

Structural Change, Urban Congestion,  
and the End of Growth

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# Structural Change, Urban Congestion, and the End of Growth

## Abstract

This paper develops a two-sector R&D-based growth model with congestion effects from increasing urban population density. We show that endogenous technological progress causes structural change if there are positive productivity spillovers from the modern to the traditional sector and Engel's law holds. In turn, urban congestion effects cause a productivity slowdown in the modern sector. Eventually, economic growth may cease in the long-run. We also show that land dilution from a higher workforce may give rise to negative scale effects on GDP per capita. Finally, we investigate how the optimal land allocation depends on the strength of urban congestion effects.

JEL-Code: O100, O300, O400.

Keywords: congestion, endogenous growth, Engel's law, structural change, urbanization.

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"Congestion is one of the single largest threats to our economic prosperity and way of life."

—U.S. Department of Transportation report released May 16, 2006

## 1 Introduction

According to United Nations projections, more than three-fifths of the world's population will live in urban areas by 2025. Urbanization gives rise to congestion effects like traffic jams, traffic accidents, crowded public transport, overcharged electricity networks, pollution, noise, crime, and communicable diseases.<sup>1</sup> For instance, Hartgen and Fields (2006) show that traffic congestion in the U.S. has grown significantly over the last decades and "if trends continue, by 2030 even small cities will be experiencing significant and noticeable congestion" (p.6). Urbanization is particularly rapid in fast-growing China and India, which causes huge problems in cities like Beijing, Shanghai, Dehli and Mumbai. For instance, exploding motorization is responsible for a surge in both traffic fatalities and air pollution. Average roadway speeds for motor vehicles substantially declined to often less than 10 km/h in central areas (Pucher et al., 2007).

Henderson (2003) provides empirical evidence that urbanization is a by-product rather than the cause of economic growth. He finds that there is an optimal degree of urbanization which maximizes productivity growth and too high an urban concentration can be very costly. A question which immediately arises from potentially severe urban congestion effects is whether productivity growth can be sustained in the long-run. Surprisingly, this question is largely under-researched in the literature on economic growth.

This paper develops a two-sector R&D-based growth model in which rising urban population density, associated with endogenous structural change, has adverse productivity effects. Structural change results from three basic features of the model: first, there is endogenous technological progress in the modern ("industrial") sector, characterized by imperfect competition and increasing returns. Second, productivity advances in the mod-

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<sup>1</sup>The WHO event World Health Day 2010 about 'Urbanization: a challenge for public health' stressed that communicable diseases like viral hepatitis, HIV/AIDS and tuberculosis are concentrated in urban areas. Moreover, cities are at particular risk of pandemic infectious diseases.

ern sector spill over to the traditional ("agricultural") sector.<sup>2</sup> Third, and consistent with a large body of empirical evidence (surveyed by Browning, 2008), the income elasticity of demand for the agricultural good is less than unity ("Engel's law"). The property is implied by the assumption that there is a subsistence level of consumption of the agricultural good.

We show that positive productivity spillovers from the modern to the traditional sector cause a reallocation of labor towards the modern sector when Engel's law holds. In turn, such structural change leads to congestion in the urban area and therefore to a productivity slowdown in the modern sector. Eventually, economic growth may cease in the long-run. The analysis thus identifies the conditions under which an inverse relationship of urban population density to the industrial productivity level implies severe consequences for long-run productivity growth. As these conditions are likely to hold, the analysis provides a pessimistic outlook for the future of economic growth.

We also address the long-standing debate in the endogenous growth literature on scale effects and show that these may not be positive. Positive scale effects are said to occur if an increase in the labor force either causes the growth rate or the level of per capita income to rise. The proposed framework belongs to the class of second-generation endogenous growth models with vertical innovations where scale effects with respect to the growth rate are removed.<sup>3</sup> Intermediate good firms can freely enter and the average quality of producer goods matters for growth-generating intertemporal R&D spillovers (Young, 1998). In standard versions of such a model, specialization gains from an increased number of firms (associated with higher population size) still cause scale effects in levels. However, at least in modern times, any kind of positive scale effect seems to be empirically refuted.<sup>4</sup> That scale effects are not necessarily positive in the model proposed in this paper naturally follows from the basic premise that land is a critical factor also for modern production.

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<sup>2</sup>Greenwald and Stiglitz (2006) develop a dynamic version of a Ricardo-trade model and base an infant-industry protection argument on the assumption that productivity advances are solely driven by the modern sector. They discuss that the modern sector is characterized by large, stable and geographically concentrated firms prone to innovation, whereas the traditional sector does not innovate but benefits from cross-sectoral spillovers.

<sup>3</sup>See Jones (1999, 2005) for discussions of the scale effect problem in endogenous growth theory.

<sup>4</sup>See Grossmann (2009) for a discussion of empirical evidence on scale effects. There I propose a model with sustained long-run growth where innovating entrepreneurs operate in perfect competition. Therefore, scale effects from specialization gains cannot arise in his framework.

Examples include access to railways, airports, rivers and roads at the location of plants as well as office space in cities. As land is a fixed factor, a larger labor force causes land dilution effects which may dominate specialization gains. Consequently, per capita income may decline, i.e., scale effects may be negative. One contribution of this paper is to conceptually separate land dilution effects from congestion effects and to show that they work independently from each other.

Finally, it is natural to investigate in a framework where scarcity of land is at the center of the analysis how the welfare-maximizing land allocation across production sectors depends on the strength of congestion effects.<sup>5</sup> The analysis suggests that a higher elasticity of industrial productivity with respect to urban population density implies that more land should be allocated to the urban area at the expense of rural land.

The paper is organized as follows. Section 2 discusses the related literature. The model is presented in section 3. Section 4 analyzes the equilibrium by distinguishing between congestion and dilution effects. In section 5, the optimal land allocation is derived. The last section concludes.

## 2 Related Literature

There is a large literature on structural change, stressing both demand and supply factors.<sup>6</sup> Our two-sector framework with non-homothetic preferences and endogenous growth may be most closely related to Matsuyama (1992). He stresses that an increase in agricultural productivity may squeeze out the manufacturing sector and therefore prevent learning-by-doing effects in an open economy. In a closed economy, by contrast, the opposite holds in his framework due to a reallocation of labor towards manufacturing. There are three key differences of our model to Matsuyama (1992). First, productivity gains in the industrial sector are driven by R&D rather than learning-by-doing effects. Second, agricultural productivity growth is linked to innovative activity in the modern sector rather than being exogenous. Third, we model urban congestion effects. Consequently, even in a

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<sup>5</sup>The land allocation may be influenced by government policy (by zoning laws, production permits, etc.).

<sup>6</sup>See Matsuyama (2008) for an overview.

closed economy an increase in agricultural productivity may be harmful for growth.

A more recent literature deals with models of non-balanced economic growth, which are, under certain conditions, consistent with the Kaldor facts in the aggregate. For instance, Kongsamut, Rebelo and Xie (2001), Föllmi and Zweimüller (2006) and Boppart (2011) develop growth frameworks where the income elasticity of demand differs across sectors. Acemoglu and Guerrieri (2008) allow for different capital intensities across sectors, whereas Ngai and Pissarides (2007) and Boppart (2011) propose models with different growth rates across sectors. The focus of these contributions on the Kaldor facts is rather different to our focus on growth slowdowns caused by urban congestion and land dilution effects.

With respect to the scale effects prediction, this paper is not the first one which suggests how positive scale effects can be entirely removed. Dalgaard and Kreiner (2001) and Strulik (2005, 2007) employ infinite-horizon growth models with ever increasing average human capital levels. They argue that faster population growth depresses the level of human capital per worker, similar to an increase in the depreciation rate of human capital. Their line of reasoning is thus different to the land dilution effects stressed here. Acemoglu and Johnson (2007) argue that higher life expectancy may lower per capita income in a Solow-type neoclassical growth model by lowering the land-labor ratio.<sup>7</sup> One contribution of the present paper is to show that a similar type of argument can remove scale effects in endogenous growth models with imperfect competition and specialization gains.

Finally, there is a literature on the role of congestion of public infrastructure for optimal linear income taxation in one-sector growth models (e.g. Barro and Sala-i-Martin, 1992; Glomm and Ravikumar, 1994, 1997; Turnovsky, 1997; Eicher and Turnovsky, 2000).<sup>8</sup> In contrast, this paper highlights the interaction between productivity growth, urban population density and structural change. This is not to deny that public infrastructure investment can mitigate urban congestion. However, this paper is based on the premise that ultimately land will be the limiting factor, whereas the previous growth literature

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<sup>7</sup>Their empirical evidence suggests that the causal effect of higher life expectancy on per capita income is negative, lending some support for land dilution effects investigated in this paper.

<sup>8</sup>See Irmen and Kuehnel (2008) for an assessment of this literature.

has assumed that public infrastructure capital can limit congestion indefinitely.<sup>9</sup> In order to focus on the implications of congestion of urban land for long-run growth, as a first step, this paper abstracts from public capital.

### 3 The Model

Consider the following Ricardo-Viner type two-sector model with one intersectorally mobile factor ("labor") and two immobile, fixed factors ("land"). Factor markets are competitive. Modern ("industrial") production is characterized by increasing returns and may suffer from congestion effects. The focus on non-accumulated immobile factors and congestion in the modern sector permits the interpretation of industrialized production taking place in the urban region. By contrast, traditional ("agricultural") production takes place in the rural region. As usual, the notion of traditional production is that there are many small, perfectly competitive firms with a constant-returns to scale technology. Goods can be costlessly transported between regions. Migration of labor across sectors is costless as well. Time is discrete and indexed by  $t = 1, 2, \dots$ . The time index is omitted whenever this does not lead to confusion.

#### 3.1 Individuals and Endowments

There are  $\bar{L}$  individuals who live one period and have one child each. They inelastically supply one unit of labor in the region they live. Moreover, each individual owns  $\bar{Z}_A/\bar{L}$  units of land located in the rural region and  $\bar{Z}_M/\bar{L}$  units of land in the urban region. That is, for simplicity, all individuals are identical in their endowments. Landholdings are passed to the offspring. With total land area  $\bar{Z}$  in the economy, we have  $\bar{Z}_A + \bar{Z}_M = \bar{Z}$ .

Each individual decides where to locate and chooses demand of a manufacturing good,  $c_M$ , and an agricultural good,  $c_A$ . Denote the rental rates of land for agricultural and manufacturing production by  $r_A$  and  $r_M$ , respectively. With analogous notation for wage rates, as labor is fully mobile, in equilibrium,  $w_A = w_M = w$  must hold. Thus, each

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<sup>9</sup>See De Haan, Romp and Sturm (2007) for some discussion on this point.

individual earns income

$$y = w + \frac{r_A \bar{Z}_A + r_M \bar{Z}_M}{\bar{L}}. \quad (1)$$

Preferences are represented by the Stone-Geary utility function

$$u(c_M, c_A) = (c_M)^\gamma (c_A - \bar{c})^{1-\gamma}, \quad (2)$$

$\gamma \in (0, 1)$ ,  $\bar{c} \geq 0$ . If there exists a subsistence consumption level of the agricultural good,  $\bar{c} > 0$ ,<sup>10</sup> consistent with Engel's law, the income elasticity of agricultural consumption is below unity whereas that of manufacturing consumption is above unity. To see this, denote by  $p_A$  the price of the agricultural good and normalize the price of the manufacturing good to unity,  $p_M = 1$ . Under utility function (2), individuals possess the following goods demand structure:

$$c_M = \gamma(y - p_A \bar{c}) \equiv \tilde{c}_M(p_A, y), \quad (3)$$

$$c_A = \frac{(1 - \gamma)y}{p_A} + \gamma \bar{c} \equiv \tilde{c}_A(p_A, y). \quad (4)$$

### 3.2 Technology

The industrial sector produces competitively in the urban region the manufacturing consumption good. It combines labor and differentiated intermediate inputs. Formally, output is given by

$$Y_M = (L_M)^{1-\alpha} \int_0^n (A_i)^{1-\alpha} (x_i)^\alpha di, \quad (5)$$

$0 < \alpha < 1$ , where  $L_M$  denotes labor input in manufacturing,  $x_i$  is the quantity of intermediate input  $i \in [0, n]$ , and  $A_i$  is a quality measure of  $i$ .

Production of an intermediate good requires a fixed number  $f > 0$  of workers each period (overhead staff). Fixed costs give rise to increasing returns and imperfect competition. Each intermediate good is produced by one monopolistically competitive firm.

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<sup>10</sup>We abstract from intertemporal decisions of consumers (e.g., on financial bequests and savings) for simplicity. However, reassuringly, Steger (2000) demonstrates in a one-sector model that subsistence consumption is an important ingredient to explain that saving rates rise with per capita income, inducing divergence in early development phases.



The mass ("number") of intermediate good firms,  $n$ , is endogenous and determined by free entry. One unit of output of an intermediate good requires one unit of urban land. That is, marginal costs are equal to the rental rate of land in the urban region,  $r_M$ .

Denote by  $\bar{A} \equiv \frac{1}{n} \int_0^n A_i di$  the average quality of intermediate goods, where initial level  $\bar{A}_0 > 0$  is given.  $\bar{A}$  may be interpreted as "knowledge stock" in the industrial sector. By employing  $l_{it}$  R&D workers in period  $t$  prior to production, intermediate good firm  $i$  can offer in period  $t$  product quality

$$A_{it} = g(\bar{A}_{t-1}, D_t)h(l_{it}), \quad (6)$$

where  $D$  is the urban population density, i.e., the number of workers in the urban region,  $\tilde{L}_M = L_M + \int_0^n (l_i + f) di$ , per unit of urban land:  $D = \tilde{L}_M / \bar{Z}_M$ . Like other aggregates,  $D$  is taken as given by firms. Function  $h$  is increasing and strictly concave with  $h(0) \geq 0$ . To ensure an interior solution for the optimal R&D choice, suppose "Inada conditions"  $\lim_{l \rightarrow \infty} h'(l) = 0$ ,  $\lim_{l \rightarrow 0} h'(l) \rightarrow \infty$  hold. Function  $g$  is increasing in  $\bar{A}$  which captures a standard "standing on shoulders effect" from access to previous knowledge ( $g_{\bar{A}} > 0$ ). Moreover, to capture congestion effects from urbanization, suppose that  $g$  is decreasing in urban population density ( $g_D < 0$ ).<sup>11</sup> Also suppose that  $g(0, D) = 0$  for all  $D < \bar{L} / \bar{Z}_M$ .

The agricultural sector is competitive. It produces by combining land and labor according to a constant-returns to scale technology. For simplicity, we focus on the Cobb-Douglas case. Output  $Y_A$  is given by

$$Y_A = B(Z_A)^\beta (L_A)^{1-\beta}, \quad (7)$$

where  $L_A$  is labor input in agriculture and  $Z_A$  is land input;  $B > 0$ .

Following Greenwald and Stiglitz (2006), there may be a cross-sectoral technological spillover effect from the manufacturing sector to the agricultural sector. The spillover takes place with a lag of one period. Formally, traditional sector's total factor productivity

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<sup>11</sup>One may argue that  $g_D \geq 0$  holds when population density  $D$  is below some threshold level, e.g. due to positive spillover (learning) effects among individuals. Suppose for simplicity that the threshold level is zero. This assumption does not affect the main insights of our analysis but avoids only mildly interesting case distinctions.

in period  $t$  is given by

$$B_t = b(\bar{A}_{t-1}), \quad (8)$$

$b' \geq 0$ . We will examine the implications of the case  $b' > 0$  vis-à-vis the case  $b' = 0$ .

We assume throughout that the subsistence level of agricultural consumption,  $\bar{c}$ , is smaller than agricultural output per worker in the case where all individuals work in the traditional sector. Formally,

$$\bar{c} < B \left( \frac{\bar{Z}_A}{\bar{L}} \right)^\beta. \quad (A1)$$

Assumption (A1) ensures the existence of an interior equilibrium.<sup>12</sup>

## 4 Equilibrium Analysis

Denote by  $p_i$  the price set by intermediate good firm  $i \in [0, n]$ . The equilibrium is defined as follows.

**Definition 1.** *An equilibrium is given by prices  $(p_A, r_M, r_A, w_M, w_A, \{p_i\}_{i \in [0, n]})$ , quantities  $(Y_M, Y_A, L_M, L_A, Z_A, \{l_i\}_{i \in [0, n]}, \{x_i\}_{i \in [0, n]})$ , quality levels  $\{A_i\}_{i \in [0, n]}$ , and a firm number  $n$  such that*

(i) *the final manufacturing goods sector, intermediate goods firms, and the agricultural sector maximize profits;*

(ii) *intermediate goods firms have zero profits (free entry condition);*

(iii) *the labor markets clears:  $\tilde{L}_M + L_A = \bar{L}$ , where  $\tilde{L}_M = L_M + \int_0^n (l_i + f) di$ ;*

(iv) *workers maximize utility; in particular, they are indifferent where to locate:  $w_M = w_A = w$ ;*

(v) *land markets clear in both regions:  $\int_0^n x_i di = \bar{Z}_M$ ,<sup>13</sup>  $Z_A = \bar{Z}_A$ ;*

(vi) *consumption goods markets clear:  $Y_M = \bar{L}\tilde{c}_M(p_A, y)$ ,  $Y_A = \bar{L}\tilde{c}_A(p_A, y)$ .*

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<sup>12</sup>Since for  $b' > 0$  productivity level  $B$  may change over time, we have to assume that (A1) holds for all  $t$ . As will become apparent, this is ensured if (A1) holds for  $t = 1$  and  $\bar{A}_0 < \bar{A}_1$ .

<sup>13</sup>Recall that one unit of output of an intermediate good requires one unit of urban land.

## 4.1 Urban Congestion Effects

Denote equilibrium values by superscript (\*). We find that the following holds.

**Lemma 1.** *There exists a symmetric and time-invariant equilibrium R&D labor input; i.e.,  $l_{it} = l^*$  and  $A_{it} = \bar{A}_t = g(\bar{A}_{t-1}, D_t)h(l^*)$  for all  $i \in [0, n_t]$  and  $t \geq 1$ .  $l^*$  is uniquely given by  $h(l^*) = h'(l^*)(l^* + f)$ .*

All proofs are relegated to the appendix. Lemma 1 is an implication of the ex-ante symmetry and free entry of intermediate goods firms. Consequently, R&D labor input per firm is independent of endowments; that is, it does neither depend on population size  $\bar{L}$  nor on land supply. The equilibrium number of intermediate goods firms increases proportionally to  $\bar{L}$  (see appendix), leaving  $l^*$  unaffected (e.g. Young, 1998).

**Proposition 1.** (Equilibrium labor allocation) *Define  $l_A \equiv L_A/\bar{L}$ .*

(a) *There exists an interior and unique equilibrium allocation of labor. The equilibrium fraction of labor in the traditional sector,  $l_A^*$ , is bounded away from zero.*

(b)  *$l_A^*$  is independent of urban land supply,  $\bar{Z}_M$ ; for a given agricultural productivity level,  $B$ ,  $l_A^*$  is also independent of the quality of intermediate goods,  $\bar{A}$ .*

(c) *If  $\bar{c} > 0$  (Engel's law holds),*

(i)  *$l_A^*$  is decreasing in rural land supply,  $\bar{Z}_A$ , and agricultural productivity,  $B$ ,*

(ii) *the impact of an increase in  $B$  on  $l_A^*$  is decreasing with  $B$  ( $\partial^2 l_A^* / \partial B^2 > 0$ ),*

(iii)  *$l_A^*$  is increasing in population size,  $\bar{L}$ .*

(d) *If  $\bar{c} = 0$  (homothetic preferences),  $l_A^*$  is independent of  $\bar{Z}_A$ ,  $B$  and  $\bar{L}$ .*

**Remark 1.** The alternative utility function  $u(c_M, c_A) = (c_M + \bar{c})^\gamma (c_A)^{1-\gamma}$ ,  $\bar{c} > 0$ , also implies that Engel's law holds. However, in that case, part (b) of Proposition 1 and the first part of (c) would be reversed; that is,  $l_A^*$  would be independent of agricultural productivity  $B$  and decreasing in the contemporaneous industrial knowledge stock  $\bar{A}$ . Thus, structural change would be induced by R&D-driven productivity advances in the industrial sector also if  $b' = 0$ . Utility specification (2) is employed to account for the plausible existence of a subsistence level of agricultural consumption.

That  $l_A^* \in (0, 1)$  holds at all times (part (a) of Proposition 1) is an implication of assumption (A1). Comparative-static results can be understood as follows. Consider first an increase in the contemporaneous knowledge stock  $\bar{A}$  or an increase in urban land supply,  $\bar{Z}_M$ , holding  $\bar{Z}_A$  constant. These changes have two counteracting effects on the incentive to work in manufacturing. On the one hand, the marginal productivity of labor in the manufacturing sector (wage rate  $w_M$ ) rises. On the other hand, for a given allocation of labor across sectors, manufacturing output goes up as. In turn, this reduces relative supply of the agricultural good and therefore raises its relative price  $p_A$ . In turn, the marginal productivity of labor in the agricultural sector (wage rate  $w_A$ ) rises as well. Both effects cancel out, such that  $w_M = w_A$  still holds (no migration of labor is induced), explaining part (b) of Proposition 1.

Regarding parts (c) and (d), an increase in rural land supply ( $\bar{Z}_A$ ) or in agricultural productivity ( $B$ ) have, analogously, counteracting effects. However, these cancel each other out if and only if  $\bar{c} = 0$  (homothetic preferences). First, the marginal productivity of labor in the agricultural sector,  $w_A$ , increases for a given relative goods price,  $p_A$ . Second, since output of the agricultural good increases, there is a negative effect on relative price  $p_A$ , leading to a decrease in  $w_A$ . If Engel's law holds ( $\bar{c} > 0$ ), then the second effect dominates the first one. Thus, in this case the incentive to work in the traditional sector is weakened.

The opposite holds when the economy's scale,  $\bar{L}$ , increases. In fact, for given  $\bar{Z}_A/\bar{L}$ , the equilibrium allocation of labor does not depend on  $\bar{L}$  (see proof of Proposition 1). Part (c) also suggests that the equilibrium fraction of labor in the traditional sector,  $l_A^*$ , approaches its lower bound in ever smaller "steps" as agricultural productivity increases ( $\partial^2 l_A^* / \partial B^2 > 0$ ).

Recall that, if  $b' > 0$ , a higher past industrial knowledge stock,  $\bar{A}_{t-1}$ , raises current agricultural productivity,  $B_t$ . Thus, the following result is implied by Proposition 1.

**Proposition 2.** (Structural change) *If  $b' > 0$  and  $\bar{c} > 0$ , an increase in the average intermediate good quality,  $\bar{A}_{t-1}$ , induces structural change, i.e., the equilibrium fraction of agricultural labor in period  $t$ ,  $l_{At}^*$ , declines. Otherwise (if  $b' = 0$  or  $\bar{c} = 0$ ), an increase in  $\bar{A}_{t-1}$  has no effect on the equilibrium allocation of labor.*

Proposition 2 suggests that R&D-related productivity progress in the manufacturing sector induces structural change and thus migration of labor in the urban area if and only if two conditions simultaneously hold: there is a subsistence level of consumption of the traditional good ( $\bar{c} > 0$ ) and there are cross-sectoral productivity spillovers ( $b' > 0$ ).

Using Proposition 1, let us write  $l_A^* = \tilde{l}(B, \bar{Z}_A, \bar{L})$  and recall that partial derivatives read  $\tilde{l}_B < (=)0$ ,  $\tilde{l}_{\bar{Z}_A} < (=)0$  and  $\tilde{l}_{\bar{L}} > (=)0$  if  $\bar{c} > (=)0$ . According to equilibrium condition (iii) in Definition 1, we also have  $\tilde{L}_M/\bar{L} = 1 - l_A$ . Recalling  $D = \tilde{L}_M/\bar{Z}_M$ ,  $\bar{Z}_M = \bar{Z} - \bar{Z}_A$  and  $B_t = b(\bar{A}_{t-1})$ , equilibrium urban population density in period  $t$  can therefore be written as

$$D_t^* = \frac{\left[1 - \tilde{l}(b(\bar{A}_{t-1}), \bar{Z}_A, \bar{L})\right] \bar{L}}{\bar{Z} - \bar{Z}_A}. \quad (9)$$

In equilibrium, the industrial knowledge stock thus evolves according to the first-order difference equation

$$\bar{A}_t = g \left( \bar{A}_{t-1}, \frac{\left[1 - \tilde{l}(b(\bar{A}_{t-1}), \bar{Z}_A, \bar{L})\right] \bar{L}}{\bar{Z} - \bar{Z}_A} \right) h(l^*) \equiv \Omega(\bar{A}_{t-1}; \bar{Z}_A, \bar{L}), \quad (10)$$

where Lemma 1 has been used. It is evident that, in the proposed simple model, all dynamics are driven by knowledge stock  $\bar{A}$ . It thus suffices to focus on (10) in the following. We have

$$\Omega_{\bar{A}} = \left[ g_{\bar{A}}(\bar{A}, D^*) - \frac{\bar{L} g_D(\bar{A}, D^*) \tilde{l}_B(b(\bar{A}), \bar{Z}_A, \bar{L}) b'(\bar{A})}{\bar{Z} - \bar{Z}_A} \right] h(l^*) \quad (11)$$

From this we obtain the following insights. In standard endogenous growth models without potential congestion ( $g_D = 0$ ) there is positive endogenous growth even in the long-run, if intertemporal knowledge spillovers are strong enough. A sufficient condition is that  $g_{\bar{A}} h(l^*) \geq 1$  for all  $\bar{A} \geq 0$ . A balanced growth equilibrium (in which all variables grow with a constant rate) exists if function  $g$  is linear in  $\bar{A}$ ; for instance, if  $A_{it} = \bar{A}_{t-1} h(l_{it})$ , then  $\bar{A}_t/\bar{A}_{t-1} = h(l^*)$  holds for all  $t \geq 1$ , i.e., the economy immediately jumps onto a balanced growth path. By contrast, if  $g_{\bar{A}\bar{A}} > 0$ , the growth rate of knowledge stock  $\bar{A}$  would be increasing over time. That is, growth would be "explosive" (e.g. Jones, 1999).

Finally, if  $g_{\bar{A}\bar{A}} < 0$  and eventually  $g_{\bar{A}}h(l^*) < 1$ , economic growth cannot be sustained in the long-run, even if there are no congestion effects.<sup>14</sup>

For the present model with congestion effects ( $g_D < 0$ ), we focus on the case which ensures positive long-run growth in standard models. The following key result of the paper emerges.

**Proposition 3.** (Evolution of the industrial knowledge stock) *Suppose that intertemporal spillovers are strong enough such that  $g_{\bar{A}}(\bar{A}, D^*)h(l^*) \geq 1$  for all  $\bar{A} \geq 0$ .*

(a) *If  $b' > 0$  and  $\bar{c} > 0$ , then economic growth may nevertheless cease in the long-run.*

(i) *In this case, adjustment to the steady state level of industrial knowledge stock, which is given by  $\bar{A}^* = \Omega(\bar{A}^*; \bar{Z}_A, \bar{L})$ , may be gradual or cyclical. It is also possible that  $\bar{A}^*$  is unstable.*

(ii) *Moreover, a decrease in rural land area  $\bar{Z}_A$  raises long-run level  $\bar{A}^*$ .*

(b) *If  $b' = 0$  or  $\bar{c} = 0$ , then the growth rate of the industrial knowledge stock is always positive.*

Proposition 3 suggests that the prospects for sustained economic growth are slim when structural change causes congestion effects. One should stress the role of cross-sectoral technology spillovers ( $b' > 0$ ) for the relationship between potential urban congestion effects and long-run economic growth in the model. Without positive R&D externalities from the industrial sector to the traditional sector ( $b' = 0$ ), advances in knowledge stock  $\bar{A}$  do not cause structural change, and therefore do not foster urban congestion, even if Engel's law holds (Proposition 2). Hence, if  $\bar{c} > 0$ , there arises the possibility that, all other things being equal, long-run growth is sustained in the case where  $b' = 0$  but not in the case where  $b' > 0$ . In other words, we obtain the – at the first glance somewhat counterintuitive – insight that positive productivity externalities in favor of the traditional sector may contribute to the end of economic growth in the long-run.

If preferences are homothetic ( $\bar{c} = 0$ ), R&D-driven productivity advances in the modern sector do not cause structural change (again, recall Proposition 2). Hence, there

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<sup>14</sup>For instance, Jones (1995) has introduced an underproportional intertemporal technology spillover effect in the horizontal innovation framework of Romer (1990), showing that long-run economic growth can only be sustained if there is positive population growth.

is positive long-run growth under the presumption of Proposition 3, irrespective of the strength of potential congestion effects.

Fig. 1 shows the two kinds of possible transitional dynamics for  $\bar{A}_0 < \bar{A}^*$  in the case where the long-run knowledge stock  $\bar{A}^*$  is a (locally) stable steady state level (such that  $|\Omega_{\bar{A}}| < 1$  at  $\bar{A}^*$ ). In panel (a), the adjustment to  $\bar{A}^*$  is gradual, whereas it is cyclical in panel (b) locally around the steady state.

It is not ensured that  $\Omega(\bar{A}, \cdot)$  is concave as a function of  $\bar{A}$ . This means that also multiple interior and stable steady states are possible. This is illustrated in panel (a) of Fig. 1, where both  $\bar{A}_1^*$  and  $\bar{A}_2^*$  are stable. In this case, the long-run position of the economy depends on initial condition  $\bar{A}_0$ .

Also note from (10) and Proposition 1 that assigning more land from the rural to the urban area (decrease in  $\bar{Z}_A$ ) shifts up the  $\Omega$ -curve in Fig. 1, by lowering the urban population density in equilibrium,  $D^*$ . Consequently, if stable, the steady state industrial knowledge stock rises. This is shown in panel (b) of Fig. 1, where a decrease in  $\bar{Z}_A$  from  $\bar{Z}_A^0$  to  $\bar{Z}_A^1 < \bar{Z}_A^0$  causes an increase in  $\bar{A}^*$  from  $\bar{A}_0^*$  to  $\bar{A}_1^*$ .

## 4.2 Dilution and Scale Effects

The previous subsection has shown that for R&D in the industrial sector to cause structural change both positive subsistence consumption of the agricultural good ( $\bar{c} > 0$ ) and cross-sectoral technology spillovers ( $b' > 0$ ) must be present; in this case, congestion effects in the urban region arise.

In this subsection, we emphasize the consequences of dilution effects from higher scale  $\bar{L}$  on the per capita income level,  $y$ , when urban land is an important factor for modern production. To distinguish the analysis from the previous subsection, and for the sake of simplicity, we now focus on the case of homothetic preferences ( $\bar{c} = 0$ ). The next result shows that scale effects may even be negative.

**Proposition 4.** (Scale effects) *Suppose that  $\bar{c} = 0$ . For given  $\bar{A}_{t-1}$ , an increase in population size  $\bar{L}$  causes a decline in per capita income  $y_t$  if  $\alpha \geq 0.5$ .*

The intuition can be seen as follows. Note that, since intermediate good firms are





symmetric and thus choose the same amount of land in the urban area as input, we have  $x_i = \bar{Z}_M/n$  for all  $i$ . Using this in (5), we find that the per capita level of manufacturing output reads

$$\frac{Y_M}{\bar{L}} = \left(\frac{\bar{Z}_M}{\bar{L}}\right)^\alpha \left(n\bar{A}\frac{L_M}{\bar{L}}\right)^{1-\alpha}. \quad (12)$$

As an implication of Proposition 1, the fraction of labor allocated to produce the manufacturing consumption good,  $L_M/\bar{L}$ , is independent of scale  $\bar{L}$  in equilibrium (if  $\bar{c} = 0$ ). Now, for a given knowledge stock  $\bar{A}$ , an increase in scale  $\bar{L}$  has two counteracting effects on  $Y_M/\bar{L}$ . First, the number of intermediate good firms,  $n$ , increases in equilibrium. In turn, due to specialization gains, equilibrium manufacturing output per capita increases. Second, however, the urban land input per head,  $\bar{Z}_M/\bar{L}$ , declines – a dilution effect with respect to a fixed production factor. If the output elasticity of urban land,  $\alpha$ , is high, then the second effect dominates the first one. The intuition applies for per capita income  $y$  as well. If there were no congestion effects from higher urban population density, then both effects would exactly cancel for  $\alpha = 0.5$ ; that is, when  $g_D = 0$ , per capita income  $y$  would decrease (increase) in  $\bar{L}$  if  $\alpha > (<)0.5$ . With congestion effects, the scale effect on per capita income may be negative even for  $\alpha < 0.5$ .

## 5 Optimal Land Allocation

Governments may be able to affect the allocation of land use by zoning laws and production permits. For instance, the government can extend the urban area at expense of the rural area by allowing industrial firms to locate near cities. It is interesting to study the optimal (welfare-maximizing) allocation of land in an economy where land scarcity and congestion effects are at the center of the analysis. Welfare is given by utility (2), which in equilibrium is the same for all individuals.

Let us again focus for simplicity on homothetic preferences,  $\bar{c} = 0$ .<sup>15</sup> Moreover, to

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<sup>15</sup>For  $\bar{c} > 0$  and  $b' > 0$  a change in the allocation of land use would change the allocation of labor (recall part (c) of Proposition 1 that, in this case,  $\partial l_A^*/\partial \bar{Z}_A < 0$ ). The implication for the optimal land use, compared to the case where  $\bar{c} = 0$ , is ambiguous and no further insight is gained.

obtain easily interpretable results, consider the following specific functional form for  $g$ :

$$g(A, D) = \frac{A^\phi}{D^\theta}, \quad (13)$$

$\phi, \theta > 0$ . That is, the elasticity of product quality of an intermediate goods firm with respect to urban population density is constant:  $-Dg_D/g = \theta$ . Parameter  $\theta$  may thus be viewed as the strength of urban congestion effects. Let  $z_M \equiv \bar{Z}_M/\bar{Z}$ . The optimal land allocation can be characterized as follows.

**Proposition 5.** (Optimal land allocation) *Suppose that  $\bar{c} = 0$  and  $g$  is given by (13). The optimal fraction of land allocated to urban use,  $z_M^{opt}$ , is time-invariant and given by*

$$z_M^{opt} = \frac{\alpha + (1 - \alpha)\theta}{\alpha + (1 - \alpha)\theta + \frac{(1-\gamma)\beta}{\gamma}} \in (0, 1); \quad (14)$$

thus,  $z_M^{opt}$  is increasing in the strength of urban congestion effects,  $\theta$ .

The comparative-static result in Proposition 5 is intuitive. The stronger are urban congestion effects on productivity, the higher is the fraction of land which should be allocated to the industrial area. We also see from (14) that  $z_M^{opt}$  rises with the output elasticity of urban land for the modern sector,  $\alpha$  (see (12)), declines with the output elasticity of rural land for the traditional sector,  $\beta$ , and increases if the manufacturing consumption good is more valuable to consumers (higher  $\gamma$ ).

It is remarkable that the optimal allocation of land is time-invariant; that is,  $z_M^{opt}$  does neither depend on the stage of economic development nor on the long-run properties of productivity growth. The result is an implication of the fact that, under specification (13), elasticity  $-Dg_D(\bar{A}, D)/g(\bar{A}, D)$  does not depend on the stock of knowledge,  $\bar{A}$ .

Using (13) in (10), we find that the knowledge stock evolves according to

$$\bar{A}_t = (\bar{A}_{t-1})^\phi \left( \frac{\bar{Z} - \bar{Z}_A}{(1 - l_{At}^*)\bar{L}} \right)^\theta h(l^*), \quad (15)$$

where the agricultural labor share in equilibrium of period  $t$ ,  $l_{At}^*$ , is independent of  $\bar{A}_t$  and  $B_t = b(\bar{A}_{t-1})$  for  $\bar{c} = 0$  (part (b) and (d) of Proposition 1, respectively). Thus, growth

ceases in the long run if  $\phi < 1$ . Moreover, there exists a balanced growth equilibrium if  $\phi = 1$ ,<sup>16</sup> whereas growth is explosive if  $\phi > 1$ .<sup>17</sup>

In the case where  $\phi < 1$ , the long-run equilibrium knowledge stock is given by<sup>18</sup>

$$\bar{A}^* = \left( \frac{z_M}{1 - l_A^*} \frac{\bar{Z}}{\bar{L}} \right)^{\frac{\theta}{1-\phi}} h(l^*)^{\frac{1}{1-\phi}}. \quad (16)$$

We thus see that  $\bar{A}^*$  may not necessarily decrease in the strength of urban congestion effects,  $\theta$ . We can also derive an optimal long-run knowledge stock, which we obtain by evaluating the right-hand side of (16) at  $z_M = z_M^{opt}$ . Since  $\bar{A}^*$  is rising in the fraction of urban land,  $z_M$ , and  $\partial z_M^{opt} / \partial \theta > 0$  (Proposition 5), we also find that the optimal steady state knowledge stock may increase in the strength of congestion effects,  $\theta$ .

## 6 Conclusion

This paper has examined the growth implications of urban congestion effects from endogenous structural change in a R&D-based growth framework with non-homothetic preferences and cross-sectoral technology spillovers. The analysis has demonstrated that there may be congestion-related limits to both urbanization and long-run economic growth. Even in the case where intertemporal knowledge spillover effects are strong, urban congestion associated with structural change may leave economic growth unsustainable in the long-run. In the model, structural change was driven by Engel's law together with R&D-driven productivity advances which spill over to the traditional sector. Paradoxically, prospects of sustained long-run growth are mitigated by cross-sectoral productivity spillovers in the proposed framework.

Moreover, the analysis has addressed the long-standing debate on scale effects in the

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<sup>16</sup>If  $\phi = 1$ , it could be the case that the long-run growth rate of the economy is positive if  $\bar{c} = 0$  but zero if  $\bar{c} > 0$  (see Proposition 3).

<sup>17</sup>Thus, with respect to the role of intertemporal knowledge spillovers, for  $\bar{c} = 0$ , we obtain long-run properties which are similar to Jones (1999). Jones discusses the "knife-edge property" of the case of a linear knowledge spillover. In fact, if  $\bar{c} = 0$ , under (13) and in absence of population growth,  $\phi = 1$  is necessary for a balanced growth path with sustained positive growth to exist also in our model with congestion effects.

<sup>18</sup>Use both  $\bar{Z}_M = \bar{Z} - \bar{Z}_A$  and  $z_M = \bar{Z}_M / \bar{Z}$  in (15) and set  $\bar{A}_t = \bar{A}_{t-1} = \bar{A}^*$  to derive (16).

endogenous growth literature. We have shown that, due to land dilution effects, the impact of an increase in population size on per capita income may be negative. Finally, the analysis suggests that the fraction of urban land should rise with the strength of urban congestion effects.

Future research should incorporate public infrastructure investment, which potentially mitigates urban congestion effects, into the model. This would allow us to look more closely at transitional dynamics. Such an extension would also enable us to investigate how the optimal path of productive public investment interacts with urban population density which therefore could provide useful policy recommendations.

## Appendix

**Proof of Lemma 1:** In the modern sector, the inverse demand schedule for intermediate good  $i$  is given by its marginal product  $p_i = \alpha(A_i L_M / x_i)^{1-\alpha} \equiv P(x_i)$ . (Recall that  $p_M = 1$ .) Monopoly profits of each firm  $i$  are given by

$$\pi_i = (p_i - r_M)x_i - w_M(l_i + f). \quad (17)$$

Profit-maximizing price-setting, when accounting for demand schedule  $p_i = P(x_i)$ , leads to mark-up factor  $1/\alpha$ . Thus,

$$x_i = \left( \frac{\alpha^2}{r_M} \right)^{\frac{1}{1-\alpha}} A_i L_M. \quad (18)$$

Using  $p_i = r_M/\alpha$ , (18) and R&D technology (6) in (17), profits of firm  $i$  in  $t$  are given by

$$\pi_{it} = (1 - \alpha) \alpha^{\frac{1+\alpha}{1-\alpha}} (r_{Mt})^{-\frac{\alpha}{1-\alpha}} \underbrace{g(\bar{A}_{t-1}, D_t) h(l_{it})}_{=A_{it}} L_{Mt} - w_{Mt}(l_{it} + f). \quad (19)$$

Now consider the R&D decision of intermediate good firms. Maximizing profits  $\pi_i$  in

(19) with respect to  $l_i$  and observing (6) yields first-order condition

$$(1 - \alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}(r_M)^{-\frac{\alpha}{1-\alpha}} A_i \frac{h'(l_i)}{h(l_i)} L_M = w_M. \quad (20)$$

Moreover, from free entry equilibrium condition (ii) in Definition 1,  $\pi_i = 0$ . Using (19) and (20), this implies that each firm  $i$  chooses a time-invariant R&D input as given by

$$1 = \frac{h'(l^*)}{h(l^*)}(l^* + f) \equiv Q(l^*, f). \quad (21)$$

Note that  $\partial Q(l^*, f)/\partial l = h''(l^*)/h'(l^*) < 0$  and Inada conditions were assumed to hold. Thus,  $l^*$  exists and is unique. ■

**Proof of Proposition 1:** The urban wage rate is given by the marginal product of labor in manufacturing,  $w_M = (1 - \alpha)Y_M/L_M$ . Substituting (18) into (5) and using the resulting expression for  $Y_M$  leads to

$$w_M = (1 - \alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(r_M)^{-\frac{\alpha}{1-\alpha}} n\bar{A}. \quad (22)$$

Now combine (20) and (22), and then use  $A_i = \bar{A}$  and (21), to find that the number of intermediate good firms is given by

$$n = \frac{\alpha L_M}{l^* + f}; \quad (23)$$

therefore,  $n$  is proportional to manufacturing labor input  $L_M$ . Thus,  $\tilde{L}_M = L_M + n(l^* + f)$  is given by

$$\tilde{L}_M = (1 + \alpha)L_M. \quad (24)$$

According to (7), the marginal productivity of agricultural labor is given by

$$w_A = Bp_A(1 - \beta) \left( \frac{\bar{Z}_A}{L_A} \right)^\beta, \quad (25)$$

where we used equilibrium condition  $Z_A = \bar{Z}_A$ . Moreover, since intermediate good firms are symmetric, equilibrium condition (v) implies that  $x_i = \bar{Z}_M/n$  for all  $i$ . Thus, using

(5), we have

$$Y_M = (\bar{Z}_M)^\alpha (n\bar{A}L_M)^{1-\alpha}. \quad (26)$$

Hence, wage rate  $w_M = (1 - \alpha)Y_M/L_M$  can be written as

$$w_M = (1 - \alpha) \left( \frac{\bar{Z}_M}{L_M} \right)^\alpha (n\bar{A})^{1-\alpha}. \quad (27)$$

Using (25), (27) and equilibrium condition  $w_M = w_A$  yields

$$p_A = \frac{(1 - \alpha) \left( \frac{\bar{Z}_M}{L_M} \right)^\alpha (n\bar{A})^{1-\alpha}}{B(1 - \beta) \left( \frac{\bar{Z}_A}{L_A} \right)^\beta}. \quad (28)$$

Using (3) and (4) in goods market clearing conditions (vi) and combining them by eliminating  $y$ , we find that

$$p_A Y_A = \frac{1 - \gamma}{\gamma} Y_M + \bar{L}\bar{c}p_A. \quad (29)$$

Substituting (7) and (26) into (29), and using  $Z_A = \bar{Z}_A$ , we obtain

$$p_A = \frac{1 - \gamma}{\gamma} \frac{(\bar{Z}_M)^\alpha (n\bar{A}L_M)^{1-\alpha}}{B(\bar{Z}_A)^\beta (L_A)^{1-\beta} - \bar{L}\bar{c}}. \quad (30)$$

Next note that substituting (24) into labor market clearing condition,  $\tilde{L}_M + L_A = \bar{L}$ , implies that

$$l_M \equiv \frac{L_M}{\bar{L}} = \frac{1 - l_A}{1 + \alpha}. \quad (31)$$

By combining (28) and (30), and using (31), we find that  $l_A^*$  is implicitly given by

$$0 = 1 - l_A^* - \chi (l_A^*)^\beta \left( (l_A^*)^{1-\beta} - \frac{\bar{c}}{B(\bar{Z}_A/\bar{L})^\beta} \right), \quad (32)$$

$\chi \equiv \frac{\gamma(1-\alpha^2)}{(1-\beta)(1-\gamma)} > 0$ . Rewriting (32) to

$$l_A^* = \frac{1 + \frac{\chi\bar{c}(l_A^*)^\beta}{B(\bar{Z}_A/\bar{L})^\beta}}{1 + \chi}, \quad (33)$$

we find that there always exists a unique, positive and finite level for  $l_A^*$  which solves (33). This level is increasing in  $\frac{\bar{c}}{B(\bar{Z}_A/\bar{L})^\beta}$  and is equal to one if  $\bar{c} = B(\bar{Z}_A/\bar{L})^\beta$ . Hence, under (A1), we have  $l_A^* < 1$  and therefore

$$(l_A^*)^{1-\beta} > \frac{\bar{c}}{B(\bar{Z}_A/\bar{L})^\beta}, \quad (34)$$

according to (32). Also note from (33) that  $l_A^* \geq (1+\chi)^{-1} > 0$ . This confirms part (a) of Proposition 1. The other results can be proven by applying the implicit function theorem to (32). For instance,

$$\frac{\partial l_A^*}{\partial B} = -\frac{\bar{c}\chi\left(\frac{\bar{L}}{\bar{Z}_A}\right)^\beta (l_A^*)^\beta B^{-2}}{1 + \chi(l_A^*)^{\beta-1} \left( (l_A^*)^{1-\beta} - \frac{\bar{c}}{B(\bar{Z}_A/\bar{L})^\beta} \right) + \chi(1-\beta)}. \quad (35)$$

Observing (34) confirms that  $\partial l_A^*/\partial B < (=)0$  if  $\bar{c} > (=)0$ . Using (35), one can also show that  $\partial^2 l_A^*/\partial B^2 > 0$  if and only if

$$\bar{c} \left[ \beta \frac{\partial l_A^*}{\partial B} - \frac{2l_A^*}{B} + \chi\beta(l_A^*)^{\beta-1} \frac{\partial l_A^*}{\partial B} \left( (l_A^*)^{1-\beta} - \frac{\bar{c}\bar{L}^\beta}{B(\bar{Z}_A)^\beta} \right) - \frac{\chi(l_A^*)^\beta}{B} \left( 2(l_A^*)^{1-\beta} - \frac{\beta\bar{c}\bar{L}^\beta}{B(\bar{Z}_A)^\beta} \right) \right] < 0. \quad (36)$$

Using  $\bar{c} > 0$ ,  $\partial l_A^*/\partial B < 0$  and (34), we find that (36) is fulfilled. The remainder of the proof is obvious. ■

**Proof of Proposition 2:** Immediately follows from the impact of an increase in  $B$  on  $l_A^*$  (parts (c) and (d) of Proposition 1) and  $dB_t/dA_{t-1} > (=)0$  if  $b' > (=)0$ . ■

**Proof of Proposition 3:** According to (11),  $\Omega_{\bar{A}} < 1$  is possible despite  $g_{\bar{A}}(\bar{A}, \cdot)h(l^*) \geq 1$  for all  $\bar{A} \geq 0$  if and only if  $\tilde{l}_B < 0$  and  $b' > 0$ ; we have  $\tilde{l}_B < 0$  if and only if  $\bar{c} > 0$  and  $b' > 0$  (Proposition 1). In this case, it is thus possible that the  $\Omega$ -curve as a function of  $\bar{A}$  crosses the 45-degree line (see Fig. 1). If it does, it may be the case that  $\Omega_{\bar{A}}(\bar{A}^*, \cdot) \in (0, 1)$ ,  $\Omega_{\bar{A}}(\bar{A}^*, \cdot) \in (-1, 0)$  or  $\Omega_{\bar{A}}(\bar{A}^*, \cdot) < -1$ , corresponding to gradual, cyclical or no adjustment to steady state level  $\bar{A}^*$  over time, respectively. ■

**Proof of Proposition 4:** According to (18), land demand in manufacturing,  $Z_M =$

$\int_0^n x_i di$ , is given by

$$Z_M = \alpha^{\frac{2}{1-\alpha}} (r_M)^{-\frac{1}{1-\alpha}} \bar{A} n L_M. \quad (37)$$

Since  $Z_M = \bar{Z}_M$  in equilibrium, we find that

$$r_M = \alpha^2 \left( \frac{\bar{A} n L_M}{\bar{Z}_M} \right)^{1-\alpha}. \quad (38)$$

The marginal product of rural land is given by

$$r_A = B p_A \beta \left( \frac{\bar{Z}_A}{L_A} \right)^{-(1-\beta)}. \quad (39)$$

Now substitute  $w = w_A$  from (25),  $r_M$  from (38) and  $r_A$  from (39) into the expression for  $y$  in (1) and use the expression for  $p_A$  in (28) to find that

$$y = (1 - \alpha) \bar{L}^{-\alpha} (\bar{A} n)^{1-\alpha} \left( \frac{\bar{Z}_M}{l_M} \right)^\alpha \left[ 1 + \frac{\beta l_A}{1 - \beta} + \frac{\alpha^2 l_M}{1 - \alpha} \right]. \quad (40)$$

Recall that

$$\bar{A}_t = g \left( \bar{A}_{t-1}, \frac{(1 - l_A) \bar{L}}{\bar{Z}_M} \right) h(l^*), \quad (41)$$

$$n = \frac{\alpha l_M \bar{L}}{l^* + f}, \quad (42)$$

according to (10) and (23), respectively. In equilibrium, neither  $l_A$  nor  $l_M$  depend on  $\bar{L}$  if  $\bar{c} = 0$ , according to part (d) of Proposition 1 and (31). According to Lemma 1, also  $l^*$  is independent of  $\bar{L}$ . Thus, for given  $\bar{A}_t$ , (40) implies that  $y_t$  is proportional to  $\bar{L}^{1-2\alpha}$ . Since  $g_D < 0$ , (41) implies that  $\bar{A}_t$  is decreasing in  $\bar{L}$ , holding  $\bar{A}_{t-1}$  constant. This concludes the proof. ■

**Proof of Proposition 5:** First, use (2)-(4) to find that indirect utility is given by

$$v(p_A, y) = \gamma^\gamma (1 - \gamma)^{1-\gamma} (p_A)^{-(1-\gamma)} (y - p_A \bar{c}). \quad (43)$$

Let us denote welfare in equilibrium by  $W$ . Substituting both the expression for  $p_A$  in



(28) and the expression for  $y$  in (40) into (43) yields, for  $\bar{c} = 0$ , equilibrium welfare

$$W = \delta \left[ 1 + \frac{\beta l_A}{1 - \beta} + \frac{\alpha^2 l_M}{1 - \alpha} \right] B^{1-\gamma} \bar{L}^{-\alpha\gamma - \beta(1-\gamma)} (\bar{A}n)^{\gamma(1-\alpha)} \left( \frac{\bar{Z}_A}{l_A} \right)^{\beta(1-\gamma)} \left( \frac{\bar{Z}_M}{l_M} \right)^{\alpha\gamma}, \quad (44)$$

$\delta \equiv (1 - \alpha)^\gamma (1 - \beta)^{1-\gamma} \gamma^\gamma (1 - \gamma)^{1-\gamma}$ . Now use both (41) and (42) and recall from part (d) of Proposition 1 and Lemma 1 that, for  $\bar{c} = 0$ , equilibrium values of  $l_A$ ,  $l_M$  and  $l$  do not depend on land endowments. Thus, with  $\bar{Z}_A = \bar{Z} - \bar{Z}_M$ , the optimal allocation of land use in period  $t$  solves

$$\max_{\bar{Z}_{Mt}} g \left( \bar{A}_{t-1}, \frac{(1 - l_A^*) \bar{L}}{\bar{Z}_{Mt}} \right)^{\gamma(1-\alpha)} (\bar{Z} - \bar{Z}_{Mt})^{\beta(1-\gamma)} (\bar{Z}_{Mt})^{\alpha\gamma}. \quad (45)$$

Using the first-order condition to maximization problem (45) and observing  $-Dg_D/g = \theta$  confirms (14). ■

## References

- [1] Acemoglu, Daron and Simon Johnson (2007). Disease and Development: The Effect of Life Expectancy on Economic Growth, *Journal of Political Economy* 115, 925-985.
- [2] Acemoglu, Daron and Veronica Guerrieri (2008). Capital Deepening and Nonbalanced Economic Growth, *Journal of Political Economy* 116, 467-497.
- [3] Barro, Robert J. and Xavier Sala-I-Martin (1992). Public Finance in Models of Economic Growth, *Review of Economic Studies* 59, 645-661.
- [4] Boppart, Timo (2010). Engel's Law and Growth with Directed Technical Change, Paper presented at the conference on Dynamics, Economic Growth, and International Trade (DEGIT XV), Frankfurt am Main, Germany.
- [5] Boppart, Timo (2011). Structural Change, Relative Price Effects and the Kaldor Facts in a Growth Model with Non-Gorman Preferences, University of Zurich, mimeo.

- [6] Browning, Martin (2008). Engel's Law, in: Steven N. Durlauf and Lawrence E. Blume (eds.), *The New Palgrave Dictionary of Economics*, Second Edition, Palgrave Macmillan.
- [7] Dalgaard, Carl-Johan and Claus T. Kreiner (2001). Is Declining Productivity Inevitable?, *Journal of Economic Growth* 6, 187-203.
- [8] De Haan, Jakob, Ward Romp and Jan-Egbert Sturm (2007). Public Capital and Economic Growth: Key Issues for Europe, *Perspektiven der Wirtschaftspolitik* 8, 6-52.
- [9] Eicher, Theo and Stephen J. Turnovsky (2000). Scale, Congestion and Growth, *Economica* 67, 325-346.
- [10] Föllmi, Reto and Josef Zweimüller (2008). Structural Change, Engel's Consumption Cycles and Kaldor's Facts of Economic Growth, *Journal of Monetary Economics* 55, 1317-1328.
- [11] Glomm, Gerhard and B. Ravikumar (1994). Public Investment in Infrastructure in a Simple Growth Model, *Journal of Economic Dynamics and Control* 18, 1173-1187.
- [12] Glomm, Gerhard and B. Ravikumar (1997). Productive Government Expenditure and Long-Run Growth, *Journal of Economic Dynamics and Control* 21, 183-204.
- [13] Greenwald, Bruce and Joseph E. Stiglitz (2006). Helping Infant Economies Grow: Foundations of Trade Policies for Developing Countries, *American Economic Review Papers and Proceedings* 96, 141-146.
- [14] Grossmann, Volker (2009). Entrepreneurial Innovation and Economic Growth, *Journal of Macroeconomics* 31, 602-613.
- [15] Hartgen, David T. and M. Gregory Fields (2006). Building Roads to Reduce Traffic Congestion in America's Cities: How Much and at What Cost?, Reason Foundation Report.

- [16] Henderson, Vernon (2003). The Urbanization Process and Economic Growth: The So-what Question, *Journal of Economic Growth* 8, 47-71.
- [17] Irmen, Andreas and Johanna Kuehnel (2008). Productive Government Expenditure and Economic Growth, *Journal of Economic Surveys* 23, 692-733.
- [18] Jones, Charles I. (1995). R&D-based Models of Economic Growth, *Journal of Political Economy* 103, 759-784.
- [19] Jones, Charles I. (1999). Growth: With or Without Scale Effects, *American Economic Review Papers and Proceedings* 89, 139-144.
- [20] Jones, Charles I. (2005). Growth and Ideas, in: P. Aghion and S. Durlauf (eds.), *Handbook of Economic Growth*, North-Holland, Amsterdam.
- [21] Kongsamut, Piyabha, Sergio Rebelo and Danyang Xie (2001). Beyond Balanced Growth, *Review of Economic Studies* 68, 869-882.
- [22] Matsuyama, Kiminori (1992). Agricultural Productivity, Comparative Advantage, and Economic Growth, *Journal of Economic Theory* 58, 317-334.
- [23] Matsuyama, Kiminori (2008). Structural Change, in: Steven N. Durlauf and Lawrence E. Blume (eds.), *The New Palgrave Dictionary of Economics*, Second Edition, Palgrave Macmillan.
- [24] Ngai, L. Rachel and Christopher A. Pissarides (2007). Structural Change in a Multi-sector Model of Growth, *American Economic Review* 97, 429-443.
- [25] Pucher, John, Zhong-Ren Peng, Neha Mittal, Yi Zhu and Nisha Korattyswaroopam (2007). Urban Transport and Policies in China and India: Impacts of Rapid Economic Growth, *Transport Reviews* 27, 379-410.
- [26] Romer, Paul M. (1990). Endogenous Technological Change, *Journal of Political Economy* 98, 71-101.
- [27] Steger, Thomas M. (2000). Economic Growth with Subsistence Consumption, *Journal of Development Economics* 62, 343-361.

- [28] Strulik, Holger (2005). The Role of Human Capital and Population Growth in R&D-based Models of Endogenous Growth, *Review of International Economics* 13, 129-145.
- [29] Strulik, Holger (2007). Too Much of a Good Thing? The Quantitative Economics of R&D-driven Growth Revisited, *Scandinavian Journal of Economics* 109, 369-386.
- [30] Turnovsky, Stephen J. (1997). Optimal Tax, Debt, and Expenditure Policies in a Growing Economy, *Journal of Public Economics* 60, 21-44.
- [31] Young, Alwyn (1998). Growth Without Scale Effects, *Journal of Political Economy* 106, 41-63.