Phi_1}{\partial n_A} = [Q(n) + (1-Q(N))Q(N-n_A)][\frac{P(n_A)}{n_A^2} + \frac{P'(n_A)}{n_A}] - (1-Q(N))Q'(N-n_A)\frac{1}{n_A}$$

and

$$\frac{\partial^2\Phi_1}{\partial n_A \partial p} = \frac{(1-p)^{n_A-1}}{(1-q)^{n_A}}\{[(1-q)^{n_A} - (1-q)^{2N}]\ln(1-p) + (1-q)^{2N}\ln(1-q)\} < 0.$$

Verify that $\partial\Phi_1/\partial n_A$ is zero at $p = 0$. Hence, $\partial\Phi_1/\partial n_A < 0$ for all $p > 0$. Similarly,

$$\frac{\partial\Phi_2}{\partial n_A} = [P(n_A) + (1-P(n_A))P(N)][-\frac{Q(N-n_A)}{(N-n_A)^2} + \frac{Q'(N-n_A)}{N-n_A}] - (1-P(N))P'(n_A)\frac{1}{N-n_A}$$

and

$$\frac{\partial^2 \Phi_2}{\partial n_A \partial q} = (1-q)^{N-n_A-1}\{(1-p)^{N+n_A}\ln(1-p) + [1-(1-p)^{N+n_A}]\ln(1-q)\} < 0.$$

Check that $\partial\Phi_2/\partial n_A$ is zero at $q = 0$. Hence, $\partial\Phi_2/\partial n_A < 0$ for all $q > 0$. To see point (ii), observe that

$$\Pi_A(N/2, N/2) - \Pi_B(N/2, N/2) = \frac{m}{N}\{P(N/2)[1-Q(N/2)]Q(N) - [1-P(N/2)]Q(N/2)P(N)\}$$

Further note that

$$\frac{P(N/2)Q(N)}{Q(N/2)P(N)} = \frac{[1-(1-p)^{N/2}]}{[1-(1-q)^{N/2}]}\frac{[1-(1-q)^N]}{[1-(1-p)^N]} = \frac{1+(1-q)^{N/2}}{1+(1-p)^{N/2}} > 1$$

Thus, $P(N/2)Q(N) > Q(N/2)P(N)$. In addition, $[1-Q(N/2)] > [1-P(N/2)]$. Therefore, $\Pi_A(N/2, N/2) - \Pi_B(N/2, N/2) > 0$. ■

Proof of Proposition 6. It is useful to rewrite the value functions as

$$V(A,A) = V(B,B) = \frac{2\delta pq\Psi(p)\Psi(q)m}{(1-p)(1-q)}, \quad V(A,B) = \frac{pq[1+\delta\Psi(p)+3\delta\Psi(q)]m}{2[1-\delta(1-p)(1-q)]}$$
$$\text{and } V(B,A) = \frac{pq[1+3\delta\Psi(p)+\delta\Psi(q)]m}{2[1-\delta(1-p)(1-q)]}$$

where

$$\Psi(x) \equiv \frac{(2-x)(1-x)}{2[1-\delta(1-x)^2]} > 0, \qquad \Psi'(x) = -\frac{3-2x+\delta(1-x)^2}{2[1-\delta(1-x)^2]^2} < 0.$$

From $\Psi'(x) < 0$ and $p > q$ follows that $V(A,B) > V(B,A)$.

1. First, we show that $V(A,B) > V(B,B)$, which means that both firms choosing the difficult project B cannot be an equilibrium. We prove this indirectly by using the following relationship,

$$\begin{aligned} V(A,B) + V(B,A) &= W(A,B) \\ &= W(B,B) + \frac{pq(1-\delta)m}{[1-\delta(1-p)^2][1-\delta(1-q)^2]} > W(B,B) = 2V(B,B). \end{aligned}$$

Prove by contradiction. Suppose that $V(A,B) \le V(B,B)$. Since $V(A,B) > V(B,A)$ it must hold that $V(B,A) < V(B,B)$. Taken together, we have $V(A,B) + V(B,A) < 2V(B,B)$, which yields a contradiction. Hence, $V(A,B) > V(B,B)$.

2. From $V(A,B) > V(B,B)$ follows that clustering in A is the unique equilibrium if and only if $V(A,A) > V(B,A)$. Otherwise, two equilibria exist in which firms choose different project. In what follows we show that there exists one $\delta^* \in (0,1)$ such that $V(A,A) > V(B,A)$ if and only if $\delta \geq \delta^*$. To see this, observe that $V(A,A)$ and $V(B,A)$ are continuous in δ,

$$\lim_{\delta \to 1} [V(A,A) - V(B,A)] = \frac{(p-q)m}{4p(1-q)+4q} > 0$$

and

$$\lim_{\delta \to 0} [V(A,A) - V(B,A)] = -\lim_{\delta \to 0} [V(B,A)] < 0$$

Thus, there exist at least one $\delta^* \in (0,1)$ such that $V(A,A) = V(B,A)$. Note that given $p, q < 1$, $V(A,A) - V(B,A)$ is differentiable with respect to δ everywhere. So, to prove the claim, it is enough to show that $\partial [V(A,A) - V(B,A)] / \partial \delta$ evaluated at $\delta = \delta^*$ is positive. Define $\Psi_\delta(x) = \partial \Psi(p)/\partial \delta$ and observe that

$$\begin{aligned}
\left.\frac{\partial V(A,A)}{\partial \delta}\right|_{\delta=\delta^*} &= pqm\frac{2\Psi(p)\Psi(q) + 2\delta^*\Psi_\delta(p)\Psi(q) + 2\delta^*\Psi(p)\Psi_\delta(q)}{(1-p)(1-q)} \\
&= \frac{V(A,A)}{\delta^*} + pqm\frac{2\delta^*\Psi_\delta(p)\Psi(q) + 2\delta^*\Psi(p)\Psi_\delta(q)}{(1-p)(1-q)} \\
&= \frac{V(A,A)}{\delta^*} + V(A,A)\left[\frac{\Psi_\delta(p)}{\Psi(p)} + \frac{\Psi_\delta(q)}{\Psi(q)}\right] \\
&= \frac{V(B,A)}{\delta^*} + V(B,A)\left[\frac{\Psi_\delta(p)}{\Psi(p)} + \frac{\Psi_\delta(q)}{\Psi(q)}\right] \qquad \text{(AA)}
\end{aligned}$$

and

$$\begin{aligned}
\left.\frac{\partial V(B,A)}{\partial \delta}\right|_{\delta=\delta^*} &= pqm\frac{[3\Psi(p) + \Psi(q) + 3\delta^*\Psi_\delta(p) + \delta^*\Psi_\delta(q)]}{2[1-\delta^*(1-p)(1-q)]} + \frac{V(B,A)(1-p)(1-q)}{1-\delta^*(1-p)(1-q)} \\
&= \frac{V(B,A)}{\delta^*} + pqm\frac{[3\delta^*\Psi_\delta(p) + \delta^*\Psi_\delta(q) - 1/\delta^*]}{2[1-\delta^*(1-p)(1-q)]} + \frac{V(B,A)(1-p)(1-q)}{1-\delta^*(1-p)(1-q)} \\
&= \frac{V(B,A)}{\delta^*} + pqm\frac{[3\delta^*\Psi_\delta(p) + \delta^*\Psi_\delta(q) - 1/\delta^*]}{2[1-\delta^*(1-p)(1-q)]} + \frac{V(A,A)(1-p)(1-q)}{1-\delta^*(1-p)(1-q)} \\
&= \frac{V(B,A)}{\delta^*} + pqm\frac{[3\delta^*\Psi_\delta(p) + \delta^*\Psi_\delta(q) - 1/\delta^*]}{2[1-\delta^*(1-p)(1-q)]} + pqm\frac{2\delta^*\Psi(p)\Psi(q)}{1-\delta^*(1-p)(1-q)} \qquad \text{(BA)}
\end{aligned}$$

Subtracting (BA) from (AA), and dividing it by $pqm/2[1-\delta^*(1-p)(1-q)]$, we obtain that $\partial\left[V(A,A)-V(B,A)\right]/\partial\delta$ is positive at $\delta=\delta^*$ if and only if

$$\frac{\Psi_\delta(p)}{\Psi(p)}+\frac{\Psi_\delta(q)}{\Psi(q)}+\frac{3\delta^*\Psi(p)\Psi_\delta(q)}{\Psi(q)}+\frac{\delta^*\Psi(q)\Psi_\delta(p)}{\Psi(p)}-4\delta^*\Psi(p)\Psi(q)+\frac{1}{\delta^*}\geq 0.$$

Substituting $\Psi(x)$, we can rewrite the first five expressions of the above condition as

$$\begin{aligned}
&\frac{\Psi_\delta(p)}{\Psi(p)}+\frac{\Psi_\delta(q)}{\Psi(q)}+\frac{3\delta^*\Psi(p)\Psi_\delta(q)}{\Psi(q)}+\frac{\delta^*\Psi(q)\Psi_\delta(p)}{\Psi(p)}-4\delta^*\Psi(p)\Psi(q)\\
&=\frac{2(1-p)^2\left[1-\delta^*(1-q)^2\right]+2(1-q)^2\left[1-\delta^*(1-p)^2\right]}{2\left[1-\delta^*(1-p)^2\right]\left[1-\delta^*(1-q)^2\right]}-\delta^*\frac{(1-p)(1-q)\left[2q(1-p)+(p+q)\right]}{2\left[1-\delta^*(1-p)^2\right]\left[1-\delta^*(1-q)^2\right]}\\
&=\frac{(1-p)^2\left[1-\delta^*(1-q)\right]}{\left[1-\delta^*(1-p)^2\right]\left[1-\delta^*(1-q)^2\right]}+\frac{2(1-q)^2\left[1-\delta^*(1-p)^2\right]-\delta^*(1-p)(1-q)(p+q)}{2\left[1-\delta^*(1-p)^2\right]\left[1-\delta^*(1-q)^2\right]}\\
&>\frac{(1-p)^2\left[1-\delta^*(1-q)\right]}{\left[1-\delta^*(1-p)^2\right]\left[1-\delta^*(1-q)^2\right]}+\frac{2(1-q)^2\left[1-\delta^*(1-p)^2\right]-2\delta^*(1-q)^2p}{2\left[1-\delta^*(1-p)^2\right]\left[1-\delta^*(1-q)^2\right]}\quad(\because p>q)\\
&=\frac{(1-p)^2\left[1-\delta^*(1-q)\right]}{\left[1-\delta^*(1-p)^2\right]\left[1-\delta^*(1-q)^2\right]}+\frac{2(1-q)^2\left[1-\delta^*(1-p(1-p))\right]}{2\left[1-\delta^*(1-p)^2\right]\left[1-\delta^*(1-q)^2\right]}>0,
\end{aligned}$$

which completes the proof. ■

Appendix B

Definition of profit functions. Suppose firms choose the same project i. Let x_k denote the success probability of firm $k=1,2$ in this project and y_k in the other project j. Then, the expected profit for firm k when clustering in i is given by

$$\begin{aligned}
\Pi_k(i,i) \quad = \quad & x_k x_{-k}\frac{1}{2}[\pi_0(y_k,y_{-k})+\pi_1(y_k,y_{-k})]m+x_k(1-x_{-k})\pi_1(y_k,y_{-k})m+\\
& x_{-k}(1-x_k)\pi_0(y_k,y_{-k})m+(1-x_k)(1-x_{-k})pq_A\frac{1}{2}m.
\end{aligned}$$

Similarly, when firm k chooses project i while firm $-k$ choose project j we have

$$\begin{aligned}
\Pi_k(i,j) \quad = \quad & x_k y_{-k}\frac{1}{2}m+x_k(1-y_{-k})\pi_1(y_k,y_{-k})m+y_{-k}(1-x_k)\pi_0(x_k,x_{-k})m+\\
& (1-x_k)(1-y_{-k})pq_A\frac{1}{2}m.
\end{aligned}$$

Proof of Remark 1. Verify that

$$\pi_0(x,y)+\pi_1(x,y)=\frac{1}{2}(x(3-2y)+y)\geq 1$$

if and only if

$$x \geq x' \equiv \frac{2}{3} + \frac{y}{3(3-2y)} = y + \frac{2(1-y)^2}{3-2y}$$

which establishes the property. ■

Proof of Lemma 5. The expected welfare difference between (A,B) and (A,A) is

$$\Pi_1(A,B)+\Pi_2(A,B)-[\Pi_1(A,A)+\Pi_2(A,A)] = pq(1-p)[(1-\alpha q)(3\alpha-1)+\alpha q(2\alpha-1)] \geq 0$$

for $\alpha \geq 1/2$. The expected welfare difference between (A,B) and (B,B) is

$$\Pi_1(A,B)+\Pi_2(A,B)-[\Pi_1(B,A)+\Pi_2(B,A)] = pq(1-p)(1-\alpha q)(3\alpha-1) \geq 0$$

for $\alpha \geq 1/2$. expected welfare difference between (A,B) and (A,A) is

$$\Pi_1(A,B)+\Pi_2(A,B)-[\Pi_1(B,A)+\Pi_2(B,A)] = pq(1-p)(2\alpha-1)(3-q-\alpha q) \geq 0$$

for $\alpha \geq 1/2$. ■

Equilibrium cut-off values. First rewrite (10) as

$$\frac{1}{4}aq[2-3p+(1-\alpha)(2p-1)q] \geq \frac{1}{4}q(2\alpha-1)(1-\alpha q)(2-3p)$$

which yields

$$p \leq p_2^A(\alpha,q) \equiv \frac{2}{3} + \frac{\alpha(1-\alpha)q}{9(2\alpha-1)(1-\alpha q)+3\alpha(3-2(1-\alpha)q)}.$$

Next consider (11). Simplifying the LHS and RHS, respectively, gives

$$(2\alpha-1)[2-p-(1-p)(3\alpha-1)] \geq (1-\alpha)[2-p-(1-\alpha)(3-2p)].$$

Both sides are linear and decreasing in p. At $p=0$, the LHS is larger than the RHS iff $(3\alpha-2)(2-(1+\alpha)q) \geq 0$ which holds if $\alpha \geq 2/3$. At $p=1$, the LHS is larger than the RHS iff $3\alpha-2+(1-\alpha)^2 q \geq 0$ which holds if $\alpha \geq \alpha'$ where $1/2 < \alpha' < 2/3$. It follows that for $\alpha \leq \alpha'$, this condition is never satisfied. For $\alpha \geq 2/3$ it is always satisfied. For $\alpha' \leq \alpha < 2/3$, there exists a $\widehat{p}_1^A(\alpha,q)$ such that (11) holds if and only if $p \geq \widehat{p}_1^A(\alpha,q)$.

Finally consider (12). Simplifying the LHS and RHS, respectively, gives

$$(1-\alpha)[2-3p+\alpha q(2p-1)] \geq (2\alpha-1)(2-3p)[1-(1-\alpha)q].$$

Both sides are linear in p. At $p=0$, the LHS is larger than the RHS iff $(2-3\alpha)(2-(1-\alpha)q) \geq 0$ or $\alpha \leq 2/3$. At $p=1$, the LHS is larger than the RHS iff $3\alpha - 2 + (1-\alpha)^2 q \geq 0$ which holds if $\alpha \geq \alpha'$ where $1/2 < \alpha' < 2/3$. It follows that for $\alpha \leq \alpha'$, there exists a $\widehat{p}_2^B(\alpha, q)$ such that (12) holds if and only if $p \leq \widehat{p}_1^A(\alpha, q)$. For $\alpha' \leq \alpha < 2/3$, the condition always holds. For $\alpha \geq 2/3$, there exists a $\widehat{p}_2^B(\alpha, q)$ such that (12) holds if and only if $p \geq \widehat{p}_1^A(\alpha, q)$.
■

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