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Abstract

We study the role of social long term care (LTC) insurance when income taxation and private insurance markets are imperfect. Policy instruments include public provision of LTC as well as a subsidy on private insurance. The subsidy scheme may be linear or nonlinear. For the linear part we consider a continuous distribution of types, characterized by earnings and survival probabilities. In the nonlinear part, society consists of three types: poor, middle class and rich. The first type is too poor to provide for dependence; the middle class type purchases private insurance and the high income type is self-insured. The main questions are at what level LTC should be provided to the poor and whether it is desirable to subsidize private LTC for the middle class. Interestingly, the results are similar under both linear and nonlinear schemes. First, in both cases, a (marginal) subsidy of private LTC insurance is not desirable. As a matter of fact, private insurance purchases should typically be taxed (at least at the margin). Second, the desirability of public provision of LTC services depends on the way the income tax is restricted. In the linear case, it may be desirable only if no demogrant (uniform lump-sum transfer) is available. In the nonlinear case, public provision is desirable when the income tax is sufficiently restricted. Specifically, this is the case when the income is subject only to a proportional payroll tax while the LTC reimbursement policy can be nonlinear.

Keywords: long term care, social insurance.

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1 Introduction

It is now widely accepted that one of the main rationales for social insurance is redistribution. The basic argument goes back to Rochet (1991) who shows that social insurance may be an effective way to supplement an optimal income tax; see also Cremer and Pestieau (1996). Roughly speaking, the intuition is the following. Consider an actuarially fair private insurance and introduce the possibility of a social insurance scheme which offers coverage at a uniform rate (irrespective of the individual risk). If there were no tax distortion the optimal policy would be to redistribute resources through income taxation, and to let individuals purchase the private insurance that fits their needs. If there are tax distortions, and if the probability of loss is inversely correlated with earnings, then social insurance becomes desirable.

While the above proposition applies to a number of life-cycle risks, it does not apply to risks which are positively correlated to earnings, as is typically the case for long term care. Dependence is known to increase with longevity and longevity with income; consequently the need for LTC is positively correlated with income. Consequently, Rochet's argument implies that social LTC insurance would not be desirable.

However, Rochet's argument also relies on a number of otherwise strong assumptions. In reality income taxation is not optimal. Furthermore, private LTC insurance is far from being actuarially fair. Loading costs are high and individuals may prefer not to buy insurance. Low income individuals will rely on family solidarity or social assistance and high income individuals on "self-insurance" (*i.e.*, finance their LTC expense from their accumulated savings).

In this paper we study the role of social LTC insurance in a setting inspired by Rochet (1991), but amended to account for the imperfection of income taxation and private insurance markets. Policy instruments include public provision of LTC as well as a subsidy on private insurance. The subsidy scheme may be linear or nonlinear. For the nonlinear part, we will look at a society made of three types: poor, middle class and rich. The first type is too poor to provide for dependence; the middle class type purchases private insurance and the high income type is self-insured. Two of the main questions are then at what level LTC should be provided to the poor and whether it is desirable to subsidize private LTC for the middle class. Interestingly, the results are similar under both linear and nonlinear schemes. First, in both cases, a (marginal) subsidy of private LTC insurance is not desirable. As a matter of fact, private insurance purchases should typically be taxed (at least at the margin). Second, the desirability of public provision of LTC services depends on the way the income tax is restricted. In the linear case, it may be desirable only if no demogrant (uniform lump-sum transfer) is available. In the nonlinear case, public provision is desirable when the income tax is sufficiently restricted. Specifically, this is the case when income is subject only to a proportional payroll tax while the LTC reimbursement policy can be nonlinear.

There exist very few studies which have addressed the issue of social insurance for LTC. Jousten *et al.* (2005) focus on families with different levels of altruism. Given the cost of public funds, the central planner tries to induce the more altruistic families to assist their dependent parents and to restrict aid to the dependent elderly whose children are less altruistic. This may imply a suboptimal quality of public LTC, compared to the first-best level. Pestieau and Sato (2008) study the problem of evenly altruistic children who differ in their earning capacities, that is in their wages and thus in the opportunity cost of the time spent assisting their dependent parents. In case of parent's dependency, the more productive children tend to provide financial help whereas the less productive children offer their time. Parents who have sufficiently large pensions or other resources and who do not expect enough assistance from their children purchase some private insurance. The social welfare maximizing government can subsidize family assistance and/or private insurance. It can also directly provide nursing services. The appropriate policy is shown to depend on the loading cost of private insurance, the cost of public funds and the wealth of the parents. Finally, Pestieau and Sato (2010) consider a society segmented into altruistic and non altruistic families and also into poor and rich families. Private insurance is available and the role of the government is restricted to a nonlinear tax/subsidy of the insurance premiums. Insurance companies provide a lump-sum reimbursement that is not equal to actual costs incurred but to an average value of these costs. They study the shape of this nonlinear subsidy in a setting where neither incomes nor altruism are observable by the social planner. The main result is that asymmetric information implies marginal taxation of all types of households but the top one in terms of self selection constraints.

2 Linear scheme

Consider a two period model where individuals work and save in the first period. Individuals differ in their wage (earning ability) w. Their probability to be alive in the second period is φ; this probability may be positively correlated with earning ability. In their second period individuals face a probability π (assumed to be uniform) of becoming dependent. An individual's expected utility is given by

$$u(x) + \varphi(1-\pi)u(d) + \varphi\pi H(m),$$

where $x = c - \ell^2/2$ denotes the first period consumption, c, net of the (quadratic) disutility of labor ℓ; d represents second period consumption, while m denotes total expenditures in case of dependency. Wage is denoted by w and pretax (first-period) income is given by $w\ell$. Private saving, s, is invested in a perfect annuity market and, with a zero interest rate, its return is s/φ. The functions u and H are strictly increasing and strictly concave. We assume that $u(x) > H(x)$ to reflect that dependence implies costly needs. The following additional assumption is used for some arguments below.

Assumption 1 *The degree of relative risk aversion associated with u is smaller than that associated with H, i.e., for all $x > 0$ we have*

$$R_u(x) = -\frac{xu''(x)}{u'(x)} < R_H(x) = -\frac{xH''(x)}{H'(x)}. \qquad (1)$$

2.1 Social LTC benefits

With the linear scheme considered in this section, a (uniform) social LTC benefit, g, and a demogrant a are financed by a proportional tax on wage income at rate τ. We thus have

$$u(x) = u[w\ell(1-\tau) + a - s - \theta - \ell^2/2]$$

such that $\ell = w(1-\tau)$ and hence $u(x) = u[w^2(1-\tau)^2/2 + a - s - \theta]$, where θ is the payment for private LTC insurance.

An individual's problem is given by

$$\max_{s,\theta} \; u\left(\frac{(1-\tau)^2 w^2}{2} - s - \theta + a\right) + \varphi(1-\pi)u\left(\frac{s}{\varphi}\right) + \varphi\pi H\left(\frac{s}{\varphi} + g + \frac{\theta\gamma_p}{\varphi\pi}\right),$$

The associated benefits (paid in case of dependency) are given by $\theta\gamma_p/\varphi\pi$ where γ_p is the (private insurance) loading factor. The corresponding first-order condition are

$$\begin{aligned} &-u\prime(x) + (1-\pi)u'(s/\varphi) + \pi H'(s/\varphi + g + \theta\gamma_p/\varphi\pi) = 0 \\ &-u\prime(x) + \gamma_p H'(s/\varphi + g + \theta\gamma_p/\varphi\pi) = 0 \end{aligned}$$

We now study the optimal linear policy that maximizes a utilitarian social welfare function. We assume a discrete wage distribution, and denote by n_i the number (proportion) of individuals who have wage w_i and survival probability φ_i. Total population size is normalized at one so that $\sum_i n_i = 1$. The Lagrangian expression associated with this problem can be written as follows

$$\begin{aligned} \pounds \;=\; & \sum_i n_i \left\{ u\left(\frac{(1-\tau)^2 w_i^2}{2} - s_i - \theta_i + a\right) + \varphi_i(1-\pi)u\left(\frac{s_i}{\varphi_i}\right) \right. \\ & \left. + \pi H\left(\frac{s_i}{\varphi_i} + g + \frac{\theta_i\gamma_p}{\varphi_i\pi}\right) + \mu\left[\tau(1-\tau)w_i^2 - a - \pi\varphi_i g\right] \right\}, \end{aligned}$$

where μ is the multiplier of the budget constraint.

Using the expectation operator E, the first-order conditions are[1]

[1] $\sum_i n_i x_i = Ex$

$$
\begin{aligned}
&-(1-\tau)Eu'(x)w^2 + \mu(1-2\tau)Ew^2 = 0 \\
&Eu'(x) - \mu = 0 \\
&\frac{\partial \pounds}{\partial g} = \pi E\varphi H'(m) - \mu\pi E\varphi
\end{aligned}
$$

To state these conditions we have assumed an interior solution for τ and a, but we do not rule out a corner solution for g. Rearranging these expression we have

$$
-(1-\tau)Eu'(x)w^2 + (1-2\tau)Eu'(x)Ew^2 = 0,
$$

and

$$
\begin{aligned}
\frac{\partial \pounds}{\partial g} &= \frac{\pi}{\gamma_p}\left[E\varphi u'(x) - E\varphi Eu'(x) + (1-\gamma_p)E\varphi Eu'(x)\right] \\
&= \frac{\pi}{\gamma_p}\left[\operatorname{cov}\left(\varphi, u'(x)\right) + (1-\gamma_p)E\varphi Eu'(x)\right].
\end{aligned}
$$

As the covariance between longevity and productivity is positive we have that $\operatorname{cov}(\varphi, u'(x)) < 0$ and if the private loading factor is close to one, social insurance is clearly not welfare improving. This can be summarized by the following proposition.

Proposition 1 *Suppose that a linear income tax is used to finance a demogrant and a LTC flat rate benefit. Suppose further that LTC private insurance loading factor is not much lower than one. Suppose also that the survival probability and thus the probability of dependence increases with income. Then a LTC social insurance is socially undesirable.*

Let us now assume that there is no demogrant. In other words we take the distribution of earnings as given. We then obtain the following expression for the flat rate social benefit

$$
\frac{\partial \pounds}{\partial g} = \frac{\pi}{\gamma_p}\left[(1-\tau)\left\{\operatorname{cov}(u'(x),\varphi)Ew^2 - \operatorname{cov}(u'(x),w^2)\bar{\varphi}\gamma_p\right\} - \tau Ew^2 Eu'(x)\varphi\right]
$$

We now see that the case for a LTC public insurance is stronger. We have

$$\left.\frac{\partial \pounds}{\partial g}\right|_{g=0} = \frac{\pi}{\gamma_p}\left\{\operatorname{cov}(u'(x),\varphi)Ew^2 - \operatorname{cov}(u'(x),w^2)\bar{\varphi}\gamma_p\right\}.$$

To give private insurance its best case, assume that $\gamma_p = 1$. then we can rewrite the above expression as:

$$\left.\frac{\partial \pounds}{\partial g}\right|_{g=0} = \pi\bar{\varphi}Ew^2\left\{\operatorname{cov}\left(\frac{u'(x)}{Eu'(x)},\frac{\varphi}{\bar{\varphi}}\right) - \operatorname{cov}\left(\frac{u'(x)}{Eu'(x)},\frac{w^2}{Ew^2}\right)\right\}. \tag{2}$$

In this new expression the variables have been normalized in such a way that the two covariances are now comparable. given the above assumptions those two covariances are negative and we can thus expect that the RHS of (2) is positive if the variance of w is higher than that of φ. In other words if income inequality dominates longevity inequalities LTC social insurance becomes socially desirable.

Proposition 2 *Suppose that a payroll tax is used to finance a LTC flat rate benefit. Suppose further that LTC private insurance loading factor is equal (or very close to) unity. Suppose finally that the survival probability and thus the probability of dependence increases with income. Then a LTC social insurance is desirable if the covariance between marginal utilities and the square of wages is in absolute value higher that the covariance between the marginal utilities and survival probabilities (after normalization of these variables).*

2.2 Social LTC benefits and subsidy of private insurance

Let us now assume that besides providing LTC benefits the government can also subsidize the purchase of private insurance. Denoting the subsidy rate by σ, the Lagrangian is now rewritten as

$$\begin{aligned}\pounds =& \sum n_i u\left((1-\tau)^2 w_i^2/2 - s_i - \theta_i(1-\sigma) + a\right) + \varphi_i(1-\pi)u(s_i/\varphi_i)\\ &+ \pi H(s_i/\varphi_i + g + \theta_i\gamma_p/\varphi_i\pi) + \mu\left[\tau(1-\tau)w_i^2 - a - \sigma\theta_i - \pi\varphi_i g\right]\end{aligned}$$

Observe that like in the previous sub-section we start with the case where a demogrant a is available. The FOC with respect to σ is

$$\frac{\partial \pounds}{\partial \sigma} = Eu'(x)\theta - \mu E\left(\sigma\frac{\partial \theta}{\partial \sigma} + \theta\right)$$

Assuming an interior solution and combining this expression with the FOC with respect to a lead to

$$\sigma = \frac{\text{cov}(u'(x), \theta)}{\mu E\frac{\partial \theta}{\partial \sigma}} < 0.$$

In the presence of a distributive demogrant and given the fact that θ is a normal good, it is socially desirable to tax θ to finance a. This is in line with the results obtained by Sato and Pestieau (2010) who also find a negative subsidy. Recall that in a setting where the demogrant is available, it is likely that $g = 0$; see Proposition 1. In other words, the government intervention is limited to redistributing disposable income so that low productivity households can buy more goods, including private LTC insurance coverage. Given that LTC purchase increases with earnings, they are also taxed. Remember that θ increases with w because it is a normal good but also because the probability of dependence increases with w.

Naturally this formula for σ only holds if there is a demogrant. In the case such a demogrant is not available, the only instruments are τ, g and σ and then we have

$$\frac{\partial \pounds}{\partial \sigma} = \bar{\theta}\left[E\left(u'(x).\frac{\theta}{\bar{\theta}}\right) - E\left(u'(x).\frac{\varphi}{\bar{\varphi}\gamma_p}\right) - \sigma\frac{\varphi}{\bar{\varphi}\bar{\theta}\gamma_p}Eu'(x)E\frac{\partial \theta}{\partial \sigma}\right].$$

Evaluating at the point where the subsidy is 0 and where γ_p is equal to 1, we have

$$\left.\frac{\partial \pounds}{\partial \sigma}\right|_{\sigma=0} = \bar{\theta}\left[\text{cov}(u'(x), \frac{\theta}{\bar{\theta}}) - \text{cov}(u'(x), \frac{\varphi}{\bar{\varphi}})\right]$$

Observe that insurance purchases θ are related to x through two channels: because it is a normal good and because longevity increases with earnings. Consequently, we can indeed expect that $\left|\text{cov}(u'(x), \theta/\bar{\theta})\right| > |\text{cov}(u'(x), \varphi/\bar{\varphi})|$, which in turn implies

$$\left.\frac{\partial \pounds}{\partial \sigma}\right|_{\sigma=0} < 0.$$

In words, one should expect a tax (negative subsidy) on private insurance. This is intuitive; from a redistributive viewpoint providing g to the households belonging to the bottom of the income distribution is more desirable than granting them a subsidy $\sigma\theta$ that increases with w.

Proposition 3 *Suppose that the government LTC policy comprises a flat benefit and a subsidy on the purchase of private insurance, both being financed by a linear earnings tax. Under reasonable assumptions private insurance should not be subsidized but taxed.*

3 Nonlinear scheme

The linear schemes studied so far have known limits. For example, one might object to the idea of offering a flat benefit to all households even the wealthiest. It is quite clear that this is hard to defend, except for reasons of political sustainability. In this section we introduce nonlinear tools that would allow for self-selecting individuals, thus avoiding that such benefit go to everyone even to those who can afford either insurance or self-insurance. Given the analytical difficulty of the problem at hand we consider a setting with three income classes indexed $i = 1, 2, 3$ so that $w_1 < w_2 < w_3$. The first class consist of low income people who are constrained in their first period consumption and their saving. The middle class can save and purchase some LTC insurance. The high-income class people does not buy LTC insurance; they prefer self-insurance due to too high loading costs of insurance. The only way to help the poor is to provide them with some public benefit that is not compatible with other type of benefits (e.g., a slot in a nursing home). The middle class individual can thus be deterred from mimicking the poor if the social benefit is less attractive than the private benefit minus the insurance premium. Moreover, the government can subsidize the purchase of private insurance by the middle-class but in a way that does not induce the rich to prefer mimicking the middle class rather than to self-insure. Given that we focus on LTC and not on the tax system we assume that the LTC policy is financed by a proportional tax on type

2 and 3 earnings. For simplicity we assume that all individuals have the same survival probability so that $\varphi_1 = \varphi_2 = \varphi_3 = \varphi$.

3.1 Individual choice and laissez-faire

With the instruments considered here, utility of an middle- or high-income individual is given by

$$U(w) = u\left(\frac{w(1-\tau)}{2} - s - \theta + \sigma(\theta)\right) + \pi\varphi H\left(\frac{s}{\varphi} + \frac{\theta\gamma_p}{\pi\varphi}\right) + (1-\pi)\varphi u\left(\frac{s}{\varphi}\right),$$

where $\sigma(\theta)$ is the nonlinear subsidy (or tax when it is negative) on private LTC insurance. Assuming an interior solution for s, we have

$$\frac{\partial U(w)}{\partial s} = -u'(x) + \pi H'\left(\frac{s}{\varphi} + \frac{\theta\gamma_p}{\pi\varphi}\right) + (1-\pi)u'\left(\frac{s}{\varphi}\right) = 0, \quad (3)$$

where $x = w(1-\tau)/2 - s - \theta + \sigma(\theta)$ and

$$m = \frac{s}{\varphi} + \frac{\theta\gamma_p}{\pi\varphi}$$

An interior solution for θ requires

$$\frac{\partial U(w)}{\partial \theta} = -(1-\sigma')u'(x) + \gamma_p H'(m) = 0,$$

which can be rearranged as

$$(1-\sigma') = \frac{\gamma_p H'(m)}{u'(x)} = \gamma_p MRS, \quad (4)$$

where $MRS = H'(m)/u'(x)$ is the marginal rate of substitution between x and m. To obtain the *laissez-faire* it is sufficient to set $\tau = \sigma = 0$ in these expressions.

The following lemma is useful to determine the demand for private LTC insurance in the *laissez-faire*.

Lemma 1 *(i) When $\gamma_p \leq \pi$ no private insurance will be bought in the laissez faire. (ii) Let $\widetilde{\gamma}_p(w)$ denote the critical level of the loading factor such that an individual with* *wage w chooses in the laissez-faire a strictly positive level of private insurance if an only if $\gamma_p > \widetilde{\gamma}_p(w)$. Under Assumption 1, $\widetilde{\gamma}_p$ increases with w.*

Proof. *With $\tau = 0$ and $\sigma = 0$, the utility of an individual with wage w is*

$$U(w) = u(w(1-\tau)/2 - s - \theta) + \pi\varphi H(s/\varphi + \theta\gamma_p/\pi\varphi) + (1-\pi)\varphi u(s/\varphi),$$

and the first-order condition with respect to θ (at $\theta = 0$) is given by

$$\left.\frac{\partial U(w)}{\partial \theta}\right|_{\theta=0} = -u'(x) + \gamma_p H'(s/\varphi)$$

Combining with (3) yields.

$$\left.\frac{\partial U(w)}{\partial \theta}\right|_{\theta=0} = -(1-\pi)u'(s/\varphi) + \left(\gamma_p - \pi\right) H'(s/\varphi). \tag{5}$$

When $\gamma_p \leq \pi$, the RHS of (5) is negative which establishes part (i) of the lemma. To establish part (ii) define $\widetilde{\gamma}_p$ such that

$$-(1-\pi)u'(s/\varphi) + \left(\widetilde{\gamma}_p - \pi\right) H'(s/\varphi) = 0 \tag{6}$$

Observe that with (6) depends on w only indirectly, through s which in turn is an increasing function of w (with separable utility all goods are normal). Totally differentiating (6) yields

$$\begin{aligned} \frac{d\widetilde{\gamma}_p}{ds} &= \frac{(1-\pi)u''(s/\varphi) - \left(\widetilde{\gamma}_p - \pi\right) H''(s/\varphi)}{\varphi H'(s/\varphi)} \\ &= \frac{\left(\widetilde{\gamma}_p - \pi\right)}{\varphi}\left[\frac{(1-\pi)u''(s/\varphi)}{\left(\widetilde{\gamma}_p - \pi\right) H'(s/\varphi)} - \frac{H''(s/\varphi)}{H'(s/\varphi)}\right] \end{aligned}$$

Using (6) to simplify the first term in bracket, rearranging and using the definition of relative risk aversion given in (1) we obtain

$$\frac{d\widetilde{\gamma}_p}{ds} = \frac{\left(\widetilde{\gamma}_p - \pi\right)}{s}\left[R_H(s/\varphi) - R_u(s/\varphi)\right],$$

which establishes part (ii). ■

The first part of this lemma is not surprising. When $\gamma_p/\pi < 1$ the LTC insurance payment, $\gamma_p/\pi\varphi$, in case of dependency is less than the return to savings, $1/\varphi$. Consequently, private insurance is dominated by savings. Part *ii)* of the lemma tells us that for higher levels of the loading factor ($\pi < \gamma_p < 1$), richer individuals are more likely to self-insure. More precisely we can have an equilibrium where type 2 individuals buy private insurance while type 3 persons don't, but the opposite pattern is not possible.

4 Optimal policy

We now turn to the problem of the utilitarian social planner concentrating on the case where rich individuals self-insure. The Lagrangian expression associated with this problem is given by

$$\begin{aligned}
\mathcal{L} = {} & n_1\left[u(\overline{x}_1) + (1-\pi)\varphi u\left(\frac{\bar{s}_1}{\varphi}\right) + \pi\varphi H\left(\frac{\bar{s}_1}{\varphi} + g_1\right)\right] + \\
& + n_2\left[u\left(\frac{w_2^2(1-\tau)^2}{2} - s_2^* - \theta_2 + \sigma_2)\right) + (1-\pi)\varphi u\left(\frac{s_2^*}{\varphi}\right) + \pi\varphi H\left(\frac{s_2^*}{\varphi} + \frac{\theta_2\gamma_p}{\pi\varphi}\right)\right] \\
& + n_3\left[u\left(\frac{w_3^2(1-\tau)^2}{2} - s_3^*\right) + (1-\pi)\varphi u\left(\frac{s_3^*}{\varphi}\right) + \pi\varphi H\left(\frac{s_3^*}{\varphi}\right)\right] \\
& + \mu\left[\tau(1-\tau)(n_2 w_2^2 + n_3 w_3^2) - n_1\pi\varphi g_1 - n_2\sigma_2\right] \\
& + \lambda_2\left[u\left(\frac{w_2^2(1-\tau)^2}{2} - s_2^* - \theta_2 + \sigma_2)\right) + (1-\pi)\varphi u\left(\frac{s_2^*}{\varphi}\right) + \pi\varphi H\left(\frac{s_2^*}{\varphi} + \frac{\theta_2\gamma_p}{\pi}\right)\right. \\
& \qquad \left. - u\left(\frac{w_2^2(1-\tau)^2}{2} - \tilde{s}_2^*\right) - (1-\pi)u\left(\frac{\tilde{s}_2^*}{\varphi}\right) - \pi\varphi H\left(\frac{\tilde{s}_2^*}{\varphi} + g_1\right)\right] \\
& + \lambda_3\left[u\left(\frac{w_3^2(1-\tau)^2}{2} - s_3^*\right) + (1-\pi)\varphi u\left(\frac{s_3^*}{\varphi}\right) + \pi\varphi H\left(\frac{s_3^*}{\varphi}\right) - u\left(\frac{w_3^2(1-\tau)^2}{2} - \tilde{s}_3^* - \theta_2 + \sigma_2\right)\right. \\
& \qquad \left. - (1-\pi)\varphi u\left(\frac{\tilde{s}_3^*}{\varphi}\right) - \pi\varphi H\left(\frac{\tilde{s}_3^*}{\varphi} + \frac{\theta_2\gamma_p}{\pi\varphi}\right)\right] \qquad (7)
\end{aligned}$$

where $\bar{x}_1$and $\bar{s}_1$ are given.

The FOC's are

$$\tau : -(1-\tau)\sum_{i=2}^{3} n_i u'(x_i) w_i^2 + \mu(1-2\tau)\sum_{i=2}^{3} n_i w_i^2$$
$$- \lambda_2(1-\tau)w_2^2\left[u'(x_2) - u'(\tilde{x}_2)\right] - \lambda_3(1-\tau)w_3^2\left[u'(x_3) - u'(\tilde{x}_3)\right] = 0, \tag{8}$$

$$\sigma_2 : u'(x_2) - \mu + \frac{\lambda_2}{n_2}u'(x_2) - \frac{\lambda_3}{n_2}u'(\widetilde{x}_3) = 0, \tag{9}$$

$$\theta_2 : -u'(x_2) + H'(m_2)\gamma_p + \frac{\lambda_2}{n_2}\left[-u'(x_2) + H'(m_2)\gamma_p\right]$$
$$+ \frac{\lambda_3}{n_2}\left[u'(\widetilde{x}_3) - H(\tilde{m}_3)\gamma_p)\right] = 0, \tag{10}$$

$$g_1 : \pi\varphi\left[H'(m_1) - \mu\right] - \frac{\lambda_2}{n_1}\pi\varphi H'(\tilde{m}_2) = 0. \tag{11}$$

4.1 Full information optimum

The true first-best, that is the utilitarian allocation achieved with full control of quantities and full information (and no loading factors) would imply

$$u'(x_i) = u'(d_i) \text{ for } i = 1,2,3 \text{ and } H'(m_1) = H'(m_2) = H'(m_3). \tag{12}$$

These conditions do not come as a surprise. On the one hand, they imply that the *MRS* between m and x of all individuals is equal to 1 so that there is full insurance. This condition holds of course for all Pareto-efficient allocations and not just for the utilitarian solution. On the other hand, condition (12) means that there is full redistribution across income classes. This property is specific to the utilitarian solution.

For our purpose it is more instructive to consider the full information solution that one obtains when policy tools are limited in the same way as in our second-best solution. In other words, the problem is the same as stated in (7), except that individual types are observable so that incentive constraints are not relevant and $\lambda_2 = \lambda_3 = 0$. The FOCs can then be rewritten as follows

$$\begin{aligned}
\tau :& -(1-\tau)\sum_{i=2}^{3} n_i u'(x_i) w_i^2 + \mu(1-2\tau)\sum_{i=2}^{3} n_i w_i^2 = 0,\\
\sigma_2 :& u'(x_2) - \mu = 0,\\
\theta_2 :& -u'(x_2) + H'(m_2)\gamma_p = 0,\\
g_1 :& \pi\varphi\left[H'(m_1) - \mu\right] = 0.
\end{aligned}$$

Summarizing these results we obtain

$$u'(x_2) = \gamma_p H'(m_2) = H'(m_1) = \mu. \tag{13}$$

The marginal rate of substitution between m and x is now higher than one; because of the loading factors there is less than full insurance. Individuals of type 3 choose self-insurance. So doing they oversave and do not consume as much LTC as they would do with fair private insurance. Their choice of saving is indeed determined by the following FOC

$$u'(x_3) = (1-\pi)u'(d_3) \ + \pi H'(m_3).$$

Furthermore in this constrained optimum, one has a limited redistribution between income classes. Individuals of type 1 keep their low level of consumption; they just benefit from the benefit g. Both types 2 and 3 contribute to financing this benefit. In other words, income inequality is unchanged. What the LTC optimal policy achieves is an equalization of the m_i's, namely the LTC spending.

$$u'(x_1) > u'(x_2) > u'(x_3).$$

4.2 Second best with $n_3 = 0$

To facilitate the exposition of the results we first focus on the case where we only have individuals of types 1 and 2. In these circumstances case the FOC's can be rewritten as

$$\tau : -(1-\tau)u'(x_2)w_2^2 + \mu(1-2\tau)w_2^2 - \frac{\lambda_2}{n_2}(1-\tau)w_2^2\left[u'(x_2) - u'(\tilde{x}_2)\right] = 0$$
$$\sigma_2 : u'(x_2) - \mu + \frac{\lambda_2}{n_2}u'(x_2) = 0$$
$$\theta_2 : -u'(x_2) + H'(m_2)\gamma_p + \lambda_2\left[-u'(x_2) + H'(m_2)\gamma_p\right] = 0$$
$$g_1 : \left[H'(m_1) - \frac{\mu}{\gamma_s}\right] - \frac{\lambda_2}{n_1}H'(\tilde{m}_2) = 0$$

Combining these conditions yields

$$H'(m_2)\gamma_p = u'(x_2) = \frac{\mu}{(1+\lambda_2/n_2)} < \mu, \tag{14}$$
$$H'(m_1) = \frac{\mu}{\gamma_s} + \frac{\lambda_2 H'(\tilde{m}_2)}{n_1} > \frac{\mu}{\gamma_s}. \tag{15}$$

Combining (4) and (14) we obtain $\sigma_2' = 0$; there is no marginal tax or subsidy on the purchases of private LTC insurance. Put differently, the asymmetric information on types here does not introduce a distortion on private LTC insurance. This is simply the traditional "no distortion at the top" property. Observe that no distortion here has to be understood as no *extra* distortion (compared to the full-information outcome); we do have a distortion associated with the loading factor. Condition (15) is best interpreted with reference to (13). Comparing these two expressions shows that g_1, the level of LTC provided in kind (and for free) is distorted downward. In other words, it is set at a lower level than under full information. This is done to relax the incentive constraint of type 2.

4.3 Second best with $n_3 > 0$

The FOC with respect to g_1 is not influenced by the presence of type 3 individuals. Consequently, expression (15) continues to describe the tradeoff for public LTC benefits and the results discussed in the previous are not modified. However, the policy applied to type 2 individuals is affected. To see this, rewrite (5) as

$$u'(x_2)\left(1+\frac{\lambda_2}{n_2}\right)-\frac{\lambda_3}{n_2}u'(\widetilde{x}_3)=H'(m_2)\gamma_p\left(1+\frac{\lambda_2}{n_2}\right)-\frac{\lambda_3}{n_2}H(\widetilde{m}_3)\gamma_p$$

dividing this expression by $u'(x_2)\,(1+\lambda_2/n_2)$, rearranging and factoring out $MRS_2 = H'(m_2)/u'(x_2)$ yields

$$\gamma_p MRS_2 = \frac{1-\frac{\lambda_3}{n_2+\lambda_2}\frac{u'(\widetilde{x}_3)}{u'(x_2)}}{1-\frac{\lambda_3}{n_2+\lambda_2}\frac{u'(\widetilde{x}_3)}{u'(x_2)}\frac{\widetilde{MRS}_3}{MRS_2}}, \tag{16}$$

where $\widetilde{MRS}_3 = H(\widetilde{m}_3)/u'(\widetilde{x}_3)$ is the marginal rate of substitution of an individual of type 3 mimicking and individual of type 2. Observe that the RHS of (16) increases with the ratio $\widetilde{MRS}_3/MRS_2$ and is equal to one when $\widetilde{MRS}_3/MRS_2$. Using (4) we then obtain that

$$\sigma_2' \lesseqqgtr 0 \qquad \Leftrightarrow \qquad \widetilde{MRS}_3 \gtreqqless MRS_2.$$

As m is a normal good, we have $\widetilde{MRS}_3 > MRS_2$ so that there is a *marginal* tax on private LTC insurance: $\sigma_2' < 0$. This is consistent with optimal taxation theory and to the findings of the previous section. Note that a *marginal* tax does not preclude an overall subsidy, namely $\sigma_2 > 0$. In other words, type 2 individuals receives cash transfers from which they can buy LTC insurance, but the transfer is not conditional on θ; it is effectively a lump-sum transfer. Recall that the income tax is restricted and the lump-sum transfer is used to replicated the optimal nonlinear tax as best as possible.

The main results of this section are summarized in the following proposition.

Proposition 4 *Consider a three-class society in which in the Laissez-Faire is as follows: the poor cannot afford private LTC insurance, the middle class purchases such insurance and the rich self-insure. An optimal nonlinear social insurance for LTC consists of a social benefit targeted towards the poor and a marginal tax (possibly along with an overall subsidy) on the purchase of private insurance by the middle class.*

5 Conclusion.

This paper was concerned by the design of a social insurance for LTC when a private LTC is available and when the objective of the government is to maximize a utilitarian social welfare. We distinguished between a linear and a nonlinear scheme. With a linear scheme it is optimal to provide a flat LTC benefit to everyone along with a tax on the purchase of private insurance. With a nonlinear scheme, we are able to target the LTC social benefit to the bottom of the income distribution. The middle class is subject to both a marginal tax and an overall subsidy for their purchase of private LTC insurance. So doing, the top income earners are prevented from mimicking the middle class and keep self-insuring.

References

[1] Cremer, H. and P. Pestieau (1996), "Redistributive taxation and social insurance", International Tax and Public Finance, 3, 281–295.

[2] Jousten, A., Lipszyc, B., Marchand, M. & Pestieau, P. (2005): Long-term care insurance and optimal taxation for altruistic children, *Finanzarchiv*, 61, 1–18.

[3] Pestieau, P. and M. Sato, (2006), Long term care: the State and the family, *Annales d'Economie et de Statistique*, 83–84, 151–166.

[4] Pestieau, P. and M. Sato, (2008), Long term care: the State, the market and the family, *Economica*, 75, 435-454, 2008.

[5] Pestieau, P. and M. Sato, (2010), Should we subsidize long term care insurance? unpublished,

[6] Rochet JC (1991) Incentives, Redistribution and Social Insurance. *Geneva Papers on Risk and. Insurance Theory* 16(2):143–165.