

Social Ideology and Taxes in a Differentiated Candidates Framework

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Abstract

How does ideological polarization on non-economic matters influence the size of government? We analyze this question using a differentiated candidates framework: Two office-motivated candidates differ in their (fixed) ideological position and their production function for public goods, and choose which tax rate to propose. We provide conditions under which a unique equilibrium exists. In equilibrium, candidates propose different tax rates, and the extent of economic differentiation is influenced by the distribution and intensity of non-economic preferences in the electorate. In turn, the extent of economic differentiation influences whether parties divide the electorate primarily along economic or social lines.

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1 Introduction

It is well known that the two major political parties in the U.S. differ significantly in their positions on both social issues as well as economic issues. As a consequence, both economic preferences as well as "ideology", which we take to mean preferences on cultural issues, influence voting behavior.

For example, Table 1 displays information from California voter exit polls in the 2008 elections. Rows correspond to information on how a voter voted on Proposition 8, a ballot measure that would outlaw gay marriage (so yes-votes are by "social conservatives"). Columns correspond to a voter's household income in the 2007. Entries in the cells are Obama's share of the two party vote for President (i.e., $\frac{\text{votes for Obama}}{\text{votes for Obama or McCain}} \times 100\%$).

	income < \$50000	≥ \$ 50000
YES on Prop. 8	41%	36%
NO on Prop. 8	90%	86%

Table 1: Cultural and economic determinants of voting behavior

The attitude toward gay marriage is a useful proxy for preferences on social policy only, as the economic effect of Proposition 8 is very limited. Household income is a plausible proxy for preferences on economic policy and the scope of government. Table 1 indicates very clearly that both economic and ideological factors influence a person's vote for an office such as the presidency that combines a role in economic policy with a strong influence on social issues (for example, via judicial appointments). Social conservatives (i.e., yes-votes on Proposition 8) are substantially less likely to vote for Obama than social liberals (by about 50 percentage points), and poorer voters are more likely to vote for Obama than richer ones (by about 5 percentage points).²

¹Data from National Election Pool state exit poll for California, available from the Roper Center (http://www.ropercenter.uconn.edu/elections/common/state_exitpolls.html)

²Of course, neither category can be expected to be a perfect measure of preferences in the respective policy area: There are several other social policy questions such as abortion or gun rights on which the two parties differ substantially and which may influence a voter's ideological preference for one of the parties' social policy positions. Similarly, a voter's economic interests in an election are not only determined by household income in any given year in the past, but also by his expectations about future income, household size and composition (as this influences both how much taxes a voter has to pay and, presumably, his consumption of public goods) and age.

This voter behavior is plausible if parties/candidates differ in both the economic and the social-cultural policies that they would implement if elected. If parties are completely identical (both on economic and social issues), then voter behavior should be random. If parties differed only in one policy area, (e.g., the social position as is the case in the probabilistic voting model, where both candidates choose identical economic platforms) voter choices should only be determined by their preferences on that area, but independent of their preferences on the other policy. Note that, if social and economic preferences are correlated (e.g., wealthier voters are more likely to be socially conservative) then voting behavior is correlated although there is no causal relation. However, once we condition on both income and social preference, as we do in Table 1, then the remaining differences must be based on causation rather than correlation.³

The extent of the policy difference between parties in each policy area will influence how strongly different preferences on a dimension translate into different voting behavior. For example, in "What's the matter with Kansas", Frank (2004) argues that Democrats' economic policy has become very similar to Republican economic policy, causing many voters who would be "natural" Democratic partisans instead follow their culturally conservative leanings and vote Republican. Similarly, the exit poll data in Table 1 appear to indicate that cultural determinants of voting behavior have a quantitatively stronger effect than economic ones.

From an economic point of view, this raises an important question: How does ideological polarization on social issues affect economic policy? While both economic and ideological factors interact in determining a voter's choice between candidates, the standard models in political economy are ill-equipped to analyze these questions. If the simple one-dimensional policy model is interpreted as one of economic policy, there is, by definition, no ideological dimension, and voters split according to their economic preferences even if there is only slight differentiation between the economic platforms proposed by the candidates.

The probabilistic voting model accommodates both an "economic" dimension on which candidates choose a policy and an "ideological" dimension which is an additive shock to the utility of voters and can be thought of as arising from cultural issues on which the candidates' positions differ. However, in the equilibrium of the standard probabilistic voting model, both candidates always propose the same economic policy, and thus the voters' preference for one of the candidates

³In fact income and the position on Proposition 8 are correlated: 44.7% of voters with income less than \$50,000 voted in favor of Proposition 8, while the percentage increased to 56.2% among those with incomes exceeding \$50,000, indicating that wealthier voters are more socially conservative. However, the numbers given in table 1 are conditioned on both income *and* social preferences, and are therefore unaffected by any correlation between the two.

is *only* determined by their ideological position and not by their economic characteristics. The data reported in Table 1 suggest that this prediction is not entirely correct, and clearly, the reason is that real life Democratic and Republican candidates differ not only in ideological positions, but also in economic policy platforms.

The main question of this paper is how ideological polarization affects the economic platforms of the parties. We approach this issue in a framework where candidates have both fixed and flexible positions. We think of the fixed position as reflecting ideological differences that the candidates do not want to or cannot credibly compromise on, just like in the citizen-candidate model. However, just like in the Downsian model, candidates in our model are office motivated, and choose position on economic policy to maximize their winning probability.

The advantage of our framework is twofold. First, since both candidates' immutable positions on social issues and their equilibrium platforms on economic issues differ, voters choose their preferred candidate based on both economic and ideological issues: Social conservatives who happen to be sufficiently keen on government spending may vote for the Democrat, and social liberals who are sufficiently opposed to high taxation may vote for the Republican. Second, candidates compete for voter support by choosing economic platforms, taking as given their ideological differences and the preference distribution in the population. Within our framework we can think of *polarization* as a measure of preference intensity on the ideological component. We analyze how increasing ideological polarization translates into changes of economic policy. In addition, we can consider the effects of shifts in the ideological composition of the electorate (say, an increase in the number of social conservatives), as well as changes in the economic preference distribution (either allowing for an on average higher demand for public goods, or for more polarization of economic preferences).

Our main results are as follows. We first show that an equilibrium is characterized by two cutoff voter types, one for each ideological type. Cutoff voters are indifferent between candidates and therefore must strictly prefer the economic platform of the candidate whose ideological position they dislike. Thus, the socially liberal cutoff voter is in favor of less government spending than the socially conservative cutoff voter. (Note that this is only true for the *cutoff* voter. It may well be the case that, on average, social conservatives prefer lower tax rates than social liberals.)

What matters for the candidates' position choice are these potentially swingable cutoff voters. In equilibrium, candidates propose tax rates that are intermediate between the rate preferred by the social liberal cutoff voter and the one preferred by the social conservative cutoff voter. A candidate

who marginally increases his proposed tax rate gains votes among social conservatives, but loses some liberals, and those gains and losses exactly balance in equilibrium for each candidate. Note that the statement that more government spending increases the set of conservatives who vote for the candidate does *not* imply that higher tax rates are on average popular with social conservatives as a group. Clearly, at least some social conservatives (and quite possibly a majority of them) dislike higher taxes, but those are not the swing voters that the candidates focus on.

Taking the opponent's tax rate as given, varying a candidate's tax rate generates a curve of cutoff voter pairs in a two-dimensional space, and a candidate chooses the best cutoff voter pair from this curve. We show that, in equilibrium, the two candidates' curves are tangent to each other at the equilibrium-induced cutoff voter pair. They are also tangent to an isoprobability curve, i.e. a curve that connects all those cutoff voter combinations that lead to the same winning probability for the Democrat.

We provide sufficient conditions for an equilibrium to exist and to be unique. The graphical characterization of the equilibrium described above can be used to study the comparative statics properties of the equilibrium, because it is relatively easy to characterize how parameter changes affect isoprobability curves.

We show that any parameter change induces the candidates to change their respective platforms in the same direction. That is, changes in cultural polarization (either in the number of liberals or conservatives, or in the intensity with which they care about non-economic issues) either lead to an increase of both the Democratic and the Republican tax rate, or to a decrease of both of them. This appears consistent with the observed recent movement of both parties' economic platforms to the right.⁴

If there are more socially conservative voters, or if socially conservative voters' emphasis on cultural issues increases, then both candidates propose more government spending, but the small-government candidate's winning probability increases. The opposite conclusions hold when there are more socially-liberal voters, or if they care more about cultural issues.

Finally, we also provide a comparative static analysis of an increase in economic polarization. Interestingly, the results here depend on the ideological composition of the electorate. If there are more social conservatives than social liberals (which appears to be the case relevant for the United

⁴For example, the health care plan of the Republican party in the 1990s involved subsidies for low income households and an individual mandate to buy health insurance. A similar plan was eventually passed in 2010 by a Democratic Congress, and against strong Republican opposition.

States), then an increase in income inequality leads to less government spending. We do not explicitly model feedback effects in a dynamic setting, but this is clearly a possibility here: More inequality leads to less government spending, and less government spending in certain areas such as education may itself lead to a more unequal income distribution, further depressing government spending, and so on. In contrast, if there are more social liberals than social conservatives, then an increase in initial inequality leads to more government spending, potentially diminishing inequality in the future. Thus, in our model, the distribution of cultural ideology in the electorate may have an important effect on how a society reacts to initial economic shocks, and whether such a reaction reinforces or mitigates the initial shock.

2 Related literature

Our model is based on the general differentiated candidates framework developed in Krasa and Polborn (2009a, 2010b, 2010a), in which the two competing candidates have some characteristics that cannot be changed, but choose a position (or "policy") in order to maximize their respective probability of winning. Voters' utility depends on both fixed characteristics and flexible policies. In this model, candidates are differentiated with respect to ideology and their ability to provide public goods, with one of the candidates having an advantage in providing a large quantity of public goods, while his opponent has an advantage in providing a lean government. Both candidates choose a tax rate in order to maximize their respective winning probability.

The advantage of the differentiated candidates framework relative to a standard probabilistic voting model (PVM) is that there is complete policy convergence in the PVM (i.e., in any equilibrium, both candidates choose the same economic policy), and thus, voting behavior is determined only by the voters' position on the "ideological" dimension in which candidates are exogenously fixed. Any observed influence of economic factors on voting behavior would have to stem from ideologically fixed positions that influence the utilities of rich and poor voters differentially.

The advantage of our model relative to a citizen-candidate model (in which candidates are fixed to their "ideal position" in every policy area) is that there is a unique equilibrium in our model, and that we can relate changes in ideological polarization of the electorate to changes in the economic policies proposed by the candidates.⁵

⁵The citizen-candidate framework can handle multidimensional policy spaces without fundamental difficulties (Osborne and Slivinski 1996, Besley and Coate 1997). However, there are generally very many equilibria that only share

In a standard one-dimensional spatial model, equilibrium policy depends only on the ideal policy position of the median voter, but is independent of the higher-order moments of the distribution of voter preferences.⁶ Lindqvist and Östling (2010) find empirical evidence that a larger degree of preference polarization is associated with a smaller size of government, but the theoretical basis for this effect remains a bit unclear.

There are a number of papers that use different variations on the spatial model to analyze how increasing diversity of voter preferences affects the size of government. Austen-Smith and Wallerstein (2006) analyze how, in a legislative bargaining model, general redistribution is affected by the existence of racial preferences. Lizzeri and Persico (2001, 2004) analyze the incentives of politicians for redistribution under different electoral systems and show that expanding the set of citizens who are eligible to vote may induce candidates to change their equilibrium platforms from patronage policies towards policies that have more general benefits. Somewhat relatedly, Fernández and Levy (2008) develop a model in which all poor voters prefer general redistributive taxation, but have conflicting interests regarding a number of local public goods that are beneficial only for a subset of them. They show that this setup leads to a non-monotonic relationship between preference fragmentation and redistribution. Preference diversity in all of these models is "economic", i.e., politicians have different types of economic policies (such as general and targeted redistribution) at their disposal, voters are interested in both general interest and (some) special interest policies, and they only care about their total economic benefit from the bundle of policies that are enacted by the election winner. In contrast, our model has a simpler economic policy (as it contains only the choice of one parameter, the tax rate), but it analyzes how this choice is affected by preference diversity in non-economic dimensions, which are non-existent in these models.

Roemer (1998) analyzes a model in which, like in our model, voters care about economic policy and about government policy along a non-economic dimension. Parties are considerably more complex in Roemer's model: They consist of some members who want to maximize the probability of winning the election and some who want to maximize the expected (policy) utility of particular party members. Since party positions on both dimensions are flexible, it is necessary to introduce this modeling of parties in order to ensure the existence of an equilibrium. Under

the property that both candidates always receive the same number of votes. Just like in the one-dimensional setup, the citizen candidate model imposes few restrictions on which policies can arise in equilibrium. Thus, no useful comparative static analysis with respect to social polarization is possible in that framework.

⁶In Meltzer and Richard (1981), a classical political-economy model of redistribution, the income distribution in society matters, but only to the extent that it influences the median voter's preferences for redistribution.

certain conditions, Roemer finds that a higher weight on non-economic policy in the voters' utility function decreases the optimal tax rate for the party that prefers more redistribution.

The topic of economic and social polarization has also attracted considerable interest in political science. McCarty, Poole, and Rosenthal (2006) show that there is considerable correlation between the development of economic inequality in the U.S. as measured by the Gini coefficient and a measure of polarization between Democrats and Republicans in the U.S. Congress. They argue that the relationship is causal: Increased economic inequality has caused the parties to choose more polarized economic platforms. However, they do not present a formal model that generates this prediction. Also the empirical conclusion that polarization is primarily along economic issues is not uncontested (see Green, Palmquist, and Schickler (2002), Lee and Roemer (2006)). Our model takes non-economic policy differences between parties as given, and analyzes how economic platforms are affected by them.

3 Model

3.1 Description of the model

Two candidates, j = D, R, compete in an election. There are two major components of policy, which Stokes (1963) calls "position issues" and "valence issues". Position issues are ideological issues such as abortion or gun control, and candidates are exogenously committed to differentiated positions; due to their own history or their party label, they cannot credibly change this position. Voters have different ideal positions on the position issue. In contrast, the valence issue is related to the management of public good provision by the office holder, and all voters prefer ceteris paribus (i.e., if costs of implementing the policy are not taken into account) a higher provision level. Candidates propose a tax rate and will then use the tax revenue to provide a public good. Candidates differ in their production function, so that, in addition to the tax rate, the identity of the office holder also matters for the quantity of public goods produced.

The modeling of the valence issue follows Krasa and Polborn (2009b). Candidate j proposes a tax rate t_j , which is applied to the average income of the population, \bar{m} , normalized to 1. Thus, tax revenue if candidate j is elected is t_j and is used to pay for government fixed cost and for the provision of a public good g. The ability to provide the public good differs among candidates, and is given by an affine linear production function, $g_j = f_j(t_j) = a_j t_j - b_j$. We analyze situations in

which candidate R has an advantage with respect to fixed cost b, while his opponent D has a higher marginal product in public good provision. Formally,

Assumption 1. Let $a_R < a_D$ and $b_R < b_D$.

The candidates' positions on the ideological position issue are fixed. Because there are only two candidates, we can, without loss of generality, assume that $q \in \{L, C\}$, where q = L ("liberal") for the Democrat and q = C ("conservative") for the Republican.

Individual voters' preferences depend on public good consumption g, their private good consumption x (determined by the tax rate), and the ideological position of the elected candidate. Formally, the utility function of a voter of type $\tau = (\eta, m, p) \in T$ is $u_{\tau}(x, g, q) = x + \eta w(g) + v_{\tau}(p, q)$, where x is the voter's private consumption; g is public consumption; w is increasing, strictly concave and differentiable, and satisfies $\lim_{x\to 0} w'(x) = \infty$ and $\lim_{x\to \infty} w'(x) = 0$ — different preferences over public good consumption are reflected in the parameter $\eta \in \mathbb{R}$, with high η -types having a stronger preference for public goods. Finally, $v_{\tau}(p,q)m$ is a measure of the ideological (dis)utility. We assume that

$$v_{\tau}(p,q) = \begin{cases} \delta m & \text{if } p = L, \ q = D \\ \rho m & \text{if } p = C, \ q = R \\ 0 & \text{otherwise} \end{cases}$$
 (1)

The parameter δ in (1) captures the ideological benefit, expressed as a percentage of income, that liberals get if the Democrat (rather than the Republican) is elected, and ρ is the same for conservatives if the Republican candidate is elected.⁷ Note that the assumption that $v_{\tau}(L,R) = v_{\tau}(C,D) = 0$ is without loss of generality because for each voter type p = L, C we have one free normalization.⁸

⁷Multiplying δ and ρ by m has the effect that the "willingness to pay" for the ideologically preferred candidate is linear in income. If, instead, this willingness to pay was constant in income, then economic concerns would trump social issues for all sufficiently wealthy individuals, and only sufficiently poor individuals vote based on ideology. The reverse would be true, i.e., wealthy citizens would only vote based on ideology if ρ and δ are multiplied with a function of m that increases at a faster than linear rate. Our assumption provides a middle ground between these two cases and simplifies the analysis — the analysis can be extended to more general cases at the cost of the two-dimensional geometric presentation provided below.

⁸Equation (1) can be derived from a spatial representation $v_{\tau}(p,q) = -|p-q|m$ (i.e., a voter's utility decreases linearly with the distance between the voter's ideal position p and the candidate's position q, and the equivalent variation for having one's ideologically favorite candidate elected is a constant fraction of the voter's income) by

At the time when candidates choose their platforms, they are uncertain about the distribution of types τ . Specifically, there is a state of the world $\omega \in \Omega$, distributed according to the cumulative distribution function β . Given ω , type $\tau \in T$ is distributed according to ν_{ω} . Let S_j be the set of all types who vote for candidate j. Then, candidate j's winning probability is given by

$$\Pi_j = \int_{\Omega} \int_{T} \xi(\nu_{\omega}(S_j)) \, d\beta(\omega),$$

where

$$\xi(x) = \begin{cases} 0 & \text{if } x < 0.5; \\ 0.5 & \text{if } x = 0.5; \\ 1 & \text{if } x > 0.5. \end{cases}$$

The timing of events is as follows:

Stage 1 Candidates j = D, R simultaneously announce tax rates $t_j \in [0, 1]$. Candidates are office-motivated (they receive utility 1 if elected, and utility 0 otherwise, independent of the implemented policy), so that their objective is to maximize their respective winning probabilities Π_D and Π_R .

Stage 2 Nature draws ω , which determines the distribution of voter preferences τ in the electorate. Each citizen votes for his preferred candidate, or abstains when indifferent. The candidate with a majority of votes wins, collects taxes and provides the public good.

3.2 Discussion of modeling choices

Differential candidate capabilities. A key assumption of the differentiated candidates model is that candidates have differential abilities, with one candidate better at providing limited government, while the other candidate is better than his competitor for large expenditures. While non-standard, this assumption appears eminently reasonable. Economists agree that workers or firms differ in their productivities, and this fact is evident as output can easily be measured in many

normalizing accordingly. Note that $\delta \neq \rho$ corresponds to cases where the distance between the Republican candidate's ideological position and the ideal position of conservatives is different from the distance between the Democrat's ideological position and the ideal position of liberals.

⁹If a voter has a strict preference, then it is a weakly dominant strategy to vote for the preferred candidate. If a voter is indifferent, he could in principle vote for any candidate or abstain, and we assume that he abstains.

private sector occupations. In contrast, the "output" of politicians in terms of public good production is significantly more difficult to measure, and thus it is tempting to use expenditures on inputs as a proxy measure for the quantity of the public good supplied. However, in reality, citizens derive utility, for example, from the quality of education in state schools and not *per se* from the money spent on education. Thus, when two competing candidates propose to spend the same amount of money on schools, this does not mean that both of them would produce the same quality of service for citizens if elected. Our model formalizes this notion.

There are several different interpretations of the candidates' differentiated production possibilities. First, there is a widespread notion that Republicans have an advantage when it comes to running a small government. For example, Egan (2008) demonstrates that Republicans have a long-run public opinion advantage over Democrats on the issue of "taxes", while simultaneously a majority of people say that they trust Democrats more than Republicans on large expenditure issues such as education and health care. Of course, revenues and expenditures are two sides of the same coin. Our preferred interpretation of these opinion polls is therefore that (many) people think that the advantage of a Republican government is that it is better in taking care of taxpayer dollars by trimming government spending to a minimum, a task in which Democrats may be hampered, for example by their connections to unions of government workers. On the other hand, Democrats are preferable for delivering a high level of public good service.

A difference between political parties can also arise as a consequence of specialization on different policy areas: Republicans may be specialized in the efficient provision of services such as law enforcement that are "basic" in the sense that every government – whether Democratic or Republican – has to provide them, while the Democrats' efficiency advantage lies in the provision of "optional" services (i.e., services that could, but need not be provided by the government) such as, for example, government provision of health care.

Alternatively, suppose that learning-by-doing increases the incumbent's marginal productivity over his challenger's one. However, incumbency also leads to entrenchment, so if the next office holder were charged with reducing bureaucracy and government spending, it may well be the case that the challenger is better able to achieve this objective.

Ideology. Economists tend to focus on economic issues as the central field of conflict in political competition. Specifically, in most political economy models, candidates choose a policy that is interpreted as a tax rate, and voters split over candidates according to their economic preferences.

In our opinion, this view is only half-right.

We agree that economic issues are the main flexible position for candidates: While it may be very difficult for a candidate to credibly change a position on a position issue such as abortion, the death penalty or gun control, there are no comparable constraints that prevent a politician who favored a 5 percent sales tax in a previous campaign to credibly advocate a 6 percent or a 4 percent rate in the current campaign. A reason for this difference is also that the optimal economic policy (for any preference type) depends on the state of the economy and thus naturally changes over time, while one's view of the desirability of gay marriage or abortion restrictions is more likely to be fairly constant over time.

The economic policy platforms of Republican and Democratic candidates usually differ in a non-trivial way, but, while economic positions clearly influence the voting choice of some voters, economic interests are far from being a perfect predictor of voting behavior. For example, according to the exit polls of the 2008 U.S. presidential election, voters making less than \$100000 went 55-43 for Obama over McCain, while they split voters making more than \$100000 49-49. This is a significant, but not overwhelmingly large difference. Non-economic social issues play a role for voting choices that is at least as important, and probably more important than economic interests. Whether a voter regularly goes to church (a proxy for attitudes towards social issues) is a strong predictor of voting intentions. For example, according to the exit polls of the 2008 U.S. presidential election, voters who attended church weekly went for McCain 55-43, while occasional church-goers went for Obama 57-42, and those who never go to church went for Obama 67-30.

These results indicate that we need a theory of candidate competition and voting that accommodates the strong role of non-economic issues on voting behavior, and helps us understand how ideological issues influence the positions that candidates take on economic issues.

Uncertainty about the voter preference distribution. Including uncertainty about the voter distribution has two objectives. First, it appears quite realistic to assume that the preference distribution in the electorate is not precisely known and that candidates have to make their choices under some uncertainty. Second, the assumption helps us to refine the set of equilibria. If the distribution of voters is known with certainty and candidate payoffs depend only on whether they win (rather than vote share), then, generically, there are many equilibria. The reason is that one candidate usually wins for sure, and thus, the policy choice of his opponent is indeterminate. Also,

¹⁰Available at http://www.cnn.com/ELECTION/2008/results/polls/#USP00p1.

the better candidate can win with a whole set of policies. Therefore, many strategies could be part of an equilibrium. Assuming uncertainty about the voter preference distribution eliminates most of these equilibria.

Ricardian equivalence. In our model, all government expenditures have to be financed by contemporaneously raised taxes, and we therefore use "higher taxes" and "more government spending" as synonymous. When the government can run a deficit, taxes and spending need not be the same in any given year, but Ricardian equivalence suggests that current government spending is the appropriate measure for the taxes that have to be raised either today or in the future to finance today's government spending. We would therefore interpret periods in which government spending increased as a percentage of GDP (such as Reagan's or George W. Bush's presidency) as periods of "higher taxes", even if nominal tax rates remained constant or even declined, while the shortfall was made up by a deficit.

4 Equilibrium

Substituting candidate j's proposed tax rate t_j into the utility function of a type τ voter, we get indirect utility $u_{\tau}((1-t_j)m, g_j, q_j) = (1-t_j)m + \eta w(g_j) + v_{\tau}(p, q_j)$. Dividing by m, we have

$$\frac{u_{\tau}((1-t_{j})m, g_{j}, q_{j})}{m} = (1-t_{j}) + \theta w(g_{j}) + \begin{cases} \delta & \text{if } p = L, \ q = D \\ \rho & \text{if } p = C, \ q = R \\ 0 & \text{otherwise} \end{cases}$$
 (2)

where $\theta = \eta/m$. With utility written in this form, the relevant type space is $\Theta \times \{L, C\}$. Distribution Φ_{ω} defines a cumulative distribution function $G_{\omega}(\theta_L, \theta_C)$.

We now show that a candidate's supporters consist of θ -types below or above a cutoff, where the location of the cutoff depends on the ideological type. Formally, the sets of a candidate's supporters are of the form $\{L\} \times (-\infty, \theta_L] \cup \{C\} \times (-\infty, \theta_C]$, or $\{L\} \times [\theta_L, \infty] \cup \{C\} \times [\theta_C, \infty)$. In the following, denote the amount of public goods provided by the two candidates by $g_D = f_D(t_D)$ and $g_R = f_R(t_R)$, respectively. Furthermore, let $v(p,q) \equiv v_\tau(p,q)/m$ denote the last term in (2). A voter of type (θ, p) prefers candidate D over candidate R if and only if

$$(1 - t_D) + \theta w(g_D) + v(p, D) \ge (1 - t_R) + \theta w(g_R) + v(p, R). \tag{3}$$

(3) is equivalent to

$$\theta \ge \frac{t_D - t_R + v(p, R) - v(p, D)}{w(q_D) - w(q_R)},\tag{4}$$

if $w(g_D) - w(g_R) > 0$, and the inequality changes its sign if $w(g_D) - w(g_R) < 0$.

Recall that $\delta = v(L, D)$ and $\rho = v(C, R)$. Then

$$\theta_L^* = \frac{(t_D - t_R) - \delta}{w(g_D) - w(g_R)},\tag{5}$$

$$\theta_C^* = \frac{(t_D - t_R) + \rho}{w(g_D) - w(g_R)} \tag{6}$$

are the voter types that are indifferent between the candidates. Higher types vote for the candidate who offers more public goods, while lower types support the other candidate. More formally, if $g_D > g_R$, then $w(g_D) > w(g_R)$ and candidate R receives the votes of all liberal voters with $\theta \le \theta_L$ and of all conservative voters with $\theta \le \theta_C$. Candidate R's winning probability is derived by integrating $\xi(G_{\omega}(\theta_L, \theta_C))$ with respect to the distribution of the state of the world ω , i.e.,

$$G(\theta_L, \theta_C) = \int_{\Omega} \xi(G_{\omega}(\theta_L, \theta_C)) \, d\beta(\omega). \tag{7}$$

If, instead, $g_D < g_R$ then candidate R receives the support of all voters (θ, P) where $\theta > \theta_p^*$, p = L, C, and the winning probability is $1 - G(\theta_L^*, \theta_C^*)$. The situation is reversed for candidate D, i.e. if $g_D > g_R$ then D's winning probability is $1 - G(\theta_L^*, \theta_C^*)$, else if $g_D < g_R$ then D's winning probability is $G(\theta_L^*, \theta_C^*)$.

Finally, suppose that $g_D = g_R$. Then (3) simplifies to $v(p, D) - t_D \ge v(p, R) - t_R$, i.e., the equation is independent of θ . As a consequence, all voters with ideology p either vote for the same candidate, or if $v(p, D) - t_D = v(p, R) - t_R$ then they are indifferent between candidates.

For the moment, focus on the case that the Democrat provides more public goods than the Republican. Within each ideological group, the highest types then vote for the Democrat, while the lowest types vote Republican. Because ideological partisans get an additional payoff from the election of their closer candidate, the cutoff voter type among conservatives, θ_C , is larger than the cutoff voter type among liberals, θ_L . This follows directly from (5) and (6). Intuitively, the social conservative who is indifferent between the Democrat and the Republican candidate is so because his preference for the Democrat's economic platform just counterbalances his cultural preference for the Republican; but a voter who prefers the Democrat's economic platform is someone with a preference for high public good provision (i.e., a voter with a relatively high θ). By an analogous argument, the culturally liberal cutoff voter is economically quite conservative (i.e., a low θ -type).

For a more formal analysis, we need the derivatives of θ_L^* and θ_C^* with respect to t_D and t_R , taking into account that g_D and g_R are functions of t_D and t_R , respectively.

$$\frac{\partial \theta_L^*}{\partial t_D} = \frac{(w(g_D) - w(g_R)) - ((t_D - t_R) - \delta) a_D w'(g_D)}{(w(g_D) - w(g_R))^2},\tag{8}$$

$$\frac{\partial \theta_C^*}{\partial t_D} = \frac{(w(g_D) - w(g_R)) - ((t_D - t_R) + \rho)a_D w'(g_D)}{(w(g_D) - w(g_R))^2},\tag{9}$$

$$\frac{\partial \theta_L^*}{\partial t_R} = -\frac{(w(g_D) - w(g_R)) - ((t_D - t_R) - \delta)a_R w'(g_R)}{(w(g_D) - w(g_R))^2},\tag{10}$$

$$\frac{\partial \theta_C^*}{\partial t_R} = -\frac{(w(g_D) - w(g_R)) - ((t_D - t_R) + \rho)a_R w'(g_R)}{(w(g_D) - w(g_R))^2},\tag{11}$$

Comparing (8) with (9), and (10) with (11), shows that $\frac{\partial \theta_L^*}{\partial t_D} > \frac{\partial \theta_C^*}{\partial t_D}$ and $\frac{\partial \theta_L^*}{\partial t_R} < \frac{\partial \theta_C^*}{\partial t_R}$. If, for example, $\frac{\partial \theta_C^*}{\partial t_D} > 0$, then the Democrat (who receives the support of all voters above the cutoffs) can move both cutoffs further to the left, and increase his vote share by decreasing taxes t_D . If, instead, $\frac{\partial \theta_L^*}{\partial t_D} < 0$, then the Democrat can gain votes by proposing higher taxes. In an optimum, raising taxes must create a trade-off between losing some socially liberal but fiscally conservative voters and gaining some social conservatives who prefer a higher consumption of public goods. Thus, in equilibrium, $\frac{\partial \theta_L^*}{\partial t_D} < 0 < \frac{\partial \theta_L^*}{\partial t_D}$. A similar argument shows that, in equilibrium, $\frac{\partial \theta_L^*}{\partial t_R} < 0 < \frac{\partial \theta_C^*}{\partial t_R}$.

In order to determine the equilibrium cutoff, it is useful to investigate how the candidates' tax rates affect the cutoff types in a $\theta_L - \theta_C$ diagram. We first define functions k_D and k_R that map the respective candidate's tax rate into a curve of the cutoff points θ_L^* and θ_C^* , taking as given the tax rate of the opponent (which we suppress in the notation). Thus, k_D describes the feasible set of cutoff voter combinations that the Democratic candidate can implement for any tax rate between 0 and 1, and k_R is the same curve for the Republican.

An important characteristic of these curves is their *signed curvature*. In general, the curvature of a two-dimensional curve $(x_1(t), x_2(t))$ is defined as

$$\kappa = \frac{x_1' x_2'' - x_2' x_1''}{(x_1'^2 + x_2'^2)^{3/2}}.$$
 (12)

The absolute value of κ at a particular point is the inverse of the radius of the circle that approximates the curve in this point; thus, a small value of κ corresponds to an almost linear curve, while a large value of κ is a strongly bent curve. A positive value of κ indicates that, as t increases, the

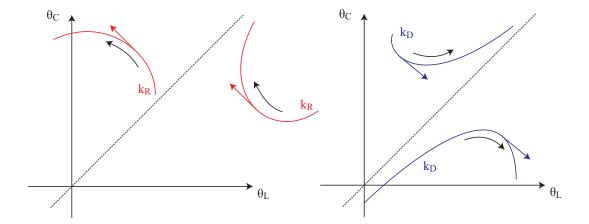


Figure 1: The curves $k_R: t_R \mapsto (\theta_L^*, \theta_C^*)$ and $k_D: t_D \mapsto (\theta_L^*, \theta_C^*)$.

cutoff point moves through the curve in a clockwise direction (and vice versa).¹¹

The following Lemma 1 characterizes the curves k_D and k_R , drawn in Figure 1.

Lemma 1.

1. The function $k_R: [0,1] \to \mathbb{R}^2$ defined by $t_R \mapsto (\theta_L^*(t_R), \theta_C^*(t_R))$ has signed curvature of

$$\kappa_R = -\frac{(\rho + \delta)a_R^2 w''(g_R)}{w(g_D) - w(g_R)} \left(\left(\frac{\partial \theta_L^*}{\partial t_R} \right)^2 + \left(\frac{\partial \theta_C^*}{\partial t_R} \right)^2 \right)^{-3/2}.$$
 (13)

2. The function $k_D: [0,1] \to \mathbb{R}^2$ defined by $t_D \mapsto (\theta_L^*(t_D), \theta_C^*(t_D))$ has signed curvature of

$$\kappa_D = -\frac{(\rho + \delta)a_D^2 w''(g_D)}{w(g_D) - w(g_R)} \left(\left(\frac{\partial \theta_L^*}{\partial t_D} \right)^2 + \left(\frac{\partial \theta_C^*}{\partial t_D} \right)^2 \right)^{-3/2}. \tag{14}$$

Lemma 1 implies that the signs of κ_D and κ_R equal the sign of the term in the denominator (because w'' < 0, and all the other terms are positive). Thus, if $w(g_D) > w(g_R)$, both curves rotate in a counterclockwise direction, and vice versa if instead $w(g_D) < w(g_R)$. Lemma A.1 in the Appendix shows the shapes of k_R and k_D are those drawn in Figure 1.

In equilibrium each candidate chooses a tax rate that maximizes his probability of winning, taking the opponent's tax rate as given. In order to characterize the equilibrium and to determine

¹¹For example, consider $t \mapsto (r\sin(t), r\cos(t))$. This is a circle with radius r, and has curvature $\kappa = -1/r$. The negative sign indicates that as we raise t, the curve is drawn clockwise. In contrast, the curvature of $t \mapsto (r\cos(t), r\sin(t))$ is $\kappa = 1/r$. The positive sign means that the rotation (as t increases) is counterclockwise.

necessary conditions for its existence, it is useful to define *isoprobability curves*, comprising all combinations of cutoff voter types that lead to the same winning probability. Formally, an isoprobability curve is a set of (θ_L, θ_C) that fulfill an equation of the form $G(\theta_L, \theta_C) = \bar{k}$, where \bar{k} is a constant. Such isoprobability curves are depicted in Figure 2. Clearly, any isoprobability curve must have a negative slope, as an increase in θ_L must be offset by a decrease in θ_C in order to keep the candidates' winning probabilities constant.

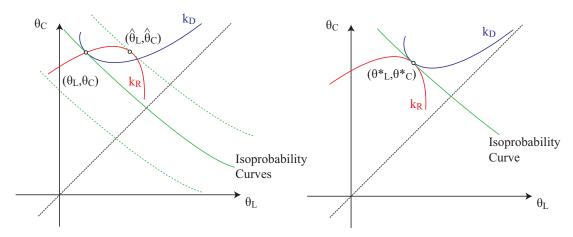


Figure 2: Necessary conditions for an equilibrium.

In the left panel of Figure 2, consider point (θ_L, θ_C) , the cutoffs implied by some tax rates (t_D, t_R) . Is (t_D, t_R) an equilibrium? Note that candidate D, who can move along the convex curve k_D by changing t_D , could increase his winning probability only if he gets to a point below the solid isoprobability curve. However, this is impossible here because k_D is tangent to the isoprobability curve at (θ_L, θ_C) . In contrast, k_R is not tangent, so that candidate R can increase his winning probability by moving to any point above the solid isoprobability curve, for example to $(\hat{\theta}_L, \hat{\theta}_C)$, which is, in fact, his optimal deviation. As indicated in Figure 1, curve k_R rotates in a counter-clockwise direction as t_R increases, so to reach $(\hat{\theta}_L, \hat{\theta}_C)$ requires a decrease in t_R . Since candidate R can improve by deviating, (θ_L, θ_C) is not an equilibrium.

For (θ_L^*, θ_C^*) to be an equilibrium, we must have a situation as drawn in the right panel where both k_D and k_R are tangent to the isoprobability curve at (θ_L^*, θ_C^*) . Here, any "small" deviation makes the deviating candidate worse off. By a small deviation, we mean one that does not change the *structure* of voter support in the sense that, for both ideological groups, it is still the case that low θ -types vote Republican and high θ -types vote Democrat (i.e., cutoff types $(\hat{\theta}_L, \hat{\theta}_C)$ remain above the 45-degree line).

Lemma 2 formally summarizes the necessary tangency condition for an equilibrium. Note that this result does not depend on Assumption 1.

Lemma 2. Let (t_D^*, t_R^*) be an equilibrium with $f(t_D^*) \neq f(t_R^*)$ and $0 < t_D^*, t_R^* < 1$. Then

1. The curves k_R and k_D are tangent to each other at t_D^* , t_R^* , which is equivalent to

$$a_D w'(f_D(t_D^*)) = a_R w'(f_R(t_R^*)).$$
 (15)

2. The isoprobability curve through (θ_L^*, θ_R^*) is tangent to k_R and k_D , i.e.,

$$D_{t_R} k_R(t_R^*) \cdot \nabla G(\theta_L^*, \theta_C^*) = D_{t_D} k_R(t_D^*) \cdot \nabla G(\theta_L^*, \theta_C^*) = 0.$$
 (16)

Condition (15) implies a general comparative static result. Since w'' < 0 it follows that w' is monotone, and consequently, any increase of t_D must result in an increase of t_R and vice versa. Thus, there are no exogenous changes (e.g., to the isoprobability curves) that lead to an increase in the equilibrium value of t_R and, at the same time, to a decrease in t_D ; whenever a parameter change affects equilibrium platforms, the change goes in the same direction. However, in contrast to the classical median voter model, or the probabilistic voting model in which candidate platforms move exactly in parallel, a parameter change may lead to more or less differentiation in economic platforms in our model, i.e., the policy difference $t_D - t_R$ may decrease or an increase. We will return to this issue in more detail in Section 5.

We now turn to sufficient conditions for existence of an equilibrium. In addition to a standard (global) Nash equilibrium, we also consider "local" equilibria. In a local equilibrium small deviations from the equilibrium strategies cannot make a candidate strictly better off. There are unmodeled, but plausible constraints that make the notion of local equilibrium particularly relevant to our model of candidate competition. Formally, candidates in our model can commit to *any* tax rate (as long as it is sufficient to pay for the candidate-specific fixed costs of bureaucracy), and voters believe that the candidate will carry out whatever he promises. However, in practice, some promises may be more credible than others. Suppose, for example, that voters ex-ante "expect" that the Democrat will announce a tax rate of 10 percent, and the Republican one of 8 percent. For this configuration to be a stable equilibrium, it appears highly desirable that deviating to any other tax rate between, say 9 and 11 percent is not profitable for the Democrat, and similarly that small deviations from 8 percent are not profitable for the Republican. In contrast, even if the Democrat could in principle gain by deviating to, say, a tax rate of 5 percent (assuming the Republican stays

at 8 percent), this may not be sufficiently credible to convince low θ types (i.e., rich voters) to vote for the Democrat. The notion of a local equilibrium captures this intuition that "big" deviations from expected behavior may not be feasible for candidates.

We also consider the notion of a semi-global equilibrium. Consider a situation in which R receives the support of all types below the cutoffs θ_C^* and θ_L^* , and D the support of everyone above. A strategy profile is a semi-global equilibrium if it is robust against all deviations that do not change the qualitative structure of the candidates' support, i.e., after the deviation, R still gets the support of sufficiently low types, and D those of sufficiently high types. Since small deviations do not change the qualitative structure of the candidates' support, a semi-global equilibrium is also a local equilibrium, but not necessarily the other way around.

- **Definition 1.** 1. (t_R^*, t_D^*) is a local equilibrium if there exist open neighborhoods $U(t_R^*)$, $U(t_D^*)$ of t_R^* and t_D^* , respectively, such that (t_R^*, t_D^*) is a Nash equilibrium if the candidates' strategies are restricted to $U(t_R^*)$, and $U(t_D^*)$, respectively.
 - 2. (t_R^*, t_D^*) is a semi-global equilibrium if and only if (t_R^*, t_D^*) is a Nash equilibrium when candidate strategies are restricted to the sets $\{t_R|f_R(t_R) < f_D(t_D^*)\}$ and $\{t_D|f_D(t_D) > f_R(t_R^*)\}$, respectively.

Consider again the right panel of Figure 2. The necessary conditions for an equilibrium are satisfied at (θ_L^*, θ_C^*) . It is also clear that at least any local deviation (in the sense of Definition 1 cannot increase the winning probability of a candidate: Any point on k_R is below, and any point on k_D is above the isoprobability curve. The reason is that the curvatures of k_R and k_D exceed that of the isoprobability curve.

The left panel of Figure 3 shows a situation in which the necessary conditions are satisfied at (θ_L^*, θ_C^*) , but where this is not a local equilibrium. The curvature of the isoprobability curve at (θ_L^*, θ_C^*) is strictly positive and larger than than that of k_D . Thus, candidate D can increase his winning probability by increasing t_D in order to move to a point below the isoprobability curve such as $(\hat{\theta}_L, \hat{\theta}_C)$. In the right panel, the curvature of the isoprobability curve is negative. Since the curvature of k_R at (θ_L^*, θ_C^*) is larger in absolute value, we have (at least) a local equilibrium. Note that in this case there is no restriction on the curvature of k_D . As a consequence, a necessary condition for a local equilibrium is that the curvature κ_G of the isoprobability curve at (θ_L^*, θ_C^*) is strictly between $-\kappa_R(t_R^*)$ and $\kappa_D(t_D^*)$. Further, if this condition holds for all points along the isoprobability curve (as in the the right panel), then we have a semi-global equilibrium. Theorem 1

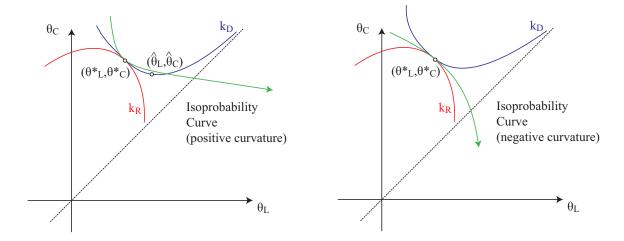


Figure 3: Necessary and Sufficient Conditions for an equilibrium.

formally states necessary and sufficient conditions for an equilibrium.

Theorem 1.

- 1. Suppose that (t_D^*, t_R^*) is a local equilibrium with $f(t_D^*) \neq f(t_R^*)$ and $0 < t_D^*, t_R^* < 1$. Then conditions (15) and (16) hold, and the curvature of the isoprobability curve is between $-\kappa_R$ and κ_D , i.e., $-\kappa_R(t_R^*) \leq \kappa_G(\theta_L^*, \theta_C^*) \leq \kappa_D(t_D^*)$.
- 2. Suppose that $0 < t_D^*, t_R^* < 1$ satisfy (15) and (16) and that the curvature of isoprobability curve is strictly between $-\kappa_R$ and κ_D , i.e., $-\kappa_R(t_R^*) < \kappa_G(\theta_L^*, \theta_C^*) < \kappa_D(t_D^*)$. Then t_R^* , t_D^* is a local equilibrium.
- 3. Same as item 2, and, in addition, suppose that the curvature of the isoprobability curve through (θ_L^*, θ_C^*) is strictly between $-k_R(t_R^*)$ and $k_D(t_D^*)$ for all points above the 45 degree line. Then t_R^* , t_D^* is a semi-global equilibrium.

Proof. See Appendix. □

5 Comparative Statics

5.1 Preliminaries

From Lemma 2, we know that in equilibrium k_D and k_R are tangent to each other. This property is equivalent to (15). If w is strictly concave, w' is strictly monotone and we can therefore solve (15) by writing t_D as a function of t_R . The set of tangency points of k_D and k_R are given by $K(t_R) = k_D(t_D(t_R))$. If Assumption 1 is satisfied, then $t_D(t_R) > t_R$, and as a consequence curve K is located above the 45-degree line. Curve K is useful for comparative statics, since it must be tangent to an isoprobability curve in any equilibrium. Lemma 3 below analyzes its key properties.

Lemma 3. 1. Let $t_D(t_R)$ be the solution of the equation (15). Then the curve of all tangency points $K: [0,1] \to \mathbb{R}^2$ defined by $t_R \mapsto (\theta_L^*(t_R, t_D(t_R)), \theta_C^*(t_R, t_D(t_R)))$ has a signed curvature of

$$\hat{\kappa} = \frac{\kappa_R}{|t_D'(t_R) - 1|}.\tag{17}$$

2.
$$D_{t_R}K(t_R) = [1 - t'_D(t_R)]D_{t_R}k_r(t_R)$$

From Lemma 1, we know that $\kappa_R > 0$ whenever $\theta_C > \theta_L$. Thus, the first point of Lemma 3 implies that the signed curvature of $K(t_R)$, is strictly positive, and therefore K turns counterclockwise. The second point determines whether the curve is convex or concave toward the origin. If $t'_D(t_R) < 1$, the derivative of K must point in the same direction as the derivative of k_R and thus, K is concave toward the origin, just like k_R . In contrast, if $t'_D(t_R) > 1$, K is convex toward the origin. Depending on whether $t'_D(t_R)$ is smaller or greater than 2, the curvature of K is larger or smaller than that of k_R .

What determines the sign of $1 - t'_D(t_R)$? We provide some examples.

Example 1 If $w(g) = -e^{-sg}$ then $t'_D(t_R) \equiv a_R/a_D < 1$.

As another example, suppose that utility is of the form $w(x) = x^{1-s}/(1-s)$. Then (15) implies that

$$t_D = \frac{b_D}{a_D} - a_D^{\frac{1-s}{s}} a_R^{-\frac{1}{s}} b_R + \left(\frac{a_R}{a_D}\right)^{\frac{s-1}{s}} t_R.$$
 (18)

Thus, $t'_D(t_R) = (a_R/a_D)^{\frac{s-1}{s}}$. Since $a_R < a_D$, this implies that $t'_D > 1$ if s < 1 and $t'_D < 1$ if s > 1.

Note that for s = 1 we have log utility, in which case t_D increases one-to-one in t_R so that the difference $t_D - t_R$ is constant at all points where k_D and k_R are tangent to each other. Furthermore,

(17) implies that, for s = 1, K has an infinite curvature so that the "curve" K is condensed into a single point. That is, only one particular pair of cutoff values θ_L^* and θ_C^* are consistent with equilibrium. This holds independent of the distribution G. In particular, changes in the distribution of preference types in the electorate (say, a higher percentage of liberals or a higher preference of all voters for public good provision) generally affect equilibrium tax rates, but do this in a way that the equilibrium cutoffs remain constant.

For the comparative static analysis, we focus on a class of voter type distributions with a known ideological composition of the electorate, but uncertainty about the economic preference distribution. This appears quite realistic, because the relative constancy of the preference distribution in important "value" issues is well documented (e.g., Fiorina, Abrams, and Pope (2006)). In contrast, exact individual preferences on the trade-off between public good provision and taxation are much harder to pinpoint in opinion polls (than, say, preferences over abortion), and economic preferences are also more likely to change over time (say, with the current or expected position in the business cycle).

Assumption 2.

- 1. The electorate consists of a fraction p of liberals, and 1 p of conservatives.
- 2. For each ideology type, θ is normally distributed with mean $\mu \omega$, and standard deviation σ .

Let $\Phi_{\mu,\sigma}(\cdot)$ denote the cdf of a normal distribution with mean μ and standard deviation σ , then the distribution of voter types given ω is $\Phi_{\mu,\sigma}(\theta - \omega)$. We now construct the isoprobability curves $G(\theta_L, \theta_R)$ by determining the collection of all θ_L , θ_R at which the election ends in a tie, for a given $\bar{\omega}$, i.e.

$$p\Phi_{\mu,\sigma}(\theta_L - \bar{\omega}) + (1 - p)\Phi_{\mu,\sigma}(\theta_C - \bar{\omega}) = 0.5, \tag{19}$$

Thus, if the cutoff types (θ_L, θ_C) are on this curve, then the candidate who gets the support of all low θ types, wins in all states $\omega < \bar{\omega}$, loses when $\omega > \bar{\omega}$, and the election ends in a tie if $\omega = \bar{\omega}$. Lemma 4 summarizes how the shape of the isoprobability curves depends on parameters.

Lemma 4. Suppose that Assumption 2 holds.

1. If p = 1/2, then all isoprobability curves are straight lines with curvature 0.

- 2. The curvature κ_G of the isoprobability curves is continuous in p. Thus, for p close to 1/2, κ_G is close to 0.
- 3. If p < 0.5 (p > 0.5), then isoprobability curves are strictly concave (strictly convex) above the 45 degree line, and strictly convex (strictly concave) below the 45 degree line.
- 4. The slope of the isoprobability curve through (θ_L, θ_C) is given by

$$\theta_C'(\theta_L) = -\frac{p}{1-p} e^{\frac{\theta_C - \theta_L}{\sigma^2} \left[\frac{\theta_L + \theta_C}{2} - (\mu + \bar{\omega}) \right]}$$
(20)

The slope $\theta'_C(\theta_L)$, is negative and decreases in p (i.e., becomes steeper), is constant in μ , and decreases (increases) in σ if p < 1/2 (p > 1/2). Moreover, $\lim_{\sigma \to \infty} \theta'_C = -p/(1-p)$.

These properties are the key to our comparative static results, so it is worthwhile discussing a few of the results briefly. If p = 1/2, then a tie requires that the Democrat wins exactly the same percentage of liberals as the Republican wins of conservatives. Thus, isoprobability curves are straight lines with slope -1. If, instead, p is small, then a tie between the candidates requires that the candidates split conservatives more-or-less 50/50, so that θ_C is close to the median of the θ -distribution, $\mu - \omega$. In contrast, θ_L will be relatively far away from this maximum. Because there is less probability mass in the tails than in the center of a normal distribution, a further decrease of θ_L is easier and easier to compensate with increases in θ_C , which implies the concave shape of the isoprobability curves above the 45-degree line. An analogous argument implies that the isoprobability curves are convex below the 45-degree line, if p < 1/2. Obviously, these results just flip around if p > 1/2.

Finally, as σ increases, the difference between the probability mass in the center and in the tails diminishes, and isoprobability curves become more like straight lines. Thus, for p < 1/2, as σ increases, isoprobability curves become steeper above the 45-degree line.

5.2 Ideological polarization

We can conceptualize "polarization" in two different ways. In the next section, we will deal with changes of the economic preference distribution (both shifts and increasing the spread in the distribution of θ for both ideological voter types. The latter captures *economic polarization*, i.e., an

increase of the number of people who either want a very strong or a very limited government spending. However, in this section, we start with the comparative static analysis of the effects of ideological polarization. This includes both changes in the composition of the electorate from social conservatives and social liberals, and in the intensity with which these groups care about the ideological differences between candidates.

Consider first an increase of the proportion of social conservatives in the electorate (i.e., p decreases). Lemma 4 implies that the isoprobability curves become flatter. In Figure 4, the original equilibrium is the tangency point of K and the solid isoprobability curve. The new, flatter, isoprobability curves are indicated by the dashed lines, and the black circle marks the new tangency point. In both cases (the left panel with $t'_D(t_R) < 1$ and the right panel with $t'_D(t_R) > 1$), the new equilibrium moves in the direction of the rotation of the curves, i.e., both t_R and t_D increase.

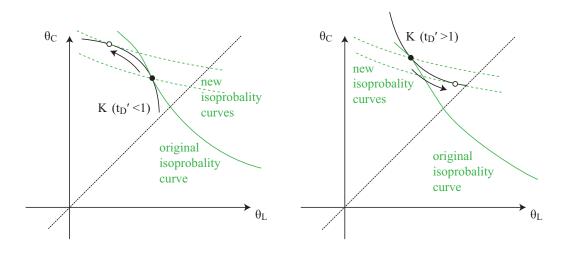


Figure 4: Comparative Statics: Increase of social conservatives (*p* decreases)

The two cases of $t'_D(t_R) < 1$ and $t'_D(t_R) > 1$ differ in what they imply about the structure of voter polarization. For $t'_D(t_R) < 1$ (the left panel) θ_L^* decreases, while θ_C^* increases. Intuitively, while both candidates increase taxes, the Democratic increase is smaller than the Republican one, and thus, parties become more similar economically. As a consequence, the two ideology cutoffs move away from the 45-degree line, and thus, society appears more polarized along cultural-ideological issues (i.e., more socially-conservative types vote for the Republican, and more socially-liberal

¹²Which of these cases obtains is determined primarily by the shape of the preferences for public goods, w(g). As shown above, if w corresponds to exponential utility or if $w(g) = g^{1-s}/(1-s)$, with s > 1 (i.e., more curvature than logarithmic utility), then $t'_D < 1$.

types vote for the Democrat). In contrast, if $t'_D(t_R) > 1$ (right panel), then the parties' economic platforms become more dissimilar (as the Democrat increases taxes by more than the Republican), and consequently, the Democrat now appeals to more socially-conservative voter types, and fewer socially-liberal types.

An increase in the number of social conservatives (i.e., of voters with a *cultural* bias for the small government party) leads both candidates to *increase* their proposed tax rate. This result may appear surprising, but the logic behind it is quite straightforward. Remember that candidates compete for the support of cutoff voters, and that cutoff voters are torn between their economic and cultural-ideological preferences in that they like the economic position of one candidate and the cultural position of the other. In particular, the socially-conservative cutoff voter prefers a higher level of government spending than is provided by both candidates, while the socially liberal cutoff voter prefers a smaller level of government spending. An increase in the number of social conservatives makes it attractive for both candidates to put more weight on the economic preferences of the conservative cutoff voter, and thus to increase the provision of public goods.

Moreover, this effect is likely to be very robust, in the sense that it does not depend on the specific setup. In our model, the productivity difference between candidates is what drives candidate differentiation. Alternatively, one could, for example, imagine a model in which the Republican candidate has a more conservative ideal position on both the cultural and the economic dimension than the Democrat, and both candidates are policy motivated and use their economic position to maximize their expected utility from the implemented policy. Whether in this or any other model in which candidates have at least some incentive to care about winning the election, they will consider the desires of swing voters more than those of core supporters (i.e. voters who either vote for the candidate or for his opponent, no matter what policies the candidates choose), and, to the extent that there are different swing voter groups, their relative sizes influence which group candidates cater to most. Socially conservative *swing voters* are necessarily economic liberals (otherwise, they would be the core supporters of the Republican), and socially liberal *swing voters* are economic conservatives. An increase in the number of a particular swing voter group means that the candidates' incentive to cater to this group (through their policy choice) is increased.

Our result appears consistent with behavior observed in the last decade in which Republicans were in control of the executive and the legislative branch for most of the time. The aftermath of the terrorist attacks of September 11 conceivably increased the proportion of voters with a non-economic preference for the Republican party (and their intensity of preference), and the Re-

publicans in spite of their small-government rhetoric, increased government spending as a fraction of GDP from 18.2 percent in 2000 to 20.7 percent in 2008.

Since 2008, Republicans have become more concerned with the deficit, but this is probably due to intraparty effects that are not present in our model: In our model, candidates take positions that maximize their winning probability in the general election, while taking as given the support of their core constituencies. While this was likely a good description until 2008, it is quite conceivable that the rise of the "Tea Party" has shifted the focus of Republican politicians from choosing the policy that would be most successful in the general election to choosing a policy that minimizes the probability of being attacked from the right in a primary.

Next, we analyze how the equilibrium is affected by changes in ρ and δ , which can be interpreted as changes in the social policy partisanship of conservatives and liberals, respectively, another measure of polarization. In general, the intuition for the effect of an intensification of cultural preferences is very transparent in our model framework: More intense non-economic preferences among social conservatives, for example, imply that the conservative cutoff type must increase (i.e., has a stronger preference for government spending than before) in order to remain indifferent between Republican and Democrat. As candidates maximize some weighted average of the economic preferences of socially liberal and socially conservative cutoff voters, they now have an incentive to propose higher government spending. For simplicity, we focus on the case where isoprobability curves are straight lines (e.g., p = 0.5, or σ sufficiently large).¹³

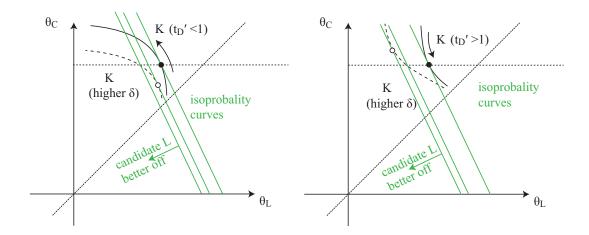


Figure 5: Comparative Statics: Increase of δ for linear isoprobability curves

¹³The extension to general isoprobability curves is available from the authors upon request.

Figure 5 shows the effect of increasing δ . The solid curves represent the original K curves (for $t'_D < 1$ in the left panel and for $t'_D > 1$ in the right panel). The dashed curves show K after δ increases. Note that $t_D(t_R)$ is independent of δ . Thus, (5) implies that θ_L^* decreases, while (6) implies that θ_C remains unchanged. Therefore points on the solid and dashed K lines that correspond to the same tax rate t_R are aligned horizontally. The curved arrow along K indicates the direction of movement as t_R increases.

Equations (31) and (32) in the Appendix imply that for given t_R , the slope of $K(t_R)$ increases (i.e., a negative slope becomes less steep) as δ increases. The comparative static result for δ now follows immediately from simple geometric observations. In the left panel, the new equilibrium point is below the horizontal line. Given the direction of rotation of K indicated by the arrow, this corresponds to a lower t_R and hence lower t_D . The cutoff θ_C decreases. θ_L may decrease (as in the graph) or increase, depending on the curvature of K.

In the right panel, the new equilibrium is above the horizontal line. Again, the rotation direction of K implies that at this new equilibrium, taxes are lower. Note that θ_C increases, while θ_L decreases — in the case of $t'_D > 1$ there is no ambiguity about the change of cutoffs.

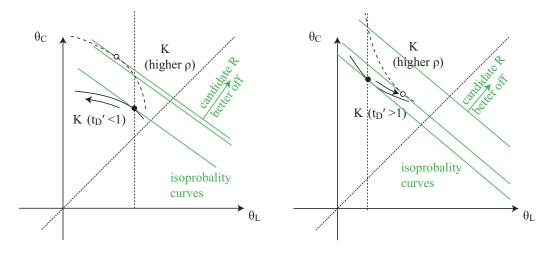


Figure 6: Comparative Statics: Increase of ρ for linear isoprobability curves

Figure 6 shows that, by analogous arguments, increasing ρ leads to the reverse effects of increasing δ : Increasing ρ moves K up and results in a steeper slope along vertical lines; note that vertical rather than horizontal lines connect points on the two K curves with the same tax rate; and in equilibrium, taxes are increased rather than decreased.

Finally, note that the effect of a parameter change on the expected economic policy does not

only depend on the effect that the parameter change has on equilibrium platforms, but also on who wins the election. As Figures 5 and 6 indicate, the winning probability of candidate D increases as δ increases, and similarly, the winning probability of candidate R rises in response to an increase of ρ . Consider, for example the case where liberals become more partisan, so that both candidates propose a lower tax rate. However, since the winning probability of candidate D increases, who proposes a higher tax rate than candidate R, the net effect on the expected tax rate is ambiguous.

We now summarize our results.

Theorem 2. Suppose that Assumptions 1 and 2 hold, and that either $t'_D(t_R) < 1$ for all t_R or $t'_D(t_R) > 1$ for all t_R , and that the isoprobability curves are straight lines. Then

- 1. Suppose that liberal ideology intensity δ increases. Then both candidates decrease their proposed tax rate. If $t'_D < 1$ then the cutoff θ_C decreases. If $t'_D > 1$, cutoff θ_C increases while θ_L decreases.
- 2. Suppose that conservative ideology intensity ρ increases. Then both candidates increase their proposed tax rate. If $t'_D < 1$ then the cutoff θ_L decreases while θ_C increases. If $t'_D > 1$, cutoff θ_L increases.
- 3. If p decreases, then t_D and t_R increase and the winning probability of candidate D decreases. If $t'_D(t_R) < 1$ then θ_L^* decreases and θ_C^* increases, while the reverse is true when $t'_D(t_R) > 1$.

If $\theta_L^* = \theta_C^*$ the economic position is a perfect predictor of voting behavior. As $\theta_L^* - \theta_C^*$ increases, ideology starts becoming a better predictor, until economics becomes irrelevant and only ideology matters (when $\theta_L^* = \infty$ and $\theta_C^* = -\infty$). One may expect that increasing δ or ρ would make ideology a better predictor of voting behavior. However, our results shows that this is not necessarily true. For example, when δ increases and $t_D' < 1$ the new equilibrium may be closer to the 45 degree line (as indicated in Figure 6), i.e., $\theta_C^* - \theta_L^*$ decreases and economics becomes more important. The reason is that as taxes are increased, $t_D' < 1$ implies that the difference between the tax rates, $t_D - t_R$ increases. Hence, the economic difference between the platforms increases which counterbalances the effect of the increased ideological differences between the candidates.

It is easy to extend the results of Theorem 2 to nonlinear isoprobability curves. Depending on whether the isoprobability curves are convex or concave above the 45 degree line, the effects described in the theorem are strengthened or weakened.

As a final comparative static exercise, consider a simultaneous increase of the ideological intensity for both liberals and conservatives (i.e., both ρ and δ increase by the same amount, h). This is particularly useful if we want to think about ideology in a spatial framework: Suppose that voter preferences are constant, but that candidate positions on the ideological dimension move away from each other. In this case, voters care more about their ideological favorite winning, i.e., both ρ and δ increase.

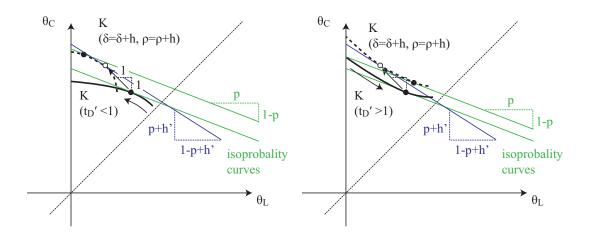


Figure 7: Comparative Statics: Both ρ and δ are increased by the same amount h.

Figure 7 considers a case with p < 1 - p, i.e., there are more conservatives than liberals, and linear isoprobability curves. Equations (5) and (6) show that if taxes remain the same, then the new cutoff value moves exactly to the northwest, along a line with slope -1 to the point indicated by the white circle. We refer to this change as the ideology effect.

At the original equilibrium cutoffs, the slope of K is -p/(1-p) because K is tangent to the isoprobability curve through (θ_L, θ_C) . Equations (31) and (32) imply that after the increase of δ and ρ by amount h, the derivative of K in the θ_L direction, which is negative, becomes more negative by some amount h'. The derivative in the θ_C direction, which is positive, increases by the same amount h'. Thus, p < 1 - p implies that the slope must become steeper. The change of the slope of K implies that taxes must change, since we do not have tangency at the white circle. We refer to the change in cutoffs because of the tax change as the tax effect.

Specifically, consider the left panel of Figure 7, where $t'_D < 1$. The concavity of K implies that

¹⁴Note that if p = 1 - p then the slope of K would not change, and we would have tangency at the point indicated by the white circle, i.e., taxes would stay the same.

the tax effect moves the new tangency point even further to the northwest. Since we move in the direction of the arrow (counterclockwise), taxes increase. Since $t'_D < 1$, the difference between the candidates' tax rates decreases. Thus, candidates differ less on economic policy, and consequently, voters separate more by ideology: The tax effect and the ideology effect reinforce each other.

The right panel of Figure 7 analyzes the case of $t'_D > 1$. Again, if tax rates were kept constant, the cutoff moves to the northwest. However, from there, the convexity of K implies that the new tangency point moves toward the southeast, again in the direction of increased taxes. For $t'_D > 1$, the tax effect and the ideology effect have opposite signs. Hence, it is possible that the difference between cutoffs $\theta_C^* - \theta_L^*$ remains almost unchanged, and thus, there is no perceived increase in the extent to which ideology rather than economic interests determine voting behavior, even after ideological polarization has increased. The reason is that when $t'_D > 1$ and taxes increase, the difference in tax rates increases, so that candidates' economic policies differ more. Hence, economic policy becomes more important for voters, too, which can countervail the increased importance of ideology.

The effects are reversed if p > 1 - p. In this case, proposed tax rates always decrease as the ideological intensity of the electorate increases. The ideology effect and the tax effect go in opposite direction when $t'_D < 1$, and they reinforce each other when $t'_D > 1$.

5.3 Economic polarization

We now consider changes in the distribution of economic preferences. Note that changes in economic preferences leave the location of the curve *K* unaffected, while generally changing the location of isoprobability curves.

Consider first what happens when the mean μ of the distribution changes. For a given point (θ_L, θ_C) , (19) remains satisfied if $\bar{\omega}$ adjusts in a way that exactly offsets the change in μ . As a consequence, the shape of isoprobability curves remains the same, but each of them corresponds to a lower realization of ω . Thus, the equilibrium does not change since the K curve remains tangent to an isoprobability curve. However, the isoprobability curve that is tangent to K now corresponds to a higher probability of winning for candidate D.

This result implies that the candidates' economic policy proposals display a remarkable rigidity. Remember that an increase in μ means that, for any given level of ω , all voters would like to have more public goods than before. Yet, this change has no effect on equilibrium policies, but instead

changes the candidates' winning probabilities. To understand this result, note first that in a standard spatial setup, both candidates can appeal equally to all voters. For this reason, both candidates cater to the median voter, the voter type who is decisive for the outcome of the election (or the expected median voter, if there is uncertainty about the distribution of voter types). This implies that, if the median voter's preferences change, the candidates' positions exactly reflect this change and adapt to the median's new preferred position.

In contrast, candidates in our model have exogenous advantages and disadvantages in appealing to particular voter types. Cutoff types are determined as those voter types to whom candidates *can* appeal equally. Remember that, in equilibrium, both candidates choose positions that appeal to some type located between the liberal and the conservative cutoff voter. Thus, in principle, each candidate could expand his set of liberal supporters relative to the equilibrium, but only at the expense of his conservative support. This trade-off is the same for both candidates, and it does not change as the likely preference distribution changes. Specifically, an increase in μ by $\Delta\mu$ increases the *expected* average value of θ in the population and thus, for a given value of ω , increases the vote share of the Democrat. Thus, the critical state of the world in which the candidates receive a vote share of exactly one-half decreases by $\Delta\mu$, and consequently, candidates face exactly the same preference distribution in the critical state. Thus, both candidates continue to maximize their respective voter support with unchanged platforms.

Now consider an increase in σ , the standard deviation of the distribution of θ . Since $\theta = \eta/m$, where η is the preference for public good provision and m the individual's income, an increase in σ can be caused by an increase in income inequality or an increase in polarization of η .

If there are as many liberals as conservatives (p = 0.5), then Lemma 4 implies that equilibrium platforms and winning probabilities do not change. If there are more conservatives than liberals (i.e., p < 0.5), then Lemma 4 implies that increasing σ results in steeper isoprobability curves above the 45-degree line. Qualitatively, the results correspond to those we derived for increasing p (the fraction of liberals), i.e., the equilibrium tax rates of both candidates decrease. Similarly, if p > 0.5, then increasing σ results in a flatter isoprobability curves and all effects are reversed.

This result establishes a relation between economic polarization and equilibrium economic platforms. For example, suppose that the income distribution becomes more unequal, which results in a larger standard deviation of θ . Consider the case where p < 1/2, which appears plausible in the U.S. where self-described conservatives usually outnumber self-described liberals by a substantial margin. Then, an increase in economic polarization (i.e., σ) increases the equilibrium spending by

both candidates. Moreover, if $t'_D(t_R) < 1$ (which happens with exponential utility, or with a utility function for which the marginal utility of public good consumption decreases sufficiently fast), then economic polarization of voters leads to convergence of the tax rates proposed by the two parties. As a consequence of this increased economic similarity between parties, citizens' votes will reflect more strongly their cultural cultural preferences. That is, θ_L^* decreases and θ_C^* increases so that more liberal voter types vote for the Democrat, and more social conservatives vote for the Republican.

To highlight the significant differences between our differentiated candidates model and previous literature, it is instructive to compare our result with the seminal model of Meltzer and Richard (1981), where economic polarization is interpreted as an increase in the difference between the income of the median voter and the average income in the economy. In their model, polarization thus means that the median voter benefits more from redistribution (so their effect goes in the opposite direction from ours if p < 1/2). It is well known that empirical tests of this prediction tend to find the opposite effect, which is a significant puzzle. For example, Lindert (1996) uses panel data from 19 OECD countries for the time period 1960-1992 and analyzes what factors influence social and other government spending. He finds that an increase in his measure of income inequality (the ratio between the average income of the top and the bottom quintile of the income distribution) significantly *decreases* total government expenditures.

The effect of the income distribution in Meltzer and Richard (1981) is driven by the gap between median and average income. If this gap increases, the median voter benefits more from redistribution (either directly, or through an increased provision of public goods). With a symmetric distribution, a change in the standard deviation σ would not affect policy in Meltzer and Richard (1981).

We consider symmetric distributions of θ (remember that $\theta = \eta/m$ so that, even if the income distribution is skewed to the right, as is always the case for empirical income distributions, the distribution of θ may be symmetric or skewed either to the left or to the right). The intuition for our result is that, if economic preferences become more polarized, voters effectively care more about economic issues and thus, in comparison, relatively less about ideology; to see this, note that

¹⁵In the economic growth literature, the seminal papers of Alesina and Rodrik (1994) and Persson and Tabellini (1994) show that, empirically, pre-tax inequality is bad for economic growth. Their theoretical models explain this by a close variation on the Meltzer and Richard (1981) model: More unequal societies have higher taxes which has a negative effect on savings and thus growth. However, neither paper tests whether pre-tax inequality actually increases government spending.

an increase of σ means that more voters have types such that they vote Democrat or Republican *independently* of their ideological disposition. As we know from the previous section, a decrease in the ideological preference intensity of voters is bad news for the party whose ideology is supported by a majority of voters (in terms of their probability of winning). The policy effect is similar to the one that would result if the majority ideological types start to care less about ideology. If p < 1/2, this means that policy reacts in the same way that it would if conservatives became less ideological, and consequently, equilibrium tax rates decrease.

Theorem 3 formally summarizes our results in this section.

Theorem 3. Suppose that Assumptions 1 and 2 hold, and that either $t'_D(t_R) < 1$ for all t_R or $t'_D(t_R) > 1$ for all t_R .

- 1. If μ increases, then the equilibrium policies are unaffected, but the winning probability of candidate D increases.
- 2. An increase of σ has the following effects.
 - (a) If p < 0.5, then equilibrium tax rates t_D and t_R decrease. If $t'_D(t_R) < 1$, then cutoff θ_L^* increases while θ_C^* decreases, and the reverse is true for $t'_D(t_R) > 1$.
 - (b) If p = 0.5, then equilibrium policies, cutoffs θ_L^* , θ_C^* and winning probabilities are unaffected.
 - (c) If p > 0.5, then equilibrium tax rates t_D and t_R increase. If $t'_D(t_R) < 1$, then θ_L^* decreases while θ_L^* increases, and the reverse is true for $t'_D(t_R) > 1$.

The prediction of our model that government spending decreases in countries with a majority of social conservatives as a consequence of an increase in inequality is compatible with the empirical result of Lindert (1996) discussed above. Moreover, it is interesting that such an effect may be reinforcing in a dynamic version of the model: In a country with more social conservatives (such as the US), an initial increase in inequality decreases government spending. If , say, education expenditures are reduced, this policy response may conceivably increase pre-tax income inequality in the future even further, which leads to more cuts in government spending and so on. In contrast, in a society with a majority of social liberals, an initial increase in inequality increases government spending, which is likely to decrease future inequality.

6 Conclusion

In this paper, we have developed a model in which voters care about both social ideology (which, in our model, is exogenously given for candidates) and the economic positions that candidates take. The interaction between these two dimensions is of first-order importance for our understanding of what determines economic policy: In reality, there are considerable differences in candidates' economic policy platforms, but voter preferences for parties and candidates appear to be influenced by both economic and, probably to an even greater extent, by cultural-ideological positions. A model that explicitly incorporates these non-economic factors provides us with a better understanding of this important interaction, and thus with a better understanding of the determinants of economic policy than a model that abstracts from cultural ideology in order to focus entirely on economic policy issues.

There is an intense and ongoing discussion in both political science and popular discourse as to whether cultural or economic factors become more important as determinants of voter behavior (Green, Palmquist, and Schickler (2002), Frank (2004), McCarty, Poole, and Rosenthal (2006)). Our paper contributes to this literature by providing a unified model framework in which voters care about both cultural-ideological positions and economic policy. Our main results are as follows: When choosing their economic platforms, candidates focus on the two swing voter groups who are close to indifferent between the two parties: Small-government/social liberals and biggovernment/social conservatives. The equilibrium is determined in a way that a marginal increase in taxes yields a candidate a larger set of supporters among social conservatives (because the cutoff type among social conservatives prefers more government spending) and a smaller set of supporters among social liberals. This result holds even if social conservatives are *on average* for a lower level of taxation than social liberals, because candidates focus their attention on the respective swing voters only.

Any change in equilibrium platforms that is brought about by changes in the voter preference distribution goes in the same direction for both candidates: Either, both candidates propose higher spending, or both propose lower spending than before. Furthermore, an increase in support for the cultural-ideological position of the small-government party (either through an increase in the number of social conservatives, or through an intensification of their cultural preference) leads to an increase in spending. The same holds when both groups become more ideologically polarized, but there are more social conservatives than social liberals. Economic polarization may also affect

equilibrium policies: If there are more conservatives than liberals, increasing economic preference polarization (brought about, for example, by an increase in economic inequality) leads to lower taxes – a result in sharp contrast with the result of the seminal work of Meltzer and Richard (1981).

7 Appendix

Proof of Lemma 1. For fixed t_D , consider the curve given by $t_R \mapsto (\theta_L^*(t_R), \theta_C^*(t_R))$. Let $S(t_R) = \frac{\partial \theta_C(t_R)}{\partial t_R} / \frac{\partial \theta_L(t_R)}{\partial t_R}$. Thus, (10) and (11) imply that

$$S(t_R) = \frac{(w(g_D) - w(g_R)) - ((t_D - t_R) + \rho)a_R w'(g_R)}{(w(g_D) - w(g_R)) - ((t_D - t_R) - \delta)a_R w'(g_R)}.$$
(21)

Let

$$A(t_R) = (w(g_D) - w(g_R)) - (t_D - t_R)a_R w'(g_R)$$
(22)

Then

$$\frac{\partial S(t_R)}{\partial t_R} = (\rho + \delta) \frac{w'(g_R) a_R \frac{\partial A(t_R)}{\partial t_R} - A(t) w''(g_R) f_R'^2(t_R)}{\left((w(g_D) - w(g_R)) - ((t_D - t_R) - \delta) a_R w'(g_R) \right)^2},\tag{23}$$

where

$$\frac{\partial A(t_R)}{\partial t_R} = -(t_D - t_R) f_R^{\prime 2}(t_R) w^{\prime \prime}(g_R). \tag{24}$$

Next, note that

$$f'_{R}(t_{R})\frac{w'(g_{R})}{w''(g_{R})} - f'_{R}^{2}(t_{R})\frac{A(t_{R})}{\frac{\partial A(t_{R})}{\partial t_{R}}} = a_{R}\frac{w'(g_{R})}{w''(g_{R})} + \frac{w(g_{D}) - w(g_{R})}{(t_{D} - t_{R})w''(g_{R})} - a_{R}\frac{w'(g_{R})}{w''(g_{R})}$$

$$= \frac{w(g_{D}) - w(g_{R})}{(t_{D} - t_{R})w''(g_{R})}.$$
(25)

Thus,

$$\frac{\partial S(t_R)}{\partial t_R} = -(\rho + \delta) \frac{w''(g_R) f_R'^2(t_R) (w(g_D) - w(g_R))}{\left((w(g_D) - w(g_R)) - ((t_D - t_R) - \delta) a_R w'(g_R) \right)^2},\tag{26}$$

The signed curvature of candidate R's response function is given by

$$\kappa_R = \frac{\frac{\partial \theta_L^*}{\partial t_R} \frac{\partial^2 \theta_L^*}{\partial t_R^2} - \frac{\partial^2 \theta_L^*}{\partial t_R^2} \frac{\partial \theta_L^*}{\partial t_R}}{\left(\left(\frac{\partial \theta_L^*}{\partial t_R}\right)^2 + \left(\frac{\partial \theta_L^*}{\partial t_R}\right)^2\right)^{3/2}}.$$
(27)

Thus,

$$\kappa_R = \frac{\partial S(t_R)}{\partial t_R} \frac{\left(\frac{\partial \theta_L^*(t_R)}{\partial t_R}\right)^2}{\left(\left(\frac{\partial \theta_L^*}{\partial t_R}\right)^2 + \left(\frac{\partial \theta_C^*}{\partial t_R}\right)^2\right)^{3/2}}.$$
(28)

As a consequence (23), (24), (25) and (28) imply (13).

Similarly, it follows that the curvature of $t_D \mapsto (\theta_L(t_D), \theta_C(t_D))$ is given by (14).

Lemma A.1 *Suppose that* $\rho + \delta > 0$ *. Then*

- 1. $k_R(t_R)$ is strictly concave toward the origin for all t_R where the slope is negative and for which $\theta_C^* > \theta_L^*$.
- 2. $k_R(t_R)$ is strictly convex toward the origin for all t_R where the slope is negative and for which $\theta_C^* < \theta_I^*$.
- 3. $k_D(t_D)$ is strictly convex toward the origin for all t_D where the slope is negative and for which $\theta_C^* > \theta_L^*$.
- 4. $k_D(t_D)$ is strictly concave toward the origin for all t_D where the slope is negative and for which $\theta_C^* < \theta_I^*$.

Proof of Lemma A.1. (10) and (11) imply that $\frac{\partial \theta_C^*}{\partial t_R} > \frac{\partial \theta_L^*}{\partial t_R}$. If the slope of the curve is negative, then $\frac{\partial \theta_L^*}{\partial t_R}$ and $\frac{\partial \theta_C^*}{\partial t_R}$ must have opposing signs. Thus, $\frac{\partial \theta_C^*}{\partial t_R} > 0$ while $\frac{\partial \theta_L^*}{\partial t_R} < 0$. The tangent vector therefore points toward the northwest.

Consider first the k_R -curve above the 45 degree line (i.e., $\theta_C^* > \theta_L^*$). (5) and (6) imply that $w(g_D) > w(g_R)$, so that D provides more public good than R. Thus, low θ types vote for R and high θ types vote for D. Lemma 1 implies that $\kappa_R > 0$, so that the curve rotates counterclockwise. Since the tangent vector for negative slopes points northwest, it follows that the curve is concave. Analogous arguments show that k_R is convex below the 45-degree line, and that k_D is convex (concave) above (below) the 45-degree line, as shown in Figure 1.

Proof of Lemma 3. Applying (10) and (11) it follows that the slope of $(\theta_L^*(t_R), \theta_C^*(t_R))$ is

$$\frac{(w(g_D) - w(g_R)) - ((t_D - t_R) - \delta)a_R w'(g_R)}{(w(g_D) - w(g_R)) - ((t_D - t_R) + \rho)a_R w'(g_R)}.$$
(29)

Similarly, (8) and (9) it follows that the slope of $(\theta_L^*(t_D), \theta_C^*(t_D))$ is

$$\frac{(w(g_D) - w(g_R)) - ((t_D - t_R) - \delta)a_D w'(g_D)}{(w(g_D) - w(g_R)) - ((t_D - t_R) + \rho)a_D w'(g_D)}.$$
(30)

Thus, (29) and (30) are the same if and only if $a_D w'(g_D) = a_R w'(g_R)$. This proves the first statement. Substituting $t_D(t_R)$ for t_D in (5) and (6), and taking the derivative with respect to t_R yields

$$\frac{\partial \tilde{\theta}_L}{\partial t_R} = (t_D' - 1) \frac{(w(g_D) - w(g_R)) - ((t_D - t_R) - \delta) a_R w'(g_R)}{(w(g_D) - w(g_R))^2} = (t_D' - 1) \frac{\partial \theta_L^*}{\partial t_R}.$$
 (31)

$$\frac{\partial \tilde{\theta}_C}{\partial t_R} = (t_D' - 1) \frac{(w(g_D) - w(g_R)) - ((t_D - t_R) + \rho) a_R w'(g_R)}{(w(g_D) - w(g_R))^2} = (t_D' - 1) \frac{\partial \theta_C^*}{\partial t_R}.$$
 (32)

This and equations (10), and (11) proves the third statement of the Lemma.

Next, note that (31), (32) and (15) imply that the slope $H(t_R) = S(t_R)$, where $S(t_R)$ is given by (21). Thus, the candidates' reaction functions have the same slope as $(\theta_L(t_R), \theta_C(t_D(t_R)))$.

Let

$$B(t_R) = (w(f_D(t_D(t_R))) - w(f(R(t_R)))) - (t_D - t_R)a_Rw'(f_R(t_R))$$
(33)

Then (21) implies that

$$\frac{\partial B(t_R)}{\partial t_R} = -(t_D - t_R) f_R^{\prime 2}(t_R) w^{\prime \prime}(f_R(t_R))). \tag{34}$$

Thus,

$$\frac{\partial H(t_R)}{\partial t_R} = \frac{\partial S(t_R)}{\partial t_R} = (\rho + \delta) \frac{w''(g_R) f_R'^2(t_R) (w(g_D) - w(g_R))}{\left((w(g_D) - w(g_R)) - ((t_D - t_R) - \delta) a_R w'(g_R) \right)^2},\tag{35}$$

Let $\hat{\kappa}$ be the signed curvature of the curve. Then as in equation (28) it follows that

$$\hat{\kappa} = \frac{\partial H(t_R)}{\partial t_R} \frac{\left(\frac{\partial \tilde{\theta}_L(t_R)}{\partial t_R}\right)^2}{\left(\left(\frac{\partial \tilde{\theta}_L}{\partial t_R}\right)^2 + \left(\frac{\partial \tilde{\theta}_C}{\partial t_R}\right)^2\right)^{3/2}}.$$
(36)

Thus, (28), (31), (32), and (36) imply

$$\hat{\kappa} = \frac{\partial S(t_R)}{\partial t_R} \frac{(t_D' - 1)^2 \left(\frac{\partial \theta_L^*(t_R)}{\partial t_R}\right)^2}{\left((t_D' - 1)^2\right)^{3/2} \left(\left(\frac{\partial \theta_L^*}{\partial t_R}\right)^2 + \left(\frac{\partial \theta_L^*}{\partial t_R}\right)^2\right)^{3/2}} = \frac{\kappa_R}{|t_D' - 1|}.$$

Proof of Lemma 2. If $f_R(t_R^*)$ $< f_D(t_D^*)$ then in equilibrium t_R^* must solve $\max_{t_R} G(k_R(t_R))$, while t_D^* solves $\min_{t_D} G(k_D(t_D))$. The first order conditions are given by $D_{t_R}k_R(t_R^*) \cdot \nabla G(\theta_L^*, \theta_C^*) = D_{t_D}k_R(t_D^*) \cdot \nabla G(\theta_L^*, \theta_C^*) = 0$.

If $f_R(t_R^*) > f_D(t_D^*)$ then in equilibrium t_R^* must solve $\min_{t_R} G(k_R(t_R))$, while t_D^* solves $\max_{t_D} G(k_D(t_D))$. The first order conditions is therefore $D_{t_R}k_R(t_R^*) \cdot \nabla G(\theta_L^*, \theta_C^*) = 0$.

Proof of Theorem 1. Lemma 2 immediately implies that (15) and (16) must hold. Next, suppose that $f_D(t_D^*) > f_R(t_R^*)$. Then $\theta_C^* > \theta_L^*$. As a consequence, Lemma 1 implies that $\kappa_R, \kappa_D > 0$.

The equation of the isoprobability curve is $(\theta_L, \theta_C(\theta_L))$, where, $\theta_C(\theta_L)$ solves $G(\theta_L, \theta_C(\theta_L)) = G(\theta_L^*, \theta_C^*)$. Thus, the curvature of the isoprobability curve can be computed using equation (12)as

$$\kappa_{G}(\theta_{L}^{*}, \theta_{C}^{*}) = \frac{\frac{\partial^{2}G(\theta_{L}^{*}, \theta_{C}^{*})}{\partial \theta_{L}^{2}} \left(\frac{\partial G(\theta_{L}^{*}, \theta_{C}^{*})}{\partial \theta_{C}}\right)^{2} - 2\frac{\partial^{2}G(\theta_{L}^{*}, \theta_{C}^{*})}{\partial \theta_{L} \partial \theta_{C}} \frac{\partial G(\theta_{L}^{*}, \theta_{C}^{*})}{\partial \theta_{L}} \frac{\partial G(\theta_{L}^{*}, \theta_{C}^{*})}{\partial \theta_{C}} + \frac{\partial^{2}G(\theta_{L}^{*}, \theta_{C}^{*})}{\partial \theta_{C}^{2}} \left(\frac{\partial G(\theta_{L}^{*}, \theta_{C}^{*})}{\partial \theta_{L}}\right)^{2}}{\left(\frac{\partial G(\theta_{L}^{*}, \theta_{C}^{*})}{\partial \theta_{L}}\right)^{2} + \left(\frac{\partial G(\theta_{L}^{*}, \theta_{C}^{*})}{\partial \theta_{C}}\right)^{2}}.$$
(37)

The isoprobability curve at (θ_L^*, θ_C^*) is concave toward the origin if $\kappa_G < 0$, and convex if $\kappa_G > 0$. Thus, if $\kappa_G < 0$ a necessary condition is that $\kappa_R \ge -\kappa_G$. If, instead, $\kappa_G > 0$ then in a local equilibrium the curvature κ_D cannot be strictly less than κ_G , i.e., $\kappa_D \ge \kappa_G$.

The argument is similar if $f_D(t_D^*) < f_R(t_R^*)$. In this case we must take into account that the curvatures κ_R and κ_D are negative. As a consequence, we get $|\kappa_R| \ge -\kappa_G$ and $|\kappa_D| \ge \kappa_G$.

To prove the reverse implication, note that if (15) and (16) hold then Lemma 2 implies that curves k_R and k_D are tangent at t_D^* and t_R^* to the isoprobability curves. If $|\kappa_R| > -\kappa_G$ and $|\kappa_D| > \kappa_G$ then above argument implies that locally the curvature of the isprobability curve is strictly less than of k_D and k_R . As a consequence, we have a local equilibrium.

The final statement of the Theorem requires the curvature condition to hold for all points above the 45 degree line if $f_D(t_D^*) > f_R(t_R^*)$, or for all points below the 45 degree axis, otherwise. Thus, k_R and k_D only touch the isoprobability curve at (θ_L^*, θ_C^*) and we have a semi-global equilibrium. \Box

Proof of Lemma 4. 1. If p=1/2 and the density function f is symmetric around some point $\bar{\theta}$, then, for a tie to occur in state $\bar{\omega}$, $\theta_L - \bar{\omega}$ and $\theta_C - \bar{\omega}$ must be located symmetrically around $\bar{\theta}$, i.e., the isoprobability curve associated with $\bar{\omega}$ are straight lines of the form $(\bar{\theta} - h + \bar{\omega})$, $\bar{\theta} + h + \bar{\omega}$), $h \in \mathbb{R}$, with curvature $\kappa_G \equiv 0$. Thus, if p=1/2, any solution to the necessary conditions of Lemma 2 is a local equilibrium and a semi-global equilibrium. Note that this result is independent of the distribution of θ -types, F.

2. If θ_C and θ_L are normally distributed, then Equations (19) and (37) imply that

$$\kappa_{G} = -\frac{2p(1-p)2e^{-\frac{(\theta_{L}-\mu-\bar{\omega})^{2}+(\theta_{C}-\mu-\bar{\omega})^{2}}{2\sigma^{2}}}}{\sigma^{2}\left(pe^{-\frac{(\theta_{L}-\mu-\bar{\omega})^{2}}{2\sigma^{2}}}+(1-p)e^{-\frac{(\theta_{C}-\mu-\bar{\omega})^{2}}{2\sigma^{2}}}\right)\left(\frac{\theta_{L}+\theta_{C}}{2}-\mu-\bar{\omega}\right).$$
(38)

If p=0.5, then (19) implies $\frac{\theta_L+\theta_C}{2}=\mu+\bar{\omega}$, and hence $\kappa_G=0$, resulting in linear isoprobability curves. If p is close to 0.5 then $\frac{\theta_L+\theta_C}{2}-\mu+\bar{\omega}$ does not differ too much from zero, and hence the curvature κ_G remains close to zero.

On the 45-degree line ($\theta_L = \theta_C$), (19) implies that $\theta_L = \theta_C = \mu + \bar{\omega}$. As a consequence, $\kappa_G = 0$ for any p. Now suppose that p > 0.5. For candidates to tie in this case, the Republican needs to attract the support of more conservatives than the Democrat attracts liberals. Since the density f is symmetric, this requirement implies that $-(\theta_L - \mu - \bar{\omega}) < \theta_C - \mu - \bar{\omega}$ for $\theta_L < \theta_C$, and the reverse inequality holds for $\theta_L > \theta_C$. As a consequence $\frac{\theta_L + \theta_C}{2} > \mu + \bar{\omega}$ and hence $\kappa_G < 0$ for $\theta_L < \theta_C$, and $\kappa_G > 0$ for $\theta_L > \theta_C$. Thus, isoprobability curves are convex above the 45 degree line, and concave below the 45 degree line. It follows immediately that this result is reversed for p < 0.5, i.e., isoprobability curves are concave above the 45 degree line and convex below.

Next, note that increasing σ will move the curvature closer to zero. In particular, as $\sigma \to \infty$ all exponentials in (38) converge to 1, and hence $\kappa_G \to 0$, i.e., isoprobability curves become straight lines. The slope of the isoprobability curve is given by

$$\theta_C'(\theta_L) = -\frac{pe^{-\frac{(\theta_L - \mu - \bar{\omega})^2}{2\sigma^2}}}{(1 - p)e^{-\frac{(\theta_C - \mu - \bar{\omega})^2}{2\sigma^2}}}.$$

which is equivalent to (20). Note that (20) converges to -p/(1-p) as $\sigma \to \infty$.

Let $\theta_L < \theta_C$. Then (19) implies that $(\theta_L + \theta_C)/2 < \mu + \bar{\omega}$ if p < 0.5. Thus, the argument of the exponential function in (20) is strictly negative. Increasing σ therefore decreases $\theta'_C(\theta_L)$, i.e., the slope becomes steeper. The reverse is true if p > 0.5.

Now suppose that the percentage of liberals, p, increases, and consider the isoprobability curve through (θ_L, θ_C) . Equation (19) implies that $\bar{\omega}$ must decrease. Thus, the argument of the exponential function in (20) increases. This and the increase of p implies that $\theta'_C(\theta_L)$ decreases, i.e., the slope of the isoprobability curve through (θ_L, θ_C) becomes steeper.

Finally, consider a change in μ . In order for (19) to hold after a change of μ , $\bar{\omega} + \mu$ must remain constant. Thus, a change in μ does not affect $\theta'_C(\theta_L)$.

8 Existence of Global Equilibria

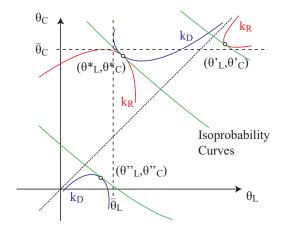


Figure 8: Existence of a global Equilibrium.

We now provide conditions for the existence of global equilibria. By Assumption 1, candidate D has higher fixed costs but lower marginal costs. Consider tax rates t_R^* , t_D^* that satisfy the first-order condition, and for which D provides more of the public good. Then the associated (θ_L^*, θ_C^*) is above the 45 degree axis. If the equilibrium is semi-global, then any deviation that remains above the 45 degree line cannot increase a candidate's probability of winning the election.

Thus, consider deviations to points below the 45-degree line. If the deviation is by candidate D, then it involves a tax rate at which he provides strictly less of the public good than candidate R. In the right panel of Figure 8, the optimal such deviation implements (θ''_L, θ''_C) — note that, for $g_D < g_R$, the k_D -curve becomes convex and D is supported by all voters below the cutoff (θ''_L, θ''_C) . Thus, he maximizes his winning probability by moving to the highest possible isoprobability curve.

Define $\bar{\theta}_C$ to be the highest possible cutoff among conservatives that R can achieve as long as he provides less of the public good than D. Similarly, let $\bar{\theta}_L$ be the lowest possible cutoff among liberals that D can achieve as long as he provides more of the public good than R (see the right panel of Figure 8). Formally,

$$\bar{\theta}_C = \max_{\{t_R | f_R(t_R) < f_D(t_D^*)\}} \theta_C^*(t_R), \text{ and } \bar{\theta}_L = \min_{\{t_D | f_D(t_D) > f_R(t_R^*)\}} \theta_L^*(t_R).$$
(39)

Theorem 4 below show that $(\theta''_L, \theta''_C) < (\bar{\theta}_L, \bar{\theta}_C)$. Candidate D's winning probability in the original equilibrium is $1 - G(\theta^*_L, \theta^*_C)$, since he receives the support of all voters above θ^*_L , and θ^*_C , respectively. After the deviation, he receives the support of all voters below θ^*_L , and θ_C *, and his winning probability is $G(\theta''_L, \theta''_C)$. Since isoprobability curves have strictly negative slope, $G(\theta''_L, \theta''_C) < G(\theta^*_L, \theta^*_C)$. Thus, if $G(\theta^*_L, \theta^*_C) = 1/2$, then it is guaranteed that D's winning probability when deviating, $G(\theta''_L, \theta''_C)$ is lower than D's equilibrium winning probability $1 - G(\theta^*_L, \theta^*_C) = 1/2$.

An analogous argument shows that deviations by candidate R are not profitable if $G(\theta_L^*, \theta_C^*)$ is sufficiently close to 1/2.

For the formal statement of this result in Theorem 4, we need to guarantee that the branches of k_R and k_D above the 45 degree line resemble those in Figure 8, that is, cutoffs $\bar{\theta}_C$ and $\bar{\theta}_L$ exists. The following assumption is sufficient for this.

Assumption 3. Let \hat{t}_R be defined by $f_R(\hat{t}_R) = f_D(t_D^*)$, and \hat{t}_D by $f_R(t_R^*) = f_D(\hat{t}_D)$. Then $\hat{t}_R > t_D^* + \rho$ and $\hat{t}_D > t_R^* + \delta$.

Note that $\hat{t}_R > t_D^*$ is the tax rate that the Republican would have to charge in order to provide the same amount of public goods as the Democrat does in equilibrium, so that candidates differ only in tax rate and ideology. Thus, all voters of the same ideology (irrespective of their θ) have the same preference over candidates. To avoid corner solutions and indeterminateness of equilibrium tax rates, Assumption 3 requires that deviating to \hat{t}_R is unattractive for the Republican (he would even lose all conservatives). The analogous condition for the Democratic candidate is $\hat{t}_D > t_R^* + \delta$. Clearly, Assumption 3 restricts the size of ρ and δ , because if ideology overwhelms all economic considerations, then all conservatives vote for candidate R and all liberals for candidate R. Further, note that ρ and δ can be larger if the difference between f_D and f_R increases, because this raises both \hat{t}_R and \hat{t}_D .

Theorem 4. Let (t_R^*, t_D^*) be a semi-global equilibrium with $f_R(t_R^*) < f_D(t_D^*)$, and suppose that Assumption 3 is satisfied. Then

- 1. $\bar{\theta}_C$ and $\bar{\theta}_D$ defined in (39) exists and $(\bar{\theta}_C, \bar{\theta}_C) > (\theta_L^*, \theta_C^*) > (\bar{\theta}_L, \bar{\theta}_L)$.
- 2. If $1 G(\bar{\theta}_L, \bar{\theta}_L) > G(\theta_L^*, \theta_C^*) > 1 G(\bar{\theta}_C, \bar{\theta}_C)$. then (t_D^*, t_R^*) is a (global) Nash equilibrium.

Proof. Note $g_D = g_R$ if candidates at t_D^* and \hat{t}_R . Let $t_R' < \hat{t}_R$ such that $t_R' > t_D^* + \rho$. Thus, (6) implies that $\theta_C^*(t_D^*, t_R') < 0$. Further, $\theta_C^*(t_D^*, 0) > 0$. This. continuity and compactness imply that

$$\bar{\theta}_C = \max_{0 \le t_D \le \hat{T}_D} \theta_C^*(t_D^*, t_R) \tag{40}$$

exists.

Next, note that $\bar{\theta}_C > \theta_C^*$. Clearly, $\bar{\theta}_C \geq \theta_C^*$. Thus, suppose that $\bar{\theta}_C = \theta_C^*$. This, however, means that $\frac{\partial \theta_C^*}{\partial t_R}(t_D^*, t_R^*) = 0$. This and (10) imply $\frac{\partial \theta_L^*}{\partial t_R}(t_D^*, t_R^*) < 0$. However, since $\frac{\partial G}{\partial t_L}(\theta_L^*, \theta_C^*) \neq 0$, this implies that statement 2 of Lemma 2 is violated. Thus, $\bar{\theta}_C > \theta_C^*$.

Equations 5 and 6 imply $(\bar{\theta}_C, \bar{\theta}_C) > (\theta_L^*, \theta_C^*)$. Since the equilibrium is local, any deviation \tilde{t}_R by candidate R with $f_R(\tilde{t}_R) \leq f_D(t_D^*)$ cannot be optimal. Thus, consider a deviation \tilde{t}_R with $f_R(\tilde{t}_R) > f_D(t_D^*)$. Let $\tilde{\theta}_C$, $\tilde{\theta}_L$ be the new cutoff voters. Now candidate R receives the support of all types $\theta_C \geq \tilde{\theta}_C$ and $\theta_L \geq \tilde{\theta}_L$, where $\tilde{\theta}_C < \tilde{\theta}_L$. We now show that $\bar{\theta}_C < \tilde{\theta}_C$.

Suppose by way of contradiction that $\bar{\theta}_C \geq \tilde{\theta}_C$. Let $\bar{t}_R \in \arg\max_{0 \leq t_R < \hat{t}_R} \theta_C^*(t_D^*, t_R)$. Then

$$\bar{t}_R \in \arg\max_{t_R < \hat{t}_R} m(1 - t_R) + \bar{\theta}_C w(f_R(t_R)) + \rho. \tag{41}$$

Else, if (41) is violated then there exists t_R' that gives type $\bar{\theta}_R$ a strictly higher utility. This, however, means that $\bar{\theta}_R$ strictly prefers candidate R to candidate D if candidates choose t_D^* and t_R' , respectively. Thus, by continuity there would exist $\theta_R' > \theta_R$ such that θ_R' strictly prefers R to D. This, however, contradicts 40. Hence, (41) must hold.

Next, note that $m(1 - t_R) + \bar{\theta}_C w(f_R(t_R)) + \rho$ is strictly concave in t_R . Thus, \bar{t} in (41) is the unique maximum, even if we eliminate the constraint that $t_R < \hat{t}_R$.

If $\bar{\theta}_C \geq \tilde{\theta}_C$ then type $\bar{\theta}_C$ is at least as well from candidate R with tax rate \tilde{t}_R than from candidate C with tax rate t_D^* . This, however, means that $\bar{\theta}_C$'s utility is at least as high from candidate R with tax rate \bar{t}_R . This, however, contradicts that \bar{t}_R is the unique solution to (41). Thus, $\bar{\theta}_C < \tilde{\theta}_C$. Hence, the winning probability from deviating is $1 - G(\tilde{\theta}_L, \tilde{\theta}_C) \leq 1 - G(\bar{\theta}_C, \bar{\theta}_C)$. Thus, the assumption that $G(\theta_L^*, \theta_C^*) > 1 - G(\bar{\theta}_C, \bar{\theta}_C)$ implies that such a deviation is not optimal.

Next, we consider deviations by candidate D. Since $\hat{t}_D > t_R^* + \delta$, equation (5) implies that $\lim_{t_D \downarrow \hat{t}_D} \theta_L^*(t_D, t_R^*) = \infty$. Thus, we can conclude that

$$\bar{\theta}_L = \min_{\hat{t}_D < t_D < 1} \theta_L^*(t_D, t_R^*) \tag{42}$$

exists.

The remainder of the argument is similar to that for $\bar{\theta}_C$. In particular, it follows that $\bar{\theta}_L < \theta_L^*$, and that consequently $(\theta_L^*, \theta_C^*) > (\bar{\theta}_L, \bar{\theta}_L)$. Similar to above it follows that the utility of type $\bar{\theta}_L$ is maximized when candidate D chooses tax rate \bar{t}_D that solves (42). As a consequence, concavity of utility implies that any deviation $t_D' \in [0, 1]$ makes type $\bar{\theta}_L$ worse off. It also follows again that, if $t_D' > \hat{t}_D$ then the new cutoff voter $\tilde{\theta}_L < \hat{\theta}_L$. Candidate D's winning probability after the deviation is $G(\tilde{\theta}_L, \tilde{\theta}_C) < G(\hat{\theta}_L, \hat{\theta}_C) < G(\hat{\theta}_L, \hat{\theta}_L)$, where the last inequality follows since $\tilde{\theta}_C < \tilde{\theta}_L$ for $t_D > t_D'$. Thus, a sufficient condition for the deviation not to be optimal is $1 - G(\theta_L^*, \theta_C^*) > G(\hat{\theta}_L, \hat{\theta}_L)$, which is equivalent to the condition $1 - G(\bar{\theta}_L, \bar{\theta}_L) > G(\theta_L^*, \theta_C^*)$ in the statement of the theorem. \Box

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