On a Three-Alternative Condorcet Jury Theorem

Johanna M. M. Goertz
François Maniquet

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Abstract

We investigate whether the simple plurality rule aggregates information efficiently in a large election with three alternatives. The environment is the same as in the Condorcet Jury Theorem (Condorcet (1785)). Voters have common preferences that depend on the unknown state of nature, and they receive imprecise private signals about the state of nature prior to voting. With two alternatives and strategic voters, the simple plurality rule aggregates information efficiently in elections with two alternatives (e.g., Myerson (1998)). We show that there always exists an efficient equilibrium under the simple plurality rule when there are three alternatives as well. We characterize the set of inefficient equilibria with two alternatives and the condition under which they exist. There is only one type of inefficient equilibrium with two alternatives. In this equilibrium, voters vote unresponsively because they all vote for the same alternative. Under the same condition, the same type of equilibrium exists with three alternatives. However, we show that the number and types of coordination failures increase with three alternatives, and that this leads to the existence of other types of inefficient equilibria as well, including those in which voters vote informatively.

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Johanna M. M. Goertz
University of Guelph
Department of Economics
Guelph, ON N1G 2W1
Canada
jgoertz@uoguelph.ca

François Maniquet
Catholic University of Louvain
CORE /ECORE
1348 Louvain-la-Neuve
Belgium
maniquet@core.ucl.ac.be

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1 Introduction

We investigate whether the simple plurality rule aggregates information efficiently in a large three-alternative election in which all voters have common preferences that depend on the unknown state of nature, and in which all voters receive imprecise private signals about the state of nature prior to voting. This environment should remind the reader of the Condorcet Jury Theorem (Condorcet (1785)). This theorem is about a (potentially large) jury with common state-dependent preferences who has to elect one of *two* alternatives. In state a, all voters prefer to elect alternative A; in state b, all voters prefer to elect alternative B. The state of nature is unknown at the time of the election, but each voter receives a private signal which is independently drawn from the same distribution. The winning alternative is elected according to the simple plurality rule (one vote per voter, and the alternative with most votes wins). Condorcet (1785) finds that a population of voters elects the correct alternative (the one preferred by all voters) with probability converging to one if the number of voters converges to infinity under the conditions that voters vote informatively and signals are more likely correct than incorrect. Informative voting means that each voter votes for his or her private signal.

However, voting informatively is not necessarily a Nash equilibrium in the environment of the Condorcet Jury Theorem (Feddersen and Pesendorfer (1998), Wit (1998), among others). With strategic voters, one can obtain the same result, even with a relaxed assumption on private signals: Myerson (1998) shows that there always exists an 'informationally efficient' equilibrium in which the correct alternative is elected with probability converging to one if the number of voters converges to infinity. However, in informationally efficient equilibria with two alternatives, voters never vote informatively (but usually mix between their signal and the other alternative).

We add a third alternative and a third state of nature to the environment of the Condorcet Jury Theorem. With the results from Goertz and Maniquet (2009, 2011), it is not obvious whether efficient equilibria exist and whether they are the unique type of equilibria with three alternatives. Goertz and Maniquet (2009, 2011) consider two different versions of a three-alternative large election with independent voters, who have common state-dependent preferences, and with partisan voters, who always prefer one of the three alternatives independent of the state of nature. If the independent voters are part of a majority in each state of nature, informational efficiency requires that the elected alternative be the same as it would be if all information were public. However, both papers find, contrary to the previous literature, that no simple scoring rule (including the simple plurality rule) except approval voting is efficient with three alternatives. This means that no simple scoring rule except approval voting has at least one efficient equilibrium.[1] Three alternatives lead to coordination failures

[1] With approval voting, there is at least one efficient equilibrium, but also inefficient equilibria.

among voters with common preferences that do not exist with two alternatives.[2]

In Goertz and Maniquet (2009, 2011), the partisan voters are 'responsible' for the fact that no simple scoring rule except approval voting is informationally efficient. So, it is interesting to investigate whether the same or other types of coordination failures exist in a model without partisans. And then we will also know whether or not the Condorcet Jury Theorem with strategic voters (Myerson (1998)) extends to more than two alternatives.

The present model is most closely related to Goertz and Maniquet (2011) because all voters receive private signals from the same information technology. It is also closely related to Myerson (1998). Compared to the former, it has no partisan voters and does not allow for abstention. Compared to the latter, it has three alternatives rather than two.

We find that an efficient equilibrium always exists. However, there are also inefficient equilibria. To compare the sets of inefficient equilibria with two alternatives and with three alternatives, we characterize the set of inefficient equilibria with two alternatives and the necessary condition for their existence. There is only one type of inefficient equilibrium with two alternatives. It is unresponsive because all voters vote for the same alternative independent of their private signals.[3] It only exists when private signals are 'not particularly informative' relative to prior probabilities. Not surprisingly, the example of an inefficient equilibrium in Myerson (1998) is of exactly this type. The same type of inefficient equilibrium under the same condition exists with three alternatives as well.

There are also other inefficient equilibria with three alternatives that do not exist with two alternatives. In Theorem 3, we give an example of an inefficient equilibrium in which voters surprisingly vote informatively, but the election outcome is inefficient nevertheless. This is a particularly interesting inefficient equilibrium because informative voting is typically not a Nash equilibrium with two alternatives. In addition, it will obviously lead to an inefficient outcome because of the relaxed assumption on private signals (they are not more likely correct than incorrect). In addition, this inefficient equilibrium is entirely different from the inefficient equilibria with three alternatives in Goertz and Maniquet (2009, 2011). In their inefficient equilibria, voters vote responsively, but not informatively. In addition, the equilibria require drastically different posterior beliefs of voters (some voters believe certain states to have zero probability, while other voters consider the same states as likely) to lead to coordination failures. In the inefficient equilibrium in Theorem 3, posterior beliefs are not so drastically different.

So, besides extending the Condorcet Jury Theorem to three alternatives, this paper shows that new coordination failures arise when the number of alternatives increases beyond two, even if there are no partisans and posterior beliefs

[2]In models with independent voters and partisan voters and two alternatives, informational efficiency of equilibria is generally satisfied (most prominently Feddersen and Pesendorfer (1996, 1997, 1999)).

[3]The definition of responsive and unresponsive strategies and equilibria is taken from Feddersen and Pesendorfer (1998).

are not drastically different. In addition, we can have inefficient equilibria in which voters vote informatively. Notice that, of course, these coordination failures are different from the typical difficulties with preference aggregation when there are more than two alternatives. In our model, all voters have the same preferences. The coordination failures stem from the difficulty of coordination among voters with common interests but imprecise private signals, and these difficulties arise only when there are more than two alternatives.

For three alternatives, we can only prove the existence of responsive inefficient equilibria by example. As we will argue later (discussion below Theorem 3), more general results (such as the characterization of the set of inefficient equilibria with three alternatives) are currently beyond reach.

In Section 2, we describe the model. In Section 3, we show that there exists an informationally efficient equilibrium for three alternatives. In Section 4, we present our results on inefficient equilibria for two and three alternatives. In Section 5, we conclude.

2 The Setting

2.1 The Model

We consider an election with three alternatives $\{A, B, C\} = \mathbf{K}$ and three states of nature $\{a, b, c\} = \mathbf{k}$ with prior probabilities $\pi_a, \pi_b, \pi_c \in (0, 1)$. Voters have common, state-dependent, and dichotomous preferences. They prefer a particular alternative in each state of nature, and are indifferent between the other two:

$$\begin{aligned} u(X|x) &= 1 \ \forall X \in \mathbf{K}, x \in \mathbf{k}, \\ u(Y|x) &= 0 \ \forall Y \neq X \in \mathbf{K}, x \in \mathbf{k}. \end{aligned}$$

Assuming dichotomous preferences simplifies the strategic analysis. With dichotomous preferences, voters care only about those election outcomes in which their vote changes the outcome from any of the two disliked alternatives to the preferred alternative; they never care about election outcomes in which they are pivotal between the two less preferred alternatives. Even in this less strategic environment, we find coordination failures among voters that lead to inefficient equilibria. The reader can easily imagine that more general assumptions on the preferences would only increase these problems.

We assume that the population of voters is large. In addition, there is uncertainty about the actual size of the population. This assumption is fairly common in the previous literature (e.g., Feddersen and Pesendorfer (1996, 1997, 1999), Myerson (1998, 2000, 2002), Goertz and Maniquet (2009, 2011), among others). In the precise way of modeling the population uncertainty, we follow Myerson (1998, 2000, 2002) and assume that the population size is Poisson-distributed with parameter n. So, the voting game is a so-called *Poisson game*.

The probability there are exactly N voters is[4]

$$P(N|n) = \frac{e^{-n}n^N}{N!}.$$

With population uncertainty, each voter is pivotal (i.e., the decisive voter) with positive probability. This is true even in an equilibrium in which all voters vote for the same alternative. Therefore, unresponsive equilibria are not (as they are in other models without population uncertainty) equilibria in weakly dominated strategies.

Prior to voting, each voter receives an informative but imprecise signal about the state of nature. Signals are drawn independently from the same distribution. In this sense, the model is similar to Feddersen and Pesendorfer (1998, 1999), Myerson (1998), and Goertz and Maniquet (2011).[5] For simplicity, we assume that there are only three signals $\{a, b, c\} = \mathbf{T}$. We denote by $\varphi_z(x)$ the probability that a voter receives signal x in state z. Ex ante, all voters are the same. They only differ after receiving the private signals. Therefore, a voter's type is defined by the signal he or she receives.

Signals are informative about the state of nature in the sense of Myerson (1998):

$$\varphi_x(x) > \varphi_z(x) \ \ \forall x \neq z \in \mathbf{k}, x \in \mathbf{T}. \tag{1}$$

In the original Condorcet Jury Theorem and in, for example, Feddersen and Pesendorfer (1998), signals are 'correct' with a probability larger than 1/2. This means that signal x is more likely in state x than any other signal. Myerson (1998) relaxes this assumption. With Eq. (1), signal x is merely more likely in state x than in any other state of nature. He shows that this condition is sufficient to ensure the existence of an efficient equilibrium in a two-alternative election if voters vote strategically and the number of voters converges to infinity. The same is true for three-alternative elections (Theorem 1 below). However, it does not preclude the existence of inefficient equilibria for two- or for three-alternative elections.

If signals satisfy Eq. (1), there is no aggregate uncertainty in the population: If all private information were public, the state of nature would be known and voters would unanimously vote for the same alternative. So, we can say that informational efficiency is satisfied if the elected alternative is the same as the one that would be elected if all information were public.

There is no abstention in our model. This makes the comparison with Myerson (1998) more straightforward. However, this assumption is not consequential for our main results on three alternatives (Theorems 1 and 3). In particular, the

[4]Notice that population uncertainty implies that a voter does not know precisely how many other voters there are in the game. In models that allow for abstention, some of those other voters might decide to abstain, while others decide to submit a particular ballot. Population uncertainty is different from uncertainty about how many voters abstain.

[5]In a second set of papers on information aggregation in large elections, voters receive signals from signal technologies that are differently precise (e.g., Feddersen and Pesendorfer (1996, 1997), or Goertz and Maniquet (2009)).

inefficient equilibrium in Theorem 3 would also exist with abstention because in the equilibrium all voters receive positive expected utility from the particular ballot they choose (which implies that they rather choose this ballot than any other action including abstention (if it was allowed)).

Each voter votes for an alternative in $\mathbf{K}$ or mixes between different alternatives. The alternative with most votes is elected. We assume a particular tie-breaking rule that is without loss of generality: Any tie involving alternative A is broken in favor of A; a tie between alternatives B and C is broken in favor B.

An economy $\mathcal{E}$ is a list (π, φ) that satisfies Eq. (1). For any expected size of the population n, a strategy is a function $\sigma_n : \mathbf{T} \rightarrow \Delta(\mathbf{K})$, associating a voter type with a probability distribution over $\mathbf{K}$.[6] Let $\sigma_n^X(t) \geq 0$ denote the probability that a voter of type t chooses to vote for alternative X. It has to be true that $\sum_{X \in \mathbf{K}} \sigma_n^X(t) = 1 \forall t \in \mathbf{T}$. Suppose that σ_n^* is a Bayesian Nash equilibrium of the voting game given n expected voters. We are interested in limit equilibria σ^* such that $\sigma_n^* \rightarrow \sigma^*$ as $n \rightarrow \infty$.

Feddersen and Pesendorfer (1998) call an equilibrium strategy profile responsive, if voters "change their vote as a function of their private information with positive probability" (p. 26). Following them, we call a limit equilibrium *responsive* if $\sigma^*(t) \neq \sigma^*(t')$ for all $t \neq t' \in \mathbf{T}$. In an unresponsive equilibrium, all voters vote the same way, independent of the signal. We call a limit equilibrium *unresponsive* if $\sigma^*(t) = \sigma^*(t')$ for all $t \neq t' \in \mathbf{T}$. If voters vote informatively, they vote for their signal. Informative voting is responsive, but responsive voting is not necessarily informative. We call a limit equilibrium *informative* if $\sigma^{X*}(x) = 1$ for all $X \in \mathbf{K}$ and $x \in \mathbf{T}$. If voters vote informatively, the election outcome accurately reflects the private information held by the population. But if signals satisfy only Eq. (1), but are not more likely correct than incorrect in each state of nature (as the were in the original Condorcet Jury Theorem), informative voting cannot be efficient. So, it is all the more surprising that it can be an equilibrium.

2.2 Voting Behavior

Before we can prove our results, we need to consider how rational voters vote in a large Poisson voting game. However, we will keep this section brief, discuss only the necessary tools, and refer the reader to more elaborate references where appropriate.

Recall that voters derive utility from the outcome of the election alone. A rational voter needs to consider pivotal events in which his or her vote changes the outcome of the election from a less preferred to a more preferred alternative. Pivotal events depend on the particular ballot that a voter wants to submit and on the tie-breaking rule. A voter who considers ballot B, for example, is pivotal if alternative A has the same number of votes as alternative B and both have

[6] In Poisson games, strategies are defined by type. This corresponds to an assumption of symmetric equilibria in games in which strategies are defined agent by agent.

at least as many votes as C. If a voter considers ballot A, however, the voter is pivotal if alternative B has one more vote than alternative A and alternative C is sufficiently behind. Generally, denote by E_z^{XY} the pivotal event in which one additional vote for alternative X changes the outcome of the election from alternative Y to alternative X in state z. And denote by piv_z^{XY} the probability of this pivotal event. The probability of a pivotal event depends on the underlying strategy profile. To save on notation, we will avoid the additional index it it is not misleading. If a voter of type t considers voting for alternative Y rather than alternatives X or Z, the expected utility of this voter can be written as

$$EU(Y|t) = -\pi_x(t)piv_x^{YX} + \pi_y(t)piv_y^{YX} + \pi_y(t)piv_y^{YZ} - \pi_z(t)piv_z^{YZ}, \quad (2)$$

where $\pi_z(t)$ denotes the posterior probability of state z conditional on receiving signal t. We need to evaluate equations such as Eq. (2) to construct strategies of the voters. In large elections, the probability of a pivotal event converges to zero, and so do entire equations such as Eq. (2). However, Myerson (2000) shows that probabilities of pivotal events do not converge to zero with the same speed. Events with probabilities that converge to zero slower than others are infinitively more likely, and so events with probabilities that converge faster can be ignored. The difference in the speed of convergence of pivotal probabilities makes comparisons between expected utilities from different ballots meaningful.

Myerson (2000) proposes the magnitude as a measure for the speed of convergence to zero of a probability in a large Poisson game. The magnitude μ of the probability of a pivotal event is defined as

$$\mu(E_z^{XY}) = lim_{n\to\infty}\frac{log(piv_z^{XY})}{n},$$

where piv_z^{XY} is the probability of event E_z^{XY} given the underlying strategy profile. Events with larger magnitude are infinitively more likely than events with smaller magnitude. The magnitude of an event can be calculated by solving a maximization problem (Magnitude Theorem (Myerson (2000))). For a more detailed discussion of magnitudes, we would like to refer the reader to Myerson (2000) or to Goertz and Maniquet (2009, 2011). Here, we will only highlight the outcome of the maximization problem that gives us a formula according to which we can calculate the magnitudes of the probabilities of pivotal events that are relevant for Theorems 2 and 3.

Each pivotal event is characterized by a system of linear equations. Consider, for example, pivotal event E_z^{BA}. This event occurs if $N_z^A = N_z^B \geq N_z^C$, where N_z^X denotes the actual number of votes for alternative X in state z. The magnitude of a pivotal event depends on the closeness of the race between the alternatives, and also on the number of alternatives that are involved in the race. In a three-alternative election, a pivotal event can either be the result of a close race between two alternatives (Case 1 below) or between three alternatives (Case 2 below). If there are only two alternatives, Case 2 below is irrelevant.

Case 1: The constraint on N_z^C is not binding, i.e., $N_z^A > N_z^C$.
In this case, the magnitude of the probability of event E_z^{AB} can be calculated

using the following formula, which is derived from the maximization problem mentioned above:

$$\mu(E_z^{BA}) = 2\sqrt{\lambda_z^A \lambda_z^B} - (\lambda_z^A + \lambda_z^B), \tag{3}$$

where λ_z^X denotes the expected fraction of votes for alternative X in state z. Of course, event E_z^{BA} is composed of many subevents of the form $N_z^A = N_z^B \geq N_z^C$ because there are many possible election outcomes in which a voter with ballot B is pivotal. However, in a large Poisson game, it is true in the limit that the probability of E_z^{BA} is equal to the probability of its most likely subevent. The solution of the maximization problem mentioned above not only gives us the magnitude of the event, but also its most likely subevent. In the case of E_z^{BA}, for example, the most likely subevent is election outcome $\tilde{N}_z^A = \tilde{N}_z^B = n\sqrt{\lambda_z^A \lambda_z^B}$ and $\tilde{N}_z^C = n\lambda_z^C$. The number of votes for the two leading alternatives in the most likely subevent is a function of the two expected fractions of votes. Given the expected fractions of votes, one can easily verify whether the constraint on the actual number of votes for alternative C is binding or not.

Case 2: The constraint on N_z^C is binding, i.e., $N_z^A = N_z^B = N_z^C$.

In this case, the magnitude of event E_z^{BA} has to be calculated using a different formula:

$$\mu(E_z^{BA}) = 3\sqrt{\lambda_z^A \lambda_z^B \lambda_z^C} - (\lambda_z^A + \lambda_z^B + \lambda_z^C). \tag{4}$$

The magnitude of the event now depends on the expected fractions of votes for all three alternative because the race involves all three alternatives. In this case, the most likely sub-event of E_z^{BA} is election outcome $\tilde{N}_z^A = \tilde{N}_z^B = \tilde{N}_z^C = n\sqrt[3]{\lambda_z^A \lambda_z^B \lambda_z^C}$.

From Eqs. (3) and (4), it is clear that the magnitude of a pivotal event depends on the expected closeness of the race between alternatives: The closer the expected fractions of votes for the alternatives with most votes are, the more likely is a pivotal event and the larger is its magnitude. Eqs. (3) and (4) suffice to calculate all necessary magnitudes in the proofs of Theorems 2 and 3.

Generally, a voter wants to submit ballot X if the most likely pivotal event occurs in state x. It can, however, be the case (as we will see in the proof of Theorem 3) that events E_x^{XY} and E_y^{XY} have the largest and the same magnitude. In this case, it is possible that both ballots X and Y yield negative expected utility. Then the voter needs to consider pivotal events with lower magnitudes to decide whether voting for alternative Z yields positive expected utility. If yes, the voter will submit ballot Z instead.

Besides the two formulas to calculate magnitudes, we need two other results from Myerson (2000) in the proof of Theorem 3. The first one is about the actual probability of a pivotal event. Recall that each pivotal event has a most likely subevent denoted by $(\tilde{N}_z^A, \tilde{N}_z^B, \tilde{N}_z^C)_{z\in\mathbf{k}}$. As mentioned above, it is true in a large Poisson game that

$$lim_{n\to\infty} piv_z^{XY} = \prod_{X\in\mathbf{K}} \frac{e^{-n\lambda_z^X}(n\lambda_z^X)^{\tilde{N}_z^X}}{(\tilde{N}_z^X)!}. \tag{5}$$

In addition, one can show that $\mu(E_z^{BA}) = \mu(E_z^{AB})$ (Offset-Theorem (Myerson (2000))). The intuition behind this result is that the most likely election outcome leading to event E_z^{AB} is only one vote different from the most likely election outcome leading to event E_z^{BA}. In a large Poisson game, the probabilities of these two events are not very different, and so they have the same magnitude. Indeed, one can use Eq. (5) to verify that the ratio of the probabilities of these two events converges to a finite number different from zero (ratios of events with different magnitudes converge to zero or to infinity).

The decision rules and tools outlined in this section suffice to evaluate any strategy we have to consider in the remainder of the paper, so that we can now prove our results.

3 Informational Efficiency with Three Alternatives

Since all voters have common preferences and there is no aggregate uncertainty, it is quite natural to call an equilibrium informationally efficient if the elected alternative in this equilibrium is the same as the one that would be elected if all private information was public. In an informationally efficient limit equilibrium, this alternative is selected with probability converging to one in each state of nature.

Definition 1. ***Informationally Efficient Equilibrium*** *A limit equilibrium σ^* of an economy $\mathcal{E}$ is informationally efficient if for all $X \in \mathbf{K}, x \in \mathbf{k}$ it is true that $P_{\sigma^*}(X|x) = 1$.*

Following Goertz and Maniquet (2009, 2011), we call a voting rule informationally efficient if all of its limit equilibria are informationally efficient for all $\mathcal{E}$. A voting rule is called weakly informationally efficient if it has at least one efficient limit equilibrium for each $\mathcal{E}$. In the presence of partisans (as in Goertz and Maniquet (2009, 2011)), the simple plurality rule is not informationally efficient and not weakly informationally efficient because there does not necessarily exist an efficient equilibrium under this rule. Theorem 1 shows that the simple plurality rule is weakly informationally efficient for three alternatives in the absence of partisans because an efficient limit equilibrium always exists.

Theorem 1. *There exists an informationally efficient equilibrium for any economy $\mathcal{E}$ with three alternatives if signals satisfy Eq. (1).*

The proof is an adaptation of the proof of Theorem 2 in Goertz and Maniquet (2011) to the current model.

Proof. The proof is divided into two steps. In step 1, we show that for any $\mathcal{E}$ with three alternatives that satisfies Eq. (1) there exists a sequence of strategy profiles σ_n such that $lim_{n\to\infty} P_{\sigma_n}(X|x) = 1$ for all $X \in \mathbf{K}, x \in \mathbf{k}$. To guarantee that $lim_{n\to\infty} P_{\sigma_n}(X|x) = 1$ it is sufficient to verify that the expected fraction of votes for alternative X in state x is larger than the expected fraction of votes

for each of the other two alternatives. As n tends to infinity, the whole mass of probability concentrates in arbitrarily close neighborhoods around the expected events (Law of Large Numbers). In step 2, we deduce from step 1 that there exists a limit equilibrium σ^* that aggregates information efficiently.
Step 1: Consider an economy $\mathcal{E}$ with three alternatives that satisfies Eq. (1). Let ϵ_a, ϵ_b, ϵ_c be small positive numbers such that

$$\epsilon_a \varphi_a(a) = \epsilon_b \varphi_b(b) = \epsilon_c \varphi_c(c).$$

Consider a sequence of strategy profiles with $\sigma_n^X(x) = \epsilon_x + \frac{1-\epsilon_x}{3}$, $\sigma_n^Y(x) = \frac{1-\epsilon_x}{3}$, $\sigma_n^Z(x) = \frac{1-\epsilon_x}{3}$ $\forall x \in \mathbf{T}$ and $\forall X, Y, Z \in \mathbf{K}$. Recall that λ_z^X denotes the expected fraction of votes for alternative X in state z. Given the strategy profile, the expected fractions of votes for each alternative in state a are

$$\begin{aligned}
\lambda_a^A &= \varphi_a(a)(\epsilon_a + \frac{1}{3}(1-\epsilon_a)) + \frac{1}{3}\varphi_a(b)(1-\epsilon_b) + \frac{1}{3}\varphi_a(c)(1-\epsilon_c), \\
\lambda_a^B &= \varphi_a(b)(\epsilon_b + \frac{1}{3}(1-\epsilon_b)) + \frac{1}{3}\varphi_a(a)(1-\epsilon_a) + \frac{1}{3}\varphi_a(c)(1-\epsilon_c), \\
\lambda_a^C &= \varphi_a(c)(\epsilon_c + \frac{1}{3}(1-\epsilon_c)) + \frac{1}{3}\varphi_a(a)(1-\epsilon_a) + \frac{1}{3}\varphi_a(b)(1-\epsilon_b).
\end{aligned}$$

It can easily be seen that $lim_{n\to\infty} P_{\sigma_n}(A|a) = 1$ as $n \to \infty$. Similarly, it can be shown that $lim_{n\to\infty} P_{\sigma_n}(B|b) = 1$ and $lim_{n\to\infty} P_{\sigma_n}(C|c) = 1$ as $n \to \infty$. Notice that ϵ_x can always be chosen so that $\sum_{X\in\mathbf{K}} \sigma^X(t) = 1 \forall t \in \mathbf{T}$. Also notice that σ_n does not depend on n.
Step 2: Let σ_n^* be a sequence of strategy profiles that maximizes the ex-ante utility of the voters. Such strategies exist, as they maximize a continuous function on a compact set. We claim that they are equilibrium strategies. Indeed, the existence of a profitable deviation would contradict the fact that $\sigma_n^*(t)$ maximizes expected utilities. Also, it is impossible that the expected utility from $\sigma_n^*(t)$ is lower than the expected utility from $\sigma_n(t)$ as defined above. The expected utility of a voter of type t tends to 1 if alternative X is elected in state x for all $X \in \mathbf{K}$. Given that $P_{\sigma_n}(X|x) \to 1$ as $n \to \infty$ for all $X \in \mathbf{K}$, so that the expected utility associated with σ tends to 1, and, given that, by construction, σ_n^* yields at least the same expected utility as σ, it has to be true that $P_{\sigma_n^*}(X|x) \to 1$ as $n \to \infty$ for all $X \in \mathbf{K}$. So, $lim_{n\to\infty}\sigma_n^*$ is a limit equilibrium that aggregates information efficiently. □

4 Informationally Inefficient Equilibria

To be able to compare the set of inefficient equilibria with two and with three alternatives, we characterize the set of inefficient equilibria with two alternatives and the necessary condition under which they exist (Theorem 2). There is only one type of inefficient equilibrium with two alternatives. In this equilibrium, all voters vote unresponsively because they all vote for the same alternative. And they do so because they all believe the same state of nature to be more likely, conditional on their signal. This is true because private signals are 'not particularly informative about the state of nature', relative to the prior probabilities. Or, in other words, in the inefficient equilibrium with two alternatives, all voters vote for the same alternative because

$$\pi_x(t) > \pi_y(t) \;\; for \;\; x \in \mathbf{k}, \;\; all \;\; y \in \mathbf{k} \setminus \{x\}, \;\; all \;\; t \in \mathbf{T}. \tag{6}$$

Theorem 2. *Suppose that there are two alternatives and that signals satisfy Eq. (1). The only inefficient equilibrium that exists is unresponsive because all voters vote for the same alternative. It only exists if signals and prior probabilities are such that posterior beliefs satisfy Eq. (6).*

Proof. Consider a two-alternative election with $\mathbf{K} = \{A, B\}$, $\mathbf{k} = \{a, b\}$, and $\mathbf{T} = \{a, b\}$. Recall from the proof of Proposition 1 that, by the Law of Large Numbers, a limit equilibrium is informationally efficient if the expected fraction of votes for alternative X is larger than the expected fraction of votes for alternative Y in state x. Consider, without loss of generality, an inefficient equilibrium in which alternative B is elected in state a, i.e., $\lambda_a^B > \lambda_a^A$.

The expected utility of voting for alternative Y, after receiving signal s, can be written as $EU(Y|s) = \pi_y(s)piv_y^{YX} - \pi_x(s)piv_x^{YX}$. Consider now signal s' so that state y is more likely with signal s' than with signal s. Then $EU(Y|s') = \pi_y(s')piv_y^{YX} - \pi_x(s')piv_x^{YX}$. With Eq. (1), $EU(Y|s') \geq EU(Y|s)$. So, type b is never less likely to vote for alternative B than type a, and type a is never less likely to vote for alternative A than type b. In addition, at most one of the two types of voters mixes between the two alternatives (if, for example, $EU(Y|s) = 0$, then necessarily $EU(Y|s') > 0$). If type a mixes, then type b votes for B, and if type b mixes, then type a votes for A. If, as assumed above, alternative B is elected in state a, then it cannot be true that both types vote for alternative A. Therefore, we have to consider four cases.

Case 1: Type a mixes and type b votes for B.
Suppose that in equilibrium, type a mixes with probabilities $\sigma^{*A}(a) > 0$ and $\sigma^{*B}(a) = 1 - \sigma^{*A}(a) > 0$. The expected fractions of votes are

$$\lambda_a^A = \sigma^{*A}(a)\varphi_a(a), \tag{7}$$

$$\lambda_a^B = (1 - \sigma^{*A}(a))\varphi_a(a) + \varphi_a(b), \tag{8}$$

$$\lambda_b^A = \sigma^{*A}(a)\varphi_b(a), \tag{9}$$

$$\lambda_b^B = (1 - \sigma^{*A}(a))\varphi_b(a) + \varphi_b(b). \tag{10}$$

Because the equilibrium is inefficient, it must be true that (8)-(7)> 0. So, it is also necessarily true that (10)-(9)>(8)-(7). With Eq. (3) we can conclude that $\mu(E_a^{AB}) > \mu(E_b^{AB})$. This implies, among others, that $EU(A|a) > EU(B|a)$, which is a contradiction to the assumption that type a mixes.

Case 2: Type a votes for A and type b votes for B.
The expected fractions of votes are

$$\lambda_a^A = \varphi_a(a), \tag{11}$$

$$\lambda_a^B = \varphi_a(b), \tag{12}$$
$$\lambda_b^A = \varphi_b(a), \tag{13}$$
$$\lambda_b^B = \varphi_b(b). \tag{14}$$

Because the equilibrium is inefficient, it must be true that (12)-(11)> 0. So, it is also necessarily true that (14)-(13)>(12)-(11). Consequently, $\mu(E_a^{AB}) > \mu(E_b^{AB})$, which implies that $EU(A|b) > EU(B|b)$, a contradiction to the fact that type b votes for B.

Case 3: Type a votes for A and type b mixes.
Suppose that in equilibrium, type b mixes with probabilities $\sigma^{*A}(b) > 0$ and $\sigma^{*B}(b) = 1 - \sigma^{*A}(b) > 0$. The expected fractions of votes are

$$\lambda_a^A = \varphi_a(a) + \sigma^{*A}(b)\varphi_a(b), \tag{15}$$
$$\lambda_a^B = (1 - \sigma^{*A}(b))\varphi_a(b), \tag{16}$$
$$\lambda_b^B = (1 - \sigma^{*A}(b))\varphi_b(b), \tag{17}$$
$$\lambda_b^A = \varphi_b(a) + \sigma^{*A}(b)\varphi_b(b). \tag{18}$$

Because the equilibrium is inefficient, it must be true that 16)>(15). So, it is also necessarily true that (17)-(18)>(16)-(15). Consequently, $\mu(E_a^{AB}) > \mu(E_b^{AB})$, which implies that $EU(A|b) > EU(B|b)$, a contradiction to the fact that type b mixes.

Case 4: Type a votes for B and type b votes for B.
The expected fractions of votes are

$$\lambda_a^A = 0,$$
$$\lambda_a^B = 1,$$
$$\lambda_b^B = 1,$$
$$\lambda_b^A = 0.$$

In this case, the magnitudes of the pivotal events are all the same and are equal to -1. The pivotal event of a voter considering voting for A is the event that one other voter votes. This voter votes according to the proposed strategy $\sigma^{*B} = 1$. The probability of this event, piv_a^{AB} or piv_b^{AB}, is the same in both states and is, according to Eq. (5), equal to $\frac{e^{-n}n}{1}$. The pivotal event of a voter considering voting for B is the event in which no other voter votes. The probability of this event, piv_a^{BA} or piv_b^{BA}, is the same in both states and is equal to $\frac{e^{-n}}{1}$.
A voter of type a votes for B only if $EU(B|a) > EU(A|a)$, or if

$$(\pi_b(a) - \pi_a(a))piv_a^{BA} > (\pi_a(a) - \pi_b(a))piv_a^{AB}.$$

However, since

$$lim_{n\to\infty}\frac{piv_a^{BA}}{piv_a^{AB}} = 0,$$

a voter of type a only votes for B if $0 > (\pi_a(a) - \pi_b(a))$. This implies that this type of inefficient equilibrium only exists if

$$\frac{\pi_a}{\pi_b} < \frac{\varphi_b(a)}{\varphi_a(a)},$$

or if

$$\pi_a(a) < \pi_b(a). \tag{19}$$

With Eq. (1), Eq. (19) implies that $\pi_a(b) < \pi_b(b)$ and that a voter of type b votes for B as well.

Therefore, the only possible inefficient equilibrium with two alternatives is the equilibrium described in Case 4. In this equilibrium, all voters vote for the same alternative because posterior beliefs satisfy Eq. (6). □

Notice that the equilibrium described in Case 4 of the proof is different from those types of equilibria in voting games without population uncertainty in which all voters voting for the same alternative is always an equilibrium because no voter is ever pivotal.[7] With population uncertainty, each voter has a positive probability of being pivotal. However, if all voters vote for the same alternative, the probability of being pivotal is the same in state a and in state b. While the probabilities of pivotal events usually convey valuable information about the likelihoods of the different states of nature, this is not true here. So, voters rely on their posterior beliefs to guide their voting behavior. If all voters consider the same state of nature to be more likely, voting for the same alternative is clearly an equilibrium. Myerson (1998) gives an example of an inefficient equilibrium with two alternatives. Not surprisingly, it is of exactly the type described in Theorem 2. Signals and prior probabilities (or, more precisely, posterior beliefs) satisfy Eq. (6) and all voters vote for the same alternative.

The arguments in Case 4 of the proof of Theorem 2 can be applied to an election with three alternatives as well. If all voters consider the same state of nature most likely, the same type of unresponsive equilibrium also exists for three alternatives.

Corrollary 1. *Suppose that there are three alternatives and that signals satisfy Eq. (1). There exists an unresponsive inefficient equilibrium in which all voters vote for the same alternative if* (φ, π) *satisfy Eq. (6).*

Any further arguments of the proof cannot be extended to three alternatives because they rely on the fact that a voter of type b is never less likely to vote for alternative B than a voter of type a and a voter of type a is never less likely to vote for alternative A than a voter of type b. In addition, only one of the two types mixes if there are only two alternatives. However, the same facts are not true for three alternatives (see discussion below Theorem 3). So, we cannot

[7] These types of equilibria exist, for example, in Feddersen and Pesendorfer (1998) or in Wit (1998).

exclude the existence of other inefficient equilibria with three alternatives. Indeed, in Theorem 3, we show by example that other types of inefficient equilibria exists with three alternatives. In the particular equilibrium we present, voters not only vote responsively, but (surprisingly) they vote informatively.

Theorem 3. *Suppose that there are three alternatives and that signals satisfy Eq. (1), but that Eq. (6) is not satisfied. There exists a responsive inefficient equilibrium in which voters vote informatively.*

We prove Theorem 3 by example. More general results (including a complete characterization of the set of inefficient equilibria) for three alternatives are currently beyond reach (see discussion below).

Proof. We construct $\mathcal{E}$ that satisfies Eq. (1) and for which there exists a limit equilibrium σ^* in which alternative A is not elected in state a. Consider $\mathcal{E}$ such that $\pi_a = \frac{1}{3}$, $\pi_b = \frac{1}{3} + \epsilon$, $\pi_c = \frac{1}{3} - \epsilon$, and ϵ sufficiently small. Also assume that $\varphi_a(a) = 0.2$, $\varphi_a(b) = \varphi_a(c) = 0.4$, $\varphi_b(a) = 0.1$, $\varphi_b(b) = 0.5$, $\varphi_b(c) = 0.4$, $\varphi_c(a) = 0.1$, $\varphi_c(b) = 0.4$, $\varphi_c(c) = 0.5$. We claim that $\sigma^{*X}(x) = 1$ is a limit equilibrium, but is not efficient. With σ^*, the expected fractions of votes are

$$\lambda_a^a = 0.2,\ \ \lambda_a^B = \lambda_a^C = 0.4,$$

$$\lambda_b^A = 0.1,\ \ \lambda_b^B = 0.5,\ \ \lambda_b^C = 0.4,$$

$$\lambda_c^A = 0.1,\ \ \lambda_c^B = 0.4,\ \ \lambda_c^C = 0.5.$$

We need to show that σ^* is an equilibrium.

With the formulas from Section 2.2, the magnitudes of the most likely pivotal events are $\mu(E_b^{BC}) = \mu(E_b^{CB}) = \mu(E_c^{BC}) = \mu(E_c^{CB}) = -5.75 * 10^{-3}$ and $\mu(E_a^{AB}) = \mu(E_a^{AC}) = -0.034$. Clearly, pivotal events in states b and c have a larger magnitude than pivotal events in state a and are therefore more likely.

Given that the most likely pivotal events in states b and c all have the same magnitude, we will also need to know the exact relationship between the pivotal probabilities. Consider first events E_b^{CB} and E_c^{CB}. Recall from Section 2.2 that the number of votes of the two leading alternatives in the most likely subevent is a function of the expected fractions of votes of these two alternatives. Since the expected fractions of votes for alternatives B and C are symmetric in states b and c, the two most likely subevents take the form $(\tilde{N}_b^A, \tilde{N}_b^B, \tilde{N}_b^C) = (\tilde{N}_c^A, \tilde{N}_c^B, \tilde{N}_c^C) = (j, k, k)$, where $k > j$. This implies that

$$piv_b^{CB} = \frac{e^{-n\lambda_b^C}(n\lambda_b^C)^k}{(k)!} * \frac{e^{-n\lambda_b^B}(n\lambda_b^B)^k}{(k)!} * \frac{e^{-n\lambda_b^A}(n\lambda_b^A)}{(n\lambda_b^A)!} \tag{20}$$

$$piv_c^{CB} = \frac{e^{-n\lambda_c^C}(n\lambda_c^C)^k}{(k)!} * \frac{e^{-n\lambda_c^B}(n\lambda_c^B)^k}{(k)!} * \frac{e^{-n\lambda_c^A}(n\lambda_c^A)}{(n\lambda_c^A)!} \tag{21}$$

So, it is clear that $piv_b^{CB} = piv_c^{CB}$.

Now consider pivotal events E_b^{BC} and E_c^{BC}. For these two events, the most likely subevents, due to arguments similar as above, take the form $(\tilde{N}_b^A, \tilde{N}_b^B, \tilde{N}_b^C) = (\tilde{N}_c^A, \tilde{N}_c^B, \tilde{N}_c^C) = (j, k, k+1)$, where $k > j$. This implies that

$$piv_b^{BC} = \frac{e^{-n\lambda_b^C}(n\lambda_b^C)^{k+1}}{(k+1)!} * \frac{e^{-n\lambda_b^B}(n\lambda_b^B)^k}{(k)!} * \frac{e^{-n\lambda_b^A}(n\lambda_b^A)}{(n\lambda_b^A)!} \tag{22}$$

$$piv_c^{BC} = \frac{e^{-n\lambda_c^C}(n\lambda_c^C)^{k+1}}{(k+1)!} * \frac{e^{-n\lambda_c^B}(n\lambda_c^B)^k}{(k)!} * \frac{e^{-n\lambda_c^A}(n\lambda_c^A)}{(n\lambda_c^A)!} \tag{23}$$

So, $piv_c^{BC} = piv_b^{BC} * \frac{\lambda_c^C}{\lambda_b^C}$. Because of the expected fractions of votes, it is slightly (in terms of a large Poisson game) more likely in state c than in state b that alternative C has one more vote than alternative B.

We can now prove that σ^* indicated above is indeed and equilibrium.

Case 1: A voter receives signal b.
Considering only the most relevant pivotal events, $EU(B|b) = \pi_b(b)piv_b^{BC} - \pi_c(b)piv_c^{BC}$ and $EU(C|b) = \pi_c(b)piv_c^{CB} - \pi_b(b)piv_b^{CB}$. Consider first $EU(B|b)$. This ballot yields positive expected utility if $\pi_b(b)piv_b^{BC} > \pi_c(b)piv_c^{BC}$, or if $\pi_b(b)\lambda_b^C > \pi_c(b)\lambda_c^C$, or if $(\frac{1}{3}+\epsilon)\varphi_b(b)\lambda_b^C > (\frac{1}{3}-\epsilon)\varphi_c(b)\lambda_c^C$, which is true. Now consider $EU(C|b)$. This ballot yields negative expected utility if $\pi_c(b)piv_c^{CB} < \pi_b(b)piv_c^{CB}$, or if $\pi_c(b) < \pi_b(b)$, or if$(\frac{1}{3}-\epsilon)\varphi_c(b) < (\frac{1}{3}+\epsilon)\varphi_b(b)$, which is true. So, this voter receives positive expected utility from ballot B and negative utility from ballot C. In addition, the expected utility from ballot B is larger than the expected utility from ballot A because of the ranking of pivotal events. Therefore, voting B is a best response for this voter.

Case 2: A voter receives signal c.
Considering only the most relevant pivotal events, $EU(B|c) = \pi_b(c)piv_b^{BC} - \pi_c(c)piv_c^{BC}$ and $EU(C|c) = \pi_c(c)piv_c^{CB} - \pi_b(c)piv_b^{CB}$. We use similar arguments as in Case 1 to show: $EU(C|c) > 0$ because $\pi_c(c) > \pi_b(c)$, or because $(\frac{1}{3}-\epsilon)\varphi_c(c) > (\frac{1}{3}+\epsilon)\varphi_b(c)$, which is indeed true for sufficiently small ϵ. And $EU(B|c) < 0$ because $\pi_b(c)piv_b^{BC} < \pi_c(c)piv_c^{BC}$, which is indeed true for sufficiently small ϵ. For similar reasons as above, a voter with signal c also does not want to submit ballot A rather than ballot C. So, the best response of a voter of type c is to vote C.

Case 3: A voter receives signal a.
Considering only the most relevant pivotal events, $EU(B|a) = \pi_b(a)piv_b^{BC} - \pi_c(a)piv_c^{BC}$ and $EU(C|a) = \pi_c(a)piv_c^{CB} - \pi_b(a)piv_b^{CB}$. Clearly, $EU(B|a) < 0$ because $\pi_b(a)\lambda_b^C < \pi_c(a)\lambda_c^C$, which is indeed true for sufficiently small ϵ. And $EU(C|a) < 0$ because $\pi_c(a) < \pi_b(a)$, which is true as well. So, ballots B and C both yield negative expected utility for this voter. Recall from Section 2.2 that a voter for whom the consideration of the most likely pivotal events leads to negative expected utility must consider the remaining ballot A. With the ranking of magnitudes, $EU(A|a) = \pi_a(a)piv_a^{AB} + \pi_a(a)piv_a^{AC} > 0$. The best response of a voter of type a is to vote for A. □

In the equilibrium presented in the proof of Theorem 3, voters vote informatively. In this equilibrium, the most likely pivotal events occur in states b and c, and they have the same magnitude, so that their probabilities are not too different in a large Poisson game. So, being pivotal is no overwhelming evidence to help discriminate between states b and c, and voters also need consider their posterior beliefs about the states of nature $\pi_x(z)$ as additional information. Receiving signal b is stronger evidence for state b, so a voter with signal b votes for alternative B. Similarly, a voter with signal c votes for C.

According to her posterior beliefs, a voter with signal a believes states b and c to be almost equally likely. This type of voter finds him- or herself in a situation similar to the swing voter's curse (Feddersen and Pesendorfer (1996)). Both ballots B and C lead to the wrong outcome almost as likely as to the right outcome. So, this voter considers ballot A instead because it yields positive expected utility.

Because voters vote informatively, but signal a is less likely in state a than signals b or c, the equilibrium is inefficient. However, voters with signals b and c do not consider state a because they are less likely to be pivotal in this state than in both states b and c. This type of coordination failure cannot happen with two alternatives, so a similar inefficient equilibrium does not exist.

Notice also that in the equilibrium the expected utilities of voters from voting for the respective ballots is always larger than zero, so the same equilibrium would also exist if one allowed for abstention. Of course, the economy in the proof also has an efficient equilibrium according to Theorem 1.

If voters vote informatively, the voting outcome accurately reflects the private information held by the population. But since Eq. (1) does not require that signals are more likely correct than incorrect, an informative vote is not desirable because it cannot efficient. Nevertheless, we have just shown that with three alternatives, it can be a Nash equilibrium.

The inefficient equilibrium in proof is quite different from those in Goertz and Maniquet (2009) and Goertz and Maniquet (2011). In their inefficient equilibria, voters vote responsively, but not informatively. In addition, these equilibria are very particular because voters with common preferences have drastically different posterior beliefs about the states of nature (some consider certain states of nature to occur with zero probability, while others consider the same states as likely) that lead to coordination failures among them. The inefficient equilibrium above shows that posterior beliefs do not need to be drastically different and that we do not need partisan voters to find coordination failures among voters with common preferences if there are more than two alternatives.

Unfortunately, more general results for elections with three alternatives are currently beyond reach. Recall, for example, Theorem 2, a general theorem about two-alternative elections. In the proof, we use the argument that voter type x is never less likely to vote for alternative X than type y, and that at most one type mixes. The same statements cannot be made for three alternatives. The formal reasoning for this is taken from Goertz and Maniquet (2011). Consider, similar to the argument the proof of Theorem 2, a voter voting for alternative Y when receiving signal s. Then

$$\pi_y(s)piv_y^{YX} + \pi_y(s)piv_yYZ - \pi_x(s)piv_x^{YX} - \pi_z(s)piv_z^{YZ} \geq 0. \tag{24}$$

Now consider signal s', which makes state y more likely. Then we *cannot* conclude that

$$\pi_y(s)piv_y^{YX} + \pi_y(s)piv_yYZ - \pi_x(s)piv_x^{YX} - \pi_z(s)piv_z^{YZ} \geq 0. \tag{25}$$

With three alternatives, it is not true that $\pi_y(s) < \pi_y(s')$ leads to both $\pi_x(s) > \pi_x(s')$ and $\pi_z(s) > \pi_z(s')$. Therefore, Eq. (25) can very well be less than zero (in which case type s' is less likely to vote for alternative Y than type s); or both Eqs. (24) and (25) can be zero, so that both types mix.

The fact that strategies lose a certain monotonicity in signals is detrimental for the general analysis of elections with three alternatives. Without any lead about general properties of strategies, it is very hard to create constructive proofs and not likely that one can prove general results.

5 Conclusion

This paper shows that new types of coordination failures arise in elections with more than two alternatives. These inefficiencies are caused by failures to coordinate votes if voters with common preferences have imprecise private signals about the state of nature.

There are efficient equilibria in elections with both two and three alternatives. However, with two alternatives, there is only one type of inefficient equilibrium in which all voters vote unresponsively (for the same alternative) because they consider the same state of nature to be more likely and therefore disregard their private signals. The same type of inefficient equilibrium also exists with three alternatives. On the other side of the spectrum, there are also inefficient equilibria with three alternatives in which voters vote entirely informatively (only for their signal). The voting outcome now correctly reflects the private information held by the electorate, but the voting outcome is inefficient nevertheless.

While more general are currently beyond reach (discussion in Section 4), there can be no doubt that increasing the number of alternatives beyond two increases the number (and types) of possible coordination failures between voters immensely and that inefficient equilibria become more and more common (almost certainly others besides those in Theorems 2 and 3 exists). This should in particular be true for environments in which voters do not have dichotomous preferences.

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