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Abstract

Increasing-returns-to-scale imperfect competition trade models predict a more than proportionate relationship between the larger country's share in world endowments and its share in producing firms: the so called home market effect (HME). While this result plays a key role in empirical testing, its theoretical foundation typically posits a linear, friction-free and perfectly competitive outside sector. Replacing this assumption with firm heterogeneity and Melitz (2003) type selection-into-exporting, we demonstrate the existence of a weak and a strong HME. The HMEs are generally non-linear; they are magnified by lower trade costs or more pronounced productivity dispersion. The weak version of the HME continues to hold for general sampling distributions and if the conventional sorting condition fails. In terms of demand shares, a HME holds if demand shocks are due to endowment shocks but reverses in the case of productivity shocks. Finally and in contrast to the model with an outside sector, trade liberalization leads to convergence of real per capita income.

JEL-Code: F120, R120.

Keywords: home market effect, monopolistic competition, heterogeneous firms, economic geography.

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1 Motivation

Policy makers and the public are concerned about the *relative size* of their economies. This is most clearly visible in the discussion about the increase of the relative weight of emerging countries.[1] New Trade Theory (Krugman, 1980; Helpman and Krugman, 1985) provides a theoretical rationale. With trade costs, product differentiation, increasing returns to scale at the firm-level, and imperfect competition, the relative size of countries (or regions) affects industrial structure and welfare. In those models, firms prefer to locate in the larger country, *ceteris paribus*, since this allows the majority of sales to be carried out without incurring transportation costs. The larger country supports, in equilibrium, the production of a more-than-proportionate number of differentiated varieties.

The prediction that the larger country produces an overproportional share of differentiated goods is known as the *Home Market Effect* (HME). Helpman and Krugman (1985) and much of the following literature derive the HME using a two-sector two-country single-factor (labor) model. The framework has a two-tier CES structure. There is a differentiated goods sector featuring increasing returns to scale, monopolistic competition, and iceberg trade costs. And there is a linear outside sector producing a freely traded homogeneous good under perfect competition, which is assumed to be active (no diversification). The HME appears robust to allowing for a non-CES demand structure (Ottaviano and van Ypersele, 2005), for oligopolistic rather than monopolistic market structure (Feenstra, Markusen and Rose, 2001), for many differentiated goods sectors (Hanson and Xiang, 2004), for the existence of multinational firms (Larch, 2007) or for more than two trading countries (Behrens, Lamorgese, Ottaviano and Tabuchi, 2009). Also, as shown by Helpman and Krugman (1985), one can have more than one factor of production if the production technology is homothetic.

The assumption of the linear outside sector is more critical. It contributes to the analytical tractability of the model. Factor prices are pinned down by technology in the outside sector. Thus, as long as the equilibrium is *diversified*, factor prices (i.e., the wage rates) are *insensitive* to changes in model parameters. The assumption may, however, not be innocuous. Davis (1998) introduces trade costs into the outside sector. He shows that, when transport costs are identical for both types of goods, the HME vanishes. The HME re-emerges only if relative costs of trading differentiated goods are unusually high. The usual modeling of the outside sector implies a perfectly elastic supply of labor to the increasing returns sector so that the higher attractiveness of the larger economy as a location of production of differentiated goods is not offset by an increase in the wage rate. If the wage does adjust, the HME can be dampened or can even disappear (Head and Mayer, 2004). Crozet and Trionfetti

[1] In a special report, The Economist (Sept 24, 2011) argues "*The shift in economic power from West to East is accelerating ... The rich world will lose some of its privileges*". It provides examples of policy makers' obsession with "*grandeur and decline*" and China "*Becoming number one*".

(2008) have qualified this prediction.[2] They introduce Armington differentiation and trade costs into the outside sector, nesting Helpman and Krugman (1985) and Davis (1998). Their numerical results suggests that the HME survives but becomes non-linear.

Due to its prevalence in models of *increasing returns to scale* models, it has been used as a discriminating criterion to test for the validity of New Trade Theory in empirical work (Davis and Weinstein, 1999, 2003).[3] Results are mixed. However, it is well possible that rejection of the HME is actually due to the empirical failure of the outside sector assumption rather than the more relevant increasing returns to scale feature.

The linear outside sector has also been used in models with heterogeneous firms.[4] It may be a perfectly harmless assumption in many applications. However, when one is interested in aggregate outcomes, such as in welfare, trade policy, or unemployment, pinning down the wage to a technological constant may have crucial bearings on the results. Using a two-country asymmetric Melitz model, Felbermayr, Jung and Larch (2011) argue that outcomes of non-cooperative tariff games depend on using a linear outside sector. Using a simulated asymmetric three-country Melitz model with search frictions on the labor market, Felbermayr, Larch and Lechthaler (2009) argue that the direction cross-country spillovers from labor market reforms also depend on that assumption. Very recently, Demidova and Rodriguez-Clare (2011) use a small economy Melitz (2003) model to show that eliminating the assumption of an outside sector reverses the result in Melitz and Ottaviano (2008) or Demidova (2008), where a country that unilaterally lowers trade costs experiences a decline in welfare.[5]

Besides its obvious lack of realism, the outside sector assumption may not be innocuous in theoretical work; it is therefore relevant for econometric testing. This paper shows that the outside sector assumption can be replaced by a Melitz (2003) type mechanism by which *heterogeneous* firms *select* into exporting according to their productivity. The country with the larger share in world endowments supports a more than proportionate share of firms: a larger fraction of firms remain purely domestic and therefore relatively smaller. This allows a larger number of them to exist in equilibrium, which, in turn, increases welfare per capita in the larger country due to the availability of a larger range of varieties. The HME arises despite the upward adjustment in wages that the *crowding* of firms in the larger country entails.

[2]In a multi-sector extension of the standard model, Krugman and Venables (1999) show that the HME continues to exist as long as there are at least some homogeneous goods sectors with zero transportation costs and some differentiated goods sectors with zero fixed costs.

[3]A number of prominent empirical papers are Feenstra et al., (2001), Head and Ries (2001), Davis and Weinstein (2003), Hanson and Xiang (2004). In their survey, Head and Mayer (2004) conclude that “The evidence on HMEs accumulated in those papers is highly mixed”. More recent research finds stronger results in favor of the HME; see Crozet and Trionfetti (2008) or Brülhart and Trionfetti (2009).

[4]Other prominent papers that have used a linear, freely traded, perfectly competitive outside sector in a Melitz-type environment are Helpman, Melitz and Yeaple (2004), Grossman, Helpman and Szeidl (2006), Chor (2009), Baldwin and Okubo (2009), Baldwin and Forslid (2010), and Ossa (2011).

[5]Demidova and Rodriguez-Clare (2009) use this framework to analyze trade policy.

Higher wages increase the larger country's *market potential.* This is particularly relevant for domestic firms whose competitive cost disadvantage relative to foreign firms is attenuated by trade costs. Importantly, while the mass of firms *attempting entry* is strictly proportional to the endowment size, the likelihood of a given firm to *successfully* cover its fixed costs is greater in the larger market.

The HME arises due to the interaction between fixed market access costs and firm-level heterogeneity in productivity. Fixed export costs alone would not suffice (Venables, 1994; Medin, 2003). In contrast, the mechanism relies on the fact that some firms do not export the share of which is endogenous and positively related to relative country size. When sorting becomes more pronounced due to larger degree of productivity dispersion, the HME becomes stronger. As in Helpman and Krugman (1985), the HME is magnified by lower trade costs. As in Crozet and Trionfetti (2008), the HME is non-linear. The result does not require to make assumptions on the sampling distribution of productivity; it continues to hold even under reverse sorting of firms (when exporting firms are on average less productive than domestic ones). Whether trade frictions turn up as variable or fixed costs does not matter. These results apply to what we term a *weak* (or static) HME, i.e., the overproprtionate relationship between the endowment share and the share of firms. We also characterize a *strong* (or dynamic) HME, which relates to an overproportionate increase in the share of firms triggered by an increase in the endowment share. While the weak HME holds over all endowment distributions, the strong HME holds only when the distribution is not too skewed.

Empirically, the literature often states the HME in demand shares: the country with the larger share in demand hosts an overproportionate share of firms. With fixed factor prices, this prediction follows from the relationship in endowment shares. When factor prices adjust, this is no longer necessary. However, the weak HME continues to hold when the underlying variation in the demand shares is due to endowment shocks; if it is due to productivity shocks, the weak HME actually reverses. For empirical work, the single-sector view has the disadvantage, that the share of a country in the value of production of differentiated goods and its share in demand are identical due to balanced trade. However, the extension to the empirically relevant multisector situation (as in Behrens et al. (2009) for the homogeneous firms plus outside sector model) is not difficult. Then, a (weak) HME exists in the relationship between the production and the demand shares on the industry level. Nonetheless, our analysis has important implications for empirical work. HME regressions should account for the non-linearity of the HME, its dependence on productivity dispersion and the average level of productivity.

Our paper is related to a growing literature on the asymmetric Melitz model. Most, but not all papers use the outside sector simplification. Arkolakis, Costinot and Rodriguez-Clare (2011) discuss a general class of models encompassing the Melitz framework for asymmetric countries. Their comparative static exercise relates to purely foreign variables; moreover, their ambition is not to fully characterize endogenous variables in terms of exogenous ones, but demonstrate the validity of a simple welfare function that is isomorphic across models with and without firm-level heterogeneity. Their argument is that heterogeneity and selection matter less for aggregate welfare than what has been hitherto believed.

In contrast, for our result, heterogeneity is absolutely crucial. We can show that in the limit, where the model converges to the homogeneous goods Krugman case, the HME disappears: When a country commands a larger share of world population, its attractiveness as a production location increases. The increased demand for labor is accommodated along two margins: first, the relative wage of the country goes up; second, the average size of firms goes down. In a model with a linear outside sector, the first channel would not be present, and the HME is of maximum size. In a model with homogeneous firms, firm size cannot adjust, and the wage needs to rise until the number of firms is exactly proportional to the labor force. The more dispersed the productivity distribution, the stronger the link between average firm size and the number of firms, and, accordingly, the more pronounced is the HME.

Finally, it is important to notice that the role of the outside sector is crucial for predictions about trade-induced welfare convergence across countries. In the Melitz (2003) model with an outside sector, trade liberalization does not entail any convergence: factor prices are insensitive and regional price levels adjust in exactly the same proportions. In the absence of an outside sector, trade liberalization results in convergence of real per capita income like in familiar models of comparative advantage. This is despite the magnification of the HME. It is a direct implication of the fact that the model with the outside sector tends to overstate the HME as wages are parametrically fixed.

The remainder of the paper is structured as follows. Chapter 2 describes the model. Chapter 3 proofs our main results: the existence of the HME, and the magnification by lower trade costs or higher productivity dispersion. Chapter 4 contains a discussion of extensions. Chapter 5 concludes.

2 The Model

2.1 Basic environment

The model is the basic extension of Melitz (2003) to the case of two large asymmetric countries, indexed by $i \in \{H, F\}$. Each country is populated with L_i identical households. Labor is the only factor of production.[6] Importantly, each household *inelastically* supplies one unit of labor.[7] We will denote wages by w_i. The representative

[6] We devise the model as a single-factor framework. Multiple (non-traded) factors can be easily accommodated if one is willing to assume that variable and fixed inputs are in terms of a composite input, which combines different factor services in a constant returns to scale fashion. That composite input takes the role of labor in our analysis; all results stated in this paper would continue to hold.

[7] In the standard Krugman (1980) framework, Fujita et al. (1999) consider a two-sector model with flexible elasticity of labor supply to the differentiated good sector. With perfectly elastic labor supply, the HME always appears, but if we approach the perfectly inelastic labor supply case, the HME will be

consumer has a standard Dixit-Stiglitz utility function defined over a continuum of differentiated varieties

$$U_i = \left[\int_{z\in\Omega_i} q\left[z\right]^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where the measure of the set Ω_i is the mass of available varieties, $q\left[z\right]$ is the quantity of variety z consumed, and $\sigma > 1$ is the elasticity of substitution.

Firms compete monopolistically in a *single* sector. After paying fixed setup costs $w_i f^e$, they obtain information about their productivity level φ which is sampled from a Pareto distribution whose c.d.f. is given by $G_i\left[\varphi\right] = 1-(b_i/\varphi)^{\beta}$. The shape parameter β is inversely related to productivity dispersion. In most of this paper we assume that the location parameter b_i is constant across countries.[8] Output is linear in φ. A firm in country i pays fixed market access costs $w_i f_{ij}$ to serve consumers in country j. Selection implies that a firm does not necessarily serve both markets. Whenever advantageous, we use $f_{ij} = f_{ji} = f^x$ and $f_{ii} = f_{jj} = f^d$. As usual, exporting involves symmetric iceberg trade costs $\tau_{ij} = \tau_{ji} = \tau \geq 1$, where $\tau_{ii} = 1$. Then, $\tau_{ij} w_i/\varphi$ is the marginal cost of producing one unit of output in i and selling to j. In the following description of equilibrium conditions, we will be very brief since the model is very standard.

2.2 Equilibrium conditions

The first set of equilibrium conditions are *zero cutoff profit conditions*. They pin down the minimum productivity level φ^*_{ij} required for a firm in country i to make at least zero profits by selling in country j. Since we have two countries, there are *four* of those conditions:

$$\frac{R_j}{\sigma}\left(\frac{\rho P_j}{\tau_{ij} w_i}\varphi^*_{ij}\right)^{\sigma-1} = w_i f_{ij}, \quad i \in \{H, F\}, j \in \{H, F\}, \tag{2}$$

where $\rho = (\sigma-1)/\sigma \in [0, 1]$ is the inverse of the mark-up.[9] Since we assume balanced trade, aggregate expenditure R_j is equal to national income $w_j L_j$. The price index is given by

$$P_i^{1-\sigma} = \theta \sum_{j\in\{H,F\}} m_{ji} M_j \left(\frac{\rho\varphi^*_{ji}}{\tau_{ji} w_j}\right)^{\sigma-1}, i \in \{H, F\}, \tag{3}$$

where M_i denotes the (endogenous) mass of firms, $m_{ij} = \left(1 - G\left[\varphi^*_{ij}\right]\right) / \left(1 - G\left[\varphi^e_i\right]\right)$ is the fraction of firms located in country i which serve market j. Note that φ^e_i de-

reversed for some level of trade costs; see Head and Mayer, 2004, p. 29f.

[8] In extensions we show that our core results hold under a general sampling distribution of productivity. Note that each variety z is produced by a single firm with productivity level φ. We henceforth index varieties by φ.

[9] The equilibrium conditions are derived in detail in (A.1) in the Appendix.

notes the productivity of the marginal firm (the least efficient operative firm).[10] $\theta \equiv \beta/(\beta-(\sigma-1))$ is a strictly positive constant.[11] The left hand side of (2) denotes profits of a firm with labor productivity φ^*_{ij}. They are proportional to aggregate profits R_j/σ. Firm-level profits increase in the foreign price level as the firm's competitive position there is improved; they decrease in w_i for the opposite reason. The right hand side denotes the value of fixed market entry costs.

The second set of equilibrium conditions are *free entry conditions*. In each country, firms invest fixes setup costs until expected profits from entering $(\theta-1)w_i \sum_j m_{ij} f_{ij}$ are equal to entry costs discounted by the probability of successful entry $p_i^{in} = 1 - G[\varphi_i^e]$ for $i \in \{H, F\}, j \in \{H, F\}$. The *two* free entry conditions therefore are

$$(\theta-1)p_i^{in} \sum_{j\in\{H,F\}} m_{ij} f_{ij} = f^e, i \in \{H, F\}. \tag{4}$$

Note that wages have dropped out from this condition.

Finally, there are *two labor market clearing conditions*. With the above equilibrium conditions and using the Pareto distribution, they simplify to

$$\frac{M_i}{p_i^{in}} \frac{\beta f^e}{\rho} = L^i, i \in \{H, F\}. \tag{5}$$

If p_i^{in} were exogenous (or, as in Krugman (1980) equal to unity), labor supply and the mass of operative firms M_i would be proportional and there could not be a HME. It is the fact that p_i^{in} is endogenous in the model that enables the existence of a HME.

Summarizing, we have four zero cutoff profit conditions (2), two free entry conditions (4) and two labor market clearing conditions (6) to pin down eight unknown endogenous variables of the model $\{\varphi^*_{HH}, \varphi^*_{FF}, \varphi^*_{HF}, \varphi^*_{FH}; M_H, M_F; w_H, w_F\}$. Knowledge of these equilibrium objects allows to determine m_{ij} and p_i^{in}. In the following, we use w_F as the numeraire and denote $w_H/w_F \equiv \omega$ as the relative wage.

2.3 Trade balance and equilibrium wage

As the Krugman (1980) model, one can reduce the equilibrium conditions to one equation in a single unknown, namely ω. Balanced trade is implicit in conditions (2), (4) and (6); it is implied by the representative agents in both countries each satisfying their respective budget constraints. Nonetheless, it is useful to make the *balanced trade* condition explicit. It can be written as

$$M_H \bar{r}_{HF} = M_F \bar{r}_{FH}, \tag{6}$$

where $M_j \bar{r}_{ji}$ denotes aggregate sales of firms located in country j in market i. One can show that $\bar{r}_{ij} = \sigma\theta w_i m_{ij} f_{ij}$. Using the definition of m_{ij} expression along with equation

[10] If conventional sorting holds, we have $\varphi_i^e = \varphi^*_{ii}$; if not, we have $\varphi_i^e = \varphi^*_{ij}$. See the discussion below for further details.

[11] This is to ensure that the variance of the size distribution is finite.

(5), the trade balance condition (6) can be rewritten so that the relative wage ω appears as a function of Home's share in the world labor endowment $\lambda \equiv L_H / (L_H + L_F)$ and the ratio of the two countries' export productivity cutoffs:

$$\omega = \frac{1-\lambda}{\lambda} \left(\frac{\varphi^*_{FH}}{\varphi^*_{HF}} \right)^{-\beta}. \tag{7}$$

So, the trade balance condition ensures that, for given λ, a shift in ω will induce opposite movements in two two countries' foreign market access threshold productivities φ^*_{FH} and φ^*_{HF}.

Using the zero cutoff profit conditions (2) to substitute out φ^*_{FH} and φ^*_{HF}, the trade balance condition becomes

$$0 = \left(\frac{1}{(1-\lambda)\,\omega^{\beta/\rho} + \lambda \omega \eta} - \frac{1}{\lambda \omega^{-(\beta-\rho)/\rho} + (1-\lambda)\,\eta} \right), \tag{8}$$

where $\eta \equiv \tau^{-\beta} \left(f^x / f^d \right)^{1-\beta/(\sigma-1)} \in (0,1)$ is a measure of the freeness of trade.

The first term in the brackets of (8) is strictly downward-sloping in ω from $+\infty$; the second term is strictly upward-sloping from 0. Hence, by (8), a unique equilibrium relative wage ω exists. If $\eta \to 1$ (free trade) or $\lambda = 1/2$ (symmetric distribution of labor endowments), $\omega = 1$ solves this equation: wages are equalized across countries. If $\eta < 1$ and $\lambda > 1/2$, one can easily show that $\omega > 1$: the large country has the higher wage. These findings are well-known from the Krugman (1980) model which is nested by equation (8) if $\beta \to \sigma - 1$. In that case, the productivity distribution exhibits maximum dispersion. Only a few very productive firms exist, and all of them export. So, the selection channel is effectively shut off and the model collapses to Krugman's model of homogeneous firms.

We summarize this finding by a first lemma.

Lemma 1 *The larger country pays the higher wage.*

Proof. In the Appendix. ■

The intuition for this result is simple: At given factor costs, firms find it more profitable to produce in the larger market as this minimizes payments of variable trade and market access costs. To keep labor employed in both countries, this advantage must be offset by a wage differential.

Empirical evidence suggests that only the most productive firms export. The model reproduces this stylized fact if parameters are such that $\varphi^*_{ij} > \varphi^*_{ii}$. We refer to this situation as to the case of *conventional sorting*, whereby $p^{in}_i = 1 - G\left[\varphi^*_{ii}\right]$. Unconventional sorting obtains if Home becomes very large relative to Foreign, for given fixed costs, the sorting condition can reverse: then, only the more productive foreign firms serve the small foreign market, so $p^{in}_i = 1 - G\left[\varphi^*_{ij}\right]$. In line with the evidence, the following analysis assumes that conventional sorting holds. This happens in equilibrium, if Home's share in the world labor endowment is not too big (i.e., if $\lambda < \bar{\lambda} \equiv \lambda\left[\eta, \beta, \rho; f^x/f^d\right]$).[12]

2.4 Market Crowding and Market Potential Curves

Instead of working with (8), it is insightful to characterize the equilibrium of the asymmetric Melitz model with the help of two separate equilibrium conditions in a diagram with $p_H^{in}/p_F^{in} \equiv \chi$ on the y-axis and ω on the x-axis. The two curves have opposite slopes and allow conducting comparative statics in an insightful and tractable manner. Equilibrium of the two-country asymmetric Melitz economy is given by the intersection of these curves.

Lemma 2 *If* $\lambda < \bar{\lambda} \equiv \lambda\left[\eta, \beta, \rho; f^x/f^d\right]$, *the equilibrium exhibits conventional sorting. There always exists a unique equilibrium at the intersection between a strictly downward-sloping convex* ***market crowding*** *curve (MCC)*

$$\chi \equiv \frac{p_H^{in}}{p_F^{in}} = \frac{1 - G\left[\varphi_{HH}^*\right]}{1 - G\left[\varphi_{FF}^*\right]} = \frac{\lambda}{1-\lambda}\omega^{-\frac{2\beta-\rho}{\rho}}. \tag{9}$$

and a strictly increasing convex (the latter under mild parameter restrictions) ***market potential*** *curve (MPC):*

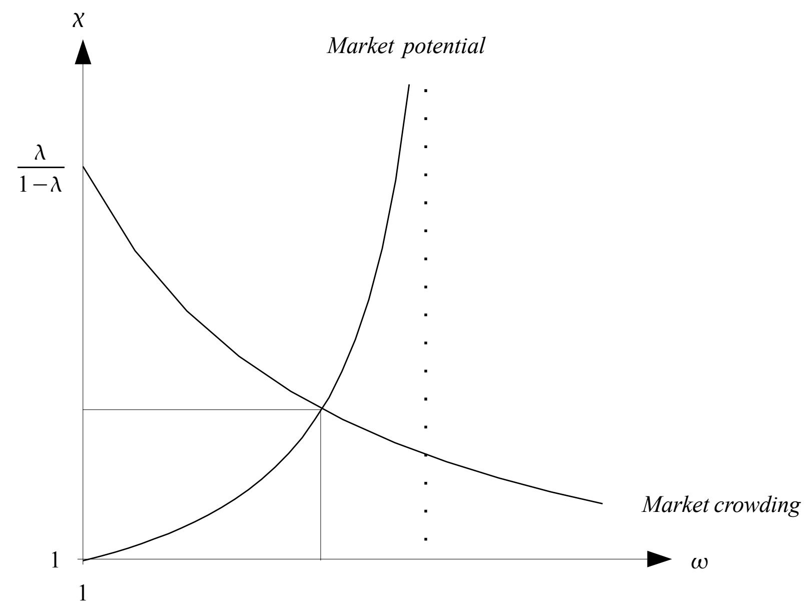
$$\chi = \frac{1 - \eta\omega^{-\frac{\beta}{\rho}}}{1 - \eta\omega^{\frac{\beta}{\rho}}}, \eta\omega^{\frac{\beta}{\rho}} \neq 1. \tag{10}$$

Proof. In the Appendix. ■

Market crowding condition. Under the assumption of conventional sorting, the market crowding curve (9) is obtained by combining all four zero cutoff profit conditions (2) and observing that balanced trade implies (7). Without loss of generality, we focus on the case where Home is large ($\lambda > 1/2$) and therefore $\omega > 1$. Hence, in the relevant space ($\omega \in [1, \infty)$), the curve is strictly downward-sloping. At $\omega = 1$, we have $\chi = \lambda/(1-\lambda)$. Moreover, χ converges to zero as $\omega \to \infty$. Figure 1 illustrates the locus. We refer to it as the "market crowding curve" (MCC): if ω increases, Home's relative labor costs go up and it becomes a less attractive location for production. In turn, the domestic entry cutoff φ_{HH}^* relative to φ_{FF}^* has to go up. So, the likelihood of successful entry falls. The MCC illustrates a dispersion force, i.e., a negative equilibrium correlation between relative costs and locational advantage. It takes an *ex post* perspective in that it summarizes firm behavior after the resolution of uncertainty about productivity.

[12] Section 4.1 contains the generalization to the case of unconventional sorting. The function $\lambda\left[\eta, \beta, \rho; f^x/f^d\right]$ is characterized in (A.3.1) in the Appendix.

Figure 1: Existence and uniqueness of equilibrium



Market potential condition. Equation (10) constitutes a second relationship between relative entry probability χ and relative wage ω. The derivation starts from the free entry conditions (4). It makes use of balanced trade (7) and employs equations (2) to eliminate productivity cutoffs. In the space $(\omega \in [1, \infty))$, the curve is strictly upward-sloping. It has an asymptote at $\omega = \eta^{-\rho/\beta} > 1$ and emerges from the point $(1, 1)$ with slope $2\beta / (\rho (1 - \eta))$. Figure 1 illustrates the locus. We refer to this schedule as to the "market potential curve" (MPC): if ω increases, Home's relative income goes up so that Home's market potential increases. This makes entry of firms more attractive, the domestic entry cutoff φ^*_{HH} relative to φ^*_{FF} has to fall. So, the likelihood of successful entry goes up. The MPC illustrates an agglomeration force, i.e., a positive equilibrium correlation between relative market potential and locational advantage. In contrast to the MCC, the MPC takes an *ex ante* perspective in that it relates to potential firms' decisions to sink setup costs and learn about their productivities. The MPC is convex if $\eta > \rho / (4\beta + \rho)$; a sufficient condition for this is $\eta > 1/5$.[13]

Figure 1 contains the same information as equation (8). In particular, it is easy to see that $\lambda > 1/2$ implies $\omega > 1$. At $\omega = 1$, the *ex ante* profitability curve implies a relative entry probability equal to unity, while the *ex post* profitability curve implies a relative entry probability equal to $\lambda/(1 - \lambda)$. Assume without loss of generality $\lambda > 1/2$. Then, $\lambda/(1 - \lambda) > 1$. Given the shape of the curves, wage equalization is an equilibrium only with $\lambda = 1/2$. When the strict inequality holds, we have $\omega > 1$.

[13]With the realistic parameterization $\rho = 0.74$ (i.e. $\sigma = 3.8$), and $\beta = 3.3$, convexity requires that $\eta > 0.053$. That, in turn, requires an ad valorem tariff equivalent of 144 percent if $f^x/f^d = 1$, and even higher if $f^x > f^d$.

3 Single-Sector Home Market Effects

3.1 Preliminaries

We are interested in the mapping between the share of firms located in the larger economy and its share in the world labor endowment. From equation (5) we know that the mass of active firms in each country is proportional to the labor force times the probability of successful entry.[14] This allows writing Home's share of firms as a function only of the relative entry probabilities χ and of Home's labor share λ

$$\phi \equiv \frac{M_H}{M_H + M_F} = \gamma\lambda \tag{11}$$

with

$$\gamma \equiv \frac{\chi}{1 + \lambda(\chi - 1)}. \tag{12}$$

Clearly, γ increases in χ and falls in λ.

Before we proceed, we need a more precise definition of the phenomenon that we are interested in:

Definition 1 *A* ***weak (static) home market effect*** *(weak HME) exists, if the share of firms located in Home is larger than its share in the world labor endowment, i.e., if*

$$\phi > \lambda.$$

A ***strong (dynamic) home market effect*** *(strong HME) exists, if an increase in Home's labor share yields a more than proportionate increase in Home's share of firms, i.e., if*

$$\phi'(\lambda) > 1.$$

The weak form of the HME is the one usually discussed in the literature. The definition of the HME used by Helpman and Krugman (1985), Hanson and Xiang (2004) or Behrens et al. (2009) coincide with it. In the standard case with a linear outside sector (with or without firm-level heterogeneity),[15] γ is equal to a constant $\bar{\gamma}$ and so $\phi = \bar{\gamma}\lambda$. The weak HME materializes if and only if $\bar{\gamma} > 1$. Linearity of ϕ in λ implies that the requirement for the strong HME is identical: $\phi'(\lambda) = \bar{\gamma} > 1$. Hence, in contrast to the single-sector model, in the presence of a linear outside sector, it is not interesting to distinguish a weak and a strong version of the HME.

[14] Note that Home's share of firms that pay entry fixed costs is proportional to relative country size: $M_H^e / (M_H^e + M_F^e) = \lambda$.

[15] See Appendix C for the model with a linear outside sector.

3.2 Home Market Effects

Using Figure 1, it is very easy to show that the model exhibits a *weak HME*. Remembering $\lambda > 1/2$, equation (12) and Definition 1 imply that

$$\chi > 1 \Leftrightarrow \gamma > 1. \tag{13}$$

So, a weak HME exists if and only if the probability of successful entry, χ is greater in the larger home economy than in Foreign, i.e., $p_H^{in}/p_F^{in} \equiv \chi > 1$. Figure 1 establishes that this is indeed the case since $\lambda/(1-\lambda) > 1$ so that all admissible realizations of χ satisfy (13) and there is indeed a weak HME. Note the crucial role of firm-level heterogeneity: if all firms were identical, and thus, in a meaningful equilibrium,[16] all of them would find it worthwhile to produce, in both countries we would have $p_i^{in} = 1$, and hence $\chi = 1$. It follows that $\gamma = 1$ and the relationship between ϕ and λ would be one-to-one: there would not be a HME.

The *strong HME* is slightly more involved. It obtains when an increase in the labor share of a country leads to a more-than-proportionate increase in its share of firms. Denote by ϵ_x the elasticity of some variable x with respect to λ. Then,

$$\epsilon_\phi = 1 + \epsilon_\gamma > 1 \Leftrightarrow \epsilon_\gamma > 0 \tag{14}$$

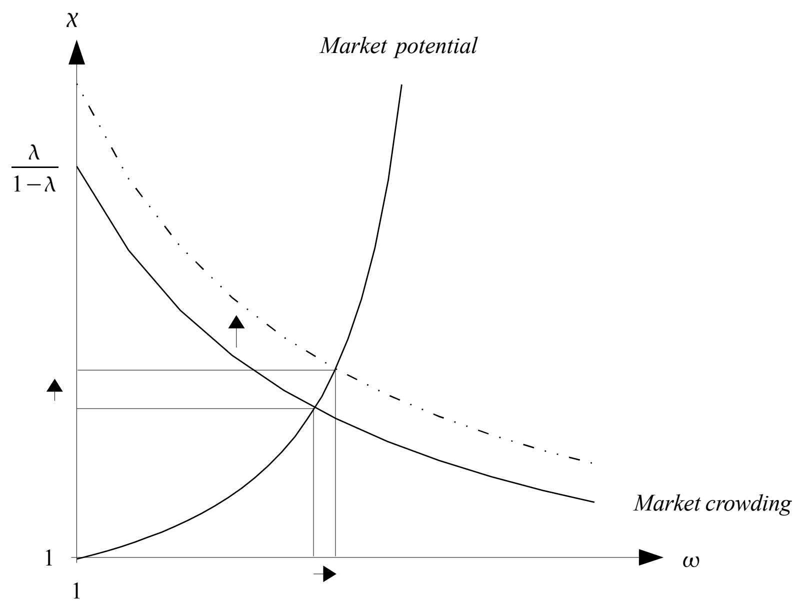
To verify the validity of the above condition, one needs to understand how γ, and hence χ, depend on λ. This can be easily seen with the help of our figure, where the effect of an increase in Home's share of labor affects only the market crowding curve. It shifts upwards if λ increases; the shift is larger, the smaller λ is initially. Figure 2 illustrates this situation. Clearly, an increase in λ leads to a higher relative wage ω and to a higher relative entry probability of Home.

So, as long as the conventional sorting condition holds, $d\chi/d\lambda > 0$ and $d^2\chi/d\lambda^2 < 0$. Using (12), it is clear that around the symmetric equilibrium $(\lambda = 1/2, \chi = 1)$, the derivative of γ with respect to λ is given by $(d\chi/d\lambda)/2$. It follows that, around the symmetric equilibrium, $\epsilon_\gamma > 0$ is positive and a strong HME exists. As λ grows away from symmetry, the positive increments to χ become smaller; moreover, a higher λ also has a direct negative effect on γ. It follows that $\epsilon_\gamma > 0$ cannot hold for all λ. Let $\bar{\lambda}$ denote the endowment share at which conventional sorting does no longer hold. It can be proved that the strong HME exists over an interval $(1/2, \lambda^*)$ with the critical value λ^* bounded by $\lambda^* < \bar{\lambda}$.

We know from equation (12), that a change in the relative entry probability χ translates into a change in the share of firms ϕ. So, a shock on λ has both a "price effect" and a "quantity effect" (Head and Mayer, 2004). The higher relative wage of Home shifts the price distribution since it affects unit labor costs. It also affects the composition of productivities and the share of firms that do not find it worthwhile to operate (besides the obvious effect of increasing the number of firms that attempt

[16] The parameter constellation could be such that no firm wants to operate $\left(p_i^{in} = 0\right)$. We exclude such shut-down equilibria.

Figure 2: Country size shock



entry.) In the one-sector Krugman (1980) model, only the price effect emerges. Resources are fixed by full-employment and balanced trade conditions, where the latter ensures that the larger country pays the higher wage. Note that the equilibrium relative entry probability χ is concave in relative size λ. The reason is that (i) the market potential curve is concave in the relative wage, (ii) it is not shifted itself by a country size shock, and (iii) the relative wage is strictly increasing in the share of consumers.

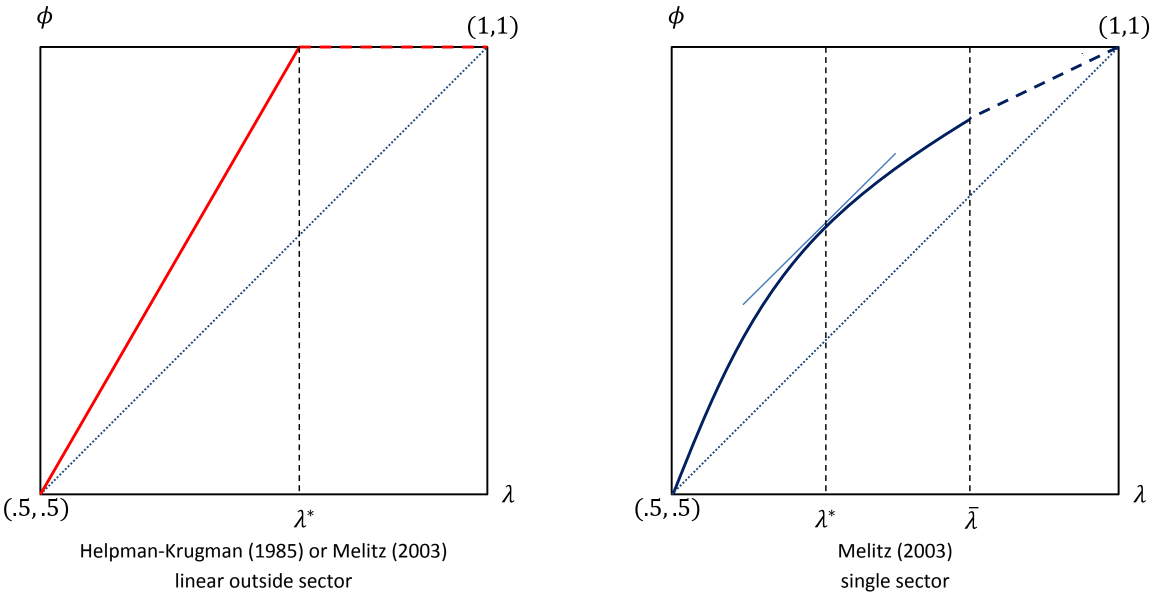
These considerations allow stating the first main result of the paper:

Proposition 1 *(**Home market effect**). Without loss of generality, assume* $\lambda > 1/2$. *Also, assume that conventional sorting holds* ($\lambda < \bar{\lambda}$). *Then,*
(a) the model exhibits a weak HME if Home's endowment share lies in the interval $(1/2, \bar{\lambda})$;
(b) and a strong HME if Home's endowment share lies in the interval $(1/2, \lambda^*)$, *where* $\lambda^* \leq \bar{\lambda}$.

Proof. In the Appendix. ■

In the asymmetric Melitz (2003) model, the larger country pays the higher wage. However, with heterogeneous firms, a deviation from wage equalization also induces selection effects which imply that the larger country hosts an overproportional share of firms. This effect is related to the "quantity effect" discussed in Head and Mayer (2004). It is still more profitable to produce in the larger market to minimize variable trade and market access costs. However, in the Melitz environment, presence in the large country is particularly valuable for firms with intermediate productivity levels. Since they do not export, the higher wage in Home puts them at a competitive disadvantage in Home but not in Foreign. The adjustment of entry margins in the Melitz

Figure 3: Home Market Effects with and without an outside sector



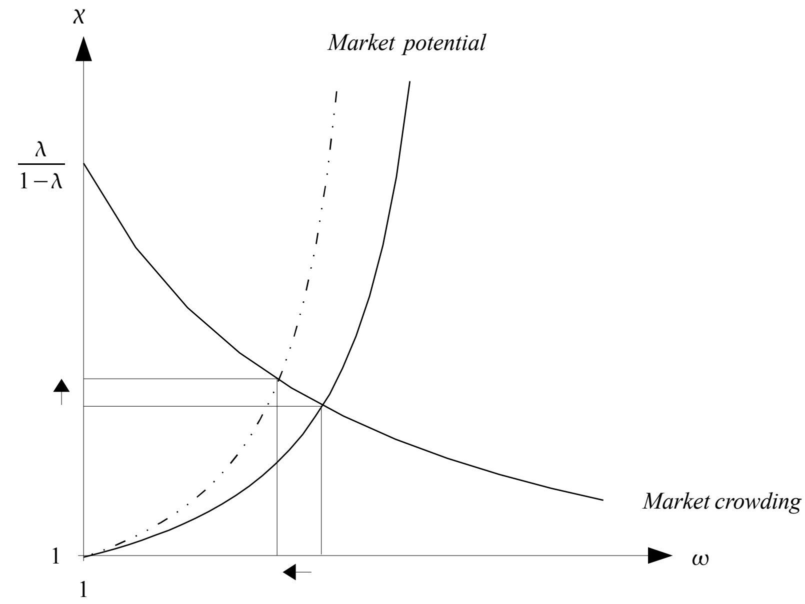
model makes it possible for the economy to host additional firms; the price effect will not crowd out the quantity effect as it would without firm heterogeneity.

Proposition 1 applies also when $\lambda < 1/2$, where Foreign would exhibit a weak HME over the interval $\left(1-\bar{\lambda}, 1/2\right)$ and a strong HME over $(1-\lambda^{*}, 1/2)$. Also note that Proposition 1 can be strengthened in so far as the weak HME can be shown to extend into the region of unconventional sorting; see the extension in section 4.1. Figure 3 anticipates this generalization and graphically illustrates the strong and the weak HMEs in models with and without a linear outside sector. The diagram on the left hand is the standard illustration of the HME in the Helpman-Krugman (1985) world. It also holds in the Melitz (2003) model with a linear outside sector.[17] In the interval $\lambda < \lambda^{*}$, there is a strong HME as the share of firms increases more than proportionately due to an increase in λ. When $\lambda > \lambda^{*}$, the large economy is fully agglomerated $(\phi = 1)$, so that the strong HME cannot hold anymore. Clearly, a weak HME exists over the entire interval. The diagram on the right-side illustrates the HMEs in the single-sector Melitz case. The functional relationship $\phi(\lambda)$ clearly is increasing from (1/2,1/2) to (1,1). It has a kink at $\lambda = \bar{\lambda}$, where conventional sorting no longer holds anymore. So, over the interval $\lambda < \lambda^{*}$ the strong HME holds, while the weak HME obtains over the full interval. Moreover, $\phi(\lambda)$ can be shown to be concave in the interval $(1/2, \bar{\lambda})$ but is convex in the interval $(\bar{\lambda}, 1)$.

Finally, one can show that the existence of a linear outside sector in the Melitz (2003) model magnifies the HME relative to the case where the outside sector is absent. In particular, in the case with the outside sector, the slope of the locus $\phi(\lambda)$ is equal to $1 + 2\eta$. In the single-sector case, it is equal to $1 + \eta/\left(2 - \rho(1-\eta)/\beta\right)$; see

[17] See Appendix C for the model with a linear outside sector.

Figure 4: Higher freeness of trade and relative entry probability



(A.5) in the Appendix for details. That the linear outside sector exaggerates the HME follows from the fact that $1 - \eta < \beta/\rho$.

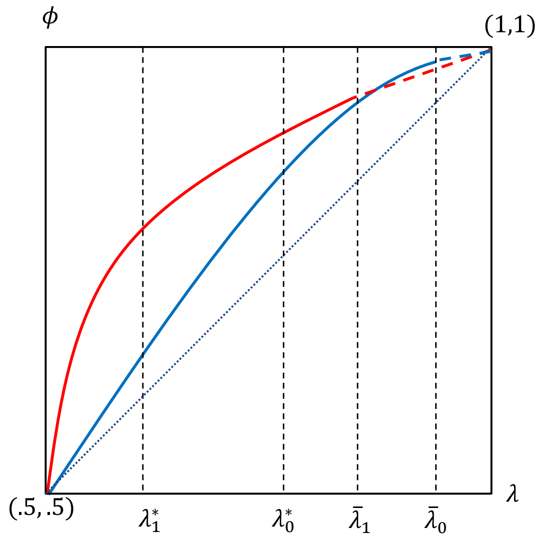
3.3 Home Market Magnification Effects

Next, we investigate how the strength of the HME is affected by the freeness of trade and by the extent of productivity dispersion. In the traditional Helpman-Krugman case with outside sector, lower variable trade costs make the HME more pronounced. They also make it more likely that the model degenerates to full agglomeration (where Home has all firms). This home market magnification effect also exists in our case; only it appears in a somewhat subtler form due to the absence of a linear outside sector. We also show that increased dispersion of productivity magnifies the HME.

The role of freeness of trade. Remember that freeness of trade is defined as $\eta \equiv \tau^{-\beta} \left(f^x/f^d\right)^{1-\beta/(\sigma-1)}$. It falls when variable trade costs τ shrink and/or when foreign entry costs relative to domestic ones f^x/f^d fall. The precise origin of a change in η does not matter for the result. Conveniently, η only appears in the market potential curve but not in the market crowding curve. Using Figure 1, the comparative statics with respect to η are therefore very easy. Figure 4 provides an illustration.

The intuition is that for a given wage (market potential), a higher freeness of trade favors the larger country since serving the smaller country through exports is now cheaper. The market crowding locus is not affected since freeness of trade rises symmetrically. Hence, the equilibrium relative entry probability goes up, which translates into a larger HME since γ rises in χ for given λ. Moreover, the equilibrium relative wage declines, so that higher freeness of trade leads to convergence of nominal

Figure 5: Home market magnification due to higher freeness of trade



wages.

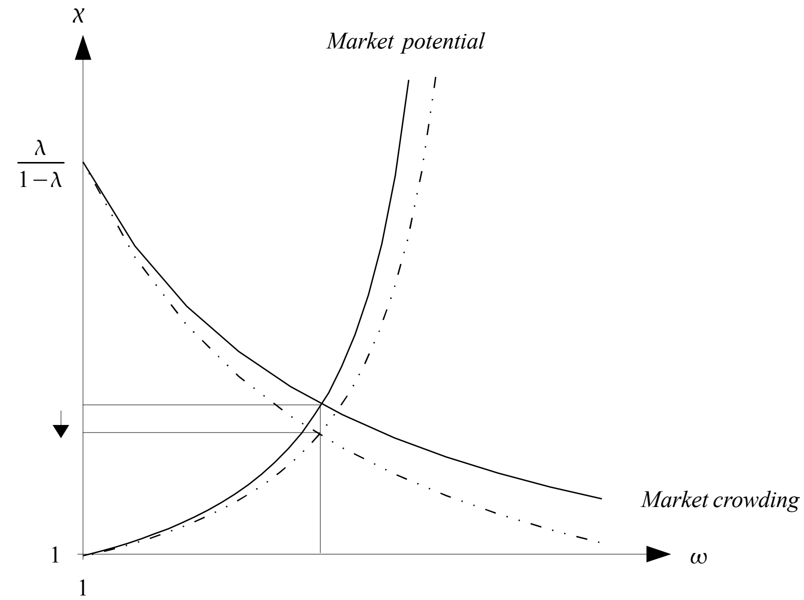
There are additional side effects. Higher η makes it more likely that the conventional sorting conditions fails to hold, so $\bar{\lambda}$ falls and the kink in the $\phi(\lambda)$ locus occurs earlier. Moreover, also the interval over which the strong HME can be observed shrinks since λ^* falls as well. Figure 5 provides an illustration in (ϕ, λ) −space. As long as conventional sorting continues to hold, one can show that γ increases so that the HME becomes stronger.[18] As a corollary, there must be some interval over which also the strong HME becomes augmented.

The role of productivity dispersion. Next, we consider the comparative statics with respect to β, the shape parameter of the Pareto distribution, which is inversely related to the variance of the sampling distribution. β appears in both the market crowding and the market potential curves. An increase in β rotates both down. This leads to an unambiguously negative effect on relative entry probabilities χ, while the effect on the wage rate depends on model parameters in a complicated fashion. Lower χ translates into lower γ so that the (weak) HME is diminished when β goes up (i.e., when the dispersion of productivity falls). The converse is true, too: higher dispersion magnifies the (weak) HME. The intuition for this result is that differences in domestic entry cutoffs due to size differentials are magnified when the productivity dispersion is higher. In other words, selection is more important. It turns out to favor the larger country *ex ante* and *ex post*. So, for the emergence of a HME in the single-sector Melitz model, productivity dispersion is important.[19]

[18] We provide a generalization of this result to the case of unconventional sorting in the extensions below.

[19] This finding has important implications for empirical studies on the HME, such as Hanson and Xiang (2004). The model suggests that one important industry characteristic that shapes the size of

Figure 6: Lower productivity dispersion and relative entry probability.



The following proposition summarizes this second key result of our analysis.

Proposition 2 *(**Home Market Magnification Effects**). Assume that conventional sorting holds ($\lambda < \bar{\lambda}$). Then the (weak) home market effect is magnified by*
(a) a higher freeness of trade and
(b) a higher degree of productivity dispersion.

Proof. See the Appendix. ■

3.4 Distribution of Endowments, Distribution of Real Income, and Trade

In this subsection we show that the distribution of labor endowments across countries matters for the distribution of per capita real income (welfare). We also investigate how cross-country income inequality is affected by trade liberalization. Importantly, the single-sector predicts convergence in real income levels while the model with a linear-outside sector does not.

Analytical results. To study *real per capita income*, one can exploit a well known feature of the Melitz (2003) model, where a country's real income per worker (welfare per worker) is proportional to its domestic entry cutoff. This directly follows from the

the HME is the degree of productivity dispersion as captured by the shape parameter of the Pareto distribution.

country's domestic zero cutoff profit condition.

$$W_i \equiv \frac{w_i}{P_i} = \rho \left(\frac{L_i}{\sigma f_{ii}} \right)^{\frac{1}{\sigma-1}} \varphi_{ii}^*. \tag{15}$$

This, in turn, allows to write *relative real per capita* income (the cross-country gap in living standards) as a function of a single endogenous variable that has played a key role in our analysis so far and whose properties are well understood, namely χ :

$$\frac{W_H}{W_F} = \left(\frac{\lambda}{1-\lambda} \right)^{\frac{1}{\sigma-1}} \chi^{-\frac{1}{\beta}}. \tag{16}$$

We know from Figure 1, that $\chi < \lambda / (1-\lambda)$. Together with the condition $\beta > \sigma - 1$, this ensures that $W_H > W_F$. So, the larger Home has the higher real per capita income.[20] For the same reason, it is easy to see that W_H/W_F increases with λ so that a more unequal distribution of the world labor endowment leads to more disparity in terms of welfare per capita.

Equation (16) is also useful to understand the fundamental drivers of international disparities. For given relative country size, an increase in the freeness of trade raises χ and therefore lowers W_H/W_F. In other words, trade liberalization leads to convergence of real income per capita across countries. The intuition is that higher freeness of trade favors the more open country, which is the small country. In the model, convergence of real income is equivalent to factor price convergence.

Importantly, trade liberalization does not lead to convergence if the model features a linear outside sector.[21] In that case, relative real per capita income is given by

$$\widetilde{\frac{W_H}{W_F}} = \left(\frac{\lambda}{1-\lambda} \right)^{\frac{\mu}{\sigma-1}}, \tag{17}$$

where $\mu \in (0,1)$ is the share of expenditure that the representative consumer allocates to the differentiated good. Clearly, there is no role for trade openness in that expression of the real income gap. The intuition for this result lies in the simple fact that fixing the wage rate also fixes mill prices in a CES environment. So, trade liberalization could affect W differently across countries, if cutoff productivity levels were differently affected by a change in η. This is, however, not possible when wages are insensitive to λ and η.

We have argued above that the presence of a linear outside sector exaggerates the importance of the HME. Interestingly, one can show that this does not imply that the cross-country welfare differential must be bigger, too. Quite the opposite is true. Comparison of (16) and (17) reveals that the welfare differential is larger in the absence of the outside sector if $\chi < [\lambda/(1-\lambda)]^{\beta(1-\mu)/(\sigma-1)}$. This inequality always holds if $\lambda > 1/2$: Figure 1 implies $\chi < \lambda/(1-\lambda)$ and $\beta(1-\mu)/(\sigma-1) > 0$. We summarize these results in the following proposition:

[20] This result extends to the case of unconventional sorting; see (A.7.3) in the Appendix.

[21] See Appendix C for a detailed description of this model.

Proposition 3 ***(Per capita income and convergence)****. Assume that conventional sorting holds ($\lambda < \bar{\lambda}$).*
(a) The larger economy exhibits the higher real per capita income.
(b) Trade liberalization leads to real per capita income convergence across countries. There is no convergence in the presence of a linear outside sector.
(c) The presence of linear outside sector reduces cross-country real income disparities.

Proof. In the text. ■

Numerical analysis. Finally, we are interested in the role of firm-level productivity dispersion for cross-country welfare differences. However, the effect of a higher degree of productivity dispersion (lower β) on relative welfare is ambiguous. Lower β raises χ, which reduces relative welfare. At the same time, the elasticity of relative welfare in χ falls (in absolute terms), which works in the opposite direction. For this reason, and to gain a rough idea of how important the quantitative importance of endowment differences is for the cross-country welfare gap, we carry out a simple numerical analysis. The calibration of the model is very standard and follows the literature; see Bernard et al. (2007) for a leading example.

We highlight the role of firm-level productivity dispersion for relative real income differences across countries by comparing the Melitz (2003) model with a Pareto sampling distribution to Krugman (1980). In the calibration, we make sure that the two scenarios yield the same relative welfare for some common λ. From Arkolakis et al. (2011), we know that this requires that, across the two models, at some λ, the endogenous degree of 'autarkiness' and the used trade elasticities are identical. For the calibration we choose $\lambda = 1/2$, and then simulate W_H/W_F for $\lambda > 1/2$. Details of the calibration are explained in Appendix B.

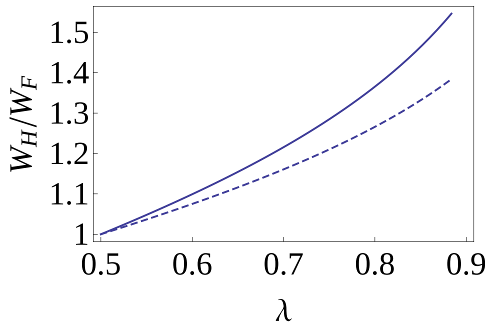
Figure 7 displays W_H/W_F for the Melitz (solid line) and the Krugman (dashed line) models.[22] Clearly, in both models cross-country welfare differences increase as the distribution of labor endowment becomes more unequal. However, the increase is stronger in the world with productivity dispersion. Figure 7 shows that in both models, a deviation from symmetry to, say, $\lambda = 0.6$ leads to a fairly substantial increase in international inequality: Home's welfare is now about 10% larger than Foreign's. So, the model suggests that cross-country endowment differences, which lead to HMEs, may be important both qualitatively and quantitatively for cross-country real income discrepancies.

Finally, trade liberalization (higher η) leads to per capita real income convergence; see Figure 8 in both the Melitz and the Krugman models. However, the amount of convergence is larger in the Krugman case. Thus, firm-level heterogeneity lowers the convergence gains that an increase in η delivers.[23]

[22]We only consider cases in which conventional sorting holds under the Melitz specification.

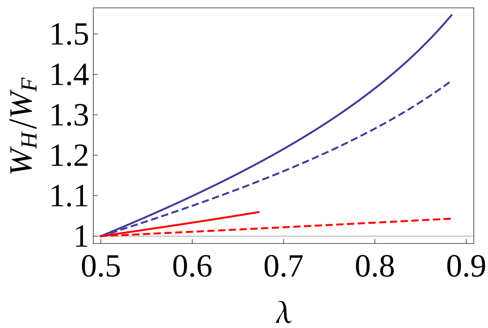
[23]Note that the range in which conventional sorting holds shrinks in response to trade liberalization.

Figure 7: Welfare differential and endowment distribution



Solid line: Benchmark specification. Dashed line: Krugman specification.

Figure 8: Welfare differential and trade liberalization



Solid line: Benchmark specification. Dashed line: Krugman specification. Blue line: Before liberalization. Red line: After liberalization

The following observation provides a summary.

Observation 1 *Firm-level productivity heterogeneity exacerbates cross-country real per capita income differences and reduces the amount of real per capita income convergence brought about by trade liberalization.*

4 Extensions and Additional Results

4.1 Unconventional Sorting

It is a well documented stylized fact that only the most productive firms engage in exporting; see Bernard et al. (2007). We have this situation "conventional sorting" and have so far assumed that it holds. In symmetric equilibrium, $f^x > f^d$ is a sufficient condition to guarantee this sorting pattern.[24] Then, only the most productive firms will generate sufficiently large sales to overcome export fixed costs. However, when countries are asymmetric in size, the sorting pattern can reverse in the smaller country. The intuition for this lies in the fact that less productive firms would make negative profits on their small domestic market, for which fixed costs are high relative to revenue, while they can make profits on the large export market, where fixed costs are lower relative to revenue. When the sorting pattern reverses, the definition of the relative entry probability χ needs to be adapted since p_F^{in} will now be given by $1 - G\left[\varphi_{FH}^{*}\right]$.

The balanced trade condition (8) is not affected by the reversal of the sorting condition. So, it is still true that the larger country commands the higher wage. Also equations (11) and (12) continue to hold. However, the *market crowding curve* (MCC) now is given by

$$\chi = \frac{f^x/f^d}{\eta}\omega^{-\frac{\beta}{\rho}}. \tag{18}$$

It follows from the relative zero cutoff profit conditions for targeting the Home's market. Clearly, the MCC is strictly decreasing in ω. In contrast to the case of conventional sorting, it is now independent of λ but depends on trade costs. The *market potential curve* (MPC) now reads

$$\chi = \frac{f^x/f^d}{1 - \frac{1-\lambda}{\lambda}\omega^{\frac{\beta-\rho}{\rho}}\left(\omega^{\beta/\rho} - \eta\right)}. \tag{19}$$

The MPC is strictly increasing in ω, and now depends on λ as long as on trade costs. Similar to the case of conventional sorting, equilibrium can be determined by the intersection of the modified MPC and MCC curves. Evaluated at $\omega = 1$, the MPC curve yields $\chi > 1$. Hence, the weak HME also occurs under unconventional sorting.

Variable trade cost liberalization, in turn, shifts both the MCC and MPC down. Lowering relative export fixed costs additionally moves both curves proportionally. Hence, a higher freeness of trade lowers the relative probability, which dampens the home market effect. Higher λ also lowers the HME. This is a necessary consequence of the fact that $\phi(1) = 1$. We summarize these results in the following proposition:

Proposition 4 ***(Unconventional sorting)****. Assume that unconventional sorting holds ($\lambda > \bar{\lambda}$).*
(a) The larger country has a higher relative wage (ω) and higher relative participation probability (χ), so that a ***weak home market effect*** *exists.*
(b) Higher freeness of trade (η) or higher asymmetries in country size (λ) reduce χ, so that the (weak) home market effect is ***diminished****.*

As a corollary, the model features a weak HME over all possible cross-country allocations of the world labor endowment. As in Crozet and Trionfetti (2008) the (weak) HME is non-linear. Their numerical exercise suggests that the HME is concave in λ

[24] In the presence of variable trade costs, this requirement can be weakened.

for values of λ around the symmetric equilibrium and convex thereafter. In our setup, the HME is concave for $\lambda < \bar{\lambda}$ and convex for $\lambda > \bar{\lambda}$.

4.2 Technology differences and the HME in demand shares

Empirical work typically employs cross-industry data to study how shocks on *demand shares* affect production patterns. We do not pursue the straight-forward extension of our model to the multi-industry case, but characterize analytically the endogenous relationship between Home's share of firms (ϕ) and its share in world demand (δ).[25] Home's share in world demand (GDP) is given by

$$\delta \equiv (1 + (1-\lambda) / (\omega\lambda))^{-1},$$

which is of course endogenous to the model. In the standard setup, with an identically parameterized linear outside sector in both countries, we would have $\omega = 1$ and therefore $\delta = \lambda$. The HME in demand shares is then identical to the HME in endowment shares. In our case, this is of course different since $\omega > 1$. The relative wage can be affected through exogenous changes in λ and/or through differences in the technology levels across countries. We model the latter by differences in the lower bound of the productivity distribution, which we again take to be the Pareto. I.e, we fix the shape parameter of the Pareto (β) in both countries, but assume that there are cross-country differences in the minimum admissibly productivity level (b_i).

We define the (weak) home market effect in demand shares as an overproportional relationship between the share of firms and the demand share, so that $\delta > 1/2 \Rightarrow \phi > \delta$. From equation (11), it is easy to see that the HME in demand shares requires $\chi > \omega$. Note that the condition for a HME in demand shares is stronger than the one for a HME in labor shares, which is $\chi > 1$.

Let $B \equiv (b_H/b_F)^{\beta}$ denote the degree of productivity differences. Then, the relative entry probability is defined as $\chi \equiv B\left(\varphi^*_{FF}/\varphi^*_{HH}\right)^{\beta}$. We show in the Appendix that the market crowding curve (MCC) can be generalized to

$$\chi = B^2 \frac{\lambda}{1-\lambda} \omega^{-\frac{2\beta-\rho}{\rho}}. \tag{20}$$

As before, the MCC is downward sloping in ω. Evaluated at $\omega = 1$, the MCC yields $\chi = B^2 \frac{\lambda}{1-\lambda}$, which is the upper bound for χ. The market potential curve (MPC) generalizes to

$$\chi = \frac{1 - B\omega^{-\frac{\beta}{\rho}}\eta}{1 - B^{-1}\omega^{\frac{\beta}{\rho}}\eta}. \tag{21}$$

The MPC is upward sloping and features an asymptote at $\omega = \left(\frac{B}{\eta}\right)^{\frac{\rho}{\beta}}$. In the supported range, the denominator is always positive. The sign of the numerator is unclear as $B > 1, \eta < 1, \omega > 1$.

[25] To work with the share of production rather than the share of firms would require a multi-industry setup. In the single-sector framework that approach would lead to an identity $(GDP = GDP)$.

First, consider labor share asymmetries under symmetric productivities, i.e., $B = 1$. While the generalized MCC and MPC curves collapse to the ones in section (2), a weak HME in demand shares requires that $\chi > \omega$ so that an parameter restriction is necessary. One can show that a weak HME arises if and only if $\eta > \rho/(2\beta + \rho)$; a sufficient condition for this is $\eta > 1/3$.[26] Next, consider a symmetric distribution of the world labor endowment, but differences in the loci of the Pareto productivity distributions across countries. Then, the necessary condition for a weak HME can never hold. Hence, there exists a weak reverse HME: the richer country hosts an underproportional share of firms. This is due to the fact that higher average productivity translates into larger average firm size so that the number of firms has to adjust downwards. Productivity growth (as modeled by an increase in b_H) therefore leads to lower prices, but also reduces the number of domestically produced varieties that are available without trade costs.

These results are summarized in the following proposition:

Proposition 5 ***(Home market effect in demand shares)****.Let δ be Home's share in GDP. Assume that conventional sorting holds $\left(\lambda < \bar{\lambda}\right)$ and that the freeness of trade is not too low $(\eta > 1/3)$.*
(a) Let $\lambda > 1/2$ and $B = 1$. Then, the larger economy exhibits a weak HME in demand shares $(\phi > \delta)$.
(b) Let $\lambda = 1/2$ and $B > 1$. Then, the richer economy exhibits a weak reverse HME in demand shares $(\phi < \delta)$.

Proof. In the Appendix. ■

If both $\lambda > 1/2$ and $B > 1$, the situation is more complicated. A weak HME obtains if the relative endowment of Home (L_H/L_F) is large enough relative to B. The finding that the underlying cause for cross-country variation in demand shares matters for whether or not a HME exists, is important for empirical work. If a researcher runs a regression of ϕ on δ, it is important to control for some measure of b, for example average TFP. Failing to do so may explain why the empirical literature has found mixed support for the (weak) HME so far.

4.3 The HME with a General Sampling Distribution

In this subsection, we generalize the argument to a situation where firms' productivity levels are sampled from a general productivity distribution. Our MCC and MPC curves hinge on the assumption of the Pareto distribution and are therefore of no help in the general case. While it is difficult to derive results on the strong HME or on magnification effects, it is possible to establish the weak HME for the case of conventional sorting.

[26] With the simple (and very standard) calibration of section 3.4, the requirement for the weak HME is $\eta > 0.1$. This would represent an valorem tariff equivalent of 95 per cent $(\tau = 1.946)$. Hence, the condition is likely to be met in all reasonable circumstances.

To this end, it appears useful to reduce the equilibrium conditions (2) - (6) to two equations in ω and Home's export cutoff φ^*_{HF}, which have opposite slopes. The two loci are substantially more complicated than our MCC and MPC schedules, but they are still useful for our purposes. Labor endowment shares affect only one of these curves, which allows inference on the effect of a country size shock on the relative wage and the various cutoffs. Moreover, drawing on a generalized labor market clearing condition, we can derive our result on the home market effect. We relegate the formal proofs to the Appendix and immediately summarize the results:

Lemma 1' *Assume that productivity levels are sampled from a general productivity distribution. Then, the larger country pays the higher wage.*

Proof. In the Appendix. ■

Proposition 2(a)' *Assume that productivity levels are sampled from a general productivity distribution. Moreover, assume that conventional sorting holds. Then, the economy exhibits a weak HME on the interval* $\left(1/2, \bar{\lambda}\right)$.

Proof. In the Appendix. ■

4.4 Margins of Trade and the HME

The conventional Helpman-Krugman model has the important prediction that the larger country should be a net exporter of the differentiated good and a net importer of the outside good. This prediction is less suited than the HME to empirically discriminate models with increasing returns to scale at the firm level from models based on comparative advantage, since the latter models can give rise to similar predictions (Helpman, 1999; Hanson and Xiang, 2004). From a theory perspective, the trade pattern result is a direct corollary of the HME if the upper tier demand structure is homothetic: An increase in Home's labor force triggers an overproportional expansion of the differentiated goods sector; this requires that the labor share of the homogeneous goods sector shrinks; expenditure shares, in contrast, remain constant.

In our simple single-sector model, one cannot discuss the sectoral trade pattern. However, it is easy to see that an asymmetric distribution of the world labor endowment has implications for the margins of trade. The balanced trade condition (6) can be alternatively written as

$$\frac{M_H}{M_F} \times \frac{\bar{r}_{HF}}{\bar{r}_{FH}} = \underbrace{\frac{m_{HF}}{m_{FH}} \frac{M_H}{M_F}}_{\text{extensive}} \times \underbrace{\omega}_{\text{intensive}} = 1. \tag{22}$$

Remember that $\bar{r}_{ij} = \sigma\theta m_{ij} w_i f_{ij}$ is the value of sales on market j that a firm located on market i can expect ex ante. We can decompose relative exports into its extensive and the intensive margin. Since the larger country has the higher wage, that is $\omega > 1$, the second equality implies $m_{HF}M_H < m_{FH}M_F$. So, there are fewer exporters in H than in F. Given that the mass of firms active in Home is larger than the mass of firms in Foreign, i.e. $M_H > M_F$, it must be case that the larger country exhibits the lower export participation rate. However, Home's exporters are larger on average. We state this testable implication of the model as a corollary to Lemma 1:[27]

Corollary 1 *Exports of the larger country are dominated by the intensive margin; exports of the smaller country are dominated by the extensive margin.*

5 Conclusion

This paper provides a tractable way to characterize a two-country single-sector asymmetric Melitz (2003) model for the purpose of conducting comparative statics. It does so without imposing a linear, perfectly competitive and frictionless outside sector, as the literature has usually chosen to do. The outside sector assumption has been criticized to be unrealistic and possibly important for aggregate results, such as welfare (Demidova and Rodriguez-Clare, 2011), or for the ability of the model to predict a Home Market Effect (HME), by which a large country attracts a more than proportionate share of producing firms (Davis, 1998).

The present analysis focuses on the HME, which has been used as a criterion to discriminate between trade models featuring increasing returns to scale and more conventional comparative advantage based setups. It shows that the unrealistic and potentially problematic outside sector assumption can be replaced by a Melitz-type selection mechanism, where only a fraction of heterogeneous firms sell to all markets. The resulting HME is non-linear, as the empirical analysis of Crozet and Trionfetti (2008) suggests. It is magnified by falling trade costs and by a higher degree of firm-level productivity dispersion. The HME translates into cross-country welfare differences. In contrast to the model with a linear outside sector, trade liberalization attenuates these cross-country differences and leads to real wage convergence. Firm-level heterogeneity is absolutely crucial for these results: in the Krugman (1980) single-sector model, no HME can arise.

The results presented in this paper are important for empirical work. First, since the outside-sector is not crucial for the existence of a HME, an empirical rejection of an overproportionate relation between a country's share of firms and its share of endowments is indeed evidence against increasing returns and not against the linear outside sector. Second, empirical tests that fail to control for the level of technology may wrongly reject the existence of an endowment-driven HME.

[27] Interestingly, the opposite result follows if one assumes that fixed market entry costs are in terms of foreign instead of domestic labor. The corollary could therefore be used to discriminate empirically between the approaches. Clearly, any empirical test would have to acknowledge that countries also differ with respect to the ex ante production distribution.

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A Proofs of Lemmas and Propositions, Details to Derivations

A.1 Derivation of equilibrium conditions

Zero cutoff profit conditions. Demand for any variety is given by

$$q[z] = R_i P_i^{\sigma-1} p[z]^{-\sigma},$$

where the price index to (1) is given by $P_i^{1-\sigma} = \int_{z\in\Omega_i} p[z]^{1-\sigma} dz$ and R_i denotes aggregate expenditure.[28] Given the demand function, the price charged at the factory gate is $w_i/(\rho\varphi)$. Then, operating profits of a firm from country i on market j are

$$\pi[\varphi] = R_j P_j^{\sigma-1} \left(\frac{\rho\varphi}{\tau_{ij} w_i}\right)^{\sigma-1} /\sigma - w_i f_{ij}.$$

The zero cutoff profit conditions follow from noting that $\pi\left[\varphi_{ij}^*\right] = 0$.

Price index. Using the zero cutoff profit condition, we can write the price level P_i as

$$\begin{aligned}
P_i^{1-\sigma} &= \sum_{j\in\{H,F\}} \int_{\varphi_{ji}^*}^{\infty} \left(\frac{\tau_{ji} w_j}{\rho\varphi}\right)^{1-\sigma} M_j m_{ji} \frac{dG[\varphi]}{1-G\left[\varphi_{ji}^*\right]} \\
&= \sum_{j\in\{H,F\}} \left(\frac{\tau_{ji} w_j}{\rho}\right)^{1-\sigma} M_j m_{ji} \theta \left(\varphi_{ji}^*\right)^{\sigma-1} \\
&= \theta \sum_{j\in\{H,F\}} m_{ji} M_j \left(\frac{\rho\varphi_{ji}^*}{\tau_{ji} w_j}\right)^{\sigma-1},
\end{aligned}$$

where $\theta \equiv \beta/(\beta-\sigma+1)$ is a positive constant.

Free entry condition. Using optimal demand and the zero cutoff profit condition, we obtain the following expression for expected profits of a firm in country i from entering

$$\begin{aligned}
\bar{\pi}_i &= \sum_{j\in\{H,F\}} \int_{\varphi_{ij}^*}^{\infty} \pi_{ij}[\varphi] \frac{dG[\varphi]}{1-G\left[\varphi_{ii}^*\right]} \\
&= \sum_{j\in\{H,F\}} m_{ij} \left(\theta \frac{R_j P_j^{\sigma-1}}{\sigma} \left(\frac{\tau_{ij} w_i}{\rho}\right)^{1-\sigma} \left(\varphi_{ij}^*\right)^{\sigma-1} - w_i f_{ij}\right) \\
&= \sum_{j\in\{H,F\}} m_{ij} \left(\theta R_j P_j^{\sigma-1} \left(\frac{\tau_{ij} w_i}{\rho}\right)^{1-\sigma} R_j^{-1} P_j^{1-\sigma} \left(\frac{\tau_{ij} w_i}{\rho}\right)^{\sigma-1} w_i f_{ij} - w_i f_{ij}\right),
\end{aligned}$$

[28]Note that each variety z is produced by a single firm with productivity level φ. We henceforth index varieties by φ.

which reduces to the expression in the text.

Labor market clearing condition. Labor market clearing is given by

$$L_i = M_i^e f^e + M_i \sum_j m_{ij} f_{ij} + M_i \sum_j \int_{\varphi_{ij}^*} \frac{\tau_{ij} q_{ij}[\varphi]}{\varphi} \frac{dG[\varphi]}{1-G\left[\varphi_{ij}^*\right]} = M_i \theta\sigma \sum_j m_{ij} f_{ij},$$

where the second equality follows from inserting $M_i^e = M_i / p_i^{in}$, using the free entry condition to substitute out f^e, and using the zero cutoff profit conditions to substitute out the cutoff productivity levels. The formula in the text follows from using the free entry condition to substitute out $\sum_j m_{ij} f_{ij}$ and noting that $\theta\sigma/(\theta-1) = \beta/\rho$.

Trade balance condition. In analogy to expected profits, we can write expected revenues of a firm in country i from selling to country j as

$$\bar{r}_{ij} = \int_{\varphi_{ij}^*}^{\infty} r_{ij}[\varphi] \frac{dG[\varphi]}{1-G\left[\varphi_{ii}^*\right]} = \sigma\theta w_i m_{ij} f_{ij}.$$

Using this expression, the labor market clearing condition, the definition of m_{ij} and exploiting symmetry of fixed cost, we obtain the balanced trade condition (7).

A.2 Proof of Lemma 1

Let α_{ij} denote the share of country i's income spent on varieties from country j. It is given by

$$\alpha_{ij} \equiv \frac{M_j \bar{r}_{ji}}{w_i L_i},$$

Using the budget constraint $w_i L_i = \sum_j M_j \bar{r}_{ji}$, we can rewrite this share as

$$\alpha_{ij} = \left(1 + \frac{M_i \bar{r}_{ii}}{M_j \bar{r}_{ji}}\right)^{-1}.$$

Using the labor market clearing condition (5) and $\bar{r}_{ij} = \sigma\theta m_{ij} w_i f_{ij}$, we obtain

$$\alpha_{ij} = \left(1 + \left(\frac{\varphi_{ji}^*}{\varphi_{ii}^*}\right)^{\beta} \frac{L_i}{L_j} \frac{w_i}{w_j} \frac{f^d}{f^x}\right)^{-1}$$

Relative zero cutoff profit conditions can be used to substitute out productivity cutoffs. Then,the balanced trade condition

$$\alpha_{HF}(1-\lambda) = \alpha_{FH}\lambda$$

implies

$$\begin{aligned} \lambda\omega^{-(\beta-\rho)/\rho} + (1-\lambda)\eta &= (1-\lambda)\omega^{\beta/\rho} + \lambda\omega\eta \\ \iff \frac{\lambda}{1-\lambda} &= \frac{\omega^{\beta/\rho} - \eta}{\omega\left(\omega^{-\beta/\rho} - \eta\right)} \end{aligned}$$

If $\lambda > 1/2$, we have

$$\omega^{\beta/\rho} - \omega^{1-\beta/\rho} > \eta (1 - \omega).$$

This equation is violated for $\omega \leq 1$ since $\beta/\rho > 1$ so that the left hand side would be negative while the right hand side would be positive. It follows that $\lambda > 1/2$ must entail $\omega > 1$.

A.3 Proof of Lemma 2

A.3.1 Derivation of conventional sorting cutoff $\bar{\lambda}$

Cutoff level $\bar{\lambda}$ up to which Foreign first serves domestic and then export market is implicitly defined by $\varphi^*_{FF} = \varphi^*_{FH}$

$$\varphi^*_{FF} = \varphi^*_{FH} \Leftrightarrow 1 = \tau^{-1} \bar{\omega}^{\frac{\rho-\beta}{\rho\beta}} \left(\frac{\bar{\lambda}}{1-\bar{\lambda}} \right)^{\frac{1}{\beta}} \left(\frac{f^x}{f^d} \right)^{\frac{1}{1-\sigma}} \tag{23}$$

Market potential and market crowding curves imply

$$\frac{\bar{\lambda}}{1-\bar{\lambda}} = \frac{\eta - \bar{\omega}^{\beta/\rho}}{\eta \bar{\omega}^{\beta/\rho} - 1} \bar{\omega}^{(\beta-\rho)/\rho}, \tag{24}$$

which is equivalent to equation (8).Using this expression to substitute out $\frac{\bar{\lambda}}{1-\bar{\lambda}}$ from equation (23) and solving for $\bar{\omega}$, we obtain

$$\bar{\omega}^{\frac{\beta}{\rho}} = \frac{\eta \tau^{-\beta} \left(\frac{f^x}{f^d} \right)^{-\frac{\beta}{\sigma-1}} + 1}{\tau^{-\beta} \left(\frac{f^x}{f^d} \right)^{-\frac{\beta}{\sigma-1}} + \eta}.$$

Using the definition of η, we can write $\bar{\omega}^{\frac{\beta}{\rho}}$ as

$$\bar{\omega}^{\frac{\beta}{\rho}} = \frac{\eta + \frac{1}{\eta} \frac{f^x}{f^d}}{1 + \frac{f^x}{f^d}}.$$

This expression can be used to back out $\bar{\lambda}$ from equation (24).

A.3.2 Derivation of market crowding curve

In order to derive the market crowding curve (MCC), we use the **zero cutoff profit conditions** in relative terms and the balanced trade condition. Taking F as the target market and using the two associated zero cutoff profit conditions

$$\left(\frac{\varphi^*_{HF}}{\varphi^*_{FF}} \right)^{\sigma-1} = \tau^{\sigma-1} \omega^{\sigma} \frac{f^x}{f^d}. \tag{25}$$

Taking H as the target market and dividing the two associated zero cutoff profit conditions

$$\left(\frac{\varphi_{FH}^*}{\varphi_{HH}^*}\right)^{\sigma-1} = \tau^{\sigma-1}\omega^{-\sigma}\frac{f^x}{f^d} \tag{26}$$

Using equations (25) and (26) together with the trade balance condition (7), we obtain

$$\chi = \frac{\lambda}{1-\lambda}\omega^{-\frac{2\beta-\rho}{\rho}},$$

where $\chi \equiv (\varphi_{FF}^*/\varphi_{HH}^*)^{\beta}$ denotes Home's relative entry probability.

A.3.3 Derivation of market potential curve

In order to derive the market potential curve (MPC), we use the **free entry conditions** in relative form along with the zero cutoff profit conditions and the balanced trade condition.

In relative form, the free entry conditions read

$$\chi = \frac{f^d + \left(\frac{\varphi_{FF}^*}{\varphi_{FH}^*}\right)^{\beta} f^x}{f^d + \left(\frac{\varphi_{HH}^*}{\varphi_{HF}^*}\right)^{\beta} f^x} \tag{27}$$

Using the zero cutoff profit conditions and the trade balance condition to eliminate φ^* terms, we obtain

$$\frac{\varphi_{FF}^*}{\varphi_{FH}^*} = \tau^{-1}\omega^{\frac{\rho-\beta}{\rho\beta}}\left(\frac{\lambda}{1-\lambda}\right)^{\frac{1}{\beta}}\left(\frac{f^x}{f^d}\right)^{\frac{1}{1-\sigma}} \tag{28}$$

from (7) and (25) and

$$\frac{\varphi_{HH}^*}{\varphi_{HF}^*} = \tau^{-1}\left(\frac{\lambda}{1-\lambda}\right)^{-\frac{1}{\beta}}\omega^{\frac{\beta-\rho}{\beta\rho}}\left(\frac{f^x}{f^d}\right)^{\frac{1}{1-\sigma}}$$

from (7) and (26). Substituting out these expressions, we get

$$\chi = \frac{1+\omega^{-\frac{\beta-\rho}{\rho}}\left(\frac{\lambda}{1-\lambda}\right)\tau^{-\beta}\left(\frac{f^x}{f^d}\right)^{-\frac{1-\sigma+\beta}{\sigma-1}}}{1+\omega^{\frac{\beta-\rho}{\rho}}\left(\frac{1-\lambda}{\lambda}\right)\tau^{-\beta}\left(\frac{f^x}{f^d}\right)^{-\frac{1-\sigma+\beta}{\sigma-1}}}.$$

Using $\eta \equiv \tau^{-\beta}\left(\frac{f^x}{f^d}\right)^{-\frac{\beta-(\sigma-1)}{\sigma-1}}$ and the market crowding curve to substitute out $\frac{\lambda}{1-\lambda}\omega^{-\frac{2\beta-\rho}{\rho}}$ in the numerator and denominator, we obtain

$$\chi = \frac{1-\eta\omega^{-\frac{\beta}{\rho}}}{1-\eta\omega^{\frac{\beta}{\rho}}}.$$

A.3.4 Characteristics of market potential and market crowding curve

Market crowding curve. The market crowding curve implies a downward-sloping and convex relationship between χ and ω as

$$\frac{\partial\chi}{\partial\omega} < 0 \text{ and } \frac{\partial^2\chi}{\partial\omega^2} > 0.$$

Evaluated at $\omega = 1$, the market crowding curve takes the value $\chi = \frac{\lambda}{1-\lambda} \geq 1$. χ is bounded from below by 0.

Market potential curve. Evaluated at $\omega = 1$, we have $\chi = 1$. Given that χ is restricted to positive values, we have $\omega < \eta^{-\frac{\rho}{\beta}}$. The market potential curve implies an upward-sloping relationship between χ and ω

$$\frac{\partial\chi}{\partial\omega} = \frac{\beta\eta}{\rho}\frac{\chi\omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}}}{\omega\left(1-\eta\omega^{\frac{\beta}{\rho}}\right)} > 0.$$

Convexity of the market potential curve requires that the freeness of trade is not too small

$$\eta > \frac{\rho}{4\beta+\rho}.$$

In order to see this,.we compute

$$\frac{\partial^2\chi}{\partial\omega^2} = \frac{\partial\chi}{\partial\omega}\frac{\omega^{-1}}{\chi\omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}}}\left(2\frac{\partial\chi}{\partial\omega}\omega^{\frac{\beta+\rho}{\rho}} + \frac{\beta-\rho}{\rho}\chi\omega^{\frac{\beta}{\rho}} - \frac{\beta+\rho}{\rho}\omega^{-\frac{\beta}{\rho}}\right).$$

The sign of the second derivative is the sign of

$$h[\omega] = (\beta+\rho)\left(\eta\chi\omega^{\frac{2\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}} + \eta\right) + 2\beta\eta + (\beta-\rho)\chi\omega^{\frac{\beta}{\rho}}.$$

Evaluating in symmetric equilibrium ($\omega = \chi = 1$), we obtain

$$2(4\beta\eta + \rho\eta - \rho) > 0 \Leftrightarrow \eta > \frac{\rho}{4\beta+\rho}.$$

Moreover, it is easy to check that $h[\omega]$ is strictly increasing in ω. The reason is that χ is increasing in ω. Moreover, $\beta > \rho$.

A.4 Proof of Proposition 1

A.4.1 Weak HME

In symmetric equilibrium ($\lambda = 1/2$), we have $\chi = \omega = \gamma = 1$. In response to a labor share shock, the market crowding curve shifts upwards

$$\frac{\partial\chi}{\partial\lambda} = \frac{\omega^{-\frac{2\beta-\rho}{\rho}}}{(1-\lambda)^2} > 0.$$

The market potential curve is unaffected. Then, in equilibrium $\chi > 1$ for $\lambda > 1/2$. γ is increasing in χ

$$\frac{\partial \gamma}{\partial \chi} = \frac{1-\lambda}{\left[1+\lambda\left(\chi-1\right)\right]^2} \geq 0,$$

where the equality occurs for $\lambda = 1$. Hence, for $\lambda > 1/2$, we have $\gamma > 1$, which is the definition of the weak HME.

A.4.2 Strong HME

Remember that

$$\phi = \gamma\lambda, \gamma = \frac{\chi}{1+\lambda\chi-\lambda}$$

with

$$\frac{\partial \gamma}{\partial \lambda} = \frac{\frac{\partial \chi}{\partial \lambda}\left[1+\lambda\left(\chi-1\right)\right] - \chi\left(\chi+\lambda\frac{\partial \chi}{\partial \lambda}-1\right)}{\left[1+\lambda\left(\chi-1\right)\right]^2}$$

Then

$$\begin{aligned} \epsilon_\gamma &\equiv \frac{\lambda\frac{\partial \gamma}{\partial \lambda}}{\gamma} = \frac{\frac{\partial \chi}{\partial \lambda}\lambda\left[1+\lambda\left(\chi-1\right)\right] - \lambda\chi\left(\chi+\lambda\frac{\partial \chi}{\partial \lambda}-1\right)}{1+\lambda\left(\chi-1\right)}\frac{1}{\chi} \\ &= \epsilon_\chi\left(1-\phi\right) - \phi\left(\frac{\chi-1}{\chi}\right), \end{aligned}$$

where $\epsilon_\chi \equiv \frac{\partial \chi}{\partial \lambda}\frac{\lambda}{\chi}$. We have $\epsilon_\gamma > 0$ if

$$\epsilon_\chi > \frac{\phi}{1-\phi}\left(\frac{\chi-1}{\chi}\right).$$

An alternative way to write this is

$$\epsilon_\chi > \frac{\frac{\chi\lambda}{1+\lambda\chi-\lambda}}{\frac{1-\lambda}{1+\lambda\chi-\lambda}}\left(\frac{\chi-1}{\chi}\right) = \frac{\lambda}{1-\lambda}\left(\chi-1\right) \Leftrightarrow \frac{\epsilon_\chi}{\chi-1} > \frac{\lambda}{1-\lambda}.$$

The conjecture is that we can identify a downward- and an upward-sloping curve such that the left hand side and the right hand side are equal for some unique $\lambda^* \in (1/2, 1)$.

Using $\epsilon_\chi = \frac{\partial \chi}{\partial \lambda}\frac{\lambda}{\chi} = \frac{\partial \chi}{\partial \omega}\frac{\omega}{\chi}\frac{\partial \omega}{\partial \lambda}\frac{\lambda}{\omega}$, we can rewrite the inequality as

$$\frac{\frac{\partial \chi}{\partial \omega}\frac{\omega}{\chi}}{\chi-1} > \frac{\lambda}{1-\lambda}\frac{1}{\frac{\partial \omega}{\partial \lambda}\frac{\lambda}{\omega}}.$$

We prove that the left hand side is downward-sloping in ω and therefore downward-sloping in λ, whereas the right hand side is increasing in λ.

The $\frac{\partial\chi}{\partial\omega}\frac{\omega}{\chi}/\left(\chi-1\right)$ locus. Recall that

$$\frac{\partial\chi}{\partial\omega}=\frac{\eta\beta}{\rho}\frac{\left(\omega^{-\frac{\beta}{\rho}}-\eta\right)+\left(\omega^{\frac{\beta}{\rho}}-\eta\right)}{\omega\left(1-\eta\omega^{\frac{\beta}{\rho}}\right)^{2}}>0.$$

The elasticity of χ in ω is then

$$\frac{\partial\chi}{\partial\omega}\frac{\omega}{\chi}=\frac{\eta\beta}{\rho}\frac{\left(\omega^{-\frac{\beta}{\rho}}-\eta\right)+\left(\omega^{\frac{\beta}{\rho}}-\eta\right)}{\left(1-\eta\omega^{\frac{\beta}{\rho}}\right)\left(1-\eta\omega^{-\frac{\beta}{\rho}}\right)},$$

where

$$\Leftrightarrow\frac{\partial\chi}{\partial\omega}\frac{\omega}{\chi}=\epsilon_{\chi}/\left(\frac{\partial\omega}{\partial\lambda}\frac{\lambda}{\omega}\right)$$

Moreover,

$$\chi-1=\eta\frac{\omega^{\frac{\beta}{\rho}}-\omega^{-\frac{\beta}{\rho}}}{1-\eta\omega^{\frac{\beta}{\rho}}}>0,$$

and therefore

$$\frac{\frac{\partial\chi}{\partial\omega}\frac{\omega}{\chi}}{\chi-1}=\frac{\beta}{\rho}\frac{\left(\omega^{-\frac{\beta}{\rho}}-\eta\right)+\left(\omega^{\frac{\beta}{\rho}}-\eta\right)}{\left(1-\eta\omega^{-\frac{\beta}{\rho}}\right)\left(\omega^{\frac{\beta}{\rho}}-\omega^{-\frac{\beta}{\rho}}\right)}$$

Slope of $\frac{\partial\chi}{\partial\omega}\frac{\omega}{\chi}/\left(\chi-1\right)$ in ω. We conjecture that $\partial\left[\frac{\partial\chi}{\partial\omega}\frac{\omega}{\chi}/\left(\chi-1\right)\right]/\partial\omega<0$. To check this, define

$$\begin{aligned}f\left[\omega\right]&=\omega^{-\frac{\beta}{\rho}}+\omega^{\frac{\beta}{\rho}}-2\eta>0\\ g\left[\omega\right]&=\left(1-\eta\omega^{-\frac{\beta}{\rho}}\right)\left(\omega^{\frac{\beta}{\rho}}-\omega^{-\frac{\beta}{\rho}}\right)\\ &=\omega^{\frac{\beta}{\rho}}-\eta-\omega^{-\frac{\beta}{\rho}}+\eta\omega^{-\frac{2\beta}{\rho}}>0\end{aligned}$$

We have to show that

$$f'\left[\omega\right]g\left[\omega\right]-f\left[\omega\right]g'\left[\omega\right]<0,$$

where

$$\begin{aligned}f'\left[\omega\right]&=-\frac{\beta}{\rho}\omega^{-\frac{\beta}{\rho}-1}+\frac{\beta}{\rho}\omega^{\frac{\beta}{\rho}-1}=\frac{\beta}{\rho}\left(\frac{\omega^{\frac{\beta}{\rho}}-\omega^{-\frac{\beta}{\rho}}}{\omega}\right)>0\\ g'\left[\omega\right]&=\frac{\beta}{\rho}\omega^{\frac{\beta}{\rho}-1}+\frac{\beta}{\rho}\omega^{-\frac{\beta}{\rho}-1}-\frac{2\beta}{\rho}\eta\omega^{-\frac{2\beta}{\rho}-1}=\frac{\beta}{\rho}\left(\frac{\omega^{\frac{\beta}{\rho}}+\omega^{-\frac{\beta}{\rho}}-2\eta\omega^{-\frac{2\beta}{\rho}}}{\omega}\right)>0.\end{aligned}$$

The last inequality follows from $\omega < \eta^{-\frac{\rho}{\beta}}$

$$\begin{aligned} & \omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}} - 2\eta\omega^{-\frac{2\beta}{\rho}} \\ > \ & \omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}} - 2\eta \left(\eta^{-\frac{\rho}{\beta}}\right)^{-\frac{2\beta}{\rho}} \\ = \ & \left(\omega^{\frac{\beta}{\rho}} - \eta^3\right) + \left(\omega^{-\frac{\beta}{\rho}} - \eta^3\right) \end{aligned}$$

We can rewrite the necessary condition as

$$\frac{\beta}{\rho}\frac{1}{\omega}\left(\omega^{\frac{\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}}\right)^2 \left(1 - \eta\omega^{-\frac{\beta}{\rho}}\right) < \frac{\beta}{\rho}\frac{1}{\omega}\left(\omega^{-\frac{\beta}{\rho}} + \omega^{\frac{\beta}{\rho}} - 2\eta\right)\left(\omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}} - 2\eta\omega^{-\frac{2\beta}{\rho}}\right).$$

Since $1 - \eta\omega^{-\frac{\beta}{\rho}} < 1$ and $\omega^{-\frac{2\beta}{\rho}} < 1$, a sufficient condition is

$$\left(\omega^{\frac{\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}}\right)^2 < \left(\omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}} - 2\eta\right)^2 \Longleftrightarrow 0 < \omega^{\frac{\beta}{\rho}} - \eta.$$

The $\frac{\lambda}{1-\lambda}\left(\frac{\partial\omega}{\partial\lambda}\frac{\lambda}{\omega}\right)^{-1}$ locus. The trade balance condition implies

$$TB\left[\lambda, \omega\right] \equiv \frac{\eta\omega^{\frac{\beta}{\rho}} - 1}{\eta - \omega^{\frac{\beta}{\rho}}}\omega^{-\frac{\beta-\rho}{\rho}} - \frac{1-\lambda}{\lambda} = 0.$$

By the implicit function theorem, we have

$$\frac{\partial\omega}{\partial\lambda} = -\frac{\partial TB}{\partial\lambda}\Big/\frac{\partial TB}{\partial\omega},$$

where

$$\frac{\partial TB}{\partial\lambda} = \frac{1}{\lambda^2}$$

and

$$\frac{\partial TB}{\partial\omega} = -\omega^{-\frac{\beta}{\rho}}\left(\frac{\beta}{\rho}\frac{1-\eta^2}{\left(\eta - \omega^{\frac{\beta}{\rho}}\right)^2}\omega^{\frac{\beta}{\rho}} + \frac{\beta-\rho}{\rho}\frac{\eta\omega^{\frac{\beta}{\rho}} - 1}{\eta - \omega^{\frac{\beta}{\rho}}}\right) < 0.$$

Then,

$$\frac{\lambda}{1-\lambda}\left(\frac{\partial\omega}{\partial\lambda}\frac{\lambda}{\omega}\right)^{-1} = -\lambda\frac{\lambda}{1-\lambda}\omega^{-\frac{\beta-\rho}{\rho}}\left(\frac{\beta}{\rho}\frac{1-\eta^2}{\left(\eta - \omega^{\frac{\beta}{\rho}}\right)^2}\omega^{\frac{\beta}{\rho}} + \frac{\beta-\rho}{\rho}\frac{\eta\omega^{\frac{\beta}{\rho}} - 1}{\eta - \omega^{\frac{\beta}{\rho}}}\right).$$

Using the trade balance condition to substitute out $\lambda/\left(1-\lambda\right)$ on the right hand side, we obtain

$$\begin{aligned} \frac{\lambda}{1-\lambda}\left(\frac{\partial\omega}{\partial\lambda}\frac{\lambda}{\omega}\right)^{-1} &= -\lambda\left(\frac{\beta}{\rho}\frac{1-\eta^2}{\left(\eta - \omega^{\frac{\beta}{\rho}}\right)\left(\eta\omega^{\frac{\beta}{\rho}} - 1\right)}\omega^{\frac{\beta}{\rho}} + \frac{\beta-\rho}{\rho}\right) \\ &= \lambda\left(\frac{\beta}{\rho}\frac{1-\eta^2}{\eta^2 + 1 - \eta\left(\omega^{-\frac{\beta}{\rho}} + \omega^{\frac{\beta}{\rho}}\right)} + \frac{\beta-\rho}{\rho}\right). \end{aligned}$$

Slope of the $\frac{\lambda}{1-\lambda}\left(\frac{\partial\omega}{\partial\lambda}\frac{\lambda}{\omega}\right)^{-1}$ locus. It is easy to check that $\frac{\lambda}{1-\lambda}\left(\frac{\partial\omega}{\partial\lambda}\frac{\lambda}{\omega}\right)^{-1}$ rises in λ, given that ω increases in λ. Ignoring

Hence, the downwards-sloping $\frac{\partial\chi}{\partial\omega}\frac{\omega}{\chi}/\left(\chi-1\right)$ locus and the upward-sloping $\frac{\lambda}{1-\lambda}\left(\frac{\partial\omega}{\partial\lambda}\frac{\lambda}{\omega}\right)^{-1}$ locus determine a unique λ^* such that for $\lambda<\lambda^*$ a strong HME occurs.

A.5 Slope of $\phi\left(\lambda\right)$ in symmetric equilibrium

Preliminaries. It is easy to check that $\partial\phi/\partial\lambda=\gamma+\lambda\partial\gamma/\partial\lambda$, where the first term is the direct effect of a country size shock on λ. The second term represents the indirect effect due to adjustments in the relative entry probability. We have already shown that

$$\frac{\partial\gamma}{\partial\lambda}=\frac{\gamma}{\lambda}\left[\epsilon_\chi\left(1-\phi\right)-\phi\left(\frac{\chi-1}{\chi}\right)\right],$$

where ϵ_χ denotes the elasticity of χ in λ. Applying the chain rule, we can rewrite ϵ_χ as $\epsilon_\chi=\frac{\partial\chi}{\partial\omega}\frac{\omega}{\chi}\frac{\partial\omega}{\partial\lambda}\frac{\lambda}{\omega}$. At symmetric equilibrium, we have $\phi=\lambda=1/2$ and $\omega=\chi=\gamma=1$. Hence, we have

$$\frac{\partial\chi}{\partial\omega}=\frac{2\eta\beta}{\rho\left(1-\eta\right)}\text{ and }\frac{\partial\omega}{\partial\lambda}=\frac{1}{\lambda^2}\frac{\rho}{2\frac{\beta}{1-\eta}-\rho}.$$

Moreover, we have

$$\epsilon_\chi=\frac{\partial\gamma}{\partial\lambda}=2\eta\left(1-\frac{\rho\left(1-\eta\right)}{2\beta}\right)^{-1}.$$

Home market effect. At symmetric equilibrium, the slope of the ϕ-curve is given by

$$\frac{\partial\phi}{\partial\lambda}=1+\eta\left(1-\frac{\rho\left(1-\eta\right)}{2\beta}\right)^{-1},$$

which is strictly smaller than the slope of ϕ in λ for the case with the outside sector, which is equal to $1+2\eta$ (see Appendix C) since $\rho\left(1-\eta\right)/\beta<1$.

Home market magnification effect. At the symmetric equilibrium, we have $\frac{\partial^2\phi}{\partial\lambda\partial\eta}=\frac{\partial\gamma}{\partial\eta}+\lambda\frac{\partial^2\gamma}{\partial\lambda\partial\eta}$,where $\frac{\partial\gamma}{\partial\eta}=\frac{\partial\gamma}{\partial\chi}\frac{\partial\chi}{\partial\eta}>0$ and $\frac{\partial^2\gamma}{\partial\lambda\partial\eta}=2\left(1-\frac{\rho}{2\beta}\right)\left(1-\frac{\rho(1-\eta)}{2\beta}\right)^{-2}$. In contrast, the model with an outside sector implies $\partial^2\phi/\partial\lambda\partial\eta=2$.

A.6 Proof of Proposition 2

A.6.1 Freeness of trade

Evaluated at $\omega=1$, the market potential curve takes the value $\chi=1$, which does not depend on η. The market potential curve rotates upwards in response to a freeness of trade shock since

$$\frac{\partial \chi}{\partial \eta} = \frac{\omega^{\frac{\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}}}{\left(1 - \eta \omega^{\frac{\beta}{\rho}}\right)^2} > 0.$$

The locus of the market crowding curve is unaffected. Then, χ has to increase, which raises γ and therefore magnifies the HME. This proves part (a) of proposition (2). The equilibrium value of ω declines in response to a freeness of trade shock.

A.6.2 Productivity dispersion

Evaluated at $\omega = 1$, the market crowding curve yields $\chi = \lambda / (1 - \lambda)$, which does not depend on β. Moreover, we have

$$\frac{\partial \chi}{\partial \beta} = -\frac{2\lambda}{\rho (1 - \lambda)} \omega^{-\frac{2\beta - \rho}{\rho}} \ln [\omega] < 0$$

since $\omega > 1$.

Evaluated at $\omega = 1$, the market potential curve yields $\chi = 1$, which does not depend on β. Moreover, we have

$$\frac{\partial \chi}{\partial \beta} = \frac{\eta}{\rho} \frac{\omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}} - 2\eta}{\left(1 - \eta \omega^{\frac{\beta}{\rho}}\right)^2} \ln [\omega] + \frac{\omega^{\frac{\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}}}{\left(1 - \eta \omega^{\frac{\beta}{\rho}}\right)^2} \frac{\partial \eta}{\partial \beta}$$

where the indirect effect through adjustment in the relative wage is non-negative since $\omega^{\frac{\beta}{\rho}} - \eta > 0$, $\omega^{-\frac{\beta}{\rho}} - \eta > 0$, and $\omega \geq 1$. The direct effect through changes in the freeness of trade is negative since

$$\frac{\partial \eta}{\partial \beta} = -\eta \left(\ln [\tau] + \frac{1}{\sigma - 1} \ln \left[\frac{f^x}{f^d} \right] \right) < 0$$

as $\tau \geq 1$ and $f^x / f^d > 1$.

The sign of $\partial \chi / \partial \beta$ is then given by the sign of

$$\frac{1}{\sigma - 1} \left(\omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}} - 2\eta \right) \ln [\omega] - \left(\omega^{\frac{\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}} \right) \left(\frac{1}{\sigma - 1} \ln \left[\frac{f^x}{f^d} \right] + \ln [\tau] \right)$$

Hence, $\partial \chi / \partial \beta < 0$ requires

$$\left(\omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}} - 2\eta \right) \ln [\omega] < \left(\omega^{\frac{\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}} \right) \left(\ln \left[\frac{f^x}{f^d} \right] + (\sigma - 1) \ln [\tau] \right),$$

Note that the expression on the left hand side and on the right hand side intersect at $\omega = 1$. We conjecture that for $\omega > 1$, the expression on the right hand side rises faster than the expression on the left hand side. Let

$$\begin{aligned}
f[\omega] &\equiv \omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}} - 2\eta, \\
g[\omega] &\equiv \ln[\omega], \\
h[\omega] &\equiv \left(\omega^{\frac{\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}}\right)\left(\ln\left[\frac{f^x}{f^d}\right] + (\sigma-1)\ln[\tau]\right).
\end{aligned}$$

Then,

$$\begin{aligned}
f'[\omega] &= \frac{\beta}{\rho}\frac{\omega^{\frac{\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}}}{\omega} > 0, \\
g'[\omega] &= \frac{1}{\omega} > 0, \\
h'[\omega] &= \frac{\beta}{\rho}\frac{\omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}}}{\omega}\left(\ln\left[\frac{f^x}{f^d}\right] + (\sigma-1)\ln[\tau]\right).
\end{aligned}$$

The slope of the left hand side is given by

$$f'[\omega]\,g[\omega] + f[\omega]\,g'[\omega] = \frac{\beta}{\rho}\frac{\omega^{\frac{\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}}}{\omega}\ln[\omega] + \frac{\omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}} - 2\eta}{\omega}.$$

Hence, we require

$$\frac{\beta}{\rho}\left(\omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}}\right)\left(\ln\left[\frac{f^x}{f^d}\right] + (\sigma-1)\ln[\tau]\right) > \frac{\beta}{\rho}\left(\omega^{\frac{\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}}\right)\ln[\omega] + \omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}} - 2\eta.$$

Noting that $\ln[\omega] < \rho\ln[\tau] + \frac{\beta-(\sigma-1)}{\beta\sigma}\ln\left[\frac{f^x}{f^d}\right]$, a sufficient condition reads

$$\begin{aligned}
&\frac{\beta}{\rho}\left(\omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}}\right)\left(\ln\left[\frac{f^x}{f^d}\right] + (\sigma-1)\ln[\tau]\right) \\
> \;&\frac{\beta}{\rho}\left(\omega^{\frac{\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}}\right)\left(\rho\ln[\tau] + \frac{\beta-(\sigma-1)}{\beta\sigma}\ln\left[\frac{f^x}{f^d}\right]\right) + \omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}} - 2\eta.
\end{aligned}$$

Rearranging terms, we get

$$\begin{aligned}
-2\eta \;<\; &\left[\frac{\beta\sigma-\sigma-1}{\sigma-1}\left(\omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}}\right) - \frac{\beta-(\sigma-1)}{\sigma-1}\left(\omega^{\frac{\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}}\right)\right]\ln\left[\frac{f^x}{f^d}\right] \\
&+ \left[(\sigma-1)\,\omega^{\frac{\beta}{\rho}} + (\sigma+1)\,\omega^{-\frac{\beta}{\rho}}\right]\beta\ln[\tau].
\end{aligned}$$

Collecting terms, we obtain

$$\left[\beta\omega^{\frac{\beta}{\rho}} + (\sigma+1)\frac{\beta - \frac{2}{\sigma+1}(\sigma-1)}{\sigma-1}\omega^{-\frac{\beta}{\rho}}\right]\ln\left[\frac{f^x}{f^d}\right] + \left[(\sigma-1)\,\omega^{\frac{\beta}{\rho}} + (\sigma+1)\,\omega^{-\frac{\beta}{\rho}}\right]\beta\ln[\tau] > -2\eta,$$

where the inequality holds since we have only positive terms on the left hand side (remember that $\sigma > 1$ and $\beta > \sigma - 1$) and a negative term on the right side.

Hence, χ rises in response to an drop in β (higher dispersion), which raises γ and magnifies the HME. This proves part (b) of proposition 2.

A.7 Proof of Proposition 4

A.7.1 Weak HME

Under unconventional sorting, the trade balance condition (8) remains unaffected. Hence, the positive relationship between the labor share and the relative wage continues to hold.

The relative entry probability χ is given by

$$\chi = \frac{1 - G\left[\varphi_{HH}^*\right]}{1 - G\left[\varphi_{FH}^*\right]} = \left(\frac{\varphi_{FH}^*}{\varphi_{HH}^*}\right)^\beta .$$

The two zero cutoff profit conditions from targeting Home imply

$$\chi = \tau^\beta \left(\frac{f^x}{f^d}\right)^{\frac{\beta}{\sigma-1}} \omega^{-\frac{\beta}{\rho}} = \frac{f^x}{f^d}\eta^{-1}\omega^{-\frac{\beta}{\rho}}, \tag{29}$$

which constitutes our market crowding curve.

The trade balance condition (8) can be rewritten as

$$\frac{\lambda}{1-\lambda} = \omega^{\frac{\beta-\rho}{\rho}} \frac{\omega^{\beta/\rho} - \eta}{1 - \eta\omega^{\beta/\rho}}.$$

We use equation (29) to substitute out $\omega^{\frac{\beta}{\rho}}$ in the denominator and solve for χ, which yields a market potential curve

$$\chi = \frac{f^x}{f^d} \frac{1}{1 - \frac{1-\lambda}{\lambda}\omega^{\frac{\beta-\rho}{\rho}}\left(\omega^{\beta/\rho} - \eta\right)}$$

The market potential curve is strictly increasing in ω.

We have $\chi > 1$ since

$$1 - \frac{1-\lambda}{\lambda}\omega^{\frac{\beta-\rho}{\rho}}\left(\omega^{\beta/\rho} - \eta\right) < 1.$$

Together with equation (12), this proves the existence of a (weak) HME under unconventional sorting.

A.7.2 Home Market Magnification Effect

Lower variable trade cost shift the down both the market crowding and the market potential curves. Lower export fixed cost additionally shift both curves proportionally. Hence, a higher freeness of trade lowers the relative entry probability under unconventional sorting, which implies that the home market effect is dampened under unconventional sorting.

A.7.3 Welfare differential

Using equations (26) and (28), we can rewrite relative welfare as

$$\frac{W_H}{W_F} = \left(\frac{\lambda}{1-\lambda}\right)^{\frac{\beta-(\sigma-1)}{\beta(\sigma-1)}} \omega^{\frac{2\beta-\rho}{\beta\rho}} > 1.$$

This is a general expression which holds under conventional and unconventional sorting. ω is increasing in λ also under unconventional sorting. The inequality follows from $\lambda > 1/2$, $\omega > 1$, $\beta > (\sigma-1)$, and $2\beta > \rho$. While under conventional sorting an increase in the freeness of trade reduces ω and therefore the welfare differential, it is unclear whether trade liberalization leads to real per capita income convergence under unconventional sorting because the effect on ω is ambiguous.

A.8 Proof of Proposition 5

We first augment the model by differences in the lower bound of the productivity distribution b_i. This leaves the zero cutoff profit conditions unchanged, but affects various equations of the model. The relative entry probability is given by

$$\chi \equiv \frac{1-G_H\left[\varphi^*_{HH}\right]}{1-G_F\left[\varphi^*_{FF}\right]} = B\left(\frac{\varphi^*_{FF}}{\varphi^*_{HH}}\right)^{\beta},$$

where $B \equiv (b_H/b_F)^{\beta}$. The export participation rate now reads

$$m_{ij} = \left(\frac{b_i}{b_i}\right)^{\beta}\left(\frac{\varphi_{ii}}{\varphi_{ij}}\right)^{\beta}.$$

The balanced trade condition becomes

$$\omega B\frac{\lambda}{1-\lambda} = \left(\frac{\varphi^*_{FH}}{\varphi^*_{HF}}\right)^{-\beta}.$$

This has an important implication for the market crowding curve, which now reads

$$\chi = B^2\frac{\lambda}{1-\lambda}\omega^{-\frac{2\beta-\rho}{\rho}}.$$

Relative free entry reads

$$\chi = \frac{f^d + \left(\frac{\varphi^*_{FF}}{\varphi^*_{FH}}\right)^{\beta} f^x}{f^d + \left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^{\beta} f^x},$$

where the b_i disappeared because it is part of χ.

Using the zero cutoff profit conditions and balanced trade, we obtain

$$\chi = \frac{1+\omega^{-\frac{\beta-\rho}{\rho}}B\frac{\lambda}{1-\lambda}\eta}{1+\omega^{\frac{\beta-\rho}{\rho}}B^{-1}\left(\frac{\lambda}{1-\lambda}\right)^{-1}\eta}.$$

Using the market crowding curve to substitute out $B^2\frac{\lambda}{1-\lambda}\omega^{-\frac{2\beta-\rho}{\rho}}$, we obtain the market potential curve

$$\chi = B\frac{1-B\omega^{-\frac{\beta}{\rho}}\eta}{B-\omega^{\frac{\beta}{\rho}}\eta}.$$

The market crowding curve is downward sloping in ω. Evaluated at $\omega = 1$, we have $\chi = B^2\frac{\lambda}{1-\lambda}$, which is the upper bound for χ. The market potential curve is upward sloping since

$$\frac{\partial\chi}{\partial\omega} = B\frac{\beta\eta\omega^{-\frac{\beta+\rho}{\rho}}\left(B^2+\omega^{\frac{2\beta}{\rho}}-2B\eta\omega^{\frac{\beta}{\rho}}\right)}{\rho\left(B-\eta\omega^{\frac{\beta}{\rho}}\right)^2} > 0,$$

where the inequality follows from rewriting the terms in brackets in the nominator as

$$B^2 - 2B\omega^{\frac{\beta}{\rho}} + \omega^{\frac{2\beta}{\rho}} + 2B\omega^{\frac{\beta}{\rho}}(1-\eta) = \left(B-\omega^{\frac{\beta}{\rho}}\right)^2 + 2B\omega^{\frac{\beta}{\rho}}(1-\eta) > 0.$$

Note that the market potential curve features an asymptote at $\omega = \left(\frac{B}{\eta}\right)^{\frac{\rho}{\beta}}$. We consider the range $\omega \in [1, \left(\frac{B}{\eta}\right)^{\frac{\rho}{\beta}})$. In the supported range, the denominator is always positive. The sign of the numerator is unclear as $B > 1, \eta < 1, \omega > 1$.

Home's share of firms is given by

$$\phi = \left(1+\frac{M_F}{M_H}\right)^{-1} = \left(1+\frac{b_F^{\beta}L_F\left(\varphi_{FF}^*\right)^{-\beta}}{b_H^{\beta}L_H\left(\varphi_{HH}^*\right)^{-\beta}}\right)^{-1} = \left(1+\frac{1-\lambda}{\lambda}\frac{1}{\chi}\right)^{-1}$$

while Home's share of demand is

$$\left(1+\frac{1-\lambda}{\lambda}\frac{1}{\omega}\right)^{-1}.$$

A weak HME in demand shares occurs if $\chi > \omega$.

A necessary condition for $\chi > \omega$ is that the value of the market potential curve exceeds the value of $\chi = B^2\frac{\lambda}{1-\lambda}\omega^{-\frac{2\beta-\rho}{\rho}}$ for $\chi = \omega$, which is given by

$$\tilde{\omega} = B^2\frac{\lambda}{1-\lambda}\tilde{\omega}^{-\frac{2\beta-\rho}{\rho}} \Leftrightarrow \tilde{\omega}^{\frac{2\beta}{\rho}} = B^2\frac{\lambda}{1-\lambda} \Leftrightarrow \tilde{\omega} = B^{\frac{\rho}{\beta}}\left(\frac{\lambda}{1-\lambda}\right)^{\frac{\rho}{2\beta}}.$$

Hence, we require

$$\chi[\tilde{\omega}] > \tilde{\omega} \Leftrightarrow \chi[\tilde{\omega}] = B\frac{1-B\tilde{\omega}^{-\frac{\beta}{\rho}}\eta}{B-\tilde{\omega}^{\frac{\beta}{\rho}}\eta} > \tilde{\omega}.$$

By rearranging terms, we have

$$B^2\left(\left(\frac{\lambda}{1-\lambda}\right)^{\frac{\beta+\rho}{2\beta}}\eta^{-1}B^{\frac{\rho}{\beta}} - \left(\frac{\lambda}{1-\lambda}\right)^{\frac{1}{2}}\eta^{-1}+1\right) < \tilde{\omega}^{\frac{2\beta+\rho}{\rho}}.$$

Inserting the expression for $\tilde{\omega}$ and solving for η, we obtain

$$\eta > \ell^{\frac{1}{2}} \frac{B^{\frac{\rho}{\beta}} \ell^{\frac{\rho}{2\beta}} - 1}{B^{\frac{\rho}{\beta}} \ell^{\frac{2\beta+\rho}{2\beta}} - 1},$$

where $\ell \equiv \lambda / (1-\lambda)$ with $\ell \geq 1$.

In the presence of symmetric country sizes, i.e. ℓ, the necessary condition for a weak HME in demand shares reduces to

$$\eta > 1.$$

This condition can never hold. To the contrary, we always have $\eta < 1$. Hence, if countries differ only in the lower productivity bound, the richer country hosts an underproportional share of firms. This proves part (b) of our proposition.

We now prove that the weak HME in demand shares arises under a mild condition if countries only differ in their country size. With $B = 1$, the necessary condition reads

$$\eta > \ell^{\frac{1}{2}} \frac{\ell^{\frac{\rho}{2\beta}} - 1}{\ell^{\frac{2\beta+\rho}{2\beta}} - 1}.$$

If ℓ increases, the numerator increases, but the denominator increases faster, such that the term on the right hand side declines. Then, a sufficient condition for a weak HME in demand shares is

$$\eta > \lim_{\ell \to 1} \ell^{\frac{1}{2}} \frac{\ell^{\frac{\rho}{2\beta}} - 1}{\ell^{\frac{2\beta+\rho}{2\beta}} - 1}.$$

Employing l'Hospital's rule, we find

$$\lim_{\ell \to 1} \ell^{\frac{1}{2}} \frac{\ell^{\frac{\rho}{2\beta}} - 1}{\ell^{\frac{2\beta+\rho}{2\beta}} - 1} = \lim_{\ell \to 1} \frac{\frac{\rho}{2\beta} \ell^{-\frac{2\beta-\rho}{2\beta}}}{\frac{2\beta+\rho}{2\beta} \ell^{\frac{\rho}{2\beta}}} = \frac{\rho}{2\beta + \rho}.$$

A.9 Proof of Lemma 1'

As proposed by Demidova and Rodriguez-Clare (2011), we reduce the model's equilibrium conditions (2), (4) and (6) to a system of two equations in ω and φ^*_{HF}, The first curve draws on zero cutoff profit and free entry conditions and is independent of the productivity distribution. The relative cutoff profit conditions for targeting Foreign is given by

$$\varphi^*_{HF} = \tau \left(\frac{f^x}{f^d} \right)^{\frac{1}{\sigma-1}} \omega^{\frac{1}{\rho}} \varphi^*_{FF}. \tag{30}$$

Foreign's domestic entry cutoff φ^*_{FF} is a function of its export cutoff φ^*_{FH} by free entry. Moreover, using the relative cutoff profit conditions for entry into Home, φ^*_{FH} can be expressed as a function of the relative ω and φ^*_{HH}. The latter is a function of φ^*_{HF} by free entry. Hence, our first equilibrium condition constitutes an upward sloping and concave relationship between the relative wage and Home's export cutoff. It is important to note that the locus of this curve is not affected by country size.

Under a general productivity distribution, the balanced trade condition can be rewritten as

$$\sigma\left(\varphi_{HH}^{*}\right)^{-\rho}\left(\psi_{H}+1\right)=\frac{\lambda}{1-\lambda}\tau^{\rho}\left(\frac{f^{x}}{f^{d}}\right)^{\frac{1}{\sigma}}\left(\varphi_{FH}^{*}\right)^{-\rho}\left(\psi_{F}+1\right), \tag{31}$$

where

$$\psi_{i}=\frac{f^{d}}{f^{x}}\left(\frac{\varphi_{ij}^{*}}{\varphi_{ii}^{*}}\right)^{\sigma-1}\frac{\int_{\varphi_{ii}^{*}}\varphi^{\sigma-1}dG_{i}\left[\varphi\right]}{\int_{\varphi_{ij}^{*}}\varphi^{\sigma-1}dG_{i}\left[\varphi\right]}.$$

The left hand side of equation (31) is independent of ω. Moreover, it rises in φ_{HF}^{*}. The right hand side can be expressed as a function of φ_{FH}^{*}. φ_{FH}^{*} has to rise in φ_{HF}^{*}. By free entry, φ_{HH}^{*} falls. Relative entry in Home reads

$$\varphi_{FH}^{*}=\tau\left(\frac{f^{x}}{f^{d}}\right)^{\frac{1}{\sigma-1}}\omega^{-\frac{1}{\rho}}\varphi_{HH}^{*}. \tag{32}$$

Since φ_{FH}^{*} rises and φ_{HH}^{*} falls, ω must fall to restore equilibrium. Hence, trade balance establishes a negative relationship between Home's export cutoff and the relative wage.

Consider a country size shock. For a given relative wage, the right hand side of the trade balance curve must be larger, which can only come about by an increase in φ_{HF}^{*}. Hence, a country size shock shifts the trade balance locus upwards. We also conclude that the larger country pays a higher wage, which extends Lemma 1 to the general case.

A.10 Proof of Proposition 2(a)'

Using the free entry conditions, we can write the ratio of active firms as

$$\frac{M_{H}}{M_{F}}=\frac{\lambda}{1-\lambda}\frac{f^{e}/p_{F}^{in}+f^{d}+f^{x}m_{FH}}{f^{e}/p_{H}^{in}+f^{d}+f^{x}m_{HF}}. \tag{33}$$

We have argued in the proof of Lemma 1' that a country size shock raises Home's export cutoff φ_{HF}^{*}. By free entry, Home's domestic entry cutoff falls. Hence, the denominator of the above expression falls. It follows from equation (32) that Foreign's export cutoff φ_{FH}^{*} falls since φ_{HH}^{*} falls and ω rises. By free entry, φ_{FF}^{*} rises. Then, the numerator of the above equation rises, which implies that the effect of the country size shock on the relative mass of firms is magnified.

It is easy to check that $M_{H}/M_{F}=\phi/\left(1-\phi\right)>\lambda/\left(1-\lambda\right)$ directly translates into $\phi>\lambda$, which constitutes a weak home market effect.

B Calibration

We use standard results from the literature to calibrate the model at symmetric equilibrium.[29] An important source of information is Bernard et al. (2003). They argue that $\sigma = 3.8$ fits their data well and report that the standard deviation of domestic US plant sales is 0.84. In terms of the Melitz (2003) model, this value has to equal $(\sigma - 1)/\beta$. With the estimate for σ at hand, we obtain $\beta = 3.3$, which meets the restriction $\beta > \sigma - 1$. Moreover, it is close to estimates from other sources.[30] According to Bernard et al. (2003), the export participation rate of US firms is about 21%.[31] Using $\tau = 1.3$, which we take from Obstfeld and Rogoff (2001), and the corresponding values for σ and β, the implied relative export fixed costs amount to $f^x/f^d = 1.8$.

Under this parameter constellation, the freeness of trade is given by $\eta \approx 0.38$. The implied conventional sorting amounts to $\bar{\lambda} \approx 0.88$. Since it is only implicitly defined, we have to back out λ^* from our simulations. The market potential curve is convex, and a weak HME in demand shares arises.

In the Krugman (1980) model, the share of expenditure on domestic varieties at symmetric equilibrium is given by $\left(1 + \tilde{\tau}^{1-\tilde{\sigma}}\right)^{-1}$. In the Krugman specification, we choose $\tilde{\sigma}$ such that observed trade elasticities are the same, i.e. $-\beta = 1 - \tilde{\sigma} \Leftrightarrow \tilde{\sigma} = 1 + \beta$. Moreover, we set $\tilde{\tau}$ such that trade openness is the same in symmetric equilibrium, i.e.

$$\tau^{-\beta} \left(\frac{f^x}{f^d}\right)^{1-\frac{\beta}{\sigma-1}} = \tilde{\tau}^{1-\tilde{\sigma}} \Leftrightarrow \tilde{\tau} = \tau \left(\frac{f^x}{f^d}\right)^{\frac{\beta-(\sigma-1)}{(\sigma-1)\beta}}$$

Our liberalization scenario considers the elimination of variable trade costs in the Melitz specification. In the Krugman specification, variable trade costs $\tilde{\tau}$ are computed according to the formula given above. Table 1 summarizes the parameter values that we use in the two specifications.

Table 1: Parametrization

Parameter	Melitz	Krugman
σ	3.8	4.3
β	3.3	
f^x/f^d	1.8	
τ	1.3	1.34
$\tau_{\text{liberalization}}$	1.0	1.03

[29] We follow Demidova (2008).

[30] Mayer and Ottaviano (2007) report that the shape parameters for total manufacturing are 3.03 and 2.55 for Italy and France, respectively.

[31] In Europe, the UK features a similar export participation rate (28%), but German and French export participation rates are higher; see Mayer and Ottaviano (2007).

C Melitz (2003) with outside sector

In this appendix, we derive equilibrium in the presence of an outside sector. We show that the economy exhibits a weak and a strong home market effect. As in Helpman and Krugman (1995), the home market effect is linear in λ. Moreover, we discuss welfare implications.

Basic environment. The model is augmented by a homogeneous good produced under constant returns to scale and perfect competition. Utility takes the Cobb-Douglas form, where μ denotes the share of expenditure spent on differentiated varieties. The outside good is freely tradable. Hence, wages are equalized and henceforth normalized to unity. Welfare per worker is given by the inverse of the *aggregate* price index. Using P_i to denote the price index of the differentiated goods sector and defining $\tilde{\mu} \equiv (1-\mu)^{1-\mu} \mu^{\mu}$, we can write welfare per worker as

$$W_i = \tilde{\mu}/P_i^{\mu}.$$

Equilibrium. The free entry conditions are unaffected. In the *zero cutoff profit conditions*, however, wages drop. In relative form, the zero cutoff profit condition becomes

$$\varphi_{ji}^* = \tau \left(\frac{f^x}{f^d}\right)^{\frac{1}{\sigma-1}} \varphi_{ii}^*.$$

Hence, we can substitute out export cutoffs from the *free entry conditions*. We obtain two equations in two unknowns, which can be used to solve for the domestic entry cutoffs as

$$(\varphi_{ii}^*)^{\beta} = (\theta-1)\frac{\left(f^d\right)^{\beta}}{f^e}(1+\eta).$$

It is important to note that the cutoff productivity levels do not depend on country size. Hence, they are symmetric across countries irrespectively of the country size distribution.

Labor market clearing implies

$$\xi_i L_i = M_i^e f^e + M_i \sum_j m_{ij} f_{ij} + M_i \sum_j \int_{\varphi_{ij}^*} \frac{\tau_{ij} q_{ij}[\varphi]}{\varphi} \frac{dG[\varphi]}{1-G\left[\varphi_{ij}^*\right]} = M_i \theta \sum_j m_{ij} f_{ij},$$

where ξ_i denotes the fraction of workers employed in the differentiated good sector.

Using the free entry condition, we obtain

$$M_i = \frac{\xi_i L_i}{\bar{r}_i};$$

where $\bar{r}_i \equiv \sum_j \bar{r}_{ij} = \frac{\beta f^e}{\rho} (\varphi_{ii}^*)^{\beta}$. Expected revenues $\bar{r}_{ij}$ are given by

$$\bar{r}_{ij} = \int_{\varphi_{ij}^*} r_{ij}[\varphi] \frac{dG[\varphi]}{1-G\left[\varphi_{ij}^*\right]} = \theta\sigma m_{ij} f_{ij}.$$

Due to the symmetry of the cutoffs, we henceforth suppress the subscripts of the revenue terms. Note that $\bar{r}^x / \bar{r}^d$ reduces to η.

Balanced trade is given by

$$M_i \bar{r}^x = M_j \bar{r}^x + (1 - \mu) L_i - (1 - \xi_i) L_i$$

where the term on the left hand side represents country i's exports of the differentiated good. The first term on the right hand side represents i's imports of the differentiated good. The remaining terms reflect i's imports of the homogeneous good (spending on the homogeneous good minus value of domestic homogeneous good production).

Substituting out M_i and $\bar{r}_{ij}$ from balanced trade and using $\bar{r}_i = \bar{r}_j$, we obtain

$$\xi_i = \mu \frac{\bar{r}}{\bar{r}^d} - \xi_j \frac{L_j}{L_i} \frac{\bar{r}^x}{\bar{r}^d}, i \in \{H, F\}.$$

Solving the system of two equations in ξ_i and ξ_j, we obtain

$$\xi_i = \mu \frac{\frac{\bar{r}}{\bar{r}^d} - \frac{L_j}{L_i} \frac{\bar{r}^x}{\bar{r}^d} \frac{\bar{r}}{\bar{r}^d}}{1 - \left(\frac{\bar{r}^x}{\bar{r}^d}\right)^2}, i \in \{H, F\},$$

Moreover, we have

$$\xi_H > \xi_F \Leftrightarrow \lambda > 1/2.$$

Hence, the smaller country is a net exporter of the homogeneous good.

Home market effect. The labor clearing conditions implies that Home's share of firms active in the differentiated good sector can be written as

$$\phi = \frac{1}{\lambda + (1 - \lambda) \frac{\xi_F}{\xi_H}} \lambda.$$

Note that entry cutoffs have dropped due to their symmetry. We have seen above that $\xi_H > \xi_F$, which constitutes a weak home market effect.

We can rewrite ϕ as

$$\phi = \frac{1}{1 + \frac{1-\lambda}{\lambda} \frac{\bar{r} - \frac{\lambda}{1-\lambda} \bar{r}^x}{\bar{r} - \frac{1-\lambda}{\lambda}^x}} = \frac{\lambda \bar{r} - (1 - \lambda) \bar{r}^x}{\bar{r}^d}$$

with

$$\frac{\partial \phi}{\partial \lambda} = 1 + 2 \frac{\bar{r}^x}{\bar{r}^d} = 1 + 2\eta > 1$$

The following observations stand out. First, the weak HME is linear in λ. Second, there exists a strong HME. Third, the HME is magnified by a reduction in trade barriers (HMME).

Welfare per capita. Using the domestic zero cutoff profit condition, we can rewrite welfare per worker as

$$W_i = \tilde{\mu} \left(\frac{\beta L_i}{\sigma f^d} \right)^{\frac{\mu}{\sigma-1}} (\rho \varphi_{ii}^*)^{\mu} ,$$

where $\tilde{\mu} \equiv (1-\mu)^{1-\mu} \mu^{\mu}$ is a constant. The following observations stand out. First, a shock on the relative country size λ affects both countries symmetrically. The reason that domestic entry cutoffs are independent of the country size distribution. Second, an increase in freeness of trade rises the domestic entry cutoff and therefore increases welfare per worker in both countries. Third, an increase in the freeness of trade leaves relative welfare per worker unaffected.