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Household Relational Contracts for Marriage, Fertility and Divorce

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Abstract

This paper applies the theory of relational contracts to a model in which a couple decides whether to marry or cohabit, how many children to have and subsequently whether to stay together or separate. We make precise the idea that cooperation in a household can be supported by self interest. Since the costs of raising children are unequally distributed among partners, there is a potential conflict between individually optimal and efficient, i.e. surplus maximizing, decisions. Side-payments are used to support cooperation but are not legally enforceable and thus have to be part of an equilibrium. This requires a stable relationship and credible punishment threats. Within this relational contracts framework, we analyze the effects of policy variables such as rights of access to children post-separation and wealth division/alimony rules, as well as the legal costs of divorce, on the interrelationships among the decisions on marriage, fertility and divorce.

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1 Introduction

Over the past four to five decades, fundamental changes have taken place in the nature of the family and the structure of family relationships.[1] These reflect complex interactions among changes in divorce laws, increasing female labour force participation, innovations in the technology of contraception and changes in social attitudes and norms. This paper is intended to contribute to the economic analysis of these phenomena by constructing a model of a couple-based household which chooses marriage vs cohabitation, its fertility, and whether to continue the relationship or to separate, in the context of given labour market conditions and a set of legal rules regulating post-separation outcomes.

Also over the past four to five decades there has been substantial development in the economic modelling of the household, and a large theoretical and empirical literature now exists dealing with issues such as marriage, fertility and divorce. A large part of this literature is based on the assumption that family members act cooperatively and necessarily achieve Pareto efficient allocations.[2] For example, the Nash bargaining models of household behavior originating with Manser and Brown (1980) and McElroy and Horney (1981) assume that household allocations are Pareto efficient and can somehow be enforced as binding agreements even in a one-shot game. Early challenges to this assumption were made by Ulph (1988), Woolley (1988) and, within the Nash bargaining framework, by Ott (1992), Konrad and Lommerud (1995), and Lundberg and Pollak (2003), among others. Applying non-cooperative game theory to household decision making in a static environment, they identify sources of inefficient behavior of household members.

In this paper, instead of *assuming* either cooperative or non-cooperative behavior, we derive conditions for cooperation in a dynamic setting where players are solely driven by their self-interest and are not able to write exogenously enforceable agreements. Thus, the present paper directly relates to the theory of relational contracts, which we believe provides an appropriate tool to gain new insights into decision-making within households, especially when this involves intertemporal choices. Relational contracts are dynamic games based on actions or outcomes that are observable but not verifiable, i.e., the associated contracts are not legally enforceable. As agreements in household relationships are to a large extent implicit and extend over quite long periods of time, they present a good subject for an analysis with a relational contracts model. Starting with Bull (1987), relational contracts were initially developed to analyze labour markets and agency situations. MacLeod/Malcomson (1989) provide a complete analysis for perfect information, while Levin (2003) explores the case of imperfect public monitoring. Baker, Gibbons, and Murphy (1994) and Schmidt and Schnitzer (1995) study the interaction between relational and formal contracts. This relates to the model in this paper since after a divorce, implicit agreements

[1] For extensive documentation and discussion of these changes see in particular the two Symposia on, respectively, Household Economics in the *Journal of Economic Perspectives*, 2007, 21(2), and Investment in Children in the *Journal of Economic Perspectives*, 2008, 22(3).

[2] See Apps and Rees (2009), Chapters 3,4 for an extensive survey and list of references.

between partners are typically replaced by formal legal arrangements that regulate such issues as wealth division, child custody, and access of the non-custodial parent to his children. Most closely related to the present analysis is the paper by Matouschek and Rasul (2008). They develop a model where ongoing cooperation within the household creates an exogenously given benefit and has to be enforced by threats of punishment for deviations. Divorce costs serve as a commitment device and thus increase the ability to generate utility by cooperation. Our model extends this approach and applies it to an analysis of a richer set of phenomena, in particular the fertility decision.

The underlying model consists of two risk-neutral players, the primary and the second earner (referred to as he and she respectively[3]), who form a – potentially – long-lasting relationship. They first decide whether to marry or cohabit and then how many children they want to have. When children are present, the second earner reduces her market labour supply. This causes current income losses as well as a reduced accumulation of human capital, thus inducing lower future wages. In later periods, the couple decides whether to remain together or to separate. Remaining together is efficient if the sum of players' payoff levels within the relationship is higher than outside. Reallocations of resources may be needed to maintain an efficient marriage. One partner might prefer a separation, while the other might want to stay together. Then, the former has to be sufficiently compensated. Such a transfer might or might not always be enforceable, depending on the underlying assumptions concerning the timing of payments. We analyze both cases, namely that the couple only breaks up if it is efficient, and the situation where inefficient separations can occur.

It is not however possible to make a formally binding commitment to a certain contingent allocation of the utility surplus arising from the relationship *ex ante*. Therefore, all related promises have to be self-enforcing and part of equilibrium strategies.[4] Cooperative behavior is only individually rational if reneging is followed by sufficient and credible punishment. A separation as punishment must be credible in the sense that it has be optimal for a player to actually terminate the relationship.

After identifying the Pareto efficient equilibrium, we analyze the situation following a divorce in more detail. While agreements during a relationship are to a large extent implicit, this changes after a divorce. When all goodwill is lost, issues like financial support or access to children are mainly governed by law. Thus, we take an institutional perspective and analyze the impact of different policy changes on fertility, marriage stability, and the propensity to get married versus cohabiting. In doing so, we want to contribute to the discussion concerning low birth rates in many countries. We argue that in addition to issues that deal with the increased opportunity costs of children mentioned earlier, a shift in the enforceability of transfers induced by legal changes can be an important factor determining fertility levels.

[3] The pronouns reflect the fact that 70-90% of second earners in North America and Europe are female. See Immervoll et al., 2009, Table 1, for country-specific numbers.

[4] Although bargining models might implicitly assume a dynamic setting to support efficient decisions, they do not make precise the conditions necessary for cooperation.

A major part of this analysis deals with divorce costs. Our model predicts that divorce costs may in general have a positive impact on fertility by increasing relationship stability and decreasing players' reservation utilities. Thus, a marriage can serve as a commitment device to enforce cooperation within a relationship. This idea has already been discussed by Becker (1991) and Rawthorn (1999), was more formally derived by Matouschek and Rasul (2008), and empirically tested by various authors (see Rasul, (2003), Stevenson, (2007), Matouschek and Rasul, (2008), and Bellido and Marcén, (2011)).However, if divorce costs reduce the credibility of an off-equilibrium divorce threat too far might they create adverse effects, so that higher divorce costs do not necessarily increase welfare in our model. Making a divorce more difficult induces couples to stay together when their match quality has become relatively bad and they would prefer to break up, absent divorce costs. If the gains from increased commitment are lower than this welfare loss, a couple might not get married but instead choose to cohabit (where separation costs are substantially lower), and in this way higher divorce costs might ultimately decrease fertility.

The result of the model,to the effect that total welfare might not necessarily increase with higher divorce costs, is in line with the empirical results presented by Alesina and Giuliano (2007) who, in contrast to Rasul (2003) and Matouschek and Rasul (2008), find that unilateral divorce, which is taken to imply a fall in divorce costs, does not imply a decrease but rather an increase in the number of marriages. Concerning the impact on fertility, Alesina and Guiliano (2007) also find that in-wedlock fertility basically remains unaffected by the adoption of unilateral divorce laws, while out-of-wedlock fertility decreases significantly and fertility rates for newly married couples go up. This supports our view that the impact of divorce costs on marriage and fertility overall is not as obvious as it might seem, since our model is rich enough to capture more aspects of the interaction than just an increased degree of commitment.

In addition to divorce costs, we analyze the impact of wealth division rules, in the form of post-separation monetary payments solely based on income differences. Although having no (direct) impact on relationship stability, they can help to increase fertility. Since raising children is typically associated with a decrease in future income for one spouse, such payments can serve as an insurance against the human capital loss that results from taking time out for child care. Both effects together – no direct impact on relationship stability in equilibrium combined with an increased slackness of the enforceability constraint – increase the relative benefits of being married compared to cohabiting for higher post-separation payments.

Finally, we model the effects of a reduction of the primary earner's access to his children following a separation. By fostering marriage stability, reducing reservation utility and therefore increasing the punishment following noncooperation, such a restriction can also help to increase fertility. It does imply however that an unintended consequence of increases in post-separation access of primary earners, though increasing the utility of both them and their children, may be an increase in the divorce rate and lower fertility.

2 The Model

Two individuals decide whether to form a household and, if so, whether to marry or cohabit. In each case a household consists of a primary and a second earner, denoted $i = 1, 2$. The time horizon is infinite[5], $t = 0, 1, 2, ...$, and players discount the future with the factor $\delta \in (0, 1)$.

In the first period of the game, i.e., in $t = 0$, the couple chooses to have $n \geq 0$ children. For convenience, we assume n is a real number. This requires the second earner to devote $g(n)$ of her total time allocation (normalized to 1) to raising children in $t = 0$, with $g(0) = 0$, $g' > 0$, $g'' \geq 0$ and $g(n) \leq 1$. In this period therefore she earns $(1 - g(n))\, w_{20}$, where w_{20} is her wage in period $t = 0$. In all future periods, the second earner supplies her total time allocation to the labour market, earning $w_{2t}(n) \geq 0$. Because of work-related human capital acquisition, the wage is an increasing concave function of her period 0 labor supply and therefore a decreasing convex function of fertility, i.e., $w'_{2t} < 0$ and $w''_{2t} \geq 0$.[6] It is assumed that human capital accumulation only occurs in the first period[7], and the wage $w_{2t}(n) \equiv \overline{w}_2(n)$ is constant for $t \geq 1$.

The primary earner works full time in every period and earns w_{1t} in period t. As his human capital accumulation is of no interest to our analysis, w_{1t} is constant over time and equals $\overline{w}_1$. Furthermore, we assume that $\overline{w}_1 > \overline{w}_2(n) \geq w_{20}$.

Per period utility functions if the household is formed are $u_{it} = x_{it} + \varphi_i(n)$, with $\varphi_i(0) = 0$, $\varphi'_i > 0$ and $\varphi''_i < 0$ for $i = 1, 2$, where x is a private consumption good. The individual consumptions are defined by:

$$x_{1t} = \overline{w}_1 - p_t \quad t = 0, 1, ... \tag{1}$$

$$x_{20} = w_{20}[1 - g(n)] + p_0 \tag{2}$$

$$x_{2t} = \overline{w}_2(n) + p_t \quad t = 1, 2, ... \tag{3}$$

where $p_t \gtreqless 0$ is a payment made from one partner to the other. If $p_t > 0$, the primary earner makes the payment.

The payment p_t need not be explicit, its value is *implied by any choice* of n *and the* x_{it}, given $\overline{w}_1, w_{20}$ and $\overline{w}_2(n)$. For analytical purposes however it is useful to treat this *as if* it were an explicit payment. On the other hand, no explicit contract on the p_t is feasible, it has to be part of an equilibrium supported by the household relational contract (HRC), defined below.

In periods $t = 1, 2, ...$, the couple makes the decision whether to separate or remain together. A separation has the following consequences:

- Each receives an exogenously given outside net utility $\tilde{v}_i$, a random variable, in every period, which reflects possibilities outside the relationship,

[5] Although not factually correct, this assumption can be rationalised by assuming individuals do not know for sure when the last periods of their lives will be.

[6] The reduced wage can also reflect difficulties in re-entering the labor market after being out of it for some time. Future non-participation can be handled by assuming $w_{2t} = 0$.

[7] This assumption has no qualitative impact on our results.

such as potential new partners, as well as those within, like love or caring for the existing partner. The common assumption that the couple receives utility just by being together is captured by making the values of these outside utilities net of any internal "relationship utilities". These outside utilities determine the "match quality" of the couple in a given period, as the difference between the sums of achieved and outside utilities. The value $\tilde{v}_i$ is drawn independently in each period from a distribution $F_i(\tilde{v}_i)$ with continuous density $f_i(\tilde{v}_i)$, strictly positive everywhere on the support $[v_i^0, v_i^1]$. Thus the $\tilde{v}_i$ are independently and identically distributed over time. Furthermore, we assume $v_i^0 \leq 0 < v_i^1$ and denote the unconditional expectation $E[\tilde{v}_i] \equiv \overline{v}_i$. For now, we do not impose further restrictions on the distributions. We will implicitly do so later in order to have second order conditions satisfied. We also assume that both outside utility realizations, v_1 and v_2, are observed by each partner.[8] If a player breaks a promise (what this means will be made precise below), the general quality of living together can be negatively affected. Thus, following a deviation, the non-reneging partner i's outside utility is increased by the amount $\Delta v_i \geq 0$ in every subsequent period.

- The utility derived from children by the primary earner after a separation is $\theta\varphi_1(n)$, $\theta \in [0, 1]$. Here, we want to allow for differences in legislation determining the access of the primary earner to his children, given the assumption that custody is granted to the second earner,[9] and this is captured in a simple way by θ.

If the couple had chosen to marry, as opposed to cohabiting, a separation is a divorce and has two further effects:

- The partners bear possibly unequal divorce costs $k_i > 0$.
- The second earner receives a monetary transfer $\phi\{w_{1t} - w_{2t}(n)\}$ from 1. We will refer to this transfer as a wealth division rule or alimony payment. Since we only consider agents who are risk neutral and saving is not considered explicitly, both terms mean the same thing in our setting. Although the transfer does not directly depend on the number of children, n enters via its impact on 2's wage. The factor ϕ is assumed to be known ex ante and is determined by divorce law. Note that we are assuming that this law takes into account the effects of the second earner's withdrawal from the labour market on her human capital, in assessing the value of the payment. Then, ϕ measures the weight given to this effect.

[8] The assumption that spouses know their partners' outside option fairly well is supported by Peters (1986)

[9] Note, we do not take account of any perceived disutility to the children arising from divorce. This could be treated as a factor, say $\rho_i \in (0, 1]$, applied to *both* utilities. Nothing in the following discussion would change qualitatively as a result, as long as the value of ρ for the second earner was not so much smaller than that for the primary earner as to outweigh the effects of θ as analyzed here.

The values for k_i, θ and ϕ remain constant over time. The separation decision is irreversible, so that a couple never gets together again. After a separation, no further voluntary transfers are made. We assume that then, all trust between (former) partners is lost, implying that transfers can no longer be self enforcing (i.e., part of an equilibrium – this is further specified below).

As a result of these assumptions, the partners' separation utilities in periods $t = 1, 2, ..$ are, if married (and omitting the redundant time subscript)

$$\tilde{U}_1(v_1) = \frac{1}{1-\delta}\left(\overline{w}_1 + \theta\varphi_1(n) - \phi[\overline{w}_1 - \overline{w}_2(n)]\right) - k_1 + v_1 + \frac{\delta}{1-\delta}\overline{v}_1 \quad (4)$$

and

$$\tilde{U}_2(v_2) = \frac{1}{1-\delta}\left(\overline{w}_2(n) + \varphi_2(n) + \phi[\overline{w}_1 - \overline{w}_2(n)]\right) - k_2 + v_2 + \frac{\delta}{1-\delta}\overline{v}_2 \quad (5)$$

We denote the expectation of separation utilities $E[\tilde{U}_i(\tilde{v}_i)] = \tilde{U}_1(\overline{v}_i)$ by $\bar{U}_i$.

If the partners are not married, we simply set $k_i = \phi = 0$. Thus we model cohabitation as essentially the decision to avoid divorce costs and dispense with legal regulation of wealth division payments. We discuss the question of child custody/access arrangements in section 8.[10]

To complete the model we specify the timing of events within one period. At the beginning of $t = 0$, the couple decides between marriage or cohabitation.[11] The spouses must both agree to whichever choice they make.[12] Then, they unanimously decide on n, the number of children. The primary earner then works full-time, while the second earner allocates her time between work and raising children, as specified above. After players receive their income, a transfer p_0 is made.

At the beginning of each subsequent period, if the couple is still together both observe the realizations of this period's outside utilities, v_{1t} and v_{2t}. Taking these into account, the spouses then decide whether to remain together or not. If they separate, they receive their separation utilities $\tilde{U}_i$. Otherwise, both work and receive their income, followed by the transfer p_t. We do not impose any exogenous bound on the transfer levels. This implicitly assumes that players can save or borrow.

Note that we abstract from consumption costs of children. If we included such costs and assumed a given allocation among partners, our results would not be affected qualitatively. The same would be true for laws providing for financially support to a parent who has custody of the children post-separation to cover child costs. Furthermore, our wealth division rules do not take the utility of children into account and are only supposed to compensate the second earner for her human capital loss. Thus, we do not consider child support laws.

[10] We ignore possible costs or utility of the act of getting married in itself.

[11] The matching process is taken as exogenously given.

[12] In contrast, we assume below that separation/divoce decisions can be made unilaterally.

These are beyond the scope of our analysis, especially as one problem associated with them is that fathers often do not pay despite the existence of a legal title (see Allen and Brinig, 2010).

Our assumptions with regard to $g(n)$, the time needed for child care, and the fact that only the second earner participates in this, require some discussion. If non-parental child-care facilities were available, $g(n)$ and the associated human capital loss could be reduced. However, this would not affect our results, as long as a substantial amount of parental time still has to be spent. Realistically of course both parents engage in child care, but as long as their input levels are substantially different, as appears to be generally the case, our qualitative results go through.

3 Household Relational Contracts

3.1 The game: formal characterization

Players have to decide on whether to form a household, and if they do so the legal form of their relationship, the number of children they want to have, payments p_t, and, in later periods, whether they separate or stay together. We assume that while together they formulate a household relational contract (HRC), which is a subgame perfect equilibrium of the game and specifies all actions players will take conditional on all possible histories. However, this cannot be a legally binding contract contingent on actions or outcomes, because of the non-verifiability of the payments p_t.[13]

We briefly give a formal characterization of actions, strategies and conditions for a subgame perfect equilibrium, i.e., an HRC. Instead of just referring to the net transfer p_t, we split it into the individual contributions of players 1 and 2, with $p_t = p_{1t} + p_{2t}$, where $p_{1t} \geq 0$ and $p_{2t} \leq 0$. Obviously, only the net transfer p_t is relevant and thus used in all other sections. However, splitting it into two components simplifies a characterization of strategies.

The number of children is determined as follows. Each player announces their preferred fertility level n_i, $i = 1, 2$. Since this decision has to be made unanimously, we assume for convenience that realized fertility[14] n is the smaller of both players' announcements if they differ, i.e., $n = \min\{n_1, n_2\}$.

Players also announce $d_{it} \in \{0, 1\}$, where $d_{it} = 1$ indicates that player i wants to remain together for period t. If at least one of them chooses $d_{it} = 0$, they irrevocably break up.[15] The variable $d_t \in \{0, 1\}$ indicates whether the relationship is still active in period t. It is defined recursively by $d_t = d_{t-1} d_{1t} d_{2t}$, with $d_0 = 1$.

[13] This is supported by the argument that individual consumptions within a household cannot be verifiably measured. In reality of course there is a far richer set of reasons for the impossibility of complete marital contracts than this.

[14] Which, we assume, is non-stochastic.

[15] We thus assume a unilateral divorce regime.

Finally, each player announces a value $m_i \in \{0,1\}$. $m_i = 1$ indicates that player i wants to get married, whereas $m_i = 0$ implies that the player prefers to cohabit. The spouses marry if and only if both agree, i.e., if $m \equiv m_1 m_2 = 1$. Otherwise, the couple cohabits.

Then, the history h_t specifies all events that occur at time t. For $t = 0$, we have $h_0 = \{m_1, m_2, n_1, n_2, d_{10}, d_{20}, p_{10}, p_{20}\}$ (note that $d_{10} = d_{20} = d_0 = 1$ by assumption). For all $t \geq 1$, the history is $h_t = \{d_{1t}, d_{2t}, p_{1t}, p_{2t}\}$. Then, $h^t = \{h_\tau\}_{\tau=0}^{t-1}$ is the history path at the beginning of period t, with $h^0 = \emptyset$. $H^t = \{h^t\}$ characterizes the set of history paths up to time t, while $H = \cup_t H^t$ is the set of all possible histories.

A strategy σ_i for player i, $i = 1, 2$, is a sequence of functions $M_i \cup N_i \cup \{P_{it}, D_{it}\}_{t=0}^{\infty}$, where $M_i : H^0 \to \{0,1\}$ specifies whether the couple gets married (if $m = m_1 m_2 = 1$) or cohabits ($m = 0$). $N_i : H^0 \cup \{m_1, m_2\} \to [0, \infty)$ describes the process determining fertility at the beginning of period $t = 0$, and $n = \min\{n_1, n_2\}$ is the realized fertility level. $D_{it} : H^t \cup \{v_{1t}, v_{2t}\} \to \{0,1\}$, $t \geq 1$, characterizes players' decisions on whether they want to remain together ($d_{it} = 1$) or separate, with $d_t = d_{t-1} d_{1,t} d_{2,t}$ and $d_0 = 1$. Finally, transfers are determined by $P_{1,t} : H^t \cup \{v_{1t}, v_{2t}, d_{1t}, d_{2t}\} \to [0, \infty)$ and $P_{2,t} : H^t \cup \{v_{1t}, v_{2t}, d_{1t}, d_{2t}\} \to (-\infty, 0]$ for periods $t \geq 1$. In $t = 0$, the functions are $P_{1,0} : H^0 \cup \{m_1, m_2, n_1, n_2\} \to [0, \infty)$ and $P_{2,0} : H^0 \cup \{m_1, m_2, n_1, n_2\} \to (-\infty, 0]$.

Denoting a player's payoffs following history h_t by $U_i(\sigma_1, \sigma_2 \mid h_t)$, a strategy profile (σ_1, σ_2) is a subgame perfect equilibrium if and only if following any history h_t,

$$\sigma_1 \in \arg\max_{\tilde{\sigma}_1} U_1(\tilde{\sigma}_1, \sigma_2 \mid h_t)$$

$$\sigma_2 \in \arg\max_{\tilde{\sigma}_2} U_1(\sigma_1, \tilde{\sigma}_2 \mid h_t)$$

3.2 Fertility, Transfers, and Constraints

The spouses will use the payments p_t as an incentive to either raise fertility or maintain the relationship.[16] This increases efficiency as absent any transfers chosen fertility, as the minimum of the individual optima, would be lower and separation probabilities higher, since separation could take place even when the sum of utilities is greater with the relationship than without. Note that we refer to efficiency as the outcome players would choose if they were able to fully commit.

Too low fertility is induced by the exogenously given distribution of the costs and benefits of having children, i.e., the fact that the second earner loses human capital when she takes time to raise them in period $t = 0$. Thus it is very likely that the individually optimal levels of n differ between spouses. Then, gains from cooperation exist which the partners can try to exploit. The partner bearing relatively higher costs (in relation to the benefits) might be willing to agree on having more children than individually optimal if compensation is (credibly) promised.

[16] Transfers can always contain a purely redistributive component as well.

Inefficient separation has to be prevented if staying together is efficient, i.e, outside utility realizations (v_1, v_2) in a period are low enough that the sum of utility streams when remaining together is higher than $\sum_i \tilde{U}(v_i)$, but one player would find it individually optimal to split. Then, a transfer exists that makes it optimal for both to remain together.

However, transfers to increase efficiency by giving incentives to raise fertility and prevent inefficient divorce must be self-enforcing, i.e., part of a subgame perfect equilibrium as specified above. For example, assume a transfer is supposed to be positive. Then, it must be in the interest of the primary earner to actually make it *ex post*, i.e., after the second earner has kept her promise, either to accept a higher fertility level or abstain from inducing a separation. Thus, although he might be willing to make that transfer *ex ante*, the limits of commitment in the absence of explicit contingent contracts might make him break this promise ex post. As this is anticipated by the second earner, her ex-ante willingness to cooperate would be limited by his credibility.

More precisely, a transfer will only be made if reneging triggers sufficient punishment. We use the standard dynamic games/relational contracts approach[17] and assume that after someone has reneged, the relationship has become strained, and any trust between the partners is lost. Thus, the harshest possible punishment is used (Abreu, 1986), implying that the equilibrium with the lowest payoff for the player that reneged (pushing that player down to their reservation utility) is played. As the only decision players can make in periods $t \geq 1$ determines whether they want to remain together, punishment here must take the form of a separation.[18] Still, this punishment threat has to be credible. Assume that a player did not act as intended and is supposed to be punished by a separation. If staying together is in the interest of both in the following period, the punishment will be postponed until a sufficiently high draw of one of the outside utilities is realized.[19] Furthermore, a separation only effectively penalizes a player if it does not occur in equilibrium anyway. Thus, transfers to reward cooperation can be enforced more easily if separation is less likely in equilibrium and if the probability that – absent transfers – one partner will want to break up is higher.

However, these factors might offset each other to some extent. As an example, take 2's divorce costs k_2. It is intuitive and will be shown below that higher divorce costs generally make divorce less likely. But they also increase the probability that the second earner is willing to stay within the marriage

[17] For example, see MacLeod and Malcomson (1989).

[18] In the household Nash bargaining literature discussed in the Introduction, considerable discussion has taken place over whether separation is too drastic a punishment for failure to disagree, and this has led to models which take as threat points non-cooperative Nash equilibria within an ongoing household. It is said, for example, that one would not threaten divorce over a failure to agree on the colour of a sofa. While we agree with that viewpoint, the class of household decisions being analyzed in this paper is we believe sufficiently fundamental that threats based on separation are the appropriate ones to assume.

[19] Formally, a player can always make sure of receiving her minmax-payoff. Furthermore, note that as reneging triggers the end of all cooperation, it does not matter whose outside utility is sufficiently high.

anyway and does not need compensation. Therefore, it is not clear whether higher divorce costs k_2 have a positive or negative impact on the enforceability of transfers.

A broken promise however can also have an impact on the general quality of living together. Thus we assume that after a player reneges, the partner‘s outside utility increases by Δv_i in every period in each state. The size of Δv_i has no qualitative impact on our results, unless it is so large that a punishment is always credible and divorce occurs immediately after a deviation. Both cases will be analyzed below.

To summarize: a spouse who wants to reward her partner for cooperation has to make a payment now, but is punished in the future for not doing so. Thus, inefficiency can still exist in equilibrium, if either a punishment cannot be enforced immediately after a deviation or if the future is sufficiently heavily discounted.

While having children is a “discrete” event in time, the same is not necessarily true for making transfers and inducing a breakup. The period between which payments are feasible could be made arbitrarily small,[20] and there are good reasons why an artificial division into fixed periods would not reflect the real life of a couple. Thus, the main part of our paper will impose the assumption that the decision whether to separate or remain together is always made efficiently in equilibrium. In the Appendix, we show that we approach this outcome arbitrarily closely by assuming that time is continuous and each period of a given (discrete) length can be divided into subperiods. If transfers can be made and a separation induced in each of these subperiods,[21] making the latter arbitrarily small lets the couple separate almost only when this is actually efficient. The reason is that reneging is almost immediately followed by a punishment. Subsequently, we take the initially assumed discrete nature of the game literally and show what happens if inefficient breakups can happen on the equilibrium path.[22]

Before going on with the formal analysis, we should briefly discuss whether a separation really is always necessary to punish a devation from cooperative behavior, as it destroys surplus (off equilibrium) and thus is not renegotiation proof. Although this issue is not relevant within the limited horizon of our model - where a deviation and thus punishment nevers occurs in equilibrium - we want to point out that it is possible to construct an off-equilibrium outcome that actually is renegotiation proof. Instead of breaking up after a deviation, the couple can continue to play a cooperative equilibrium, but where the reneging player is subsequently pushed down to their reservation utility. This is possible because of one important feature of relational contracts, namely that any surplus distribution can be induced as long as both players at least receive their reservation utilities. However, both approaches give the same equilibrium out-

[20] Wickelgren (2007), among others, makes this argument.

[21] Still, the interval between new draws of outside utilities remain fixed.

[22] However, the inability to commit to a necessary transfer might not be the only reason why an inefficient divorce can occur. Voena (2011), for example, assumes that one spouse's intertemporal budget constraint could bind.

comes, and our focus on a non-renegotiation-proof equilibrium is without loss of generalitiy.

In the following, we derive necessary (and sufficient) conditions to induce allocations that increase efficiency. These results do not depend on how the resulting surplus is shared among players. Assuming a transfer to maintain the relationship can be enforced, actually any surplus distribution is feasible - as long as both players at least receive their reservation utilities. Thus, our first objective is to characterize the set of subgame perfect equilibria that are Pareto efficient and maximize the sum of players' utilities.

4 Constraints in $t = 0$

In this section, we derive a general condition that specifies to what extent utility transfers in period $t = 0$ are enforceable. If it binds, this condition determines equilibrium fertility. If it does not bind, the efficient fertility level can be attained. Note that all results derived here hold independently of whether we assume that the separation decision is always made efficiently or not.

It will further become clear that only the (promised) allocation of utility streams matters for players' willingness to cooperate, and using an explicit formulation in terms of the transfer p_0 is just a useful tool to obtain that objective. This also implies that players are indifferent between receiving/giving current resources (via p_0) or expected future payoffs as a reward for cooperative behavior - both can be substituted arbitrarily, and we do not have to specifiy how exactly resources are redistributed.

We start with the definition of the relevant payoff streams. Define $\bar{U}_i^*$ as i's expected discounted continuation utility on the equilibrium path at $t = 1$ taking into account both non-divorce and divorce states.[23] Then define $\overline{U}_i$ as i's expected discounted continuation utility off the equilibrium path at $t = 1$.

Note that off-equilibrium or reservation utilities $\overline{U}_i$ do not necessarily coincide with expected separation utilities $\tilde{U}_i$ (defined in (4) and (5) above) because they might cover states where divorce does not occur. To what extent they differ depends on the credibility of punishment threats. Furthermore, we can omit the time subscript without loss of generality.[24]

At the beginning of period $t = 0$, both spouses unanimously decide on equilibrium fertility n^* and an associated transfer p_0, taking future utility allocations into account (which might be a function of fertility as well).[25] If they fail to reach an agreement, they have $n^{**} = \min\{n_1^{**}, n_2^{**}\}$ children and play the non-cooperative equilibrium from then on, where n_i^{**} is player i's individually preferred non-cooperative fertility level, i.e., if $p_t = 0$ for all $t \geq 0$.

Knowledge of n_i^{**} also tells us who needs to be compensated in equilibrium,

[23] Corresponding to the $\overline{U}_i^*$ will be (possibly implicit) side payments p which in general also vary across states.

[24] The reason is that current and future payoffs are perfect substitutes, and we can thus focus on stationary contracts.

[25] For a formal description see section 2.1 above.

namely the one with a lower level. To fix ideas we will generally assume that $n_1^{**} > n_2^{**}$.[26] Then, the players can (possibly tacitly) agree on the following deal at the beginning of period 0. Player 2 is willing to accept $n^* > n_2^{**}$. After the children are born, she receives a transfer $p_0(n^*)$ at the end of period 0 and/or the promise of higher continuation payoffs in the future. If she insists on any smaller number of children, there will be no transfer in period $t = 0$ as well as in any other subsequent period. If she insists on a smaller n, she will always choose n^{**}.

The opposite is true for $n_1^{**} < n_2^{**}$. Then, the primary earner needs be compensated for agreeing on a higher fertility level, either with the (now negative) transfer $p_0(n^*)$ or a higher expected continuation utility.

Two kinds of conditions have to be satisfied so that n^* can actually be part of an equilibrium. First of all, given players believe the transfers will be made, it has to be optimal for both to choose n^* rather than any other level. Furthermore, transfers have to be credible, i.e., making them has to give players a higher utility than not making them.

Making n^*optimal for both players given that promised transfers are made is captured by incentive compatibility (IC) constraints, which are

(IC1)

$$u_{11}(n^*) - p_0(n^*) + \delta\overline{U}_1^*(n^*) \geq u_{11}(n^{**}) + \delta\overline{U}_1^*(n^{**}) \tag{6}$$

for player 1, and

(IC2)

$$u_{21}(n^*) + p_0(n^*) + \delta\overline{U}_2^*(n^*) \geq u_{21}(n^{**}) + \delta\overline{U}_2^*(n^{**}) \tag{7}$$

for the second player.

Here, $u_{i1}(n)$ is player i's period-0 utility and $\overline{U}_i^*(n^*)$ player i's expected discounted equilibrium payoff stream in period 1. Which case actually holds determines whether transfers are negative or positive: $n_1^{**} > (<)\ n_2^{**} \Rightarrow p_0(n^*) > (<) 0$.[27]

Furthermore, it has to be in the interest of players to make a promised transfer. This is only the case if their utility is higher than otherwise. Thus, reneging requires a punishment. As discussed above, this punishment takes the form of pushing a player down to her reservation utility. The following dynamic enforcement (DE) constraints make these arguments precise. If a transfer is positive, the primary earner has to decide whether to make it or renege. He will only keep his promise, if

(DE1)

$$p_0(n^*) \leq \delta[\overline{U}_1^*(n^*) - \overline{U}_1(n^*)] \tag{8}$$

[26] We could support this with the argument that the introduction of the contraceptive pill gave women control over their own fertility and was associated with a significant fertility decline.

[27] If continuation utilities alone give sufficient incentives for first-best equilibrium fertility, it would even be possible that we observe $n_1^{**} > n_2^{**}$ and a negative transfer. However, $p_0(n^*)$ then is being solely used for redistributive purposes in period $t = 0$ and not to give incentives. We are not interested in this possibility, as it would imply that constraints in period $t = 0$ do not bind in equilibrium and thus are not relevant.

is satisfied. If the payment is supposed to be negative, the secondary earner makes the relevant decision. She will only cooperate if her utility after making it is at least as high as if she did not

(DE2)

$$-p_0(n) \leq \delta[\overline{U}_2^*(n^*) - \overline{U}_2(n^*)] \tag{9}$$

Note that the (DE) constraints require players to believe that future equilibrium transfers are also made.

Combining (IC) and (DE) constraints then gives just one constraint which is both necessary and sufficient for an equilibrium fertility level n^* to be enforceable.

Proposition 1:

If $n_1^{**} > n_2^{**}$ a fertility level n^* can be enforced if and only if it satisfies the **(IC-DC)** constraint condition

$$u_{21}(n^*) - u_{21}(n^{**}) + \delta\left[\overline{U}_1^*(n^*) + \overline{U}_2^*(n^*) - \left(\overline{U}_1(n^*) + \overline{U}_2(n^{**})\right)\right] \geq 0 \tag{10}$$

If $n_1^{**} < n_2^{**}$, the necessary and sufficient condition for equilibrium fertility n^* is

$$u_{11}(n^*) - u_{11}(n^{**}) + \delta\left[\overline{U}_1^*(n^*) + \overline{U}_2^*(n^*) - \left(\overline{U}_1(n^{**}) + \overline{U}_2(n^*)\right)\right] \geq 0 \tag{11}$$

The proof of Proposition 1 can be found in the Appendix[28].

The (IC-DE) constraint states that the gains from deviating today must not exceed the future surplus, i.e., equilibrium payoffs net of reservation utilities.

The chosen fertility level has a direct impact on the enforceability of transfers, a feature usually not observed in relational contracting models, where the production process tends to be independent across periods. This aspect of the problem becomes especially important when inefficient separation occurs in equilibrium, and higher fertility can help to increase relationship stability. Furthermore, although the kind of production process usually used in the literature is independent over time, it still remains identical. This implies a further dimension in which our setting differs, since incentives to increase fertility are provided by using the surplus from remaining together.

The fact that satisfying the (IC-DE) constraint is also sufficient for enforcing a fertility level n^* (Proposition 1) allows us to separate surplus distribution from incentive giving. This implies that any surplus distribution that gives players at least their reservation utilities is enforceable. Thus, we can confine our interest

[28] All proofs not presented in the text of the paper are contained in an Appendix which is available electronically from the authors on request

to the (constrained) Pareto optimal equilibrium, without having to worry about who gets what.

Denoting players' expected payoffs at the beginning of period $t = 0$ but after the marriage decision has been made U_i, equilibrium fertility n^* solves

$$\max_n U = U_1 + U_2 \tag{12}$$

subject to (IC-DE).

Depending on the realizations of outside utilities, the couple might break up in equilibrium. Thus, the enforceability of transfers crucially depends on the perceived relationship stability. If partners strongly believe that they will end up getting separated anyway, they are less willing to find ways to cooperate early on in the relationship. In the following, we therefore further analyze the determinants of relationship stability on and off the equilibrium path.

5 Efficient Separation

In this section, we derive conditions for the case in which breaking up is optimal for the couple. As already pointed out, we also assume at first that a separation in equilibrium only occurs if it is efficient (i.e., what the partners would choose if they were able to fully commit) and then allow for inefficient separations later.

This assumption somewhat neglects the discreteness of the model. As we will see later, taking the discreteness seriously will always induce situations where remaining together is efficient but not possible, as the necessary transfer cannot be enforced. Yet, it is not obvious why it should not be possible to make the separation decision - as well as corresponding transfers - at any point in time. In Appendix II, we show that if time is continuous and each original period is subdivided into sufficiently small subperiods, we can get arbitrarily close to the outcome that the couple breaks up if and only if that is actually efficient. Thus, even when this assumption is imposed, players still act within the framework of a relational contract and not within a bargaining game.

If a bargaining structure such as that in MacLeod and Malcomson (1995) or Shaked and Sutton (1984) were imposed, the surplus distribution on and off equilibrium would be the same, as would be the decision whether to remain together or break up. Then, no punishment would be feasible, making it impossible to enforce a transfer. However, as the game continues with positive probability after a transfer has been made and since trust in the partner's ongoing willingness to cooperate is necessary to sustain cooperation at any point in time, remaining within the relational contracts framework even with the assumption regarding efficient separations seems sensible. This implies two further issues. If all trust between the players is lost after one reneged, the couple will break up off equilibrium even if remaining together would be optimal. Furthermore, any surplus distribution is feasible.[29]

[29] Again, this would allow us to obtain an outcome that is renegotiation proof - even off

5.1 Some Implications

Take periods $t \geq 1$ (the first time a break-up can occur is $t = 1$) and assume that the couple has married.[30] Define

$$u_1^0 = \overline{w}_1 + \varphi_1(n) \tag{13}$$

$$u_2^0 = \overline{w}_2(n) + \varphi_2(n) \tag{14}$$

as the per-period utilities the partners would have within the relationship with $p_t = 0$ for $t \geq 1$, and U_i^0 as the respective infinite discounted payoff streams (taking into account that a divorce might occur in future periods). If $\sum_{i=1}^2 (U_i^0 - \tilde{U}_i) < 0$, a separation is optimal and will occur.

As all utility components are fixed and constant over time except for the realizations of $\tilde{v}_i$, the latter determine whether the couple should break up. More precisely, this is specified by the sum of outside utility realizations, $v_1 + v_2$, independent of the respective individual values. Thus, define

$$\tilde{v} \equiv \tilde{v}_1 + \tilde{v}_2 \tag{15}$$

where $\tilde{v}$ has distribution $F(\tilde{v})$ and continous density $f(\tilde{v})$ (specified below) and is strictly positive everywhere on the support $[v_1^0 + v_2^0, v_1^1 + v_2^1]$.

Lemma 1: *Assume the separation decision is made efficiently. Then, a divorce takes place if and only if $\tilde{v} > \hat{v}$, where $\hat{v}$ is defined by*

$$\varphi_1(n)(1-\theta) + (1-\delta)(k_1 + k_2) + \delta \int_{v_1^0 + v_2^0}^{\hat{v}} f(\tilde{v})(\hat{v} - \tilde{v}) d\tilde{v} - \hat{v} = 0 \tag{16}$$

Proof:

The assumption that the couple chooses to separate if and only if $\sum_{i=1}^2 (U_i^0 - \tilde{U}_i) < 0$ is the first component needed to establish the existence of the threshold $\hat{v}$. In addition, we need that, given that the threshold setting $\sum_{i=1}^2 (U_i^0 - \tilde{U}_i) = 0$ exists, $\sum_{i=1}^2 (U_i^0 - \tilde{U}_i)$ is decreasing in $\hat{v}$.

Finding a value $\hat{v}$ that satisfies $\sum_{i=1}^2 (U_i^0 - \tilde{U}_i) = 0$ is done recursively. First, we assume this threshold exists and that a divorce takes place if and only if $v > \hat{v}$ for any value of $\hat{v}$. Then, we derive the conditions for this behavior actually to be optimal, i.e., specify $\hat{v}$.

equilibrium, the separation decision could be made efficiently, yet pushing the player who deviated down to his/her reservation utility.

[30] The issue of marriage versus cohabitation is considered in section 8 below.

Given the threshold $\hat{v}$, the partners' expected discounted payoff streams within the relationship when $p_t = 0$ for an arbitrary period $t \geq 1$ (which also allows us to omit time subscripts) are

$$U_1^0 = \overline{w}_1 + \varphi_1(n) + \delta \left[F(\hat{v})U_1^0 + (1 - F(\hat{v}))\, E[\tilde{U}_1 \mid v \geq \hat{v}] \right] \tag{17}$$

$$U_2^0 = \overline{w}_2(n) + \varphi_2(n) + \delta \left[F(\hat{v})U_2^0 + (1 - F(\hat{v}))\, E[\tilde{U}_2 \mid v \geq \hat{v}] \right] \tag{18}$$

Furthermore, recall that the payoff streams in a period where a divorce happens equal

$$\tilde{U}_1(v_1) = \frac{1}{1-\delta} \left(\overline{w}_1 + \theta\varphi_1(n) - \phi[\overline{w}_1 - \overline{w}_2(n)] \right) - k_1 + v_1 + \frac{\delta}{1-\delta}\overline{v}_1 \tag{19}$$

$$\tilde{U}_2(v_2) = \frac{1}{1-\delta} \left(\overline{w}_2(n) + \varphi_2(n) + \phi[\overline{w}_1 - \overline{w}_2(n)] \right) - k_2 + v_2 + \frac{\delta}{1-\delta}\overline{v}_2 \tag{20}$$

where we take into account the assumption that once a couple breaks up, it will not get together again in the future. To obtain a characterization of $E[\tilde{U}_i \mid v \geq \hat{v}]$, the realizations of v_i in $\tilde{U}_i(v_i)$ only have to be replaced by $E[v_i \mid v \geq \hat{v}]$.

Note that (as v_1 and v_2 are independently distributed)

$$f(\tilde{v}) = (f_1 * f_2)(\tilde{v}) = \int_{v_1^0}^{v_1^1} f_1(v_1) f_2(\tilde{v} - v_1) dv_1 = \int_{v_2^0}^{v_2^1} f_1(\tilde{v} - v_2) f_2(v_2) dv_2 \tag{21}$$

and

$$\begin{aligned} F(\tilde{v}) &= \int_{v_1^0+v_2^0}^{\hat{v}} f(\tilde{v}) d\tilde{v} = \int_{v_1^0+v_2^0}^{\hat{v}} \left(\int_{v_1^0}^{v_1^1} f_1(v_1) f_2(\tilde{v} - v_1) dv_1 \right) d\tilde{v} \\ &= \int_{v_1^0+v_2^0}^{\hat{v}} \left(\int_{v_2^0}^{v_2^1} f_1(\tilde{v} - v_2) f_2(v_2) dv_2 \right) d\tilde{v} \end{aligned} \tag{22}$$

Thus,

$$E[v_1 \mid v \geq \hat{v}] = \frac{1}{1 - F(\hat{v})} \int_{\hat{v}}^{v_1^1+v_2^1} \left(\int_{v_1^0}^{v_1^1} f_1(v_1) f_2(\tilde{v} - v_1) v_1 dv_1 \right) d\tilde{v} \tag{23}$$

and

$$E[v_2 \quad | \quad v \geq \hat{v}] = \frac{1}{1-F(\hat{v})} \int_{\hat{v}}^{v_1^1+v_2^1} \left(\int_{v_2^0}^{v_2^1} f_1(\tilde{v}-v_2) f_2(v_2) v_2 dv_2 \right) d\tilde{v} \quad (24)$$

$$= \frac{1}{1-F(\hat{v})} \int_{\hat{v}}^{v_1^1+v_2^1} \left(\int_{v_1^0}^{v_1^1} f_1(v_1) f_2(\tilde{v}-v_1)(\tilde{v}-v_1) dv_1 \right) d\tilde{v} \quad (25)$$

Plugging all expressions into $U_1^0 + U_2^0 = \tilde{U}_1(v_1) + \tilde{U}_2(v_2)$, applying Bayes' rule and rearranging gives (16).

Finally, it remains to show that (16) is decreasing in $\hat{v}$. Differentiating (16) with respect to $\hat{v}$ gives $-(1-\delta F(\hat{v})) < 0$, which completes the proof.

Note that this proof does not require $\hat{v} \leq v_1^1 + v_2^1$, i.e., that the threshold is below the upper bound of the support of $\tilde{v}$. Thus, we also cover the case in which divorce never occurs in equilibrium.

It is then easy to prove

Proposition 2: Given that the separation decision is efficient, divorce in a period is less likely - for given distributions of outside options - the higher are divorce costs, the lower is the primary earner's post-separation right of access to the children, θ, and the higher the number of children, while it is independent of the wealth division parameter ϕ, the wage gap $\overline{w}_1 - \overline{w}_2$ and the second earner's labour supply $1 - g(n)$.

Proof: These follow straightforwardly from implicitly differentiating (16), which gives

$$\frac{d\hat{v}}{dk_1} = \frac{d\hat{v}}{dk_2} = \frac{(1-\delta)}{(1-\delta F(\hat{v}))} > 0; \; \frac{d\hat{v}}{d\theta} = \frac{-\varphi_1(n)}{(1-\delta F(\hat{v}))} < 0; \; \frac{d\hat{v}}{dn} = \frac{\varphi_1'(1-\theta)}{(1-\delta F(\hat{v}))} > 0 \quad (26)$$

These results are perfectly intuitive. Wealth division simply represents a transfer between the partners. Although it makes the primary earner less prone to file for a divorce, the opposite is true for the second earner, with a net effect of zero. Loss of the primary earner's access to the children on the other hand is a form of deadweight loss to the couple, as are divorce costs. This suggests that there is a tradeoff from society's point of view between the primary earner's post-divorce right of access to the children and the divorce rate, since increasing the former also raises the latter, other things equal.

In the restricted context of the separation decision, higher fertility leads to a lower divorce rate, since the deadweight loss from divorce increases with n, given $\varphi_1'(n) > 0$ and $\theta < 1$. Since fertility is endogenous, however, there is still much more to be said on the relationship between fertility and divorce.

Note that the results for k_i are valid for couples married at the time when the law changes. They do not imply that divorce rates have to go up in the long run (if costs are reduced and θ increased). Instead, a new institutional setting also changes incentives to actually become married, thus affecting subsequent divorce propensities. We further explore this issue in section 8 below. We just note here that short-run effects do indeed appear to differ from long-run effects. As an example, take the change to unilateral divorce laws in many US states some decades ago, which could be regarded as a reduction of divorce costs. In the short run, divorce rates went up, confirming our predictions; however, they returned essentially to their initial levels after some time (see Wolfers, (2006), Matouschek and Rasul, (2008)).

5.2 Who is more likely to initiate divorce?

There seems to be strong empirical evidence that in some countries at least, a wife is significantly more likely than a husband to be the one to initiate divorce.[31] One obvious possible explanation would be that husbands are more likely to break their promises, but that is not consistent with our model, which predicts no deviations along the equilibrium path. Given that we assume efficient divorce, our model would predict that the second earner would be the one to initiate divorce on the equilibrium path if and only if

$$\sum_{i=1}^{2}(U_i^0 - \tilde{U}_i) < 0 \text{ and } U_2^0 - \tilde{U}_2 < 0 < U_2^0 - \tilde{U}_2 \tag{27}$$

In that case the primary earner is the one who would like to keep the marriage going, but he cannot make a sufficiently large transfer to induce the second earner to agree. Using the earlier definitions of these payoffs, it is straightforward to prove:

Proposition 3: *Given that divorce takes place if and only if it is efficient, the second earner is more likely to initiate divorce than the primary earner on the equilibrium path (cet. par.):*

- *the more favourable her distribution $F_2(\hat{v}_2)$ relative to his, $F_1(\hat{v}_1)$, in the sense of First Order Stochastic Dominance*
- *the higher is ϕ, the generosity of the wealth division rule toward the second earner*
- *the larger is the gender wage gap $\overline{w}_1 - \overline{w}_2(n)$*
- *the lower is her divorce cost k_2 relative to k_1*
- *the lower is θ, the measure of post-divorce access to children for the primary earner*

[31] See Guven et al (2009), who show this using data sets for the UK, Germany and Australia.

- *the higher is fertility n and the utility the primary earner derives from his children, $\varphi_1(n)$*

6 Fertility

We now characterize equilibrium fertility and derive comparative statics results with respect to a number of divorce laws. The aim is to contribute to the public discussion on why fertility in (especially) Western countries has been falling. As already pointed out, this discussion usually restricts attention to a simple benefit-cost analysis and discusses the effectiveness of various policies to reduce various costs (also including parents' human capital losses). All these issues could also be incorporated into our model, yielding the predicted results.[32] Here, we take a different approach and show that legislation that is not directly aimed at influencing the propensity to have children might have a substantial impact as well. Since costs and benefits are at least partially exogenously given and fixed (for reasons explained above), redistribution within the household is needed to equalize the burden among spouses. However, no formal contract determining within-household allocation can be written, and all transfers have to be self-enforcing.

We assume that all cooperation ceases after a separation, and the implicit agreement is replaced by formal rules.[33] Different divorce laws have an impact on relationship stability and/or the absolute and relative welfare levels of spouses after a separation. Thus, these rules will directly affect each partner's utility as well as the enforceability of transfers, by having an impact on the credibility of punishment threats as well as the risk of being left alone. Note that the following results are true for a couple given it chooses marriage. It does not necessarily imply that divorce laws have the predicted consequences at the aggregate level. Instead, couples might also adjust their marriage-versus-cohabitation decision. We further explore this issue in section 8 below.

6.1 Equilibrium Fertility

Absent any transfers, individually optimal fertility levels n_1^{**} and n_2^{**} will generally differ. To what extent the spouses' interests can be aligned depends on the enforceability of transfers.

Recall that the couple solves

$$\max_n U = U_1 + U_2$$

subject to the (IC-DE) constraint derived above, where U_i are the expected utility streams at the beginning of period $t = 0$, i.e., when the household has just

[32] For example, providing subsidized child-care facilities would reduce $g(n)$ at each n and thus increase fertility.

[33] We will further specify below how our setting relates to the general matter of interactions between explicit and implicit contracts, as for example analyzed by Baker, Gibbons and Murphy (1994).

been formed. The objective in period 0 is set on lifetime utility streams, taking into account the utilities that will actually be chosen in each state (including divorce utilities in the corresponding states). The distributional variables are those relevant in period 0, when the allocation is being chosen. The decision must take into account the effect of the current fertility choice on all future utilities along the equilibrium path.

If the (IC-DE) constraint does not bind, we obtain the efficient outcome, and equilibrium fertility is described by

Proposition 4: *Assuming (IC-DE) does not bind, optimal fertility n^* satisfies*

$$\alpha\varphi_1'(n^*) + \varphi_2'(n^*) = (1-\delta)w_{20}g_0'(n^*) - \delta\overline{w}_2'(n^*) \tag{28}$$

where

$$\alpha \equiv \frac{1-\delta+\delta(1-F(\hat{v}))\theta}{1-\delta F(\hat{v})} \leq 1 \tag{29}$$

The proof of Proposition 4 can be found in the Appendix.

This leads to the conclusion that in the presence of a positive probability of divorce $(1 - F(\hat{V}_2) > 0)$ and less than complete access to the children after divorce for the primary earner $(\theta < 1)$ there will be a lower fertility rate than is socially optimal, since this would require $\alpha = 1$. The marginal social benefit of fertility is $\left(\varphi_1'(n^*) + \varphi_2'(n^*)\right)/1-\delta$, and the marginal social cost (recall that child consumption costs have been set to zero) is the marginal value of the time the second earner spends in child rearing in both periods, taking into account also the value in period 1 of the loss of human capital in period 0.

This immediately allows us to obtain some comparative statics predictions with respect to divorce laws when the efficient fertility level is feasible. It suffices to discuss this for the case of $n_1^{**} > n_2^{**}$.

Proposition 5: *Assume that $n_1^{**} > n_2^{**}$ and the respective (IC-DE) constraint does not bind. Then, higher divorce costs increase equilibrium fertility, a lower access of the primary earner to his children might or might not increase fertility, while wealth division laws have no impact.*

Proof:

Equilibrium fertility is characterized by (28). Note that the second order condition is satisfied by construction, and so we must have $\partial^2 U/\partial n^2 < 0$ at the optimal n^*.

Thus, we have

$$\frac{dn^*}{dk_1} = \frac{dn^*}{dk_2} = -\frac{f(\hat{v})\delta(1-\delta)^2}{(1-\delta F(\hat{v}))^3}\frac{\varphi_1'(n)(1-\theta)}{\partial^2 U/\partial n^2} > 0 \tag{30}$$

$$\frac{dn^*}{d\phi} = 0 \tag{31}$$

$$\frac{dn^*}{d\theta} = -[\frac{f(\hat{v})(1-\delta)}{(1-\delta F(\hat{v}))^2}\delta\frac{d\hat{v}}{d\theta}\varphi_1'(n)(1-\theta) + \frac{\delta(1-F(\hat{v}))\theta}{1-\delta F(\hat{v})}\varphi_1'(n)]/\frac{\partial^2 U}{\partial n^2} \lesseqgtr 0 \quad (32)$$

As wealth division rules after a divorce only redistribute funds between spouses, they cancel out when the constraint does not bind and thus have no impact on equilibrium fertility.

Condition (28) in Proposition 4 gives some intuition on whether increasing divorce costs or reducing θ could be expected to raise or lower fertility. Clearly for $\theta = 1$ we have $\alpha = 1$ and so fertility will be at its first best level, since the probability of divorce no longer plays a role in determining fertility. However, realistically we must have $\theta < 1$ if a couple ceases to cohabit after divorce and the children remain with the second earner.[34] Then, higher divorce costs always increase fertility by making divorce less likely. The probability of the efficiency loss induced by a separation is reduced, inducing the couple to have more children. Thus, divorce costs serve as a commitment device, an outcome supported empirically by Rasul (2005), Stevenson (2007), Matouschek and Rasul (2008), and Bellido and Marcén (2011).

The results of reducing θ are ambiguous because there are two opposing effects. On the one hand, the marginal return to fertility across divorce states goes down. However, the probability of no divorce increases, and the net effect depends on parameter values and the form of the distribution function $F(.)$. If θ is close to 1, the latter effect is negligible, and a reduction of a father's access to his children after a separation always leads to a fertility reduction.

Finally, note that reducing the marginal cost on the right hand side of (28) would also increase fertility, and this could be achieved by reducing the rate at which increased fertility reduces the second earner's loss of human capital, clearly strengthening the argument for policies that allow second earners to combine raising a family with pursuing a career.

The question arises of whether higher divorce costs or a decrease in θ are beneficial, especially when they increase fertility. Although low birth rates affect a society as a whole (consider for example the discussions on the financing of the welfare state), we restrict attention to the impact of divorce laws on the utilities of the partners involved. Moreover, in the long run only the couple's welfare is relevant. If their utilities are lower in the presence of divorce laws, they will simply abstain from getting married and instead cohabit (see section 8)

Then, as long as the (IC-DE) constraint is not binding, restrictions such as higher costs or a decreased access to children after a divorce reduce total utility.

Lemma 3: *Given the efficient fertility level can be enforced, higher divorce costs and a lower θ decrease total equilibrium surplus U.*

[34] Even if there are no legal restriction to a primary earner's access, the pure fact that the parents no longer live together will reduce the time he can spend with his children.

Proof:

Applying the Envelope Theorem gives

$$\frac{dU}{dk_i} = \frac{\partial U}{\partial k_i} = -\delta \frac{(1 - F(\hat{v}))}{1 - \delta F(\hat{v})} < 0 \tag{33}$$

$$\frac{dU}{d\phi} = 0 \tag{34}$$

$$\frac{dU}{d\theta} = \frac{\partial U}{\partial \theta} = \frac{\delta}{1-\delta}\varphi_1(n)\frac{1 - F(\hat{v})}{1 - \delta F(\hat{v})} > 0 \tag{35}$$

The reason for this result is that although higher costs or a lower access reduce the probability of divorce, this destroys surplus as players cannot consume outside utilities v_i where it would otherwise be optimal (note that marriage stability has no value *per se*). Thus, although divorce costs and a lower value of θ serve as a commitment device to increase fertility, the increased commitment is harmful if (IC-DE) does not bind.

6.2 A Binding (IC-DE) Constraint

If the relationship is relatively unstable or the difference between n_1^{**} and n_2^{**} large (for example because 2's marginal human capital loss is high), it is likely that the (IC-DE) constraint (10) binds and equilibrium fertility is smaller than the efficient level.

First, to see that equilibrium fertility is lower than the level implied by (28), take the Lagrange function $L = U^0 + \lambda[(IC - DE)]$. The first order condition gives

$$\frac{dU^0}{dn} + \lambda \frac{d(IC - DE)}{dn} = 0 \tag{36}$$

If (IC-DE) does not bind, $\lambda = 0$ and we are in the unconstrained case. If it binds, $d(IC - DE)/dn$ has to be negative. The reason is that otherwise, increasing fertility would relax the constraint, contradicting that we are at an optimum. Thus, $dU^0/dn > 0$ in an equilibrium with the (IC-DE) constraint binding. As d^2U^0/dn^2 must has to be negative as well, equilibrium fertility is lower when (IC-DE) binds. Furthermore, it decreases with λ.

Then, n^* is determined by the binding constraint, or, in case of $n_1^{**} > n_2^{**}$,

$$u_{21}(n^*) - u_{21}(n^{**}) + \delta\left[\overline{U}_1^*(n^*) + \overline{U}_2^*(n^*) - \left(\overline{U}_1(n^*) + \overline{U}_2(n^{**})\right)\right] = 0$$

It is worth further specifying off-equilibrium utilities $\overline{U}_1(n^*)$ and $\overline{U}_2(n^{**})$. Recall that after a player deviates, no further transfers are made. The couple breaks up if inducing a divorce is optimal for at least one player, which depends on the realizations of the outside utilities. If these are not sufficiently high, both just receive their relationship-utilities $w_i + \varphi_i(n)$ in the respective period and wait for the next draw of $\tilde{v}_i$. In addition, the partner's failure to make an agreed payment has a negative impact on the overall quality of the relationship and thus increases the partner's outside utility by $\Delta v_i \geq 0$ in every subsequent period. This has an impact on the couple's ability to redistribute resources, but only as it affects the likelihood of a separation off equilibrium.

In the Appendix we derive thresholds v_i^* and v_i^{**} (depending on who deviated) for individual outside utilities. Only if either one of these thresholds is exceeded does a separation occur in the respective period. Otherwise, if both realizations of outside utilities are below these thresholds, both players prefer to stay together for at least one more period even though the partnership is no longer really working properly.[35] Then, they just wait until at least one partner's outside utility is sufficiently high as to end the relationship.

The actual levels of Δv_i have no qualitative impact on comparative statics results unless they are so high that both thresholds are below the lower bound of the support of outside utilities, i.e., if $v_i^*/v_i^{**} \leq v_i^0$. Then, a deviation is immediately followed by a separation in any case. This changes the impact of divorce laws on fertility if (IC-DE) binds, as increasing marriage stability then does not make a divorce threat less credible, and thus has an unambiguously positive effect. Therefore, we treat the cases where the Δv_i are high enough as to always induce a punishment, and that in which they are not, separately, still focusing on the situation with $n_1^{**} > n_2^{**}$.

In the first case, we have

Proposition 6 : *Assume $n_1^{**} > n_2^{**}$, the (IC-DE) constraint binds and both values of Δv_i are sufficiently high that any deviation is immediately followed by a separation for all realizations of $\tilde{v}_i$. Then, (cet. par.)*

- *higher divorce costs*
- *higher alimony payments*
- *lower access of the primary earner to his children following a divorce*

increase fertility .

Proof:

As $v_i^*/v_i^{**} \leq v_i^0$, we have $\overline{U}_i = \tilde{U}_i$. Plugging all values into the binding (IC-DE) constraint gives

(IC-DE)

$$w_{20}\left(g_0(n^{**}) - g_0(n^*)\right) + \varphi_2(n^*) - \varphi_2(n^{**})$$
$$+\frac{\delta}{1-\delta}\left(\overline{w}_2(n^*) + \varphi_2(n^*) - \overline{w}_2(n^{**}) - \varphi_2(n^{**})\right)$$

[35] This gives a kind of counterpart in this model to the "non-cooperative equilibrium within an ongoing marriage" idea applied for example by Lundberg and Pollak (2003).

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$$+\delta \int_{v_1^0+v_2^0}^{\hat{v}} f(\tilde{v})(\hat{v}-\tilde{v})d\tilde{v} + \frac{\delta}{1-\delta}\phi\left(\overline{w}_2(n^{**})-\overline{w}_2(n^*)\right) \geq 0 \tag{37}$$

If this binds in equilibrium, $\partial(IC-DE)/\partial n < 0$, since otherwise, higher fertility would relax the constraint, contradicting that it binds and fertility is too low at the same time. We then have, since $n^{**} \leq n^*$:

$$\frac{dn^*}{dk_i} = -\frac{d\hat{v}}{dk_i}\frac{\delta F(\hat{v})}{\partial(IC-DE)/\partial n} > 0; \ \frac{dn^*}{d\theta} = -\frac{d\hat{v}}{d\theta}\frac{\delta F(\hat{v})}{\partial(IC-DE)/\partial n} \leq 0 \tag{38}$$

$$\frac{dn^*}{d\phi} = -\frac{\delta}{1-\delta}\frac{\overline{w}_2(n^{**})-\overline{w}_2(n^*)}{\partial(IC-DE)/\partial n} \geq 0 \tag{39}$$

As higher divorce costs reduce the likelihood of a divorce without decreasing the severity of punishment, the range of states where surplus can be redistributed and used to provide incentives becomes larger. As empirically established, divorce costs thus also serve as a commitment device when the (IC-DE) constraint binds.

Higher alimony payments partially compensate the secondary earner for her human capital loss and thus reduce her marginal costs of having children. For a given fertility level the difference between her on- and off-equilibrium fertility increases as $\overline{w}_2(n^{**}) > \overline{w}_2(n^*)$. Thus, more redistribution between the spouses can be enforced, allowing them to increase n^*. Note that the impact of higher alimony is not driven by reducing 1's reservation equilibrium utility, as this cancels out against 2's increased reservation utility. Although having no direct impact on relationship stability, alimony payments thus make a separation less likely in equilibrium, as the probability of a divorce decreases in equilibrium fertility n^*.

A reduction of θ now has an unambiguosly positive impact on fertility. As fertility is too low, the utility reduction in case of a separation as a factor reducing fertility is obviously not taken into account. Thus, lower access of the primary earner to his children increases fertility by relaxing the (IC-DE) constraint.

As fertility is inefficiently low when (IC-DE) binds, making divorce more costly or difficult to obtain can even increase the total relationship surplus. This is always the case for alimony payments, which would have no impact on the surplus if the efficient fertility level were enforceable.

Lemma 4: *Assume $n_1^{**} > n_2^{**}$, the (IC-DE) constraint binds and that both values of Δv_i are sufficiently high that any deviation is immedialety followed by a separation for all realizations of $\tilde{v}_i$. Then, higher divorce costs and a lower θ might or might not increase the relationship surplus. Higher alimony payments* *always increase the surplus*

Proof:
As fertility is inefficiently low $\partial U(n^*)/\partial n > 0$. Thus,

$$\frac{dU(n^*)}{dk_i} = \frac{\partial U(n^*)}{\partial n}\frac{\partial n^*}{\partial k_i} + \frac{\partial U(n^*)}{\partial k_i} = \frac{\partial U(n^*)}{\partial n}\frac{\partial n^*}{\partial k_i} - \delta(1 - F(\hat{v})) \lesseqgtr 0 \quad (40)$$

$$\frac{dU(n^*)}{d\phi} = \frac{\partial U(n^*)}{\partial n}\frac{\partial n^*}{\partial \phi} + \frac{\partial U(n^*)}{\partial \phi} = \frac{\partial U(n^*)}{\partial n}\frac{\partial n^*}{\partial \phi} > 0 \quad (41)$$

$$\frac{dU(n^*)}{d\theta} = \frac{\partial U(n^*)}{\partial n}\frac{\partial n^*}{\partial \theta} + \frac{\partial U(n^*)}{\partial \theta} = \frac{\partial U(n^*)}{\partial n}\frac{\partial n^*}{\partial \theta} + \frac{\delta}{1-\delta}\varphi_1(n^*)\frac{1 - F(\hat{v})}{1 - \delta F(\hat{v})} \lesseqgtr 0 \quad (42)$$

If a player's deviation is not triggered by a sufficiently high increase of the partners outside utilities and thus not immediately followed by a separation (i.e., $v_i^*/v_i^{**} > v_i^0$), the impact of divorce laws is less obvious. Now, they not only affect divorce utilities but also the likelihood that punishment can actually be carried out. Thus, we have

Proposition 7: *Assume $n_1^{**} > n_2^{**}$ and $v_i^*/v_i^{**} > v_i^0$. Then, the impact of divorce laws on fertility is ambiguous.*

The proof for Proposition 7 can be found in the Appendix.

The effects of higher divorce costs or a lower level of θ are ambiguous, as these policies not only lower utilities in case of a divorce (which helps to enforce transfers) but also make it less likely that a punishment is actually carried out, because it becomes more attractive for players to remain within a marriage even if partners no longer cooperate. Concerning wealth division rules, we have as before increasing equilibrium fertility. Furthermore, 2's willingness to actually induce a divorce in each period offßequilibrium increases, while the primary earner is less likely to do that. Which effect dominates depends on the distributions of outside utilities. If these are for example uniformly distributed on the same support, both effects cancel out, and the impact of alimony on fertility is unambiguously positive.

The effect of divorce laws on total efficiency is also ambiguous, and we omit a formal analysis. When they increase fertility, they might increase total surplus for the same reason as above.

7 Inefficient Separation

In the preceding section, we assumed that the separation decision is always made efficiently in equilibrium. We further show in Appendix II that this outcome can be approximated arbitrarily closely in a continuous-time setting where the period length between decisions diminishes. Now, we take a different approach and take the discreteness of the game literally, although we do think assuming an efficient decision is more persuasive. For example, the fact that initiating a divorce takes time does not play a role here. This does indeed create a time-lag, namely between the decision to break up and the time from which the institutional changes - costs, wealth division, and a reduced access - come into force. However, this just requires us to discount the relevant parameters accordingly, which will affect the threshold $\hat{v}$, but give the same qualitative results as before.

The main difference is that a separation can occur even if it is efficient to remain together. The reason is that not all necessary transfers are enforceable, their enforceability only depends on expectations about future payoffs. Still, the results are not too different from those given above; the possibility of inefficient divorce is just always taken into account. Furthermore, fertility is affected, as children do not just provide utility *per se* but can also have an impact on the likelihood of a separation. If this likelihood is reduced, fertility can even be above the efficient level.

To start with the formal analysis, we briefy recall the timing within a period $t \geq 1$:

- The realizations of players' outside utilities $\tilde{v}_i$ are revealed to both. Then, they continue the relationship or break up. After a separation, players immediately receive their reservation utilities $\tilde{U}_i(v_i)$.
- If they remain together, they work and receive their wages. Then, the transfer p_t (also allowed to be negative) is made from 1 to 2.
- Finally, they consume the private consumption good x_t and enjoy utility from their children.

Furthermore, once the couple separates, it is assumed that the partners never come together again. Note that the timing of the transfer is not important - if it can already be made after outside utilities are revealed but before players have to make the separation decision, no additional stability is created.

Thus the $\tilde{v}_i$ are independently and identically distributed over time. Obtaining the states in which a separation occurs is slightly more involved now, as there does not exist just one value for the outside utilities above which the couple breaks up. Instead we have three thresholds, one for $\tilde{v}_1$, one for $\tilde{v}_2$ and one for the sum $\tilde{v}$. If any one of these thresholds is exceeded, a separation will occur. The reason is that we not only have to worry about efficiency, but also about the enforceability of transfers to maintain the relationship. As the latter only depends on expectations about future payoffs, thresholds for enforceability and efficiency generally will not coincide.

For concreteness, again assume that the couple chooses to marry. Recall the definitions $u_i^0 = \overline{w}_i + \varphi_i(n)$ as the per-period utilities within the relationship with $p_t = 0$ for any $t \geq 1$. Then, player i must be compensated to be willing to stay in the relationship if $u_i^0 + \delta\overline{U}_i^* - \tilde{U}_i(v_i) < 0$, where $\overline{U}_i^*$ is the expected equilibrium utility stream of player i (including transfers)[36] and $\tilde{U}_i(\tilde{v}_i)$ is player i's utility if filing for divorce in the respective period. Any transfer $p(v_i)$ must make it optimal for both players to stay within the relationship. Furthermore, it has to be in the interest of player 1 to provide a positive payment and for 2 to provide a negative payment. Concerning off-equilibrium payoffs, we assume for concreteness that if any player does not keep a promise, the increase in the other's outside utility, i.e., Δv_i, is large enough to immediately induce a subsequent divorce.

To derive the relevant constraints, let us first assume that $u_2^0+\delta\overline{U}_2^*-\tilde{U}_2(v_2) < 0$, i.e., the secondary earner needs a transfer to remain within the relationship. This transfer, denoted $p(v_2)$, must be large enough to satisfy 2's individual rationality (IR) constraint

$$u_2^0 + p(v_2) + \delta\overline{U}_2^* - \tilde{U}_2(v_2) \geq 0 \tag{43}$$

Furthermore, 1's (IR) constraint must hold, which is obviously the case if breaking up would be inefficient, i.e., if $v_1 + v_2 \leq \hat{v}$.[37] If this is the case, the primary earner must actually be willing to make the transfer $p(v_2)$. This is captured by 1's dynamic enforcement (DE) constraint,

$$p(v_2) \leq \delta[\overline{U}_1^* - \overline{U}_1] \tag{44}$$

Obviously, the right hand side of (44) is independent of current realizations of outside utilities. Thus, the maximum feasible value of $p(v_2)$ is the same in each period. Denoting this maximum feasible transfer by $\max p \equiv \delta[\overline{U}_1^* - \overline{U}_1]$, a transfer that keeps 2 in the relationship and satisfies 1's (DE) constraint exists if $u_2^0 + \max p + \delta\overline{U}_2^* - \tilde{U}_2(v_2) \geq 0$. Whether it is actually made also depends on 1's (IR) constraint, i.e., whether v_1 is sufficiently small that $v \leq \hat{v}$.

Concluding the previous arguments, a transfer that keeps 2 in the relationship and satisfies 1's (DE) constraint exists if $v_2 \leq v_2^{max}$, where v_2^{max} is defined by

$$u_2^0 + \delta\overline{U}_2^* - \tilde{U}_2(v_2^{max}) + \delta[\overline{U}_1^* - \overline{U}_1] = 0$$

Equivalent considerations help us to define the threshold v_1^{max}, stating when a negative transfer exists that keeps player 1 within the relationship and satisfies 2's (DE) constraint as long as $v_1 \leq v_1^{max}$:

$$u_1^0 + \delta\overline{U}_1^* - \tilde{U}_1(v_2^{max}) + \delta[\overline{U}_2^* - \overline{U}_2] = 0$$

Therefore, a separation can never be prevented if either $v_1 > v_1^{max}$ or $v_2 > v_2^{max}$. As already pointed out, this does not imply that if $v_1 \leq v_1^{max}$ and $v_2 \leq v_2^{max}$

[36] Note that we omit a time subscript and thus do not allow expected equilibrium payoffs to change over time. This is just done for convenience and without loss of generality, as - as we will see when computing the relevant constraints - the enforceability of any transfer as well as the efficiency of a separation always depends on the sum of players' utilities in equilibrium and not on the surplus distribution.

[37] Note that $\hat{v}$ here does not coincide with its value above, as separation probabilities will be different. Just the meaning is identical, namely that in a given period a separation is efficient if $v > \hat{v}$.

are satisfied, the couple remains together. It still has to be in the interest of a player to make a transfer. More precisely, the other's (IR) constraint must be satisfied as well, which will be the case if staying together is efficient. Concluding, a couple will not break up in any period t, if at the same time $v_1 \leq v_1^{max}$, $v_2 \leq v_2^{max}$ and $v_1 + v_2 = v \leq \hat{v}$, where $\hat{v}$ is characterized by all combinations of (v_1, v_2) that satisfy $u_1^0 + u_2^0 + \delta\overline{U}_2^* + \delta\overline{U}_1^* = \tilde{U}_1(v_1) + \tilde{U}_2(v_2)$.

In any period, the likelihood of remaining together therefore is $\Pr(\tilde{v}_1 \leq v_1^{max} \cap \tilde{v}_2 \leq v_2^{max} \cap v \leq \hat{v})$. In the Appendix, we give an explicit formulation of this probability and also prove

Proposition 8: *The divorce probability is higher than when this decision is made efficiently*

The proof of Proposition 8 can be found in the Appendix.

Proposition 8 is very intuitive. Because a transfer necessary to maintain a relationship might not be enforceable, the couple can also break up in states where this is not efficient. What we also show in the proof to proposition 8 is that the divorce probability is strictly lower (unless $k_i = \phi = (1 - \theta) = 0$) than in the case of efficient divorce, i.e., that $v_1^{max} \geq \hat{v}$ and $v_2^{max} \geq \hat{v}$ cannot both be satisfied.

In the following proposition, we state the impact of divorce laws and fertility on marriage stability.

Proposition 9: *The probality of a divorce decreases with higher divorce costs and a lower value of θ. The impact of alimony payments and more children on marriage stability is ambiguous.*

The proof of Proposition 9 can be found in the Appendix.

As before, divorce becomes more likely for lower divorce costs and a higher θ. They increase the efficient threshold $\hat{v}$ but also make transfers to maintain the relationship easier to enforce (recall that we focus on the case with sufficiently high Δv_i). Higher alimony payments can have a positive or negative impact on marriage stability. On the one hand, they make it more difficult to enforce a positive transfer, as the secondary earner already gets alimony in the period of divorce, while the primary earner only takes the future into account when considering whether to make the transfer. The opposite is true for negative payments. Which effect dominates depends on the exact specifications of the distribution functions of players' outside utilities. More children, on the one hand, make a marriage more stable by reducing utility after a separation if $\theta < 1$. Still, children also affect stability via alimony payments. Thus, if those make a divorce more likely (recall that their impact on stability is ambiguous), the total effect of a higher fertility level on stability might be negative. Yet, the net impact of alimony payments on stability should not be too high as it consists of two countervailing effects which might cancel out depending on the $F_i(.)$ distributions. Thus, it seems more convincing that children generally have a positive impact on relationship stability, by increasing the gap between the primary earner's utility within and outside the relationship.

Some characterstics of equilibrium fertility are given in

Proposition 10: *Assume the respective (IC-DE) constraint does not bind. If the impact of children on marriage stability is positive, equilibrium fertility might be higher than under full commitment. Otherwise it is lower. The impact of divorce laws on equilibrium fertility is ambiguous.*

The proof of Proposition 10 can be found in the Appendix.

Two issues are different compared to before, when the divorce decision was always made efficiently. As the marriage is less stable, the couple's propensity to have children is lower, because the likelihood of the utility loss induced by $\theta < 1$ is higher. Yet, if more children increase marriage stability, a countervailing effect exists, and each of them can dominate. In addition to providing utility, children might thus be "used" as a commitment device that makes a separation less likely.

Concerning comparative statics, higher divorce costs and a lower value of θ still might increase fertility by increasing relationship stability. Yet, if children also make a separation less likely, some substitution between these two instruments takes place. As the commitment role of children is less necessary, due to a higher k_i and lower θ, a countervailing effect decreasing fertility exists, and it is not clear which one dominates.

When the constraint binds, the situation changes, and the impact of a change in costs and θ again becomes unambiguous.

Proposition 11: *Assume $n_1^{**} > n_2^{**}$ and that the respective (IC-DE) constraint binds. Then, higher divorce costs and a lower level of θ increase equilibrium fertility. The impact of alimony payments is ambiguous.*

The proof of Proposition 11 can be found in the Appendix.

As fertility is too low anyway, potential substitution effects between fertility and higher costs and a lower access play no role. Thus, these forms of increased regulation unambiguously increase fertility as they increase maximum enforceable transfers by reducing off equilibrium utilities. Higher alimony payments have a positive impact on fertility if their effect on marriage stability is not too large. Then, a higher ϕ increases the secondary earner's compensation for her human capital loss in divorce states, induced by the wage difference $\overline{w}(n^{**}) - \overline{w}(n^{*})$.

8 Marriage Versus Cohabitation

Until now, the major part of our analysis, especially when considering the impact of divorce laws, assumed that the couple marries. In this section, we explore conditions for the optimality of marriage as opposed to cohabitation. If there are no divorce costs $k_i = 0$, then the model implies that the couple will choose marriage if, following divorce, the regulation of the wealth division/alimony, summarized by ϕ, and of the primary earner's child access, θ, increases their utilities ex ante, relative to what they expect them to be if they cohabit. To fix ideas, we assume firstly that it is sufficient that the sum of utilities is higher under marriage, i.e., we do not have to focus on individual utility levels, and secondly that child access under divorce and cohabitation will be the same. This latter is a strong assumption, since in many countries the rights of access of a previously cohabiting partner to his children post-separation may be legally undefined, thus introducing an important degree of uncertainty into the cohabitation relationship. This would be an important subject for further work, in the present paper it implies that the analysis is biased in favour of the cohabitation decision.

If the separation decision is always made efficiently in equilibrium, we can state a first rather strong result, namely that a marriage will not take place if the (IC-DE) constraint does not bind under cohabitation.

Proposition 12: *Assume the separation decision is always made efficiently in equilibrium and that the relevant (IC-DE) does not bind for* $m = 0$*. Then, the couple will not marry if* $k_i > 0$*,* $i = 1, 2$*. The spouses are indifferent if* $k_i = 0$ *and* $\phi \geq 0$*.*

Proof: Follows from Lemma 2, which states that U is monotonically decreasing in k and independent of ϕ if (IC-DE) does not bind.

Since fertility is at its efficient level, divorce costs are not needed as a commitment device. Furthermore, they decrease utility in case of a divorce and unnecessarily increase relationship stability. This can change if the separation decision is not always made efficiently and if the associated efficiency loss is sufficiently large. We do not further explore the case of inefficient separation in this section, but simply note that in that case the increased relationship stability induced by a marriage can be beneficial.

Thus, marriage can only be a useful institution in our setting if under cohabitation, fertility is too low because of commitment problems. Of course, the present analysis only captures a limited part of potential benefits of a marriage. For some couples, marriage might have a value per se,[38] and young adults may still face more or less pressure, arising out of social norms and attitudes, to get married rather than cohabit in some societies. Furthermore, the tax system

[38] Which could be captured in our model by assuming different distributions of outside utilities for spouses that are cohabiting and those that are married.

can favour marriage, especially if joint instead of individual taxation is applied. Finally, relationship stability can have a value per se. Compensation for ongoing household production might have to be self enforcing, and higher stability increases the scope for cooperation. Also the welfare of children - which is left aside in the current analysis - might be negatively affected by a separation.[39] This result points to the importance of post-separation child access arrangements if these were to differ in favour of marriage.

In our setting, the institutional framework a marriage provides can only be beneficial if the (IC-DE) constraint binds for a cohabiting couple. To simplify the analysis, we now focus on the case where Δv_i, i.e., the increase in outside utilities after a partner reneges is sufficiently high for a divorce to occur immediately after someone deviates.[40] Then, if divorce costs are negligible and a wealth division rule $\phi > 0$ is in place, a marriage will always be optimal if the primary earner's desired fertility level is higher than that of the second earner, $n_1^{**} > n_2^{**}$.[41]

Proposition 13: *Assume the separation decision is always made efficiently in equilibrium, that $n_1^{**} > n_2^{**}$, and that (IC-DE) binds under cohabitation. Then, the couple will marry if $k_i = 0$, $i = 1, 2$ and $\phi > 0$. If $k_i > 0$, the couple might or might not get married*

Proof: Follows from Lemma 3, which establishes $dU(n^*)/d\phi > 0$ for $n_1^{**} > n_2^{**}$ and a binding (IC-DE) constraint. Furthermore, Lemma 3 establishes that $dU(n^*)/dk_i = (\partial U(n^*)/dn)(\partial n^*/\partial k_i) - \delta(1 - F(\hat{v})) \lesseqgtr 0$.

A generous wealth division rule combined with laws making a divorce very easy will thus induce the couple to marry, since ϕ increases the enforceability of transfers without having a negative impact on efficiency. However, we have to recognise that we only consider the impact of divorce laws on the sum of payoffs. If an ex-ante redistribution is not feasible, things might be different. Despite an inefficiently low fertility level, it can then be in the interest of the primary earner to abstain from a marriage.

To our knowledge, the impact of wealth division rules on marriage rates has received only limited attention in the empirical literature. One exception is Rasul (2003), who finds a negative effect on marriage rates of a change to an equal division of property after a divorce. However, his results have to be treated with care when comparing them to our analysis. Table 14 of his paper shows that the explained negative impact is strongly and significantly negative only for spouses contemplating a second marriage. For those not previously married,

[39] However, if spouses care about their children's welfare, they will take the negative impact of a separation into account and should not need the commitment induced by divorce costs.

[40] If this is not the case, potential benefits of divorce costs and thus a marriage are even lower, since they increase marriage stability not only in but also off equilibrium.

[41] To be fully precise, we would have to take into account the possibility that for example under cohabitation, $n_1^{**} > n_2^{**}$, while $n_1^{**} < n_2^{**}$ after a marriage. This could be induced by substantial alimony payments, which increase the second earner's benefits from children in case of a divorce. However, we restrict our attention to the case where $n_1^{**} > n_2^{**}$, whether the couple is cohabiting or married.

the coefficient is not significant at the 10% level, and positive for men and negative for women. Since our prediction of a positive impact of an equitable wealth division after divorce is only due to a subsequent increase of fertility rates, his results might be driven by spouses who do not consider having any more children. Thus, wealth division rules should be analyzed empirically in more detail, especially in connection to fertility levels.

The impact of divorce costs on marriage rates has received more attention in the empirical literature, mainly due to the replacement of consent with unilateral divorce laws in many US states some decades ago, which is taken as implying a reduction in divorce costs,[42] However, the empirical results are ambiguous. Whereas Rasul (2003) and Matouschek and Rasul (2008) observe a decline in marriage rates, Alesina and Giuliano (2007) do not find this effect, but to the contrary find that there is an increase. This supports our claim that the greater degree of commitment induced by higher divorce costs is not automatically preferred by couples, since divorce utilities are reduced as well as the option to utilize relatively high realizations of outside utilities. Only if the utility loss induced by the binding (IC-DE) constraint is sufficiently high can the existence of divorce costs make marriage optimal. If divorce costs are relatively high, their reduction might make more couples willing to use them as a commitment device to increase fertility. Alesina and Guiliano's (2007) results are perfectly in line with this interpretation. In wedlock fertility basically remains unaffected by the adoption of unilateral divorce laws, while out of wedlock fertility decreases significantly and fertility rates for newly married couples go up.

In the remainder of the section, we consider conditions that actually make the (IC-DE) constraint bind and thus increase a couple's propensity to get married when substantial divorce costs are present. For $n_1^{**} > n_2^{**}$ and with Δv_1 sufficiently large, the (IC-DE) constraint for a cohabiting couple is

$$w_{20}\left(g_0(n^{**}) - g_0(n^*)\right) + \varphi_2(n^*) - \varphi_2(n^{**})$$

$$+\frac{\delta}{1-\delta}\left(\overline{w}_2(n^*) - \overline{w}_2(n^{**}) \quad + \varphi_2(n^*) - \varphi_2(n^{**})\right) + \delta \int\limits_{v_1^0+v_2^0}^{\hat{v}} f(\tilde{v})(\hat{v}-\tilde{v})d\tilde{v} \geq 0 \tag{45}$$

where $\hat{v}$ is defined by $\varphi_1(n)(1-\theta) + \delta \int\limits_{v_1^0+v_2^0}^{\hat{v}} f(\tilde{v})(\hat{v}-\tilde{v})d\tilde{v} - \hat{v} = 0$.

Note that the first line of the expression above is negative because $n_1^{**} > n_2^{**}$. Furthermore, since $n^* \geq n_2^{**}$ it is decreasing in n^*.

Then, the (IC-DE) constraint is more likely to bind if more time is needed to raise an additional child ($g_0(n)$ is steeper) and if more children imply a higher human capital loss for the second earner ($\overline{w}_2(n^*)$ is steeper). This will also make it more likely that the desired transfer is larger, implied by a larger difference

[42] This is supported by the fact that divorce rates immediately went up after the introduction of unilateral divorce, see Friedberg (1998) or Matouschek and Rasul (2008).

between n_1^{**} and n_2^{**}. Furthermore, a lower relationship stability (captured by a smaller value of $\int_{v_1^0+v_2^0}^{\hat{v}} f(\tilde{v})(\hat{v}-\tilde{v})d\tilde{v}$) decreases the value of the left hand side of the condition for given values of n^* and n^{**}.

Although the first components are exogenously given within our model, they deserve some attention here. For example, $g_0(n)$ could be steeper if fewer child-care facilities were available at reasonable costs. Thus, couples might be more inclined to marry. This point is also important for the second aspect, a higher human capital loss associated with children. There, recall our assumption that the second earner alone is responsible for raising the couple's children. As argued above, it seems unlikely that the fact that women still assume major parts of the responsibilities associated with having children is purely driven by an optimal (in an economic sense) allocation of tasks, but could also be influenced by factors outside our model, for example cultural norms and values.

If men were willing to substantially participate in child-rearing and if jobs were sufficiently flexible, i.e., if the couple were able to commit to any allocation of $g(n)$, it would be possible to obtain efficient fertility without the need of additional transfers. Thus, if the couple is closer to an optimal time allocation, the (IC-DE) is less likely to bind. Therefore, couples with more traditional views should be more likely to get married, a claim that is supported by empirical evidence[43].

9 Conclusion

Making precise the conditions under which cooperation within a relationship can be enforced, this paper has shown how the institutional setting following a separation can make it easier or more difficult to allocate resources within a household and thus compensate a partner for the human capital loss associated with having children. However, our approach is only a first step in the analysis of the (often unintended) consequences of legislative changes. Future research could evolve along three lines.

Further empirical research is needed to test our predictions. Whereas the impact of divorce laws on divorce rates, the propensity to marry and fertility has been extensively tested – especially using the natural experiment of a switch from consensual or no-fault to unilateral divorce – this still remains to be done for wealth division rules and a reduced access of one partner to his children after a separation.

Furthermore, our model is general enough to incorporate further laws that are important for a marriage. The impact of different forms of income taxation – for example joint versus individual taxation – could be analyzed. The model is also precise enough to look at the question of consent versus unilateral divorce in more depth. Of course, a unilateral divorce is very likely to be associated with

[43] Kaufmann (2004) for example finds that men with egalitarian counterparts are more likely to cohabit than those with more traditional views

lower divorce costs, as shown by the large empirical literature that found an immediate increase in divorce rates following its introduction. However, taking this matter literally and noting that under a consent divorce regime, one partner alone cannot easily induce a divorce, might allow us to explain some empirical results that canot be explained by a change in commitment power alone.

For example, Alesina and Giuliano (2007) find that after the introduction of unilateral divorce laws, fertility rates for newly married couples went up, while out-of-wedlock fertility decreased. They claim that lower divorce costs induce couples to enter marriage more readily, without however further elaborating on this explanation. Our model also provides an explanation. A consent divorce regime taken literally makes it much more difficult to enforce payments between spouses. The reason for this is that transfers are no longer needed to keep the partner within a marriage, and allocations on and off the equilibrium path do not differ.[44]

The model setting itself can be extended, for example by taking children's welfare into account and by assuming risk-averse players. In the latter case, it will be necessary to analyze wealth division laws separately, since saving will become an important endogenous variable. In addition, these laws might not only compensate the secondary earner for her human capital loss, but also help to equalize income across states. Furthermore, relationship stability - a reduction of uncertainty - might have a value *per se*. Then, laws reducing the attractiveness of a divorce could be welfare enhancing in comparison to the present setting, and the role of children as a device to increase stability (via the access parameter θ) would have to be reassessed.

In conclusion, we hope to have shown that an approach to the issues concerning marriage, fertility and divorce based on the theory of relational contracts can make a new and useful contribution to our understanding of the complex interrelationships among these institutional features of a modern society.

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