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Should Pensions be Progressive? Yes, at least in Germany!

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Abstract

Recent reforms that aim at reducing the upcoming burdens of population ageing might seriously harm low income individuals. An increase in old-age poverty and disability will be the result. Under this prospect, the present paper quantitatively characterizes the optimal progressivity of unfunded pension systems in an overlapping generations model with idiosyncratic income, disability and longevity risk as well as endogenous labor supply at the intensive and extensive margin. Focusing on the German pension system, our model features the most recent demographic projections and distinguishes three skill classes with skill-dependent risk profiles. Starting from a baseline path that reflects a purely earnings related pension system, we increase the degree of progressivity and compute the resulting macroeconomic, welfare and efficiency effects. For our most preferred parametrization we find an optimal flat-rate pension share of 40 percent. This indicates that in Germany recent reforms that aim at raising retirement age and cutting benefit levels should be complemented by increases in pension progressivity, since improved insurance provision dominates higher labor supply distortions. In addition, we also find that reductions in the benefit level (i.e. privatization) will only reduce economic efficiency.

JEL-Code: C680, H550, J110, J260.

Keywords: stochastic OLG model, tax-benefit linkage, endogenous retirement, population ageing.

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1 Introduction

Population ageing will put severe pressure on public pension systems in Western OECD countries in the future. Since most of these systems will not be fiscally sustainable, various reforms have been legislated in these countries in recent years. These mainly follow two central targets: the prolongation of working life and the reduction of labor supply distortions induced by the pension system. To achieve the former of these goals, countries like Denmark, Germany, Norway and the US already have increased or will increase statutory retirement age from 65 to 67 while Austria, Belgium, Portugal and the UK will equalize normal retirement ages for men and women. In order for this reforms to have a significant effect, most of these countries also reduced incentives for early retirement. Consequently, effective retirement age is expected to increase in the near future.

The reduction of labor supply distortions is mainly targeted using two strategies. On the one hand, pension benefits are adapted to life-expectancy to prevent strong increases in future contribution rates. Germany, Finland and Portugal e.g. established pension calculation formulas that will decrease pension benefits in the case the population ages. Italy, Poland and Sweden changed to so-called notional defined contribution (NDC) systems in which the notional capital accrued during working life will be converted into an annuity at the date of retirement. Obviously, when individuals tend to become older, annual annuity payments will decrease. The other means to reduce labor supply distortions is to strengthen the tax-benefit linkage. Therefore, countries like France and Austria, that were calculating pension benefits on the basis of only a few years of working life, now extended their calculation periods by 15 and 25 years, respectively. The most drastic change, however, took place in Sweden, where the two tier pension system consisting of a basic and an earnings-related part was replaced by an NDC system with a small means-tested minimum pension. As a result of these reform efforts, the pension progressivity index declined noticeably in most OECD countries in recent year, confer OECD (2005) and OECD (2009).

Yet, the effects of the reforms described above are twofold. Obviously, they will keep the pension system fundamentally sound and decrease labor supply distortions. However, they might also come at costs. Hagen et al. (2010) e.g. argue that disability risk tends to rise exponentially with age and is higher for low than for high income households. Hence, when the number of working years rises significantly, especially poorer individuals will be exposed to higher disability risk for a longer time. Furthermore, when policy makers aim at strengthening the relation between earnings and pension benefits and cut benefit levels at the same time, it might be that low income households run short of resources during their retirement phase. As a consequence, old-age poverty could increase significantly, so that recent pension reforms that aim at reducing the burdens from population ageing may come at huge costs for low income individuals. Hence, policy makers should ask themselves some very important questions: Is it really efficient to maximize tax-benefit linkage in the pension system? Under the current pension reforms that target the problem of population ageing, shouldn't we rather make the pension system more progressive in order to prevent old-age poverty and guarantee low income households a reasonable amount of resources?

The literature identifies two main effects of an increase in pension progressivity. On the one hand, when the link between pension contributions and benefits becomes weaker, contributions to the system will more and more be perceived as taxes. Hence, labor supply distortions will increase. On the other hand, especially in the presence of uncertainty about individual productivity, more progressive pension arrangements also provide insurance against labor market risk. Obviously, the latter and the former effects work in opposite directions in terms of efficiency. A popular means to identify which of the two dominates are calibrated overlapping generations models in the spirit of Auerbach and Kotlikoff (1987). This framework was already used by Huggett and Ventura (1999) as well as Nishiyama and Smetters (2008) to quantify the long-run macroeconomic and welfare effects of different pension designs. Fehr and Habermann (2008) in addition compute transition paths between long-run equilibria and therefore are able to quantify the efficiency effects of different degrees of progressivity in the pension formula. However, this study neglects the problem of population ageing as well as related burdens and upcoming reforms. Yet, in our view, this is a very important point since, on the one hand, the increasing ratio between retirees and workers will itself increase labor market distortions and, on the other hand, old-age poverty and therefore the need for redistribution might rise from the reform packages discussed above.

The present paper aims at targeting the question of optimal pension progressivity under the problem of population ageing using the example of the German economy. Germany is a rather interesting country to study, since it already has brought on the way several pension reforms that will increase statutory retirement ages and reduce future pension benefits. In addition, the German pension system features a nearly perfect earnings relation and pension benefits are calculated using the complete working period. Therefore, labor supply distortions already tend to be very low in this country. As argued in Breyer and Hupfeld (2009) and Arent and Nagl (2010), Germany expects a significant increase in old-age poverty. Therefore the current reform debate calls for at least some redistribution within the pension system, see Breyer and Hupfeld (2009).

The model we use here extends the work of Nishiyama and Smetters (2007) and Fehr and Habermann (2008) in several ways. First, we incorporate a reasonable forecast of survival probabilities, fertility rates and migration to derive a suitable projection of the future population structure in Germany. With this approach we are able to incorporate the upcoming burdens of population ageing. Next, in order to quantify all consequences of rising labor market distortions, we consider labor supply decisions not only at the intensive but also the extensive margin like in Sánchez-Martín (2010) or Jaag, Keuschnigg and Keuschnigg (2010), i.e. retirement is an active choice. Finally, a detailed modeling of household labor market risk is necessary to quantify insurance effects of progressive pension arrangements. We therefore introduce permanent and transitory wage shocks, the former of which are identified as household's skill level. In addition, disability risk is a major risk factor especially for low income households. In Germany in 2009, about 20 percent of new entries into the pension system were due to disability, see DRV (2010). Hence, following Díaz-Giménez and Díaz-Saavedra (2009), individuals will face exogenous disability shocks that force them to retire early and receive disability pensions. With this approach and a detailed modeling of early retirement rules for different types of agents, we feel that our model features all the important exit pathways to retirement and is therefore able to adequately capture both the determinants of household labor supply decisions and income risk factors.

In order to quantify the macroeconomic, welfare and efficiency consequences of different degrees of progressivity, we will proceed as follows: We first compute a baseline simulation starting from 2008 onwards. Our base year 2008 will be calibrated to the German economy and the baseline path features adequate population forecasts and reflects the upcoming pressure on the pension system due to population ageing. The baseline scenario already will include recent pension reforms that aim at both increasing retirement ages and reducing pension benefits in Germany. We then will compute reform scenarios in which the degree of progressivity in the pension system varies and quantify the optimal progressivity in our model setup. We find that increasing progressivity of the pension system will significantly increase economic efficiency, since insurance effects outweigh the costs of labor market distortions. A further reduction in benefit levels, however, will only decrease economic efficiency.

The remainder of the paper is arranged as follows: the next section describes the general equilibrium model we use in our quantitative analysis. Sections 3 discusses the calibration of the baseline scenario. Our simulation results are presented in Section 4, Section 5 concludes.

2 The model economy

This section provides a description of the simulation model we use to quantify our results. A more equation-based description of our model and a definition of an equilibrium path can be found in Appendix A.

2.1 Demographics

Our model is populated by J overlapping generations. At any discrete point t in time, a new generation is born, the mass of which grows with rate n_t compared to the previous one. At the beginning of life, individuals are (exogenously) assigned a skill level $s \in \mathcal{S}$. During their life-cycle, they only survive from period to period with the skill-level, time, and age dependent survival probabilities $\psi_{t,j,s}$, where $\psi_{t,J,s} = 0$. Since our model abstracts from annuity markets, individuals that die before the maximum age of J may leave accidental bequests that will be distributed in a lump-sum fashion across all working individuals. In the following, we will, for the sake of simplicity, omit the indices t and s wherever possible.

2.2 Endowments and intra-cohort heterogeneity

Individuals start their life with zero assets $a_1 = 0$. Note that we restrict assets to be greater or equal to zero throughout the whole life cycle, i.e. agents might be liquidity constrained. During their working phase, they may work up to the maximum time endowment of 1 in each period. Time that is not devoted to working is consumed as leisure. Labor productivity e_j depends on individual's skill level s as well as idiosyncratic shocks $\eta_j \in \mathcal{E}$. Individuals are assigned a skill level s according to the probability distribution ϖ_s at the beginning of their life and remain in it. Hence, skill level may be interpreted as a permanent shock. When working in the market, agents will accumulate pension claims $ep_j \in \mathcal{P}$ that define their pension payments when retired. In addition to labor market shocks, they are exposed to the risk of becoming severely disabled and therefore unable to work. Consequently, individuals' state is characterized by

$$z_j = (s, a_j, ep_j, \eta_j, d_j, o_j) \in \mathcal{Z},$$

where $d_j = 1$ indicates disability. The state o_j changes from 0 to 1 in the moment the agent chooses to retire and therefore to become a regular old-age pensioner.

2.3 The laws of motion for individual states

The budget constraint is

$$a_{j+1} = (1+r)a_j + y_j(1-\tau) + b_j + p_j - T(y_j, p_j, ra_j) - (1+\tau_c)c_j,$$

where future assets a_{j+1} are derived from current assets (including interest), gross income from labor $y_j = we_j(1-\ell_j)$ (which is due to the wage rate for effective labor, individual productivity, and hours worked), accidental bequests b_j and pensions p_j, net of payroll taxes τy_j, income taxes $T(\cdot)$ and consumption expenditures c_j (including consumption taxes).

Accumulated pension claims consist of both a flat and a perfectly earnings related part. Specifically we let

$$ep_{j+1} = ep_j + \mu\left[(1-\lambda)\frac{y_j}{\bar{y}} + \lambda\right],$$

where $\bar{y}$ indicates the average labor income of the economy.[1] When $\lambda = 0$ and $\mu = 1$, agents face a perfectly earnings related system, whereas $\lambda = 1$ means that the pension system is completely flat. Setting $\mu = 0$ we can shut down the accumulation of earning points and therefore completely abolish the pension system. Note that $ep_{j+1} = ep_j$ holds after retirement.

We assume productivity shocks to be independent across individuals and to be identically distributed across individuals of a specific skill level. They follow a time and age independent Markov process, the conditional distribution of which is given by $\pi(\eta_{j+1}|\eta_j)$.

Beneath labor market shocks, individuals are exposed to disability risk. As with productivity shocks, disability risk depends on individual skill level. At each age j during the life-cycle an agent may get a disability or bad health shock with the probability π_j^d. Having received this shock, household's status changes to $d_j = 1$ and $o_j = 1$, since he will not be able to generate labor market income anymore. A disability pension system described in more detail below will care for the sustenance of disabled agents. In order to account for utility costs of this bad health shock, we restrict individual leisure consumption to a value of $h < 1$ that is lower than the maximum time endowment. This reflects the time cost of health care, e.g. visiting a doctor, or the utility costs of a reduced quality of life.

2.4 Preferences and the decision to retire

Preferences over consumption c_j and and leisure ℓ_j are assumed to be representable by a time-separable utility function of the form

$$E\sum_{j=1}^{J}\Psi_j\beta^{j-1}u(c_j,\ell_j),$$

with Ψ_j being the unconditional probability to survive until age j. Due to this separability, we can define the individual optimization problem recursively by

$$V(z_j) = \max_{c_j,\ell_j}\left\{u(c_j,\ell_j) + \psi_{j+1}\beta E_j\left[V(z_{j+1})\right]\right\}.$$

[1] This implies that the average earning point accumulated per year is equal to 1. Note that in the German pension system and in our model the amount of earning points that can be accumulated per year is limited to 2.

Starting at age 60, agents may choose to exit the labor force and retire. Retirement is driven by two factors in our model. On the one hand, the percentage change in welfare from exiting the labor force is the most important driver. On the other hand, an individual taste parameter $\nu = \nu(\eta_j)$ that is related to own labor market status η_j captures non-pecuniary or psychological costs of working. This way of modeling is based on the idea that bad labor market shocks additionally give rise to earlier retirement beyond the incentives induced by low labor income, i.e. $\nu'(\eta_j) < 0$.[2] Specifically, an agent chooses to retire if

$$V(z^r_{j+1}) \geq V(z^w_{j+1})(1 - \nu),$$

where z^r_{j+1} and z^w_{j+1} indicate the pensioner and the worker state, respectively, and ν is the psychological costs of working measured as a percentage of individual utility.

2.5 Technology

We let the production technology in our model be represented by a Cobb-Douglas production function $Y = \theta K^{\epsilon} L^{1-\epsilon}$, where K measures aggregate capital and L aggregate labor in efficiency units. The parameter ϵ denotes the share of capital in production and θ is a technology parameter. We assume capital to depreciate at the constant rate δ_k and firms to pay corporate taxes on output net of labor and depreciation costs at rate τ_k. Since a continuum of firms produces with the same technology under perfect competition, net marginal products of capital and labor equal the interest rate for capital r and the wage rate for effective labor w. Finally, in order to account for technological progress, we follow Kotlikoff, Smetters and Walliser (2007) and assume time augmenting technological change.[3] Consequently, individual time endowment increases by κ for any individual from period to period.

2.6 Government activity

The government in our model splits into a tax and a pension system. The budgets of both systems are closed separately. While the consumption tax rate is used to balance the tax system, pension contributions are chosen in a way that pension contributions equal pension payments.

The tax system In each period t, the government issues new debt and collects taxes on consumption τ_c as well as labor and capital income. The income tax code is oriented towards the German tax system where labor is taxed progressively and capital at a constant rate (after a basic allowance). The taxation of pensions reflects the current transition towards a front-loaded taxation system in Germany. Therefore, in 2008, 66 percent of pension contributions are exempt from tax, while 56 percent of received pension payments have to be taxed. These rates gradually increase over time until in 2040, pension contributions are completely deductible and benefits are fully taxed. The revenues from taxation and debt are used to finance public expenditure and interest payments on existing debt.

2 A detailed mathematical formulation of our retirement decision can be found in Fehr, Kallweit and Kindermann (2011).

3 Note that, due to the utility function not being of Cobb-Douglas type, we can not assume labor augmenting technological change, since this would not be consistent with a balanced growth path.

The pension system The pension system is run on a pay-as-you-go basis. It collects contributions at a rate τ from all working households and pays pension benefits to both old-age and disability pensioners. There are three factors determining the size of a regular old-age pension:

1. The sum of earning points accumulated during the working periods.
2. The adjustment factor that reduces pension payments in the case of early retirement.
3. The actual pension amount (APA) which reflects the monetary value of one earning point.

When a worker retires early, i.e. before the so-called normal retirement age, his earning points will be adjusted in order to account for the prolongation of the retirement period. According to German law, they are reduced by 3.6 percent for any year the agent retires before the normal retirement age. A special rule applies to individuals that get disabled – and therefore have to exit the labor force – before the age of 60, the sustenance of which will be taken care of by the disability pillar of the pension system. In order to derive their pension payments in line with German law, it is assumed that they had worked up to the age of 60 with their average productivity. Hence, their pension gets subsidized in order to correct for the missing years of work.

Individual pension benefits are computed from

$$p_j = ep_j \times APA$$

where ep_j denotes accumulated earning points at retirement after adjustment and APA the actual pension amount. Over time, APA grows with gross labor earnings. In addition, a so-called sustainability factor adjusts APA downwards in the case the number of retirees rises in relation to the number of workers. This factor is a result of the German pension reforms in 2001 and 2004 an aims at coping with the problems of an ageing society.[4]

2.7 Equilibrium conditions

Given a specific fiscal policy, an equilibrium path of the economy has to solve the household decision problem, reflect competitive factor prices, and balance aggregate inheritances with unintended bequests. Furthermore aggregation must hold, and consumption tax and pension contribution rate have to balance the tax and pension system's budgets. Since we assume a closed economy setting, output has to be completely utilized for private consumption C_t, public consumption G_t and investment purposes, i.e.

$$Y_t = C_t + G_t + (1 + n_{t+1})(1 + \kappa)K_{t+1} - (1 - \delta_k)K_t.$$

3 Calibration

Since we use realistic demographic forecasts in our model, our base year 2008 will not be a long-run equilibrium and the benchmark simulation will be a baseline path simulated under demographic transition and including recent reforms of the tax and pension system. In the following, we will

[4] For a more detailed discussion of this adjustment formula see Fehr et al. (2011).

discuss how we calibrated the initial year of our simulation model and how our baseline path will be determined. Then we move on to policy reforms and their macroeconomic, welfare and efficiency consequences.

3.1 Demographic projections

In our model, one period covers one year. We assume agents to start their economically relevant life at the age of 20 and to live up to a maximum of 100 years. In order to get a reasonable classification of skills, we use the International Standard Classifaction of Education (ISCED) of the UNESCO. We thereby merge levels 0 to 2 (primary and lower secondary education), levels 3 and 4 (higher secondary education) and levels 5 and 6 (tertiary education) in order to receive 3 skill levels, i.e. $\mathcal{S} = \{1, 2, 3\}$. The initial probability distribution ϖ_s is calculated using data from the German Socio-Economic Panel (SOEP), a description of which can be found in Wagner, Frick and Schupp (2007). In this representative data set, low-, medium- and high-skilled individuals represent 26, 55 and 19 percent of the population, respectively. Survival probabilities $\psi_{j,2}$ for skill class 2 are taken from the 2000 Life Tables for Germany reported in Bomsdorf (2003). Taking into account the positive correlation between skill level and life expectancy, we compute probabilities for the low and the high skill class from $\psi_{j,2}$ in a way that life expectancy between those two differs by 5 years, see von Gaudecker and Scholz (2007). Consequently, initial life expectancies are 77.1, 79.6 and 82.1 for the three different skill classes, respectively.

To account for a reasonable demographic transition, we set the population structure in 2008 to the one observed in Germany. SB (2009) compares 15 alternative combinations of assumptions with respect to fertility, life expectancy and migration. Taking one of their "benchmark scenarios", we assume the total fertility rate to remain constant at 1.4 children per woman, life expectancy to increase linearly for any skill class by 7.3 years until 2060 and net migration to rise gradually from 50.000 to 100.000 until 2014. For the sake of simplicity, we assume migrants to also have the skill composition ϖ_s and to enter the economy at age 20. In order to converge to a stable population and be able to compute a long-run equilibrium, we set the population growth rate n_t to 0 and keep life-expectancy and migration unchanged from 2060 onwards. Figure 1 reports the resulting population structure from 2008 to 2100. As can be seen, we start with a population size of 82 million in 2008 which declines to 65.6 million in 2060. This is quite close to the numbers reported in SB (2009) who project a population size of 64.7 million. With the rise in life expectancy and low fertility rates, the old age dependency ratio – which is defined as the number of individuals aged 60 or older over the number of people between ages 20 and 59 – increases from currently 46.1 percent to 89.4 in 2060.

3.2 Labor productivity, idiosyncratic risk and the distribution of incomes

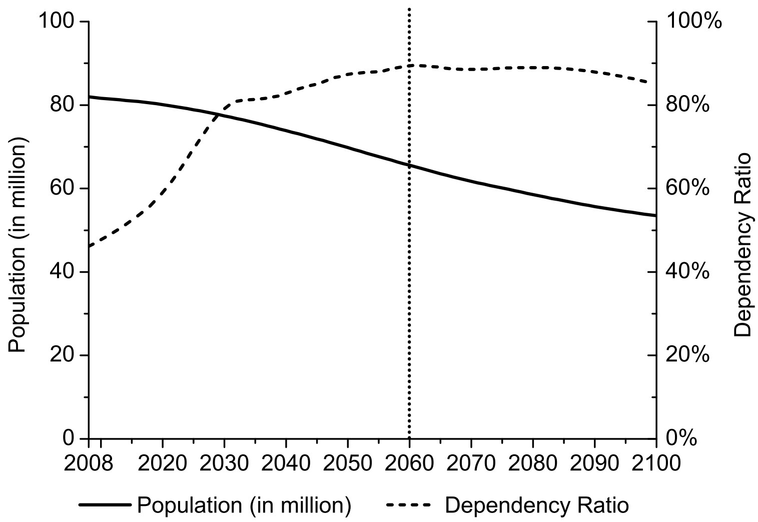
Following Love (2007), we let log-productivity for an individual of skill class s evolve over the life-cycle according to

$$\log e_j = \varsigma_0 + \varsigma_1 \cdot j + \varsigma_2 \cdot j^2 / 100 + \eta_j$$

where η_j follows an AR(1) process of the form

$$\eta_j = \varrho \eta_{j-1} + \varepsilon_j \quad \text{with} \quad \varepsilon_j \sim N(0, \sigma_\varepsilon^2).$$

Figure 1: Projected population structure



In order to estimate productivity profiles for the three skill levels, we use inflated labor income data of full-time primary household earners from the SOEP. Our unbalanced data set covers workers between ages 20 to 60 of the years 1984 to 2006 that were again divided into three skill classes according to the ISCED standard. This leads us to a total of 83893 observations with 11789, 55015, and 17089 in the three different classes, respectively. With this data set, we estimate the above equation separately for any skill class s, using some dummy variables reflecting job type and family status as well as an individual persistent effect. The results of the estimation process can be found in Table 1, standard errors are reported in parenthesis.

Table 1: Parameter estimates for individual productivity

	Low-skilled	Middle-skilled	High-skilled
Intercept and type ς_0	9.6207	9.4190	8.6649
	(0.2662)	(0.1494)	(0.3116)
age term ς_1	0.0437	0.0579	0.1025
	(0.0041)	(0.0025)	(0.0064)
age^2 term ς_2	-0.0500	-0.0649	-0.1090
	(0.0052)	(0.0031)	(0.0074)
AR(1) correlation ϱ	0.7244	0.7826	0.7770
	(0.0119)	(0.0046)	(0.0088)
transitory variance σ_ε^2	0.0646	0.0737	0.0790
	(0.0056)	(0.0039)	(0.0076)

There are two things to notice. First, we find a strong AR(1) correlation of around 0.8 for the error term η_j, which lies in the range of typical values for these types of models, see e.g. Love (2007) or İmrohoroğlu and Kitao (2009). Second, except for high-skilled, we see a small persistent variance, which means that our groups are strongly homogeneous. In the highest educational group, however, there is a certain chance of climbing up into the area of extraordinary high salaries. This makes the group somewhat more heterogeneous and explains a higher variance of the individual effect.

In our model, we use the estimated profiles in between the ages 20 and 65. Note that, with rising age, the estimates become more and more biased upwards, since individuals with low productivity, e.g. due to bad health, might already have become retired. Hence, from the age of 65 onwards, we assume in line with Eisensee (2005), that productivity depreciates quadratically until it finally reaches zero at age 75. The productivity profiles can be seen in Figure 2.

Figure 2: Productivity throughout the life cycle

In terms of income uncertainty, we discretize the estimated AR(1) process, using the procedure described in Tauchen and Hussey (1991) with 3 approximation points, since this algorithm delivers the most accurate results in terms of income distribution, see Kindermann (2010). Table 2 compares some model outcomes with income and asset inequality measures in Germany for 2007 taken from SVR (2009). We find quite a good match of both the lowest and highest percentile of the income distribution as well as the Gini index. Of course, since individuals are not allowed to run into debt and we do not explicitly account for very high income earners and self-employed, the asset distribution is more equal in our model than in reality.

Table 2: Measures of income inequality

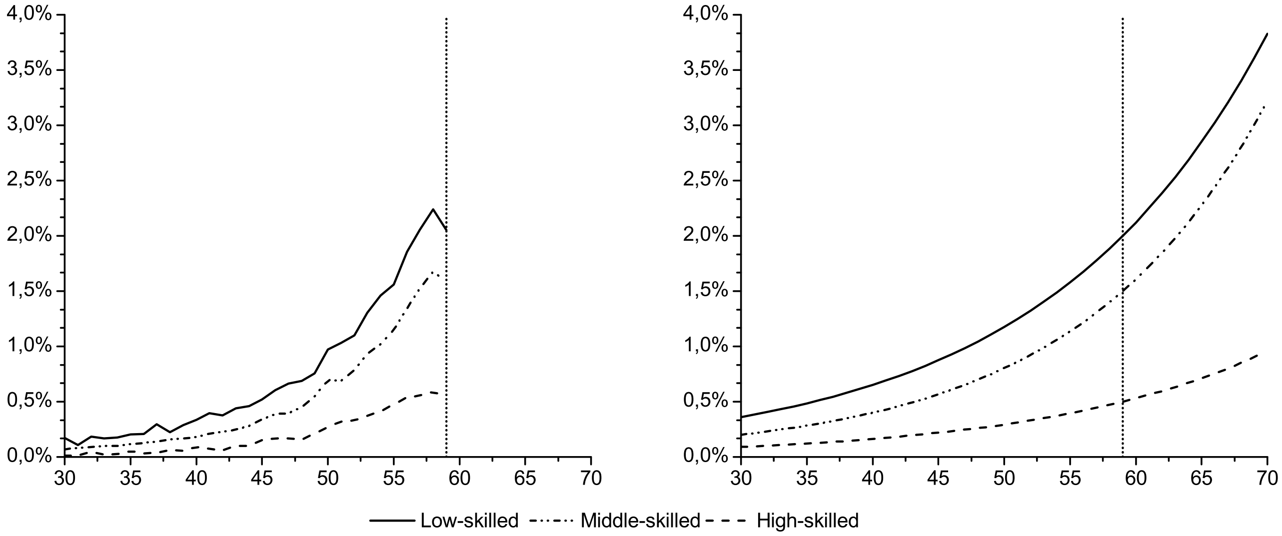
		Percentage share of Lowest 10 %	Percentage share of Highest 10 %	Gini index
Net Income	Model	2.7	22.8	0.284
	Data*	3.6	24.0	0.290
Assets	Model	0.0	39.9	0.617
	Data*	-1.1	61.0	0.799

*Source: SVR (2009)

3.3 Disability risk

Starting from age 30, we assume the probabilities of becoming disabled to be positive. Hagen et al. (2010) report disability risk for workers with different skill levels between the ages 30 and 59. They find that the probability to become disabled increases exponentially with age. Consequently, we also assume exponential growth of disability risk throughout the working life and extrapolate the probabilities mentioned above from age 59 up to age 70. Figure 3 compares the evolvement of disability risk in Hagen et al. (2010) on the left hand side with the probabilities used in our model on the right hand side. Interestingly, disability risk seems to decrease from age 58 to 59. According

Figure 3: Disability risk for different skill levels

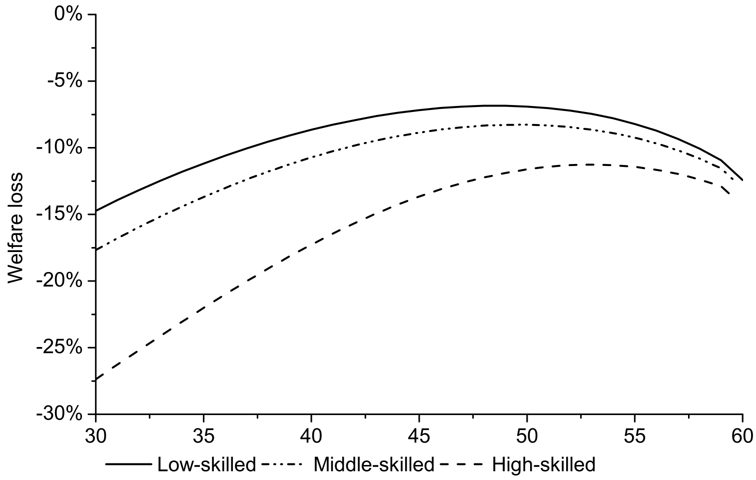


to Hagen et al. (2010), this however is due to the fact that agents facing disability shocks so late in life might also be eligible to a regular old-age pension. Since becoming eligible for disability pensions is quite some effort – one e.g. needs detailed medical examinations and to fill out many forms – individuals may choose the easier way of becoming regular old-age pensioners. Hence, the probability estimates tend to be biased downwards towards age 59.

As consequence of disability, we force workers to retire and let them suffer from a reduction in their time endowment of 20 percent, i.e. $h = 0.8$. This reflects the time cost of health care, e.g. visiting a doctor, or the utility costs of a reduced quality of life. Figure 4 shows the loss in income related to disability for individuals becoming disabled at different points in time.[5] Interestingly, the higher an agent's skill level, the higher his losses from disability. This is not intuitive in a perfectly earnings related pension system. However, it becomes clear from the fact that initial income is very similar in the three skill classes, but the higher skilled face the steeper increase in labor productivity throughout the life cycle, see Figure 2. Consequently, their forgone earnings from disability are especially large at the beginning of the life cycle, while at older ages this difference shrinks noticeably. Furthermore, utility costs are lowest around the age of 50. This is due to the fact that average productivity is the highest at this point in the life cycle. Since earning points of a disability pensioner are calculated by assum-

[5] The figures reflect compensating variation à la Hicks of becoming disabled. We talk about this variation in more detail in the next section.

Figure 4: Welfare loss from disability by age



ing he had worked the remaining years until age 60 with his average productivity, benefits from this disability subsidy reach their maximum at age 50 and decline with a decreasing labor productivity afterwards. According to Torrance, Boyle and Horwood (1982), direct utility losses from disabilities that make individuals unable to work may amount from 30 up to 70 percent. Consequently, we feel that a maximum average welfare loss of 27 percent is a quite conservative estimate. To further clarify the impact of our choice for h on efficiency effects of our reform, we will also assume different values for h in a sensitivity analysis.

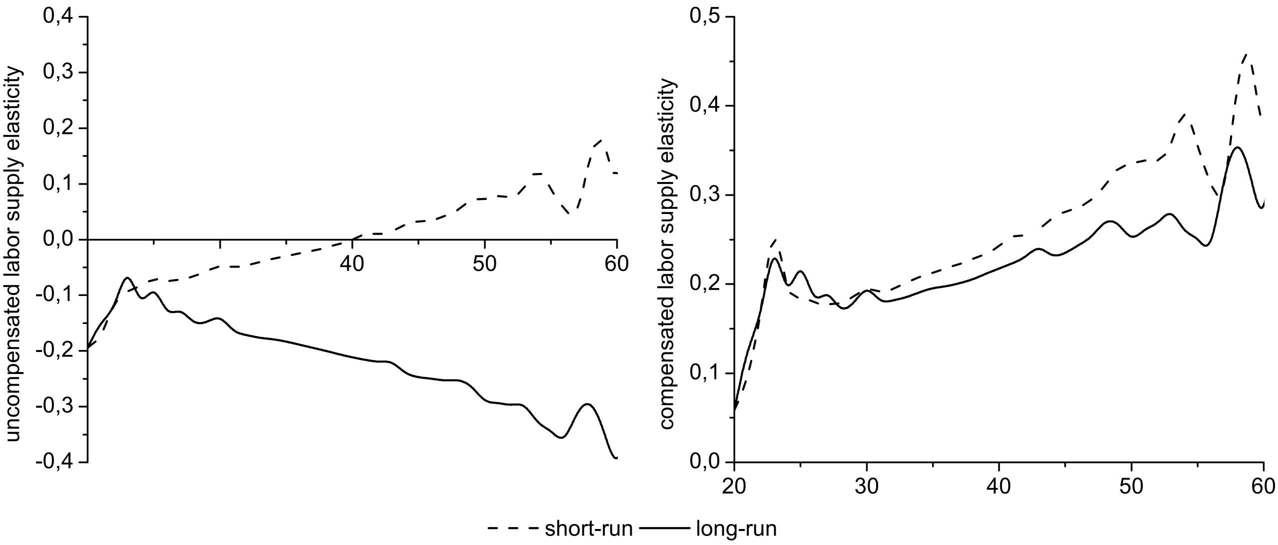
3.4 Preferences and the decision to retire

We let individual preferences over consumption and leisure be represented by the instantaneous CRRA utility function

$$u(c,\ell) = \frac{1}{1-\frac{1}{\gamma}} \left\{ c^{1-\frac{1}{\rho}} + \alpha \ell^{1-\frac{1}{\rho}} \right\}^{\frac{1-\frac{1}{\gamma}}{1-\frac{1}{\rho}}},$$

where ρ denotes the intra-temporal elasticity between consumption and leisure and α is a taste parameter for leisure consumption. γ represents the inter-temporal elasticity of substitution between consumption in different years. In order to calibrate the parameters of the utility function we first set γ at 0.5, which is in the range of commonly used parameters in these types of models, see İmrohoroğlu and Kitao (2009). We then chose $\rho = 0.6$ in order to obtain realistic labor supply elasticities. In order to set average hours worked in the economy at 0.4, which implies a 40 hours work week length, we let $\alpha = 1.6$. Finally, we chose a value of 0.985 for β to obtain a capital to output ratio of 3.5, which is close to the one observed in Germany in 2008. Figure 5 reports wage elasticities of labor supply for individuals between ages 20 and 60. We thereby distinguish between the short- and the long-run perspective. While short-run elasticities are calculated within a cross section, long-run ones are determined throughout the life-cycle of an individual. For example, the short-run labor supply elasticity of an individual aged 40 is computed from his relative change in labor supply resulting from an unexpected once and for all marginal increase in wages. The long-run one, on the other hand, will result from the behavior of an agent aged 40 that already benefited from wage increases

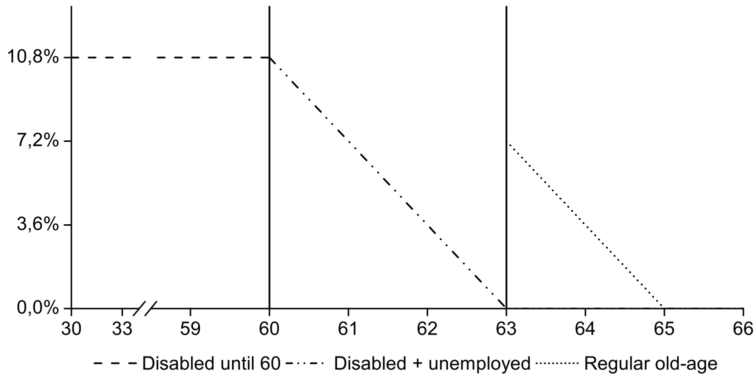
Figure 5: Uncompensated and compensated labor supply elasticities



throughout his whole life cycle. The left part of Figure 5 shows uncompensated elasticities, i.e. the relative changes in labor supply resulting from a marginal increase in wages. At young ages, increasing wages leads to a decrease in labor supply. Hence, the income effect dominates the substitution effect. Note that the very low values and the steep increase at the beginning of the life cycle are due to the fact that a major part of the population in that cohort is liquidity constrained. Liquidity constraints force individuals to increase their hours worked. Consequently, a rise in wages may loosen liquidity constraints and in turn make labor supply of liquidity constrained agents decline. Hence, the income effect is much stronger for these cohorts. Looking at later ages in life, from a cross sectional perspective, individuals will tend to increase their labor supply, since they can only benefit for a shorter period from the increase in wages and therefore will try to make the most out of it. Taking a look at the individual life-cycle, however, individuals will start saving more at younger ages. This causes an additional income effect at later periods in life which leads to a declining labor supply elasticity throughout the working period. The compensated elasticities, i.e. where additional lump-sum transfers neutralize the income effect of the initial wage increase, always have to be positive in our model. In addition, since households have to work a lot in order to be able to form both life-cycle and precautionary savings due to labor productivity and disability risk, elasticities are much lower at the beginning of the working phase than towards the end. Overall, the short-run elasticities are close to the values reported in Fenge, Übelmesser and Werding (2006) for different age groups.

The German pension system actually has two normal retirement ages, i.e. the first age at which one will receive the full pension payment without adjustment. The regular normal retirement age is 65. However, a reduced age of 63 applies to both disability pensioners and the unemployed. Since we explicitly account for disability risk, normal retirement age is set to 63 for these individuals. Note that in consequence, a disabled worker who has to retire before age 60, will have his earning points adjusted as if he had worked until age 60, but will also face an adjustment factor of $(63 - 60) \cdot 3.6 = 10.8$ percent. In addition, we interpret our worst labor market shock as old-age unemployment and let individuals with this shock also retire with a normal retirement of 63. Furthermore, the age at which individuals can make their first retirement choice differs across different groups. Whereas disabled individuals always retire immediately, the unemployed may retire from age 60 onwards while regular workers may start retiring from 63. Figure 6 summarizes the factors by which pensions will be adjusted for different types of retirees.

Figure 6: Adjustment factors by groups of pensioners

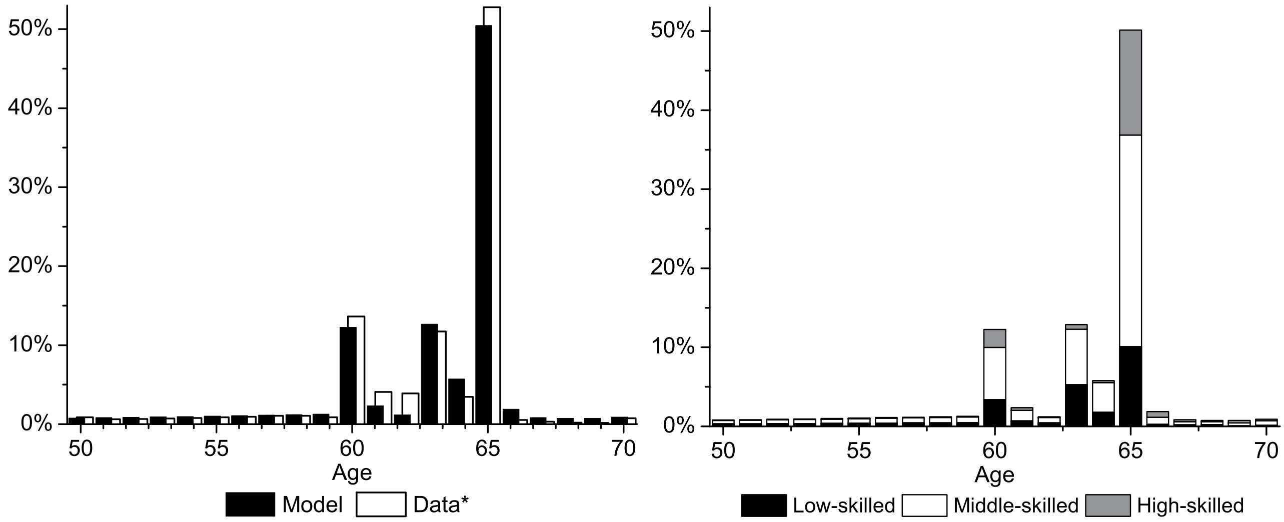


As mentioned above, psychological costs of working ν in our model depend on the own labor market status. With these cost parameters, we target a good match of the retirement pattern currently observed in Germany. Nevertheless, we also aim at minimizing the influence of this (exogenous) parameter on the decision to retire. Hence, we let ν have an expected value of 1.17 percent, a standard deviation of 1.27 percent and assume a correlation between ν and η_j of -0.9, i.e. the higher labor market income, the lower utility costs from working. This results in the retirement pattern reported in the left part of Figure 7. Due to the very detailed modeling of the German pension law, we find a good match of the model generated retirement pattern with German data. Obviously, workers retiring before age 60 have to be disabled. After age 60 we see three major peaks at ages 60, 63 and 65. The first of these is due to retirement of the unemployed. The second peak, on the other hand, is formed by individuals having had a lot of luck during their time of work and having accumulated a lot of assets and earnings points. Since those will not work anymore due to income effects, they may retire early. The last peak captures all regular old-age retirees. The right part of Figure 7 shows the retirement pattern for individuals of different skill classes. Since the risk of getting disabled decreases with skill level, the lower skilled tend to retire earlier than the higher skilled.

3.5 Technology

IdW (2009) reports a wage share in production of 65.2 percent for Germany in 2008. Since the wage share in our model is given by $1 - \epsilon$, we set ϵ at 0.35. In addition, we choose a value of 1.3 for the technology parameter θ in order to normalize the wage rate for effective labor to 1. We let the depreciation rate δ_k on capital be 4.2 percent. This guarantees investment to amount to 20.3 percent of GDP, which is slightly higher than the value of 19.3 reported in IdW (2009) for Germany. Finally, we assume a growth rate of 1.3 percent for individual time endowment, which is in line with the long run average growth rate for Germany reported in Erber and Fritsche (2009).

Figure 7: Retirement pattern in model and data

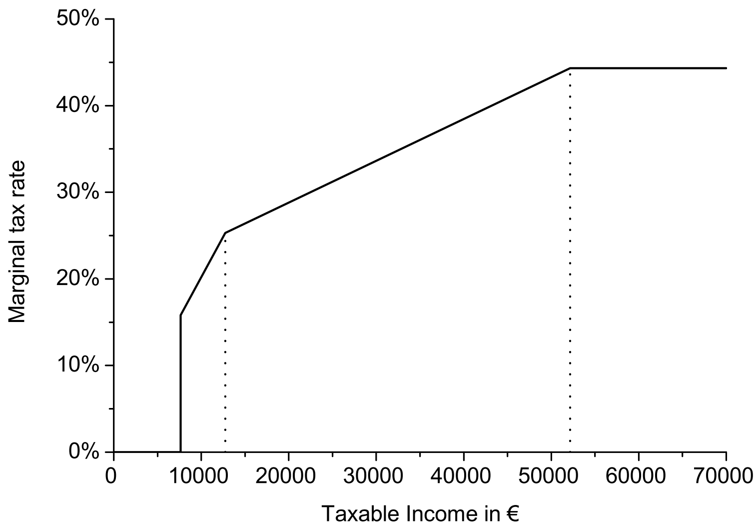


*Source: DRV (2010). Own calculations for year 2009.

3.6 Government policy

The tax system Government tax policy in our model reflects quite well the German tax system. Specifically, we set the debt to output ratio at 60 percent and fix the consumption tax rate at 17 percent, which guarantees a consumption tax revenue to output share of 9.9 percent. This share is slightly lower than the value of 10.7 percent reported in IdW (2009). We apply the German income tax code of the year 2005 to labor and pension income, the marginal tax rate schedule of which is displayed in Figure 8. In addition we tax returns from savings linearly at the rate 26.4 percent. This

Figure 8: Marginal tax rates for labor and pension income



reflects the recent reform of capital income taxation in Germany. Finally, we set the corporate tax rate τ_k at 15 percent which yields a revenue to output ratio of 3 percent that is slightly higher than the value of 2.1 percent reported in IdW (2009).

The pension system A much more detailed focus lies on the pension system, since this is the one this paper is interested in. As already mentioned above, we set normal retirement ages, ages of first retirement choice and the adjustment factor for early retirement according to German pension law. We also use realistic disability risk processes derived from Hagen et al. (2010). Finally we set the pension contribution rate to 19.9 percent, which since 2007 is the legal contribution rate in Germany. This leads us to a gross replacement rate of 48 percent, which is close to the one reported for Germany in OECD (2009), and an overall amount of pension payments of 12 percent of GDP. Thereby, 1.5 percent of GDP are spent on disability pensions, a value slightly higher than the amount of 0.9 percent reported for Germany by IdW (2009). As already discussed before, changes in the actual pension amount are derived from the evolution of aggregate household income corrected for changes in the old age dependency ratio.

3.7 The baseline path

Except for the demographic projections, all the figures above were calculated to calibrate our base year 2008. Table 3 again summarizes our calibrated parameters and the respective calibration targets. However, since we assume a demographic transition, we need a complete baseline scenario from 2008 to a long-run equilibrium as basis for our computational experiment. In order to calculate this baseline path, we have to make some specific assumptions about future public policy. We therefore consider three reform packages already initiated by the German government:

1. *"Retirement with 67":* This reform package successively increases normal retirement ages of the cohorts born 1947 and later from 65 to 67 and the reduced age from 63 to 65.

2. *"Front-loaded taxation of pension benefits:"* In 2005, the Government passed a law on the taxation of pension benefits. As a consequence, in 2008, 66 percent of pension contributions are exempt from tax, while 56 percent of received pension payments have to be taxed. These rates gradually increase over time until in 2040, pension contributions are completely deductible and benefits are fully taxed.

3. *"Targets for the pension contribution rate:"* For the evolvement of the pension contribution rate, the government has issued the targets of 20 percent for 2020 and 22 percent for 2030. There are several factors determining the size of pension payments, i.e. the actual pension amount. On the one hand, a rising old age dependency ratio leads to a cut in benefits, on the other hand, the actual pension amount grows at the same rate as wages. This will, however, not be enough in our model. Consequently, we additionally adjust the actual pension amount in each year between 2008 and 2030 to match the Government's targets.

Table 4 reports our simulated baseline path from 2008 until 2060. There are two main driving forces determining these results. On the one hand, the strong increase in the old age dependency ratio induced by a rise in longevity and low fertility rates causes a reduction in pension benefit levels. On the other hand, the reform "Retirement with 67" forces individuals to work longer. In consequence, employment as well as aggregate savings and therefore capital strongly increase. Since the latter outweighs the former, wages increase throughout the transition while the interest rate falls. Due

Table 3: Calibration targets

Parameter		Value	Target
Demographics			
Population growth	n_t		Population projections by SB (2009)
Survival probabilities	ψ_j		Bomsdorf (2003), SB(2009), von Gaudecker and Scholz (2007)
Skill distribution	ϖ_s		estimated from SOEP data
Labor productivity and disability risk			
Labor productivity	e_j, η_j		estimated from SOEP data
Disability risk	π_j^d		Hagen et al. (2010)
Time loss from disability	h	0.80	welfare loss from disability
Preferences and retirement			
Inter-temporal elasticity of substitution	γ	0.50	İmrohoroğlu and Kitao (2009)
Intra-temporal elasticity of substitution	ρ	0.60	labor supply elasticities
Leisure preference	α	1.60	average hours worked 0.4
Time discount factor	β	0.985	capital output ratio 3.5
psychological costs of working	ν		retirement pattern
Technology and government policy			
Capital share in production	ϵ	0.35	IdW (2009)
Technology parameter	θ		wage rate for effective labor of 1
Depreciation of capital	δ_k	0.042	investment to GDP ratio 19.3%
Debt to GDP ratio		0.60	target value issued by government
Consumption tax rate	τ_c	0.17	revenue to output share 10.7%
Income tax code			German tax law
Corporate tax rate	τ_k	0.15	revenue to output share 2.1%
Technological progress	κ	0.013	Erber and Fritsche (2009)
Pension system design			German pension law

to population ageing the size of the work force decreases much faster than the size of the overall population. In addition, public goods provision is constant per capita. Consequently, revenues from income taxation decline much faster than public expenditure which results in increasing consumption taxes. The policy reform "Retirement with 67" obviously causes average retirement age to rise throughout the transition. In 2060 this age is 2.4 years higher than in 2008, which is more than the two year increase in normal retirement age. This further rise is mainly due to the reduction in pension benefits resulting from population ageing, see row "Replacement rate".[6] Although the replacement rate declines throughout the transition, the budget of the pension system has to be enlarged. This is because the cut in replacement rates can not completely absorb the rising expenditure resulting from population ageing. Consequently, the pension contribution rate has to be successively adjusted until it reaches a value of 25.7 percent in 2060. Note that due to the additional reduction in pension benefits we are nearly able to match the targets for the contribution rate of 20 percent in 2020 and 22 percent in 2030 issued by the German government.

[6] Unlike in Heijdra and Romp (2009), population ageing per se does not cause increases in mean retirement in our model since we abstract from annuity markets. As shown in Kalemli-Ozcan and Weil (2010), the effect of decreasing mortality rates on the timing of retirement is ambiguous when there are unintended bequests.

Table 4: Baseline path of the economy

Year	2008	2020	2030	2040	2050	2060
Macroeconomic aggregates						
Employment	100.0	109.7	114.3	119.6	127.7	135.9
Capital	100.0	118.3	131.0	139.1	147.1	156.5
Prices						
Wage	100.0	102.7	104.9	105.4	105.1	105.1
Interest rate	4.9	4.5	4.2	4.1	4.2	4.2
Consumption tax rate	17.0	20.1	22.4	24.3	25.2	25.3
Pension system						
Mean retirement age	59.8	60.9	62.5	61.9	62.2	62.2
Replacement rate	48.0	43.9	41.0	39.6	39.5	39.2
Expenditure (in % of GDP)	12.0	12.5	13.5	14.8	15.2	15.5
Contribution rate	19.9	20.6	22.5	24.6	25.2	25.7
Poverty measures						
Elderly poverty rate	5.0	7.9	10.0	10.6	10.1	9.0
Elderly poverty risk	17.4	19.5	20.2	20.6	20.8	20.4

Finally, we report some poverty measures amongst the elderly in our baseline scenario. According to SVR (2007), a household is considered to be poor if his after tax income lies below 40 percent of the median income in the population. In addition, according to the definition of the European Union, one is at risk of poverty, if income is not higher than 60 percent of median income. In the last part of Table 4, we therefore report poverty rates and rates of poverty risk for pensioners. We find that about 5 percent of retirees are poor and 17.4 are at the risk of being so. The main reason why this numbers are higher than the 2.4 percent and 15.0 percent, reported for Germany in SVR (2007), is that we do not account for a social protection system. An individual living on social assistance receives about 54 percent of median income as benefits, hence, could not be accounted for as poor in Germany. Nevertheless, we think that these two measures still are a good indicator for the evolution of old-age income in our model. Obviously, since the replacement rate of the pension system declines by about 9 percent throughout the transition, the elderly become increasingly poor. Consequently, the poverty rate more than doubles until 2040. These projections give rise to some very important questions: Is our fully earnings related pension system really able to deal with all the problems arising from both population ageing and our future pension reform packages? Would a (at least partly) progressive system do better or would it just lead to efficiency losses due to labor supply distortions? The following sections aim at addressing this issue.

4 Simulation results

The remainder of this paper will mainly focus on the optimal progressivity of the pension system. In order to quantify the macroeconomic, welfare and efficiency effects of progressive pension arrangements, we change in year 2009 the parameter λ – in some reforms we also alter μ – in the accumulation equation for earning points ep_{j+1} and compute a new "reform path" up to the long-run equilibrium. In the first subsection we explain how welfare and efficiency effects are computed given this reform path. The next part considers a rather extreme scenario, namely the change from a fully earnings related to a fully flat pension system. Precisely because this reform is so drastic, it is the best one to show all the economic consequences of pension progression. A next step will then be to determining the optimal degree of progression in the pension system and examine the sensitivity of our result with respect to some central model assumptions. A final side note will shed light on the consequences of changes in the system size rather than its progression.

4.1 Computation of welfare and efficiency effects

The concept we apply to quantify welfare effects is compensating variation à la Hicks. Due to the homogeneity of our utility function,

$$u\left[(1+\phi)c_j, (1+\phi)\ell_j\right] = (1+\phi)^{1-\frac{1}{\gamma}} u\left[c_j, \ell_j\right]$$

holds for any c_j, ℓ_j and ϕ. In consequence, since utility is additively separable with respect to time, if consumption and leisure were simultaneously increased by the factor $1+\phi$ at any age, life-time utility would increase by the factor $(1+\phi)^{1-\frac{1}{\gamma}}$. With this considerations lets again turn to our simulation model. Assume an individual at state z_j had utility $V^b(z_j)$ in the baseline path and $V^r(z_j)$ in the reform path. The compensating variation between the baseline and the reform scenario for the individual characterized by z_j is then given as

$$\phi = \left\{\frac{V^r(z_j)}{V^b(z_j)}\right\}^{\frac{1}{1-\frac{1}{\gamma}}} - 1.$$

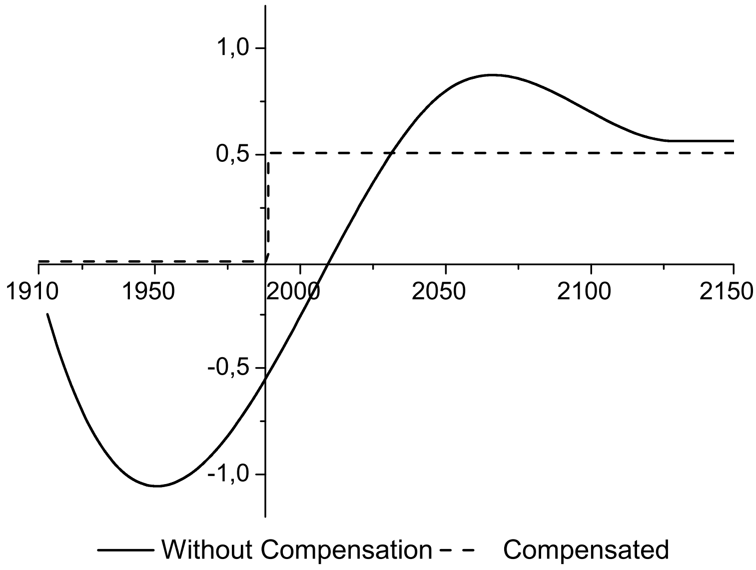
ϕ then indicates the percentage change in both consumption and leisure individual z_j would require in the baseline path in order to be as well of as in the reform scenario. The other way round we may say that an individual is ϕ better (or worse) off in terms of resources in the reform path than in the baseline scenario. If $\phi > 0$, the reform is therefore welfare improving for this individual and vice versa.

A special rule applies to individual not having entered their economically relevant phase of life in 2008 (the so-called future generations), the year before we conduct our pension reforms. We evaluate their utility behind the Rawlsian veil of ignorance, i.e. from an ex-ante perspective where neither their skill level nor any labor market shock has been revealed. The concept of compensating variation thereby applies likewise.

Figure 9 shows the possible individual welfare and aggregate efficiency consequences resulting from a generic reform experiment. For the sake of simplicity, we only consider a representative individual for each cohort. The numbers on the abscissa denote birth years of different cohorts. Since households become economically active at age 20, the last cohort that was already participating in markets in year 2008 was born in 1988. This point is indicated by the intersection of the two axes. Consequently, when talking about future generations in the following, we mean all cohorts born after 1989.

The solid line in Figure 9 indicates cohort-specific welfare consequences. As can be seen, the considered reform redistributes from currently living to future cohorts. In order to isolate the pure efficiency effects of the reform, we apply the hypothetical concept of a Lump-Sum Redistribution Authority

Figure 9: Generic welfare consequences of a reform



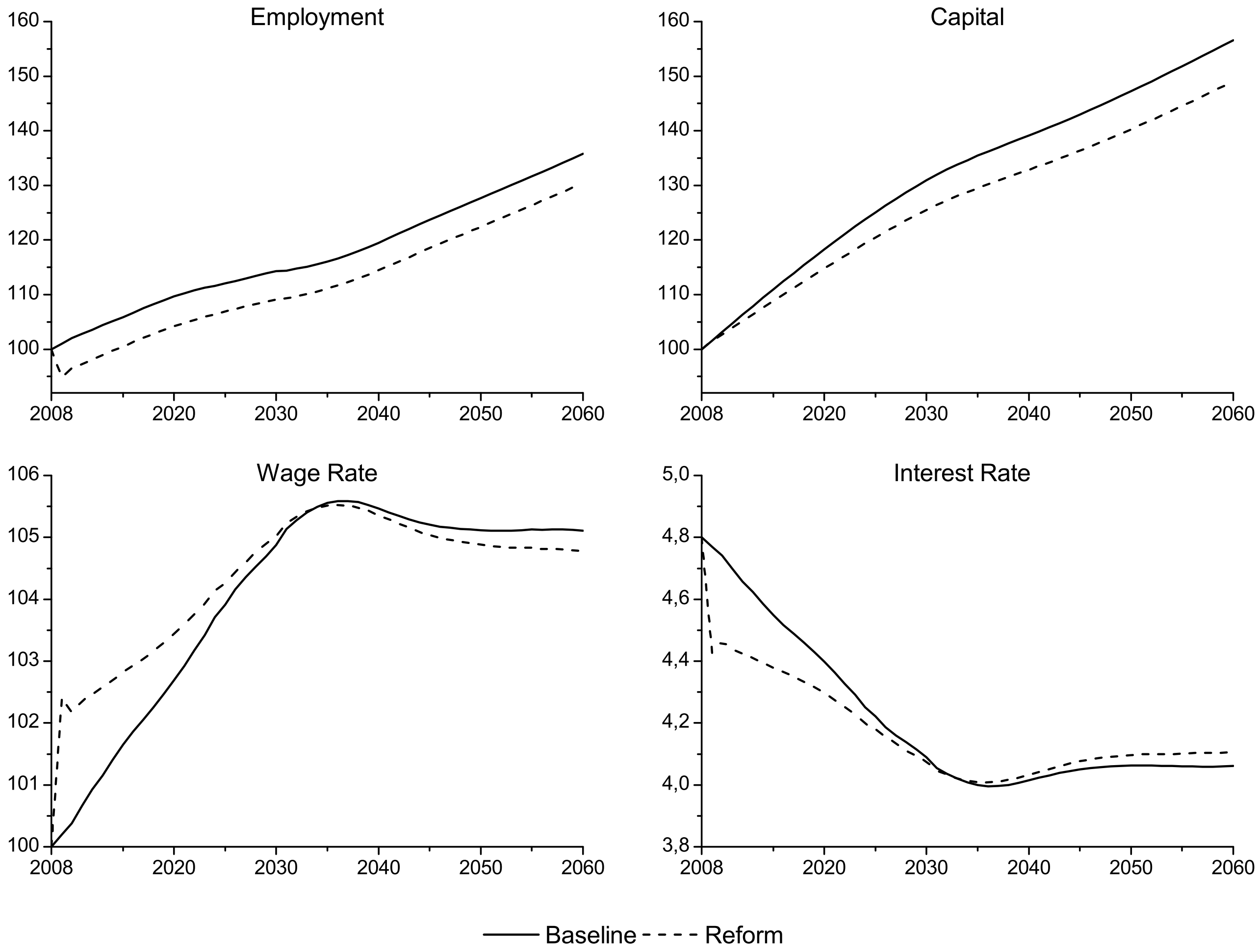
(LSRA) used by Auerbach and Kotlikoff (1987) in a separate simulation. The LSRA thereby proceeds as follows: to all generations already being economically active in 2008 it pays lump-sum transfers or levies lump-sum taxes in order to make them as well off in the reform path than in the baseline path. Consequently their compensating variation amounts to zero. Having done that, the LSRA might have run into debt or build up some assets. It now redistributes this debt or assets across all future generations in a way that they all face the same compensating variation, confer the dashed line in Figure 9. This variation can be interpreted as a measure of efficiency. Consequently, if the variation is greater than zero, the reform is Pareto improving after compensation and vice versa. With this concepts in hand, we can now proceed to our simulation results and the question of optimal progressivity of the pension system.

4.2 Flat pensions

In this section, we will show some detailed simulation results coming from the transition towards a fully flat pension system, i.e. we set $\lambda = 1$ from year 2009 onwards. The introduction of flat pensions comes with two reverse effects. On the one hand, whereas under the earnings related system a part of pension contributions was recognized as implicit savings, with flat pensions the complete contributions works as implicit tax, see below for a much more detailed analysis. Hence, the change to a flat pension benefit system increases labor supply distortions. On the other hand, a non-earnings-related pension also provides insurance against labor market risk.

Macroeconomic implications At first we want to take a look at the macroeconomic implications of our pension reform. Remember that the reform is introduced in year 2009. To clarify the impact of the reform on factor markets, Figure 10 compares in the upper part the evolution of employment and capital in the baseline simulation (solid line) and the reform scenario (dashed line). Since λ changes from 0 to a value of 1 immediately in year 2009, suddenly any worker accumulates one earning point per year and in consequence the whole contribution to the pension system is perceived as a

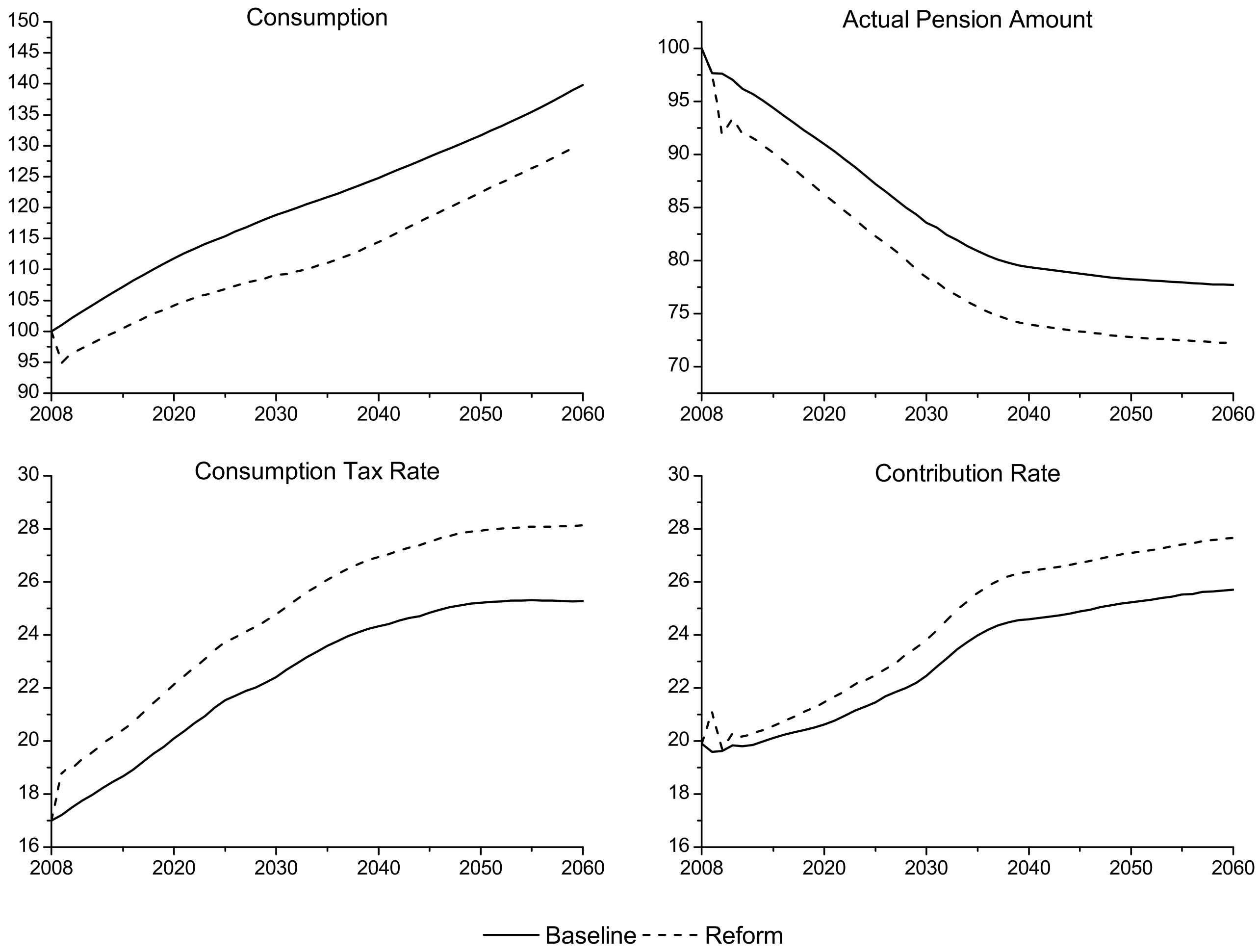
Figure 10: Factor markets before and after the reform



tax. This severely distorts employment downwards. Note that the change in the tax structure takes place in the reform year 2009 and is kept once and for all. Therefore, employment jumps in this year and then moves on in parallel with the baseline employment path. The decline in household labor supply comes along with a sharp increase in wages and a drop in interest rates. In opposite to employment, capital usually needs a longer time to adjust. Since by the drop in employment individual income shrinks, capital will successively decline in the long run, as people tend to save less on average. Obviously, individuals of different labor market status will react differently to this reform. A detailed analysis will clarify this later on. With the decline in capital and the following reduction in individual assets, factor markets again have to adjust and therefore wages decline and interest rates increase with time. Interestingly, the reduction in household asset supply outweighs the effects of employment decreases, which causes wages to be lower in the long run than in the baseline scenario.

Another interesting point to look at is the government sector analyzed in Figure 11. Since employment drops sharply in the short-run, output, household income and therefore aggregate consumption fall. The long-run decline in capital causes consumption in the baseline and the reform scenario to even diverge further. As government expenditure is held fix per capita and not in relation to output, the consumption tax rate has to increase in order to balance the tax system's budget. In the pension

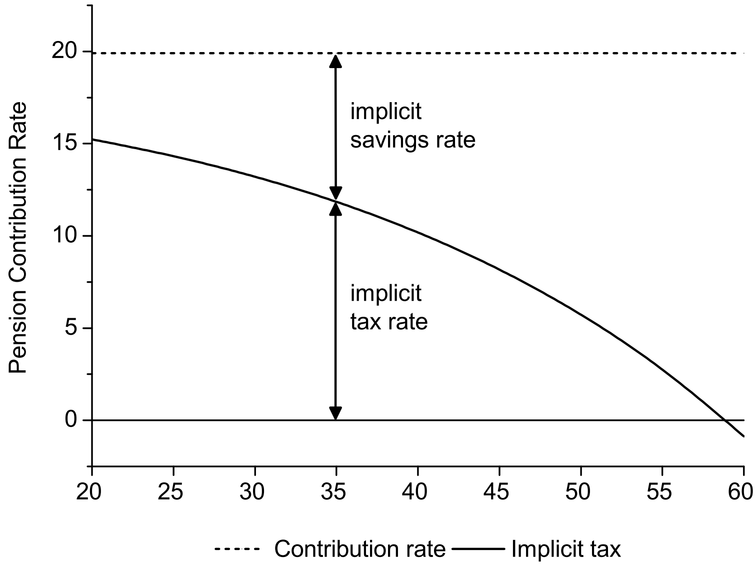
Figure 11: Consumption ans government sector before and after the reform



system, the actual pension amount, i.e. the monetary value of one earning point, is coupled with the evolution of lagged aggregate household income. Consequently, changes in this amount can first be seen in year 2010, where it decreases sharply induced by the fall in employment and the resulting change in household income. For the pension contribution rate, we therefore observe a sharp increase in 2009 due to the reduction in the pension contribution base paired with a reverse movement in 2010 resulting from the decrease in pension payments. In the long-run, the decline in wages narrows both the contribution base and pension expenditure. However, the former effects tends to be stronger than the latter, which is why the pension contribution rate again increases compared to the baseline scenario.

Implicit taxes and inequality So far, we only looked at the evolution of macroeconomic aggregates over time. However, we are also interested in the intra- and intergenerational welfare consequences of the flat pension reform. To understand these, we have to further examine the implicit tax and savings structure of the German pension system. Since pensions are earnings related in the baseline path, only a part of pension contributions is perceived as tax by individuals. The other part corresponds to implicit savings in the system. The implicit savings share can be computed from comparing the change in pension contributions implied by a marginal increase in labor supply at a specific age with the resulting increase in the present value of pension benefits, see Fehr and Kindermann (2010) for further details on the computation. Since earning points do not pay any interest, it is clear that the savings share must be lower at the beginning of the working life than shortly before retirement. However, due to the perfect tax benefit linkage, the share is the same for any individual of a cohort irrespective of his income. Figure 12 shows implicit tax and savings share over the life cycle in our model. At the beginning of the life-cycle the tax share amounts to roughly 75 percent

Figure 12: Implicit tax and savings over the life-cycle

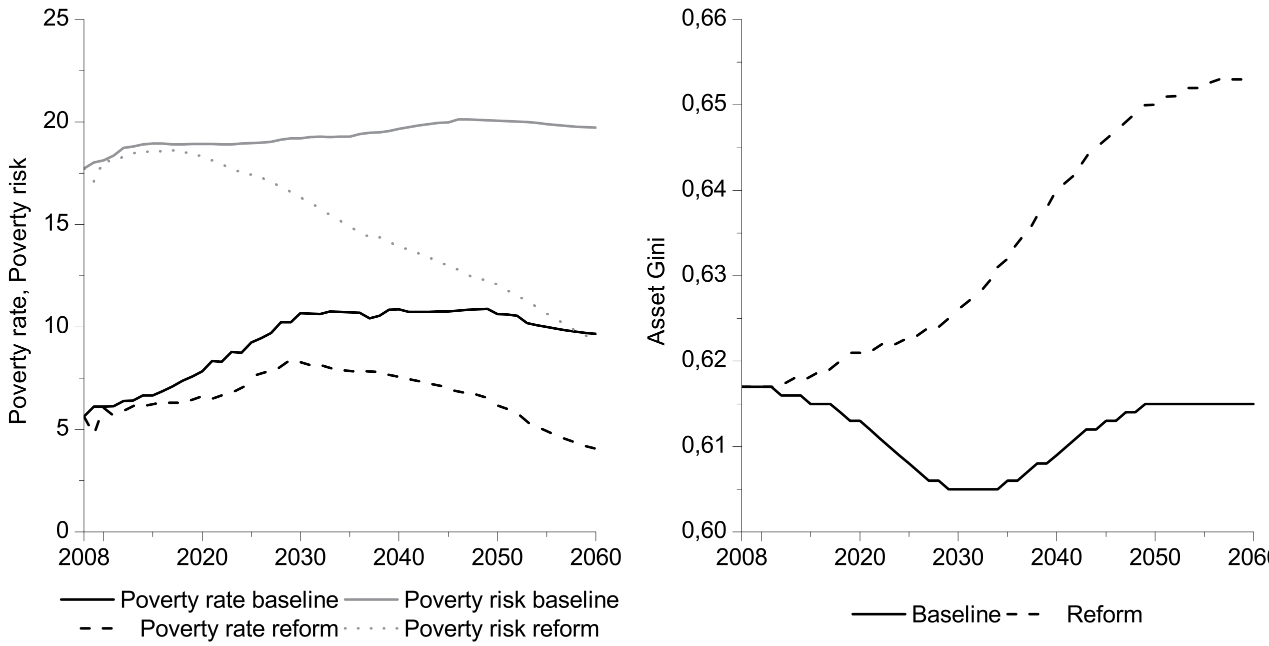


of the contribution. Throughout the working phase it successively increases. Interestingly, the tax share even turns negative shortly before retirement. During this time, individuals actually receive a savings subsidy from the pension system. Hence, contributions are very valuable during that time. This implicit tax structure will have a significant impact on individual welfare for different cohorts, see below.

Another important thing to look at, in order to understand intra-generational redistribution induced by the introduction of flat pensions, is inequality measures. Therefore, Figure 13 shows on the left hand side the evolution of poverty rates and poverty risk among the elderly generations. Obviously, since due to the reform everyone will receive the same pension independent of his former labor market income, our reform induces a strong redistribution in income from the richer to the poorer. In consequence of that, both the poverty rate and poverty risk of the elderly decreases dramatically from the introduction of flat pensions. However, the reform has another important effect. Since individuals that were facing bad labor market shocks now in proportion receive very high pension payments while individuals with high income get a relative low pension, agents have to adjust their savings to compensate for possible income gains or losses at retirement. Therefore, the gap in asset holdings between the richer and the poorer will increase from the reform. This can be seen from the right hand side of Figure 13 showing the evolution of the Gini coefficient for assets over time.

Welfare and efficiency With the above discussion in mind, we can now turn to the welfare effects of our reform. Therefore Table 5 summarizes welfare consequences measured in compensating variation for different cohorts. For agents already taking economic decisions in the reform year, we

Figure 13: Poverty measures on the baseline and the reform path



disentangle welfare effects in several ways. The left part of the Table reports for agents already retired in the reform year average welfare changes by their status in the pension system, i.e. regular or disability pensioner. The working cohorts are, beneath explicitly accounting for welfare of the disabled, grouped by their productivity level. We thereby merge the lower one sixth and the higher

Table 5: Welfare effects of flat pensions*

Birth year	Age in 2009	without LSRA: by status				without LSRA: by skill level			with LSRA
Retirees			*regular*		*disab.*	*low*	*middle*	*high*	
1920	89		-2.48		-2.46	-2.56	-2.52	-2.34	0.00
1940	69		-2.55		-2.72	-2.70	-2.65	-2.45	0.00
Workers		*low*	*medium*	*high*	*disab.*	*low*	*middle*	*high*	
1960	49	6.01	-0.39	-1.76	-3.20	-0.37	0.21	0.57	0.00
1980	29	6.69	-0.16	-1.31	–	0.15	0.99	1.07	0.00
Future Generations									
2000	9		0.62						-1.03
2020	–		0.11						-1.03
2040	–		-0.03						-1.03
2060	–		-0.08						-1.03

*In percent of initial resources.

one sixth of the overall productivity distribution in the states "low" and "high". The remaining part of the distribution is captured in state "medium".[7] In the right part of the table, we report average welfare changes by skill class. For future generations, we apply the concept of ex ante welfare and therefore only report one aggregate number per cohort. The first two columns indicate birth year of the respective cohorts and their age in the reform year 2009. Retirees face tremendous welfare losses from the reform. These are due to the strong increase in consumption taxes resulting from a decline in employment and the downward adjustment of the actual pension amount from year 2010 onwards. Obviously, the younger the cohort, the higher welfare losses, since consumption taxes rise stronger over times. For the working cohorts in the reform year, welfare effects are not so clear-cut. Noticeably, welfare changes decrease with age. This is a result of the implicit tax structure of the pension system discussed above. As older individuals were facing a very high savings share in pension contributions, they lose most when all contributions are turned into taxes by the introduction of flat pensions. However, intra-generational redistribution effects are even more interesting. What can be seen immediately is the redistribution from the richer towards the poorer induced by the progressive pension formula. This causes welfare of individuals in the bottom one sixth of the income distribution to rise by up to 6.7 percent, while average welfare especially of the highest one sixth of the productivity distribution decreases. An interesting effect can be seen when looking at the column "disab." denoting welfare changes of disabled agents. Like old-age pensioners, these individuals suffer from the increase in consumption taxes and the cut in the actual pension amount in the future.

The right panel of Table 5 indicates welfare changes for different skill levels. Among the elderly, the numbers are quite similar to the left panel. However, for working individuals, the lower educated are worse off from the reform than the higher skilled. This interesting result is due to two things: first, the income risk process accounts for a much larger fraction of heterogeneity in incomes than the distribution of skills. Second, and more important, disability risk is highest among individuals of low skills. Since disability pensioners loose from the reform, the average welfare change in this class is lowest and might even turn negative.

The short-run future generations partly gain from the introduction of flat pensions. This is mainly due to the sharp increase in wages and the insurance provision through the pension system outweighing the losses from labor market distortions. However, through time welfare decreases, since individual assets and therefore accidental bequests decline, which redistributes towards the earlier cohorts.

Finally, lets turn to welfare effects after LSRA compensation payments. As mentioned above, the LSRA makes all existing cohorts as well off as in the benchmark simulation and redistributes resources across future generations to make them all face the same welfare changes. The evolution of welfare after compensation is depicted in the right part of Table 5. We find that the reform induces losses for any future generation of about one percent of initial resources. The introduction of flat pensions comes along with two major efficiency consequences: on the one hand, insurance provision against labor market risk causes efficiency to rise while, on the other hand, increasing labor market distortions reduce it. We find in this reform scenario that the latter outweighs the former and therefore the introduction of completely flat pension benefits is Pareto inferior.

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[7] We chose exactly this partition of the work force due to computational reasons.

4.3 Optimal progressivity of the pension system

While it turned out in the previous subsection that a completely flat benefit system would reduce aggregate efficiency, it is not clear yet, whether a partly progressive pension system will also lead to efficiency losses. Having in mind that usually labor market distortions rise quadratically in tax rates, it might be that there is a combination of flat and earnings-related pensions that improves efficiency. The aim of this section is to clarify whether there is an optimal mix of flat and earnings-related pensions in our simulation model that improves aggregate efficiency. Therefore, Table 6 shows aggregate efficiency gains and losses for different choices of λ. This table reveals an interesting insight into the

Table 6: Efficiency effects of different values for λ^*

λ	0.00	0.20	0.40	0.60	0.80	1.00
ϕ	0.00	0.10	**0.48**	0.31	-0.03	-1.03

*In percent of initial resources.

opponent efficiency effects of pension progressivity. With an increasing flat pillar in the pension system, obviously both insurance provision and labor market distortions rise. However, at lower values of λ, the insurance effect clearly dominates labor supply distortions. Nevertheless, around a value of 40 percent, efficiency again starts to decline as labor supply distortions start to outweigh the insurance effects. Finally, a pension system in which 80 percent of pension payments are not related to earnings generates efficiency losses. As can be seen from the above table, we find a pension system with a 40 percent flat and a 60 percent earnings related pension to be optimal in our model.

Insurance provision versus labor market distortions As stated above, the efficiency gains arising from increasing the progressivity of the pension system are due to positive insurance and negative distortion effects. In order to disentangle the impact of these two, Table 7 reports welfare and efficiency effects in alternative model specifications. In the upper left panel, we show the effects of introducing the optimal pension system (i.e. $\lambda = 0.4$) in the regular setup used in the previous section.[8] Since the same driving forces like in the fully flat pension reform are at work in this case, welfare results are of same quality than those reported in Table 5, but less intense. On the right hand side, we assume labor supply to be fixed at the value of 0.4, i.e. the target value for average hours worked. In order to receive the same capital to output ratio in 2008, we have to adapt the time discount factor β and set it at 0.991. In the case of completely inelastic labor supply, there obviously is no employment effect in 2009, when the reform takes place. In consequence, the consumption tax rate as well as the actual pension amount remain nearly unaltered compared to the baseline scenario. Hence, welfare changes of already retired individuals (including the disabled) are nearly zero. Welfare for the working and future cohorts, on the other hand, is significantly higher than in the benchmark scenario, since labor supply distortions are absent. In total, converting 40 percent of the pension system into a flat pension pillar comes with an efficiency effect of 2.17 percent of initial resources, which reflects the gains from insurance provision against labor market risk.

[8] We will refer to this setup as our benchmark calibration.

Table 7: Welfare and efficiency of the mixed pension scheme ($\lambda = 0.4$)*

Birth year	Age in 2009	without LSRA by status				with LSRA	without LSRA by status				with LSRA
		Benchmark calibration					*Fixed labor supply ($\beta = 0.991$)*				
Retirees			*regular*		*disab.*			*regular*		*disab.*	
1920	89		-1.00		-1.00	0.00		-0.01		-0.01	0.00
1940	69		-1.01		-1.08	0.00		-0.06		-0.06	0.00
Workers		*low*	*medium*	*high*	*disab.*		*low*	*medium*	*high*	*disab.*	
1960	49	2.15	-0.13	-0.71	-1.24	0.00	4.01	0.27	-0.50	-0.08	0.00
1980	29	3.06	-0.06	-0.49	–	0.00	5.85	0.69	-0.35	–	0.00
Future Gen.											
2000	9		0.59			0.48		1.84			2.17
2020	–		0.38			0.48		1.63			2.17
2040	–		0.30			0.48		1.35			2.17
2060	–		0.27			0.48		1.22			2.17
		Risk neutral individuals ($\beta = 0.990$)					*High risk aversion ($\beta = 0.985$)*				
Retirees			*regular*		*disab.*			*regular*		*disab.*	
1920	89		-0.91		-0.91	0.00		-1.19		-1.19	0.00
1940	69		-0.91		-0.97	0.00		-1.20		-1.28	0.00
Workers		*low*	*medium*	*high*	*disab.*		*low*	*medium*	*high*	*disab.*	
1960	49	1.99	-0.13	-0.71	-1.16	0.00	2.24	-0.17	-0.78	-1.43	0.00
1980	29	1.65	-0.11	-0.57	–	0.00	4.81	0.29	-0.44	–	0.00
Future Gen.											
2000	9		-0.11			-0.56		2.65			2.45
2020	–		-0.28			-0.56		2.55			2.45
2040	–		-0.34			-0.56		2.45			2.45
2060	–		-0.35			-0.56		2.42			2.45

*In percent of initial resources.

The third specification reported in the lower left part of Table 7 is a scenario in which individuals are completely risk neutral. We therefore assume preferences to be of Epstein and Zin (1991) type and set risk aversion to zero. Note that in our benchmark calibration, the CES utility function implies a risk aversion of 2. The resulting welfare and efficiency effects are depicted in the lower part of the table. In the case of risk neutral individuals, obviously there are only labor supply distortions but no positive insurance effects. Consequently, welfare effects for the generations in the reform year are very similar to those in the upper left panel of the table, since individuals are confronted with a similar change in consumption taxes, actual pension amount and redistribution. The lack of gains from insurance provision can be seen best from the welfare losses of future generations. The decrease in welfare due to declining bequests, however, is still present. Overall, we find an efficiency loss of 0.56 percent of initial resources. Since especially the scenario where labor supply is fixed exhibits some systemic changes compared to the benchmark scenario, efficiency gains and losses of the two scenarios cannot just be added up. However, these sensitivity results give an insight into the model dynamics and emphasize the role of insurance provision and labor supply distortions. Finally, the lower right part assumes a coefficient of risk aversion of 4, which is still in the range of plausible values.[9] Of course now, we can see the opposite compared to the previous simulation. Therefore, future cohorts will value the insurance provision by the pension system much stronger, so that welfare gains are much higher especially in the long-run compared to the benchmark calibration. As a consequence, aggregate efficiency increases from 0.48 to 2.45 percent of aggregate resources. Since our benchmark assumption about risk aversion seems rather modest, this result strongly supports a more progressive pension system.

Time shocks for the disabled As already mentioned in the calibration section, our choice of h is quite conservative. To clarify to which extend this time cost influences our simulation results, we also ran a simulation in which we introduced a 40 percent flat pillar into the pension system an let $h = 0.6$. The respective efficiency gain was 0.70. This is due to the fact that in the long run, disability pensioners gain from redistribution. These gains are obviously higher the higher utility losses from disability, since marginal utility of consumption increases with h. However, the magnitude of this influence is rather small.

4.4 A note on the optimal system size

Finally, we want to use our model to make a short statement on the optimal size of the pension system. Since our disability pension system is directly coupled with regular old-age pensions, it is not surprising that a complete privatization would lead to strong efficiency losses in absence of private disability insurance. Consequently, we only simulate slight changes in the pension size parameter μ from 2009 onwards. The results are shown in Table 8. In addition to the effects from the disability

Table 8: The size of the system*

μ	0.80	0.90	1.00	1.10	1.20
ϕ	-0.34	-0.14	0.00	**0.12**	0.00

*In percent of initial resources.

insurance, altering the size of a paygo pension system comes along with different effects, a detail analysis of which can e.g. be found in Fehr, Habermann and Kindermann (2008). Summing up their arguments, the move towards a more funded system induces the following effects:

1. The switch from a front- to a back-loaded taxation resulting from the reduction in pension contributions enforces labor market distortions but, under a progressive tax system, increases insurance provision against labor market risk.
2. The loss of annuitization eliminates longevity insurance but also increases accidental bequests.

9 Typically, values between 1 and 5 are perceived as reasonable in the literature, see Cecchetti et al. (2000, p.792) for a discussion.

3. The abolition of mandatory savings relaxes liquidity constraints at the beginning of the life-cycle.

As a result of these effects, Fehr et al. (2008) find an efficiency loss from the complete privatization of the German pension system of about 0.6 percent of initial resources. In our present model, the pension system has one additional positive feature, namely it cares for the sustenance of the disabled. Hence, we would expect efficiency losses from pension privatization to be even stronger in our model. Table 8 supports this view. It shows that the current size of the pension system in Germany is close to optimal. An increase of 10 percent only comes at very small efficiency gains while a small reduction in size already leads to a decline in aggregate efficiency. Summing up, our results indicate the fact that the change towards a more progressive pension system and no reduction in its size seems to be the optimal choice for the German economy.

5 Discussion

This paper aims at determining the optimal progressivity of the pension system under the problem of population ageing. In order to clarify this question, we construct a model of overlapping generations which features realistic demographic projections, labor income, longevity and disability risk and endogenous labor supply at both the intensive and the extensive margin. We calibrate our base year to the German economy and consider recent pension and tax reforms in our baseline path. In this setup, we study the transition from the German fully earnings related pension regime towards systems with a flat pillar. We find for our most preferred parametrization that a pension system consisting of 40 percent flat and 60 percent earning related benefits is optimal for the German economy, since the positive insurance effect in this setup is stronger than the efficiency losses from labor supply distortions. Furthermore, a reduction in size only comes at efficiency costs.

The demand for more progressivity in the pension system is not new to the literature on simulated overlapping generations models. Fehr and Habermann (2008) already quantify the efficiency effects of progressive pension arrangements in an overlapping generations model. However, their findings and argumentation differ substantially from the present paper. Since their model does not feature changes in the population structure and takes the date of retirement as given, they tend to find a higher progressivity to be optimal especially in the case without basic allowances. The increasing burden from the rise in the old age dependency ratio in our model however causes pension contributions to become increasingly distortive even in a perfectly earnings related setup. Consequently, rising distortions from a reduction in this linkage weigh much stronger and we find a significantly lower degree of progressivity to be optimal in terms of aggregate efficiency. Furthermore, a detailed description of income risk factors and exit pathways to retirement makes the present model much more reliable in terms of the quantification of insurance and distortion effects.

In our model every individual that receives a disability shock will be eligible to disability insurance benefits. Yet, one might argue that in reality there is a serious information problem which not only causes healthy individuals to get access to disability pensions but also some disabled to be rejected. This problem is mainly discussed in the theoretical and empirical literature, see e.g. Diamond and Sheshinski (1995) or Duggan, Singleton and Song (2007). However, since the focus of this paper does not lie on disability risk and this type of uncertainty is not the major source of income risk, including such information constraints would only increase computational time but is very unlikely to reveal new insights. Furthermore, we completely neglected means-tested minimum income benefits. At the moment, only about 2.5 percent of retirees receives these benefits. However, since old-age poverty is expected to rise substantially in the future, this fraction would increase significantly. Obviously, a means-tested minimum income guarantee also provides insurance against labor market risk and old-age poverty. Hence, it might be that the efficiency gains from insurance are damped when such benefits are incorporated. Nevertheless, means-tested minimum income guarantees also severely distort old-age savings. Hence, considering such an income guarantee should be left to future research.

A Technical appendix

In this appendix we want to give a more equation based definition of our model and define what an equilibrium path is.

A.1 Detailed formulation of the household problem

In order to state the household optimization problem in a recursive fashion, we first want to define in detail what the household state vector is.

Definition 1 (Household state vector) *Let*

$$z = (j, s, a, ep, \eta, d, o) \in \mathcal{Z} = \mathcal{J} \times \mathcal{S} \times \mathcal{A} \times \mathcal{P} \times \mathcal{E} \times \mathcal{D} \times \mathcal{R}$$

where $\mathcal{J} = \{1, 2, \ldots, J\}$, $\mathcal{S} = \{1, 2, \ldots, S\}$, $\mathcal{A} = [0, \infty]$, $\mathcal{P} = [0, \infty]$, $\mathcal{E} = [-\infty, \infty]$, $\mathcal{D} = \{0, 1\}$, *and* $\mathcal{R} = \{0, 1\}$. *Then z completely describes the individual state of a household. In the following we will use the abbreviation*

$$z_j = (s, a, ep, \eta, d, o)$$

for the sake of simplicity.

With this definition of the household state, we can now turn to the description of the household optimization problem. We define this problem separately for working and retired agents. The decision problem of an individual with state $z_j = (s, a_j, ep_j, \eta_j, 0, 0)$ at age j and time t, i.e. an individual that is still participating in the labor market, is given by

$$V_t(z_j) = \max_{c,\ell} \Bigg\{ u(c, \ell) + \beta\psi_{t,j+1,s} \Bigg[(1 - \pi^d_{j+1,s}) \int_{\mathcal{E}} \Big((1 - o_{j+1}) V_{t+1}(z^w_{j+1}) + o_{j+1} V_{t+1}(z^r_{j+1}) \Big) \pi(\eta'|\eta, s)\, d\eta' + \pi^d_{j+1,s} V_{t+1}(z^d_{j+1}) \Bigg] \Bigg\},$$

with the terminal condition $V_t(z_{J+1}) = 0,\ \forall t$. The three different combinations for z_{j+1} define the states in which the agent is still working in the next period, in which he chooses to retire and in which he receives a disability shock, i.e.

$$z^w_{j+1} = (s, a_{j+1}, ep_{j+1}, \eta_{j+1}, 0, 0) \quad , \quad z^r_{j+1} = (s, a_{j+1}, ep_{j+1}, 0, 0, 1) \quad \text{and} \quad z^d_{j+1} = (s, a_{j+1}, ep_{j+1}, 0, 1, 1).$$

The continuous state variables a, ep and η are subject to the following laws of motions: assets evolve according to

$$a_{j+1} = (1 + r_t)a_j + y + b - \tau_t y - T_t(y, 0, r_t a_j) - (1 + \tau_{c,t})c,$$

with labor income $y = w_t e_j(1 - \ell)$[10], accidental bequests b, the pension contribution rate τ_t, the tax schedule $T_t(\cdot)$ and the consumption tax rate $\tau_{c,t}$. As already mentioned above, individuals accumulate earning points according to the formula

$$ep_{j+1} = ep_j + \mu_t \left[(1 - \lambda_t)\frac{y}{\bar{y}_t} + \lambda_t \right]$$

with the economy wide average labor income $\bar{y}_t$ at time t. Finally, log labor productivity shock evolves according to the AR(1) process

$$\eta_{j+1} = \varrho \eta_j + \varepsilon \quad \text{with} \quad \varepsilon \sim N(0, \sigma_\varepsilon^2).$$

According to this process, we can calculate the distribution function $\pi(\cdot|\cdot,\cdot)$ of η_{j+1} conditional on the realization η_j and the individual skill level s. Note that the labor productivity shock does not play a role for retired agents. Hence, we set η_j to zero when an agent chooses to retire.

An important factor for calculating expected future utility is the retirement decision in the next period o_{j+1}. Note that the retirement decision is made after all current shocks are revealed, i.e. when disability status and labor productivity are known. The individual then makes a retirement decision via comparing utilities from working for another period $o_{j+1} = 0$ and switching to the state of a pensioner $o_{j+1} = 1$, i.e.

$$o_{j+1} = \begin{cases} 1 & \text{if} \quad V(z^r_{j+1}) > V(z^w_{j+1})(1 - \nu) \quad \text{and} \\ 0 & \text{otherwise.} \end{cases}$$

For the already retired, the optimization problem is much simpler. Since they do not work anymore, their leisure consumption is already determined and they therefore only have to decide about regular consumption via

$$V_t(z_j) = \max_c \left\{ u(c, \ell) + \beta \psi_{t,j+1,s} V_{t+1}(z_{j+1}) \right\}$$

where $\ell = 1$ for old-age and $\ell = h$ for disability pensioners. Next periods state vector is then given by

$$z_{j+1} = (s, a_{j+1}, ep_j, 0, d_j, 1),$$

where assets evolve according to

$$a_{j+1} = (1 + r_t)a_j + p - T_t(0, p, r_t a_j) - (1 + \tau_{c,t})c.$$

Thereby

$$p(z_j) = \begin{cases} ep_j \times APA_t & \text{if} \quad o_j = 1 \quad \text{and} \\ 0 & \text{otherwise} \end{cases}$$

[10] Thereby, $e_j = e(z_j)$ denotes labor productivity that depends on age, skill level and labor market shock η_j.

defines pension payments that depend on the amount of earning points accumulated during the working periods. Note that individuals above age 60 will not receive any accidental bequests in our model.

Having specified the household decision problem in detail, we finally want to define household choices and the measure of households. The former thereby defines the decisions of households at a specific state, the latter at which states households are located.

Definition 2 (Optimal household decisions and measure of households) *For any $t \in \mathcal{T} = \{1, 2, \ldots, \infty\}$ let*

$$\mathbb{H}_t = \{c_t(\cdot), \ell_t(\cdot), o_t(\cdot), a'_t(\cdot), ep'_t(\cdot)\}$$

with functions

$$\begin{aligned} &c_t(\cdot) : \mathcal{Z} \rightarrow [0, \infty] \quad , \quad \ell_t(\cdot) : \mathcal{Z} \rightarrow [0, 1] \quad , \quad o_t(\cdot) : \mathcal{Z} \rightarrow \{0, 1\} \quad , \\ &a'_t(\cdot) : \mathcal{Z} \rightarrow [0, \infty] \quad \text{and} \quad ep'_t(\cdot) : \mathcal{Z} \rightarrow [0, \infty].^{11} \end{aligned}$$

Furthermore let for any $t \in \mathcal{T}$

$$\xi_t(\cdot) : \mathcal{Z} \rightarrow [0, \infty].$$

be a measure on the measurable space

$$\Big(\mathcal{Z}\ ,\ P(\mathcal{J}) \times P(\mathcal{S}) \times B(\mathcal{A}) \times B(\mathcal{P}) \times B(\mathcal{E}) \times P(\mathcal{D}) \times P(\mathcal{R})\Big),$$

where B is the Borel σ-algebra of a continuous set and P the power set of a discrete set. We say that $\mathbb{H}_t$ is a set of optimal household decisions, if – given prices and public policy – $\mathbb{H}_t$ satisfies the above optimization problem. Furthermore, we denote by ξ_t the measure of households at time t.

A.2 Firms behavior

Next, we want to define firms behavior.

Definition 3 (Production plan and price set) *For any $t \in \mathcal{T}$ let $\mathbb{Q}_t = \{K_t, L_t\}$ be the production plan for the firms and denote by $\mathbb{P}_t = \{r_t, w_t\}$ the set of prices for capital and labor.*

The factor prices are set in a competitive way, given the production plan $\mathbb{Q}_t$.

Definition 4 (Competitive factor prices) *We say that factor prices are competitive, if*

$$r_t = (1 - \tau_k)\left(\frac{\partial Y_t}{\partial K_t} - \delta_k\right)^{12} \quad \text{and} \quad w_t = \frac{\partial Y_t}{\partial L_t}.$$

[11] a'_t and ep'_t thereby define future assets a_{j+1} and earning points ep_{j+1} depending on the current state and consumption decisions.

[12] Note that, due to the perfect competition assumption, corporate taxes in our model works as additional taxes to capital income of households.

A.3 Governmental activity

The last sector to specify is the government sector.

Definition 5 (Tax and pension policy) *Let* $\mathbb{G}_t = \{G_t, \tau_k, T_t(\cdot,\cdot,\cdot), B_t, \tau_{c,t}, \mu_t, \lambda_t, APA_t, \tau_t\}$ *define governmental policy at time t, where* G_t *is public expenditure,* τ_k *the corporate tax rate,* T_t *the income tax schedule,* B_t *governmental debt holdings and* $\tau_{c,t}$ *the consumption tax rate at time t.* APA_t *and* τ_t *define the actual pension amount per earnings points and* τ_t *the contribution rate to the pension system.*

The government adjusts the the consumption tax rate in order to balance the tax system.

Definition 6 (Budget balance of the tax system) *Given a set of household decisions* $\mathbb{H}_t$*, a measure of households* ξ_t *and a set of production plans* $\mathbb{Q}_t$ *and prices* $\mathbb{P}_t$*, we say that the tax system is balanced at time* $t \in \mathcal{T}$*, if*

$$G_t + (1 + r_t)B_t = \tau_c \int_{\mathcal{Z}} c_t(z)\ d\xi_t + T_y + \frac{\tau_k}{1-\tau_k} r_t K_t + (1 + n_{t+1})(1+\kappa)B_{t+1}$$

with

$$T_y = \int_{\mathcal{Z}} T_t\big(w_t e(z)(1 - \ell_t(z))\ ,\ p(z)\ ,\ r_t a_t(z)\big)\ d\xi_t$$

defines revenue from income taxation.

On the other hand, the pension contribution rate is adjusted to balance the pension budget.

Definition 7 (Budget balance of the pension system) *Given a set of household decisions* $\mathbb{H}_t$*, a measure of households* ξ_t *and a set of prices* $\mathbb{P}_t$*, we say that the tax system is balanced at time* $t \in \mathcal{T}$*, if*

$$\tau_t \int_{\mathcal{Z}} w_t e(z)(1 - \ell_t(z))\ d\xi_t = \int_{\mathcal{Z}} p(z)\ d\xi_t.$$

A.4 Equilibrium definition

Having specified the behavior of households, firms and the government, we can now turn to the definition of a competitive equilibrium in our model. Before we do that, we want to specify the economic environment households, firms and the government face.

Definition 8 (Economic environment) *The economic environment in our model is defined as the set*

$$\mathbb{E} = \Big(\{n_t\}_{t\in\mathcal{T}}\ ,\ \{\psi_{t,j,s}\}_{t\in\mathcal{T},j\in\mathcal{J},s\in\mathcal{S}}\ ,\ \{\pi^d_{j,s}\}_{j\in\mathcal{J},s\in\mathcal{S}}\ ,\ \pi(\cdot|\cdot,\cdot)\ ,\ \Gamma(\cdot)\ ,\ \theta\ ,\ \epsilon\ ,\ \delta_k\ ,\ \kappa\Big)$$

of population growth rates n, individual survival probabilities ψ*, disability risk probabilities* π^d*, the conditional distribution function* π *for the evolvement of* η*, a distribution function for bequests* $\Gamma : \mathcal{Z} \to [0,1]$ *that satisfies*

$$\int_{\mathcal{Z}} \Gamma(z)\ d\xi_t = 1.$$

and production technology θ*,* ϵ*,* δ_k *and* κ*.*

Obviously, when we denote individual bequests as $b(z) = \Gamma(z)Q_t$, where

$$Q_t = \frac{1}{(1+n_t)(1+\kappa)} \int_{\mathcal{Z}} (1+r_t)a'_{t-1}(z)(1-\psi_{t-1,j,s})\, d\xi_t,$$

we assure that no resources are wasted. Finally, we define an equilibrium path.

Definition 9 (Competitive equilibrium) *Given an economic environment $\mathbb{E}$ and initial conditions K_1 and ξ_1, a competitive equilibrium is a set of optimal household decisions $\{\mathbb{H}_t\}_{t\in\mathcal{T}}$, production plans $\{\mathbb{Q}_t\}_{t\in\mathcal{T}}$, competitive factor prices $\{\mathbb{P}_t\}_{t\in\mathcal{T}}$ and budget balancing tax and pension policies $\{\mathbb{G}_t\}_{t\in\mathcal{T}}$ that satisfy the following conditions:*

1. *Market clearance:*

$$\begin{aligned} K_t &= \int_{\mathcal{Z}} a_t(z)\, d\xi_t - B_t, && \textit{(Capital market)} \\ L_t &= \int_{\mathcal{Z}} (1-\ell_t(z))e(z)\, d\xi_t, && \textit{(Labor market)} \\ \theta K_t^{\epsilon} L_t^{1-\epsilon} &= \int_{\mathcal{Z}} c_t(z)\, d\xi_t + G_t + (1+n_{t+1})(1+\kappa)K_{t+1} - (1-\delta_k)K_t && \textit{(Goods market).} \end{aligned}$$

2. *Law of motion: For any subset $C \subseteq \{2,\dots,J\} \times \mathcal{S} \times \mathcal{A} \times \mathcal{P} \times \mathcal{E} \times \mathcal{D} \times \mathcal{R}$,*

$$\xi_{t+1}(C) = \int_{\mathcal{Z}} \frac{\psi_{t,j+1,s}}{1+n_{t+1}} P_t(z,C)\, d\xi_t,$$

where

$$P_t(z,C) = \begin{cases} (1-\pi^d_{j+1,s})\pi(\eta'|\eta,s) & \textit{if } (j{+}1, s, a'_t(z), ep'_t(z), \eta', 0, 0) \in C \textit{ and } z = (j,s,a,ep,\eta,0,0), \\ 1-\pi^d_{j+1,s} & \textit{if } (j{+}1, s, a'_t(z), ep'_t(z), 0, 0, 1) \in C \textit{ and } z = (j,s,a,ep,\eta,0,0), \\ \pi^d_{j+1,s} & \textit{if } (j{+}1, s, a'_t(z), ep'_t(z), 0, 1, 1) \in C \textit{ and } z = (j,s,a,ep,\eta,0,0), \\ 1 & \textit{if } (j{+}1, s, a'_t(z), ep'_t(z), 0, d, 1) \in C \textit{ and } z = (j,s,a,ep,\eta,d,1), \\ 0 & \textit{otherwise.} \end{cases}$$

For any $C = C_J \times C_S \times C_A \times C_P \times C_E \times C_D \times C_R \subseteq \{1\} \times \mathcal{S} \times \mathcal{A} \times \mathcal{P} \times \mathcal{E} \times \mathcal{D} \times \mathcal{R}$,

$$\xi_{t+1}(C) = \begin{cases} \sum_{s\in C_s} \varpi_s \left[\int_{C_E} \varphi_{0,\sigma_\varepsilon^2}(\eta)\, d\eta\right] & \textit{if } 0 \in C_A,\ 0 \in C_P,\ 0 \in C_D \textit{ and } 0 \in C_R, \\ 0 & \textit{otherwise.} \end{cases}$$

Hereby, $\varphi_{0,\sigma_\varepsilon^2}$ denotes the probability density function of the normal distribution with mean 0 and variance σ_ε^2.

Definition 10 (Stationary equilibrium) *A stationary equilibrium is a competitive equilibrium in which per capital variables and functions, as well as prices are constant over times and aggregate variables grow at the constant rate $n + \kappa + \kappa n$.*

B Computational appendix

This section gives an overview over the solution methods used to solve our model numerically. We distinguish between a micro- and a macroeconomic solution method. The former is used to solve the household problem, while the letter serves to compute equilibrium prices and quantities.

B.1 Solving the household problem

In order to compute a solution of the complex household problem, we discretize the continuous elements of our state space $\mathcal{Z}$. We therefore choose $\hat{\mathcal{A}} = \{a^1, \ldots, a^{n_A}\}$, $\hat{\mathcal{P}} = \{ep^1, \ldots, ep^{n_P}\}$, and $\hat{\mathcal{E}} = \{\eta^1, \ldots, \eta^{n_E}\}$. We use the algorithm described in Tauchen and Hussey (1991) to obtain an approximation to the distribution of η with our set $\mathcal{E}$ and a suitable probability function $\hat{\pi}(\cdot|\cdot, \cdot)$. For all the resulting discrete values of z_j we compute the optimal decision of households from the household optimization problem described above. Since $V_{t+1}(\cdot)$ consequently is also only known in a discrete set of points z_{j+1}, this maximization problem can not be solved analytically. Therefore we have to use the following numerical maximization and interpolation algorithms to compute households optimal decision:

1. Compute household decisions at the last possible age J for all possible z_J. Note that the terminal condition $V_t(z_{J+1}) = 0$, households are not allowed to work anymore and they die for sure in the next period. Hence, they consume all their resources.

2. Find the solution to the household optimization problem for all possible z_j recursively using Powell's algorithm, see Press et al. (2001, 406ff.). Since this algorithm requires a continuous function, we have to interpolate $V_{t+1}(z_{j+1})$. Having computed the data $V_{t+1}(z_{j+1})$ for all $z_{j+1} \in \mathcal{S} \times \mathcal{A} \times \mathcal{P} \times \mathcal{E} \times \mathcal{D} \times \mathcal{R}$ in the last step, we can now find a piecewise polynomial function $sp_{t+1,j+1}$ that satisfies the interpolation conditions

$$sp_{t+1,j+1}(\mathrm{a}^k, ep^l) = EV(z_{j+1}) \tag{1}$$

   for all $k = 1, \ldots, n_A$, $l = 1, \ldots, n_P$. In this paper we use multidimensional spline interpolation, see Habermann and Kindermann (2007).

B.2 The macroeconomic computational algorithm

The computation method for the macroeconomic model follows the Gauss-Seidel procedure of Auerbach and Kotlikoff (1987). We start with a guess for quantities and government policy. Then we compute prices, optimal household decisions, and value functions. This involves a discretization of the state space which is explained in the previous section. Next we obtain the measure of households and new macroeconomic quantities as well as the social security tax rate and the consumption tax rate that balances government's budgets. This information allows us to update the initial guesses. The procedure is repeated until the initial guesses and the resulting values for quantities, prices and public policy have sufficiently converged.

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