

The Cash-In-Advance Constraint in Monetary Growth Models

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Abstract

In most monetary models of economic growth, higher long-run inflation is associated with a decline in the growth rate and employment. We show that this result is sensitive with respect to the specification of the cash-in-advance constraint. We consider three types of endogenous growth models: 1) the AK-model, 2) the Lucas (1990) supply-side model, and 3) the two-sector model of Jones and Manuelli (1995). With the standard cash-in-advance constraint on consumption, higher inflation results in lower growth and employment in all three models, while, in the cash-credit good economy of Dotsey and Ireland (1996), the effect is the exact opposite.

JEL-Code: O420.

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1 Introduction

Inflation and economic growth are a central subject in the literature both on growth and monetary economics. Most empirical cross-country studies support the fact that inflation has a negative effect on growth.¹ However, this evidence is much less clear-cut for countries characterized by low inflation so that inflation appears to have a non-linear effect on growth.²

Early monetary growth models emphasize the effect of inflation on savings, while the growth rate of technology is exogenous so that the rate of money growth has only transitional effects on the growth of per-capita income.

However, the results on the relation between inflation and savings implied by these and more recent models are ambiguous. In his pioneering work, Sidrauski (1967) studies a general equilibrium model with money in the utility function³ and finds that the rate of money growth has no effect on the capital-labor ratio and, hence, on per-capita income in the steady state. Money is superneutral and inflation, thus, does not affect the savings rate in the long-run.⁴ Stockman (1981) shows that the same result is also obtained if money holdings are motivated via a cash-in-advance (CIA) constraint on consumption. However, if the CIA constraint applies to investment, a higher money growth rate reduces the savings rate. Den Haan (1990) considers a shopping-time model where inflation distorts the allocation of time on shopping, leisure, and labor. For higher inflation the opportunity costs of money increase, and agents reallocate more time to shopping activities. As a consequence, savings decrease. In Heer and Süßmuth (2007), inflation reduces savings through the “Feldstein Channel”.

¹See, among others, Barro (2001), Kormendi and Mequire (1985), McCandless and Weber (1995), Bruno and Easterly (1998), and Gillman and Kejak (2005), and Walsh (2010, Chapter 1) for a survey.

²In a recent study using cross-country panel data in a dynamic GMM model, López-Villavicencio and Mignon (2011) find that there exist a threshold level of inflation below which higher inflation results in higher economic growth.

³In previous work on the effect of inflation on the portfolio allocation, Tobin (1965) assumed a constant savings rate.

⁴If labor supply is elastic, inflation may increase or decrease the savings rate depending on the functional form of utility.

As argued by Feldstein (1982), loose monetary policy can increase the real capital income tax burden in a nominally based tax system. In a similar vein, Brock and Turnovsky (1981) focus on the financing decision of the firm. As the consequence of different tax treatment of bonds and equity, money growth and, hence, inflation have different effects on the steady-state capital intensity depending on the capital structure employed by the firm. In particular, higher inflation reduces the capital intensity under equity financing, while the opposite result holds under bond financing, if the corporate income tax rate is higher than the household income tax rate.

There is also a variety of studies that analyze the effects of inflation in models of endogenous growth. In this vein, Jones and Manuelli (1995) review several growth models, including the AK-model and the model with human capital accumulation. A role of money is introduced with the help of a CIA constraint (on consumption). In their models, they find relative modest effects of inflation on economic growth. Inflation distorts the leisure-consumption choice of the households so that labor decreases. The same mechanism is at work in Gomme (1993) and Wu and Zhang (1998) where higher inflation results in lower employment and, hence, less economic growth. In the two-sector monetary growth model of Maußner (2004), the effect of inflation on the growth rate depends on the value of the intertemporal rate of substitution. A value smaller than one implies a negative relation between the growth rate of money supply and the growth rate of per-capita income. Gillman and Kejak (2005) study the sensitivity of two-sector endogenous growth models with respect to the specification of the CIA. They find a robust negative effect of inflation on growth.

In our analysis, we study the sensitivity of this result with respect to the specification of the CIA constraint in three endogenous growth models: 1) the AK-model, 2) the Lucas (1990) supply-side model, and 3) the model with human capital accumulation as in Jones and Manuelli (1995). All models are calibrated for the US economy. We are able to replicate the above finding that inflation reduces growth when we use the standard CIA constraint on consumption. However, if we consider the cash-credit good economy of Dotsey and Ireland (1996), we find a positive effect of inflation on growth. Our results are summarized in Table 1.1.

Table 1.1

Summary: Effects of money growth on per-capita income growth

	CIA		DI	
	$\pi = 0\%$	$\pi = 10\%$	$\pi = 0\%$	$\pi = 10\%$
AK-model	2.00%	1.96%	2.00%	2.01%
Lucas (1990)	2.00%	1.98%	2.00%	2.01%
Jones and Manuelli (1995)	2.00%	1.93%	2.00%	2.02%

Notes: The two columns with caption CIA display the effect of the growth rate of money on the growth rate on per-capita output in the case of the standard cash-in-advance constraint (CIA). We compare two steady states: one with zero inflation and one with ten percent annual inflation. The columns under the caption DI present the results, if money holdings are motivated as in Dotsey and Ireland (1996) (DI).

In the case of the standard CIA constraint, higher inflation introduces an inflation tax on the labor supply. For an increase of inflation from zero to ten percent, employment declines by approximately one percent in the AK-growth mode, while the growth rate decreases from 2.0% to 1.96%. Inflation has a smaller quantitative effect on the growth rate in the Lucas (1990) model, but a sizable effect in the Jones and Manuelli (1995) model, where the growth rate declines by 0.07 percentage points.

In the case of the CIA constraint in Dotsey and Ireland (1996), consumers purchase consumption goods with either cash or credit where the latter is subject to transaction costs in the financial market. If inflation increases, households buy a smaller number of goods with cash. As a consequence, an increase of inflation from zero to ten percent results in a (very small) rise of the growth rate from 2.00% to 2.01%. To understand the mechanism behind this result, consider the labor market clearing condition in the baseline AK-model:

$$w = [1 + (r + \pi)]MRS,$$

where w , r , π , and MRS denote the real wage, the real interest rate, the rate of inflation, and the marginal rate of substitution between hours and consumption, respectively. The nominal interest rate factor $1 + r + \pi$ drives a wedge between the real

wage and the marginal rate of substitution (the inflation tax). Therefore, a higher rate of inflation reduces employment, and, in the end, economic growth. In the Dotsey and Ireland (1996) model the labor market clearing condition becomes

$$w = [1 + (r + \pi)(1 - \zeta)]MRS.$$

ζ is the endogenously determined number of goods purchased on credit. With increasing inflation the costs of holding real money balances increase and the household accumulates less money. The CIA constraint $m \leq (1 - \zeta)c$ becomes more severe. If ζ were fixed, the household would have to reduce consumption. Instead, he can acquire more goods on credit and can, thus, relax the CIA constraint. Depending on the relative strength of the impact of π on the wedge and the ensuing increase in the share of credit goods, employment may either decline or increase.

In addition, we study whether the latter result also holds up in a labor market that is subject to frictions in the matching of workers and vacancies.⁵ Our results are summarized in Table 1.2. In accordance with our previous results, we find that higher inflation results in higher growth and employment in all three models of endogenous growth. However, effects are quantitatively small, except for the unemployment rate, which declines by 0.21 percentage points in all three models.

In our models, the relation between unemployment and inflation is u-shaped. The rate of unemployment declines with increasing inflation attaining a minimum at an annual inflation rate of about ten percent and increases for inflation rates beyond this point. The empirical evidence on this effect is mixed. While some studies find no evidence for a favorable long-run trade-off between inflation and unemployment as King and Watson

⁵To the best of our knowledge, there does not exist any theoretical model of long-run effects of inflation on economic growth that departs from the assumption of Walrasian labor markets. This stands in deep contrast to the literature that studies short-run effects of inflation and monetary policy in dynamic general equilibrium models. Recent models that stress the importance of labor market frictions for the propagation of monetary (and technology) shocks include Christiano et al. (2005), Walsh (2005), or Heer and Maußner (2010), among others. In this work, frictions in the labor markets have been shown to be central to the inflation-employment trade-off and the persistence of inflation and output responses.

(1994, 1997) for the US, other studies find that inflation reduces unemployment, as, for example, in Setterfield and LeBlond (2003) for the US, Shadman-Mehta (2001) for the UK, and Karanassou et al. (2003) for various European countries.

Table 1.2
Sensitivity: Models with labor market search

	Growth Rate		Unemployment Rate		Employment Rate	
	$\pi = 0\%$	$\pi = 10\%$	$\pi = 0\%$	$\pi = 10\%$	$\pi = 0\%$	$\pi = 10\%$
AK-model	2.00%	2.037%	6.00%	5.79%	64.0%	64.2%
Lucas (1990)	2.00%	2.004%	6.00%	5.79%	64.0%	64.1%
Jones and Manuelli (1995)	2.00%	2.003%	6.00%	5.79%	64.0%	64.1%

Notes: The results presented in this table refer to the models with costly credit as in Dotsey and Ireland (1996), i.e., the DI case of Table 1.1.

The paper is structured as follows. In Section 2, we present the AK-model with two different specifications for the cash-in-advance constraint. In Section 3, we study the balanced growth equilibrium. In Section 4, we analyze two models of human capital accumulation. In the first model, human capital increases with the time spent on learning as in Lucas (1990). In the second model, we follow Jones and Manuelli (1995) where the households use goods in order to increase human capital. Section 5 extends the consideration to an economy with labor market frictions. Section 6 concludes. Equilibrium conditions, the system of the dynamic equations of all three model variants with a Walrasian labor market on the one hand, and search frictions on the other hand, are described in more detail in the Appendix.

2 The model

The model introduces money demand in the endogenous growth model of Romer (1986). Money is incorporated by assuming that households finance part of their consumption

by using cash. The other part of consumption is financed by credit that is either free of transactions costs or costly as in the economy of Dotsey and Ireland (1996).

The economy consists of four sectors: households, production firms, financial intermediaries, and the monetary authority. The representative household maximizes his expected intertemporal utility subject to his budget constraint and a CIA. Firms produce a consumption-investment good using capital and labor. Financial markets provide credit services, while the central bank supplies money.

2.1 Households

Households are of measure one and maximize

$$\int_0^{\infty} u(c, n) e^{-\rho t} dt. \quad (2.1)$$

We parameterize the current-period utility function as:

$$u(c, n) = \ln c - \beta \frac{n^\eta}{\eta}. \quad (2.2)$$

c , n , and ρ denote consumption, labor supply, and the discount rate of the household, respectively.

Consumption financing. Consumers can purchase consumption with either cash or credit as in Schreft (1992), Gillman (1993), or Dotsey and Ireland (1996). The consumption goods are indexed by $i \in [0, 1]$, and the consumption aggregator is given by $c = \inf_i \{c(i)\}$. Therefore, the individuals will consume the same amount of all goods.

In the first specification of the monetary economy, the fraction ζ of consumption goods is financed on credit, while the fraction $1 - \zeta \in [0, 1]$ of goods is financed with cash:

$$c(1 - \zeta) \leq \frac{M}{P}, \quad (2.3)$$

where M and P denote nominal money and the price level, respectively. The inflation rate is defined by $\pi \equiv \frac{\dot{P}}{P}$.⁶ In this case, the credit costs are zero and the fraction ζ is given exogenously.

In the second specification that follows Dotsey and Ireland (1996), the consumer chooses the fraction $\zeta \in [0, 1)$ of goods i he purchases on credit. In order to buy an amount c of good i on credit, the household, however, must purchase $\kappa(i)$ units of financial services. The function $\kappa(i)$ is strictly increasing in i , and satisfies $\lim_{i \rightarrow 1} \kappa(i) = \infty$. According to the latter assumption, some goods will be purchased with cash, and the demand for money is well defined. In particular, we parameterize the transaction technology as:

$$\kappa(i) = \kappa_0 \left(\frac{i}{1-i} \right)^x. \quad (2.4)$$

Intermediation of credit services is subject to perfect competition, and in order to produce one unit of service one unit of labor is used. In equilibrium, the financial service companies make zero profit, and the fees per unit of financial service sold are equal to the wage rate w .

Budget constraint. In addition to the CIA constraint (2.3), households face a budget constraint. They receive income from capital k , labor n , profits Ω , and real lump-sum transfers τ from the monetary authority. Real assets a consist of capital k and real money balances $m \equiv M/P$ and accumulate according to:

$$\dot{a} = \dot{k} + \dot{m} = \begin{cases} wn + rk + \Omega + \tau - c - \pi m & \text{case 1} \\ wn + rk + \Omega + \tau - c - \pi m - w \int_0^\zeta \kappa(i) di & \text{case 2} \end{cases} \quad (2.5)$$

The initial endowments at time zero k_0 and m_0 are given.

The household maximizes (2.1) subject to (2.3)-(2.5). The first-order conditions of the household are derived in the Appendix.

⁶Since the analysis only considers the situation where π is larger than the negative real interest rate r , $\pi \geq 0 > -r$, equation (2.3) will always hold as an equality at an optimum.

2.2 The monetary authority

The economy-wide nominal money supply $M = Pm$ grows at the rate μ :

$$\frac{\dot{M}}{M} = \mu. \quad (2.6)$$

The seignorage obtained from money creation is paid to the households as a lump-sum transfer implying:

$$\tau = \mu m. \quad (2.7)$$

2.3 Firms

Firms are identical and of measure one. They use labor n and capital k in order to produce the consumption-investment good y with the technology $f(k, \bar{k}, n)$. The externality in aggregate capital accumulation \bar{k} (which equals k in equilibrium) results in constant returns to capital as in Romer (1986):

$$y = Ak^\alpha n^{1-\alpha} \bar{k}^{1-\alpha}. \quad (2.8)$$

y can either be consumed by the households or accumulated.

Firms maximize discounted profits:

$$\int_0^\infty \Omega e^{-\int_0^t r(h) dh} dt, \quad (2.9)$$

where profits are given by:

$$\Omega = y - (r + \delta)k - wn. \quad (2.10)$$

Firms take the interest rate r and the wage rate w as given. Capital depreciates at the rate δ . In factor market equilibrium, the factor prices are equal to their marginal products:

$$r + \delta = \alpha An^{1-\alpha} k^{\alpha-1} \bar{k}^{1-\alpha}, \quad (2.11a)$$

$$w = (1 - \alpha) An^{-\alpha} k^\alpha \bar{k}^{1-\alpha}. \quad (2.11b)$$

2.4 Stationary competitive equilibrium

Definition. The competitive search equilibrium is a collection of decision rules $\{c, n, \zeta, k\}$ and prices $\{w, r, \pi\}$ such that

1. Individual variables equal aggregate variables.
2. Households maximize their utility (2.1) subject to (2.3)-(2.5).
3. Firms maximize profits (2.9) subject to (2.8) and (2.10).
4. Wages and interest rates are given by (2.11b) and (2.11a), respectively.
5. Assets accumulates according to (2.5).
6. Nominal money grows at the exogenous rate μ .
7. The goods market clears:

$$\dot{k} = \begin{cases} f(k, n, \bar{k}) - \delta k - c & \text{case 1} \\ f(k, n, \bar{k}) - \delta k - c - w \int_0^\zeta \kappa(i) di & \text{case 2} \end{cases} \quad (2.12)$$

8. The externality in capital accumulation is equal to the aggregate capital stock, $\bar{k} = k$.

2.5 Calibration

The effects of a change in the inflation rate (as resulting from a change in the growth rate of money supply) cannot be studied analytically but only numerically. For this reason, the model is calibrated in order to match characteristics of the US economy. The unit time length corresponds to one quarter. In case 1 (case 2), we have to find the values of 8 (9) parameters. For a subset of 5 parameters, $\{\alpha, \eta, \rho, \delta, \mu\}$, we use observations to calibrate them individually. For the remaining parameters, i.e. either $\{\beta, A, \zeta\}$ in case 1 or $\{\beta, A, \kappa_0, \chi, \}$ in case 2, we calibrate them simultaneously such that both a set of empirical observations and the equilibrium conditions hold.⁷

⁷The GAUSS computer programs are available from the authors upon request.

We set the production elasticity of private capital equal to $\alpha = 0.36$ and the quarterly rate of capital depreciation equal to $\delta = 0.025$. The household discounts future utilities at the rate $\rho = 0.01$. $\eta = 3.5$ implies a labor supply elasticity of $\epsilon_{n,w} = 1/(\eta - 1) = 0.4$, and the value of μ follows from our inflation target, which is either 0 or 10 percent p.a.

β , A , and (in case 1) ζ are set so that

- i. the household works $n = 1/3$ hours of the time endowment of unity,
- ii. the annual growth rate of output $(1 + g)^4 - 1$ (where g is the quarterly rate) is equal to 2 percent,
- iii. the share of cash goods in total consumption is equal to 82 percent.⁸

In case 2, we use a further observation in addition to iii to simultaneously determine κ_0 and χ : the semi-interest elasticity of the income velocity of money $\epsilon_{v,r+\pi}$ implied by the model must equal 5.95 as estimated by Dotsey and Ireland (1996) for the US economy during 1959-1991. Table 2.1 summarizes our parameter settings.

Table 2.1
Calibration of the AK model

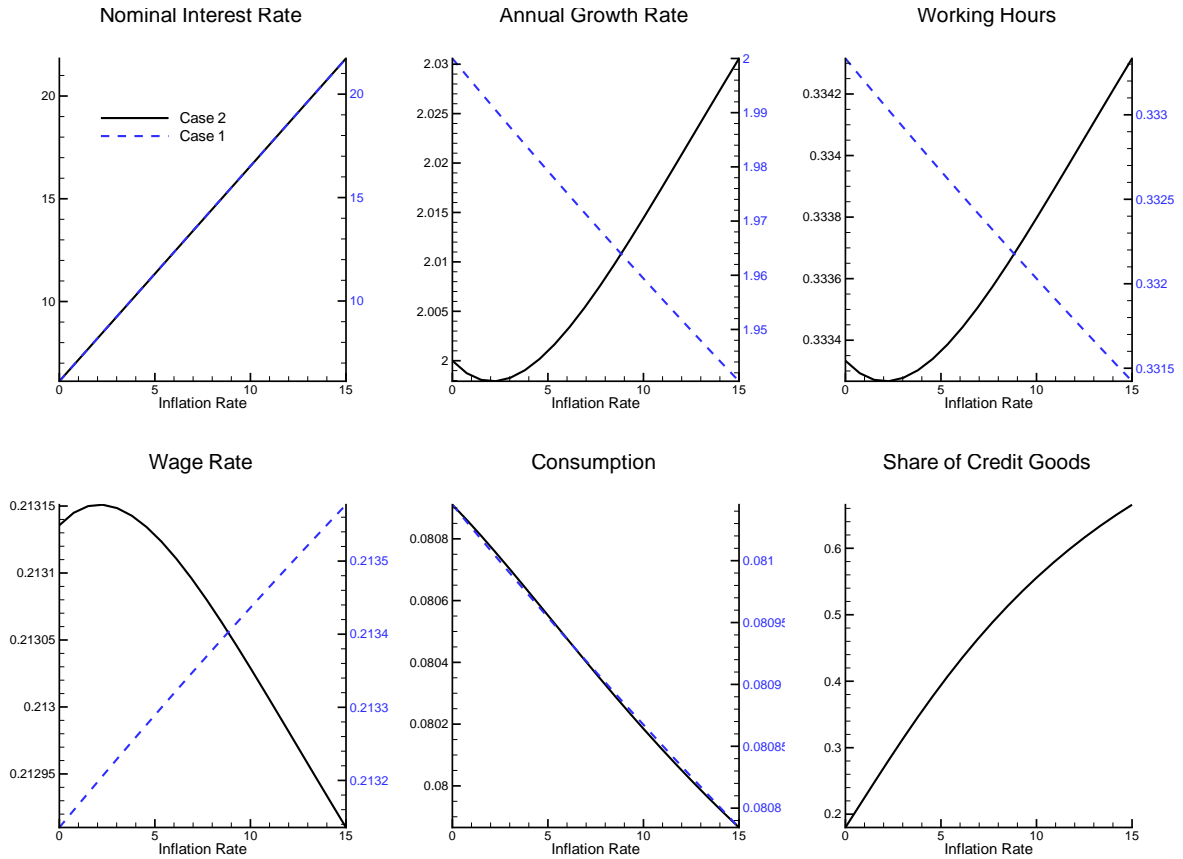
Preferences	$n=1/3$	$\eta=3.5$	$\rho=0.01$
Production	$\alpha=0.37$	$\delta=0.025$	
CIA	$\zeta=0.18$	$\epsilon_{v,r+\pi}=5.95$	
Output growth	$(1 + g)^4 - 1=0.02$		
Money supply	$(1 + \pi)^4 - 1=0$		

3 Inflation and economic growth

Figure 3.1 displays the relation between the annual rate of inflation and various endogenous variables along a balanced growth path, if the inflation rate increases from

⁸This value for the US economy is found by Avery et al. (1987).

Figure 3.1: Balanced Growth Path: AK-model



zero to 15 percent p.a.⁹ As a consequence, the nominal interest rate $r + \pi$ increases from 6.1 percent to 21.7 percent p.a. in the case of a constant share of cash goods and to 21.8 percent if the share of cash goods is endogenous. The nominal interest rate drives a wedge between the real wage and marginal rate of substitution between consumption and working hours. This can be seen from the first-order condition on labor supply:

$$w = [1 + (r + \pi)(1 - \zeta)]\beta n^{\eta-1}c.$$

In the case of a constant share of cash goods ζ , inflation increases this wedge so that the household works less and the rate of capital accumulation and, hence, per-capita

⁹To understand this figure, please note that there are different scales on the respective ordinates, corresponding to case 1 and case 2, respectively. The left (right) ordinate corresponds to the solid (broken) line.

growth slows down. The consumption-capital ratio (displayed in the panel labeled "Consumption") and the wage rate per efficiency unit of labor decline.

Now, consider case 2 and the first-order condition for the optimal share of credit goods:

$$c(r + \pi) = w\kappa_0 \left(\frac{\zeta}{1 - \zeta} \right)^x.$$

The left-hand side are the savings that result from buying less goods by cash, the right-hand-side are the costs of acquiring an additional unit of credit services. These costs increase both in the real wage w and the share of credit goods. If ζ is large, a further increase of the wage rate would force the household to buy more goods by cash and the opportunity costs of holding money would further increase. As a consequence, there is a critical share of cash goods beyond which the wage rate falls and labor supply increases. This explains the u-shaped behavior of working hours, the real wage, and the growth rate of per-capita income.

4 Endogenous growth and human capital

In the following, we examine the sensitivity of our results with respect to the specification of the engine of growth. In Lucas (1990) hours spent on learning increase the stock of human capital, which is used along with raw labor and physical capital to produce consumption and investment goods. Jones and Manuelli (1995) assume that goods rather than time are used to accumulate human capital.

4.1 The Lucas (1990) supply-side model

We introduce money in the model of Lucas (1990) in the same way as in the AK-model presented in Section 2., i.e. we distinguish between the standard CIA on consumption (case 1) and the CIA with a variable share of credit goods ζ (case 2). To keep the description of the model as brief as possible, we just present the maximization problems of the household and the firm sector, respectively. Everything else remains unchanged.¹⁰

¹⁰For a more detailed presentation of the model, see Lucas (1990) and Grüner and Heer (2000).

Households. The representative household can allocate his time endowment 1 to work n , learning l , or leisure x :

$$1 = n + l + x. \quad (4.1)$$

He maximizes his intertemporal utility

$$\int_0^{\infty} U(c, x) e^{-\rho t} dt \quad (4.2)$$

where instantaneous utility is given by

$$U(c, x) = \ln c - \beta \frac{(1-x)^\eta}{\eta}. \quad (4.3)$$

Human capital of the individual h is determined by the time l he allocates to learning:

$$\dot{h} = Dh l^\nu. \quad (4.4)$$

The individual's labor income is given by the product of the wage, w , and his effective labor, hn . In case 2, costs per unit of credit service are proportional to the costs of labor given by $w\bar{h}$, where \bar{h} is the stock of human capital of the average worker.¹¹ The household's budget constraint, thus, is:

$$\dot{a} = \dot{k} + \dot{m} = \begin{cases} wn h + rk + \Omega + \tau - c - \pi m & \text{case 1} \\ wn h + rk + \Omega + \tau - c - \pi m - w\bar{h} \int_0^\zeta \kappa(i) di & \text{case 2} \end{cases}. \quad (4.5)$$

The household maximizes 4.2 subject to (2.3), (2.4), (4.4), and (4.5). The first-order conditions and equilibrium conditions of the model are derived in the Appendix.

Firms. Output is produced using capital k and effective labor, hn , according to:

$$y = Ak^\alpha (hn)^{1-\alpha}. \quad (4.6)$$

Firm's profits are now given by

$$\Omega = y - (r + \delta)k - wn h. \quad (4.7)$$

Otherwise, the production sector is identical to that in the model of Section 2.

¹¹Note that in equilibrium $\bar{h} = h$.

Calibration. As compared to the AK-model of Section 2 the model has two additional parameters, D and ν . In the Appendix we demonstrate that the parameter D takes the position of the parameter A in the AK-model. Therefore, we put $A = 1$ and set D so that the annual growth rate of output is two percent p.a. With respect to the second parameter we follow Lucas (1990) and choose $\nu = 0.8$. The remaining parameters are either set equal to the values presented in Table 2.1 or chosen to meet the calibration targets explained in Section 2.

Results. As is evident from Figure 4.1, the same mechanism as in the previous model holds. Case 1 delivers a negative relation between the rate of inflation and the rate of economic growth, while the opposite holds in case 2. Note, however, that the effects of inflation on the rate of growth are quantitatively smaller than in the AK-model. If inflation increases from zero to ten percent p.a., the annual growth rate rises from 2 to 2.006 percent in case 2 and declines to 1.975 percent in case 1. The decisive parameter in this respect is elasticity of the growth rate with respect to learning ν . For higher ν the growth effect becomes larger.

4.2 The model of Jones and Manuelli (1995)

The model. As a second variation of the endogenous growth model with human capital, we assume that human capital accumulation requires goods rather than time:

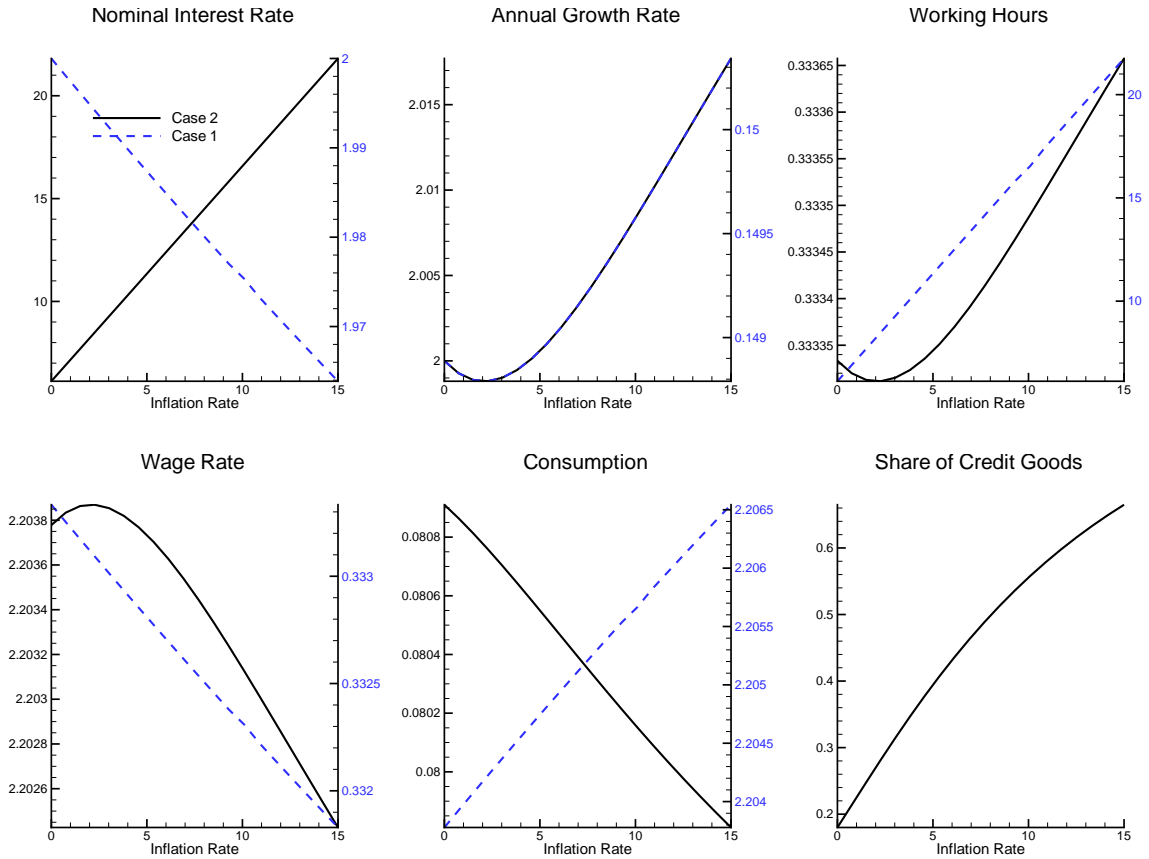
$$\dot{h} = i_h - \delta_h h, \quad (4.8)$$

where i_h and δ_h denote investment in human capital and the rate of human capital depreciation, respectively. As a consequence, the household's budget constraint is modified to:

$$\dot{a} = \dot{k} + \dot{m} = \begin{cases} wnh + rk + \Omega + \tau - c - \pi m - i_h & \text{case 1} \\ wnh + rk + \Omega + \tau - c - \pi m - i_h - w\bar{h} \int_0^\zeta \kappa(i) di & \text{case 2} \end{cases}. \quad (4.9)$$

Households maximize (2.1) (with the current-period utility function specified by (2.2)) subject to (2.3), (4.8) and (4.9).

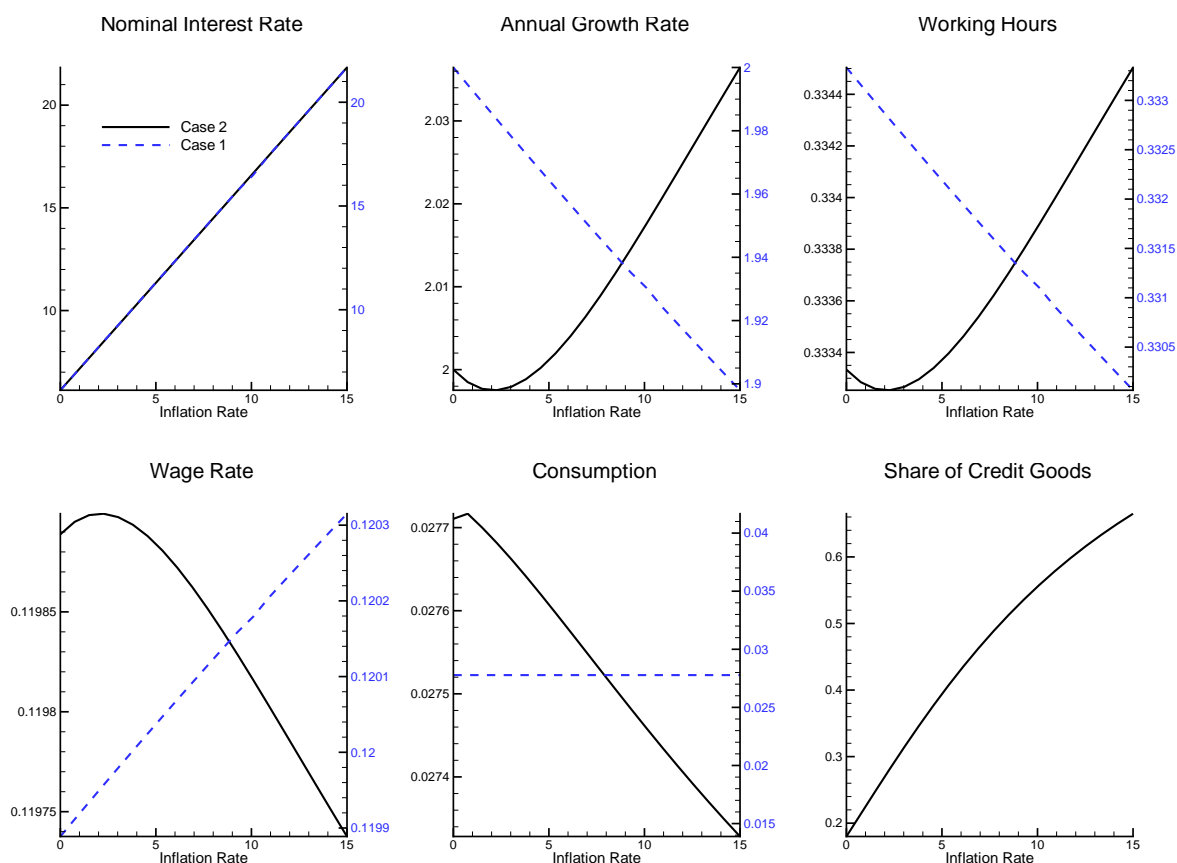
Figure 4.1: Balanced growth path: Human capital accumulation with learning



The stationary equilibrium conditions are described in more detail in the Appendix. We follow Jones and Manuelli (1995) and set $\delta_h = \delta$. The remaining parameters are either set to the values presented in Table 2.1 or obtained in the same way as explained in Section 2.

Results. Our results are presented in Figure 4.2. As in the previous two models, inflation and growth are negatively related for a fixed share of credit goods. A u-shaped relation emerges, if this share is endogenous. In case 1, the effect of inflation on consumption depends on the relative magnitudes of the rates of depreciation of physical and human capital, δ and δ_h , respectively. If the former is smaller (larger) than the latter, consumption decreases (increases) with inflation. In the knife-edge case $\delta = \delta_h$ considered here, consumption is independent of inflation, as can be seen from panel

Figure 4.2: Balanced growth path: Human capital accumulation with goods



five in Figure 4.2. An increase of inflation from zero to ten percent p.a. lowers the growth rate of per-capita income from 2% p. a. to 1.93 % in case 1. In case 2 the same change of inflation raises growth from 2% to 2.017%.

5 Search and matching in the labor market

In this section we consider frictions in the labor market as in Shi and Wen (1997, 1999). This allows us to study both the effects of inflation on the rate of growth and on the rate of unemployment. In addition, this exercise reveals whether or not the results obtained so far are sensitive with regard to the modeling of the labor market. We will focus on the case of an endogenous share of credit goods ζ , keep the exposition as brief as possible, and refer the interested reader to the Appendix for the detailed set up,

calibration, and solution of the respective models. To distinguish the different models we use the shorthands AK, L, and JM for the extended models of Sections 2, 4.1, and 4.2, respectively.

5.1 Households

A single household $h \in [0, 1]$ consists of different members who are either employed, search for a job, or enjoy leisure. In the Lucas (1990) model they also spend time on learning to build up human capital. The members pool their income. Let n and s denote the fraction of employed and searching household members, respectively. An employed person loses his job with an exogenously given probability of θ . Searching households will find a job with probability q , so that the share of employed households (both for each $h \in [0, 1]$ and for the unit mass of households) evolves according to

$$\dot{n} = qs - \theta n. \quad (5.1)$$

Households maximize

$$\int_0^\infty \left[\ln c - \beta \frac{(1-x)^\eta}{\eta} \right] e^{-\rho t} dt, \quad x = \begin{cases} 1 - n - s & \text{AK} \\ 1 - n - s - l & \text{L} \\ 1 - n - s & \text{JM} \end{cases} \quad (5.2)$$

subject to the budget constraint

$$\dot{a} = \dot{k} + \dot{m} = \begin{cases} wn + rk + \Omega + \tau - c - \pi m - w\kappa_0 \int_0^\zeta \left(\frac{i}{1-i}\right)^x di & \text{AK} \\ whn + rk + \Omega + \tau - c - \pi m - w\bar{h}\kappa_0 \int_0^\zeta \left(\frac{i}{1-i}\right)^x di & \text{L} \\ whn + rk + \Omega + \tau - c - i_h - \pi m - w\bar{h}\kappa_0 \int_0^\zeta \left(\frac{i}{1-i}\right)^x di & \text{JM} \end{cases} \quad (5.3)$$

the CIA (2.3), the dynamics of human capital, (4.4) in model L and (4.9) in model JM, and the evolution of employment (5.1).

5.2 Firms

Workers separate from the representative firm with probability θ . To attract new workers the firm posts vacancies v at cost ϕwhv (or ϕwv in the AK-model). The

probability that a vacancy is filled is ϑ . Therefore, employment at the firm level (and, in equilibrium, in the entire economy) evolves according to

$$\dot{n} = \vartheta v - \theta n. \quad (5.4)$$

The firm maximizes

$$\int_0^\infty \Omega e^{-\int_0^t r(\xi) d\xi} dt, \quad \Omega = \begin{cases} y - wn - (r + \delta)k - \phi wv & \text{AK} \\ y - whn - (r + \delta)k - \phi whv & \text{L and JM} \end{cases} \quad (5.5)$$

subject to (5.4) and the specification of the production function in (2.8) for the AK-model and in (4.6) for the L and JM model, respectively.

5.3 Matching and bargaining

At the aggregate level, the mass of successfully filled vacancies M is determined by

$$M = v^\gamma s^{1-\gamma}, \quad (5.6)$$

where v and s denote the mass of vacancies and the share of searching households. Since households cannot search while being employed, the unemployment rate u is given by

$$u = \frac{s}{n + s}. \quad (5.7)$$

The probability for a household to find a job q equals

$$q = \frac{M}{s} = (v/s)^\gamma \quad (5.8)$$

and the probability that an open vacancy is filled follows from

$$\vartheta = \frac{M}{v} = (v/s)^{\gamma-1}. \quad (5.9)$$

Wages result from decentralized Nash bargaining between the firm and the marginal worker. In particular, the wage per unit of raw labor \tilde{w} maximizes

$$\max_{\tilde{w}} [MPL - \tilde{w}]^{1-\lambda} [\tilde{w} - MRS]^\lambda \quad (5.10)$$

where MPL denotes the marginal product of labor and MRS the marginal rate of substitution between working hours and consumption.

5.4 Calibration

As compared to the models with Walrasian labor markets the models considered in this section have four additional parameters, θ , γ , λ , and ϕ . We choose the former three parameters as in Shi and Wen (1999) and determine ϕ indirectly by setting the equilibrium unemployment rate equal to six percent. In addition, we have to recalibrate the parameter β in the current-period utility function so that the labor force participation rate $n + s$ equals 68 percent.¹² The values of the exogenously given parameters are summarized in Table 5.1.

Table 5.1
Calibration of the search models

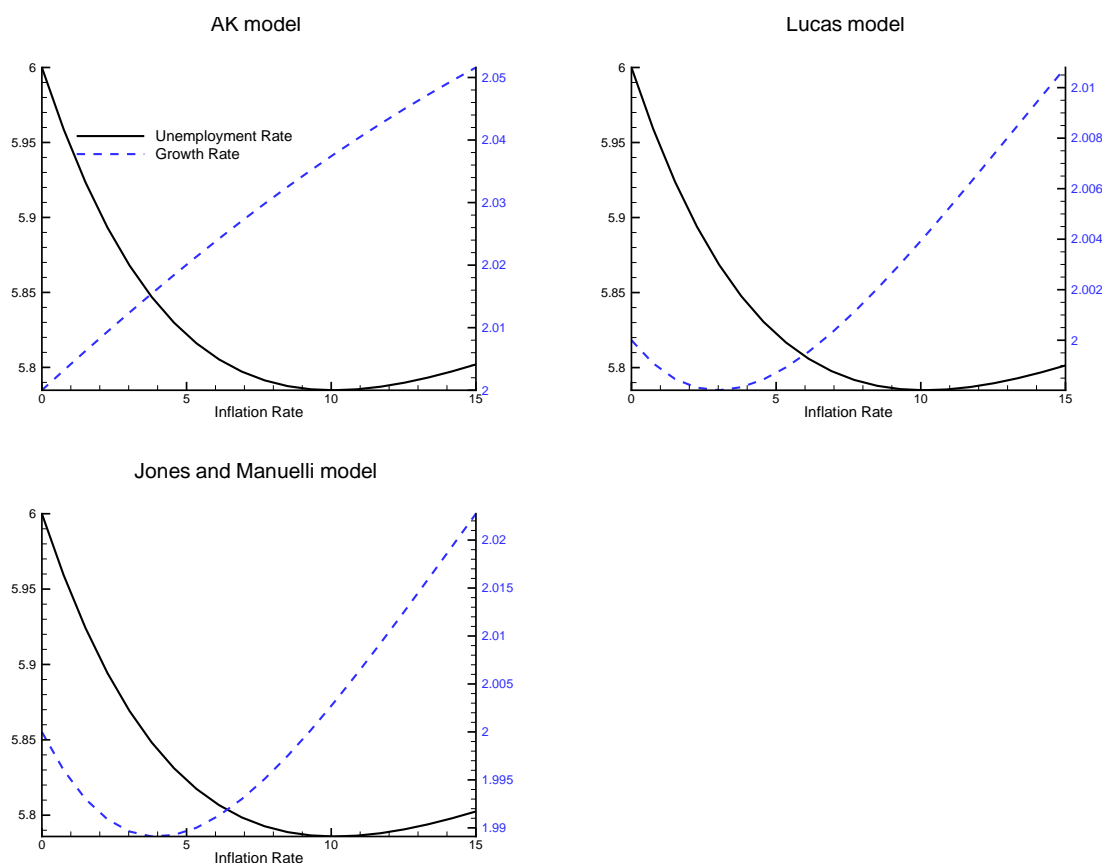
Preferences	$n + s=0.68$	$\eta=3.5$	$\rho=0.01$
Production	$\alpha=0.36$	$\delta = \delta_h=0.025$	
CIA	$\zeta=0.18$	$\epsilon_{v,r+\pi}=5.95$	
Labor market	$\theta=0.05$	$\gamma=0.6$	$\lambda=0.6$
	$u=0.06$		
Output growth	$(1 + g)^4 - 1=0.02$		
Money supply	$(1 + \pi)^4 - 1=0$		

5.5 Results

Figure 5.1 displays the relation between the rate of inflation, the unemployment rate, and the rate of per-capita growth obtained from simulations of our three models. In all three models the equilibrium unemployment rate is a u-shaped function of the rate of inflation with a minimum at around 10 percent annual inflation. The driving force behind this result is the behavior of wages. With increasing inflation households decrease their cash balances and, thus, consumption. As a consequence, the marginal rate of substitution between consumption and working hours decreases, which in turn reduces the reservation wage. Firms find it more profitable to post vacancies. In

¹²Both, the value for u and for $n + s$ are taken from Shi and Wen (1999).

Figure 5.1: Unemployment and growth in models with labor market frictions



addition, the households reduce their search effort in order to increase leisure time. Both effects increase v/s and, thus, the probability of finding a job, so that employment increases. Yet, with increasing employment the marginal product of labor falls and firms find it less profitable to post vacancies. Therefore, v/s does not increase further for high levels of inflation, but attains a maximum at about 10 percent annual inflation. The behavior of the unemployment rate mimics this pattern.

But note, the relation between inflation and the growth rate does not quite reflect the inflation-unemployment trade-off. In the AK-model, and for inflation rates between zero and 15 percent, the relation is monotonically increasing. In the L and JM model it is u-shaped as in the models of Section 2 and 4 attaining a minimum for an inflation rate between three and four percent p.a. Yet, as documented in Table 1.2 the quantitative effects are small in the AK-model – where the annual growth rate increases from 2

percent to 2.037 percent p.a. if inflation increases from zero to 10 percent p.a. – and negligible in the L- and JM-model.

6 Conclusion

In our models of economic growth, we have emphasized the costly-credit channel of Dotsey and Ireland (1996). As opposed to models with a given share of cash goods, where higher inflation reduces growth, there is a non-linear, u-shaped relation between inflation and the rate of per-capita growth. In models with a Walrasian labor market this relation rests on a non-linear behavior of the wedge between the marginal rate of substitution between consumption and working hours and the real wage. This wedge increases with nominal interest rates and declines with a smaller share of cash goods. In models with labor market frictions, this wedge interacts with a second wedge that stems from wage bargaining. As a result of this interaction, inflation also has an u-shaped effect on the rate of unemployment, which is decreasing for moderate rates of inflation and increasing with annual inflation rates beyond ten percent. In these latter models, the positive effects of inflation on the rate of growth are small or even negligible.

At this point, let us mention one word of caution. We have focused our analysis and also our discussion on models within a cash-in-advance context. Our results may be sensitive with respect to the introduction of other monetary frictions and, in particular, those that become more important for higher rates of inflation or during the process of economic development. For example, with high rates of inflation, transaction costs and employment in the financial sector as emphasized by Ireland (1994) are likely to result in lower employment and less growth. In a similar vein, inflation may increase transaction costs and therefore reduces growth in the models of De Gregorio (1993) and Jha et al. (2002). Furthermore, with higher inflation, the volatility of inflation is likely to increase. We intend to study the sensitivity of our results with respect to other monetary frictions in the future.

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Appendix

The following sections present the mathematical details of the models considered in the body of the paper.

A.1 The AK-model

A.1.1 Walrasian labor markets

A.1.1.1 Equilibrium conditions

Households. The households in this economy solve the problem

$$\begin{aligned} \max \quad & \int_0^\infty \left[\ln c - \beta \frac{n^\eta}{\eta} \right] e^{-\rho t} dt, \\ \text{subject to} \quad & \\ & \dot{k} = wn + rk + \Omega + \tau - \pi m - w\kappa_0 \int_0^\zeta \left(\frac{i}{1-i} \right)^x di - z - c, \\ & \dot{m} = z, \\ & m \leq (1 - \zeta)c. \end{aligned} \tag{A.1.1}$$

The Lagrangian of this problem in current shadow prices reads:

$$\mathcal{L} = \bar{H} + \psi (m - (1 - \zeta)c),$$

where \bar{H} , the current-value Hamiltonian, is given by:

$$\begin{aligned} \bar{H} = & \ln c - \beta \frac{n^\eta}{\eta} \\ & + \lambda_k \left(wn + rk + \Omega + \tau - \pi m - w\kappa_0 \int_0^\zeta \left(\frac{i}{1-i} \right)^x di - z - c \right) + \lambda_m z. \end{aligned}$$

The first-order conditions for an interior solution with $0 < n < 1$ and $0 < \zeta < 1$ are:

$$\frac{\partial \mathcal{L}}{\partial c} = \frac{1}{c} - \lambda_k - \psi(1 - \zeta) = 0, \tag{A.1.2a}$$

$$\frac{\partial \mathcal{L}}{\partial z} = -\lambda_k + \lambda_m = 0, \tag{A.1.2b}$$

$$\frac{\partial \mathcal{L}}{\partial n} = -\beta n^{\eta-1} + \lambda_k w = 0, \tag{A.1.2c}$$

$$\frac{\partial \mathcal{L}}{\partial \zeta} = -\lambda_k w \kappa_0 \left(\frac{\zeta}{1-\zeta} \right)^x + \psi c = 0, \tag{A.1.2d}$$

$$\dot{\lambda}_k = \rho \lambda_k - \frac{\partial \mathcal{L}}{\partial k} = (\rho - r) \lambda_k, \tag{A.1.2e}$$

$$\dot{\lambda}_m = \rho \lambda_m - \frac{\partial \mathcal{L}}{\partial m} = \rho \lambda_m + \lambda_k \pi - \psi. \tag{A.1.2f}$$

Since (A.1.2b) implies that the shadow prices λ_k and λ_m must grow at the same rate, (A.1.2e) and (A.1.2f) can be used to eliminate ψ :

$$\frac{\dot{\lambda}_k}{\lambda_k} = (\rho - r) = \frac{\dot{\lambda}_m}{\lambda_m} = \rho + \pi - \frac{\psi}{\lambda_k} \Rightarrow \psi = \lambda_k(r + \pi). \quad (\text{A.1.3})$$

Hence, if $\pi > -r$ the multiplier of the CIA is positive and the CIA binds in equilibrium. Using this result in (A.1.2a) and (A.1.2d) implies

$$\frac{1}{c} = \lambda_k(1 + (1 - \zeta)(r + \pi)), \quad (\text{A.1.4a})$$

$$c(r + \pi) = w\kappa_0 \left(\frac{\zeta}{1 - \zeta} \right)^x. \quad (\text{A.1.4b})$$

Differentiating (A.1.4a) with respect to time and using (A.1.4a) as well as (A.1.2c) to simplify the ensuing expression yields

$$\frac{\dot{c}}{c} = r - \rho - \frac{(1 - \zeta)\pi}{1 + (1 - \zeta)(r + \pi)} \frac{\dot{\pi}}{\pi} - \frac{(1 - \zeta)r}{1 + (1 - \zeta)(r + \pi)} \frac{\dot{r}}{r} + \frac{\zeta(r + \pi)}{1 + (1 - \zeta)(r + \pi)} \frac{\dot{\zeta}}{\zeta}. \quad (\text{A.1.5a})$$

Differentiating equation (A.1.4b) with respect to time gives:

$$\frac{\dot{c}}{c} + \frac{\pi}{r + \pi} \frac{\dot{\pi}}{\pi} + \frac{r}{r + \pi} \frac{\dot{r}}{r} = \frac{\dot{w}}{w} + \frac{\chi}{1 - \zeta} \frac{\dot{\zeta}}{\zeta}. \quad (\text{A.1.5b})$$

Firms. Firms maximize profits

$$\Omega = Ak^\alpha n^{1-\alpha} \bar{k}^{1-\alpha} - (r + \delta)k - wn, \quad (\text{A.1.6})$$

implying the first-order conditions

$$r = \alpha Ak^{\alpha-1} n^{1-\alpha} \bar{k}^{1-\alpha} - \delta, \quad (\text{A.1.7a})$$

$$w = (1 - \alpha) Ak^\alpha n^{-\alpha} \bar{k}^{1-\alpha}. \quad (\text{A.1.7b})$$

Monetary authority. The central bank increases money M at a constant rate μ and uses seignorage M/P to finance transfers τ to the household sectors. Thus,

$$\tau = \dot{m} + \pi m, \quad \pi \equiv \frac{\dot{P}}{P}. \quad (\text{A.1.8})$$

Stationary equilibrium. In equilibrium $\bar{k} = k$. Equations (A.1.6) and (A.1.8) imply that the households budget constraint reduces to the economy's resource restriction:

$$\frac{\dot{k}}{k} = An^{1-\alpha} - \delta - \tilde{w}\kappa_0 \int_0^\zeta \left(\frac{i}{1-i} \right)^x di - \tilde{c},$$

where the variables with a tilde $\tilde{\cdot}$ are scaled by the stock of capital k , i.e. $\tilde{w} = w/k$ etc.

On a balanced growth path employment n , the fraction of credit goods ζ , the real interest rate r , and the rate of inflation π must be constant. Equation (A.1.5a), thus, implies that the growth rate of consumption $g \equiv \dot{c}/c$ equals

$$g = r - \rho. \quad (\text{A.1.9a})$$

According to equation (A.1.5b) the real wage must grow at the same rate as consumption. If c and w growth at the same constant rate, the resource restriction implies that the stock of capital must also grow at this rate:

$$g = \alpha An^{1-\alpha} - \delta - \tilde{w}\kappa_0 \int_0^\zeta \left(\frac{i}{1-i} \right)^x di - \tilde{c}. \quad (\text{A.1.9b})$$

With constant ζ the CIA implies that the real stock of money must grow at the same rate as consumption:

$$g = \mu - \pi. \quad (\text{A.1.9c})$$

Substituting the solution for λ_k implied by (A.1.4a) in condition (A.1.2c) gives

$$\tilde{w} = [1 + (1 - \zeta)(r + \pi)]\beta n^{\eta-1}\tilde{c}. \quad (\text{A.1.9d})$$

Substituting ψ in condition (A.1.2d) and dividing both sides by k yields:

$$\tilde{c}(r + \pi) = \tilde{w}\kappa_0 \left(\frac{\zeta}{1-\zeta} \right)^x. \quad (\text{A.1.9e})$$

Equilibrium on the markets for labor and capital services implies that the firm's first-order conditions (A.1.7) hold. Using $\bar{k} = k$ equations (A.1.7) imply

$$r = \alpha An^{1-\alpha} - \delta, \quad (\text{A.1.9f})$$

$$w = (1 - \alpha)An^{-\alpha}. \quad (\text{A.1.9g})$$

The system of seven equations (A.1.9) determines the unknowns g , π , \tilde{c} , n , ζ , r , and \tilde{w} . The model with a given share of credit goods ζ is a simplified version of this system of equations: without condition (A.1.9e) and without the term $\tilde{w}\kappa_0 \int_0^\zeta \left(\frac{i}{1-i} \right)^x di$ in equation (A.1.9b).

A.1.1.2 Calibration

The system of equations (A.1.9) consists of nine parameters (in the order of their appearance in (A.1.9)):

$$\{\rho, A, \alpha, \delta, \kappa_0, \chi, \mu, \beta, \eta\}.$$

For four of these parameters, ρ , α , δ , and η , we use values commonly found in the literature. These values are presented in Table 2.1 in the body of the paper. In order to determine the values of the remaining five parameters, we employ the following calibration targets:

1. The share of cash goods must equal 82%, implying $\zeta = 0.18$.
2. Working hours equal one third of the households time endowment, $n = 1/3$.
3. The annual rate of output growth must equal 2%, i.e. $(1 + g)^4 - 1 = 0.02$.
4. The semi elasticity of the income velocity of money with respect to the annual nominal interest rate must equal 5.95.
5. To compute this elasticity we solve the model for an annual inflation rate of zero and an annual inflation rate of ten percent, which delivers a further initial condition.¹³

A.1.2 Search frictions

A.1.2.1 Equilibrium conditions

Households. In the case of search friction we include hours spent searching s in the current-period utility function and add equation (5.1) to the set of constraints in (A.1.1). The current-value Lagrangian of the respective optimization problem reads:

$$\begin{aligned} \mathcal{L} = & \left[\ln c - \beta \frac{(n + s)^\eta}{\eta} \right] \\ & + \lambda_k \left(wn + rk + \Omega + \tau - \pi m - w\kappa_0 \int_0^\zeta \left(\frac{i}{1-i} \right)^x di - z - c \right) + \lambda_m z \\ & + \lambda_n (qs - \theta n) + \psi (m - (1 - \zeta)c). \end{aligned}$$

The first-order conditions for an interior solution $0 < n + s < 1$ and $0 < \zeta < 1$ are:

$$\frac{\partial \mathcal{L}}{\partial c} = \frac{1}{c} - \lambda_k - \psi(1 - \zeta) = 0, \quad (\text{A.1.10a})$$

$$\frac{\partial \mathcal{L}}{\partial z} = -\lambda_k + \lambda_m = 0, \quad (\text{A.1.10b})$$

$$\frac{\partial \mathcal{L}}{\partial s} = -\beta(n + s)^{\eta-1} + \lambda_n q = 0, \quad (\text{A.1.10c})$$

$$\frac{\partial \mathcal{L}}{\partial \zeta} = -\lambda_k w \kappa_0 \left(\frac{\zeta}{1 - \zeta} \right)^x + \psi c = 0, \quad (\text{A.1.10d})$$

$$\dot{\lambda}_k = \rho \lambda_k - \frac{\partial \mathcal{L}}{\partial k} = (\rho - r) \lambda_k, \quad (\text{A.1.10e})$$

$$\dot{\lambda}_m = \rho \lambda_m - \frac{\partial \mathcal{L}}{\partial m} = \rho \lambda_m + \lambda_k \pi - \psi, \quad (\text{A.1.10f})$$

$$\dot{\lambda}_n = \rho \lambda_n - \frac{\partial \mathcal{L}}{\partial n} = \rho \lambda_n + \beta(n + s)^{\eta-1} - \lambda_k w + \lambda_n \theta. \quad (\text{A.1.10g})$$

¹³Let v_i and R_i denote the income velocity and the nominal interest rate implied by the model's solution for zero ($i = 1$) and ten ($i = 2$) percent annual inflation. We compute the elasticity from

$$\epsilon_{v,r+\pi} \equiv \frac{\ln(4v_2) - \ln(4v_1)}{R_2 - R_1}.$$

Conditions (A.1.10a), (A.1.10b) as well as (A.1.10d)-(A.1.10f) coincide with the first-order conditions (A.1.2a), (A.1.2b), (A.1.2d)-(A.1.2f) of the model with Walrasian labor markets. As a consequence, equations (A.1.4a), (A.1.4b), (A.1.5a) and (A.1.5b) continue to hold.

Firms. Let $\bar{r}(t) \equiv (1/t) \int_0^t r(\xi) d\xi$. Firms solve

$$\begin{aligned} & \max \int_0^\infty \Omega e^{-\bar{r}(t)} dt, \\ & \text{subject to} \\ & \Omega = Ak^\alpha n^{1-\alpha} \bar{k}^{1-\alpha} - (r + \delta)k - wn - \phi wv, \\ & \dot{n} = \vartheta v - \theta n. \end{aligned}$$

The current value Hamiltonian for this problem, $\bar{H} \equiv e^{\bar{r}(t)t} H$ (where H denotes the present value Hamiltonian), is

$$\bar{H} = \Omega + \lambda_F (\vartheta v - \theta n)$$

so that the first-order conditions read:

$$\frac{\partial \bar{H}}{\partial k} = \alpha Ak^{\alpha-1} n^{1-\alpha} \bar{k}^{1-\alpha} - (r + \delta) = 0, \quad (\text{A.1.11a})$$

$$\frac{\partial \bar{H}}{\partial v} = -\phi w + \lambda_F \vartheta = 0, \quad (\text{A.1.11b})$$

$$\dot{\lambda}_F = r\lambda_F - \frac{\partial \bar{H}}{\partial n} = (r + \theta)\lambda_F - ((1 - \alpha)Ak^\alpha n^{-\alpha} \bar{k}^{1-\alpha} - w). \quad (\text{A.1.11c})$$

Wage setting. The Nash-bargaining solution solves

$$\max_w [(1 - \alpha)Ak^\alpha n^{-\alpha} \bar{k}^{1-\alpha} - w]^{1-\lambda} [w - \beta(n + s)^{\eta-1} c]^\lambda$$

implying

$$w = \lambda(1 - \alpha)Ak^\alpha n^{-\alpha} \bar{k}^{1-\alpha} + (1 - \lambda)\beta(n + s)^{\eta-1} c. \quad (\text{A.1.12})$$

Stationary equilibrium. In the stationary equilibrium, employment n , search s , vacancies v , the fraction of credit goods ζ , the inflation rate π , and the real interest rate r are constant. As in the model of the previous section, this implies that the rate of growth g satisfies:

$$g = r - \rho. \quad (\text{A.1.13a})$$

The household's budget constraint now implies:

$$g = An^{1-\alpha} - \delta - \tilde{w} \left(\phi v + \kappa_0 \int_0^\zeta \left(\frac{i}{1-i} \right)^x di \right) - \tilde{c}. \quad (\text{A.1.13b})$$

The CIA implies

$$g = \mu - \pi. \quad (\text{A.1.13c})$$

Since $\bar{k} = k$, the wage equation can be written as:

$$\tilde{w} = \lambda \alpha n^{-\alpha} + (1 - \lambda) \beta (n + s)^{\eta-1} \tilde{c}. \quad (\text{A.1.13d})$$

As in the Walrasian setting, equation (A.1.5b) implies that consumption and the real wage will grow at the same rate so that the term $\lambda_k w$ in equation (A.1.10g) will be constant on the balanced growth path implying $\dot{\lambda}_n = 0$. Therefore, equations (A.1.10c), (A.1.10g), (A.1.4a), and $q = L/s = L_0(v/s)^\gamma$ (from equation (5.8)) imply

$$\tilde{w} = [1 + (r + \pi)(1 - \zeta)] \beta (n + s)^{\eta-1} \tilde{c} ((\theta + \rho)(v/s)^{-\gamma} + 1). \quad (\text{A.1.13e})$$

Thus, in addition to the wedge between the real wage and the marginal rate of substitution between consumption and working hours introduced by inflation, i.e., $[1 + (r + \pi)(1 - \zeta)]$, wage bargaining creates a second wedge represented by the term $(\theta + \rho)(v/s)^{-\gamma}$. Since n is constant, equations (5.1) and (5.8) imply

$$(v/s)^\gamma = \theta(n/s). \quad (\text{A.1.13f})$$

According to equation (A.1.11b), the real wage w and the multiplier λ_F must grow at the same rate g . Therefore, equation (A.1.11c) implies a further condition for the growth rate of the economy:

$$g = \frac{\dot{\lambda}_F}{\lambda_F} = r + \theta - \frac{(1 - \alpha) A k^\alpha n^{-\alpha} \bar{k}^{1-\alpha} - w}{(\phi/\vartheta)w}.$$

Substituting ϑ from equation (5.9) yields

$$g = r + \theta - \frac{(v/s)^{\gamma-1}}{\phi} \left(\frac{(1 - \alpha) A n^{-\alpha}}{\tilde{w}} - 1 \right). \quad (\text{A.1.13g})$$

The bargaining solution (A.1.13d) implies that the wage rate falls short of the marginal product of labor. Thus, the term in parenthesis in equation (A.1.13g) is positive so that (ceteris paribus) the growth rate g increases with labor market tightness v/s .

The remaining equations that characterize the balanced growth path derive from (A.1.10d) and (A.1.11a), and are, thus, identical to equations (A.1.9e) and (A.1.9f):

$$\tilde{c}(r + \pi) = \tilde{w} \kappa_0 \left(\frac{\zeta}{1 - \zeta} \right)^x, \quad (\text{A.1.13h})$$

$$r = \alpha n^{1-\alpha} - \delta. \quad (\text{A.1.13i})$$

The nine equations (A.1.13) determine the unknown variables r , g , n , s , v , ζ , π , \tilde{w} , and \tilde{c} .

A.1.2.2 Calibration

The model in the system of equations (A.1.13) has four additional parameters as compared to the model with Walrasian labor markets:

$$\{\theta, \phi, \gamma, \lambda\}.$$

Three parameters are set to the values presented in Table 5.1 and ϕ is implied by setting the equilibrium unemployment rate equal to $u = 0.06$.¹⁴

A.2 The Lucas growth model

A.2.1 Walrasian labor markets

A.2.1.1 Equilibrium conditions

Households. The households in this economy solve the problem

$$\begin{aligned} \max \quad & \int_0^\infty \left[\ln c - \beta \frac{(n+l)^\eta}{\eta} \right] e^{-\rho t} dt, \\ \text{subject to} \quad & \\ & \dot{k} = whn + rk + \Omega + \tau - \pi m - w\bar{h}\kappa_0 \int_0^\zeta \left(\frac{i}{1-i} \right)^x di - z - c, \\ & \dot{m} = z, \\ & \dot{h} = Dhl^\nu, \\ & m \leq (1 - \zeta)c. \end{aligned} \tag{A.2.1}$$

The Lagrangian of this problem in current shadow prices reads:

$$\begin{aligned} \mathcal{L} = & \left[\ln c - \beta \frac{(n+l)^\eta}{\eta} \right] \\ & + \lambda_k \left(whn + rk + \Omega + \tau - \pi m - w\bar{h}\kappa_0 \int_0^\zeta \left(\frac{i}{1-i} \right)^x di - z - c \right) \\ & + \lambda_m z + \lambda_h Dhl^\nu + \psi(m - (1 - \zeta)c). \end{aligned}$$

¹⁴We also have to recalibrate the parameter β so that the stationary rate of labor force participation $n + s$ equals 0.68.

The first-order conditions for an interior solution with $0 < n + l < 1$ and $0 < \zeta < 1$ are:

$$\frac{\partial \mathcal{L}}{\partial c} = \frac{1}{c} - \lambda_k - \psi(1 - \zeta) = 0, \quad (\text{A.2.2a})$$

$$\frac{\partial \mathcal{L}}{\partial z} = -\lambda_k + \lambda_m = 0, \quad (\text{A.2.2b})$$

$$\frac{\partial \mathcal{L}}{\partial n} = -\beta(n + l)^{\eta-1} + \lambda_k w h = 0, \quad (\text{A.2.2c})$$

$$\frac{\partial \mathcal{L}}{\partial \zeta} = -\lambda_k w \bar{h} \kappa_0 \left(\frac{\zeta}{1 - \zeta} \right)^x + \psi c = 0, \quad (\text{A.2.2d})$$

$$\frac{\partial \mathcal{L}}{\partial l} = -\beta(n + l)^{\eta-1} + \lambda_h \nu D h l^{\nu-1} = 0, \quad (\text{A.2.2e})$$

$$\dot{\lambda}_k = \rho \lambda_k - \frac{\partial \mathcal{L}}{\partial k} = (\rho - r) \lambda_k, \quad (\text{A.2.2f})$$

$$\dot{\lambda}_m = \rho \lambda_m - \frac{\partial \mathcal{L}}{\partial m} = \rho \lambda_m + \lambda_k \pi - \psi, \quad (\text{A.2.2g})$$

$$\dot{\lambda}_h = \rho \lambda_h - \frac{\partial \mathcal{L}}{\partial h} = \rho \lambda_h - \lambda_k w n - \lambda_h D l^\nu. \quad (\text{A.2.2h})$$

Equations (A.2.2a), (A.2.2b), (A.2.2f), and (A.2.2g) imply the same condition on ψ and λ_k as in the AK-model, so that (A.1.4a) as well as (A.1.5a) still hold. Condition (A.1.4b) is replaced by:

$$(r + \pi)c = w \bar{h} \kappa_0 \left(\frac{\zeta}{1 - \zeta} \right)^x. \quad (\text{A.2.3})$$

Thus, different from the AK-model, the wage rate will be constant on the balanced growth path whereas \bar{h} (which is equal to h in equilibrium) and c will grow at the same rate.

Firms. Firms maximize profits

$$\Omega = A k^\alpha (h n)^{1-\alpha} - (r + \delta)k - w h n, \quad (\text{A.2.4})$$

implying the first-order conditions

$$r = \alpha A k^{\alpha-1} (h n)^{1-\alpha} - \delta, \quad (\text{A.2.5a})$$

$$w = (1 - \alpha) A k^\alpha (h n)^{-\alpha}. \quad (\text{A.2.5b})$$

Stationary equilibrium. In equilibrium $\bar{h} = h$. Equations (A.2.4) and (A.1.8) imply that the households budget constraint reduces to the economy's resource restriction:

$$\frac{\dot{k}}{k} = A (h n)^{1-\alpha} - \delta - w h \kappa_0 \int_0^\zeta \left(\frac{i}{1 - i} \right)^x di - \tilde{c}.$$

On a balanced growth path employment n , learning l , the fraction of credit goods ζ , the real interest rate r , and the rate of inflation π must be constant. As in the models of the previous section, equation (A.1.5a), thus, implies that the growth rate of consumption $g \equiv \dot{c}/c$ equals

$$g = r - \rho. \quad (\text{A.2.6a})$$

The stock of capital must also grow at this rate so that

$$g = A(\tilde{h}n)^{1-\alpha} - \delta - w\tilde{h}\kappa_0 \int_0^\zeta \left(\frac{i}{1-i}\right)^x di - \tilde{c}. \quad (\text{A.2.6b})$$

Furthermore, from the CIA:

$$g = \mu - \pi. \quad (\text{A.2.6c})$$

Substituting the solution for λ_k in (A.1.4a) into equation (A.2.2c) yields the labor market clearing condition:

$$w\tilde{h} = [1 + (1 - \zeta)(r + \pi)]\beta(n + l)^{\eta-1}\tilde{c}, \quad (\text{A.2.6d})$$

where the left-hand-side is the wage rate per unit of raw labor n . Using the solution for ψ as well as $\bar{h} = h$, condition (A.2.3) can be written as:

$$\tilde{c}(r + \pi) = w\tilde{h}\kappa_0 \left(\frac{\zeta}{1 - \zeta}\right)^x. \quad (\text{A.2.6e})$$

In equilibrium the firm's first-order conditions (A.2.5) reduce to

$$r = \alpha(\tilde{h}n)^{1-\alpha} - \delta, \quad (\text{A.2.6f})$$

$$w = (1 - \alpha)(\tilde{h}n)^{-\alpha}. \quad (\text{A.2.6g})$$

Two further equations characterize the balanced growth path. The production function for human capital $\dot{h} = Dh l^\nu$ implies

$$g = D l^\nu \quad (\text{A.2.6h})$$

and the final equation derives from condition (A.2.2h). For $\dot{\lambda}_h/\lambda_h$ to be constant, both λ_h and λ_k must grow at the same rate. According to (A.1.4a), $\dot{\lambda}_k/\lambda_k = -\dot{c}/c = -g$. Using this, equations (A.2.2h), (A.2.2c), and (A.2.2e) can be arranged to yield:

$$\rho = \nu D l^{\nu-1} n. \quad (\text{A.2.6i})$$

The nine equations (A.2.6) determine g , π , r , w , n , l , \tilde{c} , \tilde{h} , and ζ . The simpler model with a given share of cash goods $1 - \zeta$ derives from this system if one deletes equation (A.2.6e) and cancels the credit cost term in (A.2.6b).

A.2.1.2 Calibration

As compared to the AK-model the Lucas model has two additional parameters, D and ν . We employ the value of $\nu = 0.8$ from Lucas (1990). As in the AK-model we set hours supplied to the market equal to $n = 1/3$. For our target rate of growth $g = 1.02^{1/4} - 1$, equations (A.2.6h) and (A.2.6i) can then be solved for D and l . There is, thus, one degree of freedom in the choice of the remaining parameters, which we close by setting $A = 1$. The remaining free parameters are determined in the same way as in the AK-model.

A.2.2 Search frictions

A.2.2.1 Equilibrium conditions

Households. In the case of search frictions we include hours spent searching s in the current-period utility function and add equation (5.1) to the set of constraints in (A.2.1). The current-value Lagrangian of the respective optimization problem reads:

$$\begin{aligned} \mathcal{L} = & \left[\ln c - \beta \frac{(n + s + l)^\eta}{\eta} \right], \\ & + \lambda_k \left(whn + rk + \Omega + \tau - \pi m - w\bar{h}\kappa_0 \int_0^\zeta \left(\frac{i}{1-i} \right)^x di - z - c \right) + \lambda_m z \\ & + \lambda_n (qs - \theta n) + \lambda_h Dh l^\nu + \psi (m - (1 - \zeta)c). \end{aligned}$$

The optimality conditions for an interior solution $0 < n, s, l, \zeta < 1$ are:

$$\frac{\partial \mathcal{L}}{\partial c} = \frac{1}{c} - \lambda_k - \psi(1 - \zeta) = 0, \quad (\text{A.2.7a})$$

$$\frac{\partial \mathcal{L}}{\partial z} = -\lambda_k + \lambda_m = 0, \quad (\text{A.2.7b})$$

$$\frac{\partial \mathcal{L}}{\partial s} = -\beta(n + s + l)^{\eta-1} + \lambda_n q = 0, \quad (\text{A.2.7c})$$

$$\frac{\partial \mathcal{L}}{\partial \zeta} = -\lambda_k w\bar{h}\kappa_0 \left(\frac{\zeta}{1-\zeta} \right)^x + \psi c = 0, \quad (\text{A.2.7d})$$

$$\frac{\partial \mathcal{L}}{\partial l} = -\beta(n + s + l)^{\eta-1} + \nu \lambda_h Dh l^{\nu-1} = 0, \quad (\text{A.2.7e})$$

$$\dot{\lambda}_k = \rho \lambda_k - \frac{\partial \mathcal{L}}{\partial k} = (\rho - r) \lambda_k, \quad (\text{A.2.7f})$$

$$\dot{\lambda}_m = \rho \lambda_m - \frac{\partial \mathcal{L}}{\partial m} = \rho \lambda_m + \lambda_k \pi - \psi, \quad (\text{A.2.7g})$$

$$\dot{\lambda}_h = \rho \lambda_h - \frac{\partial \mathcal{L}}{\partial h} = \rho \lambda_h - \lambda_k w n - \lambda_h D l^\nu, \quad (\text{A.2.7h})$$

$$\dot{\lambda}_n = \rho \lambda_n - \frac{\partial \mathcal{L}}{\partial n} = \rho \lambda_n + \beta(n + s + l)^{\eta-1} - \lambda_k w h + \lambda_n \theta. \quad (\text{A.2.7i})$$

Firms. In the Lucas model, the vacancy costs must rise with the human capital of the workers in the financial sector implying the profits

$$\Omega = Ak^\alpha(nh)^{1-\alpha} - (r + \delta)k - wnh - \phi whv. \quad (\text{A.2.8})$$

Firms maximize

$$\int_0^\infty \Omega e^{-\bar{r}(t)} dt$$

subject to (A.2.8) and $\dot{n} = \vartheta v - \theta n$. The current-value Hamiltonian of this problem is:

$$\bar{H} = Ak^\alpha(nh)^{1-\alpha} - (r + \delta)k - wnh - \phi whv + \lambda_F(\vartheta v - \theta n),$$

from which we derive the necessary conditions

$$\frac{\partial \bar{H}}{\partial k} = \alpha Ak^{\alpha-1}(nh)^{1-\alpha} - r - \delta = 0, \quad (\text{A.2.9a})$$

$$\frac{\partial \bar{H}}{\partial v} = \lambda_F \vartheta - \phi wh = 0, \quad (\text{A.2.9b})$$

$$\dot{\lambda}_F = r\lambda_F - \frac{\partial \bar{H}}{\partial n} = (r + \theta)\lambda_F - ((1 - \alpha)Ak^\alpha(nh)^{-\alpha} - w)h. \quad (\text{A.2.9c})$$

Wage setting. The Nash-bargaining solution solves

$$\max_{wh} [(1 - \alpha)Ak^\alpha(hn)^{-\alpha}h - wh]^{1-\lambda} [wh - \beta(n + s + l)^{\eta-1}c]^\lambda,$$

implying

$$wh = \lambda(1 - \alpha)k^\alpha(hn)^{-\alpha}h + (1 - \lambda)\beta(n + s + l)^{\eta-1}c. \quad (\text{A.2.10})$$

Stationary equilibrium. In the stationary equilibrium, employment n , search s , learning l , the mass of vacancies v , the wage rate per effective working hours w , the real interest rate r , and the rate of inflation π are constant, while output y , consumption c , physical capital k , and human capital h grow at the same rate g . As in the previous models, conditions (A.2.7a), (A.2.7b), (A.2.7f), and (A.2.7g) imply

$$g = r - \rho, \quad (\text{A.2.11a})$$

and together with (A.1.8) and the definition of profits in (A.2.8) the household's budget constraint supplies a further condition on the growth rate of output:

$$g = A(\tilde{h}n)^{1-\alpha} - \delta - w\tilde{h} \left(\phi v + \kappa_0 \int_0^\zeta \left(\frac{i}{1-i} \right)^x di \right) - \tilde{c}. \quad (\text{A.2.11b})$$

As in the models before, the CIA adds a third condition on the growth rate:

$$g = \mu - \pi. \quad (\text{A.2.11c})$$

The wage equation (A.2.10) can be written as:

$$w = \lambda(1 - \alpha)(\tilde{h}n)^{-\alpha} + (1 - \lambda)\beta(n + s + l)^{\eta-1}(\tilde{c}/\tilde{h}). \quad (\text{A.2.11d})$$

Therefore, w is stationary and since consumption c and human capital h grow at the same rate, the term $\lambda_k wh$ in equation (A.2.7i) is constant implying $\dot{\lambda}_n = 0$. Thus, equations (A.2.7c), (A.2.7i) and the solution for λ_k provided in (A.1.4a) (which is valid in all models) can be solved for

$$w\tilde{h} = [1 + (1 - \zeta)(r + \pi)]\beta(n + s + l)^{\eta-1}\tilde{c}((\theta + \rho)(v/s)^{-\gamma} + 1). \quad (\text{A.2.11e})$$

Since n is constant, (5.1) and (5.4) yield

$$(v/s)^\gamma = \theta(n/s). \quad (\text{A.2.11f})$$

According to (A.2.9c) the firm's shadow price of employment λ_F and the stock of human capital h must grow at the same rate. Thus,

$$g = \frac{\dot{\lambda}_F}{\lambda_F} = r + \theta - \frac{(1 - \alpha)A(\tilde{h}n)^{-\alpha}h - wh}{\phi wh/\vartheta}.$$

Substituting for ϑ from (5.9) yields:

$$g = r + \theta - \frac{(v/s)^{\gamma-1}}{\phi} \left(\frac{(1 - \alpha)(\tilde{h}n)^{-\alpha}}{w} - 1 \right). \quad (\text{A.2.11g})$$

Replacing ψ and λ_k in (A.2.7d) and recognizing $\bar{h} = h$ gives:

$$(r + \pi)\tilde{c} = w\tilde{h}\kappa_0 \left(\frac{\zeta}{1 - \zeta} \right)^x. \quad (\text{A.2.11h})$$

The firm's first-order condition (A.2.9a) and the production function for human capital provide two further conditions for the balanced growth path:

$$r = A(\tilde{h}n)^{1-\alpha} - \delta, \quad (\text{A.2.11i})$$

$$g = Dl^\nu. \quad (\text{A.2.11j})$$

A final equation derives from the first-order condition (A.2.7h). For $\dot{\lambda}_h/\lambda_h$ to be constant, both λ_h and λ_k must grow at the same rate. Since $\lambda_k/\lambda_k = -\dot{c}/c = -g$ (see (A.1.4a)), equations (A.1.4a), (A.2.7h), and (A.2.7e) can be arranged to yield:

$$\rho = \frac{\nu D\tilde{h}l^{\nu-1}}{\beta[1 + (1 - \zeta)(r + \pi)]\tilde{c}}(n + s + l)^{1-\eta}wn. \quad (\text{A.2.11k})$$

The eleven equations (A.2.11) determine the variables r , g , n , s , v , l , ζ , π , w , \tilde{c} , and \tilde{h} .

A.2.2.2 Calibration

The system of equations in (A.2.11) consists of 15 parameters,

$$\{A, \alpha, \delta, \phi, \kappa_0, \chi, \rho, \beta, \eta, \theta, \gamma, \lambda, D, \nu, \mu\},$$

and the 11 unknowns referred to above. We directly fix the 8 parameters

$$\{\alpha, \delta, \rho, \eta, \theta, \gamma, \lambda, \nu\}$$

as explained in the body of the paper¹⁵ and determine the values of 6 additional parameters indirectly. Our six calibration targets are

1. the unemployment rate $u = s/(n + s) = 0.06$,
2. the labor force participation rate $n + s = 0.68$,
3. the inflation rate $\pi = 0$,
4. the share of cash goods $1 - \zeta = 0.82$,
5. the annual growth rate $(1 + g)^4 - 1 = 0.02$,
6. the semi interest rate elasticity of the income velocity of money $\epsilon_{v,r+\pi} = 5.95$.

This leaves us one undetermined parameter. We are, however, not free in the choice of the remaining parameters and variables. Given u and $n + s$ we can determine s and n so that equation (A.2.11f) can be solved for v . Given these solutions, equations (A.2.11e), (A.2.11j), and (A.2.11k) determine l : Dividing (A.2.11e) by (A.2.11k) yields:

$$\frac{1}{\nu D l^{\nu-1} n} = \frac{1}{\rho} ((\theta + \rho)(v/s)^{-\gamma} + 1)$$

Since $l^\nu = g/D$ (from (A.2.11j)) we can solve this equation for l :

$$l = \frac{\nu g n}{\rho} \left[\frac{(\theta + \rho)s}{\theta n} + 1 \right]$$

where we have substituted for $(v/s)^{-\gamma} = s/(\theta n)$ from equation (A.2.11f). Therefore, the solution for l depends on g , n and the parameters θ , ν and ρ .

Equations (A.2.11d), (A.2.11e), and (A.3.3g) can be solved for the parameter ϕ . Equation (A.2.11e) can be written as:

$$\beta(n + s + l)^{\eta-1} \frac{\tilde{c}}{\tilde{h}} = \frac{w}{1 + (1 - \zeta)(r + \pi)} \left[\frac{(\theta + \rho)s}{\theta n} + 1 \right]^{-1}$$

¹⁵See Table 5.1 for the respective values.

so that (A.2.11d) can be arranged to read:

$$w = \lambda(1 - \alpha)A \left(n\tilde{h}\right)^{-\alpha} \left\{ 1 - \frac{1 - \lambda}{1 + (1 - \zeta)(r + \pi)} \left[\frac{(\theta + \rho)s}{\theta n} + 1 \right]^{-1} \right\}^{-1}. \quad (\text{A.2.12})$$

Substituting this expression for w in (A.3.3g) and solving for ϕ yields:

$$\phi = \frac{1 - \lambda}{\lambda(r + \theta - g)} \left(\frac{v}{s}\right)^{\gamma-1} \left\{ 1 - \frac{1}{1 + (1 - \zeta)(r + \pi)} \left[\frac{(\theta + \rho)s}{\theta n} + 1 \right]^{-1} \right\}.$$

Given this solution for ϕ it is easy to see that the cost share $(\phi w \tilde{h} v) / \tilde{y}$ does not depend on the parameter A (and, thus, cannot be used to determine this parameter):

$$\begin{aligned} \frac{\phi w \tilde{h} v}{A \left(n\tilde{h}\right)^{1-\alpha}} &\stackrel{(\text{A.2.12})}{=} \frac{\phi v}{n} \lambda(1 - \alpha) \left\{ 1 - \frac{1 - \lambda}{1 + (1 - \zeta)(r + \pi)} \left[\frac{(\theta + \rho)s}{\theta n} + 1 \right]^{-1} \right\}^{-1}, \\ &= \frac{v(1 - \alpha)(1 - \lambda)}{n(r + \theta - g)} \left(\frac{v}{s}\right)^{\gamma-1}. \end{aligned}$$

As a result of these considerations, we choose $A = 1$ to close the model. In the baseline solution (i.e. with $\pi = 0$), this choice delivers a reasonable share of consumption in output of 67 percent, whereas the US average consumption share between 1995-2005 is 68 percent.¹⁶

A.3 The Jones-Manuelli model

A.3.1 Walrasian labor markets

A.3.1.1 Equilibrium conditions

Households. Different from the Lucas model, households finance investment into human capital accumulation i_h out of their earnings from labor, capital services, and profits. They solve

$$\max \int_0^\infty \left[\ln c - \beta \frac{\dot{n}^\eta}{\eta} \right] e^{-\rho t} dt,$$

subject to

$$\dot{k} = whn + rk + \Omega + \tau - \pi m - w\bar{h}\kappa_0 \int_0^\zeta \left(\frac{i}{1-i}\right)^x di - z - c - i_h, \quad (\text{A.3.1})$$

$$\dot{m} = z,$$

$$\dot{h} = i_h - \delta_h h,$$

$$m \leq (1 - \zeta)c.$$

¹⁶The data on personal consumption and gross domestic product - both seasonally adjusted and in 2005 chained dollars - were taken from Tables B-9 and B-17 of the Economic Report of the President 2011.

The Lagrangian of this problem in current shadow prices reads:

$$\begin{aligned}\mathcal{L} = & \left[\ln c - \beta \frac{n^\eta}{\eta} \right] \\ & + \lambda_k \left(wnh + rk + \Omega + \tau - \pi m - w\bar{h}\kappa_0 \int_0^\zeta \left(\frac{i}{1-i} \right)^x di - z - c - i_h \right) \\ & + \lambda_m z + \lambda_h (i_h - \delta_h h) + \psi (m - (1 - \zeta)c).\end{aligned}$$

The first-order conditions for an interior solution $0 < n < 1$ and $0 < \zeta < 1$ are:

$$\frac{\partial \mathcal{L}}{\partial c} = \frac{1}{c} - \lambda_k - \psi(1 - \zeta) = 0, \quad (\text{A.3.2a})$$

$$\frac{\partial \mathcal{L}}{\partial z} = -\lambda_k + \lambda_m = 0, \quad (\text{A.3.2b})$$

$$\frac{\partial \mathcal{L}}{\partial n} = -\beta n^{\eta-1} + \lambda_k w h = 0, \quad (\text{A.3.2c})$$

$$\frac{\partial \mathcal{L}}{\partial \zeta} = -\lambda_k w \bar{h} \kappa_0 \left(\frac{\zeta}{1-\zeta} \right)^x + \psi c = 0, \quad (\text{A.3.2d})$$

$$\frac{\partial \mathcal{L}}{\partial i_h} = -\lambda_k + \lambda_h = 0, \quad (\text{A.3.2e})$$

$$\dot{\lambda}_k = \rho \lambda_k - \frac{\partial \mathcal{L}}{\partial k} = (\rho - r) \lambda_k, \quad (\text{A.3.2f})$$

$$\dot{\lambda}_m = \rho \lambda_m - \frac{\partial \mathcal{L}}{\partial m} = \rho \lambda_m + \lambda_k \pi - \psi, \quad (\text{A.3.2g})$$

$$\dot{\lambda}_h = \rho \lambda_h - \frac{\partial \mathcal{L}}{\partial h} = \rho \lambda_h - \lambda_k w n + \lambda_h \delta_h. \quad (\text{A.3.2h})$$

Conditions (A.3.2a), (A.3.2b), (A.3.2f), and (A.3.2g) can be solved for ψ and λ_k with solutions given in (A.1.3) and (A.1.4a), respectively. Using these solutions in (A.3.2d) implies the same condition on the share of credit goods as in the Lucas model (see equation (A.2.3)).

Firms. Firms solve the same problem as in the Lucas model, i.e. they maximize profits defined in (A.2.4), so that the first-order conditions presented in (A.2.5) continue to hold.

Stationary equilibrium On the balanced growth path output y , consumption c , physical capital k , and human capital h grow at the same rate g , while the rate of inflation, working hours n , the share of credit goods ζ , the real wage per efficiency unit of labor w , and the real interest rate r are constant. Furthermore $\bar{h} = h$. As in all models the long-run rate of growth derives from (A.1.5a) and equals

$$g = r - \rho. \quad (\text{A.3.3a})$$

Via equations (A.1.8) and (A.2.4) the household's budget constraint implies:

$$g = A(\tilde{h}n)^{1-\alpha} - \delta - w\tilde{h}\kappa_0 \int_0^\zeta \left(\frac{i}{1-i}\right)^x di - \tilde{c} - \tilde{i}_h. \quad (\text{A.3.3b})$$

Since the CIA binds and ζ is constant, real money balances grow at the same rate as output, so that

$$g = \mu - \pi. \quad (\text{A.3.3c})$$

The first-order condition (A.3.2c) and (A.1.4a) imply:

$$w\tilde{h} = [1 + (1 - \zeta)(r + \pi)]\beta n^{\eta-1}\tilde{c}, \quad (\text{A.3.3d})$$

and (A.3.2d) yields:

$$\tilde{c}(r + \pi) = w\tilde{h}\kappa_0 \left(\frac{\zeta}{1-\zeta}\right)^x. \quad (\text{A.3.3e})$$

The firm's first-order conditions (A.2.5) reduce to

$$r = \alpha(\tilde{h}n)^{1-\alpha} - \delta, \quad (\text{A.3.3f})$$

$$w = (1 - \alpha)(\tilde{h}n)^{-\alpha}. \quad (\text{A.3.3g})$$

The dynamics of human capital accumulation, $\dot{h} = i_h - \delta_h h$, implies:

$$g\tilde{h} = \tilde{i}_h - \delta_h \tilde{h}. \quad (\text{A.3.3h})$$

According to (A.3.2e) the multipliers λ_h and λ_k must grow at the same rate $-g$ so that (A.3.2h) yields:

$$g = wn - \rho - \delta_h. \quad (\text{A.3.3i})$$

The nine equations (A.3.3) determine g , π , r , w , n , \tilde{c} , \tilde{h} , \tilde{i}_h , and ζ . The model with an exogenously given share of cash goods $1 - \zeta$ is a simplified version of the system (A.3.3) in which equation (A.3.3d) is missing and in which credit costs $w\tilde{h}\kappa_0 \int_0^\zeta \left(\frac{i}{1-i}\right)^x di$ drop out of equation (A.3.3b).

A.3.1.2 Calibration

The system (A.3.3) consists of ten parameters and nine variables. The parameters

$$\{\alpha, \eta, \delta, \rho\}$$

are set to the values presented in Table 2.1 and δ_h is set equal to δ as in Jones and Manuelli (1995). The remaining parameters are implied by the model's equilibrium conditions and the following five calibration targets:

1. working hours $n = 1/3$,
2. the inflation rate $\pi = 0$,
3. the share of cash goods $1 - \zeta = 0.82$,
4. the annual growth rate $(1 + g)^4 - 1 = 0.02$,
5. the semi interest rate elasticity of the income velocity of money $\epsilon_{v,r+\pi} = 5.95$.

A.3.2 Search frictions

A.3.2.1 Equilibrium conditions

Households. In the case of search frictions we include hours spent searching s in the current-period utility function of the household and add equation (5.1) to the set of constraints in (A.3.1). The current-value Lagrangian of this modified optimization problem reads:

$$\begin{aligned} \mathcal{L} = & \left[\ln c - \beta \frac{(n+s)^\eta}{\eta} \right] \\ & + \lambda_k \left(wnh + rk + \Omega + \tau - \pi m - w\bar{h}\kappa_0 \int_0^\zeta \left(\frac{i}{1-i} \right)^x di - z - c - i_h \right) \\ & + \lambda_m z + \lambda_n (qs - \theta n) + \lambda_h (i_h - \delta_h h) + \psi (m - (1 - \zeta)c). \end{aligned}$$

The first-order conditions for an interior solution $0 < n + s < 1$ and $0 < \zeta < 1$ are:

$$\frac{\partial \mathcal{L}}{\partial c} = \frac{1}{c} - \lambda_k - \psi(1 - \zeta) = 0, \quad (\text{A.3.4a})$$

$$\frac{\partial \mathcal{L}}{\partial z} = -\lambda_k + \lambda_m = 0, \quad (\text{A.3.4b})$$

$$\frac{\partial \mathcal{L}}{\partial s} = -\beta(n+s)^{\eta-1} + \lambda_n q = 0, \quad (\text{A.3.4c})$$

$$\frac{\partial \mathcal{L}}{\partial \zeta} = -\lambda_k w\bar{h}\kappa_0 \left(\frac{\zeta}{1-\zeta} \right)^x + \psi c = 0, \quad (\text{A.3.4d})$$

$$\frac{\partial \mathcal{L}}{\partial i_h} = -\lambda_k + \lambda_h = 0, \quad (\text{A.3.4e})$$

$$\dot{\lambda}_k = \rho \lambda_k - \frac{\partial \mathcal{L}}{\partial k} = (\rho - r)\lambda_k, \quad (\text{A.3.4f})$$

$$\dot{\lambda}_m = \rho \lambda_m - \frac{\partial \mathcal{L}}{\partial m} = \rho \lambda_m + \lambda_k \pi - \psi, \quad (\text{A.3.4g})$$

$$\dot{\lambda}_h = \rho \lambda_h - \frac{\partial \mathcal{L}}{\partial h} = \rho \lambda_h - \lambda_k w n + \lambda_h \delta_h, \quad (\text{A.3.4h})$$

$$\dot{\lambda}_n = \rho \lambda_n - \frac{\partial \mathcal{L}}{\partial n} = \rho \lambda_n + \beta(n+s)^{\eta-1} - \lambda_k w h + \lambda_n \theta. \quad (\text{A.3.4i})$$

Firms. Firms are modeled as in the Lucas model, i.e. profits are defined as in (A.2.8) and the first-order conditions (A.2.9) apply.

Wage setting. The Nash-bargaining solution solves

$$\max_{wh} [(1 - \alpha)Ak^\alpha (hn)^{-\alpha} h - wh]^{1-\lambda} [wh - \beta(n + s)^{\eta-1}c]^\lambda,$$

implying

$$wh = \lambda(1 - \alpha)k^\alpha (hn)^{-\alpha} h + (1 - \lambda)\beta(n + s)^{\eta-1}c. \quad (\text{A.3.5})$$

Stationary equilibrium. Note that the first-order conditions (A.3.4a), (A.3.4b), (A.3.4d)-(A.3.4h) correspond to (A.3.2a), (A.3.2b), (A.3.2d)-(A.3.2h) of the model with Walrasian labor markets. Therefore, the equilibrium conditions that derive from those equations are the same in both models, and we will just repeat them below without further discussion.

The economy's growth rate satisfies

$$g = r - \rho. \quad (\text{A.3.6a})$$

The household's budget constraint implies

$$g = A(\tilde{h}n)^{1-\alpha} - \delta - w\tilde{h} \left(\phi v + \kappa_0 \int_0^\zeta \left(\frac{i}{1-i} \right)^\chi di \right) - \tilde{c} - \tilde{i}_n, \quad (\text{A.3.6b})$$

and from the CIA we derive

$$g = \mu - \pi. \quad (\text{A.3.6c})$$

The bargaining solution (A.3.5) can be written as:

$$w = \lambda A(\tilde{h}n)^{-\alpha} + (1 - \lambda)\beta(n + s)^{\eta-1}(\tilde{c}/\tilde{h}). \quad (\text{A.3.6d})$$

For $\dot{\lambda}_n = 0$ equation (A.3.4h) can be solved for λ_n . Given this solution, the first-order condition (A.3.4c), (A.1.4a), and (5.8) imply

$$w\tilde{h} = [1 + (1 - \zeta)(r + \pi)]\beta(n + s)^{\eta-1}\tilde{c}((\theta + \rho)(v/s)^{-\gamma} + 1). \quad (\text{A.3.6e})$$

On the balanced growth path the flows in and out of the labor force must balance, $qs = \theta n = \vartheta v$, so that (from (5.1) and (5.4))

$$(v/s)^\gamma = \theta(n/s). \quad (\text{A.3.6f})$$

The next equation equals (A.3.3g) in the Lucas model since it rests on the first-order conditions of the firm (A.2.9b) and (A.2.9c):

$$g = r + \theta - \frac{(v/s)^{\gamma-1}}{\phi} \left(\frac{(1 - \alpha)(\tilde{h}n)^{-\alpha}}{w} - 1 \right). \quad (\text{A.3.6g})$$

The first-order conditions (A.3.4d) and (A.2.9a) deliver two further equations also known from the Lucas model:

$$(r + \pi)\tilde{c} = w\tilde{h}\kappa_0 \left(\frac{\zeta}{1 - \zeta} \right)^x, \quad (\text{A.3.6h})$$

$$r = A(\tilde{h}n)^{1-\alpha} - \delta. \quad (\text{A.3.6i})$$

The remaining two conditions follow from human capital accumulation and the first-order condition (A.3.4h), respectively and, thus, coincide with (A.3.3h) and (A.3.3i):

$$g\tilde{h} = \tilde{i}_h - \delta_h\tilde{h}, \quad (\text{A.3.6j})$$

$$g = wn - \rho - \delta_h. \quad (\text{A.3.6k})$$

The eleven equations (A.3.6) determine g , π , r , w , n , s , v , \tilde{c} , \tilde{h} , \tilde{i}_h , and ζ .

A.3.2.2 Calibration

The system (A.3.6) consists of 14 parameters and eleven variables. The eight parameters

$$\{\alpha, \eta, \delta, \delta_h, \rho, \theta, \gamma, \lambda\}$$

are set to the values presented in Table 5.1. The remaining six parameters are implied by the model's equilibrium conditions and the same calibration targets as specified in subsection A.2.2.2.