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Abstract

There is clear evidence that fairness plays a role in redistribution. Individuals want to compensate others for their misfortune, while they allow them to enjoy the fruits of their effort. This paper introduces fairness in a tax-benefit scheme that is based on several characteristics in order to study the design of optimal taxes where people have what we call '*partial control*'. For some characteristics like sex, age and inborn handicaps the degree of control is zero (i.e., these characteristic are exogenous tags fully defined by the individual's type), while for other characteristics, think of education and family composition, the degree of control is positive, i.e. it can be changed by exerting effort. We derive the fair tax benefit formula as well as two testable predictions. We provide the first estimates of implicit tax rates for different characteristics in 26 European countries (using EU-SILC data) and the US (using CPS data) and find a robust tendency in all countries to compensate more for uncontrollable characteristics compared to the partially controllable ones. We then attempt to calculate which countries currently have fair tax systems. Only the Continental countries France and Luxembourg pass the fairness test, whereas the Baltic and Anglo-Saxon countries (including the US) perform worst. Our paper provides a new way to formalize the old intuition that, in a fair society, people should be allowed to benefit more from their own efforts than from exogenous characteristics like their genetic endowment.

JEL-Code: D600, H200, I300.

Keywords: fairness, redistribution, tax-benefit schemes, tagging, optimal taxation.

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1 Introduction

Economic models are often based on the premise that individuals are motivated only by their material self-interest. But experiments systematically reject the pure self-interest hypothesis; see, e.g., Fehr and Schmidt (2006) for an overview. Other considerations, like fairness, do play a role for redistribution. If earnings are a combination of luck (drawn by nature) and effort (chosen by the agent), then fairness urges to compensate individuals for unlucky draws by nature, while it allows individuals to enjoy the fruits of their effort. Empirical evidence shows that (the belief about) the relative importance of effort and luck in the determination of income systematically correlates with people's preferences for redistribution. The more (they believe that) income is determined by luck, the more redistribution is preferred; see Alesina and Giuliano (2010) for an overview of evidence based on social survey data, Gaertner and Schokkaert (2010) for an overview of experimental tests using structured questionnaires, and Konow (2003) for an overview of experimental laboratory evidence.

Fairness considerations have been introduced in political economy and optimal income tax models. Alesina et al. (2001) show that different beliefs about the importance of luck for income acquisition can help explain the divergence in the levels of redistribution in different democratic societies. The political economy models of Piketty (1995), Alesina and Angeletos (2005) and Bénabou and Tirole (2006) show that multiple equilibria can arise in such a way that stronger beliefs in the role of effort coincide with lower levels of redistribution. Under the influence of Rawls' (1971) seminal work, a similar notion of fairness has been introduced in the literature on distributive justice; see, e.g., Kymlicka (2002) for an overview. All these studies share a selective-egalitarian viewpoint: some inequalities in outcomes are justifiable (and should not be corrected), while others cannot be justified (and should be eliminated as much as possible). This fairness notion has been used to refine optimal income tax schemes in the (so-called) fair income tax literature.[1]

Although earnings have been the main focus in the previous political economy and fair income tax models, actual tax-benefit schemes are based on much more information than earnings only. And on average this non-income information turns out to be a more important source of variation in tax payments in Europe and the US. Different theoretical reasons have been put forward in the optimal income tax literature since Mirrlees' (1971) seminal contribution.[2] If externalities exist, then there is a role for government to subsidize or tax these activities à la Pigou (1920) to restore efficiency. If there exist tags—observable, usually exogenous characteristics that correlate with unobserved abilities or tastes—then Akerlof (1978) shows that differentiating the tax-benefit system on the basis of these tags ('tagging') can also enhance efficiency. Equity considerations can provide another rationale to differentiate tax-benefit schemes. The work of Mirrlees (1972) and Boskin and Sheshinsky (1983) discuss the optimal income tax treatment of family size and couples, respectively.

In this paper we want to *derive* and *test* a *fair and efficient* tax-benefit scheme that is based on *several characteristics*; and each characteristic can be different in terms of the *degree of control*, i.e., the extent to which it can be changed by exerting effort. We preview the core ingredients:

1. **Fair and efficient taxation.** In the standard optimal income tax problem, individual heterogeneity is usually due to unobservable differences in productivities (or types). The fair income tax literature

[1] See Roemer et al. (2003), Schokkaert et al. (2004), Fleurbaey and Maniquet (2006, 2007), Luttens and Ooghe (2007), and Jacquet and Van de gaer (2010).

[2] See, e.g., Salanié (2003) and Mankiw et al. (2009) for overviews.

adds unobservable differences in tastes for effort as a second, but normatively distinct, source of heterogeneity. These taste differences make the interpersonal comparison of utilities difficult and bring the question of fairness—which inequalities are justifiable and which are not—to the fore. To deal with this, we follow Fleurbaey and Maniquet (2006) and keep individuals responsible for their tastes, but not for their types. Two plausible fairness principles, compensation and responsibility, result. If outcome differences between two individuals are only due to differences in their types, then compensation approves of a transfer from the better off to the worse off. Responsibility demands that the laisser-faire is selected if all individuals have the same type. Indeed, in such a case all remaining differences in outcomes can only be due to differences in tastes for which individuals are (held) responsible. These fairness principles, in conjunction with efficiency, constitute the core properties of a fair and efficient planner.

2. **Partial control.** Individuals differ in several characteristics, each of which we model as a weighted combination of type (drawn by nature) and effort (chosen by the individual). We refer to this weight as the degree of control. For some characteristics like sex, age and inborn handicaps the degree of control is zero (i.e., these characteristic are exogenous tags fully defined by the individual's type), while for other characteristics, think of education and family composition, the degree of control is positive and thus partial control applies. The degree of control will play a crucial role in the shape of the resulting fair and efficient tax-benefit scheme.

3. **Theory.** The complexity of multidimensional screening exercises forces us to simplify some aspects of the model to keep analytical tractability. In section 2 we set up a model, assuming a linear production technology, quasi-linear preferences defined over consumption and multidimensional effort, independent multivariate normal distributions for types and tastes, and linear tax rates on the different characteristics. Despite being linear in each characteristic, the introduction of non-income characteristics is an important source of non-linearity in tax-benefit schemes. We show that the tax rates should decrease with the degree of control. Two testable predictions result. First, fairness and efficiency require that the tax rate on partially controllable characteristics should be lower compared to the tax on non-controllable characteristics. Second, tax rates should be set to guarantee that the total effect of the non-controllable characteristics on the post-tax outcome equals zero. That is, outcome differences due to exogenous tags should be fully compensated by the tax-benefit system.

4. **Evidence.** In section 3 we estimate and discuss the implicit tax rates for a number of characteristics in 26 European countries (using the 2007 EU-SILC data) and the US (using the CPS data). We find a robust tendency in all countries to compensate more for the non-controllable characteristic (a composite based on sex, age and disability in our study) compared to the partially controllable one (based on family composition, immigration status, unemployment and education level). We also estimate the degree of fairness of the different tax-benefit schemes: how close to zero is the total effect of the non-controllable composite on the post-tax outcome? The Baltic States (Latvia, Estonia and Lithuania) and the Anglo-Saxon countries (the United Kingdom, Ireland and the United States) are least fair. Although the Northern countries (Sweden, Denmark, Norway, Finland and Iceland) do better in terms of fairness, they are in turn outperformed by some Central Eastern and Southern countries (Poland, Hungary, Slovenia, Czech Republic, Slovakia and Italy) as well as by most continental countries (France, Luxembourg, Austria, Germany, the Netherlands and Belgium).

Among the latter, only France and Luxembourg pass the fairness test.

2 Theory

In the first part of this section, we describe the basic building blocks: production technology, individual preferences, and the social preference of a fair and efficient planner. In the second part we describe and discuss the theoretical results, with a focus on two special cases: the 'Mirrlees'-case (one taxable characteristic, say, earnings) and the 'Akerlof'-case (two taxable characteristics, say, earnings and an exogenous tag).

2.1 Model

To keep things simple, the model is additive: output is linear in characteristics and characteristics are linear in effort and type, preferences are quasi-linear in (net) output, and the welfare function will average (a concave transformation of) utilities. The additive specification is convenient in terms of interpretation, and it allows capturing the essence of a second-best information problem. We show in the appendix that a multiplicative variant of the model leads to the same qualitative results.

Production technology. Individuals (or households) can be described by a vector $x \in \mathbb{R}^J$, with J a finite set of characteristics. Although we present the model for an arbitrary number of characteristics, we often focus on the case with one or two characteristics.[3] The pre-intervention or gross outcome is denoted y. It can be anything at this stage, e.g., earnings, life-time income, or a broader concept of welfare. The general model is therefore not necessarily restricted to labor taxation only. Output is assumed to be a linear function of the different characteristics of the individual, or formally:

$$y = \beta_0 + \textstyle\sum_{j \in J} \beta_j x_j. \tag{1}$$

Without loss of generality, we assume $\beta = \left(\beta_j\right)_{j \in J} \gg 0$, and 0 denotes a vector of zeros of appropriate length. Characteristics are a combination of effort $e \in \mathbb{R}^J$ and type $\theta \in \mathbb{R}^J$, i.e., for each j in J we assume

$$x_j = \alpha_j e_j + (1 - \alpha_j)\, \theta_j. \tag{2}$$

The weights of effort—one weight for each characteristic—are collected in a vector $\alpha \in (0,1)^J$. This vector is the same for all individuals and defines the '*degree of control*' for each characteristic in between the extremes of no control ($\alpha_j \to 0$; the characteristic is pure type) and full control ($\alpha_j \to 1$; the characteristic is pure effort). In contrast to the characteristics, effort and type are not observable to the planner (but the multivariate type distribution is known).

Some special cases arise. First, if there is only one characteristic, say earnings x_1, and assuming $\beta_0 = 0$ and $\beta_1 = 1$, then $y = x_1 = \alpha_1 e_1 + (1 - \alpha_1)\, \theta_1$, and we obtain an additive version of what we call the 'Mirrlees'-case.[4] Next, if there are two characteristics, individual earnings $x_1 = \alpha_1 e_1 + (1 - \alpha_1)\, \theta_1$ and a tag, an exogenous characteristic denoted $x_2 \to \theta_2$ (given $\alpha_2 \to 0$) and if $\beta_0 = 0$ and $\beta_1 = 1$, then

[3] In the empirical part we will partition all characteristics into two groups such that the theory for two characteristics applies to these two groups as a whole.

[4] Since we focus on linear taxes as in Sehshinski (1972), a better term might be the 'Sheshinski'-case.

$y = x_1 + \beta_2 x_2 \rightarrow (\alpha_1 e_1 + (1-\alpha_1)\,\theta_1) + \beta_2\theta_2$, and we arrive in the so-called 'Akerlof'-case. Note that the tag $x_2 \rightarrow \theta_2$ can both correlate with the earnings ability θ_1 and affect well-being directly (via $\beta_2 > 0$).[5]

Preference technology. Individual utility is equal to the net outcome c (to be defined later) minus the cost of effort; no externalities occur. We assume:

$$U(c, e; \gamma, \delta) = c - \sum_{j\in J} \frac{\delta_j}{\exp(\gamma_j)} \exp\left(\frac{e_j}{\delta_j}\right), \tag{3}$$

with $\gamma \in \mathbb{R}^J$ a vector of taste parameters which defines the disutility of effort, and $\delta \in \mathbb{R}^J$, with $\delta \gg 0$, a vector controlling the degree of convexity of the cost of effort. This is a multidimensional version of the classical quasi-linear preferences which are often used in optimal tax theory to simplify the theoretical analysis by excluding income effects (see , e.g., Diamond, 1998). As usual, higher values for γ correspond with lower disutility of effort, which can be thought of as more ambitious individuals; higher values for δ correspond with more elastic responses to effort and can be interpreted as the cost of taxation for the different characteristics. In contrast to the taste vector γ, the elasticity vector δ is assumed to be the same for all individuals.

Net outcomes and behavior. The instruments of the social planner[6] are restricted to 'basic income-flat tax' schemes.[7] However, the introduction of non-income characteristics is a far more important source of non-linearity in tax-benefit schemes. In the countries we analyze in the empirical part, the variation in taxes is mainly explained by non-income characteristics (49% on average) and income (30%), while higher-order terms for income do not play an important role (5%).[8] Therefore, despite being linear in each characteristic, the resulting tax-benefit scheme is non-linear in income – depending on the correlation and distribution of each characteristic. Finally, we interpret these tax rates as implicit tax rates driving behavior, irrespective of whether or not these rates are explicit in a legal sense. For example, although there is no explicit tax on education, a more progressive tax scheme might reduce the returns to education and could therefore change educational choices. Formally, the net outcome c satisfies

$$c \leq y - t_0 - \sum_{j\in J} t_j x_j, \tag{4}$$

with $t_0 \in \mathbb{R}$ controlling the overall level of the net outcome, and $t \in \mathbb{R}^J$ the tax rates applied to the different (observable) characteristics.

Types and tastes are private information; in particular, we assume that individuals know their type when choosing effort. However, all results would remain the same if individuals only knew the distribution of types and effort choices were modeled via expected utility maximization.[9] Lemma 1 summarizes behavior, i.e., choice and indirect utility.

[5] Note also that the 'Boskin-Sheshinsky'-model for the optimal taxation of couples can be derived as a special case: choosing $\beta_0 = 0$ and $\beta_1 = \beta_2 = 1$ we have $y = x_1 + x_2$ with $x_1 = \alpha_1 e_1 + (1-\alpha_1)\,\theta_1$ and $x_2 = \alpha_2 e_2 + (1-\alpha_2)\,\theta_2$ the earnings of the partners in a couple. We do not further discuss this case here.

[6] Note that the fiction of a 'social planner' is a proxy for a more complex political model; see, e.g., Coughlin (1992), who shows equivalence between a planner with a weighted social welfare function and a probabilistic voting model with two candidates competing for votes.

[7] Although restrictive compared to non-linear tax instruments, linear schemes could be close to optimal, at least for income taxation (see, e.g., Mankiw et al., 2009 for a discussion).

[8] The remaining part is either unexplained (12%) or due to covariances between the observed characteristics (4%).

[9] Types can thus also be interpreted as representing good or bad luck for which individuals ought to be compensated.

Lemma 1. Maximization of (3) with respect to (1), (2) and (4), leads to an effort choice[10]

$$e_j^* = \delta_j \left(\ln\left(\left(\beta_j - t_j\right)\alpha_j\right) + \gamma_j\right) \text{ for all } j \text{ in } J, \tag{5}$$

which results in the characteristics

$$x_j^* = \alpha_j e_j^* + (1-\alpha_j)\,\theta_j = \alpha_j \delta_j \left(\ln\left(\left(\beta_j - t_j\right)\alpha_j\right) + \gamma_j\right) + (1-\alpha_j)\,\theta_j, \tag{6}$$

and the corresponding indirect utility $V(t_0, t; \alpha, \beta_0, \beta, \delta; \gamma, \theta)$ equals

$$\kappa(t_0, t; \alpha, \beta_0, \beta, \delta) + \textstyle\sum_{j\in J}\left(\beta_j - t_j\right)\alpha_j\delta_j\gamma_j + \sum_{j\in J}\left(\beta_j - t_j\right)(1-\alpha_j)\,\theta_j, \tag{7}$$

with

$$\kappa(t_0, t; \alpha, \beta_0, \beta, \delta) = \beta_0 - t_0 + \textstyle\sum_{j\in J}\left(\beta_j - t_j\right)\alpha_j\delta_j\left[\ln\left(\left(\beta_j - t_j\right)\alpha_j\right) - 1\right]. \tag{8}$$

A fair and efficient planner. The planner observes the multivariate type distribution F which is assumed to be independent from the multivariate taste distribution G.[11] For analytical tractability, we use normal distributions, or

$$\theta \sim N\left(\mu^\theta, \Sigma^\theta\right) \quad \text{and} \quad \gamma \sim N\left(\mu^\gamma, \Sigma^\gamma\right), \tag{9}$$

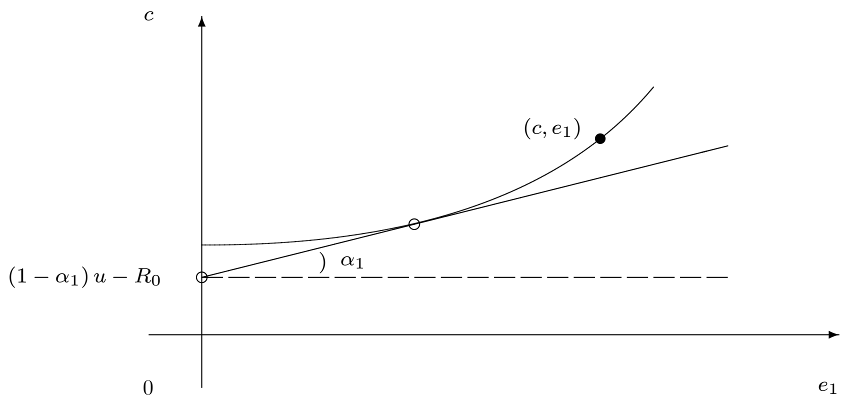
with $\mu = \left(\mu_j\right)_{j\in J}$ a vector of means and $\Sigma = \left(\sigma_{ij}\right)_{ij\in J^2}$ a variance-covariance matrix with $\sigma_{jj} > 0$ for all j in J and $\left(\sigma_{ij}\right)^2 < \sigma_{ii}\sigma_{jj}$ for all i, j in J (excluding perfect correlation). The social planner sets taxes t_0 and t to maximize welfare—to be introduced next—subject to a budget constraint, denoted by

$$t_0 + \textstyle\int\int \left(\sum_{j\in J} t_j x_j^*\right) dF(\theta)\, dG(\gamma) \geq R_0, \tag{10}$$

with R_0 an exogenous (per-capita) revenue requirement, x_j^* defined in equation (6), and the distributions F and G defined in equation (9). In order to define aggregate welfare, we assume that the planner balances efficiency and fairness. Efficiency is operationalized via the Pareto principle, while fairness is defined as selective egalitarianism: individuals are held responsible for their tastes, but not for their type. We discuss efficiency and fairness in an informal way in the next paragraph; see Fleurbaey and Maniquet (2006) for a formal discussion.[12]

A Pareto efficient planner defines welfare as an increasing function of individual well-being, and well-being is a specific cardinalization of utility defined in (3). But which cardinalization is normatively interesting? Fairness considerations can guide us. A selective egalitarian planner is egalitarian, but only with respect to those outcome differences that are caused by differences in type for which individuals are not (held) responsible. We select two plausible principles, compensation and responsibility. If two individuals have the same tastes and make exactly the same effort choices, then any remaining outcome differences can be traced back to differences in type, which are deemed relevant for redistribution. In this case the compensation principle approves of progressive Pigou-Dalton transfers, i.e., mean-preserving transfers from the richer to the poorer individual. If *all* individuals have the same type, then outcome differences in the laisser-faire allocation—i.e., the allocation which would be chosen by individuals in the absence of taxation—are only due to differences in tastes, which are deemed irrelevant for redistribution.

Figure 1: direct well-being u in the additive 'Mirrlees'-case



So, if *all* individuals have the same type, there is no reason to redistribute and the responsibility principle requires that the laisser-faire allocation should result.

We define the social planner's objective first, and link it back to efficiency and fairness afterwards. The social planner maximizes a Kolm-Pollak welfare function, i.e., welfare is a sum of increasing and concave exponential functions of well-being. Well-being is defined as a specific cardinalization of utility. More precisely, the (direct) well-being in a given bundle (c, e), denoted $u\,(c, e; \alpha, \beta_0, \beta, \delta; \gamma, \theta)$, is implicitly defined as the hypothetical type $\theta^H = (u, u, \ldots, u)$ which makes an individual indifferent between (1) the actual received bundle (c, e) and (2) the bundle the individual would choose—with her own tastes, but with this hypothetical type θ^H—in the laisser-faire, here defined as $(t_0, t) = (R_0, 0)$. Figure 1 illustrates the construction of direct well-being for the Mirrlees-case (with $y = x_1 = \alpha_1 e_1 + (1-\alpha_1)\,\theta_1$), obtained by changing the intercept of the laisser-faire budget set (a budget line with intercept $(1-\alpha_1)\,u - R_0$ and slope α_1) such that it is tangent to the indifference curve through the bundle (c, e_1); Lemma 2 derives the corresponding direct well-being index formally.

Lemma 2. Given a bundle (c, e), direct well-being u is implicitly defined by

$$U\,(c, e; \gamma, \delta) = V\,(R_0, 0; \alpha, \beta_0, \beta, \delta; \gamma, (u, u, \ldots, u))$$

with V the indirect utility function defined in Lemma 1. This results in $u\,(c, e; \alpha, \beta_0, \beta, \delta; \gamma)$ equal to

$$\frac{c - \sum_{j \in J} \frac{\delta_j}{\exp(\gamma_j)} \exp\left(\frac{e_j}{\delta_j}\right) - \kappa\,(R_0, 0; \alpha, \beta_0, \beta, \delta) - \sum_{j \in J} \alpha_j \beta_j \delta_j \gamma_j}{\sum_{j \in J} (1-\alpha_j)\,\beta_j}. \tag{11}$$

A social planner who maximizes a sum of increasing and concave exponential functions of well-being—with well-being defined in Lemma 2—is both Pareto efficient and selective egalitarian. Pareto efficiency

[10] We define $e_j^* \to -\infty$ for all tax levels $t_j > \beta_j$.

[11] Independence avoids the philosophical problem of whether we can hold individuals responsible for their tastes, if the latter correlate with type.

[12] A similar proposal has been made by Atkinson and Stiglitz (1976).

follows from the observation that welfare is increasing in well-being and well-being is a specific cardinalization of utility. For the compensation principle, note that direct well-being does not depend on type θ such that well-being differences between individuals with the same tastes and the same effort can only be due to differences in their net outcome c. Since welfare is a concave function of well-being and well-being is linear in net outcome c, Pigou-Dalton transfers increase welfare. To see why the responsibility principle holds, it is more convenient to work with the corresponding indirect well-being function, i.e., well-being measured at the bundle chosen by an individual for a given tax-benefit scheme (t_0, t). Lemma 3 provides us with the indirect well-being formula.

Lemma 3. Given a tax-benefit scheme (t_0, t), indirect well-being v is implicitly defined by

$$V(t_0, t; \alpha, \beta_0, \beta, \delta; \gamma, \theta) = V(R_0, 0; \alpha, \beta_0, \beta, \delta; \gamma, (v, v, \ldots, v))$$

with V the indirect utility function defined in Lemma 1.[13] This results in $v(t_0, t; \alpha, \beta, \delta; \gamma, \theta)$ equal to

$$\frac{\kappa(t_0, t; \alpha, \beta_0, \beta, \delta) - \kappa(R_0, 0; \alpha, \beta_0, \beta, \delta) - \sum_{j \in J} t_j \alpha_j \delta_j \gamma_j + \sum_{j \in J} \left(\beta_j - t_j\right)(1 - \alpha_j)\theta_j}{\sum_{j \in J}(1 - \alpha_j)\beta_j}. \quad (12)$$

From lemma 3 it follows that if all individuals have the same type, then they all obtain the same well-being level in the laisser faire defined by $(t_0, t) = (R_0, 0)$. As a consequence, deviating from $t = 0$ would decrease welfare, since both average well-being would decrease due to the efficiency cost of taxation and well-being inequality would increase.

2.2 Results

2.2.1 General result

The program of the social planner is to choose a tax-benefit scheme (t_0, t) in order to maximize welfare, a sum of increasing and concave exponential transformations of (indirect) well-beings, subject to a budget constraint; formally:

$$\max_{t_0, t} \; -\frac{1}{r} \ln \int\int \exp\left[-rv(t_0, t; \alpha, \beta, \delta; \gamma, \theta)\right] dF(\theta)\, dG(\gamma), \quad (13)$$

subject to the budget constraint (10), with $r > 0$ the inequality aversion parameter, R_0 the exogenous (per-capita) revenue requirement, indirect well-being $v(t_0, t; \alpha, \beta, \delta; \gamma, \theta)$ defined in lemma 3, and the distributions F and G defined in equation (9). Proposition 1 characterizes the general solution.

Proposition 1. The solution to the social planner's problem is characterized as follows:

1. the budget constraint (and efficiency) leads to

$$t_0^* = R_0 - \sum_{j \in J} t_j \alpha_j \delta_j \ln\left(\left(\beta_j - t_j\right)\alpha_j\right) - \sum_{j \in J} t_j \alpha_j \delta_j \mu_j^\gamma - \sum_{j \in J} t_j (1 - \alpha_j)\mu_j^\theta,$$

which can be plugged in in the welfare function to obtain welfare as a function of $\left(t; \alpha, \beta, \delta; r, R_0; \mu^\theta, \Sigma^\theta, \Sigma^\gamma\right)$ as defined in the appendix;

[13] Note that $V(R_0, 0; \alpha, \beta_0, \beta, \delta; \gamma, (v, v, \ldots, v))$ is strictly increasing in v, so v is well-defined and unique.

2. maximizing the previous welfare function w.r.t. t leads to a system of first-order conditions (one for each j in J) defined as

$$-\alpha_j\delta_j\frac{\zeta t_j}{\beta_j - t_j} - r\alpha_j\delta_j \sum_{k\in J} t_k\alpha_k\delta_k\sigma^{\gamma}_{kj} + r\left(1-\alpha_j\right)\sum_{k\in J}\left(\beta_k - t_k\right)\left(1-\alpha_k\right)\sigma^{\theta}_{kj} = 0,$$

with $\zeta = \sum_{j\in J}\left(1-\alpha_j\right)\beta_j > 0$. The solution t^* satisfies $t^* \ll \beta$ and is a global maximum.

Proof. See appendix.

There is little we can say in general. If the planner does not care about compensation ($r \to 0$) or if compensation is an empty requirement due to type homogeneity ($\Sigma^{\theta} \to 0$), then the laisser-faire results, i.e., $(t_0^*, t^*) = (R_0, 0)$, in the optimum. In the sequel we discuss two specific cases: the 'Mirrlees'-case, in which the outcome is defined by one endogenous characteristic (income), and the 'Akerlof'-case with an endogenous and an exogenous (non-controllable) characteristic (a tag). Especially the second case will provide us with testable hypotheses that do not depend on the (perceived) degree of control α or the inequality aversion r. This makes it particularly suitable for cross-country comparisons.

2.2.2 The 'Mirrlees'-case

To set the stage, we start with the simplest case possible. Suppose the outcome y is defined by one characteristic only, say earnings x_1, with $y = x_1 = \alpha_1 e_1 + (1-\alpha_1)\theta_1$. The system of first-order conditions in proposition 1 reduces to

$$-\alpha_1\left(1-\alpha_1\right)\delta_1\frac{t_1}{1-t_1} - r\left(\alpha_1\delta_1\right)^2 t_1\sigma^{\gamma}_{11} + r\left(1-\alpha_1\right)^2\left(1-t_1\right)\sigma^{\theta}_{11} = 0.$$

We sum up the different theoretical results here; formal derivations can be found in the appendix. The tax rate t_1^* on earnings x_1:

1. lies in between the extremes of no taxation and complete taxation, i.e., $0 < t_1^* < 1$;

2. decreases with the elasticity of effort δ_1, ranging from complete taxation, in the case of perfect inelastic effort ($t_1^* \to 1$ if $\delta_1 \to 0$), to no taxation, in the case of perfect elastic effort ($t_1^* \to 0$ if $\delta_1 \to +\infty$);

3. increases with the inequality aversion r, ranging from no taxation if the planner is inequality neutral ($t_1^* \to 0$ if $r \to 0$) to partial taxation if the planner only cares about inequality ($t_1^* \to \frac{(1-\alpha_1)^2\sigma^{\theta}_{11}}{(\alpha_1\delta_1)^2\sigma^{\gamma}_{11} + (1-\alpha_1)^2\sigma^{\theta}_{11}}$ if $r \to +\infty$);

4. increases with type heterogeneity σ^{θ}_{11}, ranging from no taxation if everyone has the same type ($t_1^* \to 0$ if $\sigma^{\theta}_{11} \to 0$) to complete taxation if types become very heterogeneous ($t_1^* \to 1$ if $\sigma^{\theta}_{11} \to +\infty$);

5. decreases with taste heterogeneity σ^{γ}_{11}, ranging from partial taxation if everyone has the same taste ($0 < t_1^* < 1$ if $\sigma^{\gamma}_{11} \to 0$) to zero taxation if tastes become very heterogeneous ($t_1^* \to 0$ if $\sigma^{\gamma}_{11} \to +\infty$);

6. decreases with the degree of control α_1, ranging from complete taxation if earnings cannot be controlled ($t_1^* \to 1$ if $\alpha_1 \to 0$) to no taxation if earnings are fully controlled ($t_1^* \to 0$ if $\alpha_1 \to 1$).

The first four results are standard in the optimal tax literature (see e.g., Mankiw et al., 2009, for a recent overview). The fifth result appears in Su and Judd (2006) and Weinzierl (2009). To compare with the results in political economy models, note that the fourth and fifth result can be combined to obtain a tax rate that increases with the signal-to-noise ratio $(\sigma_{11}^{\theta}/\sigma_{11}^{\gamma})$ (see Alesina and Angeletos, 2005). The sixth result, which is new in optimal tax models, mirrors the political economy equilibria of Piketty (1995), Alesina and Angeletos (2005) and Bénabou and Tirole (2006) where a higher belief in control coincides with a lower tax rate.

2.2.3 The 'Akerlof'-case

Suppose that there exist two characteristics, earnings $x_1 = \alpha_1 e_1 + (1-\alpha_1)\theta_1$ and an exogenous tag $x_2 = \theta_2$ and suppose output can be written as $y = x_1 + \beta_2 x_2$.[14] The system of first-order conditions reduces to

$$\begin{aligned} -\alpha_1\delta_1\frac{\zeta t_1}{1-t_1} - r(\alpha_1\delta_1)^2 t_1\sigma_{11}^{\gamma} + r(1-\alpha_1)\left((1-t_1)(1-\alpha_1)\sigma_{11}^{\theta} + (\beta_2 - t_2)\sigma_{21}^{\theta}\right) &= 0, \\ (1-t_1)(1-\alpha_1)\sigma_{12}^{\theta} + (\beta_2 - t_2)\sigma_{22}^{\theta} &= 0, \end{aligned}$$

with $\zeta = (1-\alpha_1) + \beta_2$ here. The complete comparative statics results can be found in the appendix. Here we highlight that the tax rate on earnings t_1^* also satisfies points 1-6 as described in the previous Mirrlees-case.[15] In addition, in the limiting case of perfect type correlation $\left(\left(\sigma_{12}^{\theta}\right)^2 \to \sigma_{11}^{\theta}\sigma_{22}^{\theta}\right)$ the tax rate on earnings t_1^* reduces to zero and all taxation can be done via the tax t_2^* on the tag, since the latter is a perfect signal of earnings ability and it can be taxed at no cost.

More interesting for our purposes is that the second of the first-order conditions can be rewritten as

$$(\beta_2 - t_2) + (\sigma_{12}^{\theta}/\sigma_{22}^{\theta}) \times (1-t_1)(1-\alpha_1) = 0. \tag{14}$$

Two special cases are immediately clear from equation (14). In the absence of a needs effect of the tag $(\beta_2 \to 0)$, only the signal can play a role. Then the optimal (read: efficient) tax on the tag reduces to $t_2 = (\sigma_{12}^{\theta}/\sigma_{22}^{\theta})(1-t_1)(1-\alpha_1)$, which is positive (negative) if the tag signals a higher (lower) ability to earn. In the absence of a signal $(\sigma_{12}^{\theta} = 0)$, only the needs play a role. In this case, the optimal (read: equitable) tax on tag t_2 equals β_2, i.e., the gross effect of the tag should be taxed away. In general, equity and efficiency together require $t_2 = \beta_2 + (\sigma_{12}^{\theta}/\sigma_{22}^{\theta})(1-t_1)(1-\alpha_1)$, i.e., tax away the needs effect, corrected for the signal value of the tag.

Another way to interpret equation (14) is that the (expected) total marginal effect of the tag θ_2 on the net outcome c should be equal to zero in a *fair* tax-benefit system, i.e., outcome differences due to non-controllable characteristics (exogenous tags) should be fully compensated. To see this, note that this total effect consists of two parts. The first part $(\beta_2 - t_2)$ is the direct marginal effect of θ_2 on the net outcome c. The second part can be interpreted as the indirect marginal effect of θ_2 on c: it is equal to $\sigma_{12}^{\theta}/\sigma_{22}^{\theta}$, the (expected) marginal effect of θ_2 on θ_1,[16] multiplied by $(1-t_1)(1-\alpha_1)$, the marginal effect of θ_1 on c.

[14] Besides Akerlof (1978), the theoretical use of tags in optimal taxation schemes has been analyzed by, among others, Immonen et al. (1998) and Salanié (2002, 2003). While the previous authors do not have a specific tag in mind, for instance, Blomquist and Micheletto (2008) and Weinzierl (2010) consider age tags, Mankiw and Weinzierl (2008) study height, and Alesina et al. (2008) and Cremer et al. (2010) focus on gender.

[15] Except for a different limit if the inequality aversion r becomes large $(r \to +\infty)$

[16] Note that $\sigma_{12}^{\theta}/\sigma_{22}^{\theta}$ is the OLS-estimate when regressing θ_2 on θ_1.

To test equation (14), we must be able to rewrite it in terms of empirically observable quantities. Fortunately, we can use lemma 1 to see that

$$\begin{aligned} x_1^* &= \alpha_1 \delta_1 \left(\ln \left(\left(\beta_1 - t_1 \right) \alpha_1 \right) + \gamma_1 \right) + \left(1 - \alpha_1 \right) \theta_1, \\ x_2^* &= \theta_2, \end{aligned}$$

which implies that $\sigma_{12}^x = (1 - \alpha_1) \sigma_{12}^\theta$ and $\sigma_{22}^x = \sigma_{22}^\theta$. Using these formulas, we obtain the empirical counterpart of the theoretical formula (14):

$$(\beta_2 - t_2) + (\sigma_{12}^x / \sigma_{22}^x) \times (1 - t_1) = 0. \tag{15}$$

Note that neither the degree of control α_1 nor the inequality aversion r have to be observed to test it.

Equation (15) is the theoretical basis for two hypotheses that we will derive next and test in the empirical part later on.[17] Rejection of the hypotheses would indicate that the planner is either inefficient or unfair in the above sense (or both). Note first that the expected net outcome, conditional on the tag, can be written as

$$E(c|x_2) = (1 - t_1) E(x_1|x_2) + (\beta_2 - t_2) x_2 - t_0.$$

The derivative of $E(c|x_2)$ w.r.t. x_2 equals

$$\begin{aligned} \frac{\partial E(c|x_2)}{\partial x_2} &= (1 - t_1) \frac{\partial E(x_1|x_2)}{\partial x_2} + (\beta_2 - t_2) \\ &= (1 - t_1) \frac{\sigma_{12}^x}{\sigma_{22}^x} + (\beta_2 - t_2). \end{aligned}$$

This allows us to rewrite equation (15) as

$$\frac{\partial E(c|x_2)}{\partial x_2} = (\beta_2 - t_2) + (\sigma_{12}^x / \sigma_{22}^x) \times (1 - t_1) = 0. \tag{16}$$

A first empirical hypothesis is whether the actual tax rates guarantee that the marginal effect of the tag on the conditional expectation of the net outcome is equal to zero. To derive a second and weaker hypothesis, we choose $\beta_2 = 1$ in the sequel; this will be true in the empirics later on by construction. In this case, we have

$$E(y|x_2) = E(x_1|x_2) + x_2,$$

and as before,

$$\frac{\partial E(y|x_2)}{\partial x_2} = \frac{\sigma_{12}^x}{\sigma_{22}^x} + 1.$$

Now, $t_2 \geq t_1$ must hold, if and only if

$$t_2 = 1 + (\sigma_{12}^x / \sigma_{22}^x) \times (1 - t_1) \geq t_1,$$

where the equality follows from rewriting equation (15) in terms of t_2. Using the fact that $1 - t_1 > 0$ in the optimum, we get that

$$t_2 \geq t_1 \text{ holds if and only if } \frac{\partial E(y|x_2)}{\partial x_2} = \sigma_{12}^x / \sigma_{22}^x + 1 \geq 0. \tag{17}$$

A second weaker hypothesis is whether the actual tax rate on the tag is larger than the actual tax rate on earnings whenever the the tag has a positive marginal effect on the conditional expectation of the gross outcome (and vice-versa, whether the tax on the tag is smaller in case of a negative effect).

[17] An interesting open question is whether these hypotheses can also be derived in a more general tax reform approach.

3 Evidence

3.1 Model

Before setting up the empirical model, we start with four remarks. *First*, in the empirical analysis, unfortunately, type and effort are not directly observable. Hence, the identification comes from the differences in the degree of control of our characteristics. However, we do not observe the precise degree of control for each characteristic. Therefore, we distinguish between non-controllable characteristics beyond individual control (we use age, sex and disability later on) and all other characteristics which are deemed partially controllable. Therefore, in this section, we refine the more general theoretical model in order to accommodate the data. *Second*, we make a distinction between covariates and characteristics: a characteristic can consist of several covariates, but not vice-versa. We provide two examples. The covariates for the characteristic 'education' are the different education dummies. The covariates for the characteristic 'no control' will consist of all covariates of the characteristics which are deemed beyond individual control. The last example illustrates that it is possible to create two composite characteristics, 'partial control' and 'no control', out of a finite set of covariates. Such a partitioning will allow us to test the theoretical predictions of the 'Akerlof'-case in equation (15) later on. *Third*, an error term is inevitable in empirical work. It will play the role of an additional 'unobserved' characteristic in the sequel. Since the error term is by assumption independent of the other covariates, adding it in the theoretical model as a third independent characteristic would not have changed the theoretical results. *Fourth*, many observable characteristics are not explicitly taxed in practice, but only implicitly, e.g., due to the progressivity of the tax system. For a correct interpretation later on, we define the implicit tax rate on a characteristic as the tax rate on that part of the outcome that can be attributed to the characteristic.

Let z denote a vector of covariates, which can be decomposed as $z = (z_j)_{j \in J}$, with z_j the covariates for characteristic j in J. Let '$\cdot$' denote a vector product; the gross output regression can be written as

$$\begin{aligned} y &= w_0 + w \cdot z + \epsilon \\ &= w_0 + \textstyle\sum_{j \in J} w_j \cdot z_j + \epsilon \\ &\equiv \beta_0 + \beta \cdot x, \end{aligned} \tag{18}$$

which brings us back to the theoretical model (equation (1)), defining $\beta_0 \equiv w_0$, $\beta \equiv 1$ (a vector of ones) and $x \equiv ((w_j \cdot z_j)_{j \in J}, \epsilon)$ the vector of characteristics, including the unobserved one. The tax (or subsidy, if negative) equals

$$\tau = y - c = t_0 + t \cdot x. \tag{19}$$

Equations (18)-(19) directly suggest a simple two-step approach to estimate the tax rates t_0 and t. First, estimate equation (18) by multivariate OLS, which provides us with estimates $\widehat{w}_j$. We then build predicted values $\widehat{x} = ((\widehat{w}_j \cdot z_j)_{j \in J}, \widehat{\epsilon})$ for each individual. The term $\widehat{w}_j \cdot z_j$ is the estimated contribution of characteristic j to the gross outcome y. Second, estimate equation (19) by multivariate OLS, replacing x by the prediction $\widehat{x}$ and correcting the standard errors for these added regressors (Maddala, 2001, p360). This multivariate regression gives us the implicit tax rates on the parts of gross outcome that can be attributed to the different characteristics.

3.2 Data

We use the 2007 EU-SILC data (European Union - Statistics on Income and Living Conditions), whose aim is to collect harmonized and comparable multidimensional micro data on income poverty and social exclusion for 24 EU member states (all 2006 EU member states, except Malta) as well as Norway and Iceland. Our analysis is based on the 2007 EU-SILC wave, which is the first to include gross income information for all countries. The sample size varies from 3,505 households in Cyprus to 20,982 households in Italy.[18] In the remainder we sometimes classify countries in groups and talk about the Continental,[19] the Northern,[20] the Southern,[21] the Anglo-Saxon,[22] the Central Eastern,[23] and the Baltic[24] countries.

In addition to the EU-SILC, we use data from IPUMS-CPS (King et al., 2010) which is an integrated dataset of the March Current Population Survey (CPS). The CPS is a monthly US household survey conducted jointly by the US Census Bureau and the Bureau of Labor Statistics. Our analysis is based on the 2007 wave and the variable values and definitions are adapted to follow the EU-SILC standard. We provide a definition of the income components and summary statistics in the data appendix.[25]

We select single and couple households with or without children in households attached to the labor market. In our preferred specification we estimate a joint model on the pooled data. As a robustness check, we will conduct separate estimations for singles and couples; see appendix. We also trim the top and bottom 1% of the income distribution in order to avoid estimation problems due to extreme outliers. Since needs (e.g., the number of children) are a crucial determinant of existing tax-benefit systems, we use equivalent gross household income as our preferred outcome measure; again, robustness checks will be provided in the appendix. To make incomes comparable across countries, we adjust national income amounts by the multilateral current purchasing power parities provided by Eurostat. The analysis only allocates those taxes and benefits that can be reasonably attributed to households. Therefore, corporate taxes as well as some types of government expenditures, such as expenditure on defense, are not considered. Due to data limitations, indirect taxes and in-kind benefits cannot be taken into account either. Thus, in the remainder we merely focus on cash benefits when speaking of social benefits and on personal income taxes in the case of taxes.

We construct the following characteristics. The characteristic 'sex' contains a gender dummy, 'age' contains several dummies for different age classes, 'disability' is constructed using information on disability status and the receipt of certain disability benefits, 'foreign' contains two dummies for born outside of the country but within the EU and born outside the EU. The covariates for the characteristic 'education' simply consist of different education dummies (4 levels according to the ISCED definition), 'needs' contains information about the number of children (in three age groups) together with the number of additional adults, 'couple' is a dummy for living as a couple, and 'unemployed' contains a dummy for not working. In our preferred specification, we use individual level covariates and characteristics, but again, as a robustness check, we will also perform and report the estimations on the household level in the appendix (using averages of the individual covariates of the head of the household and, eventually, his or her partner).

[18] The survey is representative for the whole population in each country due to the construction of population weights.
[19] Austria (AT), Belgium (BE), Germany (DE), France (FR), Luxembourg (LU) and the Netherlands (NL).
[20] Denmark (DK), Finland (FI), Iceland (IS), Sweden (SE), and Norway (NO).
[21] Cyprus (CY), Spain (ES), Greece (GR), Italy (IT) and Portugal (PT).
[22] Ireland (IE), the United Kingdom (UK) and the United States (US).
[23] The Czech Republic (CZ), Hungary (HU), Poland (PL), Slovenia (SI) and the Slovak Republic (SK).
[24] Estonia (EE), Lithuania (LT) and Latvia (LV).
[25] See also Fuest et al. (2010) for more details.

3.3 Results

We start with estimating the implicit tax rates for the different determinants of outcomes. Although our theory reveals little about the levels of compensation, the answer to the question how much countries compensate for the effect of different characteristics is, we believe, interesting in its own right. Afterwards, we return to the theory and derive and test two hypotheses: do countries compensate more for non-controllable characteristics compared to (partially) controllable ones and is the total effect of the non-controllable characteristics equal to zero?

3.3.1 How much do we compensate for different characteristics?

We estimate the implicit tax rates for each characteristic (i.e., 'age', 'sex', 'disability', 'couple', 'needs', 'foreign', 'unemployed', and 'education') separately.[26] Recall that we use a two-step estimation procedure based on (18)-(19) to estimate the implicit tax rates. The implicit tax rate for a characteristic that consists of a single dummy only is equal to β_τ/β_y, with β_τ the effect of the dummy on the tax paid (or subsidy received) in the second step and β_y the effect of the dummy on the gross outcome in the first step. As a consequence, the implicit tax rate can become very unstable if the first step estimate of β_y is close to zero. Therefore, Table (1) only reports estimates for the implicit tax rates of those characteristics which were significantly different from zero (at the 95%-level) in the first step of the estimation procedure; the complete first- and second-step regression results are reported in the appendix.

We order characteristics (the columns of Table 1) on the basis of the average implicit tax rate over the different countries (reported in the last row), while we order countries (the rows of Table 1) on the basis of their average implicit tax rate over the different characteristics (reported in the last column). First, the estimated implicit tax rates are typically positive. This means that outcome differences due to the different characteristics are reduced in most of the countries, e.g., due to progressive taxes and/or specific benefits. Second, in some countries the tax rates are higher compared to others. Generally speaking, we find the Southern and Anglo-Saxon countries as well as the Baltic states at lower levels of compensation and the Continental, Central Eastern and Northern countries at higher levels. Third, the tax rates are also different for different characteristics. We find the following order of compensation: there is most support for the elderly, followed by the disabled, the unemployed and families with children, less support towards foreigners and the educated, and finally, least to women and singles.[27] This revealed order of compensation is, generally speaking, in line with sociological research on attitudes on social spending, where the typical order of deservingness is old people, the sick and disabled, needy families with children, and the unemployed; see the seminal work of Coughlin (1980).

3.3.2 Back to theory

The novelty of the theoretical part is the introduction of partial control. Although we do not observe the precise degree of control in reality, the least we can say is whether some characteristics are beyond control or not. To proceed, we partition the set of observable characteristics J into a set of characteristics with no control (N) and a set with partial control (P). For the 'no control' composite we choose the covariates underlying the characteristics 'age', 'sex', and 'disability', whereas the 'partial control' composite

[26] Note that we do not include the implicit tax rate for the unobserved part. Since the unobserved part is independent of the other characteristics by assumption, its implicit tax rate is always close to the overall tax rate.

[27] In the appendix we discuss the robustness of this order of solidarity w.r.t. the empirical specification.

Table 1: Implicit tax rates for different characteristics in different countries

	AGE	DIS	UNOBSERVED	UNEMP	NEEDS	EDUC	SEX	IMMI	COUPLE	Mean
CY	0.71	0.63	0.41	0.41	0.05	0.10	0.16	0.29	-0.01	0.31
US	0.70	0.54	0.34	0.60	0.14	0.24	0.41	-0.10	0.15	0.34
PL	0.98	0.58	0.52	0.43	0.14	0.27	0.24	.	-0.04	0.39
LV	0.65	0.62	0.42	0.35	0.55	0.40	0.20	0.29	0.12	0.40
IT	0.89	0.89	0.54	0.40	0.32	0.27	-0.40	0.32	0.38	0.40
LT	0.74	0.70	0.51	0.35	0.49	0.38	-0.04	.	0.12	0.41
IE	0.71	0.57	0.50	0.51	0.44	0.30	0.26	0.36	0.35	0.45
EE	0.73	0.57	0.46	0.39	0.56	0.44	0.50	0.27	0.12	0.45
ES	0.79	0.82	0.47	0.47	0.21	0.19	.	0.24	0.53	0.47
SK	0.80	0.73	0.56	0.46	0.44	0.35	.	.	-0.01	0.48
GR	0.83	0.87	0.60	0.38	0.05	0.46	0.54	0.27	.	0.50
AT	0.92	0.78	0.62	0.62	0.53	0.21	0.51	0.16	0.30	0.51
LU	0.99	0.82	0.51	0.53	0.54	0.37	.	0.05	0.31	0.52
SI	0.90	0.78	0.65	0.53	0.47	0.41	0.71	0.50	-0.31	0.52
NO	0.76	0.86	0.58	0.54	0.43	0.43	0.54	0.42	0.21	0.53
UK	0.74	0.75	0.50	0.53	0.49	0.31	0.51	.	0.45	0.54
DE	0.84	0.76	0.58	0.64	0.54	0.30	0.08	0.77	0.32	0.54
IS	0.74	0.86	0.48	0.75	0.37	0.48	.	0.36	0.29	0.54
FR	1.01	0.90	0.66	0.67	0.51	0.34	0.16	0.26	0.39	0.54
CZ	0.80	0.82	0.60	0.58	0.53	0.44	0.53	.	0.06	0.54
HU	0.93	0.73	0.66	0.50	0.65	0.40	.	.	0.13	0.57
FI	0.79	0.86	0.60	0.65	0.46	0.48	0.61	0.49	0.31	0.58
PT	0.97	0.56	0.62	0.63	.	0.20	.	.	0.54	0.59
BE	0.85	0.79	0.67	0.75	0.58	0.39	0.52	0.48	0.37	0.60
SE	0.79	0.85	0.65	0.66	0.45	0.58	.	0.56	0.30	0.61
DK	0.78	0.95	0.65	0.74	0.40	0.50	0.58	0.57	0.40	0.62
NL	0.88	0.80	0.67	0.75	0.51	0.47	.	.	0.53	0.66
Mean	0.82	0.76	0.56	0.55	0.42	0.36	0.35	0.35	0.24	0.50

Source: Own calculations based on EU-SILC and IPUMS-CPS and IPUMS-CPS.

contains 'couple', 'needs', 'foreign', 'unemployed', and 'education'.[28] The most controversial issue is certainly age. As earnings are usually increasing over the working-life, our cross-sectional design captures redistribution from older individuals earning more to younger individuals eraning less depending on the progressivity of the tax system. With a sufficiently long panel, it would be possible to take a more appropriate life-time perspective.[29] We keep the residual error term, labelled 'unobserved', as a separate

[28] The assignment of 'foreigner' and 'disability' to either category can be disputed. Our choice can be justified in the following way: We do not observe whether it was an individual's choice to move to a foreign country or not. Hence, we consider this characteristic as (potentially) partial controllable. For disability status, we try to focus on inborn handicaps which are beyond individual control. However, our main results remain unaffected when altering these choices.

[29] As an alternative, we could assume that younger cohorts will receive in the future what older cohorts receive now. This would allow to construct a life-time well-being index.

independent characteristic.[30] We can use equation (18) again, with x now decomposed as $(x_N, x_P, x_U) = \left(\sum_{j\in N} w_j \cdot z_j, \sum_{j\in P} w_j \cdot z_j, \epsilon\right)$.

The weak hypothesis in equation (17) tells us when the non-controllable characteristics should be taxed more compared to the partially controllable ones. In the current notation, we have

Weak hypothesis: $t_N \geq t_P$ holds if and only if $\frac{\partial E(y|x_N)}{\partial x_N} = \sigma^x_{PN}/\sigma^x_{NN} + 1 \geq 0$.

Whether $\frac{\partial E(y|x_N)}{\partial x_N} \geq 0$ holds can be tested in a straightforward way, since the OLS-estimate of b in the regression

$$x_P = a + bx_N + \eta \tag{20}$$

is equal to $\sigma^x_{PN}/\sigma^x_{NN}$. Next, we define

$$FM = \frac{\partial E(c|x_N)}{\partial x_N} = (1 - t_N) + (\sigma^x_{PN}/\sigma^x_{NN}) \times (1 - t_P) \tag{21}$$

as a fairness measure: it is the total marginal effect of the non-controllable characteristics on the conditional net outcome. The closer to zero, the fairer the tax-benefit system is, because in the optimum, outcome differences due to exogenous tags should be fully compensated. The following stronger hypothesis deals with the fairness of tax-benefit systems in different countries and follows directly from equation (16):

Strong hypothesis: $FM = \frac{\partial E(c|x_N)}{\partial x_N} = 0$.

But how can we estimate FM? The net outcome c equals

$$c = (\beta_0 - t_0) + (1 - t_P)\, x_P + (1 - t_N)\, x_N + (1 - t_U)\, x_U. \tag{22}$$

Plugging (20) into (22), and replacing by b by $\sigma^x_{PN}/\sigma^x_{NN}$, we get

$$\begin{aligned} c &= (\beta_0 - t_0) + (1 - t_P)\,(a + bx_N + \eta) + (1 - t_N)\, x_N + (1 - t_U)\, x_U \\ &= \underbrace{(\beta_0 - t_0) + (1 - t_P)\, a}_{\text{constant}} + \underbrace{[(1 - t_P)\, \sigma^x_{PN}/\sigma^x_{NN} + (1 - t_N)]}_{FM} x_N + (1 - t_U)\, x_U + (1 - t_P)\, \eta. \end{aligned} \tag{23}$$

Equation (23) provides us with a two-step procedure: first, estimate equation (18) as before by OLS, which provides us with $\widehat{x} = (\widehat{x}_N, \widehat{x}_P, \widehat{x}_U)$; second, estimate equation (23) by plugging in the estimated $(\widehat{x}_N, \widehat{x}_U)$, which provides us with an estimate $\widehat{FM}$ as well as a confidence interval (again we correct standard errors for the added regressors).

3.3.3 Does compensation depend on the degree of control?

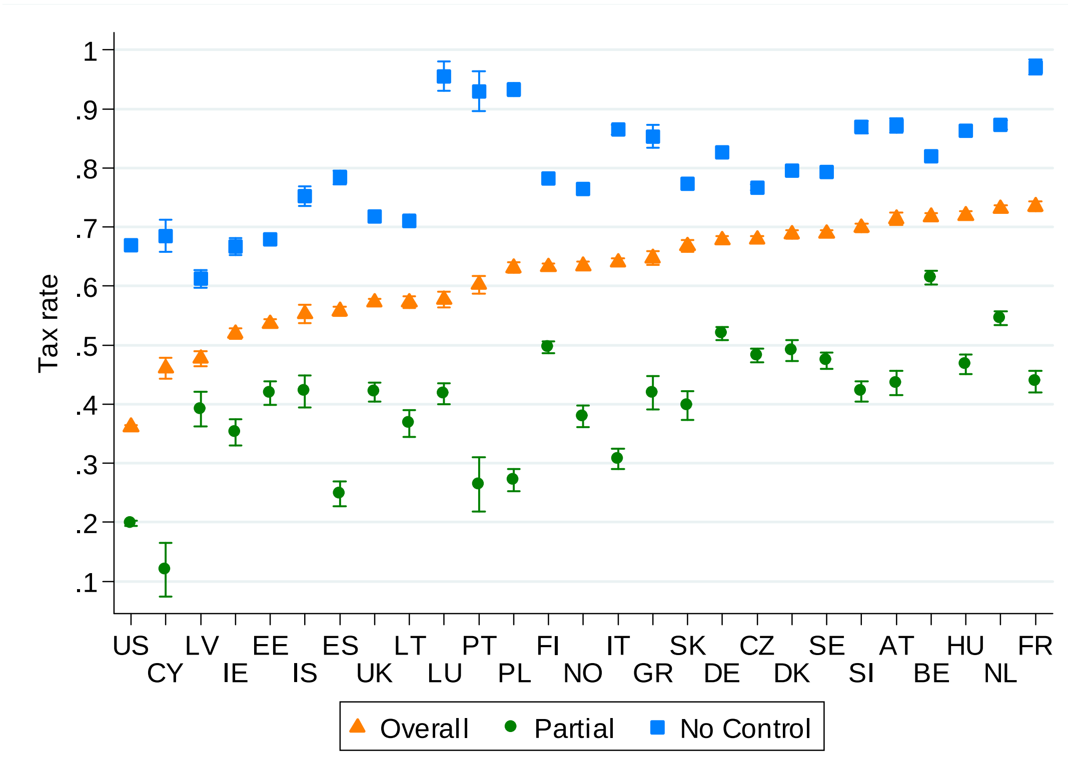
We want to test the weak hypothesis here. Table (7) in the appendix reports $\frac{\partial E(y|x_N)}{\partial x_N} = \sigma^x_{PN}/\sigma^x_{NN} + 1$ as well as the p-value of testing $\frac{\partial E(y|x_N)}{\partial x_N} < 0$. The null is rejected for each country. As a consequence, hypothesis 1 predicts that $t_N \geq t_P$ should hold, i.e. the implicit tax rate for the no-control composite should be larger than the one for the partial control composite in each country.

To check whether this prediction is true, Figure 2 shows the implicit tax rates for all countries for the 'no control' and the 'partial control' composite along with the 95% confidence bands. Countries are ordered

[30] Neither the theoretical results nor the empirics change. First, if we add a third characteristic to our theoretical model with an underlying taste and type distribution which is pairwise independent of the underlying tastes and types of the other characteristics, then the theoretical relation between the first two remains unchanged. Second, one could think of adding the error term to either the 'no control' or 'partial control' composite, but, again due to its independence, this does not change the empirical results.

on the basis of the overall tax rate.[31] Countries with higher overall tax rates also tend to compensate more for both composites, but the link is far from perfect: Luxembourg, Portugal and Poland have moderate overall tax rates, but among the highest implicit tax rates for characteristics beyond control.

Figure 2: Implicit tax rates for the different composite characteristics



Source: Own calculations based on EU-SILC and IPUMS-CPS

In line with the theoretical part, the implicit tax rate on (partially) controllable factors is always significantly below the tax rate for non-controllable factors in all countries. On average, we obtain a tax rate equal to 0.80 and 0.40 for non-controllable and partially controllable characteristics, respectively. We show in the appendix that this result is very robust with respect to the chosen empirical specification. As can be seen in Table (1), this result also holds if we compare the non-controllable characteristics age and disability separately with each of the partially controllable ones. Still, it would not hold for the characteristic sex in some countries. If we look at the dispersion in the implicit tax rates for the characteristic sex in the different countries, it turns out to be the most disputed characteristic.

For the interpretation of these results, note again that we look at implicit tax rates on the parts of the outcome that can be attributed to the different characteristics. Since income is positively correlated with age, the progressivity of the tax-benefit system in each country leads to high implicit tax rates on outcome differences due to age. Countries with lower progressivity (especially the Baltic flat tax countries) have lower implicit tax rates on age. A similar rationale applies to disability: its income differences are usually

[31] Note that the overall tax rate is an imperfect indicator of the degree of redistribution in a country. In a regression of taxes on gross incomes, the constant plays a role as well. For example, Luxembourg has a moderate overall tax rate, but a large (negative) constant such that it probably belongs to the group of highly redistributive countries.

compensated through rather generous disability benefit schemes. Hence, countries with lower benefit levels (mainly from Eastern Europe) have lower implicit tax rates on disability. The rather low implicit tax rates on sex imply that the tax-benefit system does not reduce the gender wage gap other than through the progressivity of the system. Due to anti-discrimination laws, explicit gender based taxation is prohibited in the EU and the US. Implicit tax differences arise through the definition of tax units (individual vs. joint taxation) and the definition of rules and conditions such that individual with certain circumstances are more likely to be eligible for it.

In order to better understand the role played by taxes, contributions and benefits separately, we decompose the total tax amount in equation (19) as

$$\tau = y - c = \tau_y + \tau_{ss} - b, \tag{24}$$

with τ_y (equivalized) income taxes, τ_c (equivalized) social security contributions and b (equivalized) benefits (and tax credits). We can do the second step estimation separately for each component, i.e.,

$$\tau_y = t_{y,0} + t_y \cdot x \ , \ \tau_{ss} = t_{ss,0} + t_{ss} \cdot x \ , \text{ and } \ -b = t_{b,0} + t_b \cdot x, \tag{25}$$

again with $x = (x_N, x_P, x_U)$. We obtain

$$\tau = (t_{y,0} + t_{ss,0} + t_{b,0}) + (t_y + t_{ss} + t_b) \cdot x,$$

with $t = t_y + t_{ss} + t_b$ a vector of tax rates, one rate for each composite characteristic in $x = (x_N, x_P, x_U)$, which can now be decomposed over income taxes, social security contributions and benefits. The estimated tax rates for t_y, t_{ss} and t_b, expressed as shares of the overall tax rate t, are reported in Figure 3 for 'no control' (upper panel) and 'partial control' (lower panel); countries are again sorted on the basis of their overall tax rate.

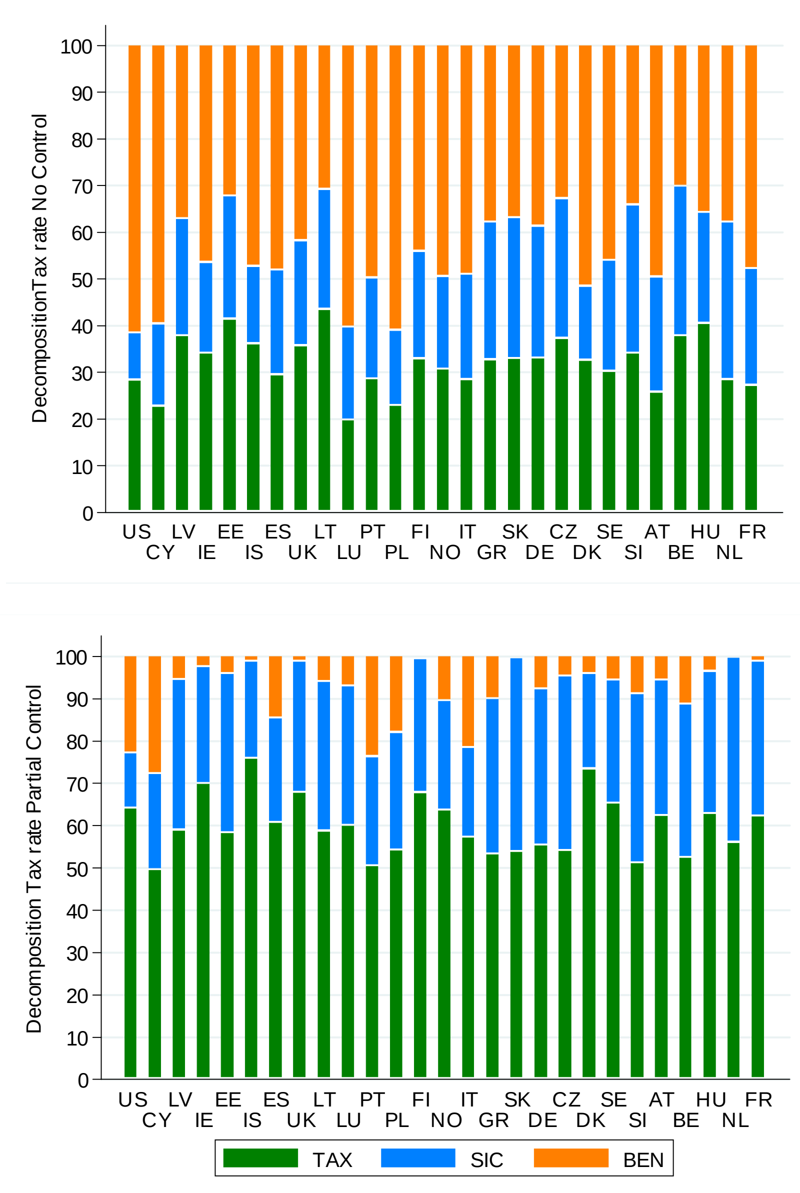
Not surprisingly, benefits tend to be relatively more important compared to taxes in the compensation for non-controllable characteristics. Still, half of the compensation for non-controllable characteristics is due to taxes, e.g., because earnings, and thus also taxes in a progressive tax scheme, tend to increase with age. In the 'partial control'-case, taxes have the highest relative importance in all countries. Both cases together indicate that benefits are mainly used to compensate for non-controllable factors whereas taxes are mainly used for compensating the non-controllable part in partially controllable characteristics. In the appendix we provide the same decomposition for each characteristic separately. If we look at the non-controllable factors (age, disability and sex), this figure confirms that pensions and disability benefits play a big role in compensating the income effect of age and disability, while progressive taxes tend to compensate for sex.

3.3.4 How fair are tax-benefit systems?

To test the stronger hypothesis, the point estimates and confidence intervals for the fairness measure FM defined in (21) are plotted in Figure 4 for each country. A value of this 'fairness measure' greater than zero implies that the compensation for the 'no control' characteristics is too low relative to the 'partial control' composite and vice-versa. The greater the distance from zero, the less fair a country.

Generally speaking, the figure shows three chains of countries: a first group with a fair tax-benefit system (or close to it), with values for the fairness measure between 0 and 0.1, a large intermediate group around 0.1 and 0.3, and a group of four countries which is further away from the fairness ideal, roughly in between

Figure 3: Decomposition implicit tax rates on composite characteristics

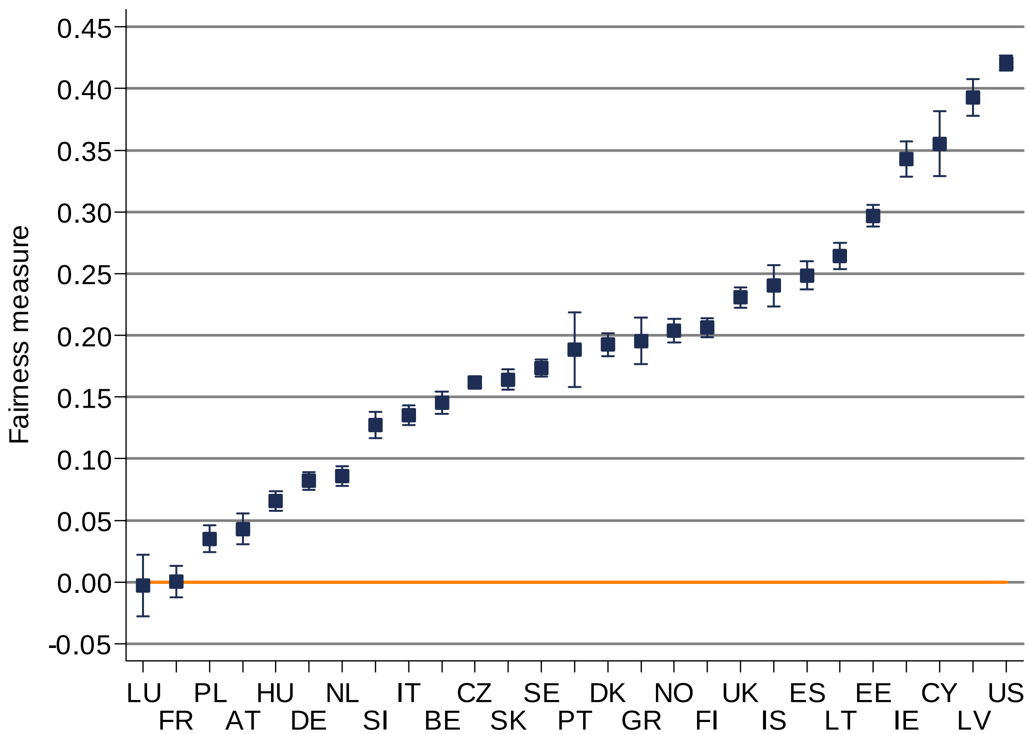


Source: Own calculations based on EU-SILC and IPUMS-CPS

0.35 and 0.45. In the appendix we show that the empirical specification does not matter for the ranking of the countries (although the numbers can be different, especially when using income rather than equivalent income).

In contrast with the weak hypothesis, the strong hypothesis can be rejected for all countries except France and Luxembourg. France and Luxembourg have a high implicit tax rate for non-controllable characteristics in common, but clearly note that their overall tax rate is not necessarily high compared to other countries. Note also that some other Continental countries (Austria, Germany and the Netherlands) as well as Hungary and Poland come close to being fair. If we only look at the countries with a good perfor-

Figure 4: Fairness measure (Akerlof case)



Source: Own calculations based on EU-SILC and IPUMS-CPS.

mance, the degree of compensation for the non-controllable characteristics seems to be the crucial factor. We also know that age is by far the most important factor among the non-controllable ones; note, for instance, that the variation of the non-controllable composite due to age accounts on average for more than 80% of the explained variation. This might also explain why the Northern countries (Sweden, Denmark, Norway, Finland and Iceland), with a moderate to low public spending on public Pensions, can be found among the worst performers in the intermediate group (see OECD, 2009). More generally, it begs the question whether the second- and third-pillar contributions and benefits should also be taken up in the output definition.[32]

The way to improve fairness can be rather different in different countries. Recall equation (21) and Figure (4). To improve fairness, all countries must lower FM, the total effect that the non-controllable characteristics have on net outcome. According to the decomposition in Figure (3) they can do so by changing the benefits (which mainly impacts t_N) and by changing (the progressivity of) income taxes (which changes t_P and t_N). For the worst performing countries (Ireland, Cyprus, Latvia and the United States), the ratio $\sigma^x_{PN}/\sigma^x_{NN}$ is positive and, as a consequence, t_N and/or t_P should be lowered. The fairness gains of increasing (the progressivity of) income taxes are triple. It directly increases t_N and t_P, and, if additional tax revenues result, benefits can also be raised to further increase t_N. The positive sign of $\sigma^x_{PN}/\sigma^x_{NN}$ is also true for some of the Southern countries (Spain, Greece and Portugal), but the

[32] The second-pillar variables are missing for all countries. The third-pillar data are present but difficult to introduce, since benefits are typically paid lump sum in most countries.

margins for increasing taxes could be more limited. For most of the other countries, the sign of $\sigma^x_{PN}/\sigma^x_{NN}$ is negative. Figure 3 suggests that changing (the progressivity of) income taxes has a bigger impact on t_P compared to t_N. Therefore, lowering (the progressivity of) income taxes could be helpful to improve fairness in these countries.

4 Conclusion

There is ample evidence from surveys and experiments that fairness plays a role in redistributive issues. Individuals want to compensate others for their misfortune, while they allow them to enjoy the fruits of their effort. Such fairness considerations have been introduced in political economy and optimal income tax models. We introduce fairness as a device to select among efficient tax-benefit schemes that are based on several characteristics. In addition, we introduce partial control: characteristics differ in the degree of control, i.e., the extent to which they can be changed by exerting effort. We derive two testable predictions. The tax rate on partially controllable characteristics should be lower compared to the tax rate on non-controllable characteristics, and the total effect of non-controllable characteristics on the post-tax outcome should be equal to zero.

We estimate implicit tax rates for a set of characteristics in 26 European countries (using the 2007 EU-SILC data) and the US (using the CPS data). We find a robust tendency in all countries to compensate more for the uncontrollable composite characteristic (based on sex, age and disability) compared to the partially controllable one (based on family composition, immigration status, unemployment and education level). We also estimate the total effect of the non-controllable composite on the post-tax outcome and test whether it is equal to zero. Only France and Luxembourg pass the fairness test. Although this result is sensitive to the empirical specification, the ranking of countries in terms of fairness tends to be robust. The way in which countries can improve fairness depends on the variance-covariance structure of the characteristics. For the worst performing countries (Ireland, Cyprus, Latvia and the United States), the analysis suggests that increasing (the progressivity of) income taxes could increase fairness considerably. For most of the other countries, the opposite is probably true. One caveat applies. Age is an important factor in the non-controllable composite. Since we can only include first-pillar pensions, the fairness measure is biased to the advantage of the (continental) countries with a generous public pension scheme. However, as we focus on households attached to the labor market, the bias is probably not that large.

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Proof of proposition 1

The planner solves

$$\max_{t_0,t} W = -\frac{1}{r}\ln\int\int \exp\left[-rv\left(t_0,t;\alpha,\beta,\delta;\gamma,\theta\right)\right]dF\left(\theta\right)dG\left(\gamma\right),$$

subject to the budget constraint

$$t_0 + \int\int\left(\sum_{j\in J} t_j x_j^*\right)dF\left(\theta\right)dG\left(\gamma\right)\geq R_0,$$

and well-being of an individual $v\left(t_0,t;\alpha,\beta,\delta;\gamma,\theta\right)$ is defined as

$$\frac{\kappa\left(t_0,t;\alpha,\beta_0,\beta,\delta\right)-\kappa\left(R_0,0;\alpha,\beta_0,\beta,\delta\right)-\sum_{j\in J}t_j\alpha_j\delta_j\gamma_j+\sum_{j\in J}\left(\beta_j-t_j\right)\left(1-\alpha_j\right)\theta_j}{\sum_{j\in J}\left(1-\alpha_j\right)\beta_j},$$

with $\kappa\left(t_0,t;\alpha,\beta_0,\beta,\delta\right)-\kappa\left(R_0,0;\alpha,\beta_0,\beta,\delta\right)$ equal to

$$R_0-t_0+\sum_{j\in J}\left(\beta_j-t_j\right)\alpha_j\delta_j\left[\ln\left(\left(\beta_j-t_j\right)\alpha_j\right)-1\right]-\sum_{j\in J}\alpha_j\beta_j\delta_j\left[\ln\left(\alpha_j\beta_j\right)-1\right],$$

while

$$x_j^*=\alpha_j\delta_j\left(\ln\left(\left(\beta_j-t_j\right)\alpha_j\right)+\gamma_j\right)+\left(1-\alpha_j\right)\theta_j.$$

Before analyzing the solution, notice that the optimal tax rates t^* must satisfy $t^*\ll\beta$. As defined before, x_j^* remains the same for all tax levels $t_j\geq\beta_j$, so it suffices for the planner to look at tax rates $t_j<\beta_j$ and $t_j=\beta_j$. In addition, a solution with $t_j^*=\beta_j$ (and $t_0\rightarrow+\infty$) can never be efficient (the laisser faire is better for everyone), leaving us with $t_j<\beta_j$ for each j in J, as required.

First, efficiency requires that the budget constraint is satisfied with equality. Given independent (multivariate normal) distributions for θ and γ, we simply get

$$t_0=R_0-\sum_{j\in J}t_j\alpha_j\delta_j\ln\left(\left(\beta_j-t_j\right)\alpha_j\right)-\sum_{j\in J}t_j\alpha_j\delta_j\mu_j^\gamma-\sum_{j\in J}t_j\left(1-\alpha_j\right)\mu_j^\theta.$$

We can plug in this equation in the expression $\kappa\left(t_0,t;\alpha,\beta_0,\beta,\delta\right)-\kappa\left(R_0,0;\alpha,\beta_0,\beta,\delta\right)$, to get

$$\sum_{j\in J}\alpha_j\beta_j\delta_j\ln\left(\frac{\beta_j-t_j}{\beta_j}\right)+\sum_{j\in J}t_j\alpha_j\delta_j\left(1+\mu_j^\gamma\right)+\sum_{j\in J}t_j\left(1-\alpha_j\right)\mu_j^\theta$$

and we can rewrite welfare $W=A+B+C$ with

$$\begin{aligned}
A &= \frac{\sum_{j\in J}\alpha_j\beta_j\delta_j\ln\left(\frac{\beta_j-t_j}{\beta_j}\right)+\sum_{j\in J}t_j\alpha_j\delta_j\left(1+\mu_j^\gamma\right)+\sum_{j\in J}t_j\left(1-\alpha_j\right)\mu_j^\theta}{\sum_{k\in J}\left(1-\alpha_k\right)\beta_k},\\
B &= -\frac{1}{r}\ln\int\exp\left(\sum_{j\in J}\frac{rt_j\alpha_j\delta_j}{\sum_{k\in J}\left(1-\alpha_k\right)\beta_k}\gamma_j\right)dG\left(\gamma\right),\\
C &= -\frac{1}{r}\ln\int\exp\left(\sum_{j\in J}\frac{-r\left(\beta_j-t_j\right)\left(1-\alpha_j\right)}{\sum_{k\in J}\left(1-\alpha_k\right)\beta_k}\theta_j\right)dF\left(\theta\right).
\end{aligned}$$

Given a multivariate normal distribution for an arbitrary vector, say z with $z\tilde{}N\left(\mu^z,\Sigma^z\right)$, we can use the following result

$$\ln\int\exp\left(\sum_{j\in J}a_jz_j\right)dF\left(z\right)=\sum_{j\in J}a_j\mu_j^z+\frac{1}{2}\sum_i\sum_j a_ia_j\sigma_{ij}^z,$$

to rewrite $W = A + B + C$ with

$$\begin{aligned}
A &= \frac{\sum_{j\in J} \alpha_j \beta_j \delta_j \ln\left(\frac{\beta_j - t_j}{\beta_j}\right) + \sum_{j\in J} t_j \alpha_j \delta_j \left(1+\mu_j^\gamma\right) + \sum_{j\in J} t_j \left(1-\alpha_j\right) \mu_j^\theta}{\sum_{k\in J} \left(1-\alpha_k\right) \beta_k} \\
B &= -\sum_{j\in J} \frac{t_j \alpha_j \delta_j \mu_j^\gamma}{\sum_{k\in J} \left(1-\alpha_k\right) \beta_k} - \frac{r \sum_i \sum_j t_i \alpha_i \delta_i t_j \alpha_j \delta_j \sigma_{ij}^\gamma}{2\left(\sum_{k\in J} \left(1-\alpha_k\right) \beta_k\right)^2} \\
C &= \sum_{j\in J} \frac{\left(\beta_j - t_j\right)\left(1-\alpha_j\right) \mu_j^\theta}{\sum_{k\in J} \left(1-\alpha_k\right) \beta_k} - \frac{r \sum_i \sum_j \left(\beta_i - t_i\right)\left(1-\alpha_i\right)\left(\beta_j - t_j\right)\left(1-\alpha_j\right) \sigma_{ij}^\theta}{2\left(\sum_{k\in J} \left(1-\alpha_k\right) \beta_k\right)^2}
\end{aligned}$$

Maximizing welfare leads to a system of equations, one for each j in J, defined as $\frac{\partial W}{\partial t_j} = \frac{\partial A}{\partial t_j} + \frac{\partial B}{\partial t_j} + \frac{\partial C}{\partial t_j} = 0$. Using the fact that

$$\frac{\partial}{\partial t_j}\left(\sum_i \sum_j \eta_i\left(t_i\right) \eta_j\left(t_j\right) \sigma_{ij}^z\right) = 2\frac{\partial \eta_j\left(t_j\right)}{\partial t_j} \sum_i \eta_i\left(t_i\right) \sigma_{ij}^z,$$

we get

$$\begin{aligned}
\frac{\partial A}{\partial t_j} &= \frac{-\frac{\alpha_j \beta_j \delta_j}{\beta_j - t_j} + \alpha_j \delta_j \left(1+\mu_j^\gamma\right) + \left(1-\alpha_j\right) \mu_j^\theta}{\sum_{k\in J} \left(1-\alpha_k\right) \beta_k}, \\
\frac{\partial B}{\partial t_j} &= -\frac{\alpha_j \delta_j \mu_j^\gamma}{\sum_{k\in J} \left(1-\alpha_k\right) \beta_k} - \frac{r \alpha_j \delta_j \sum_k t_k \alpha_k \delta_k \sigma_{kj}^\gamma}{\left(\sum_{k\in J} \left(1-\alpha_k\right) \beta_k\right)^2}, \\
\frac{\partial C}{\partial t_j} &= -\frac{\left(1-\alpha_j\right) \mu_j^\theta}{\sum_{k\in J} \left(1-\alpha_k\right) \beta_k} + \frac{r\left(1-\alpha_j\right) \sum_k \left(\beta_k - t_k\right)\left(1-\alpha_k\right) \sigma_{kj}^\theta}{\left(\sum_{k\in J} \left(1-\alpha_k\right) \beta_k\right)^2}.
\end{aligned}$$

Putting everything together (and multiplying by $\zeta^2 := \left[\sum_{k\in J} \left(1-\alpha_k\right) \beta_k\right]^2 > 0$), we get

$$-\alpha_j \delta_j \frac{t_j}{\beta_j - t_j} \zeta - r \alpha_j \delta_j \sum_k t_k \alpha_k \delta_k \sigma_{kj}^\gamma + r\left(1-\alpha_j\right) \sum_k \left(\beta_k - t_k\right)\left(1-\alpha_k\right) \sigma_{kj}^\theta = 0,$$

for each j in J.

Finally, to establish concavity, we directly focus on the case of two characteristics; the case of one characteristic can be seen from it as well:

$$\begin{aligned}
-\alpha_1 \delta_1 \frac{t_1}{\beta_1 - t_1} \zeta - r \alpha_1 \delta_1 \sum_k t_k \alpha_k \delta_k \sigma_{k1}^\gamma + r\left(1-\alpha_1\right) \sum_k \left(\beta_k - t_k\right)\left(1-\alpha_k\right) \sigma_{k1}^\theta &= 0 \\
-\alpha_2 \delta_2 \frac{t_2}{\beta_2 - t_2} \zeta - r \alpha_2 \delta_2 \sum_k t_k \alpha_k \delta_k \sigma_{k2}^\gamma + r\left(1-\alpha_2\right) \sum_k \left(\beta_k - t_k\right)\left(1-\alpha_k\right) \sigma_{k2}^\theta &= 0.
\end{aligned}$$

The Hessian matrix $H = \left(1/\zeta^2\right) \Upsilon$, and Υ has the following entries:

$$\begin{aligned}
\Upsilon_{11} &= -\alpha_1 \delta_1 \frac{\beta_1}{\left(\beta_1 - t_1\right)^2} \zeta - r\left(\alpha_1 \delta_1\right)^2 \sigma_{11}^\gamma - r\left(1-\alpha_1\right)^2 \sigma_{11}^\theta, \\
\Upsilon_{12} &= \Upsilon_{21} = -r \alpha_1 \delta_1 \alpha_2 \delta_2 \sigma_{12}^\gamma - r\left(1-\alpha_1\right)\left(1-\alpha_2\right) \sigma_{12}^\theta, \\
\Upsilon_{22} &= -\alpha_2 \delta_2 \frac{\beta_2}{\left(\beta_2 - t_2\right)^2} \zeta - r\left(\alpha_2 \delta_2\right)^2 \sigma_{22}^\gamma - r\left(1-\alpha_2\right)^2 \sigma_{22}^\theta.
\end{aligned}$$

To show that the Hessian matrix is negative semi-definite, we must have $\Upsilon_{11} \leq 0$, $\Upsilon_{22} \leq 0$ (which are true) and $|\Upsilon| = \Upsilon_{11}\Upsilon_{22} - \left(\Upsilon_{12}\right)^2 \geq 0$. To show that $|\Upsilon| = \Upsilon_{11}\Upsilon_{22} - \left(\Upsilon_{12}\right)^2 \geq 0$, note that the term $\Upsilon_{11}\Upsilon_{22}$ does not depend on the covariances and that Υ_{12} does not depend on β; therefore the worst-case (read: smallest $|\Upsilon|$ possible) is obtained for $\sigma_{12}^\gamma = \sqrt{\sigma_{11}^\gamma \sigma_{22}^\gamma}$, $\sigma_{12}^\theta = \sqrt{\sigma_{11}^\theta \sigma_{22}^\theta}$ (maximal $\left(\Upsilon_{12}\right)^2$) and $\beta \to 0$

(minimal $(\Upsilon_{11}\Upsilon_{22})$). Plugging in these values and manipulating the expression, we get a lower bound L for $|\Upsilon|$, with

$$\begin{aligned} L &= \left(r\left(\alpha_1\delta_1\right)^2\sigma_{11}^{\gamma}+r\left(1-\alpha_1\right)^2\sigma_{11}^{\theta}\right)\left(r\left(\alpha_2\delta_2\right)^2\sigma_{22}^{\gamma}+r\left(1-\alpha_2\right)^2\sigma_{22}^{\theta}\right) \\ &\quad -\left(r\alpha_1\delta_1\alpha_2\delta_2\sqrt{\sigma_{11}^{\gamma}\sigma_{22}^{\gamma}}+r\left(1-\alpha_1\right)\left(1-\alpha_2\right)\sqrt{\sigma_{11}^{\theta}\sigma_{22}^{\theta}}\right)^2 \\ &= r^2\left(\alpha_1\delta_1\left(1-\alpha_2\right)\sqrt{\sigma_{11}^{\gamma}\sigma_{22}^{\theta}}-\alpha_2\delta_2\left(1-\alpha_1\right)\sqrt{\sigma_{11}^{\theta}\sigma_{22}^{\gamma}}\right)^2 \end{aligned}$$

which is non-negative, as required.

A multiplicative model

We outline a multiplicative (i.e., log-linear) variant of our model and show that the resulting optimal tax formula remains the same. We stick to the same notation as in the main text.

Production technology. The pre-intervention or gross outcome is denoted y and is assumed to be a log-linear function of the different characteristics of the individual; formally:

$$\ln y = \ln\beta_0 + \textstyle\sum_{j\in J}\beta_j \ln x_j,$$

with $\beta_0 > 0$ and $\beta = \left(\beta_j\right)_{j\in J} \gg 0$. Characteristics are a combination of effort $e \in \mathbb{R}^J$ and type $\theta \in \mathbb{R}^J$ in a multiplicative Cobb-Douglas way, i.e., for each j in J we assume

$$\ln x_j = \alpha_j \ln e_j + (1-\alpha_j)\ln\theta_j.$$

The weights of effort—one weight for each characteristic—define the '*degree of control*' for each characteristic. The multiplicative Mirrlees model can be obtained by choosing $|J| = 1$, $\beta_0 = 1$, $\beta_1 = 2$ and $\alpha_1 = 1/2$ which leads to $y = \theta_1 e_1$. Choosing $|J| = 2$, $\beta_0 = 1$, $\beta_1 = 2$, $\alpha_1 = 1/2$ and $\alpha_2 = 0$, the multiplicative version of Akerlof's model equals $y = (\theta_1 e_1)/(\theta_2)^{\beta_2}$ where $(\theta_2)^{\beta_2}$ is a relative equivalence scale factor that adjusts income $\theta_1 e_1$ for needs (in case $\beta_2 \neq 0$).

Preference technology. We assume quasi-loglinear preferences, or:

$$\ln U\left(c, e; \gamma, \delta\right) = \ln c - \textstyle\sum_{j\in J}\dfrac{\delta_j}{\gamma_j}\left(e_j\right)^{\frac{1}{\delta_j}},$$

with $\gamma \in \mathbb{R}_{++}^J$ a vector of taste parameters which defines the disutility of effort, and $\delta \in \mathbb{R}_{++}^J$ a vector controlling the degree of convexity of the cost of effort.

Net outcomes and behavior. The instruments of the social planner are restricted to log-linear schemes: net outcome c satisfies

$$\ln c \leq \ln y - t_0 - \textstyle\sum_{j\in J} t_j \ln x_j,$$

with t_0 controlling the overall level of the net outcome, and $t \in \mathbb{R}^J$ the tax rates applied to the different (logarithmic) characteristics. The optimal effort equals

$$e_j^* = \left(\alpha_j\left(\beta_j - t_j\right)\gamma_j\right)^{\delta_j} \text{ for all } j \text{ in } J,$$

or, equivalently,

$$\ln e_j^* = \delta_j\ln\alpha_j + \delta_j\ln\left(\beta_j - t_j\right) + \delta_j\ln\gamma_j \text{ for all } j \text{ in } J.$$

This results in characteristics

$$\ln x_j^* = \alpha_j \left(\delta_j \ln \alpha_j + \delta_j \ln \left(\beta_j - t_j\right) + \delta_j \ln \gamma_j\right) + \left(1 - \alpha_j\right) \ln \theta_j, \tag{26}$$

and the (logarithm of the) corresponding indirect utility $V\left(t_0, t; \alpha, \beta_0, \beta, \delta; \gamma, \theta\right)$ equals

$$\ln V = \kappa\left(t_0, t; \alpha, \beta_0, \beta, \delta\right) + \sum_{j \in J} \left(\beta_j - t_j\right) \alpha_j \delta_j \ln \gamma_j + \sum_{j \in J} \left(\beta_j - t_j\right) \left(1 - \alpha_j\right) \ln \theta_j,$$

with

$$\kappa\left(t_0, t; \alpha, \beta_0, \beta, \delta\right) = \ln \beta_0 - t_0 + \sum_{j \in J} \left(\beta_j - t_j\right) \alpha_j \delta_j \left[\ln\left(\left(\beta_j - t_j\right) \alpha_j\right) - 1\right].$$

A fair and efficient planner. For analytical tractability, we use log-normal distributions here, or

$$\ln \theta \sim N\left(\mu^{\ln \theta}, \Sigma^{\ln \theta}\right) \quad \text{and} \quad \ln \gamma \sim N\left(\mu^{\ln \gamma}, \Sigma^{\ln \gamma}\right),$$

with $\mu = \left(\mu_j\right)_{j \in J}$ a vector of means and $\Sigma = \left(\sigma_{ij}\right)_{ij \in J^2}$ a variance-covariance matrix with $\sigma_{jj} > 0$ for all j in J. The social planner sets taxes t_0 and t to maximize an iso-elastic (Kolm-Atkinson-Sen) concave welfare function subject to the budget constraint; formally, for a given $r > 0$:

$$\max_{t_0, t} \left[\int\int \left[v\left(t_0, t; \alpha, \beta, \delta; \gamma, \theta\right)\right]^{-r} dF(\theta)\, dG(\gamma)\right]^{-\frac{1}{r}}$$

subject to a (logarithmic) budget constraint[33]

$$\int\int \left(\ln y^* - \ln c^*\right) dF(\theta)\, dG(\gamma) \geq R_0.$$

Indirect well-being v is again defined as a specific cardinalization of indirect utility, i.e.,

$$V\left(t_0, t; \alpha, \beta_0, \beta, \delta; \gamma, \theta\right) = V\left(R_0, 0; \alpha, \beta_0, \beta, \delta; \gamma, (v, v, \ldots, v)\right),$$

which leads to $\ln v\left(t_0, t; \alpha, \beta, \delta; \gamma, \theta\right)$ being equal to

$$\frac{\kappa\left(t_0, t; \alpha, \beta_0, \beta, \delta\right) - \kappa\left(R_0, 0; \alpha, \beta_0, \beta, \delta\right) - \sum_{j \in J} t_j \alpha_j \delta_j \ln \gamma_j + \sum_{j \in J} \left(\beta_j - t_j\right)\left(1 - \alpha_j\right) \ln \theta_j}{\sum_{j \in J} \beta_j \left(1 - \alpha_j\right)}.$$

Givne efficiency, the budget constraint must hold with equality, which allows us to derive t_0 as

$$t_0 = R_0 - \sum_{j \in J} t_j \left(\alpha_j \left(\delta_j \ln \alpha_j + \delta_j \ln\left(\beta_j - t_j\right) + \delta_j \mu_j^{\ln \gamma}\right) + \left(1 - \alpha_j\right) \mu_j^{\ln \theta}\right)$$

We can define $\Delta\kappa = \kappa\left(t_0, t; \alpha, \beta_0, \beta, \delta\right) - \kappa\left(R_0, 0; \alpha, \beta_0, \beta, \delta\right)$, and rewrite it, given the formula for t_0 as

$$\Delta\kappa = \sum_{j \in J} t_j \alpha_j \delta_j \left(1 + \mu_j^{\ln \gamma}\right) + \sum_{j \in J} t_j \left(1 - \alpha_j\right) \mu_j^{\ln \theta} + \sum_{j \in J} \beta_j \alpha_j \delta_j \ln \frac{\left(\beta_j - t_j\right)}{\beta_j}.$$

The government's maximand can be equivalently written as

$$\max_{t_0, t} \ln \left[\int\int \exp\left(-r \ln v\left(t_0, t; \alpha, \beta, \delta; \gamma, \theta\right)\right) dF(\theta)\, dG(\gamma)\right]^{-\frac{1}{r}}$$

and using the expression for $\Delta\kappa$ in $\ln v$ the problem reduces to

$$\max_{t} A + B + C,$$

[33] Although somewhat artificial—R_0 does not have a money interpretation—, higher values for R_0 still corresponds to a higher government requirement. In addition, note that $\ln y^* - \ln c^* \approx \left(y^* - c^*\right)/c^*$, so R_0 can be interpreted as a minimal requirement on the mean average tax rate in society.

with

$$
\begin{aligned}
A &= \frac{\sum_{j\in J} t_j\alpha_j\delta_j + \sum_{j\in J}\beta_j\alpha_j\delta_j \ln\frac{(\beta_j - t_j)}{\beta_j}}{\sum_{j\in J}\beta_j(1-\alpha_j)} \\
B &= \frac{-r\sum_i\sum_j t_i\alpha_i\delta_i t_j\alpha_j\delta_j\sigma_{ij}^{\ln\gamma}}{2\left(\sum_{j\in J}\beta_j(1-\alpha_j)\right)^2} \\
C &= \frac{\sum_{j\in J}\beta_j(1-\alpha_j)\mu_j^{\ln\theta}}{\sum_{j\in J}\beta_j(1-\alpha_j)} + \frac{-r\sum_i\sum_j(\beta_i - t_i)(1-\alpha_i)\left(\beta_j - t_j\right)(1-\alpha_j)\sigma_{ij}^{\ln\theta}}{2\left(\sum_{j\in J}\beta_j(1-\alpha_j)\right)^2}.
\end{aligned}
$$

Taking partial derivatives (and defining $\zeta := \sum_{k\in J}(1-\alpha_k)\beta_k$), we get

$$
\begin{aligned}
\frac{\partial A}{\partial t_j} &= \frac{-\alpha_j\delta_j\frac{t_j}{\beta_j - t_j}}{\zeta} \\
\frac{\partial B}{\partial t_j} &= -\frac{r\alpha_j\delta_j\sum_k t_k\alpha_k\delta_k\sigma_{kj}^{\ln\gamma}}{\zeta^2} \\
\frac{\partial C}{\partial t_j} &= \frac{r(1-\alpha_j)\sum_k(\beta_k - t_k)(1-\alpha_k)\sigma_{kj}^{\ln\theta}}{\zeta^2}
\end{aligned}
$$

adding up and multiplying with $\zeta^2 > 0$ brings us back to the same system of first-order conditions

$$
-\alpha_j\delta_j\frac{t_j}{\beta_j - t_j}\zeta - r\alpha_j\delta_j\sum_k t_k\alpha_k\delta_k\sigma_{kj}^{\ln\gamma} + r(1-\alpha_j)\sum_k(\beta_k - t_k)(1-\alpha_k)\sigma_{kj}^{\ln\theta} = 0,
$$

for each j in J, as in Proposition 1.

The Mirrlees-case

In case of one characteristic and $\beta_0 = 0$ and $\beta_1 = 1$, we get

$$
-\alpha_1\delta_1\frac{t_1(1-\alpha_1)}{1-t_1} - r(\alpha_1\delta_1)^2 t_1\sigma_{11}^{\gamma} + r(1-\alpha_1)^2(1-t_1)\sigma_{11}^{\theta} = 0. \tag{27}
$$

Point 1. The optimal tax rate t_1^* on earnings x_1 lies in between the extremes of no taxation and complete taxation, i.e., $0 < t_1^* < 1$.

We know from proposition 1 that $t_1^* < 1$. In addition, also $t_1^* > 0$ must hold, since $t_1 \leq 0$ cannot satisfy the first-order condition.

Point 2. The optimal tax rate t_1^* on earnings x_1 decreases with the elasticity δ_1 from complete taxation if the elasticity approaches zero ($t_1^* \to 1$ if $\delta_1 \to 0$) to no taxation if the elasticity becomes very high ($t_1^* \to 0$ if $\delta_1 \to +\infty$).

If $\delta_1 \to 0$, the first-order condition reduces to

$$
r(1-\alpha_1)^2(1-t_1)\sigma_{11}^{\theta} = 0,
$$

which is satisfied for $t_1 \to 1$. If $\delta_1 \to +\infty$, the first-order condition reduces to (divide by $(\delta_1)^2 > 0$ and consider the limiting case $\delta_1 \to +\infty$)

$$
-r(\alpha_1)^2 t_1\sigma_{11}^{\gamma} = 0,
$$

which is satisfied for $t_1 \to 0$. The comparative statics show that taxes decrease with δ_1, since

$$\frac{dt_1}{d\delta_1} = -\frac{\frac{\partial(27)}{\partial\delta_1}}{\frac{\partial(27)}{\partial t_1}} = -\frac{-\alpha_1 \frac{t_1}{1-t_1}(1-\alpha_1) - 2r\delta_1(\alpha_1)^2 t_1 \sigma_{11}^{\gamma}}{-\alpha_1\delta_1(1-\alpha_1)\left(\frac{1}{1-t_1}\right)^2 - r(\alpha_1\delta_1)^2\sigma_{11}^{\gamma} - r(1-\alpha_1)^2\sigma_{11}^{\theta}} < 0,$$

given $0 < t_1 < 1$.

Point 3. The optimal tax rate t_1^* on earnings x_1 increases with the inequality aversion parameter r from no taxation if the planner is inequality neutral ($t_1^* \to 0$ if $r \to 0$) to partial taxation if income is fully controlled ($t_1^* \to \frac{(1-\alpha_1)^2\sigma_{11}^{\theta}}{(\alpha_1\delta_1)^2\sigma_{11}^{\gamma}+(1-\alpha_1)^2\sigma_{11}^{\theta}}$ if $r \to +\infty$).

If there is no inequality aversion ($r \to 0$), then the first-order condition equals

$$-\alpha_1\delta_1\frac{t_1}{1-t_1}(1-\alpha_1) = 0,$$

which is satisfied for $t_1 \to 0$. The other case ($r \to +\infty$) leads to (divide by $r > 0$ and take the limit)

$$-(\alpha_1\delta_1)^2 t_1\sigma_{11}^{\gamma} + (1-\alpha_1)^2(1-t_1)\sigma_{11}^{\theta} = 0,$$

which can be solved to get

$$t_1^* = \frac{(1-\alpha_1)^2\sigma_{11}^{\theta}}{(\alpha_1\delta_1)^2\sigma_{11}^{\gamma} + (1-\alpha_1)^2\sigma_{11}^{\theta}}.$$

The comparative statics are

$$\frac{dt_1}{dr} = -\frac{\frac{\partial(27)}{\partial r}}{\frac{\partial(27)}{\partial t_1}} = -\frac{-(\alpha_1\delta_1)^2 t_1\sigma_{11}^{\gamma} + (1-\alpha_1)^2(1-t_1)\sigma_{11}^{\theta}}{-\alpha_1\delta_1(1-\alpha_1)\left(\frac{1}{1-t_1}\right)^2 - r(\alpha_1\delta_1)^2\sigma_{11}^{\gamma} - r(1-\alpha_1)^2\sigma_{11}^{\theta}}.$$

Using the first order condition, we can replace the numerator, to get

$$\frac{dt_1}{dr} = -\frac{\frac{1}{r}\alpha_1\delta_1\frac{t_1}{1-t_1}(1-\alpha_1)}{-\alpha_1\delta_1(1-\alpha_1)\left(\frac{1}{1-t_1}\right)^2 - r(\alpha_1\delta_1)^2\sigma_{11}^{\gamma} - r(1-\alpha_1)^2\sigma_{11}^{\theta}},$$

which is positive, given $0 < t_1 < 1$.

Point 4. The optimal tax rate t_1^* on earnings x_1 increases with type heterogeneity σ_{11}^{θ} from no taxation if everyone has the same type ($t_1^* \to 0$ if $\sigma_{11}^{\theta} \to 0$) to complete taxation if types become very heterogeneous ($t_1^* \to 1$ if $\sigma_{11}^{\theta} \to +\infty$).

If $\sigma_{11}^{\theta} \to 0$, the first-order condition reduces to

$$-\alpha_1\delta_1\frac{t_1}{1-t_1}(1-\alpha_1) - r(\alpha_1\delta_1)^2 t_1\sigma_{11}^{\gamma} = 0,$$

which is satisfied for $t_1 \to 0$. If $\sigma_{11}^{\theta} \to +\infty$, the first-order condition reduces to (divide by $\sigma_{11}^{\theta} > 0$ and consider the limiting case $\sigma_{11}^{\theta} \to +\infty$)

$$r(1-\alpha_1)^2(1-t_1) = 0,$$

which is satisfied for $t_1 \to 1$. The comparative statics are

$$\frac{dt_1}{d\sigma_{11}^{\theta}} = -\frac{\frac{\partial(27)}{\partial\sigma_{11}^{\theta}}}{\frac{\partial(27)}{\partial t_1}} = -\frac{r(1-\alpha_1)^2(1-t_1)}{-\alpha_1\delta_1(1-\alpha_1)\left(\frac{1}{1-t_1}\right)^2 - r(\alpha_1\delta_1)^2\sigma_{11}^{\gamma} - r(1-\alpha_1)^2\sigma_{11}^{\theta}} > 0,$$

given $0 < t_1 < 1$.

Point 5. The optimal tax rate t_1^* on earnings x_1 decreases with taste heterogeneity σ_{11}^{γ} from some taxation if everyone has the same taste ($0 < t_1^* < 1$ if $\sigma_{11}^{\gamma} \rightarrow 0$) to zero taxation if tastes become very heterogeneous ($t_1^* \rightarrow 0$ if $\sigma_{11}^{\gamma} \rightarrow +\infty$).

If there is no taste heterogeneity ($\sigma_{11}^{\gamma} \rightarrow 0$), then

$$-\alpha_1 \delta_1 \frac{t_1}{1-t_1} (1-\alpha_1) + r (1-\alpha_1)^2 (1-t_1) \sigma_{11}^{\theta} = 0,$$

which can lead to any tax rate satisfying $0 < t_1^* < 1$. The other case ($\sigma_{11}^{\gamma} \rightarrow +\infty$) leads to (divide by $\sigma_{11}^{\gamma} > 0$ and consider the limiting case $\sigma_{11}^{\gamma} \rightarrow +\infty$)

$$-r (\alpha_1 \delta_1)^2 t_1 \sigma_{11}^{\gamma} = 0,$$

which holds for $t_1 \rightarrow 0$. Taxes decrease with σ_{11}^{γ}, since

$$\frac{dt_1}{d\sigma_{11}^{\gamma}} = -\frac{\frac{\partial(27)}{\partial \sigma_{11}^{\gamma}}}{\frac{\partial(27)}{\partial t_1}} = -\frac{-r(\alpha_1\delta_1)^2 t_1}{-\alpha_1\delta_1(1-\alpha_1)\left(\frac{1}{1-t_1}\right)^2 - r(\alpha_1\delta_1)^2 \sigma_{11}^{\gamma} - r(1-\alpha_1)^2 \sigma_{11}^{\theta}} < 0$$

given $0 < t_1 < 1$.

Point 6. The optimal tax rate t_1^* on earnings x_1 decreases with the degree of control α_1 from complete taxation if earnings cannot be controlled ($t_1^* \rightarrow 1$ if $\alpha_1 \rightarrow 0$) to no taxation if income is fully controlled ($t_1^* \rightarrow 0$ if $\alpha_1 \rightarrow 1$).

If $\alpha_1 \rightarrow 0$, the first-order condition reduces to

$$r (1-\alpha_1)^2 (1-t_1) \sigma_{11}^{\theta} = 0,$$

which is satisfied for $t_1 \rightarrow 1$. If $\alpha_1 \rightarrow 1$, the first-order condition reduces to

$$-r (\delta_1)^2 t_1 \sigma_{11}^{\gamma} = 0,$$

which is satisfied for $t_1 \rightarrow 0$. The comparative statics are

$$\frac{dt_1^*}{d\alpha_1} = -\frac{\frac{\partial(27)}{\partial \alpha_1}}{\frac{\partial(27)}{\partial t_1}} = -\frac{\delta_1 \frac{t_1}{1-t_1}(2\alpha_1 - 1) - 2r\alpha_1(\delta_1)^2 t_1 \sigma_{11}^{\gamma} - 2r(1-\alpha_1)(1-t_1)\sigma_{11}^{\theta}}{-\alpha_1\delta_1(1-\alpha_1)\left(\frac{1}{1-t_1}\right)^2 - r(\alpha_1\delta_1)^2\sigma_{11}^{\gamma} - r(1-\alpha_1)^2\sigma_{11}^{\theta}}.$$

Dividing both sides by $(1-\alpha_1)\alpha_1 > 0$ and using the first-order condition to replace $\delta_1 \frac{t_1}{1-t_1}$, we get

$$\begin{aligned}
\frac{dt_1}{d\alpha_1} &= -\frac{(1-\alpha_1)\alpha_1 \left\{\left[-\frac{r\alpha_1(\delta_1)^2 t_1 \sigma_{11}^{\gamma}}{1-\alpha_1} + \frac{r(1-\alpha_1)(1-t_1)\sigma_{11}^{\theta}}{\alpha_1}\right](2\alpha_1 - 1) - 2r\alpha_1(\delta_1)^2 t_1\sigma_{11}^{\gamma} - 2r(1-\alpha_1)(1-t_1)\sigma_{11}^{\theta}\right\}}{(1-\alpha_1)\alpha_1\left(-\alpha_1\delta_1(1-\alpha_1)\left(\frac{1}{1-t_1}\right)^2 - r(\alpha_1\delta_1)^2\sigma_{11}^{\gamma} - r(1-\alpha_1)^2\sigma_{11}^{\theta}\right)} \\
&= -\frac{-\left(r(\alpha_1\delta_1)^2 t_1\sigma_{11}^{\gamma} + r(1-\alpha_1)^2(1-t_1)\sigma_{11}^{\theta}\right)}{-(1-\alpha_1)\alpha_1\left(\alpha_1\delta_1(1-\alpha_1)\left(\frac{1}{1-t_1}\right)^2 + r(\alpha_1\delta_1)^2\sigma_{11}^{\gamma} + r(1-\alpha_1)^2\sigma_{11}^{\theta}\right)},
\end{aligned}$$

which is negative, given point 1 ($0 < t_1 < 1$).

The Akerlof-case

Suppose there are two variables, earnings x_1 and an exogenous tag x_2 (thus, $\alpha_2 \to 0$). The first-order conditions reduce to

$$\begin{aligned} -\alpha_1\delta_1\frac{\zeta t_1}{1-t_1} - r\left(\alpha_1\delta_1\right)^2 t_1\sigma_{11}^{\gamma} + r\left(1-\alpha_1\right)\left(\left(1-t_1\right)\left(1-\alpha_1\right)\sigma_{11}^{\theta} + \left(\beta_2 - t_2\right)\sigma_{21}^{\theta}\right) &= 0, \\ \left(1-t_1\right)\left(1-\alpha_1\right)\sigma_{12}^{\theta} + \left(\beta_2-t_2\right)\sigma_{22}^{\theta} &= 0, \end{aligned}$$

with $\zeta = (1-\alpha_1)+\beta_2$. The second of the first-order conditions can be rewritten as

$$\begin{aligned} t_2 &= \beta_2 + \left(1-t_1\right)\left(1-\alpha_1\right)\frac{\sigma_{12}^{\theta}}{\sigma_{22}^{\theta}} \qquad (28) \\ &= \beta_2 + \rho_{12}^{\theta}\sqrt{\frac{\sigma_{11}^{\theta}}{\sigma_{22}^{\theta}}}\left(1-t_1\right)\left(1-\alpha_1\right), \end{aligned}$$

with $\rho_{12}^{\theta} = \frac{\sigma_{12}^{\theta}}{\sqrt{\sigma_{11}^{\theta}\sigma_{22}^{\theta}}}$ the type correlation. This can be plugged in in the other first-order condition to get

$$-\alpha_1\delta_1\frac{\zeta t_1}{1-t_1} - r\left(\alpha_1\delta_1\right)^2 t_1\sigma_{11}^{\gamma} + r\left(1-\alpha_1\right)^2\left(1-t_1\right)\sigma_{11}^{\theta}\left(1-\left(\rho_{12}^{\theta}\right)^2\right) = 0, \qquad (29)$$

The latter equation does not depend on t_2 and therefore completely describes the solution for t_1, which can afterwards be plugged in in (28) to obtain a solution for t_2. Before proceeding, note that we consider $\sigma_{11}^{\theta}, \sigma_{22}^{\theta}$ and $\rho_{12}^{\theta} = \frac{\sigma_{12}^{\theta}}{\sqrt{\sigma_{11}^{\theta}\sigma_{22}^{\theta}}}$ as primitives of the model and $\sigma_{12}^{\theta} = \rho_{12}^{\theta}\sqrt{\sigma_{11}^{\theta}\sigma_{22}^{\theta}}$ adjusts.[34]

Point 1. From proposition 1, we already know that $t_1 < 1$ in the optimum, and it is easy to verify that $t_1 < 0$ cannot satisfy equation (29). To summarize, we must have $0 < t_1 < 1$. As a consequence, we also have $t_2 \gtreqless \beta_2$ if $\rho_{12}^{\theta} \gtreqless 0$.

Point 2. The tax rate on earnings t_1^* decreases with the degree of control α_1, ranging from full taxation if earnings cannot be controlled ($t_1^* \to 1$ if $\alpha_1 \to 0$) to no taxation if income is fully controlled ($t_1^* \to 0$ if $\alpha_1 \to 1$); the tag is fully taxed, both if there is no control over earnings and if there is full control over earnings ($t_2^* \to \beta_2$, if either $\alpha_1 \to 0$ or $\alpha_1 \to 1$), but the change is undefined in general. We only know that, at $\alpha_1 \to 0$, the tax rate t_2^* increases (resp. decreases) with α_1 if the type correlation is positive (resp. negative) and vice-versa at $\alpha_1 \to 1$.

If $\alpha_1 \to 0$, condition (29) reduces to

$$\left(1-t_1\right)\left(1-\left(\rho_{12}^{\theta}\right)^2\right) = 0,$$

which implies $t_1 \to 1$ and, using $t_1 \to 1$ in in (28), we get $t_2 \to \beta_2$. If $\alpha_1 \to 1$, condition (29) reduces to

$$-\delta_1\frac{\zeta t_1}{1-t_1} - r\left(\delta_1\right)^2 t_1\sigma_{11}^{\gamma} = 0,$$

which is satisfied for $t_1 \to 0$ and this leads to $t_2 \to \beta_2 + \left(1-\alpha_1\right)\frac{\sigma_{12}^{\theta}}{\sigma_{22}^{\theta}}$. The comparative statics for t_1 w.r.t. α_1 are

$$\frac{dt_1^*}{d\alpha_1} = -\frac{\frac{\partial eq(29)}{\partial\alpha_1}}{\frac{\partial eq29}{\partial t_1}} = -\frac{-\delta_1\frac{t_1}{1-t_1}\left(1-2\alpha_1+\beta_2\right) - 2r\alpha_1\left(\delta_1\right)^2 t_1\sigma_{11}^{\gamma} - 2r\left(1-\alpha_1\right)\left(1-t_1\right)\sigma_{11}^{\theta}\left(1-\left(\rho_{12}^{\theta}\right)^2\right)}{-\frac{\alpha_1\delta_1\left(1-\alpha_1+\beta_2\right)}{\left(1-t_1\right)^2} - r\left(\alpha_1\delta_1\right)^2\sigma_{11}^{\gamma} - r\left(1-\alpha_1\right)^2\sigma_{11}^{\theta}\left(1-\left(\rho_{12}^{\theta}\right)^2\right)}.$$

[34] The constraints on σ_{11}^{θ} ($-\sqrt{\sigma_{11}^{\theta}\sigma_{22}^{\theta}} \le \sigma_{12}^{\theta} \le \sqrt{\sigma_{11}^{\theta}\sigma_{22}^{\theta}}$) depend on (and thus move with) changes in σ_{11}^{θ} and σ_{22}^{θ}, which could complicate the comparative statics. This is not true for the constraints on ρ_{12}^{θ} (i.e., $-1 \le \rho_{12}^{\theta} \le 1$).

We can divide both sides by $\alpha_1\zeta = \alpha_1(1-\alpha_1+\beta_2) > 0$ and using the first-order condition to replace $-\alpha_1\delta_1\frac{\zeta t_1}{1-t_1}$, we get (after some manipulation) that

$$\frac{dt_1^*}{d\alpha_1} = -\frac{-(1+\beta_2)\,r\,(\alpha_1\delta_1)^2\,t_1\sigma_{11}^\gamma - [(1-\alpha_1)(1+\beta_2)+2\alpha_1\beta_2]\,r\,(1-\alpha_1)(1-t_1)\,\sigma_{11}^\theta\left(1-\left(\rho_{12}^\theta\right)^2\right)}{-\alpha_1(1-\alpha_1+\beta_2)\left(\frac{\alpha_1\delta_1(1-\alpha_1+\beta_2)}{(1-t_1)^2}+r\,(\alpha_1\delta_1)^2\,\sigma_{11}^\gamma + r\,(1-\alpha_1)^2\,\sigma_{11}^\theta\left(1-\left(\rho_{12}^\theta\right)^2\right)\right)}$$

which is negative, given $0 < t_1 < 1$. The comparative statics for t_2 w.r.t. α_1 are

$$\frac{dt_2^*}{d\alpha_1} = \frac{\partial eq(28)}{\partial\alpha_1} + \frac{\partial eq(28)}{\partial t_1}\frac{dt_1}{d\alpha_1} = -\sigma_{12}^\theta\left(\frac{1-t_1}{\sigma_{22}^\theta} + \frac{1-\alpha_1}{\sigma_{22}^\theta}\frac{dt_1}{d\alpha_1}\right),$$

which is not defined in general. At $\alpha_1 \to 0$ (& thus, $t_1 \to \beta_1$), the derivative $\frac{dt_2}{d\alpha_1}$ equals $-\sigma_{12}^\theta\left(\frac{1}{\sigma_{22}^\theta}\frac{dt_1}{d\alpha_1}\right)$, so $\frac{dt_2}{d\alpha_1}$ is positive (resp. negative) if the type covariance/correlation is positive (resp. negative), while at $\alpha_1 \to 1$ (& thus, $t_1 \to 0$), $\frac{dt_2}{d\alpha_1}$ equals $-\sigma_{12}^\theta\left(\frac{1}{\sigma_{22}^\theta}\right)$ which leads to the opposite sign.

Point 3. The tax rate on earnings t_1^* decreases with the cost of taxation δ_1, ranging from $t_1 \to 1$ (if $\delta_1 \to 0$) to $t_1 \to 0$ (if $\delta_1 \to +\infty$). The tax rate t_2^* on the tag increases (resp. decreases) with the earnings elasticity $\delta_1$if the type correlation is positive (resp. negative), ranging from $t_2 = \beta_2$ (if $\delta_1 \to 0$) to $\beta_2 + \rho_{12}^\theta\sqrt{\frac{\sigma_{11}^\theta}{\sigma_{22}^\theta}}(1-\alpha_1)$ (if $\delta_1 \to +\infty$).

If $\delta_1 \to 0$, condition (29) reduces to

$$(1-t_1)\left(1-\left(\rho_{12}^\theta\right)^2\right) = 0,$$

which implies $t_1 \to 1$ and $t_2 \to t_2 = \beta_2$. If $\delta_1 \to +\infty$, we get

$$-r\,(\alpha_1)^2\,t_1\sigma_{11}^\gamma = 0,$$

which implies $t_1 \to 0$ and this leads to $t_2 = \beta_2 + \rho_{12}^\theta\sqrt{\frac{\sigma_{11}^\theta}{\sigma_{22}^\theta}}(1-\alpha_1)$. The comparative statics for t_1 w.r.t. δ_1 are

$$\frac{dt_1^*}{d\delta_1} = -\frac{-\alpha_1\frac{\zeta t_1}{1-t_1} - 2r\delta_1(\alpha_1)^2\,t_1\sigma_{11}^\gamma}{-\frac{\alpha_1\delta_1(1-\alpha_1+\beta_2)}{(1-t_1)^2} - r\,(\alpha_1\delta_1)^2\,\sigma_{11}^\gamma - r\,(1-\alpha_1)^2\,\sigma_{11}^\theta\left(1-\left(\rho_{12}^\theta\right)^2\right)},$$

which is negative, i.e., the more elastic the lower the tax. The comparative statics for t_2 w.r.t. δ_1 are

$$\frac{dt_2^*}{d\delta_1} = \frac{\partial eq(28)}{\partial\delta_1} + \frac{\partial eq(28)}{\partial t_1}\frac{dt_1^*}{d\delta_1} = -\sigma_{12}^\theta\frac{1-\alpha_1}{\sigma_{22}^\theta}\frac{dt_1}{d\delta_1},$$

the sign of which corresponds with the sign of the correlation.

Point 4. The tax rate on earnings t_1^* increases with the type heterogeneity σ_{11}^θ for earnings, from $t_1 \to 0$ to $t_1 \to 1$; the tax rate on the tag t_2^* equals β_2 if there is no type heterogeneity σ_{11}^θ for earnings, while the comparative statics are undefined.

If $\sigma_{11}^\theta \to 0$ (and recall that $\sigma_{12}^\theta = \rho_{12}^\theta\sqrt{\sigma_{11}^\theta\sigma_{22}^\theta}$ adjusts to 0, leaving ρ_{12}^θ unchanged) then condition (29) reduces to

$$-\alpha_1\delta_1\frac{\zeta t_1}{1-t_1} - r\,(\alpha_1\delta_1)^2\,t_1\sigma_{11}^\gamma = 0,$$

which leads to $t_1 \to 0$ and $t_2 \to \beta_2$. If $\sigma_{11}^\theta \to +\infty$, then condition (29) reduces to

$$r\,(1-\alpha_1)^2(1-t_1)\left(1-\left(\rho_{12}^\theta\right)^2\right) = 0,$$

which implies $t_1 \to 1$ and t_2 undefined (since both $\sigma^\theta_{11} \to +\infty$ and $1-t_1 \to 0$). The comparative statics for t^*_1 w.r.t. σ^θ_{11} are equal to

$$\frac{dt^*_1}{d\sigma^\theta_{11}} = -\frac{r\left(1-\alpha_1\right)^2\left(1-t_1\right)\left(1-\left(\rho^\theta_{12}\right)^2\right)}{-\frac{\alpha_1\delta_1(1-\alpha_1+\beta_2)}{(1-t_1)^2} - r\left(\alpha_1\delta_1\right)^2\sigma^\gamma_{11} - r\left(1-\alpha_1\right)^2\sigma^\theta_{11}\left(1-\left(\rho^\theta_{12}\right)^2\right)},$$

which is positive. The comparative statics for t_2 w.r.t. δ_1 are

$$\frac{dt^*_2}{d\sigma^\theta_{11}} = \frac{\partial eq(28)}{\partial\sigma^\theta_{11}} + \frac{\partial eq(28)}{\partial t_1}\frac{dt^*_1}{d\sigma^\theta_{11}} = \rho^\theta_{12}\left(1-\alpha_1\right)\sqrt{\frac{\sigma^\theta_{11}}{\sigma^\theta_{22}}}\left(\frac{(1-t_1)}{2\sigma^\theta_{11}} - \sqrt{\frac{\sigma^\theta_{11}}{\sigma^\theta_{22}}}\frac{dt^*_1}{d\sigma^\theta_{11}}\right),$$

the sign of which is not defined.

Point 5. The tax rate on earnings t^*_1 does not change with σ^θ_{22}. The tax rate on the tag t^*_2 increases (resp. decreases) with σ^θ_{22} if the type correlation is negative (resp. positive).

Condition (29) does not change with σ^θ_{22}, indicating that t^*_1 remains unchanged as well, thus $\frac{dt^*_1}{d\sigma^\theta_{22}} = 0$. The tax rate on the tag increases (resp. decreases) with σ^θ_{22} if the correlation is negative (resp. positive), which can be seen from

$$\frac{dt^*_2}{d\sigma^\theta_{22}} = \frac{\partial eq(28)}{\partial\sigma^\theta_{22}} + \frac{\partial eq(28)}{\partial t_1}\frac{dt^*_1}{d\sigma^\theta_{22}} = -\frac{\rho^\theta_{12}}{2\sigma^\theta_{22}}\sqrt{\frac{\sigma^\theta_{11}}{\sigma^\theta_{22}}}\left(1-t_1\right)\left(1-\alpha_1\right),$$

the sign of which is the opposite to the sign of the type correlation ρ^θ_{12}.

Point 6. The tax rate on earnings t^*_1 increases with ρ^θ_{12} if ρ^θ_{12} is negative, and t^*_1 decreases with ρ^θ_{12} if ρ^θ_{12} is positive. At the extremes ($\left(\rho^\theta_{12}\right)^2 = 1$) the same tax rate $t^*_1 = 0$ on earnings applies; the tax rate on the tag t^*_2 increases from $\beta_2 - \sqrt{\frac{\sigma^\theta_{11}}{\sigma^\theta_{22}}}\left(1-\alpha_1\right)$ to $\beta_2 + \sqrt{\frac{\sigma^\theta_{11}}{\sigma^\theta_{22}}}\left(1-\alpha_1\right)$;

At the extremes ($\rho^\theta_{12} = \pm 1$), condition (29) reduces to

$$-\alpha_1\delta_1\frac{\zeta t_1}{1-t_1} - r\left(\alpha_1\delta_1\right)^2 t_1\sigma^\gamma_{11} = 0,$$

which implies $t_1 \to 0$. Note that

$$\frac{dt^*_1}{d\rho^\theta_{12}} = -\frac{-2r\left(1-\alpha_1\right)^2\left(1-t_1\right)\sigma^\theta_{11}\rho^\theta_{12}}{-\frac{\alpha_1\delta_1(1-\alpha_1+\beta_2)}{(1-t_1)^2} - r\left(\alpha_1\delta_1\right)^2\sigma^\gamma_{11} - r\left(1-\alpha_1\right)^2\sigma^\theta_{11}\left(1-\left(\rho^\theta_{12}\right)^2\right)},$$

the sign of which is inversely related to ρ^θ_{12}. The comparative statics for the tax rate on the tag equals

$$\frac{dt^*_2}{d\rho^\theta_{12}} = \frac{\partial eq(28)}{\partial\rho^\theta_{12}} + \frac{\partial eq(28)}{\partial t_1}\frac{dt^*_1}{d\rho^\theta_{12}} = \sqrt{\frac{\sigma^\theta_{11}}{\sigma^\theta_{22}}}\left(1-\alpha_1\right)\left(\left(1-t_1\right) - \rho^\theta_{12}\frac{dt^*_1}{d\rho^\theta_{12}}\right),$$

which is positive (since $\rho^\theta_{12}\frac{dt^*_1}{d\rho^\theta_{12}} \leq 0$), increasing from $\beta_2 - \sqrt{\frac{\sigma^\theta_{11}}{\sigma^\theta_{22}}}\left(1-\alpha_1\right)$ to $\beta_2 + \sqrt{\frac{\sigma^\theta_{11}}{\sigma^\theta_{22}}}\left(1-\alpha_1\right)$.

Point 7. The tax rate on earnings t^*_1 and the tax rate on the tag t^*_2 do not depend on σ^γ_{22} and ρ^γ_{12}, but decreases with taste heterogeneity for earnings σ^γ_{11}; the tax rate for the tag t^*_2 increases (resp. decreases in case $\rho^\theta_{12} < 0$) with σ^γ_{11} to reach $\beta_2 + \rho^\theta_{12}\sqrt{\frac{\sigma^\theta_{11}}{\sigma^\theta_{22}}}\left(1-\alpha_1\right)$ if $\sigma^\gamma_{11} \to +\infty$.

If $\sigma_{11}^{\gamma} \to 0$, then condition (29) reduces to

$$-\alpha_1 \delta_1 \frac{\zeta t_1}{1-t_1} + r\left(1-\alpha_1\right)^2 \left(1-t_1\right) \sigma_{11}^{\theta} \left(1-\left(\rho_{12}^{\theta}\right)^2\right) = 0,$$

which does not give a clear prescription. If $\sigma_{11}^{\gamma} \to +\infty$, then condition (29) reduces to

$$-r\left(\alpha_1 \delta_1\right)^2 t_1 = 0,$$

which implies $t_1 \to 0$ and $t_2 \to \beta_2 + \rho_{12}^{\theta} \sqrt{\frac{\sigma_{11}^{\theta}}{\sigma_{22}^{\theta}}} \left(1-\alpha_1\right)$. Comparative statics are

$$\frac{dt_1^*}{d\sigma_{11}^{\gamma}} = -\frac{-r\left(\alpha_1\delta_1\right)^2 t_1}{-\frac{\alpha_1\delta_1\left(1-\alpha_1+\beta_2\right)}{\left(1-t_1\right)^2} - r\left(\alpha_1\delta_1\right)^2 \sigma_{11}^{\gamma} - r\left(1-\alpha_1\right)^2 \sigma_{11}^{\theta}\left(1-\left(\rho_{12}^{\theta}\right)^2\right)},$$

which is negative, as required, and

$$\frac{dt_2^*}{d\sigma_{11}^{\gamma}} = \frac{\partial eq(28)}{\partial \sigma_{11}^{\gamma}} + \frac{\partial eq(28)}{\partial t_1}\frac{dt_1^*}{d\sigma_{11}^{\gamma}} = -\rho_{12}^{\theta}\sqrt{\frac{\sigma_{11}^{\theta}}{\sigma_{22}^{\theta}}}\left(1-\alpha_1\right)\frac{dt_1^*}{d\sigma_{11}^{\gamma}}$$

the sign of which is the same as the sign of ρ_{12}^{θ}.

Point 8. The tax rate on earnings t_1^* increases with the inequality aversion r, from $t_1 \to 0$ to $t_1 \to \frac{\left(1-\alpha_1\right)^2\sigma_{11}^{\theta}\left(1-\left(\rho_{12}^{\theta}\right)^2\right)}{\left(\alpha_1\delta_1\right)^2\sigma_{11}^{\gamma}+\left(1-\alpha_1\right)^2\sigma_{11}^{\theta}\left(1-\left(\rho_{12}^{\theta}\right)^2\right)}$; the tax rate on the tag increases (resp. decreases) with r from $\beta_2 + \rho_{12}^{\theta}\sqrt{\frac{\sigma_{11}^{\theta}}{\sigma_{22}^{\theta}}}\left(1-\alpha_1\right)$ to $\beta_2 + \rho_{12}^{\theta}\sqrt{\frac{\sigma_{11}^{\theta}}{\sigma_{22}^{\theta}}}\frac{\left(\alpha_1\delta_1\right)^2\sigma_{11}^{\gamma}}{\left(\alpha_1\delta_1\right)^2\sigma_{11}^{\gamma}+\left(1-\alpha_1\right)^2\sigma_{11}^{\theta}\left(1-\left(\rho_{12}^{\theta}\right)^2\right)}$ if the correlation is positive (resp. negative).

If $r \to 0$, then condition (29) reduces to

$$-\alpha_1\delta_1\frac{\zeta t_1}{1-t_1} = 0,$$

which implies $t_1 \to 0$ and $t_2 \to \beta_2 + \rho_{12}^{\theta}\sqrt{\frac{\sigma_{11}^{\theta}}{\sigma_{22}^{\theta}}}\left(1-\alpha_1\right)$. If $r \to +\infty$, then condition (29) directly implies

$$t_1 = \frac{\left(1-\alpha_1\right)^2\sigma_{11}^{\theta}\left(1-\left(\rho_{12}^{\theta}\right)^2\right)}{\left(\alpha_1\delta_1\right)^2\sigma_{11}^{\gamma}+\left(1-\alpha_1\right)^2\sigma_{11}^{\theta}\left(1-\left(\rho_{12}^{\theta}\right)^2\right)},$$

and t_2 equals

$$\beta_2 + \rho_{12}^{\theta}\sqrt{\frac{\sigma_{11}^{\theta}}{\sigma_{22}^{\theta}}}\frac{\left(\alpha_1\delta_1\right)^2\sigma_{11}^{\gamma}}{\left(\alpha_1\delta_1\right)^2\sigma_{11}^{\gamma}+\left(1-\alpha_1\right)^2\sigma_{11}^{\theta}\left(1-\left(\rho_{12}^{\theta}\right)^2\right)}.$$

Comparative statics are

$$\frac{dt_1^*}{dr} = -\frac{-\left(\alpha_1\delta_1\right)^2 t_1\sigma_{11}^{\gamma} + \left(1-\alpha_1\right)^2\left(1-t_1\right)\sigma_{11}^{\theta}\left(1-\left(\rho_{12}^{\theta}\right)^2\right)}{-\frac{\alpha_1\delta_1\left(1-\alpha_1+\beta_2\right)}{\left(1-t_1\right)^2} - r\left(\alpha_1\delta_1\right)^2 \sigma_{11}^{\gamma} - r\left(1-\alpha_1\right)^2 \sigma_{11}^{\theta}\left(1-\left(\rho_{12}^{\theta}\right)^2\right)},$$

and using condition (29), we get

$$\frac{dt_1^*}{dr} = -\frac{\alpha_1\delta_1\frac{\zeta t_1}{1-t_1}\frac{1}{r}}{-\frac{\alpha_1\delta_1\left(1-\alpha_1+\beta_2\right)}{\left(1-t_1\right)^2} - r\left(\alpha_1\delta_1\right)^2 \sigma_{11}^{\gamma} - r\left(1-\alpha_1\right)^2 \sigma_{11}^{\theta}\left(1-\left(\rho_{12}^{\theta}\right)^2\right)},$$

which is positive, as required, and

$$\frac{dt_2^*}{dr} = \frac{\partial eq(28)}{\partial r} + \frac{\partial eq(28)}{\partial t_1}\frac{dt_1^*}{dr} = -\rho_{12}^{\theta}\sqrt{\frac{\sigma_{11}^{\theta}}{\sigma_{22}^{\theta}}}\,(1-\alpha_1)\,\frac{dt_1^*}{dr}$$

the sign of which is the same as the sign of ρ_{12}^{θ}.

Data

1. Pre-tax household income is the sum (at household level) of the remuneration of labor (earnings) and capital (rents), more precisely, the sum of

   (a) (gross) employee cash or near cash income,

   (b) (gross) non-cash employee income,[35]

   (c) employer's social insurance contributions,[36]

   (d) (gross) cash benefits or losses from self-employment,

   (e) (gross) rental income,

   (f) (gross) interest, dividends and profit from capital investments in unincorporated business;

2. Post-tax household income is the pre-tax household income + the sum of (gross) benefits - taxes and social insurance contributions, more precisely, pre-tax household income

   PLUS

   (a) (gross) unemployment benefits,

   (b) (gross) old-age and survivor benefits,

   (c) (gross) sickness and disability benefits,

   (d) (gross) education-related allowances,

   (e) (gross) child allowances,

   (f) (gross) other benefits (e.g., guaranteed minimum income),

   MINUS

   (a) employer's social insurance contributions,

   (b) tax on income (including taxes on holdings and tax reimbursements) and (employee's) social security contributions.

3. To obtain equivalent (pre- or post-tax) income, we divide (pre- or post-tax) income by the (modified) OECD scale, i.e., $1 + 0.5 \times$ (# of additional adults ($age \geq 14$)) $+ 0.3 \times$ (# of children ($age < 14$)).

[35] Imputed for the Netherlands on the basis of EU-SILC 2006 data.
[36] Imputed for Germany, Latvia and the UK.

Table 2: Income concepts

Concept	Definition / Imputation
Wages and Salaries	Gross employee cash or near cash income (including e.g. holiday payments, pay for overtime, bonuses etc.) plus non-cash employee income (e.g. company car, free or subsidized meals etc.).
Self-employment Income	Net operating profit or loss accruing to working owners of, or partners in, an unincorporated enterprise less interest on business loans; royalities earned on writing and inventions as well as rentals from business buildings, vehicles, equipment etc.
Capital Income	Imputed rent; income from rental of a property or land; interest, dividends, profits from capital investment in an unicorporated business; regular inter-household cash transfers received.
Social Insurance Contributions Employer	Payments made by the employers for the benefits of their employees to insurers (social security funds and private funded schemes) covering statutory, convential or contractual contributions in respect of insurance against social risks. Information on the amount of social insurance contributions paid by the employer is not reported for DE, LT and the UK. In these cases, we use country-specific legal rules to impute the SIC paid by the employer based on the corresponding employee income.
Public Pensions	Old-age benefits (any replacement income when the aged person retires from the labor market, care allowances etc.) and survivor´s benefits (such as survivor´s pension and death grants).
Cash Benefits	Unemployment benefits, sickness benefits, disability benefits, education-related allowances; family/children related allowances, housing allowances, benefits for social exclusion not elsewhere classified (periodic income support for people with insufficient resources and other related cash benefits).
Income taxes	Taxes on income, profits and capital gains, assessed on the actual or presumed income of individuals, households or tax-units. EU-SILC only reports income taxes and employee SIC as an aggregated value. We subtract imputed SIC to isolate income tax payments as a single variable.
Total Social Insurance Contributions	Employer´s SIC (see above) and employees' SIC (any contributions to either mandatory government or employer-based social insurance schemes. EU-SILC does not report SIC paid by the employee as a separate variable, therefore values are imputed (see above) applying the appropriate legal rules of each country.

Table 3: Mean statistics for different income-related concepts

	number	eq. scale	gross	eq. gross	net	eq. net	tax	eq. tax
AT	8318	1.6	34655.3	20082.0	32686.0	20071.6	1969.3	10.4
BE	8307	1.7	43194.0	24444.0	31959.1	18994.8	11234.8	5449.2
CY	4191	1.9	30744.1	15526.3	31246.0	16608.3	-502.0	-1082.0
CZ	12459	1.6	9913.8	5633.0	9199.8	5618.1	714.0	14.9
DE	19444	1.6	34776.2	20584.8	31769.7	19744.8	3006.5	840.1
DK	8527	1.7	69006.7	38351.4	48043.6	27608.0	20963.1	10743.5
EE	6029	1.7	9756.8	5291.7	8086.0	4592.2	1670.8	699.5
ES	13464	1.7	25064.6	13767.1	22414.1	13011.6	2650.4	755.5
FI	14432	1.7	49408.9	27486.2	38015.5	21884.7	11393.4	5601.4
FR	14213	1.7	37253.7	20769.3	32729.7	19363.9	4524.1	1405.3
GR	5348	1.7	24753.0	13443.0	19924.7	11647.9	4828.3	1795.1
HU	10162	1.7	6783.4	3809.9	6841.2	4155.9	-57.8	-345.9
IE	6536	1.6	37483.5	20709.8	40664.2	24115.0	-3180.8	-3405.2
IS	3838	1.8	83562.0	44325.9	60503.3	32887.2	23058.7	11438.7
IT	21976	1.7	31537.4	17788.4	27959.3	16886.4	3578.1	902.0
LT	5995	1.6	7197.7	4063.6	6105.5	3637.9	1092.2	425.7
LU	5297	1.7	62378.3	35747.7	57040.1	33279.6	5338.1	2468.0
LV	4721	1.6	5788.6	3386.9	5361.1	3294.6	427.5	92.3
NL	15263	1.7	54028.7	30428.4	37355.5	22039.3	16673.2	8389.1
NO	8534	1.7	66752.0	37481.4	55744.7	32251.8	11007.4	5229.7
PL	14184	1.7	7094.8	3769.7	7128.9	4165.5	-34.1	-395.8
PT	4345	1.7	17657.4	9640.8	16194.9	9546.4	1462.5	94.4
SE	10380	1.7	45325.4	25836.6	34950.6	20595.9	10374.8	5240.8
SI	8702	1.9	23314.9	11552.2	20117.4	10741.3	3197.5	810.8
SK	5153	1.8	7687.1	3975.4	7119.0	4024.9	568.2	-49.5
UK	12108	1.6	43820.4	25788.2	38768.2	23902.0	5052.2	1886.2
US	115650	1.8	59663.1	32526.2	52771.2	29425.9	6892.0	3100.3

Source: Own calculations based on EU-SILC and IPUMS-CPS.

Table 4: Descriptive statictics covariates

	sex	age2635	age3645	age4655	age5665	age6675	age76plus	disabled	educ2	educ3	educ4	nmonth un	nsmallchildren	nmiddlechildren	nbigchildren	naddadults	couple	eu birth	oth birth
AT	0.50	0.14	0.23	0.17	0.17	0.14	0.10	0.04	0.19	0.52	0.28	0.47	0.09	0.09	0.30	0.19	0.57	0.05	0.08
BE	0.30	0.17	0.22	0.18	0.16	0.14	0.09	0.04	0.13	0.31	0.34	1.43	0.11	0.09	0.30	0.23	0.60	0.06	0.05
CY	0.21	0.15	0.24	0.18	0.16	0.16	0.10	0.03	0.08	0.33	0.26	0.26	0.10	0.12	0.44	0.41	0.75	0.05	0.05
CZ	0.35	0.16	0.16	0.15	0.20	0.19	0.13	0.09	0.14	0.72	0.13	0.32	0.07	0.07	0.22	0.24	0.57	0.03	0.01
DE	0.41	0.11	0.22	0.18	0.19	0.21	0.07	0.04	0.08	0.43	0.49	0.81	0.05	0.07	0.24	0.18	0.61	0.00	0.09
DK	0.53	0.16	0.23	0.20	0.19	0.10	0.07	0.07	0.24	0.44	0.30	0.22	0.10	0.11	0.36	0.22	0.75	0.01	0.03
EE	0.52	0.13	0.21	0.18	0.17	0.16	0.11	0.07	0.17	0.46	0.32	0.24	0.07	0.07	0.26	0.44	0.57	0.00	0.18
ES	0.43	0.15	0.25	0.17	0.13	0.16	0.12	0.02	0.20	0.18	0.25	0.63	0.11	0.11	0.31	0.28	0.67	0.01	0.05
FI	0.51	0.15	0.20	0.22	0.20	0.09	0.06	0.09	0.08	0.41	0.34	0.73	0.10	0.09	0.33	0.26	0.72	0.01	0.02
FR	0.41	0.16	0.21	0.18	0.16	0.12	0.10	0.04	0.12	0.42	0.25	0.67	0.11	0.11	0.32	0.24	0.63	0.04	0.07
GR	0.26	0.13	0.22	0.16	0.13	0.17	0.14	0.02	0.11	0.29	0.22	0.43	0.10	0.11	0.30	0.24	0.65	0.01	0.06
HU	0.39	0.14	0.17	0.17	0.19	0.18	0.13	0.09	0.20	0.49	0.22	0.42	0.07	0.08	0.27	0.30	0.54	0.00	0.01
IE	0.59	0.10	0.19	0.16	0.17	0.18	0.17	0.10	0.17	0.16	0.30	0.37	0.09	0.10	0.33	0.23	0.51	0.08	0.03
IS	0.50	0.20	0.23	0.20	0.14	0.10	0.07	0.05	0.27	0.35	0.34	0.11	0.15	0.13	0.43	0.36	0.75	0.03	0.02
IT	0.32	0.13	0.23	0.16	0.14	0.17	0.16	0.03	0.26	0.28	0.16	0.31	0.10	0.08	0.26	0.24	0.60	0.01	0.04
LT	0.58	0.10	0.18	0.18	0.19	0.21	0.12	0.07	0.12	0.24	0.48	0.35	0.05	0.05	0.23	0.32	0.59	0.00	0.07
LU	0.35	0.26	0.26	0.17	0.14	0.08	0.04	0.04	0.09	0.30	0.32	0.44	0.17	0.14	0.36	0.22	0.68	0.47	0.08
LV	0.68	0.10	0.17	0.15	0.18	0.22	0.13	0.05	0.23	0.41	0.30	0.42	0.07	0.05	0.20	0.33	0.43	0.00	0.19
NL	0.50	0.17	0.25	0.20	0.18	0.11	0.07	0.07	0.19	0.35	0.36	0.47	0.13	0.13	0.35	0.17	0.69	0.01	0.03
NO	0.49	0.19	0.21	0.20	0.16	0.10	0.07	0.16	0.20	0.44	0.34	0.16	0.11	0.10	0.35	0.24	0.66	0.03	0.04
PL	0.42	0.15	0.20	0.20	0.17	0.16	0.10	0.06	0.00	0.56	0.21	0.55	0.09	0.08	0.32	0.36	0.62	0.01	0.01
PT	0.29	0.11	0.21	0.17	0.16	0.20	0.14	0.03	0.13	0.10	0.09	0.56	0.07	0.07	0.29	0.27	0.68	0.01	0.01
SE	0.52	0.18	0.19	0.17	0.17	0.11	0.09	0.09	0.08	0.42	0.35	0.28	0.13	0.09	0.30	0.22	0.69	0.05	0.07
SI	0.56	0.13	0.26	0.21	0.15	0.15	0.08	0.10	0.20	0.55	0.21	0.62	0.08	0.09	0.29	0.56	0.71	0.00	0.11
SK	0.41	0.11	0.21	0.19	0.18	0.18	0.11	0.08	0.14	0.65	0.19	0.32	0.05	0.06	0.26	0.47	0.57	0.02	0.00
UK	0.44	0.13	0.21	0.15	0.18	0.16	0.14	0.04	0.27	0.41	0.30	0.19	0.07	0.09	0.30	0.16	0.59	0.01	0.07
US	0.56	0.18	0.23	0.22	0.14	0.09	0.08	0.11	0.11	0.49	0.37	0.15	0.12	0.13	0.41	0.47	0.59	0.01	0.15

Source: Own calculations based on EU-SILC and IPUMS-CPS.

Regression results

Table 5: Results from first stage regression

	(1) AT	(2) BE	(3) CY	(4) CZ	(5) DE	(6) DK	(7) EE	(8) ES	(9) FI	(10) FR	(11) GR	(12) HU	(13) IE
cons	25546.922	27604.738	11775.981	9121.824	11635.050	21989.403	7819.094	14632.920	15217.289	16767.163	1852.114	5573.965	16214.242
sex	-2023.928	-3750.082	-1897.651	-663.226	-985.753	-1906.926	-762.853	-315.891	-2343.329	-1521.499	-1533.016	-126.518	-2973.823
age2635	5430.937	7566.488	6762.648	1314.249	13081.065	15251.672	911.910	3468.073	11228.432	7185.995	11727.804	1526.108	11157.349
age3645	7522.615	7651.221	6394.034	544.436	16695.645	21706.784	-477.525	2991.261	14534.171	8957.920	15731.765	973.570	14795.719
age4655	7123.261	6980.908	6263.924	-20.924	17124.971	23416.004	-1265.536	1982.675	15889.529	9866.349	15571.291	488.787	8366.525
age5665	-12896.360	-6686.064	1772.193	-5072.404	-341.439	12642.902	-3745.530	-5075.524	3181.686	-4910.749	8673.288	-2947.352	-1882.460
age6675	-25585.638	-28650.537	-10851.031	-9066.009	-18870.382	-20037.237	-6865.825	-14741.098	-19743.811	-18139.652	-953.270	-5486.774	-17537.120
age76plus	-24310.723	-27246.933	-10785.029	-8832.612	-19023.006	-21911.294	-7208.144	-14783.804	-18896.539	-17093.671	-1657.852	-5610.980	-16128.706
disabled	-15516.956	-19563.981	-8387.434	-2847.697	-18490.979	-19519.017	-3345.806	-9028.464	-12255.999	-10700.672	-9068.543	-2905.538	-10010.723
educ2	-1987.404	158.908	352.221	-210.597	4904.663	-3623.966	-463.675	910.734	1347.209	179.633	801.642	-492.606	503.996
educ3	558.645	2850.492	2987.787	205.304	5061.088	393.013	324.333	3764.982	2144.563	2204.859	3929.896	-129.050	6793.667
educ4	6999.053	10052.954	11185.447	3360.305	11550.280	10803.598	1858.626	9515.308	14931.591	11851.126	12828.814	2876.261	17884.790
nmonth_un	-1716.017	-2000.262	-991.287	-508.863	-2046.799	-2174.510	-412.606	-723.187	-1828.112	-1279.445	-1026.487	-350.733	-1878.430
nsmallchildren	-8352.094	-5675.255	-2788.660	-3329.567	-7398.067	-5705.994	-1781.639	-3058.979	-7708.518	-4598.204	-1047.134	-1943.037	-3578.840
nmiddlechildren	-5648.061	-3166.564	-1741.622	-1922.904	-5586.642	-4180.185	-786.125	-2312.458	-3939.539	-4002.338	-1272.235	-976.920	-4353.029
nbigchildren	-5333.246	-3544.717	-1727.777	-1372.474	-5572.125	-5090.791	-871.101	-2160.889	-4090.316	-3274.972	-1854.383	-1036.011	-5884.709
naddadults	-2471.722	-4093.699	-1207.926	-596.434	-4781.843	-4367.473	-420.351	-1341.946	-2620.992	-2569.075	-1177.160	-220.226	-979.068
couple	4329.557	3853.186	1626.223	974.421	5110.406	10443.145	1044.644	1476.534	8165.101	5465.491	1128.953	590.033	7017.157
eu_birth	-1202.151	669.003	-4299.538	-53.260	0.000	-3940.890	0.000	-3088.936	-3162.852	-458.352	-1279.173	-199.072	-4634.057
oth_birth	-4034.371	-7917.095	-5712.640	-774.117	-2659.346	-5380.936	-786.925	-2339.509	-4480.387	-2838.096	-3532.376	-66.456	-5953.281
Observations	5285	5198	2396	7953	12110	4866	3848	8053	8405	8746	3241	6608	4329
Adjusted R^2	0.736	0.780	0.795	0.749	0.729	0.819	0.717	0.756	0.757	0.745	0.696	0.691	0.662

	(14) IS	(15) IT	(16) LT	(17) LU	(18) LV	(19) NL	(20) NO	(21) PL	(22) PT	(23) SE	(24) SI	(25) SK	(26) UK	(27) US
cons	29376.106	17452.863	7010.652	21166.012	5272.041	14892.053	20800.671	3816.564	7411.043	13468.461	10442.496	6529.677	24923.787	12996.334
sex	-1123.843	-1436.930	-297.401	-1321.400	-537.789	-390.481	-2068.961	-475.251	-823.815	-529.337	-532.864	-128.727	-2826.715	-3181.138
age2635	11261.282	7181.390	298.038	10544.024	253.935	18249.762	17635.492	1736.125	2707.753	13376.291	3598.382	133.512	12912.774	9199.451
age3645	20204.257	8715.435	-1496.777	15973.884	-980.446	22314.580	22647.489	1629.186	3977.640	17275.971	3610.472	-462.668	15235.512	14477.318
age4655	21379.093	7932.194	-1700.132	15522.072	-1004.282	20868.972	24227.752	1216.616	4598.299	19226.984	3322.808	-657.406	11760.889	15918.309
age5665	18501.911	-5640.958	-4026.983	-2827.893	-2721.057	3013.810	15508.584	-2094.337	-1052.638	13735.337	-6620.813	-4454.938	-5515.441	8332.468
age6675	-17717.211	-15772.238	-7029.630	-27862.436	-4781.550	-19167.616	-18079.738	-3984.814	-7409.936	-16388.328	-10985.515	-6609.376	-24228.813	-8948.187
age76plus	-29055.578	-16164.824	-6946.660	-27323.716	-5228.329	-17020.321	-22579.035	-3860.253	-7768.437	-14860.275	-10739.164	-6531.714	-23310.872	-11212.453
disabled	-21282.019	-6658.895	-2525.640	-19002.966	-1884.065	-16143.149	-21091.751	-2935.210	-4857.217	-18130.129	-3538.733	-2024.023	-20467.064	-15173.720
educ2	-1874.752	327.873	-455.907	8987.047	-67.142	-1297.521	-1026.637	361.423	2948.759	489.960	387.598	206.300	-3208.715	502.749
educ3	587.193	6464.542	-89.027	11538.902	637.632	2741.395	2511.886	243.210	8423.142	1815.389	1824.128	298.080	562.728	7026.470
educ4	15411.235	12784.993	1730.697	32104.095	2142.758	12620.303	11046.939	2971.511	16797.411	7436.642	9815.110	1766.020	9603.024	23913.661
nmonth_un	-1403.858	-1333.253	-324.339	-2834.603	-194.040	-1945.435	-1674.728	-313.565	-804.223	-1197.353	-619.228	-359.426	-2384.867	-1033.872
nsmallchildren	-12095.117	-3588.698	-1616.324	-8961.596	-1117.910	-5557.881	-10023.363	-839.009	-629.316	-6749.576	-2782.041	-2236.359	-6988.060	-4684.577
nmiddlechildren	-4579.861	-3321.279	-1193.732	-7119.881	-613.570	-5699.962	-5892.108	-449.321	266.054	-5077.995	-868.085	-720.945	-7000.605	-4196.865
nbigchildren	-6016.548	-3196.804	-726.250	-6411.734	-538.773	-6218.212	-5915.701	-564.168	-738.450	-4034.798	-1082.885	-501.765	-7164.865	-4161.533
naddadults	-3983.378	-1555.534	-393.986	-5895.808	-1.944	-5072.264	-4316.707	-241.856	-232.498	-4297.894	-368.823	-451.797	-3567.803	-5537.737
couple	9771.613	795.821	893.917	5342.414	743.926	7527.399	11555.631	626.622	1265.948	8045.701	1196.037	519.841	7455.361	10668.889
eu_birth	-4225.428	-2317.148	-235.532	-1408.691	0.000	1135.863	3280.608	-176.295	-3335.950	-1162.937	0.000	-139.648	4847.473	-2149.491
oth_birth	-8051.068	-5228.669	-47.496	-8275.508	-337.974	-993.851	-5273.779	-241.072	-1411.582	-7359.617	-1371.469	1680.673	51.741	-2940.493
Observations	2188	13766	3764	3144	3307	9023	5134	8745	2580	6143	5082	3273	7619	72746
Adjusted R^2	0.807	0.728	0.674	0.760	0.660	0.794	0.804	0.678	0.697	0.796	0.763	0.775	0.691	0.649

italic $p < 0.10$, ***bold italics*** $p < 0.05$, **bold** $p < 0.01$

Source: Own calculations based on EU-SILC and IPUMS-CPS.

Table 6: Results from second stage regression

	(1) AT	(2) BE	(3) CY	(4) CZ	(5) DE	(6) DK	(7) EE	(8) ES	(9) FI	(10) FR	(11) GR	(12) HU	(13) IE	(14) IS	(15) IT	(16) LT	(17) LU	(18) LV	(19) NL	(20) NO	(21) PL	(22) PT	(23) SE	(24) SI	(25) SK	(26) UK	(27) US
SEX	0.507	0.521	0.162	0.525	0.076	0.578	0.498	-0.876	0.606	0.158	0.541	-0.690	0.263	0.172	-0.397	-0.044	1.225	0.200	0.088	0.543	0.244	-0.877	0.879	0.712	0.224	0.506	0.409
AGE	0.916	0.855	0.710	0.796	0.842	0.777	0.734	0.792	0.793	1.014	0.831	0.932	0.712	0.736	0.895	0.742	0.994	0.654	0.880	0.758	0.977	0.969	0.787	0.899	0.801	0.742	0.703
DIS	0.781	0.793	0.632	0.817	0.761	0.946	0.568	0.825	0.855	0.900	0.868	0.732	0.573	0.861	0.894	0.698	0.825	0.616	0.803	0.863	0.584	0.561	0.848	0.785	0.733	0.755	0.543
IMMI	0.158	0.483	0.291	0.351	0.774	0.574	0.275	0.244	0.490	0.256	0.267	0.987	0.364	0.358	0.316	2.181	0.054	0.293	0.517	0.415	1.003	-0.029	0.563	0.502	0.888	0.414	-0.099
EDUC	0.212	0.392	0.097	0.443	0.302	0.504	0.443	0.194	0.479	0.339	0.465	0.399	0.298	0.478	0.272	0.382	0.373	0.399	0.472	0.434	0.266	0.203	0.577	0.414	0.351	0.306	0.245
NEEDS	0.525	0.580	0.054	0.535	0.542	0.397	0.563	0.208	0.460	0.512	0.047	0.651	0.438	0.373	0.321	0.486	0.545	0.548	0.511	0.433	0.145	-0.394	0.452	0.475	0.442	0.492	0.143
COUPLE	0.298	0.372	-0.013	0.064	0.319	0.403	0.118	0.530	0.305	0.389	-0.600	0.126	0.351	0.289	0.378	0.119	0.309	0.119	0.527	0.212	-0.040	0.543	0.304	-0.309	-0.011	0.447	0.155
UNEMP	0.618	0.749	0.405	0.578	0.644	0.736	0.389	0.470	0.651	0.673	0.382	0.502	0.512	0.754	0.405	0.351	0.529	0.345	0.755	0.542	0.426	0.635	0.665	0.528	0.463	0.534	0.595
Unobserved	0.617	0.667	0.408	0.598	0.581	0.653	0.456	0.472	0.601	0.656	0.603	0.663	0.496	0.479	0.539	0.515	0.511	0.420	0.668	0.583	0.516	0.623	0.647	0.651	0.558	0.504	0.341
Constant	-7556	-6020	-2604	-1684	-8145	-8213	-1506	-760	-7348	-8301	-6484	-2106	-11236	-8230	-4150	-387	-16444	-1335	-10675	-11551	-504	2425	-8012	-3069	-590	-9677	-6728
Observations	5285	5198	2396	7953	12110	4866	3848	8053	8405	8746	3241	6608	4329	2188	13766	3764	3144	3307	9023	5134	8745	2580	6143	5082	3273	7619	72746
Adjusted R^2	0.868	0.930	0.658	0.951	0.868	0.941	0.921	0.800	0.926	0.855	0.842	0.916	0.808	0.890	0.835	0.901	0.841	0.792	0.928	0.905	0.844	0.780	0.942	0.924	0.921	0.864	0.708

italic $p < 0.10$, ***bold italics*** $p < 0.05$, **bold** $p < 0.01$

Source: Own calculations based on EU-SILC and IPUMS-CPS.

Testing the if-condition in the weak hypothesis

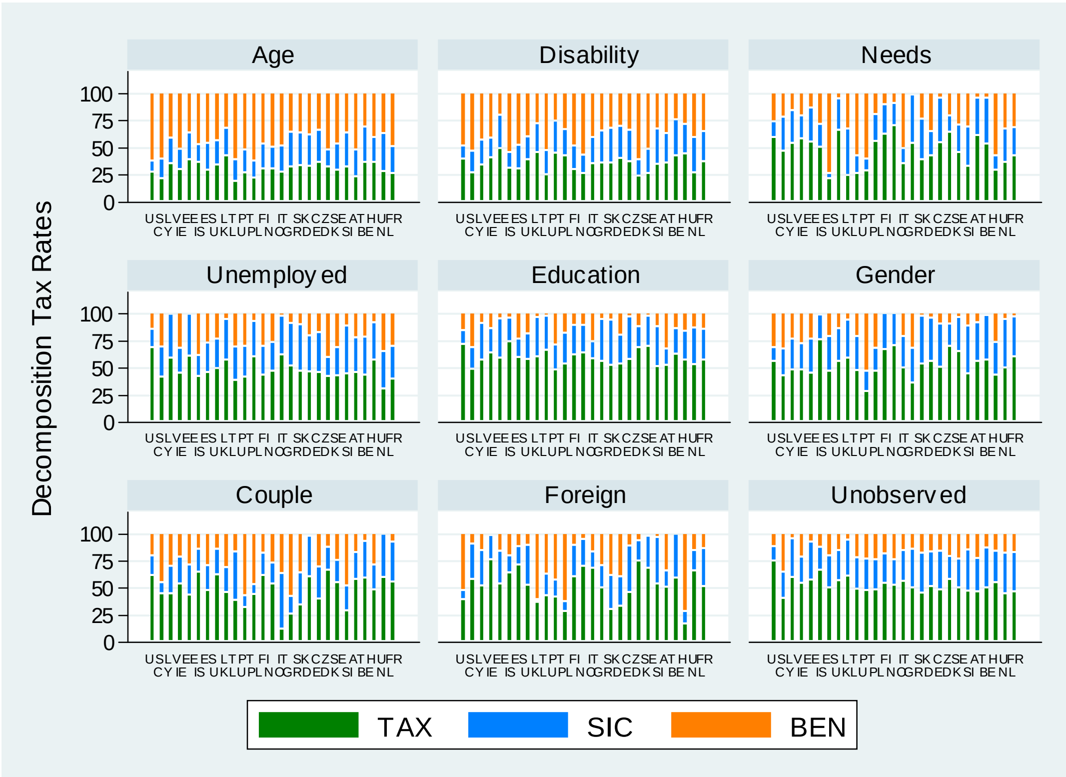
Table 7: Testing the if-condition in the WEAK HYPOTHESIS

	$\frac{\partial E(y\|x_N)}{\partial x_N} = \sigma^x_{PN}/\sigma^x_{NN} + 1$	p-value for $H_0 : \frac{\partial E(y\|x_N)}{\partial x_N} < 0$
AT	0.86	0.00
BE	0.93	0.00
CY	1.05	0.00
CZ	0.86	0.00
DE	0.81	0.00
DK	0.98	0.00
EE	0.96	0.00
ES	1.05	0.00
FI	0.98	0.00
FR	0.95	0.00
GR	1.09	0.00
HU	0.87	0.00
IE	1.02	0.00
IS	0.98	0.00
IT	1.00	0.00
LT	0.95	0.00
LU	0.92	0.00
LV	1.01	0.00
NL	0.91	0.00
NO	0.95	0.00
PL	0.96	0.00
PT	1.17	0.00
SE	0.94	0.00
SI	0.99	0.00
SK	0.90	0.00
UK	0.91	0.00
US	1.11	0.00

Source: Own calculations based on EU-SILC and IPUMS-CPS.

Decomposition for the different implicit tax rates

Figure 5: Decomposition implicit tax rates on characteristics



Source: Own calculations based on EU-SILC and IPUMS-CPS

Robustness checks

The benchmark results are based on equivalent incomes, estimated at the individual level for singles and couples together (while including a couple dummy). We think this is a good specification: needs are crucial in all tax-benefit systems and individual estimations are standard practice. Still, it is possible to come up with other specifications, leading to 8 different combinations (our preferred specification is highlighted in italics):

- 2 output definitions: income versus *equivalent income*,
- 2 estimation levels: *purely individual* versus household averages,
- 2 estimation methods: singles and couples separately versus *joint* estimation.

We look at the sensitivity of our results for these alternative specifications. First, the order of solidarity found in Table 1 is more or less robust. Table 8 reports the average tax rate for the different characteristics for the preferred specification (first column) and three other possible specifications.[37] The estimation level (individual or household level) does not induce big changes. However, if we change from equivalent income (first two columns) to income (last two columns), then the tax rate for needs goes down. And somewhat more surprisingly, the compensation rate for sex increases. To summarize, only the outcome specification could change the order of compensation, and only for the characteristics needs and sex. Next, Figure 7 shows that the different specifications do affect the implicit tax rates for the 'partial control' and the 'no control' composite both in the upper panel (joint estimation) and lower panel (separate estimation). However, more important for our purposes is the fact that the tax rates for the 'no control' composite (the squares and triangles) always remain significantly higher than for the 'partial control' characteristics (the dots and diamonds) in each alternative specification. Finally, when looking at the fairness measure in Figure 6, the main difference is again due to the choice of output definition. When using income instead of equivalent income, the value of the fairness measure is on average about 0.25 higher. As a consequence, if we do not account for economies of scale within households, we must reject the hypothesis that there exist countries with a fair tax benefit system. Still, the ranking of countries in terms of fairness turns out to be robust, irrespective of the choices made.

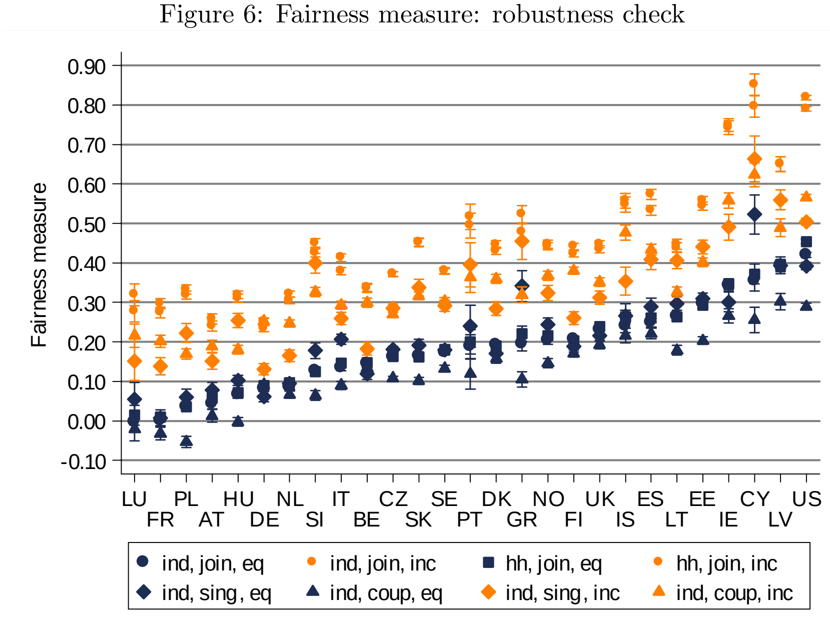
[37] Note that separate estimation for singles and couples does not allow to estimate the tax rate for couple and is therefore discarded here.

Table 8: Mean implicit tax rates for different characteristics and different methods

	eq. income		income	
	ind	hh	ind	hh
age	0.82	0.83	0.78	0.78
disability	0.76	0.77	0.76	0.74
unemployed	0.55	0.55	0.55	0.55
needs	0.42	0.43	0.20	0.23
immigration	0.38	0.34	0.41	0.41
education	0.36	0.36	0.39	0.39
sex	0.29	0.34	0.53	0.40
couple	0.21	0.09	0.11	0.15

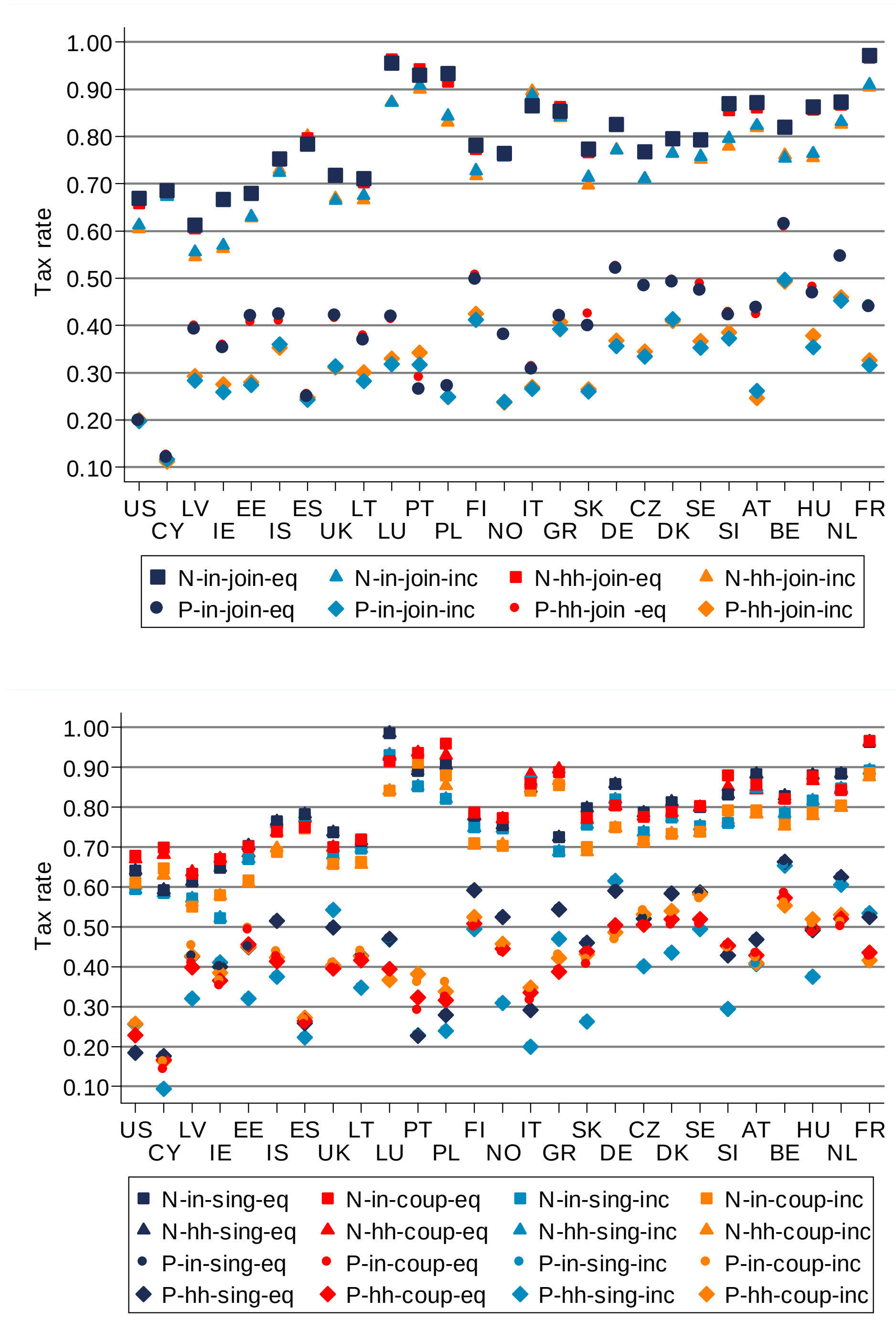
Source: Own calculations based on EU-SILC and IPUMS-CPS.

Figure 6: Fairness measure: robustness check



Source: Own calculations based on EU-SILC and IPUMS-CPS. Notes: 'hh' ('in') indicates the output level: household (individual), 'join' ('sing'/ 'coup') the estimation method: joint (single/couple) and 'inc' ('eq') the output concept: (equivalent) income.

Figure 7: Implicit tax rates: robustness check



Source: Own calculations based on EU-SILC and IPUMS-CPS. Notes: 'P' ('N') indicates the implicit tax rate for partial (no) control, 'hh' ('in') the output level: household (individual), 'join' ('sing'/'coup') the estimation method: joint (single/couple) and 'inc' ('eq') the output concept: (equivalent) income.