Capacity Constraining Labor Market Frictions in a Global Economy

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Abstract

Convex vacancy creation costs shape firms' responses to trade liberalization. They induce capacity constraints by increasing firms' cost of production, leading a profit maximizing firm not to fully meet the increased foreign demand. Hence, firms will only serve a few export markets. More productive firms will export to more countries and charge higher or similar prices compared to less productive firms. Trade liberalization also affects labor market outcomes. Increased profits by exporting firms triggers firm entry, reduces unemployment and increases wage dispersion in the on-the-job search model with monopolistic competition.

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1 Introduction

Apple launched his new iPad in the US on April 3, 2010. The pictures are well known: Large queues in a lot of cities all over the US. The iPad was then consecutively launched in different countries.[1] One may argue from different perspectives why this ordering of countries was chosen by Apple. But two things are striking: (i) It did not sell to all countries at the same time, and (ii) the countries that were served are very similar concerning their level of development and size. These export strategies are quite common. Many empirical studies, e.g., Eaton, Kortum, and Kramarz (2004), show that most exporting firms sell to only one foreign market and that the frequency of firms' selling to multiple markets declines with the number of destinations. More recent evidence by Lawless (2009) using firm-level export destination data of Irish firms and Eaton, Kortum, and Kramarz (2011) of French firms show that firms do not enter markets according to a common hierarchy. Arkolakis (2010) and Eaton, Kortum, and Kramarz (2011) explain this pattern by assuming not only market but also firm-specific heterogeneity in entry costs and market size. However, these approaches are not able to explain why the export strategy of one and the same firm varies widely across countries with similar characteristics.[2] We show that no additional heterogeneity is necessary to explain why the same firm behaves differently in similar export markets and why similar firms with similar products serve different export markets.

We propose an answer based on capacity constraining labor market frictions. We merge a generalized version of the on-the-job search model by Burdett and Mortensen (1998) with the new trade, monopolistic competition model with heterogenous firms by Melitz (2003) and show how convex vacancy creation costs lead to capacity constraints. As a result exporting firms do not grow in order to fully serve foreign demand. Given entry costs to each export market, they rather react by selling only to a few markets at a higher price. Thus, even if only symmetric countries trade, exporting firms sell – depending on their productivity – to only part of the countries. More productive firms export to more countries. This export

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[1]On May 28, 2010 in Australia, Canada, France, Germany, Italy, Japan, Spain, Switzerland and the UK. On July 23, 2010 in Austria, Belgium, Hong Kong, Ireland, Luxembourg, Mexico, Netherlands, New Zealand and Singapore. On September 17, 2010 in China. Source: http://www.apple.com/pr/products/ipad/ipad.html.

[2]Or in the words of Eaton, Kortum, Kramarz (2011): "In particular, it leaves the vastly different performance of the same firm in different markets as a residual. Our analysis points to the need for further research into accounting for this variation."

reaction also implies a different price structure. In contrast to Melitz (2003) more productive exporting firms might charge higher prices in the domestic (and the export) market than less productive non-exporting firms.

This predicted trade and price patterns can be caused by any type of capacity constraints resulting from labor market frictions, credit constraints or any other friction that induces increasing marginal costs of production. While many empirical studies have emphasized the importance of capacity constraints in determining the export behavior of firms, almost no study has convincingly identified a specific channel.[3] The only exception know to us is Manova (2008), who nicely isolates the effect of equity market liberalization on export behavior using panel data for 91 countries.[4] Still, Manova's (2008) analysis does not rule out that labor market frictions might also be an important contributing factor. Labor market frictions, more specifically high costs of hiring qualified workers – a phenomena, which is often referred to as "labor shortage" in the popular press – are often blamed for reducing firms' ability to meet their demand. The ManpowerGroup provides extensive evidence of "labor shortage", specifically of highly qualified workers in the "2011 Talent Shortage Survey" based on nearly 40,000 surveys of employers in 39 countries. There is also a heavy debate about the effects of "labor shortage" on the global competitiveness of China. The New-York-Times wrote on April 3, 2006 that "data from officials suggest that major export industries are looking for at least one million additional workers, and the real number could be much higher". A Chinese supplier survey by Global Sources (2011) reports that "the persistent labor shortage has nearly driven growth in China's export industries to a halt". Evidence is also provided from the IT branch, where Apple belongs to.[5] Lately *The National Business Review* wrote about the IT professional shortage in New Zealand[6] and Webmaster Europe, the International-

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[3]Magnier and Toujas-Bernate (1994) and Madden, Savage, and Thong (1994) argue that exporting firms may not always be able to meet the demands for its goods due to a capacity constraints. Ruhl and Willis (2008), Eaton, Eslava, Krizan, Kugler and Tybout (2009), and Fajgelbaum (2011) point out that firms need time to grow in order to be large enough to export. Redding and Venables (2004) and Fugazza (2004) find that country specific supply-side conditions can explain part of the differences in export performance.

[4]Blum, Claro, and Horstmann (2010) assume that capital capacity constraints are responsible for fluctuations in export behavior of Chilean firms. Their structural estimation is not able to identfy a certain channel since their theory does not allow for different channels.

[5]In an interview on June 1, 2010 Apple Inc. CEO Steve Jobs said that the idea for the iPad came before the iPhone. However, "...*I put the tablet project on a shelf, because the phone was more important.*"

[6]See http://www.nbr.co.nz/article/it-professional-shortage-continues-survey-118981.

European labor union for Internet professionals, stated that the IT professional shortage will continue in 2010 in Germany.[7]

In this paper we also show that trade liberalization increases firms' expected profits and triggers not only firm growth, but in contrast to Melitz (2003) also firm entry. This is well in line with recent empirical findings by Eaton, Kortum, and Kramarz (2004, 2011) that suggests that a large fraction of the adjustment in market shares comes from changes in the number of firms and not from the adjustments of the amount sold by existing firms. At the same time, opening up to trade still forces less productive firms to leave the market like in Melitz (2003).

By allowing firms with different productivities to pay different wages the on-the-job search model by Burdett and Mortensen (1998) offers a natural environment to study the effects of trade liberalization on wage dispersion and unemployment.[8] Trade liberalization increases wage dispersion since search frictions pin down the lowest wage at the level of unemployment benefits, while increased profits of exporting firms increase wages at the top of the distribution. Higher profits of exporting firms also increase job creation (both at the extensive and the intensive margin), which implies a lower unemployment rate. The effects of trade liberalization on unemployment and wage inequality have already been analyzed using the Krugman (1979, 1980) and Melitz (2003) framework. Helpman, Itskhoki, and Redding (2009, 2010) allow firms to screen workers of different abilities. They find that lower variable trade costs shift the industry composition from low- to high-productivity firms, increase wage inequality and can increase or decrease unemployment. The wage dispersion in their framework arises from the assumed heterogeneity of workers. However, empirical results show that even very similar workers are paid different wages (Abowd, Kramarz and Margolis, 1999; Abowd and Kramarz, 1999). Egger and Kreickemeier (2008) explain intra-group wage inequality among ex ante identical workers due to a fair wage-effort mechanism and suggest that trade liberalization increases wage dispersion and can increase unemployment if wage demands exceed increases in profits. Similarly, Amiti and Davis (2011) assume a fair wage constraint and show that a fall in output tariffs lowers wages at import-competing firms, but boost wages at exporting firms. Felbermayr, Prat, and Schmerer (2011) show that unemployment falls if

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[7]http://www.webmasters-europe.org/modules/news/article.php?storyid=95.

[8]Fajgelbaum (2011) uses an on-the-job search equilibrium model based on Postel-Vinay and Robin (2002) to investigate how labor market frictions influence the growth path and export decision of firms.

trade is liberalized.

The paper is structured as follows. In the next section we present the general framework that links the new trade model by Melitz (2003) with the on-the-job search model by Burdett and Mortensen (1998). In section 3 we analyze the equilibrium in a closed economy. In section 4 we investigate the effects of trade liberalization and compare the results with the literature focussing particularly on the comparison with Melitz (2003). Section 5 introduces vacancy creation with convex vacancy creation costs. We then simulate the model with convex vacancy creation costs and show that our main effects prevail. Section 6 concludes and sets out future research objectives.

2 Framework

The model merges the new trade model of Melitz (2003), where firms face monopolistic competition with perfect labor markets, with the on-the-job search model by Burdett and Mortensen (1998), where firms have local monopsony power to set wages. This setup naturally incorporates wage dispersion into the trade model of Melitz (2003) and allows us to study the effects of trade liberalization on the wage distribution. Labor market frictions also change the pattern of trade since they impose a capacity constraint on firms.

2.1 Labor market and workers' search strategy

The model has an infinite horizon, is set in continuous time and concentrates on steady states. The measure of firms M in the economy will be endogenously determined in the product market. In the basic framework, we assume that all firms face the same fixed contact rate $\eta \overline{v}$. This is identical to assuming that all firms open the same number of vacancies $\overline{v}$ due to zero vacancy creation cost for $v \leq \overline{v}$ and infinitely high vacancy creation costs for $v > \overline{v}$. In section 5 we allow firms with different productivities to decide on the number of vacancies. In the basic model the total number of contacts made by active firms is given by $\eta M \overline{v}$.

Workers are risk neutral and infinitely lived. The measure of workers is normalized to one. Workers can either be unemployed receiving unemployment benefits z or employed at a wage w that might differ across firms. Both unemployed and employed workers are searching for a job with the same intensity. Following Burdett and Mortensen (1998) the probability of a worker to meet a firm follows a Poisson process with the rate $\lambda(M)$. The contact rate of

a worker depends on the number of firms M in the market. Since aggregation requires that the total number of firm contacts equals the total number of worker contacts, we get

$$\lambda\left(M\right) = \eta M \overline{v}. \tag{1}$$

The contact rate of a worker is therefore increasing in the number of active firms in the economy.

Production starts if a worker accepts the wage offer made by a firm. $F\left(w\right)$ denotes the wage offer distribution of employers. The employment relationship ends if either the worker quits to work for a better paying job, which happens at rate $\lambda\left(M\right)\left[1 - F\left(w\right)\right]$, or the worker quits for an exogenous reason, which happens at rate $\varkappa$, or the entire firm has to close for an exogenous reason, which happens at rate δ. If the worker quits for an exogenous reason or if the firm closes, workers become unemployed.

Given the wage offer distribution $F\left(w\right)$, a worker's (flow) value rU of being unemployed consists of unemployment benefits z, plus the expected gain from searching. The latter depends on the contact rate $\lambda\left(M\right)$ and the surplus of being employed rather than unemployed. The (flow) value of being employed $rV(w)$ is given by the current wage plus the expected surplus from finding a better paid job and the expected loss from becoming unemployed, i.e.,

$$
\begin{aligned}
rU &= z + \lambda\left(M\right) \int_{\underline{w}}^{\overline{w}} \max\left[V\left(\widetilde{w}\right) - U, 0\right] dF\left(\widetilde{w}\right), &\text{(2a)} \\
rV\left(w\right) &= w + \lambda\left(M\right) \int_{\underline{w}}^{\overline{w}} \max\left[V\left(\widetilde{w}\right) - V\left(w\right), 0\right] dF\left(\widetilde{w}\right) \\
&\quad + \left(\varkappa + \delta\right)\left[U - V\left(w\right)\right], &\text{(2b)}
\end{aligned}
$$

where r is the interest rate with which workers discount future payments. The lowest and the highest wage paid in the economy are denoted by $\underline{w}$ and $\overline{w}$, respectively.

As shown by Mortensen and Neumann (1988) the optimal search strategy for a worker is characterized by a reservation wage w^r, where an unemployed worker is indifferent between accepting or rejecting a wage offer, i.e., $U = V(w^r)$. Using the above value functions it is straight forward to show that w^r is independent of $F\left(w\right)$ and given by $w^r = z$. Thus, only wages that are at least as high as unemployment benefits z are acceptable for unemployed workers. Equivalently, employed workers will only change employers if the wage $\widetilde{w}$ offered by the outside firm exceeds the current wage w.

2.2 Product market and firms' decisions

Firms are risk neutral and the life of a firm is exponentially distributed with parameter δ. Following Ethier (1982), Ludema (2002), Melitz (2003) and Helpman and Itskhoki (2010) the final output is a CES-aggregate[9], i.e.,

$$Y = q_0 + \frac{1}{\rho} \left[\int_{\omega \in \Omega} q(\omega)^\rho \, d\omega \right], \text{ where } 0 < \rho < 1. \tag{3}$$

The measure of the set Ω equals the mass of available intermediate goods, and q_0 is an outside good serving as numéraire. Each intermediate good ω is produced by a single firm in a monopolistic competitive market. Thus, the mass of intermediate goods producers is equal to the number of active firms M in the market.

We assume perfect competition in the final goods market. Profit maximization of competitive final goods producers leads to the following demand for intermediate good ω,

$$q(\omega) = p(\omega)^{-\frac{1}{1-\rho}}. \tag{4}$$

Labor $l(\omega)$ is the unique factor of production. Firms differ in labor productivity such that the output of a firm that produces intermediate good ω is given by $q(\omega) = \varphi(\omega) l(\omega)$, where $\varphi(\omega)$ denotes the labor productivity of intermediate input producer ω. As it is standard in the literature we use φ to index intermediate input producers. The productivity φ is drawn from a continuous distribution with c.d.f. $\Gamma(\varphi)$ and p.d.f $\gamma(\varphi)$ and support $[\underline{\varphi}, \overline{\varphi}]$.

The size $l(w(\varphi))$ of the labor force employed by a firm with productivity φ depends on its wage $w(\varphi)$. Unlike in the competitive labor market assumed in Melitz (2003) firms are not able to adjust their labor input freely to produce the output they want. The size of a firm's labor force $l(w(\varphi))$ and consequently the output a firm produces is determined by the firm's position in the wage offer distribution $F(w(\varphi))$, which detemines the hiring rate and the rate at which workers quit to better paying firms.

Following Burdett and Mortensen (1998) we assume for simplicity that the discount rate is set to zero. Thus, the firm closure rate δ acts as discount rate. The firms' optimization problem is equivalent to maximizing steady state profit flows[10], i.e.,

$$\delta \Pi(\varphi) = \max_w \left[p(\varphi) q(\varphi) - w(\varphi) l(w(\varphi)) - f \right], \tag{5}$$

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[9]The numéraire good q_0 in the production technology in (3) absorbs all changes in aggregate demand.

[10]Coles (2001) and Moscarini and Postel-Vinay (2010) analyze the out of steady state dynamics of the Burdett-Mortensen model and show that the position of a firm within the wage distribution equals the position in the productivity distribution even if firm size is out of steady state.

where $p(\varphi)$ denotes the price that the firm will charge in the monopolistic competitive market and the fixed costs f reflects a per period cost component that is required to serve consumers. It is assumed to be identical for all firms.

Given its expected profit a firm decides whether it will enter the market depending on its productivity φ. A firm will start to produce if profits are positive, i.e., $\delta\Pi(\varphi) \geq 0$. The cutoff productivity φ^* is therefore defined by

$$\delta\Pi(\varphi^*) = 0. \tag{6}$$

Thus, only firms with productivity $\varphi \in [\varphi^*, \overline{\varphi}]$ will be active in the market, assuming that $\varphi^* \geq \underline{\varphi}$.

Since all active firms – except the cutoff firm – make positive profits, the expected discounted profit of a firm before knowing its productivity type φ is also positive, i.e.,

$$\Pi_e = [1 - \Gamma(\varphi^*)]\overline{\Pi} = [1 - \Gamma(\varphi^*)]\int_{\varphi^*}^{\overline{\varphi}} \Pi(\varphi)\frac{\gamma(\varphi)}{1 - \Gamma(\varphi^*)}d\varphi > 0,$$

where $1 - \Gamma(\varphi^*)$ equals the probability that a firm will draw a productivity level φ high enough to ensure that production is profitable. $\overline{\Pi}$ equals the average discounted profit of all active firms.

Before a firm gets to know its productivity, it has to pay the fixed investment cost f_e. Free entry of firms ensures that firms enter the market until the expected discounted profit before entering the market equals the fixed investment cost f_e, i.e.,

$$[1 - \Gamma(\varphi^*)]\overline{\Pi} = \int_{\varphi^*}^{\overline{\varphi}} \Pi(\varphi)\gamma(\varphi)\,d\varphi = f_e. \tag{7}$$

The zero-cutoff condition (6) and the free entry condition (7) determine the number of active firms M and the labor productivity φ^* of the active firm with the lowest productivity in an economy.

2.3 Aggregation and steady state conditions

Aggregation requires that total output produced per period equals the payments made by firms, i.e.,

$$Y = q_0 + (1 - u)\int_{\varphi^*}^{\overline{\varphi}} w(\varphi)\,dG(w(\varphi)) + fM + M\delta\overline{\Pi}. \tag{8}$$

Thus, wage payments plus aggregate profit per period have to equal total value of output produced each period. Aggregate profits are used to finance the fixed investment cost of new

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incumbent firms that attempt entry, i.e.,

$$M\delta\overline{\Pi} = f_e M_e,\tag{9}$$

where M_e is the total mass of firms that attempt entry and pay the fixed investment costs f_e each period. A large unbounded set of potential new firms ensures an unlimited supply of potential entrants that is able to replace firms that go out of business. Steady state requires that the flow into the pool of active firms is equal to the flow out of this pool, i.e.,

$$[1 - \Gamma(\varphi^*)] M_e = \delta M.\tag{10}$$

It is straight forward to show that the free entry condition (7) guarantees that steady state conditions (9) and (10) hold.

In steady state in- and outflows into and out of employment offset each other such that the unemployment rate and the distribution of employment over firms are stationary. Equating the flows in and out of unemployment gives the steady state measure of unemployed, i.e.,

$$u = \frac{\varkappa + \delta}{\varkappa + \delta + \lambda(M)}.\tag{11}$$

Noting that the aggregate number of matches in the economy depends on the number of active firms M in the economy (as stated in equation (1)), the unemployment rate decreases if the number of active firms in the economy increases.

Equating the inflow and outflow into the group of workers employed at a wage $w(\varphi)$ or less gives the steady-state measure of employed workers earning a wage less than $w(\varphi)$, i.e.,

$$\lambda(M) F(w(\varphi)) u = G(w(\varphi))(1-u)[\varkappa + \delta + \lambda(M)[1 - F(w(\varphi))]]\tag{12}$$

$$\implies G(w(\varphi)) = \frac{(\varkappa + \delta) F(w(\varphi))}{\varkappa + \delta + \lambda(M)[1 - F(w(\varphi))]}.\tag{13}$$

Labor market frictions constrain the number of workers a firm can recruit. This is the reason why we call these labor market frictions capacity constraining. Like in Burdett and Mortensen (1998) the average number of workers employed by a firm equals the number of workers employed at a certain wage, i.e., $g(w(\varphi))(1-u)$, divided by the number of firms paying a certain wage, i.e., $f(w(\varphi)) M$. Differentiating equation (12) with respect to the $w(\varphi)$ and substituting the aggregate matching condition (1), the steady state measure of unemployment (11) and employment (13) allows us to write the steady state labor force of a

firm paying the wage $w(\varphi)$ as

$$l(w(\varphi)) = \frac{\eta \bar{v}(\varkappa + \delta)}{[\varkappa + \delta + \eta M \bar{v}[1 - F(w(\varphi))]]^2}. \tag{14}$$

Like in Burdett and Mortensen (1998) equation (14) implies that the size of a firm's labor force $l(w(\varphi))$ is increasing in the wage $w(\varphi)$ since on-the-job search implies that a high wage firm attracts more employed workers from firms paying lower wages and loses less workers to employers paying higher wages. Equation (14) also shows that a higher number of active firms M results in additional competition between firms and decreases the size of each firm's labor force. As shown in section 5, this effect is still present but does not necessarily dominate if we endogenize the recruiting rate $\eta \bar{v}$, i.e., if we allow firms to grow by posting vacancies.

3 Equilibrium in a closed economy

3.1 Equilibrium definition

We start by defining the product and labor market equilibrium in a closed economy. A labor market equilibrium is characterized by the contact rate of unemployed workers, the unemployment rate, the wage offer and wage earnings distribution, i.e., the set $\{\lambda(M), u, F(w(\varphi)), G(w(\varphi))\}$. Firms offer wages $w(\varphi)$ that maximize profits given their productivity φ, the demand function they face in the monopolistic competitive market $q(\varphi) = p(\varphi)^{-\frac{1}{1-\rho}}$, the wage offer distribution $F(w(\varphi))$ posted by other firms and the optimal search strategy of workers, i.e., $w^r = z$. The contact rate of unemployed workers $\lambda(M)$, the unemployment rate u and the wage earnings distribution $G(w(\varphi))$ in equations (1), (11) and (13) have to be consistent with steady state turnover given the productivity distribution $\Gamma(\varphi)$, the cutoff productivity φ^* and the wage offer distribution $F(w(\varphi))$ posted by firms.

A product market equilibrium is a set $\{\varphi^*, M\}$ such that intermediate good producers enter the product market if their productivity ensures a positive profit, i.e., $\varphi \geq \varphi^*$, where the zero-cutoff productivity has to satisfy equation (6). The number of active firms M in the product market has to be consistent with profits of active firms being used to finance the fixed investment cost f_e of potential new market entrants necessary to replace the firms exiting the product market. Thus, the number of active firms M has to satisfy the free entry condition (7).

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3.2 Firms' wage offers

Since each firm has some monopoly power in the product market and some monopsony power in the labor market, a firm will chose its wage offer such that the marginal revenue of increasing labor input with a higher wage is offset by the additional wage cost. The optimality condition for a firm with productivity φ, given the distribution of wages offered by all other firms $F(w(\varphi))$, is given by

$$\frac{\partial \delta \Pi(\varphi)}{\partial w(\varphi)} = \left[\varphi^\rho \rho l(w(\varphi))^{(\rho-1)} - w(\varphi) \right] \frac{\partial l(w(\varphi))}{\partial w(\varphi)} - l(w(\varphi)) \stackrel{!}{=} 0. \tag{15}$$

As in Mortensen (1990) more productive firms will pay higher wages if the marginal revenue is higher than the wage, i.e., $\varphi^\rho \rho l(w(\varphi))^{(\rho-1)} - w(\varphi) > 0$ (see Appendix A).

If the marginal product is lower than the wage, firms will reduce their wage in order to reduce their labor force $l(w(\varphi))$ and to increase their marginal revenue. Thus, the optimality condition (15) is only satisfied for all active firms with productivity $\varphi \geq \varphi^*$ if the marginal revenue of the least productive firm φ^* is higher than the level of unemployment benefits z. We therefore assume:

Assumption 1: The marginal revenue of the least productive firm φ^* is higher than the level of unemployment benefits z, i.e.,

$$\rho [\varphi^*]^\rho \left[\frac{\eta \overline{v}(\varkappa + \delta)}{[\varkappa + \delta + \eta M \overline{v}]^2} \right]^{\rho-1} > z. \tag{16}$$

If Assumption 1 is violated, the wage distribution will be characterized by a mass point at the level of unemployment benefits since firms with a marginal revenue below z will not find it optimal to increase their labor input by increasing wages (see Appendix A).

Given Assumption 1 wages $w(\varphi)$ increase with productivity φ like in Mortensen (1990). Thus, the wage offer distribution $F(w(\varphi))$ has to satisfy

$$F(w(\varphi)) = \frac{\Gamma(\varphi) - \Gamma(\varphi^*)}{1 - \Gamma(\varphi^*)} \quad \text{for all } \varphi \in [\varphi^*, \overline{\varphi}]. \tag{17}$$

The position of a firm in the wage offer distribution $F(w(\varphi))$ is equivalent to its position in the productivity distribution of active firms. Thus, the position of a firm in the productivity distribution of active firms determines a firm's labor input. The fact that a firm cannot adjust the size of its labor force freely to changes in output demand leads to a capacity constraint that implies that a firm will adjust its output price to reflect changes in demand.

Since a firm can only recruit workers if it pays at least a wage that equals the level of unemployment benefits z, the least productive firm that is active in the market will offer a wage equal to unemployment benefits z. The optimal wage $w(\varphi)$ posted by a firm with productivity $\varphi > \varphi^*$ is given by

$$w(\varphi) = \frac{1}{l(w(\varphi))} \left[[\varphi l(w(\varphi))]^\rho - \int_{\varphi^*}^\varphi \frac{\rho}{\widetilde{\varphi}} [\widetilde{\varphi} l(w(\widetilde{\varphi}))]^\rho \, d\widetilde{\varphi} - f \right]. \tag{18}$$

The derivation can be found in Appendix B. Multiplying equation (18) by $l(w(\varphi))$ reveals that total wage payments are given by revenues (the first term on the rhs in brackets) minus total profits of a firm with productivity φ (the second term on the rhs in brackets), minus fixed costs. Note that in contrast to Melitz (2003), total profits are a function of wages, which reflects the fact that firms have not only monopoly power on the product market but also monopsony power on the labor market.

3.3 Firm entry decision

Free entry of potential firms ensures that the expected discounted profit from entering the product market $[1 - \Gamma(\varphi^*)] \overline{\Pi}$ equals the fixed investment cost f_e as stated in equation (7). Substituting per period profit (5) and the optimal wage (18) implies the following *free entry condition*

$$f_e = \frac{1}{\delta} \int_{\varphi^*}^{\overline{\varphi}} \left[\int_{\varphi^*}^\varphi \frac{\rho}{\widetilde{\varphi}} [\widetilde{\varphi} l(w(\widetilde{\varphi}))]^\rho \, d\widetilde{\varphi} \right] \gamma(\varphi) \, d\varphi. \tag{19}$$

The expected discounted profit decreases with the number of active firms because the size of a firm's labor force $l(w(\varphi))$ is a decreasing function of the number of active firms M. At the same time the expected discounted profit increases if the cutoff productivity decreases because the likelihood of having a productivity draw that is sufficiently high to make profits increases. Using the implicit function theorem, we show in Appendix C that the free entry condition defines a decreasing relation between the zero-cutoff productivity φ^* and the number of active firms M in the market.

A firm has to offer at least the level of unemployment benefit z in order to attract any worker. Given this lower bound of the support of the wage offer distribution $F(w(\varphi))$, the zero-cutoff productivity firm φ^* employs $l(z) = l(w(\varphi^*))$ workers. Utilizing the per period profit definition (5) implies that the *zero-cutoff productivity* level φ^* is defined by $\delta\Pi(\varphi^*) = 0$.
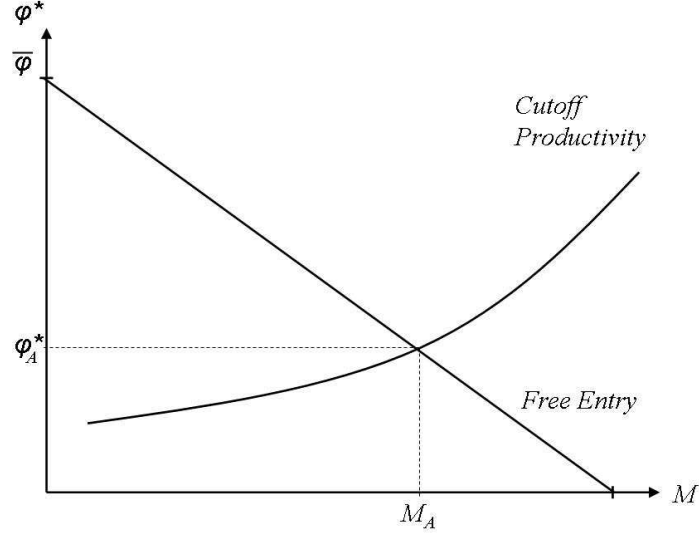
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Figure 1: Number of firms and cutoff productivity

Substituting $l\left(w\left(\varphi^{*}\right)\right)$ gives

$$\left[\varphi^{*}\frac{\eta\overline{v}\left(\varkappa+\delta\right)}{\left[\varkappa+\delta+\eta M\overline{v}\right]^{2}}\right]^{\rho}=z\frac{\eta\overline{v}\left(\varkappa+\delta\right)}{\left[\varkappa+\delta+\eta M\overline{v}\right]^{2}}+f. \qquad (20)$$

Since the zero-cutoff productivity firm pays the wage z, it will only attract unemployed workers and lose its workers to all other firms that pay higher wages. Consequently, a higher number of active firms M increases the number of quits at the zero-cutoff productivity firm and, therefore, reduces its steady state labor input. This decreases the firm's net revenue. The firm will subsequently no longer be able to cover the wage payments and the fixed costs f. Thus, only more profitable firms will be able to survive in the market, which increases the zero-cutoff productivity. Using the implicit function theorem and Assumption 1 we show in Appendix C that the zero profit condition defines an increasing relation between the zero-cutoff productivity φ^{*} and the number of active firms M in the market. Thus, the free entry condition and the zero-cutoff condition determine a unique equilibrium as shown in Figure 1, as long as unemployment benefits z and fixed costs f are low enough to ensure that an equilibrium exists.

4   Open economy

Assume that there are $n+1$ identical countries that differ only in the variety Ω of goods that they produce. Given that final output producers love variety, they are interested in trading with other countries. Due to the symmetry of countries, intermediate goods producers face the same demand curve in the export market as they face in the domestic market, i.e., $q(\varphi) = p(\varphi)^{1/(\rho-1)}$. Serving an export market involves some fixed costs $f_x \geq f$ per period and some proportional shipping costs per good shipped to the export market. Thus, the price of an export good at the factory gate is given by $p_x(\varphi)/\tau = p_d(\varphi)$, where $p_d(\varphi)$ denotes the price in the domestic market.

Given that an exporting firm with productivity φ can only produce the fixed output $\varphi l(w(\varphi))$, it will chose the number of export markets j such that the output sold in j export markets and the domestic market maximizes profits. Splitting the output for all export markets is not profit-maximizing given the capacity constraints and exporting fixed costs. Thus, a firm that decided to serve a subset $j \leq n$ of foreign markets maximizes its profits if it sells

$$q_d(\varphi) = \frac{1}{1+j\tau^{\rho/(\rho-1)}}q(\varphi), \quad \text{and} \tag{21}$$

$$q_x(\varphi) = \frac{\tau^{\rho/(\rho-1)}}{1+j\tau^{\rho/(\rho-1)}}q(\varphi), \tag{22}$$

at the domestic and at each export market, respectively (see derivation in Appendix D). The profit of a firm serving j export markets is therefore given by

$$\delta\Pi_{d+j}(\varphi) = \max_w \left[\left[1+j\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)} [\varphi l(w(\varphi))]^{\rho} - w(\varphi) l(w(\varphi)) - f - jf_x \right]. \tag{23}$$

In addition to the closed economy a firm with productivity φ decides not only on the wage $w(\varphi)$ but also on the number j of countries it wants to export to. Hence, it will choose the number of export markets such that profits are maximized, i.e., $\Pi^{\max}(\varphi) = \max_j \Pi_{d+j}(\varphi)$.

4.1   Trade pattern

Denote by φ_x^j the export cutoff productivity for a firm that decides to export to $j \leq n$ countries. Firms with $\varphi \geq \varphi_x^j$ find it optimal to export to j or more countries while firms with $\varphi < \varphi_x^j$ will only serve less than j foreign markets and the domestic market (or only the domestic market). Wages chosen by firms have to satisfy the first order condition like in

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a closed economy. The non-exporting firm with the lowest productivity level φ^* will pay the reservation wage z such that unemployed workers are willing to start working. As shown in Appendix D the wage equation $w(\varphi)$ for exporting firms is given by

$$w(\varphi) = \frac{1}{l(w(\varphi))} \left[\left[1 + j\tau^{\frac{\rho}{\rho-1}} \right]^{(1-\rho)} [\varphi l(w(\varphi))]^\rho - f - jf_x \right] \tag{24}$$
$$- \frac{1}{l(w(\varphi))} \sum_{i=1}^{j+1} \left[1 + (i-1)\tau^{\frac{\rho}{\rho-1}} \right]^{(1-\rho)} \int_{\varphi_x^{i-1}}^{\varphi_x^i} \frac{\rho}{\widetilde{\varphi}} [\widetilde{\varphi} l(w(\widetilde{\varphi}))]^\rho d\widetilde{\varphi},$$

where $\varphi_x^{j+1} = \varphi$ and $\varphi_x^0 = \varphi^*$.

Note that profit maximization ensures that the wage function does not jump upward at φ_x^j, i.e., that the support of the wage distribution is connected. To see this suppose the opposite, i.e., that the exporting firm with the lowest productivity φ_x^j were to pay a wage $w\left(\varphi_x^j\right) = w(\varphi) + \Delta$, where $\Delta > 0$ denotes the jump at $w(\varphi)$ where productivity is given by $\varphi = \varphi_x^j - \varepsilon$ for any small $\varepsilon > 0$. The wage jump does not increase the number of workers of firm φ_x^j since it has the same position in the wage distribution as before. It is, therefore, optimal for the firm to pay a wage that is only slightly above $w(\varphi)$ and save the wage costs Δ per worker. Thus, the wage function is continuous on $[\varphi^*, \overline{\varphi}]$.

For low productive firms it is optimal to serve only the domestic market, while more productive firms will export. At the export-cutoff productivity φ_x^j the firm is indifferent between serving j export markets and the domestic market or serving $j-1$ export markets and the domestic market, i.e., $\delta\Pi_{d+j}\left(\varphi_x^j\right) = \delta\Pi_{d+j-1}\left(\varphi_x^j\right)$. As proven in Appendix D more productive firms will export to more countries. Specifically, the export cutoff productivity is given by

$$\left[\varphi_x^j l\left(w\left(\varphi_x^j\right)\right)\right]^\rho = \frac{f_x}{\left[\left[1 + j\tau^{\rho/(\rho-1)}\right]^{(1-\rho)} - \left[1 + (j-1)\tau^{\rho/(\rho-1)}\right]^{(1-\rho)}\right]}. \tag{25}$$

Proposition 1 *The number of export markets $j \leq n$ served by a firm is increasing in its productivity, i.e., the export cutoff productivity φ_x^j is increasing in j.*

Proof. See Appendix D. ∎

Proposition 1 implies that more productive firms will serve more markets, and that an exporting firm may not serve all markets, even if the destinations are very similar. These predictions are well in line with recent empirical findings by Lawless (2009) and by Eaton, Kortum, and Kramarz (2011) that firms do not enter export markets according to a common

hierarchy that is determined by export destination characteristics. Our theory therefore nicely complements the explanation given by Arkolakis (2010) and Eaton, Kortum, and Kramarz (2011), which is based on market as well as firm-specific heterogeneity in entry costs or market size. In addition Proposition 1 is able to explain why the export strategy of one and the same firm varies widely across countries with similar characteristics. This cannot be explained by Eaton, Kortum, Kramarz (2011), who state: "In particular, it leaves the vastly different performance of the same firm in different markets as a residual. Our analysis points to the need for further research into accounting for this variation."

Our explanation of these trade patterns is based on capacity-constraining labor market frictions, which implies that – all else equal – firms that recruit their workers in more flexible labor markets will serve more export markets. In order to identify the link between the capacity constraining effect of labor market frictions and exports, we would need variation in labor market frictions over time and across countries that does not coincide with other trade liberalization policies. Testing the predictions of our model would require a very rich data-set with export destinations given on the firm-level for a large set of countries. This country-panel would then have to be linked to labor market indicies that characterize labor market frictions.[11]

4.2 Firm structure and prices

Given the export decisions of firms with different productivities, we are able to determine the expected profit of active firms in an economy $[1 - \Gamma(\varphi^*)]\overline{\Pi}$. The free entry condition requires that expected profit equals the fixed investment cost f_e, i.e.,

$$f_e = \frac{1}{\delta} \int_{\varphi^*}^{\overline{\varphi}} \left[\sum_{i=1}^{j+1} \left[1 + (i-1)\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)} \int_{\varphi_x^{i-1}}^{\varphi_x^i} \frac{\rho}{\widetilde{\varphi}} [\widetilde{\varphi} l\,(w\,(\widetilde{\varphi}))]^\rho \, d\widetilde{\varphi} \right] \gamma(\varphi)\, d\varphi, \qquad (26)$$

where $\varphi_x^{j+1} = \varphi$ and $\varphi_x^0 = \varphi^*$. The derivation of equation (26) is given in Appendix D. Since average profits increase due to the additional foreign demand, the free entry curve in Figure 2 rotates outward if trade is liberalized. The zero cutoff condition (20) remains unchanged

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[11]The dataset that comes closest to fulfill these requirements is the "EFIGE - European Firms in a Global Economy" data-set for European countries, which was compiled with an enormous amount of effort by Halti-wanger, Scarpetta and Schweiger (2010) for 16 developed and emerging economies (see for more details at http://www.efige.org/). However, even this data-set would have to be enriched by detailed firm information, specifically, about whether a firm exports or not, and if, in which countries it exports.

by opening up to trade since the lowest productivity firm will pay the reservation wage z and only sells at the domestic market (compare Figure 2).

The higher expected profit in an open economy compared to the closed economy, which can be seen by comparing equations (26) and (19), triggers entry and increases the number of active firms M for a given cutoff productivity φ^*. Given the increased number of active firms in the economy, potential entrants realize that their labor force will be lower than in the closed economy and that they will not be able to produce enough to pay the per period fixed costs f. Thus, the zero-cutoff productivity increases and low productivity firms do not enter the market.
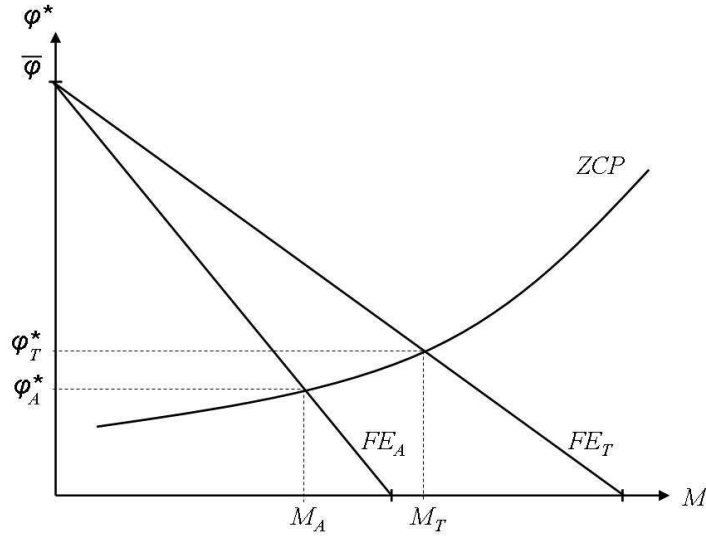
Figure 2: Number of firms and zero-cutoff productivity in an open economy

Proposition 2 *Given Assumption 1, the zero-cutoff productivity φ^* and the number of active firms M in an open economy is higher than in autarky. The size of all firms $l(w(\varphi))$ decreases.*

Proof. See Appendix D. ∎

The monopsonistic labor market changes firms reactions to trade liberalization compared to the reaction of firms in a frictionless labor market like in Melitz (2003). In a perfect labor market exporting firms increase labor input until their marginal product is reduced to equal the market wage. The higher demand for labor by exporting firms is meet at the cost of

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a lower labor input at non-exporting firms. In a frictional labor market without vacancy creation the size of a firm's labor force is determined by the position of a firm in the wage offer distribution. Thus, exporting firms are not able to increase their output since their labor input is given by their position in the wage distribution. Their position in the wage distribution decreases because the cutoff-productivity increases. Hence, opening up to trade will decrease a firm's labor force. In addition it triggers entry of new firms that compete for the same number of workers reducing the number of workers per firm even further. In contrast to Melitz (2003), the increased foreign demand therefore leads to entry of additional firms and not to growth of existing exporting firms. Figure 3 shows the firm size reactions in a frictional labor market and compares it to the perfect labor market environment of Melitz (2003). This result is specific to the simple case of no vacancy creation. In section 5, where we allow for vacancy creation, highly productive exporting firms will grow while less productive firms will shrink.

a) Perfect labor market (Melitz, 2003)   b) Frictional labor market
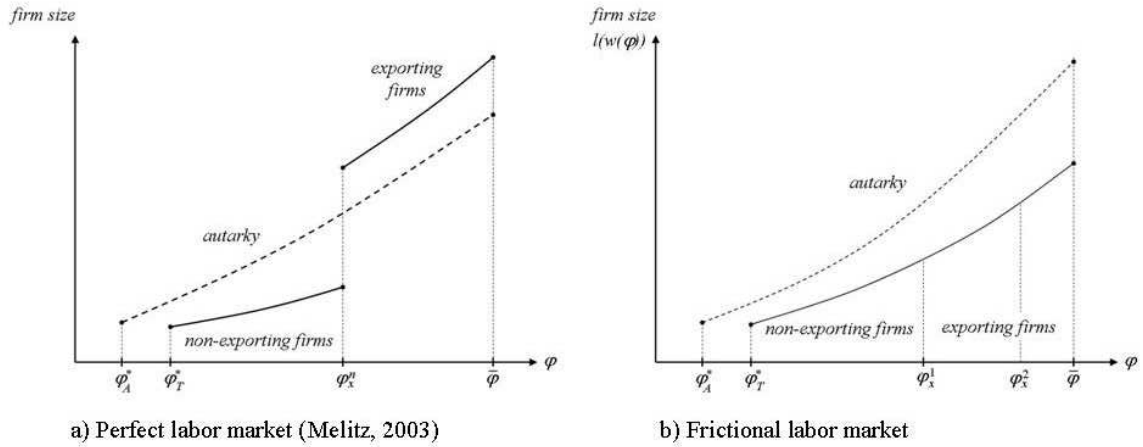
Figure 3: Firm size in autarky and in an open economy

Since exporting firms find it very costly to increase their output in response to the increase in foreign demand, they respond to the increased demand by increasing their prices. The prices charged by exporting firms in the domestic market are no longer lower for exporting firms compared to domestic firms like in Melitz (2003). As Figure 4 suggests, they are in the same range as the prices of domestic firms. The exact relation depends on the quantities of output sold as stated in the following Proposition.

Proposition 3 *Given Assumption 1, the highest domestic price of firms that export to* $0 \leq j \leq n$ *countries is higher than the highest price of firms exporting to* $j - 1$ *countries if and only if*

$$\frac{\varphi_x^{j-1} l\left(w\left(\varphi_x^{j-1}\right)\right)}{1 + (j-1)\,\tau^{\rho/(\rho-1)}} > \frac{\varphi_x^{j} l\left(w\left(\varphi_x^{j}\right)\right)}{1 + j\tau^{\rho/(\rho-1)}},$$

where $\varphi_x^0 = \varphi^*$ *and* $w\left(\varphi_x^0\right) = z$.

Proof. The firm that charges the highest price of all firms exporting to j countries is the firm with export-cutoff productivity φ_x^j. It produces and sells the smallest quantity of the good and therefore charges the highest price of all firms exporting to j countries, i.e., $\varphi_x^j = \arg\max_{\varphi \in [\varphi_x^j, \varphi_x^{j+1})} p_d\left(\varphi\right)$. Due to the downward sloping demand functions we know that $p_d\left(\varphi_x^j\right) < p_d\left(\varphi_x^{j-1}\right)$ holds if and only if $q_d\left(\varphi_x^j\right) > q_d\left(\varphi_x^{j-1}\right)$. ∎
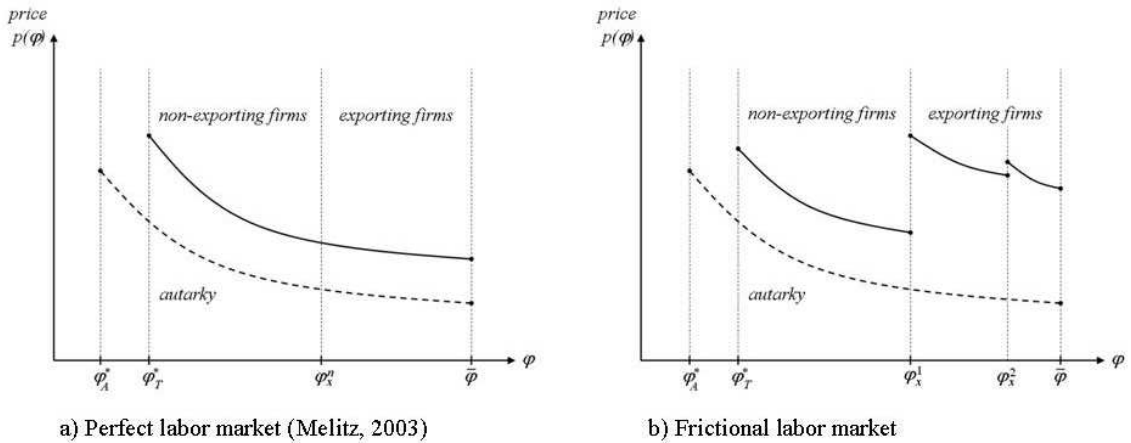
Figure 4: Prices in autarky and in an open economy

While in the perfect labor market environment prices decrease with productivity, in a frictional environment exporting firms charge similar prices compared to domestic firms because capacity constraining labor market frictions induce exporting firms to maximize their profits by selling only a limited quantity per market. These price patterns are well supported by the empirical findings of Bughin (1996) and De Loecker and Warzynski (2009). Bughin (1996) finds that the markup charged by firms increases with capacity constraints and boosts export prices and De Loecker and Warzynski (2009) find that exporters charge on average higher markups and that markups increase upon export entry.

Another explanation for similar domestic prices of non-exporting and exporting firms that the empirical literature has suggested is the higher quality of the goods produced by exporting firms (compare Kugler and Verhoogen, 2011, Coşar, Guner, and Tybout, 2011, and Fajgelbaum, 2011).

4.3 Unemployment and wages dispersion

Opening up to trade increases expected profits, triggers firm entry and reduces unemployment. Like in Felbermayr, Prat, and Schmerer (2011), Helpman and Itskhoki (2010) and Helpman, Itskhoki, and Redding (2009, 2010) additional demand from abroad increases firms' revenue and their demand for labor. While firms in Felbermayr, Prat, and Schmerer (2011), Helpman and Itskhoki (2010) and Helpman, Itskhoki, and Redding (2009, 2010) create additional vacancies in order to increase their employment, in our frictional environment additional firms enter the market since the simple framework does not allow them to increase their recruitment rate by opening new vacancies.

In the given context, opening up to trade still leads the lowest productivity firm to pay the level of unemployment benefits z like in autarky. Since the zero-cutoff productivity increases compared to autarky, i.e., $\varphi_T^* > \varphi_A^*$, some firms with a productivity $\varphi \geq \varphi_T^* > \varphi_A^*$ that paid a wage above z will decrease their wages since they now occupy a lower position within the wage offer distribution. However, if we hold the position of a firm in the wage distribution constant, trade liberalization increases wages because the marginal revenue of all firms increases due to the lower number of employees that they are able to recruit. Exporting firms experience even a higher increase in their marginal revenues since they can now charge higher prices by serving not only the domestic but also foreign markets. Thus, two counteracting effects drive wage changes: (i) the positive effect of an increase in the marginal revenue of a firm and, (ii) the negative effect of a lower position in the wage distribution. Of course, the negative effect is zero for the highest productivity firm $\overline{\varphi}$, such that wages increase at the upper end of the wage distribution. Since wages at the bottom of the wage distribution are held constant by the level of unemployment benefits z, it follows that the dispersion of wages measured as difference between the highest and the lowest wage is higher in an open economy than in autarky. The effect on the average wage is ambiguous and depends on the shape of the productivity distribution as well as the job finding and job destruction rate that translate the wage offer distribution into the wage earnings distribution as stated in equation

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(13). Proposition 4 summarizes the effect of trade liberalization on unemployment and wage dispersion.

Proposition 4 *Given Assumption 1, opening up to trade reduces the unemployment rate u and increases wage dispersion, i.e., increases $w(\overline{\varphi}) - z$, compared to autarky.*

Proof. See Appendix E. ■
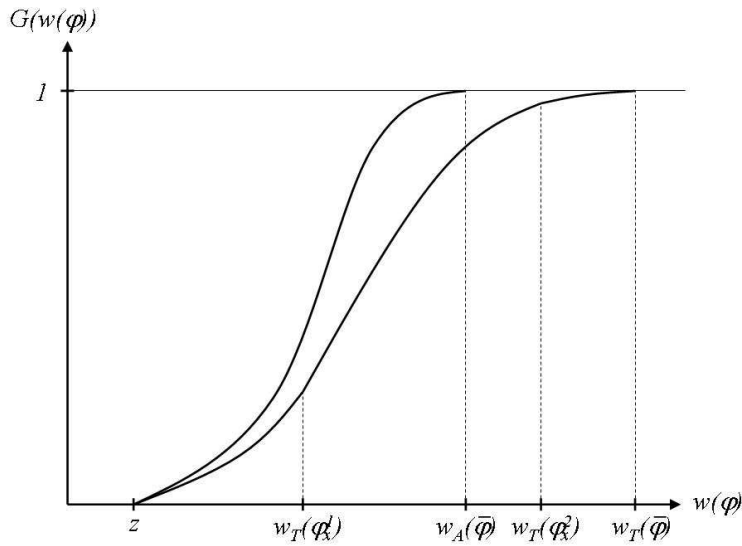
Figure 5: Wages in autarky and in an open economy

The results concerning the effects of trade liberalization on the wage distribution of ex-ante identical workers are similar to the papers by Egger and Kreickemeier (2008), Amiti and Davis (2011) and Helpman, Itskhoki, and Redding (2009, 2010). However, in our context wage inequality is not the result of exogenously given fair-wage preferences[12] or the result of monitoring or screening costs, but rather the result of continuous search for better jobs of workers, as introduced in Burdett and Mortensen (1998).

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[12]Whereas in Egger and Kreickemeier (2009) fair-wage preferences are linked to productivity differences between firms, they are based on profits of firms in Egger and Kreickemeier (2008) and Amiti and Davis (2011).

5 Vacancy creation in an open economy

5.1 The matching technology

In previous sections the analysis was based on the assumption that all firms have a constant recruitment rate $\eta\overline{v}$ and cannot expand their production by opening new vacancies in response to an increase in foreign demand. In this section we allow firms to influence their contact rate by posting vacancies like in Mortensen (2003). The contact rate of a firm with productivity φ depends on the number of vacancies $v(\varphi)$ and is given by $\eta v(\varphi)$. The total number of contacts in an economy (and the contact rate of workers) is therefore given by

$$\lambda(M\widetilde{v}) = \eta M \int_{\varphi^*}^{\overline{\varphi}} \frac{v(\varphi)}{1-\Gamma(\varphi^*)} d\Gamma(\varphi) = \eta M\widetilde{v}. \tag{27}$$

The per period cost of vacancy creation is an increasing function of the vacancies opened, i.e., $c(v) = \frac{c}{\alpha} v(\varphi)^\alpha$. This cost function allows us to compare our results with the case of constant vacancy creation cost, $\alpha = 1$, like in Felbermayr, Prat and Schmerer (2011), who link the Diamond-Mortensen-Pissarides model (see Pissarides, 2000) model with the Melitz (2003) model.

Convex vacancy costs are crucial for our results. The empirical evidence on the shape of the vacancy cost function is small. Abowd and Kramarz (2003) and Kramarz and Michaud (2010) use French firm level data and Blatter, Mühlemann, and Schenker (2009) use Swiss firm level data to look at the shape of the hiring cost function. However, hiring cost functions do not necessarily have the same shape as vacancy cost functions because the hiring rate per vacancy is generally not constant but increasing in the size of a firm and therefore increasing in the number of workers hired. This property holds in the Burdett-Mortensen model like in any monopsony wage model as shown by Manning (2006). Using firm level data from the Labour Turnover Survey in the UK, Manning (2006) shows that there are increasing marginal costs of recruitment. Similarly, using quarterly, corporate sector data for the US economy Merz and Yashiv (2007) show that convex adjustment costs for labor and capital is able to account for the data much better than formulations ignoring hiring costs. At the end of section 5.5, where we simulate the general model with vacancy creation, we show that a convex vacancy cost function is consistent with the mildly concave hiring cost function found by Abowd and Kramarz (2003) and Kramarz and Michaud (2010) for France as well as the convex hiring cost function found by Blatter, Mühlemann, and Schenker (2009) for

Switzerland.

5.2 Labor market and trade pattern

In an open economy a firm with productivity φ chooses its wage $w(\varphi)$ and its number of vacancies $v(\varphi)$ such that per period profits are maximized for a given number of export markets j, i.e.,

$$\delta\Pi_{d+j}(\varphi) = \max_{w,v} \left[\left[1 + j\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)} \left[\varphi l(\varphi,v)\right]^{\rho} - w(\varphi) l(\varphi,v) - \frac{c}{\alpha}v^{\alpha} - f - jf_x \right]$$

$$\text{s.t.} \quad l(\varphi,v) = \frac{\eta v \left[\varkappa + \delta\right]}{\left[\varkappa + \delta + \lambda(M\widetilde{v})\left[1 - F(w(\varphi))\right]\right]^{2}}. \tag{28}$$

The number of employees $l(\varphi)$ working for a firm with productivity φ increases proportionally with the number of vacancies like in Mortensen (2003) and with the wage like in Burdett and Mortensen (1998). Thus, firms can increase their labor input by increasing their wage and by opening more vacancies.

As long as the marginal revenue of a firm is higher than its wage, i.e., as long as Assumption 1 holds, more productive firms will pay higher wages. The reason is the same as in the original Burdett-Mortensen model. If a measure of firms pays the same wage, paying a slightly higher wage only marginally increases the cost per worker, while the additional revenue generated by the significantly higher labor force increases profits significantly. Thus, more productive firms will pay higher wages.

Because the contact rate between a worker and a specific firm is proportional to the number of vacancies posted by the firm and because wage offers are increasing in the productivity of a firm, the wage offer distribution is the vacancy weighted distribution of productivities, i.e.,

$$F(w(\varphi)) = \frac{\int_{\varphi^*}^{\varphi} v(\widetilde{\varphi}) \, d\Gamma(\widetilde{\varphi})}{\int_{\varphi^*}^{\overline{\varphi}} v(\widetilde{\varphi}) \, d\Gamma(\widetilde{\varphi})}. \tag{29}$$

Firms choose wages such that the resulting increase in labor balances marginal revenue with marginal labor cost. The number of vacancies are chosen such that the marginal net revenue generated by the last opened vacancy equals the marginal cost of creating the vacancy. The optimality conditions for wages and vacancies are therefore given by

$$\left[\rho\left[1 + j\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)} \varphi^{\rho} l(\varphi,v)^{(\rho-1)} - w(\varphi)\right] \frac{\partial l(\varphi,v)}{\partial v} = cv^{\alpha-1}, \tag{30}$$

$$\left[\rho\left[1 + j\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)} \varphi^{\rho} l(\varphi,v)^{(\rho-1)} - w(\varphi)\right] \frac{\partial l(\varphi,v)}{\partial \varphi} = l(\varphi,v) \frac{\partial w(\varphi)}{\partial \varphi}, \tag{31}$$

where the differential equation (31) follows from the fact that more productive firms pay higher wages. Substituting $F\left(w\left(\varphi\right)\right)$ according to equation (29) and (27) in equation (28) yields

$$l\left(\varphi, v\right) = \frac{\eta v\left(\varkappa + \delta\right)}{\left[\varkappa + \delta + \eta M \int_{\varphi}^{\overline{\varphi}} \frac{v(\widetilde{\varphi})}{1 - \Gamma(\varphi^*)} d\Gamma\left(\widetilde{\varphi}\right)\right]^2}. \tag{32}$$

Inserting into the first order conditions implies the following first order differential wage equation

$$\frac{\partial w\left(\varphi\right)}{\partial \varphi} = \left[\rho \left[1 + j\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)} \varphi^\rho l\left(\varphi, v\right)^{(\rho-1)} - w\left(\varphi\right)\right] \frac{\partial l\left(\varphi, v\right)}{\partial \varphi} \frac{1}{l\left(\varphi, v\right)}, \tag{33}$$

with the terminal condition $w\left(\varphi^*\right) = z$.

The number of vacancies created by the firm is implicitly defined by the vacancy creation condition (30), where the wage $w\left(\varphi\right)$ is given by the solution to the differential equation (33). The average number of vacancies $\widetilde{v}$ per active firm is obtained by integrating the vacancies created by active firms, i.e.,

$$\widetilde{v} = \int_{\varphi^*}^{\overline{\varphi}} \frac{v\left(\widetilde{\varphi}\right)}{1 - \Gamma\left(\varphi^*\right)} d\Gamma\left(\widetilde{\varphi}\right). \tag{34}$$

The number of export countries a firm is willing to enter depend – like in the simple model – on the comparison of profits from exporting to j or $j - 1$ countries, i.e.,

$$\Pi_{d+j}\left(\varphi\right) \gtrless \Pi_{d+j-1}\left(\varphi\right). \tag{35}$$

5.3   Product market

The product market equilibrium is defined by two conditions, the free entry condition that determines the number of active firms in the economy M given the vacancy creation decision in the labor market (that determines the average number of vacancies $\widetilde{v}$ per active firm) and the zero-cutoff productivity condition that determines the productivity level φ^* that guarantees non-negative profits.

Firms only enter the market if the profits they are able to generate are positive. Using the vacancy creation condition (30) one can write total profits of a firm serving the home market and j export markets as

$$\delta\Pi_{d+j}\left(\varphi\right) = \left(1 - \rho\right) \left[1 + j\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)} \left[\varphi l\left(\varphi\right)\right]^\rho + \left(1 - \frac{1}{\alpha}\right) cv\left(\varphi\right)^\alpha - f - jf_x. \tag{36}$$

Since the firm with the lowest productivity pays a wage equal to unemployment benefits, the zero-cutoff productivity φ^*, defined as $\delta\Pi_d\left(\varphi^*\right) = 0$, is given by the solution to the system

of two equations determining the zero-cutoff productivity φ^* and the number of vacancies $v(\varphi^*)$ created by the zero-cutoff productivity firm, i.e.,

$$(1-\rho)\left[\varphi^* l\left(\varphi^*\right)\right]^\rho + \left(1-\frac{1}{\alpha}\right) cv\left(\varphi^*\right)^\alpha \;=\; f, \tag{37}$$

$$\rho\left[\varphi^* l\left(\varphi^*\right)\right]^\rho - zl\left(\varphi^*\right) \;=\; cv\left(\varphi^*\right)^\alpha. \tag{38}$$

The labor force size of the zero-cutoff productivity firm is according to equation (28) given by

$$l\left(\varphi^*\right) = \frac{\eta v\left(\varphi^*\right)\left(\varkappa+\delta\right)}{\left[\varkappa+\delta+\eta M\widetilde{v}\right]^2}.$$

The free entry condition ensures that the profits generated by all firms are used to pay the investment cost f_e of potential market entrants. The expected discounted profit of exporting and non-exporting firms can be written as follows

$$f_e = \int_{\varphi^*}^{\overline{\varphi}} \Pi^{\max}\left(\varphi\right)\gamma\left(\varphi\right)d\varphi, \tag{39}$$

where $\Pi^{\max}\left(\varphi\right) = \max_i \Pi_{d+i}\left(\varphi\right)$ denotes the maximum profits attainable by a firm with productivity φ.

5.4 The case of linear vacancy creation costs ($\alpha = 1$)

If vacancy creation costs are linear, i.e., $\alpha = 1$, the vacancy creation condition reveals that firms increase their number of vacancies such that the marginal revenues are equalized across productivity levels (derivation is given in Appendix F), i.e.,

$$\rho\left[1+j\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)}\varphi^\rho l\left(\varphi,j\right)^{(\rho-1)} = c\frac{v\left(\varphi^*\right)}{l\left(\varphi^*\right)} + z. \tag{40}$$

Firms choose the number of export markets j such that profits are maximized, i.e., $\max_j \Pi_{d+j}\left(\varphi\right)$. Since marginal revenues are equalized across firms, all exporting firms increase their production in order to serve all export markets.

Proposition 5 *If vacancy creation costs are linear, then all exporting firms serve all n foreign markets. The unique export cutoff is given by*

$$\varphi_x = \frac{\tau}{\rho}\left[\frac{f_x}{(1-\rho)}\right]^{\frac{1-\rho}{\rho}}\left[c\frac{v\left(\varphi^*\right)}{l\left(\varphi^*\right)} + z\right]. \tag{41}$$

Proof. Comparing the profits of exporting to j or $j - k$ countries, i.e., $\delta\Pi_{d+j}(\varphi) = \delta\Pi_{d+j-k}(\varphi)$, gives

$$
\begin{aligned}
kf_x &= (1-\rho)\left[1 + j\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)}[\varphi l(\varphi, j)]^{\rho} \\
&\quad - (1-\rho)\left[1 + (j-k)\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)}[\varphi l(\varphi, j-k)]^{\rho}.
\end{aligned}
\tag{42}
$$

Using equation (40) to substitute the labor input $l(\varphi, j)$ into the profit comparison condition (42) gives the desired result. ∎

Thus, with linear vacancy creation costs exporting firms create so many vacancies that their output is large enough to meet the additional demand of all n export countries like in Felbermayr, Prat, Schmerer (2011), Helpman and Itskhoki (2010) and Helpman, Itskhoki, and Redding (2009, 2010).

Wages are still dispersed although marginal revenues are constant across productivities (derivation is given in Appendix F), i.e.,

$$
w(\varphi) = c\left[\frac{v(\varphi^*)}{l(\varphi^*)} - \frac{v(\varphi)}{l(\varphi)}\right] + z.
$$

The reason is the same as in the simple Burdett-Mortensen model. If firms paid the same wage, each firm would have an incentive to deviate and offer a slightly higher wage since it will then be able to recruit also workers employed at other firms and would therefore be able to recruit additional workers at no extra cost (i.e., could save on vacancy creation cost). Thus, in equilibrium high productivity firms pay high wages and have low turnover, while low productivity firms pay low wages and have high turnover.

5.5   The case of convex vacancy creation costs ($\alpha > 1$)

As shown by the following simulation, in the case of convex vacancy creation costs and on-the-job search labor market frictions all our propositions hold with one exception: Sufficiently productive exporting firms will be larger in a global economy compared to autarky.

5.5.1   Simulation method

As the model with endogenous vacancy creation can no longer be solved analytically, we rely on numerical solutions. We assume productivity to be Pareto distributed

$$
\Gamma(\varphi) = \frac{[\underline{\varphi}]^{-\gamma} - \varphi^{-\gamma}}{[\underline{\varphi}]^{-\gamma} - [\overline{\varphi}]^{-\gamma}} \text{ and } \gamma(\varphi) = \frac{\gamma\varphi^{-\gamma-1}}{[\underline{\varphi}]^{-\gamma} - [\overline{\varphi}]^{-\gamma}}.
$$

In order to simulate the model, we proceed as follows.[13] First we construct a grid of φ, running from $\underline{\varphi}$ to $\overline{\varphi}$ in equal steps. Afterwards we specify a starting value for φ^* somewhere above $\underline{\varphi}$ and below $\overline{\varphi}$. For each element of the vector φ we check whether the value of φ is greater than φ^*. If not, we assign the value zero to the vector of φ.

Next, we multiply the values of this vector with the step size of φ and initialize the vector for the vacancies $v(\varphi)$. Later in subsequent loops the vacancies $v(\varphi)$ are determined according to equation (30) given wages $w(\varphi)$ and labor inputs $l(\varphi)$. We then construct a vector that contains the integral $\int_{\underline{\varphi}}^{\overline{\varphi}} \frac{v(\widetilde{\varphi})}{1-\Gamma(\varphi^*)} d\Gamma(\widetilde{\varphi}) = \widetilde{v}$ for each value of φ.

To obtain the wages for each value of φ, we start with the value z at φ^*. Then, we add $\partial w(\varphi)/\partial \varphi \times$ step size of φ to the previous wage, where $\partial w(\varphi)/\partial \varphi$ is given by (33). Labor input per firm is calculated using equation (32).

Given wages $w(\varphi)$ and labor inputs $l(\varphi)$ the next steps within the same loop are to recalculate vacancies $v(\varphi)$ according to (30) and labor inputs $l(\varphi)$ according to (32). We then calculate the sum over the changes in $v(\varphi)$ from the previous and current calculation in the loop. If this change is positive, we increase every element in $v(\varphi)$ by multiplying the old values by 0.9999, and otherwise by 1.0001. We repeat this inner loop until the sum of the changes of $v(\varphi)$ is smaller than 0.01. We then recalculate the integral $\int_{\underline{\varphi}}^{\overline{\varphi}} \frac{v(\widetilde{\varphi})}{1-\Gamma(\varphi^*)} d\Gamma(\widetilde{\varphi})$ for each value of φ.

Given the values of $\widetilde{v}$, $v(\varphi)$, $w(\varphi)$, and $l(\varphi)$, we construct a matrix of size grid size$\times$(number of countries), where we calculate for each value of φ the (potential) total profit if the firm would export to zero, one, two countries, and so on, up to the maximum number of trading partners. Profits are given by equation (36). Given this matrix we next construct a vector that contains the number of countries that a firm with productivity φ should export to in order to maximum profits. The zero-cutoff productivity φ^* is given by the value of φ where total profits are equal to zero and where it is profit maximizing for a firm to serve only the home market.

After a first initialization of a chosen value of M, we calculate the free entry condition as given in equation (39). If this value is negative, we reduce the number of firms M by 0.1%; otherwise we increase it by 0.1%. We then repeat the whole process with the new value of

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[13]We solve our model using Matlab Release R2009b. The m-file is available upon request from the authors.

M until M converges.[14]

For the simulations we have chosen the following parameter values, $\chi = 0.05$, $\eta = 0.01$, $\delta = 0.02$, $\rho = 0.75$, $\tau = 1.8$, $c = 1000000$, $\alpha = 5$, $f = 0.0002$, $f_x = 35f$, $f_e = 10$, $\gamma = 3.4$, $\underline{\varphi} = 10$, $\overline{\varphi} = 100$ and $z = 1$. For the case with trade we assume 99 trading partners, i.e., 100 countries.[15]

5.5.2 Results

Throughout this section we focus on two scenarios: A world where the country is in autarky and a world where there are 99 symmetric trading partners.[16]

In Figure 6 we plot the number of vacancies created (left panel) and the number of export markets served by a firm with productivity φ (right panel). In line with Proposition 1 the number of export markets served is an increasing function of productivity. We calibrated the model such that no firm is willing to export to all foreign markets. Firms with the highest productivities enter 42 out of the 50 markets. Like in the model with fixed vacancies the level of productivity where firms can successfully survive φ^* is higher in the open economy than in autarky.
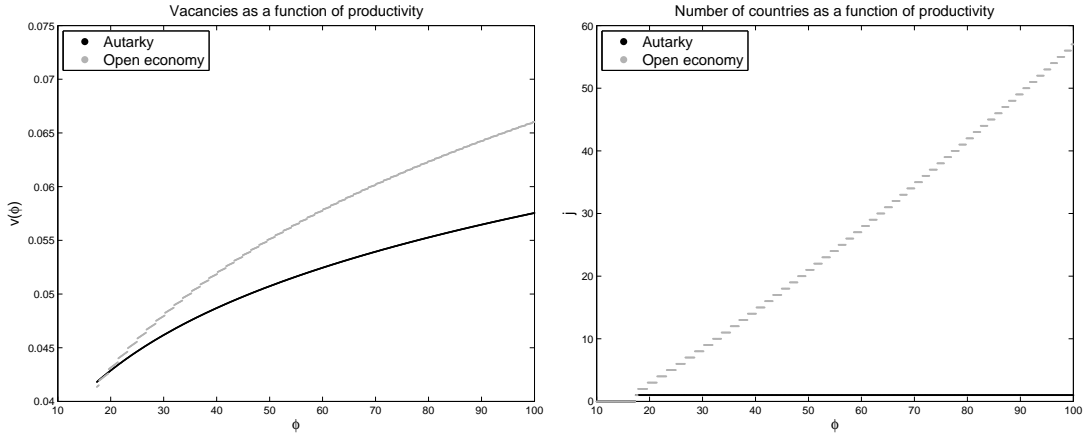
Figure 6: Vacancies and number of countries served in autarky and in an open economy with endogenous vacancies

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[14]Our convergence criterion is $\left| \left(\int_{\varphi^*}^{\overline{\varphi}} \Pi^{\max}(\varphi)\,\gamma(\varphi)\,d\varphi \right) - f_e \right| < 0.01$.

[15]The grid size is chosen to be 1000. However, results do not depend on the chosen grid size.

[16]The number of (potential) trading partners is not crucial for the basic qualitative results.

With trade the number of vacancies per firm is lower than in autarky for low-productivity firms, but higher for high productivity firms. Additionally, the number of vacancies are increasing with productivity in both scenarios. More importantly, the number of vacancies jumps up at each export-cutoff because firms increase their labor input in response to additional demand from abroad. Convex vacancy creation costs, however, restrict firms in their ability to grow.

Figure 7 plots labor inputs (left panel) and outputs (right panel) per firm. The pattern of vacancies translates into labor input and output pattern. Labor input and output per firm is lower in the open economy as in autarky for low-productivity firms and higher for high-productivity firms. High productivity firms grow at the expense of low productivity firms because the additional revenues from exports allow them to create more vacancies. Unlike in Melitz (2003) not all exporting firms grow because the increased competition in the labor market due to the increased number of vacancies has a negative effect on employment per firm, similar to the negative impact that the increased number of active firms M has on labor input in the basic framework without vacancy creation. Hence, the basic results of Proposition 2 for the case of fixed vacancies survive with the qualification that only less productive firms shrink when opening up to trade.
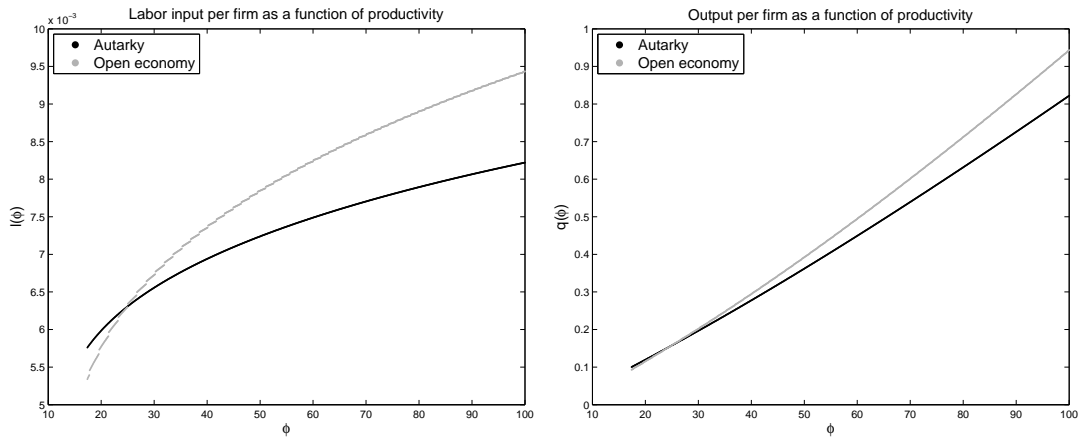

Figure 7: Firm size (labor input and output) in autarky and in an open economy with endogenous vacancies

Let us now investigate domestic prices and quantities under autarky and in an open economy. Like in Melitz (2003) domestic variety prices are a monotonically falling function

of φ under autarky (Figure 8). However, with trade the domestic price profile of firms looks very different. First, firms that only sell domestically charge a slightly higher price as firms under autarky because the increased competition in the labor market reduces their output (see Figure 7). The firm that exports to one trading partner charges a higher price in the domestic market than the firm selling only locally. The domestic price of the least productive firm in the group of firms that export to more than one country is slightly lower than the price charged by the least productive firm that exports to only one country. However, the price is still higher than the domestic price of the firm that only serves the local market.[17] These results are similar to our results shown in Figure 4b.
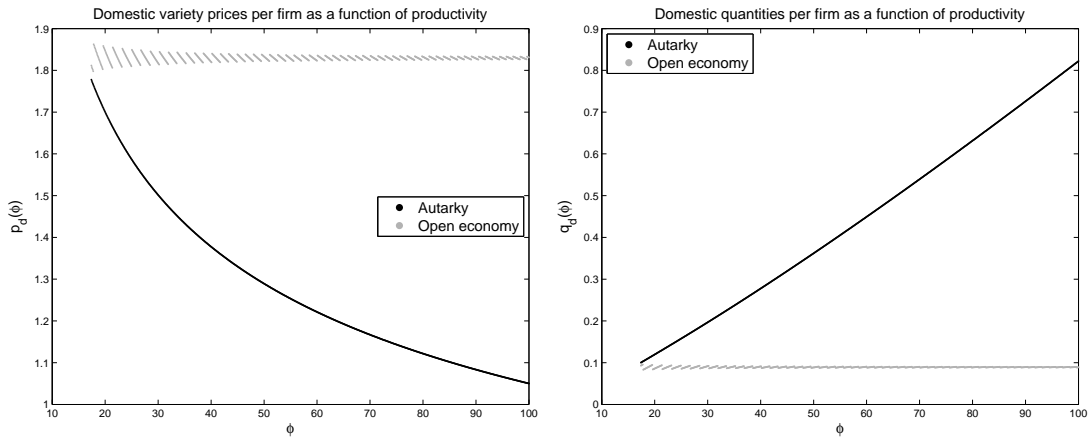
Figure 8: Domestic quantities and prices in autarky and in an open economy with endogenous vacancies

Quantities are just the reverse image of prices charged in the domestic market. The right panel shows that the domestically sold quantities are much higher under autarky than in an open economy, specifically for very productive firms. The quantity of the least productive firm, i.e., the firm with productivity φ^*, is higher than the quantity of the least productive firm serving in addition to the domestic market one foreign market. Hence, the results that we derived in Proposition 3 survive under endogenous vacancy creation.

Figure 9 shows total profits of firms as a function of productivity. In both scenarios,

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[17]We set the number of (potential) trading partners large enough so that even the most productive firm does not serve all foreign markets. If we would allow a firm to hit the boundary for expanding the number of markets to be served, this firm can only expand by lowering the prices. This would be reflected by a fall of the price line at the right.

autarky and trade, profits are increasing in productivity. Even though there are jumps in prices and quantities there are no jumps in the profit function. The extra revenues from exporting are used to pay for the foreign market entry costs. This is equivalent to the export-cutoff condition, where the least productive firm entering j markets has to be indifferent between entering j markets or only serving $j-1$ markets.
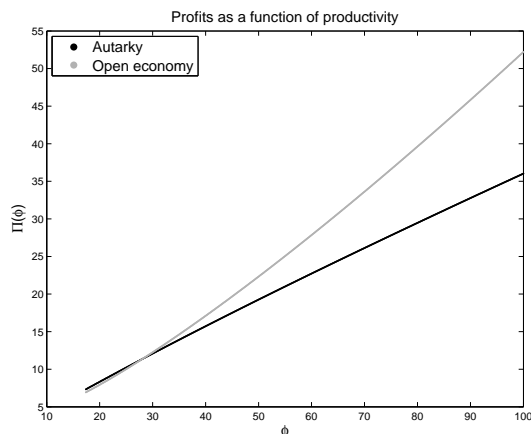
Figure 9: Profits as a function of productivity in autarky and in an open economy with endogenous vacancy creation.

If we compare the profits of firms in autarky and in an open economy, we see that the profit function under trade is much steeper than under autarky. The reason is that by serving more than one market, a firm can demand higher prices in every market and therefore generate higher profits with the same output. Furthermore, like in Melitz (2003) there are some low productivity firms that make lower profits in an open economy than under autarky because the increased competition on the labor market reduces low productivity firms' labor input and thus the output necessary to generate higher profits.

In Figure 10 we plot wages as a function of productivity (left panel) and the wage distribution (right panel). Wages are an increasing function of productivity under both, autarky and trade. Interesting are the following three observations: (i) The wage distribution starts at lower productivity values in autarky than in an open economy. This reflects the fact that only more productive firms can survive in an open economy, i.e., the zero-cut-off productivity φ^* increases when opening up to trade.[18] (ii) Wages are at least as high as unemployment

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[18]The effect is very small, though. Hence, it can not be seen in the figure.

benefits z. (iii) The wage function is much steeper in an open economy because exporting generates higher profits and opens up the opportunity for firms to pay higher wages.
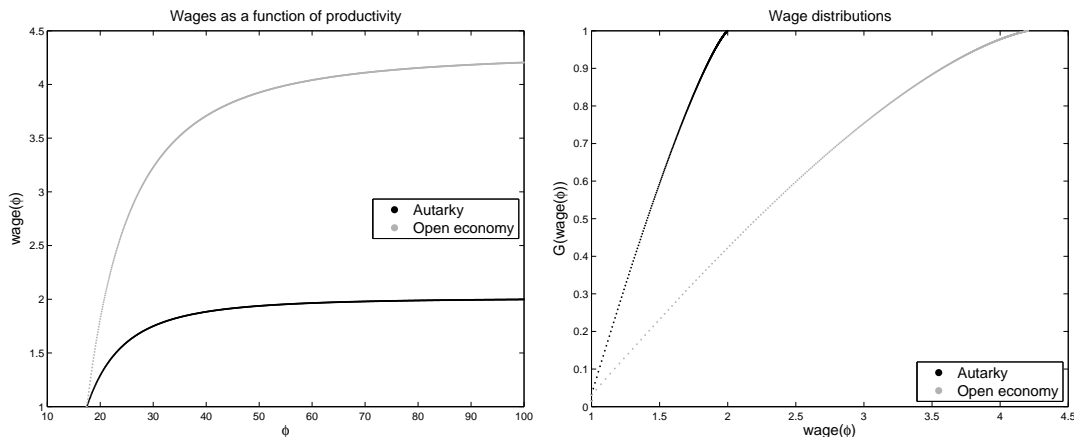
Figure 10: Wages and wage distributions in autarky and in an open economy with endogenous vacancy creation.

We can also compare the wage distribution in autarky and in an open economy. The right panel of Figure 10 shows that in both situations the lowest wage is given by z. Since wages increase at exporting firms, opening up to trade leads to a much larger wage dispersion as predicted in Proposition 4. Hence, allowing for vacancy creation does not lead to different conclusions regarding the effects of trade on the wage distribution. Note, that with endogenous vacancy creation it still holds that in an open economy the number of firms is higher and the unemployment rate lower compared to autarky.

5.5.3 Convex vacancy costs and concave hiring costs

Abowd and Kramarz (2003) and Kramarz and Michaud (2010) have shown that the shape of the hiring cost function for French firms is mildly concave, while Blatter, Mühlemann and Schenker (2009) have shown that the shape of the hiring cost function for Swiss firms is convex. In this section we show that a convex vacancy cost function is consistent with a concave and a convex hiring cost function. Hiring cost functions have the same shape as the vacancy cost functions if the hiring rate $h(v)$ per vacancy is the same for all firms. However, the hiring rate per vacancy is increasing in the wage because job offers made by high wage firms are accepted by more employed workers. This property holds in the Burdett-Mortensen

model like in any monopsony wage model as shown by Manning (2006).[19] It can also be seen by looking at the equation for the hiring rate per vacancy given by

$$h\left(v\right) = \eta\left[u + \left(1 - u\right)G\left(w\right)\right].$$

In addition the number of vacancies are an increasing function of the wage paid by firms, i.e.,

$$\frac{\partial v\left(w\right)}{\partial w} > 0.$$

Thus, the total number of workers hired $H = h\left(v\right)v$ increase with the wage for two reasons: (i) the number of vacancies created increase with the wage and (ii) the hiring rate per vacancy increases with the wage.

Now consider the shape of the hiring cost function $K\left(H\right)$ given any convex vacancy cost function $c\left(v\right)$ with $c_v'\left(v\right) > 0$ and $c_{vv}''\left(v\right) > 0$. Using the inverse function of $H = h\left(v\right)v$ and $v\left(w\right)$, the first derivative of the hiring cost function is given by

$$\frac{\partial K\left(H\right)}{\partial H} = c_v'\left(v\right)\frac{\partial v}{\partial H} = c_v'\left(v\right)\frac{1}{h\left(v\right) + v\dfrac{\partial h\left(v\right)}{\partial w}\dfrac{\partial w\left(v\right)}{\partial v}} > 0,$$

where the inequality follows from

$$\frac{\partial h\left(v\right)}{\partial w} = \left(1 - u\right)g\left(w\right) > 0 \quad \text{and} \quad \frac{\partial w\left(v\right)}{\partial v} > 0.$$

The second derivative that determines the shape of the hiring cost function is given by

$$\begin{aligned}
\frac{\partial^2 K\left(H\right)}{\partial H^2} &= c_{vv}''\left(v\right)\left(\frac{\partial v}{\partial H}\right)^2 \\
&\quad - c_v'\left(v\right)\frac{2\dfrac{\partial h\left(v\right)}{\partial w}\dfrac{\partial w\left(v\right)}{\partial v} + v\dfrac{\partial^2 h\left(v\right)}{\partial w^2}\left(\dfrac{\partial w\left(v\right)}{\partial v}\right)^2 + v\dfrac{\partial h\left(v\right)}{\partial w}\dfrac{\partial^2 w\left(v\right)}{\partial v^2}}{\left(h\left(v\right) + v\dfrac{\partial h\left(v\right)}{\partial w}\dfrac{\partial w\left(v\right)}{\partial v}\right)^2}\frac{\partial v}{\partial H},
\end{aligned}$$

where

$$\frac{\partial^2 h\left(v\right)}{\partial w^2} = \left(1 - u\right)g_w'\left(w\right) \gtrless 0 \quad \text{and} \quad \frac{\partial^2 w\left(v\right)}{\partial v^2} \gtrless 0.$$

Thus, a convex vacancy cost function implies a concave hiring cost function, if and only if

$$c_{vv}''\left(v\right) < c_v'\left(v\right)\left(2\frac{\partial h\left(v\right)}{\partial w}\frac{\partial w\left(v\right)}{\partial v} + v\frac{\partial^2 h\left(v\right)}{\partial w^2}\left(\frac{\partial w\left(v\right)}{\partial v}\right)^2 + v\frac{\partial h\left(v\right)}{\partial w}\frac{\partial^2 w\left(v\right)}{\partial v^2}\right)\frac{\partial v}{\partial H},$$

---

[19]Using firm level data from the Labour Turnover Survey in the UK, Manning (2006) shows that there are increasing marginal costs of recruitment, i.e., that the vacancy cost function is convex.
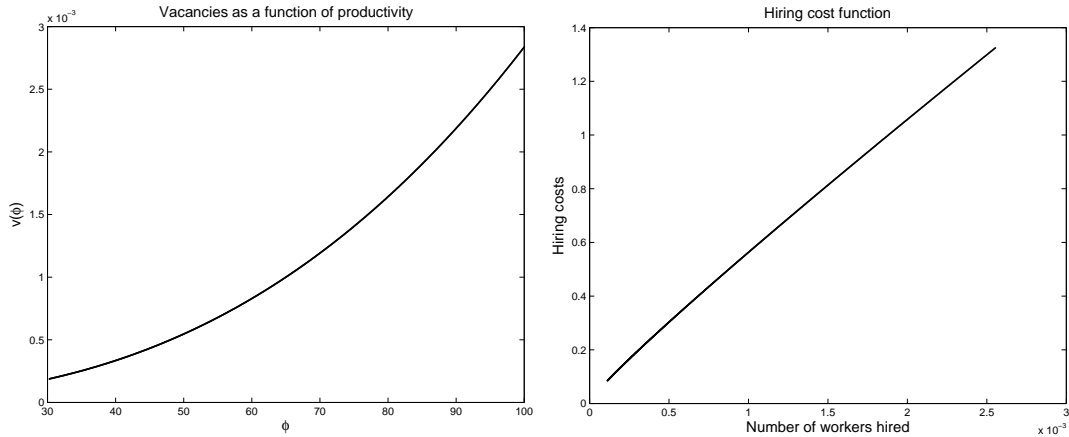
Figure 11: Hiring cost function for the convex vacancy cost function

which is feasible since $\partial h\left(v\right)/\partial w > 0$ and $\partial w\left(v\right)/\partial v > 0$. Thus, a convex vacancy cost function is consistent with a concave hiring cost function as found by Abowd and Kramarz (2003) and Kramarz and Michaud (2010) for French firms as well as a convex hiring cost function as found by Blatter, Mühlemann and Schenker (2009) for Swiss firms.

Our simulations also provide an example that a convex vacancy cost function leads to a concave hiring cost function as shown in the following Figure.[20]

6   Conclusions

The implications of trade liberalization on wages and unemployment is one of the most heavily discussed consequences of increasing globalization. Recent evidence suggests that overall trade reduces unemployment, but has heavily asymmetric distributional consequences. Most recent models of trade and unemployment emphases the role of trade on unemployment, while little is known about the consequences of labor market frictions on the structure of trade. We use the on-the-job search model from Burdett and Mortensen (1998) and combine it with the Melitz (2003) trade model in order to investigate the effects of capacity constraining labor market frictions in a global economy.

We show that capacity constraints heavily alter the results compared to models with perfect labor markets or imperfect labor markets without capacity constraining effects, such

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[20]The parametrization is as follows: $\chi = 0.02$, $\eta = 0.9$, $\delta = 0.02$, $\rho = 0.75$, $c = 500$, $\alpha = 1.01$, $f = 0.0001$, $f_e = 5$, $\gamma = 3.2$, $\underline{\varphi} = 30$, $\overline{\varphi} = 100$ and $z = 1$. We only focus on the case of autarky here.

as the recent works by Felbermayr, Prat, and Schmerer (2011), Helpman and Itskhoki (2010) and Helpman, Itskhoki, and Redding (2009, 2010). With capacity constraining labor market frictions not all firms will serve all export markets, even when export markets are similar. Rather the number of export markets served by a firm is increasing in its productivity. Even though exporting firms are more productive, they do not necessarily charge lower prices as in the Melitz (2003) model. Rather, they maximize profits by serving only part of the export markets and by charging the monopolistic price in each market. Given the capacity constraints that firms face if they want to recruit more workers in their domestic country, an obvious extension of our model is to allow for foreign direct investment since it would allow firms to relax their capacity constraints by recruiting and producing in a foreign country.

Concerning trade liberalization we find that unemployment falls and wage dispersion increases with trade liberalization. Note that in our context wage inequality is the result of continuous search for better jobs and not of fair-wage preferences or the result of monitoring or screening.

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Appendix A: Wages offers

Wages increase with productivity

To show that wages increase with productivity we follow Mortensen (1990). On the support of the wage offer distribution it must be that

$$
\begin{aligned}
\pi\left(\varphi\right) &= \varphi^{\rho} l\left(w\left(\varphi\right)\right)^{\rho} - w\left(\varphi\right) l\left(w\left(\varphi\right)\right) - f \quad \text{for all} \quad w\left(\varphi\right) \in \operatorname{supp}\left(F\right), \\
\pi\left(\varphi\right) &\geq \varphi^{\rho} l\left(w\left(\varphi'\right)\right)^{\rho} - w\left(\varphi'\right) l\left(w\left(\varphi'\right)\right) - f \quad \text{for all} \quad w\left(\varphi'\right) \notin \operatorname{supp}\left(F\right).
\end{aligned}
$$

These equilibrium conditions imply for $\varphi > \varphi'$

$$
\begin{aligned}
\varphi^{\rho} l\left(w\left(\varphi\right)\right)^{\rho} - w\left(\varphi\right) l\left(w\left(\varphi\right)\right) &\geq \varphi^{\rho} l\left(w\left(\varphi'\right)\right)^{\rho} - w\left(\varphi'\right) l\left(w\left(\varphi'\right)\right) \\
&> \left(\varphi'\right)^{\rho} l\left(w\left(\varphi'\right)\right)^{\rho} - w\left(\varphi'\right) l\left(w\left(\varphi'\right)\right) \\
&\geq \left(\varphi'\right)^{\rho} l\left(w\left(\varphi\right)\right)^{\rho} - w\left(\varphi\right) l\left(w\left(\varphi\right)\right).
\end{aligned}
$$

The difference of the first and the last term of this inequality is greater than or equal the difference of its middle terms, i.e.,

$$
\left[\varphi^{\rho} - \left(\varphi'\right)^{\rho}\right] l\left(w\left(\varphi\right)\right)^{\rho} \geq \left[\varphi^{\rho} - \left(\varphi'\right)^{\rho}\right] l\left(w\left(\varphi'\right)\right)^{\rho}.
$$

Since $l\left(w\left(\varphi\right)\right)$ is an increasing function of the wage, it implies that wages are weakly increasing in productivity. Since firms always have an incentive to deviate if other firms offer the same wage, it follows that wages strictly increase with productivity.

Wage offers, if Assumption 1 does not hold

The optimality condition (15) implies

$$l\left(w\left(\varphi\right)\right) = \left[\varphi^{\rho}\rho l\left(w\left(\varphi\right)\right)^{(\rho-1)} - w\left(\varphi\right)\right]\frac{\partial l\left(w\left(\varphi\right)\right)}{\partial w\left(\varphi\right)}.$$

This equation can only hold for $\varphi^{\rho}\rho l\left(w\left(\varphi\right)\right)^{(\rho-1)} - w\left(\varphi\right) > 0$ since $\partial l\left(w\left(\varphi\right)\right)/\partial w\left(\varphi\right) > 0$. If this condition, i.e., Assumption 1 is not satisfied, firms with productivity $\varphi \in [\varphi^{*}, \widehat{\varphi}]$ pay $w\left(\varphi\right) = z$, where the productivity $\widehat{\varphi}$ is defined such that $\rho\left[\widehat{\varphi}l\left(\widehat{\varphi}\right)\right]^{\rho} = zl\left(\widehat{\varphi}\right)$. The size of the firm's labor force $l\left(\widehat{\varphi}\right)$ is given by

$$l\left(\widehat{\varphi}\right) = \eta\overline{v}\frac{u + \left(1 - u\right)G\left(w^{-}\left(\varphi\right)\right)}{\varkappa + \delta + \lambda\left(M\right)\left[1 - F\left(w\left(\varphi\right)\right)\right]},$$

where $G\left(w\left(\varphi\right)\right) = G\left(w^{-}\left(\varphi\right)\right) + \mu\left(w\left(\varphi\right)\right)$ and $\mu\left(w\left(\varphi\right)\right)$ denotes the mass of workers employed at firms offering the wage $w\left(\varphi\right)$, and $G\left(w^{-}\left(\varphi\right)\right)$ are all the workers getting a wage lower than $w(\varphi)$. $G\left(w\left(\varphi\right)\right) = G\left(w^{-}\left(\varphi\right)\right) + \mu\left(w\left(\varphi\right)\right)$ states that the whole wage distribution is given by all the workers earning wages lower than $w(\varphi)$ plus the once earning exactly wages $w(\varphi)$.

Since firms with a marginal revenue below z will not find it optimal to increase their labor input by increasing wage, the mass point is at the lower bound of the wage distribution. Thus, $G\left(w^{-}\left(\varphi\right)\right) = 0$. Using equations (11) and (17) we get

$$l\left(\widehat{\varphi}\right) = \frac{\eta\overline{v}\left[1 - \Gamma\left(\varphi^{*}\right)\right]}{\left(\varkappa + \delta\right)\left[1 - \Gamma\left(\varphi^{*}\right)\right] + \eta M\overline{v}\left[1 - \Gamma\left(\widehat{\varphi}\right)\right]}\frac{\varkappa + \delta}{\varkappa + \delta + \eta M\overline{v}}.$$

We proceed by assuming that the marginal revenue of a firm exceeds the level of unemployment benefits, i.e., assume that Assumption 1 holds. If Assumption 1 holds, more productive firms offer higher wages and the first order condition (15) holds for all $\varphi \in [\varphi^{*}, \overline{\varphi}]$. Hence, the labor force of all firms operating is given by equation (14).

Appendix B: Derivation of the wage function $w\left(\varphi\right)$

The first order condition (15) implies

$$1 = \left[\varphi^{\rho}\rho l\left(w\left(\varphi\right)\right)^{(\rho-1)} - w\left(\varphi\right)\right]\frac{\partial l\left(w\left(\varphi\right)\right)}{\partial w\left(\varphi\right)}\frac{1}{l\left(w\left(\varphi\right)\right)}.$$

Substituting $F\left(w\left(\varphi\right)\right)$ using equation (17) gives

$$\frac{\partial w\left(\varphi\right)}{\partial\varphi} = \left[\varphi^{\rho}\rho l\left(\varphi\right)^{(\rho-1)} - w\left(\varphi\right)\right]\frac{\partial l\left(\varphi\right)}{\partial\varphi}\frac{1}{l\left(\varphi\right)}.$$

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Define

$$T\left(\varphi\right) = \ln\left[l\left(\varphi\right)\right] \quad \text{and} \quad T'\left(\varphi\right) = \frac{\partial l\left(\varphi\right)}{\partial \varphi} \frac{1}{l\left(\varphi\right)}.$$

Substitution simplifies the above differential equation to

$$\frac{\partial w\left(\varphi\right)}{\partial \varphi} + w\left(\varphi\right) T'\left(\varphi\right) = \rho\varphi^{\rho}\left[e^{T(\varphi)}\right]^{\rho-1} T'\left(\varphi\right).$$

Any solution to this differential equation has to satisfy

$$w\left(\varphi\right) e^{T(\varphi)} = \int_{\varphi^*}^{\varphi} \rho\left[\widetilde{\varphi}e^{T(\widetilde{\varphi})}\right]^{\rho} T'\left(\widetilde{\varphi}\right) d\widetilde{\varphi} + A, \tag{43}$$

where A is the constant of integration. Note that

$$\frac{d\left[\varphi e^{T(\varphi)}\right]^{\rho}}{d\varphi} = \rho\left[\varphi e^{T(\varphi)}\right]^{\rho} T'\left(\varphi\right) + \rho\varphi^{\rho-1}\left[e^{T(\varphi)}\right]^{\rho}.$$

The integral can thus be written as

$$
\begin{aligned}
\int_{\varphi^*}^{\varphi} \rho\left[\widetilde{\varphi}e^{T(\widetilde{\varphi})}\right]^{\rho} T'\left(\widetilde{\varphi}\right) d\widetilde{\varphi} &= \int_{\varphi^*}^{\varphi} \left[\frac{d\left[\widetilde{\varphi}e^{T(\widetilde{\varphi})}\right]^{\rho}}{d\widetilde{\varphi}} - \rho\widetilde{\varphi}^{(\rho-1)}\left[e^{T(\widetilde{\varphi})}\right]^{\rho}\right] d\widetilde{\varphi} \\
&= \left[\varphi e^{T(\varphi)}\right]^{\rho} - \left[\varphi^* e^{T(\varphi^*)}\right]^{\rho} - \int_{\varphi^*}^{\varphi} \rho\widetilde{\varphi}^{(\rho-1)}\left[e^{T(\widetilde{\varphi})}\right]^{\rho} d\widetilde{\varphi}.
\end{aligned}
$$

Substituting into the wage equation (43) gives

$$
\begin{aligned}
w\left(\varphi\right) e^{T(\varphi)} &= \left[\varphi e^{T(\varphi)}\right]^{\rho} - \left[\varphi^* e^{T(\varphi^*)}\right]^{\rho} - \int_{\varphi^*}^{\varphi} \rho\widetilde{\varphi}^{(\rho-1)}\left[e^{T(\widetilde{\varphi})}\right]^{\rho} d\widetilde{\varphi} + A \\
w\left(\varphi\right) &= \frac{\left[\varphi e^{T(\varphi)}\right]^{\rho} - \left[\varphi^* e^{T(\varphi^*)}\right]^{\rho}}{e^{T(\varphi)}} - \int_{\varphi^*}^{\varphi} \rho\left[\widetilde{\varphi}e^{T(\widetilde{\varphi})}\right]^{(\rho-1)} \frac{e^{T(\widetilde{\varphi})}}{e^{T(\varphi)}} d\widetilde{\varphi} \\
&\quad + z\frac{e^{T(\varphi^*)}}{e^{T(\varphi)}} \tag{44}
\end{aligned}
$$

since

$$w\left(\varphi^*\right) e^{T(\varphi^*)} = ze^{T(\varphi^*)} = A.$$

Substituting $e^{T(\varphi)} = l\left(w\left(\varphi\right)\right)$ and z by using the zero-cutoff condition (20) gives the wage equation (18).

Appendix C: Equilibrium in autarky

Applying the implicit function theorem to the free entry condition (19) implies

$$\frac{d\varphi^*}{dM} = -\frac{\int_{\varphi^*}^{\overline{\varphi}}\left[\int_{\varphi^*}^{\varphi} \rho^2\left[\widetilde{\varphi}l\left(w\left(\widetilde{\varphi}\right)\right)\right]^{(\rho-1)} \frac{\partial l(w(\widetilde{\varphi}))}{\partial \varphi^*} d\widetilde{\varphi} - \frac{\rho}{\varphi^*}\left[\varphi^* l\left(w\left(\varphi^*\right)\right)\right]^{\rho}\right]\gamma\left(\varphi\right) d\varphi}{\int_{\varphi^*}^{\overline{\varphi}}\left[\int_{\varphi^*}^{\varphi} \rho^2\left[\widetilde{\varphi}l\left(w\left(\widetilde{\varphi}\right)\right)\right]^{(\rho-1)} \frac{\partial l(w(\widetilde{\varphi}))}{\partial M} d\widetilde{\varphi}\right]\gamma\left(\varphi\right) d\varphi} < 0,$$

where the inequality follows because $\partial l\left(w\left(\widetilde{\varphi}\right)\right)/\partial\varphi^{*}<0$ and $\partial l\left(w\left(\widetilde{\varphi}\right)\right)/\partial M<0$. Thus, the free entry condition defines a decreasing relation between the zero-cutoff productivity φ^{*} and the number of active firms M in the market.

Applying the implicit function theorem to the zero-profit condition (20) implies

$$\frac{d\varphi^{*}}{dM}=\frac{\left[\rho\left[\varphi^{*}\right]^{\rho}\left[l\left(z\right)\right]^{(\rho-1)}-z\right]\left[\dfrac{2\eta\overline{v}}{\varkappa+\delta+\eta M\overline{v}}\right]l\left(z\right)}{\rho\left[\varphi^{*}\right]^{\rho-1}\left[l\left(z\right)\right]^{\rho}}>0,$$

where Assumption 1 ensures an increasing relation between the zero-cutoff productivity φ^{*} and the number of active firms M in the market.

An equilibrium only exists if unemployment benefits z and the fixed costs f are low enough.

Appendix D: The open economy

Quantities sold in the domestic and each export market

An exporting firm that decided to serve j foreign countries maximizes its profits by equalizing marginal revenues across markets. Revenues of an exporting firm are given by

$$
\begin{aligned}
R\left(\varphi\right) &= p_{d}\left(\varphi\right)q_{d}\left(\varphi\right)+j\frac{p_{x}\left(\varphi\right)}{\tau}q_{x}\left(\varphi\right)\\
&= p_{d}\left(\varphi\right)\left[q\left(\varphi\right)-jq_{x}\left(\varphi\right)\right]+j\frac{p_{x}\left(\varphi\right)}{\tau}q_{x}\left(\varphi\right)\\
&= \left[q\left(\varphi\right)-jq_{x}\left(\varphi\right)\right]^{\rho}+j\left[\frac{q_{x}\left(\varphi\right)}{\tau}\right]^{\rho}.
\end{aligned}
$$

By choosing its domestic and export sells according to equalization of marginal revenues

$$
\begin{aligned}
\frac{\partial R\left(\varphi\right)}{\partial q_{x}\left(\varphi\right)} &= 0\\
\rho j\left[q\left(\varphi\right)-jq_{x}\left(\varphi\right)\right]^{\rho-1} &= \rho j\frac{1}{\tau}\left[\frac{q_{x}\left(\varphi\right)}{\tau}\right]^{\rho-1}\\
q\left(\varphi\right)-jq_{x}\left(\varphi\right) &= \frac{1}{\tau^{\rho/(\rho-1)}}q_{x}\left(\varphi\right)\\
\tau^{\rho/(\rho-1)}q\left(\varphi\right) &= \left[1+j\tau^{\rho/(\rho-1)}\right]q_{x}\left(\varphi\right).
\end{aligned}
$$

Rearranging and using the fact that $q_{d}\left(\varphi\right)=q\left(\varphi\right)-jq_{x}\left(\varphi\right)$ implies equations (21) and (22).

The revenue of an exporting firm is, therefore, given by

$$
\begin{aligned}
R\left(\varphi\right) &= \left[q\left(\varphi\right) - jq\left(\varphi\right)\frac{\tau^{\rho/(\rho-1)}}{1+j\tau^{\rho/(\rho-1)}}\right]^{\rho} + j\left[\frac{q\left(\varphi\right)}{\tau}\frac{\tau^{\rho/(\rho-1)}}{1+j\tau^{\rho/(\rho-1)}}\right]^{\rho} \\
&= \left[1+j\tau^{\rho/(\rho-1)}\right]\left[\frac{q\left(\varphi\right)}{1+j\tau^{\rho/(\rho-1)}}\right]^{\rho} \\
&= \left[1+j\tau^{\rho/(\rho-1)}\right]^{(1-\rho)}\left[q\left(\varphi\right)\right]^{\rho}.
\end{aligned}
$$

Proof of Proposition 1: Export-cutoffs

The export-cutoff productivity φ_x^n is defined by $\delta\Pi_{d+j}\left(\varphi_x^j\right) = \delta\Pi_{d+j-1}\left(\varphi_x^j\right)$, where

$$
\delta\Pi_{d+j}\left(\varphi\right) = \left[1+j\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)}\left[\varphi l\left(w\left(\varphi\right)\right)\right]^{\rho} - w\left(\varphi\right)l\left(w\left(\varphi\right)\right) - f - jf_x.
$$

Since profit maximization implies that the wage is continuous at φ_x^j, i.e., that both wages are the same, and since the same wage implies that the number of workers employed by both type of firms are identical and given by $l\left(w\left(\varphi_x^j\right)\right)$ we may write

$$
\begin{aligned}
&\left[1+j\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)}\left[\varphi_x^j l\left(w\left(\varphi_x^j\right)\right)\right]^{\rho} - w\left(\varphi_x^j\right)l\left(w\left(\varphi_x^j\right)\right) - f - jf_x \\
&= \left[1+(j-1)\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)}\left[\varphi_x^j l\left(w\left(\varphi_x^j\right)\right)\right]^{\rho} - w\left(\varphi_x^j\right)l\left(w\left(\varphi_x^j\right)\right) - f - (j-1)f_x.
\end{aligned}
$$

Thus, the export-cutoff condition (25) can be derived

$$
\left[\varphi_x^j l\left(w\left(\varphi_x^j\right)\right)\right]^{\rho} = \frac{f_x}{\left[\left[1+j\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)} - \left[1+(j-1)\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)}\right]}.
$$

The rhs of the last equation (and therefore the export-cutoff productivity φ_x^j) is increasing in j, i.e.,

$$
\begin{aligned}
&\left[\varphi_x^j l\left(w\left(\varphi_x^j\right)\right)\right]^{\rho} - \left[\varphi_x^{j-1} l\left(w\left(\varphi_x^{j-1}\right)\right)\right]^{\rho} \\
&= f_x\frac{2\left[1+(j-1)\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)} - \left[1+(j-2)\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)} - \left[1+j\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)}}{\left[\left[1+j\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)} - \left[1+(j-1)\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)}\right]\left[\left[1+(j-1)\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)} - \left[1+(j-2)\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)}\right]} > 0,
\end{aligned}
$$

where the last inequality follows from Jensen's inequality, i.e.,

$$
\begin{aligned}
&\frac{1}{2}\left[1+(j-2)\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)} + \frac{1}{2}\left[1+j\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)} \\
&< \left[\frac{1}{2}\left[1+(j-2)\tau^{\frac{\rho}{\rho-1}}\right] + \frac{1}{2}\left[1+j\tau^{\frac{\rho}{\rho-1}}\right]\right]^{(1-\rho)} \\
&= \left[1+\left[\frac{1}{2}(j-2)+\frac{1}{2}j\right]\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)} \\
&= \left[1+(j-1)\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)}.
\end{aligned}
$$

Wages in an open economy

The wage equation for exporting firms follows from the first order condition and the equilibrium condition (17), leading to the following differential equation

$$\frac{\partial w\left(\varphi\right)}{\partial\varphi}+w\left(\varphi\right)T'\left(\varphi\right)=\left[1+j\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)}\rho\varphi^{\rho}\left[e^{T(\varphi)}\right]^{\rho-1}T'\left(\varphi\right),$$

where $T\left(\varphi\right)$ and $T'\left(\varphi\right)$ are defined in Appendix A. The solution to this differential equation is obtained by following the same steps as in Appendix A, and given by

$$w\left(\varphi\right)e^{T(\varphi)}=\left[1+j\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)}\left[\left[\varphi e^{T(\varphi)}\right]^{\rho}-\left[\varphi_{x}^{j}e^{T\left(\varphi_{x}^{j}\right)}\right]^{\rho}-\int_{\varphi_{x}^{j}}^{\varphi}\frac{\rho}{\widetilde{\varphi}}\left[\widetilde{\varphi}e^{T(\widetilde{\varphi})}\right]^{\rho}d\widetilde{\varphi}\right]+A,$$

where

$$
\begin{aligned}
A &= w\left(\varphi_{x}^{j}\right)e^{T\left(\varphi_{x}^{j}\right)} \\
&= \left[1+(j-1)\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)}\left[\left[\varphi_{x}^{j}e^{T\left(\varphi_{x}^{j}\right)}\right]^{\rho}-\left[\varphi_{x}^{j-1}e^{T\left(\varphi_{x}^{j-1}\right)}\right]^{\rho}-\int_{\varphi_{x}^{j-1}}^{\varphi_{x}^{j}}\frac{\rho}{\widetilde{\varphi}}\left[\widetilde{\varphi}e^{T(\widetilde{\varphi})}\right]^{\rho}d\widetilde{\varphi}\right] \\
&\quad +w\left(\varphi_{x}^{j-1}\right)e^{T\left(\varphi_{x}^{j-1}\right)},
\end{aligned}
$$

and

$$w\left(\varphi_{x}^{1}\right)e^{T\left(\varphi_{x}^{1}\right)}=\left[\varphi_{x}^{1}e^{T\left(\varphi_{x}^{1}\right)}\right]^{\rho}-\left[\varphi^{*}e^{T(\varphi^{*})}\right]^{\rho}-\int_{\varphi^{*}}^{\varphi_{x}^{1}}\frac{\rho}{\widetilde{\varphi}}\left[\widetilde{\varphi}e^{T(\widetilde{\varphi})}\right]^{\rho}d\widetilde{\varphi}+ze^{T(\varphi^{*})}.$$

Since A depends on the wage payments of those firms with export-cutoff productivities $\varphi_{x}^{j}<\varphi$ we need to rewrite the wage equation as follows

$$
\begin{aligned}
w\left(\varphi\right)e^{T(\varphi)} &= \left[1+j\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)}\left[\left[\varphi e^{T(\varphi)}\right]^{\rho}-\left[\varphi_{x}^{j}e^{T\left(\varphi_{x}^{j}\right)}\right]^{\rho}-\int_{\varphi_{x}^{j}}^{\varphi}\frac{\rho}{\widetilde{\varphi}}\left[\widetilde{\varphi}e^{T(\widetilde{\varphi})}\right]^{\rho}d\widetilde{\varphi}\right] \\
&\quad +\left[1+(j-1)\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)}\left[\left[\varphi_{x}^{j}e^{T\left(\varphi_{x}^{j}\right)}\right]^{\rho}-\left[\varphi_{x}^{j-1}e^{T\left(\varphi_{x}^{j-1}\right)}\right]^{\rho}-\int_{\varphi_{x}^{j-1}}^{\varphi_{x}^{j}}\frac{\rho}{\widetilde{\varphi}}\left[\widetilde{\varphi}e^{T(\widetilde{\varphi})}\right]^{\rho}d\widetilde{\varphi}\right] \\
&\quad +...+ \\
&\quad +\left[\varphi_{x}^{1}e^{T\left(\varphi_{x}^{1}\right)}\right]^{\rho}-\left[\varphi^{*}e^{T(\varphi^{*})}\right]^{\rho}-\int_{\varphi^{*}}^{\varphi_{x}^{1}}\frac{\rho}{\widetilde{\varphi}}\left[\widetilde{\varphi}e^{T(\widetilde{\varphi})}\right]^{\rho}d\widetilde{\varphi}+ze^{T(\varphi^{*})},
\end{aligned}
$$

or

$$
\begin{aligned}
w\left(\varphi\right)e^{T(\varphi)} &= \left[1+j\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)}\left[\left[\varphi e^{T(\varphi)}\right]^{\rho}-\int_{\varphi_x^j}^{\varphi}\frac{\rho}{\widetilde{\varphi}}\left[\widetilde{\varphi}e^{T(\widetilde{\varphi})}\right]^{\rho}d\widetilde{\varphi}\right] \quad (45)\\
&\quad -\sum_{i=1}^{j}\left[\left[1+i\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)}-\left[1+(i-1)\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)}\right]\left[\varphi_x^i e^{T(\varphi_x^i)}\right]^{\rho}\\
&\quad -\sum_{i=2}^{j}\left[1+(i-1)\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)}\left[\int_{\varphi_x^{i-1}}^{\varphi_x^i}\frac{\rho}{\widetilde{\varphi}}\left[\widetilde{\varphi}e^{T(\widetilde{\varphi})}\right]^{\rho}d\widetilde{\varphi}\right]\\
&\quad -\left[\left[\varphi^* e^{T(\varphi^*)}\right]^{\rho}+\int_{\varphi^*}^{\varphi_x^1}\frac{\rho}{\widetilde{\varphi}}\left[\widetilde{\varphi}e^{T(\widetilde{\varphi})}\right]^{\rho}d\widetilde{\varphi}\right]+ze^{T(\varphi^*)}.
\end{aligned}
$$

Substituting $e^{T(\varphi)}=l\left(w\left(\varphi\right)\right)$, z by using the zero-cutoff condition (20) and $\left[\varphi_x^j l\left(w\left(\varphi_x^j\right)\right)\right]^{\rho}$ using the export cutoff condition (25) gives

$$
\begin{aligned}
w\left(\varphi\right)l\left(w\left(\varphi\right)\right) &= \left[1+j\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)}\left[\varphi l\left(w\left(\varphi\right)\right)\right]^{\rho}-jf_x\\
&\quad -\left[1+j\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)}\int_{\varphi_x^j}^{\varphi}\frac{\rho}{\widetilde{\varphi}}\left[\widetilde{\varphi}l\left(w\left(\widetilde{\varphi}\right)\right)\right]^{\rho}d\widetilde{\varphi}\\
&\quad -\sum_{i=2}^{j}\left[1+(i-1)\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)}\int_{\varphi_x^{i-1}}^{\varphi_x^i}\frac{\rho}{\widetilde{\varphi}}\left[\widetilde{\varphi}l\left(w\left(\widetilde{\varphi}\right)\right)\right]^{\rho}d\widetilde{\varphi}\\
&\quad -\int_{\varphi^*}^{\varphi_x^1}\frac{\rho}{\widetilde{\varphi}}\left[\widetilde{\varphi}l\left(w\left(\widetilde{\varphi}\right)\right)\right]^{\rho}d\widetilde{\varphi}-f.
\end{aligned}
$$

The wage equation (24) follows immediately by defining $\varphi=\varphi_x^{j+1}$ and $\varphi^*=\varphi_x^0$.

Average profits in an open economy

Rearranging the wage equation (24) implies that the profit of an exporting firm is given by

$$
\begin{aligned}
\delta\Pi_{d+j}\left(\varphi\right) &= \left[1+j\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)}\left[\varphi l\left(w\left(\varphi\right)\right)\right]^{\rho}-w\left(\varphi\right)l\left(w\left(\varphi\right)\right)-f-jf_x\\
&= \sum_{i=1}^{j+1}\left[1+(i-1)\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)}\int_{\varphi_x^{i-1}}^{\varphi_x^i}\frac{\rho}{\widetilde{\varphi}}\left[\widetilde{\varphi}l\left(w\left(\widetilde{\varphi}\right)\right)\right]^{\rho}d\widetilde{\varphi},
\end{aligned}
$$

where $\varphi_x^{j+1}=\varphi$ and $\varphi_x^0=\varphi^*$. Since free entry implies $f_e=\Pi_e\left(\varphi^*\right)=\int_{\varphi^*}^{\overline{\varphi}}\Pi\left(\varphi\right)\gamma\left(\varphi\right)d\varphi$, integrating over all firms with productivity $\varphi\in[\varphi^*,\overline{\varphi}]$ implies the free entry condition for an open economy as stated in equation (26).

Proof of Proposition 2: Upward rotation of the free entry condition

We need to show that for each $\varphi^*\in[0,\overline{\varphi})$ the number of active firms increases, i.e., $M_T>M_A$. Suppose the opposite, i.e., $M_T\leq M_A$. Thus, labor input for a firm with productivity φ is

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given by $l_T(w(\varphi)) \geq l_A(w(\varphi))$ according to equation (14). Since $\left[1 + (i-1)\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)} > 1$, it follows that $\Pi_e(\varphi^*)|_{Trade} > \Pi_e(\varphi^*)|_{Autarky} = f_e$. This contradicts, however, the free entry condition in an open economy. Thus, $M_T > M_A$. The increase in the number of active firms M increases the zero-cutoff productivity φ^*. It is easy to verify from equation (14) that the size of all firms $l(w(\varphi))$ decreases.

Appendix E, *Proof of Proposition 4:* Unemployment and wage dispersion in an open economy

It follows from equation (11) that the unemployment rate decreases as the number of active firms M increases in response to opening the economy for trade.

The increase in the number of active firms M and the increase in the zero-cutoff productivity φ^* in response to opening up to trade implies that wages increase if the position of a firm in the wage offer distribution, i.e., $F(w)$, is kept constant. This can be proven by using equation (24), i.e.,

$$
\begin{aligned}
w(\varphi) l(w(\varphi)) &= \left[1 + j\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)} \varphi^\rho l(w(\varphi))^\rho - f - jf_x \\
&\quad - \sum_{i=1}^{j+1} \left[1 + (i-1)\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)} \int_{\varphi_x^{i-1}}^{\varphi_x^i} \rho \left[\widetilde{\varphi} l(w(\widetilde{\varphi}))\right]^\rho d\widetilde{\varphi},
\end{aligned}
$$

where $\varphi_x^{j+1} = \varphi$ and $\varphi_x^0 = \varphi^*$. First, keep $l(w(\varphi))$ constant and notice that for a given labor input $l(w(\varphi))$ wages increase when opening up to trade, because $\left[1 + j\tau^{\rho/(\rho-1)}\right] > \left[1 + i\tau^{\rho/(\rho-1)}\right]$ for all $i < j$. Second, an increase in M leads to a lower labor size $l(w(\varphi))$. This reduces the wage cost, i.e., (lhs) $w(\varphi) l(w(\varphi))$, more than the revenues, i.e., (rhs) $\left[1 + j\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)} \varphi^\rho l(w(\varphi))^\rho$, as can be seen from the FOC, i.e.,

$$
\left[1 + j\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)} \rho\varphi^\rho \left[l(w(\varphi))\right]^\rho - w(\varphi) l(\varphi) = \frac{\partial w(\varphi)}{\partial l(\varphi)} l(\varphi)^2 > 0.
$$

Thus, the wage must increase since a decrease in labor input $l(w(\varphi))$ decreases the lhs more then the rhs. Third, an increase in φ^* while keeping $F(w)$ constant reduces the integral in the second line of the above equation and thereby increases the wage. Since the position of the wage distribution of the highest productivity firm, i.e., $F(w(\overline{\varphi})) = 1$, remains unchanged, it follows that the highest wage increases. Since the lowest wage equals the level

of unemployment benefit z in the closed and open economy, it follows that wage dispersion $w\left(\overline{\varphi}\right) - z$ is higher in an open economy than in a closed economy.

Appendix F: Vacancy creation condition for linear vacancy costs

The optimality condition for vacancies (30) for $\alpha = 1$ and wages (31) imply the equality of marginal costs, i.e.,

$$cv\left(\varphi\right)\frac{\dfrac{\partial l\left(\varphi\right)}{\partial w\left(\varphi\right)}\dfrac{\partial w\left(\varphi\right)}{\partial\varphi}}{l\left(\varphi\right)} = \frac{\partial w\left(\varphi\right)}{\partial\varphi}l\left(\varphi\right), \tag{46}$$

where we used $\partial l(\varphi, v)/\partial v = l(\varphi, v)/v$. Similar to Appendix A we define,

$$l\left(\varphi\right) = \eta v\left(\varphi\right)\left(\varkappa + \delta\right)e^{T(\varphi)},$$

where

$$T\left(\varphi\right) = -\log\left[\left[\varkappa + \delta + \eta M\int_{\varphi}^{\bar{\varphi}}\frac{v\left(\widetilde{\varphi}\right)}{1 - \Gamma\left(\varphi^{*}\right)}d\Gamma\left(\widetilde{\varphi}\right)\right]^{2}\right],$$

and

$$T'\left(\varphi\right) \equiv \frac{2\eta Mv(\varphi)\gamma(\varphi)}{\left[\varkappa + \delta + \eta M\int_{\varphi}^{\bar{\varphi}}\frac{v(\widetilde{\varphi})}{1 - \Gamma(\varphi^{*})}d\Gamma(\widetilde{\varphi})\right]} = \frac{\frac{\partial l(\varphi)}{\partial w(\varphi)}\frac{\partial w(\varphi)}{\partial\varphi}}{l(\varphi)}.$$

Equation (46) can therefore be written as

$$\frac{\partial w\left(\varphi\right)}{\partial\varphi} = c\frac{T'\left(\varphi\right)}{e^{T(\varphi)}}.$$

Integration gives

$$\begin{aligned} w\left(\varphi\right) &= c\int_{\varphi^{*}}^{\varphi}\frac{T'\left(\widetilde{\varphi}\right)}{e^{T(\widetilde{\varphi})}}d\widetilde{\varphi} + A \\ &= c\left[e^{-T\left(\varphi^{*}\right)} - e^{-T\left(\varphi\right)}\right] + A \\ &= c\left[\frac{v\left(\varphi^{*}\right)}{l\left(\varphi^{*}\right)} - \frac{v\left(\varphi\right)}{l\left(\varphi\right)}\right] + z, \end{aligned}$$

where $A = z$ follows from $w\left(\varphi^{*}\right) = z$. Substituting the wage $w\left(\varphi\right)$ into the optimality condition for vacancies (30) for $\alpha = 1$ gives the stated result.