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Voting as a Signaling Device

R. Emre Aytimur
Aristotelis Boukouras
Robert Schwager

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Abstract

In this paper, citizens vote in order to influence the election outcome and in order to signal their unobserved characteristics to others. The model is one of rational voting and generates the following predictions: (i) The paradox of not voting does not arise, because the benefit of voting does not vanish with population size. (ii) Turnout in elections is positively related to the size of the local community and the importance of social interactions. (iii) Voting may exhibit bandwagon effects and small changes in the electoral incentives may generate large changes in turnout due to signaling effects. (iv) Signaling incentives increase the sensitivity of turnout to voting incentives in communities with low opportunity cost of social interaction, while the opposite is true for communities with high cost of social interaction. Therefore, the model predicts that smaller communities have more volatile turnout than larger communities.

JEL-Code: C700, D720, D800.

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R. Emre Aytimur
Chair of Public Economics
Georg-August-University
Göttingen / Germany
raytimu@gwdg.de

Aristotelis Boukouras
Courant Research Centre Poverty, Equity and Growth, Georg-August-University
Göttingen / Germany
aboukou@gwdg.de

Robert Schwager
Chair of Public Economics
Georg-August-University
Platz der Göttinger Sieben 3
Germany – 37073 Göttingen
rschwag@uni-goettingen.de

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1 Introduction

What motivates citizens to vote is one of the fundamental questions of political science and public economics. Since the early writings of Downs (1957) and later on Ledyard (1984), the rational-choice theory puts the desire of citizens to affect the election outcome as the main driving factor of their voting behavior. But, since the probability to actually change the outcome is very small, the instrumental view of voting generates the **paradox of not voting**[1], which has led many researchers to propose different reasons that drive voting incentives.

The purpose of this paper is to provide a formal model of voting as a signaling device, and, in doing so, to provide a rational choice model which does not generate the paradox of not voting. The main idea is that citizens possess unobserved characteristics, such as their preferences for public goods or their degree of altruism, which they signal to others through voting. If informative, these signals benefit both the sender and the receiver, because they facilitate the creation of mutually beneficial cooperations or because they increase the trust in an already given relation. Examples of cooperations are exchanging information about job opportunities, helping each other to take care of daily issues, etc.

More specifically, we consider a two-period extension of Börgers (2004) model in a finite-agent economy, which is divided into neighborhoods. In the first period citizens decide to vote or not and they also observe whether their neighbors voted. In the second period, after mutual agreement, each citizen can form partnerships with any of her neighbors[2]. Citizens derive utility from both the outcome of the election, as in the instrumental view, and the formation of partnerships in the second stage. Their utilities, however, depend on two unobservable characteristics: (i) their cost of voting, (ii) a preference parameter, the latter denoting the utility from both the election outcome and the partnership. The parameter can be interpreted as either representing the intensity of preferences for public goods (both global and local) or as representing the degree of one's altruism. The crucial assumption is that the utility of the election outcome is correlated with the utility of the partnership (the assumption that this correlation is perfect in our model is not so important and can be relaxed). Because it is costly to form partnerships, a citizen is willing to cooperate with her neighbor only if the

[1]For a formal treatment, see Palfrey and Rosenthal (1985).

[2]An alternative interpretation of the second stage is that each citizen has already a network of friends and each one of them decides whether to increase the degree of interaction with her friends or not.

latter has a high intensity of preferences for public goods. As a result, citizens' voting incentives are enhanced by their willingness to signal their preferences for cooperation to their neighbors.

We find the perfect Bayesian equilibria of the game and we analyze the most interesting case: stable interior equilibria with signaling, that is stable equilibria where a fraction of agents from every type votes. We show that such equilibria exist and we compute their comparative statics. The main results are as follows:

1. The presence of signaling strictly increases voting incentives and electoral turnout when compared to models without signaling effects, like Börgers (2004). This is a direct implication of the value of signaling and the utility that citizens receive from social interactions.

2. Even in economies with very large populations, the value of signaling does not tend to zero and therefore the paradox of not voting does not arise (or more precisely the set of parameter values, according to which non-voting equilibria exist, shrinks).

3. Communities with closer personal ties and higher level of social interaction present higher turnout.

4. Due to signaling, electoral incentives may exhibit "bandwagon" effects: the benefit of voting may increase with turnout, so that one's willingness to vote increases if the expected participation rate increases. To the best of our knowledge, this is in contrast to existing papers on rational voting, where the benefit of voting is always decreasing with turnout due to the decreasing pivotal probability.

5. Signaling incentives interact with direct electoral incentives so that even a small change in the importance of the election may generate a sizable increase in turnout. This is because turnout may be highly sensitive to signaling effects. In particular, in countries with low cost of social interaction (low opportunity costs of time, bad substitutes to social interaction), the presence of signaling increases the sensitivity of turnout to electoral incentives. On the other hand, in countries with high cost of social interaction, the presence of signaling decreases the sensitivity of turnout to electoral incentives. In terms of empirical predictions, the model suggests that communities with high cost of social interaction should have *lower* volatility of turnout in response to changes in the importance

of elections than communities with lower costs of social interactions. In terms of policy, the model predicts that increasing the value of the election (through increasing the awareness of citizens about the policy agenda or through political advertising) has a higher impact on electoral turnout in communities with lower interaction costs and closer community ties.

The model captures in a simple way the interaction between electoral and social incentives, which we believe is an important driving force of voting incentives. A growing number of empirical papers (Gerber, Green, and Larimer (2008), Funk (2010), see also below) show that social considerations and pressures play an important role in citizens' voting decisions. Also our study is motivated by common experience and intuition. Neighbors often cooperate to provide local public goods, like taking kids to school or taking care of communal spaces, while friends and colleagues engage in mutually beneficial interactions, like information sharing and undertaking of small favors. Signaling one's good will, trustworthiness, or interest in joint undertakings can thus have significant value when compared with relatively low cost activities, like voting. Our model formalizes this intuition and shows that the effects on direct electoral incentives can actually be large.

There are many strands of the literature which are related to our paper. Overbye (1995), Posner (1998) and Bufacchi (2001) also argue that reputation and signaling reasons can account for the voting behavior of citizens in modern democracies, but they provide no formal analysis. By constructing a rigorous model formalizing this idea, we are able to make testable predictions which relate the voting behavior to the social conditions of individuals.

Also, Funk (2005) analyzes a voting model with signaling incentives. However, there are two main differences between her paper and ours. First, in her model voting takes place in order to signal one's willingness to comply with social norms, while in our model the signaling concerns one's ability to cooperate in mutually beneficial interactions. Second, the main focus of our analysis is the interaction between electoral and signaling incentives, while Funk (2005) ignores electoral incentives and focuses on the impact of new technologies, which reduce the cost of voting, to signaling incentives and turnout.

Other papers, such as Edlin, Gelman, and Kaplan (2007), Fowler (2006) and Rotemberg (2009), argue that social preferences and altruism are the main driving forces of voting behavior. While our model does not focus on this explanation, one of the interpretations of the citizens' unobserved parameter is that it represents social preferences. However, this parameter generates two voting effects in our model: one direct and one indirect, through signaling. The second channel, which is our main focus of study, is absent from the social preferences literature.

There exist several other theoretical approaches to voting incentives. According to the *ethical voting* literature (Harsanyi (1980), Coate and Conlin (2004), Feddersen and Sandroni (2006)) voters decide on the ground of moral principles and they derive utility from adhering to them. The *leader-follower* theories (Uhlaner (1989), Morton (1991), Shachar and Nalebuff (1999), Herrera and Martinelli (2006)) emphasize the role of leaders and their ability to impose sanctions or to provide rewards in motivating social groups to participate in elections. Castanheira (2003) argue that voting benefit can be high, since the implemented platform after the elections depends not only on the winner, but also on the margin of victory. Papers on *expressive voting* (Brennan and Hamlin (1998), Engelen (2006)) assert that voting is a consumption good in itself, because it allows individuals to affirm their own beliefs and values. Contributions to the literature on *social norms* (e.g., Coleman (1990)) point out that voting is a public good in itself and show how social norms are used to overcome the associated free-rider problem.[3]

We do not question the relevance of these approaches. Rather, the theory presented here provides an additional rationale for voting, which may complement the arguments put forward in existing literature, and which has not been analyzed so far.

Finally, our model generates predictions which are consistent with empirical and experimental results. An increasing number of papers finds that social pressure, close community ties and voter participation increase the voting incentives for community members. Gerber, Green, and Larimer (2008) show as a result of a large-scale field experiment that turnout was substantially higher among people who received a letter before the elections, which was explaining that whether they voted or not would be made public among the neighbors. Funk (2010) finds that voter turnout was negatively affected in small communities of Switzerland after the introduction of postal voting. Her explanation is that although postal voting decreased the voting costs, it also removed signaling benefit of voting, which was substantial in small communities. Gerber and Rogers (2009) find that a message publicizing high expected turnout is more effective at motivating people to vote than a message publicizing low expected turnout. This

[3]For more complete surveys, see Aldrich (1993), Blais (2000), Dhillon and Peralta (2002), Feddersen (2004).

result in spite of lower pivotal probability with higher turnout is consistent with the signaling benefit and the bandwagon effect of our model. Similarly, an experiment of sequential voting by Großer and Schram (2006) shows that high turnout of early voters increases late voters' turnout.[4]

The paper proceeds as follows. Section two presents the model, section three provides the equilibrium analysis, section four presents the main comparative statics and results and section five includes the final comments and conclusions. Most proofs are relegated to the appendix.

2 The Model

There are N individuals, $i = 1, 2, ..., N$, and two political parties, A and B. Each individual is summarized by three characteristics. The first one is the preferred party of the individual i: $R_i \in \{A, B\}$. The second one is her cost of voting, $c_i \in [c_{min}, c_{max}]$ with $0 \leq c_{min} < c_{max}$. The last characteristic is whether she is of high or low type, $\tau(i) \in \{H, L\}$, which refers to the importance the individual i attaches to decisions taken in the public domain. Each characteristic of any individual i is a random variable. All the three characteristics are stochastically independent of each other and between individuals. The preferred party of any individual i is A with probability $1/2$ and B with probability $1/2$. The cost of voting c_i of any individual i is distributed according to the cdf F on the support $[c_{min}, c_{max}]$ with the pdf f which is positive on all of the support. Finally, any individual is of high type, $\tau(i) = H$, with probability q and of low type, $\tau(i) = L$, with probability $1 - q$. Each individual privately knows her characteristics. The distributions of individuals' characteristics are common knowledge.

There are two periods. In the first period, the election occurs in which an individual chooses to vote for her preferred party or to abstain.[5] The winner is determined by a simple majority rule. In case of a tie, each party wins with probability $1/2$.

An individual i's payoff from the first period is as follows: Her benefit is $w_1 \alpha_{\tau(i)}$ if her preferred party wins and 0 otherwise. w_1 is a parameter which measures the importance of the election. We assume that both types care about the result of the election, as measured by the parameter $\alpha_{\tau(i)}$, and that a high type individual cares more about it than a low type individual, i.e. $\alpha_H > \alpha_L > 0$. Her cost is c_i if she votes and 0

[4]For other papers which study the relation between social interactions and political participation, see for instance Schlozman, Verba, and Brady (1995) and Schram and Sonnemans (1996).

[5]Since voting for the other party is a weakly dominated strategy, we do not consider this strategy.

otherwise. Hence, if she votes and her preferred party wins, her payoff is $w_1 \alpha_{\tau(i)} - c_i$. If she abstains and her preferred party wins, her payoff is $w_1 \alpha_{\tau(i)}$. If she votes and her preferred party loses, her payoff is $-c_i$. If she abstains and her preferred party loses, her payoff is 0.

In the second period, social interactions occur in neighborhoods composed of n individuals in the form of pairwise matches. After observing whether each one of her neighbors voted or not, an individual i chooses to match or not with each individual $j = 1, 2, ..., n$, $j \neq i$. If both i and j agree to match with each other, they match together. Otherwise, a match does not occur.

An individual i's payoff from a match with an individual j depends both on her own type and her neighbor's type and we adopt the following simple interaction payoff:

$$w_2 \alpha_{\tau(i)} (\alpha_{\tau(j)} - d) \tag{1}$$

where d is the matching cost and w_2 measures the importance of social interactions. Equation (1) provides i's payoff from a match with j, when j's type is known to i. However, since j's type is private information, i needs to evaluate her expected payoff, after she has updated her belief about j's type, given j's voting choice. The formulation of the expected payoff of i and the analysis of her best response are provided in the following section. We assume that $\alpha_L < d < \alpha_H$. Hence, in the perfect information case, an individual would agree (respectively, would not agree) to match with a high (respectively, low) type individual. Moreover, since $\alpha_H > \alpha_L$, if a match has a positive expected payoff, a high type individual has a higher expected payoff from this match than a low type individual. These "matches" or "interactions" are independent and non-exclusive, meaning that each agent can potentially interact with all of her neighbors if they also want to interact with her and the utility of each match is not affected by the other matches.

As in Börgers (2004), we make two symmetry assumptions about the voting strategy. We assume that it does not depend on the individual's preferred party and that all individuals play the same strategy of the form $s : \{H, L\} \times [c_{min}, c_{max}] \to \{0, 1\}$ where $s_i(\tau(i), c_i) = 0$ (respectively 1) means that an individual i abstains (respectively votes) if she is of type $\tau(i)$ and her cost of voting is c_i.

Similarly to the voting strategy, we assume that the matching strategy does not depend on the individual's and on her potential partner's preferred parties and that every individual i plays the same strategy of the form[6] $I : \{0,1\} \to \{0,1\}$ with regards to an individual j, $i \neq j$. $I(s_j) = 0$ (respectively 1) means that an individual i does not agree (respectively agrees) to match with an individual j if the individual j's voting decision is s_j. Hence, a match between individuals i and j occurs if and only if $I(s_j) \cdot I(s_i) = 1$.

The formulation in (1) captures, in a stylized way, essential features of social interactions with a public good character. We interpret a high type individual as one who has a higher preference for the activity procured by the social interaction, and hence is more willing to contribute to the "public good" than a low type individual. That's why, agents want to match with a high type individual, and not with a low type individual. Interpreting the result of the election and social interaction processes as public goods, a high type individual gets a higher benefit in case of the victory of her preferred party and in case of a match. Therefore, her voting behavior can be a signal about her type, given that she is more likely to vote than a low type individual. The signal can be valuable since agents interact with each other if and only if they have posterior beliefs that the other one is of high type with a high enough probability.

Our equilibrium concept is subgame perfect Bayesian equilibrium. Hence, we proceed by backward induction.

3 Equilibrium Analysis

We first analyze the second stage of the game where agents decide whether to interact with each of their neighbors or not, after observing their voting behavior. Subsequently, we will use the equilibria of the second stage in order to analyze the first stage.

3.1 Second-Stage Equilibrium

Recall from the previous section that equation (1) provides i's payoff from a match with j. The expected payoff of i from a match with j, when j's type is private information and conditional on j's voting decision, is given by:

$$EP_{ij} = w_2 \alpha_{\tau(i)} \left[\lambda(s_j)(\alpha_H - d) + (1 - \lambda(s_j))(\alpha_L - d)\right] I(s_i) I(s_j) \tag{2}$$

[6]We show in the appendix that the assumption that the matching strategy depends only on the potential partner's voting decision, and not on the individual's own type and own voting decision is not a restriction.

Here, $\lambda(s_j)$ is the posterior belief that a neighbor who voted ($s_j = 1$) or did not vote ($s_j = 0$) is of type H. For later use, we define $\lambda(1) = \lambda_H$, which is the posterior belief that a neighbor, who voted, is of high type, and $1 - \lambda(0) = \lambda_L$, which is the posterior belief that a neighbor, who did not vote, is of low type. $I(s_j)$, as given in the previous section, denotes the decision of agent i (who has type $\tau(i)$) whether to match with neighbor j or not, conditional on the latter's voting behavior (s_j). The overall second stage utility of i from all her neighbors is simply the summation over all possible interactions in her neighborhood:

$$TEP_i = \sum_{j=1, j\neq i}^{n} \left\{ w_2 \alpha_{\tau(i)} \left[\lambda(s_j)(\alpha_H - d) + (1 - \lambda(s_j))(\alpha_L - d) \right] I(s_i) I(s_j) \right\} \qquad (3)$$

The best-response of i in the second stage of the game depends on the voting behavior of her neighbors and her posterior beliefs regarding their type. By (2), it is clear that the best-response for i is to match with every neighbor who generates a positive interaction payoff and not to interact if the expected payoff is negative. Therefore, her best response depends on the sign of $\Lambda = \left[\lambda(s_j)(\alpha_H - d) + (1 - \lambda(s_j))(\alpha_L - d) \right] I(s_i)$:

$$I(s_j) = \begin{cases} 1 & \text{if } \Lambda > 0 \\ 0 & \text{if } \Lambda < 0 \\ \{0, 1\} & \text{if } \Lambda = 0 \end{cases} \qquad (4)$$

We examine pure-strategy symmetric equilibria, which do not exhibit coordination failures[7]. We ignore the latter type of equilibria for two reasons. First, they are not robust to small changes of the solution concept or the structure of the game. For example, if we eliminate weakly dominated strategies or use the trembling hand perfect equilibrium concept these coordination failures disappear from the set of equilibria. Second, they are not interesting in terms of empirical implications and provide no insight to the issue at hand. Focusing on pure-strategy symmetric equilibria with no coordination failures allows us to generate the following lemma[8]:

[7]Because of the structure of the game, there are equilibria of the subgame where, irrespectively of the posterior beliefs, nobody interacts with anyone else. This is a simple case of coordination failure, where every individual is indifferent between interacting or not with her neighbors because she anticipates that none of her neighbors will interact with her, even if the interaction would generate positive payoff for both sides.

[8]The proof is included in the appendix.

Lemma 1. *When restricting attention to pure-strategy symmetric equilibria, the following equilibria can result in the second stage:*

1. *Agents choose to interact with both voters and non-voters. This is an equilibrium if and only if the expected payoff from matching with both types is non-negative, which is the case when* $\lambda_H \alpha_H + (1 - \lambda_H)\alpha_L \geq d$ *and* $(1 - \lambda_L)\alpha_H + \lambda_L \alpha_L \geq d$.

2. *Agents choose to interact with only voters. This is an equilibrium if and only if* $\lambda_H \alpha_H + (1 - \lambda_H)\alpha_L \geq d$ *and* $(1 - \lambda_L)\alpha_H + \lambda_L \alpha_L \leq d$.

3. *Agents choose to interact with only non-voters. This is an equilibrium if and only if* $\lambda_H \alpha_H + (1 - \lambda_H)\alpha_L \leq d$ *and* $(1 - \lambda_L)\alpha_H + \lambda_L \alpha_L \geq d$.

4. *Agents choose to interact with neither voters nor non-voters. This is an equilibrium if and only if* $\lambda_H \alpha_H + (1 - \lambda_H)\alpha_L \leq d$ *and* $(1 - \lambda_L)\alpha_H + \lambda_L \alpha_L \leq d$.

3.2 First-Stage Equilibrium

In this subsection, we compute the expected benefit of voting, depending on the second period equilibrium. Then, we prove that, for an expected turnout strictly between 0% and 100%, we cannot have a second period equilibrium where only non-voters match among each other. After making a list of possible types of first period equilibria, we show the existence of one of the most interesting equilibria. We begin by the following remark:

In equilibrium, an individual votes if her expected benefit from voting exceeds her cost of voting. Since the benefit of voting is independent of the cost, an equilibrium voting strategy must be a threshold strategy like in Börgers (2004). So, there is some c_H^* such that $s(H, c_i) = 1$ if $c_i < c_H^*$ and $s(H, c_i) = 0$ if $c_i > c_H^*$. Similarly, there is some c_L^* such that $s(L, c_i) = 1$ if $c_i < c_L^*$ and $s(L, c_i) = 0$ if $c_i > c_L^*$. Hence, the ex ante probability that any individual votes is $p = qF(c_H^*) + (1 - q)F(c_L^*)$. For $0 < p < 1$, the posterior beliefs that a neighbor who voted is of high type, i.e. λ_H, and that a neighbor who did not vote is of low type, i.e. λ_L, are then given as follows by Bayes' rule:

$$\lambda_H = \frac{qF(c_H^*)}{qF(c_H^*) + (1 - q)F(c_L^*)} \tag{5}$$

$$\lambda_L = \frac{(1 - q)(1 - F(c_L^*))}{(1 - q)(1 - F(c_L^*)) + q(1 - F(c_H^*))} \tag{6}$$

The expected benefit of voting is the sum of two terms. The first one which we call the expected *electoral* benefit arises because one's vote can possibly change the electoral outcome. This is the standard benefit of voting in the literature. The second one which we call as the expected *signaling* benefit arises because one's vote can possibly change one's outcome from the social interaction stage.

The expected electoral benefit of voting of an individual i is equal to $\frac{1}{2}\alpha_{\tau(i)}w_1\Pi(p)$, where $\Pi(p)$ is the probability that individual i is pivotal. This happens if her preferred party receives either the same number of votes as the other party or receives one less vote than the other party among the voters but her. In both cases, by voting for her preferred party, she increases the probability that her preferred party wins by 1/2. Taking into account that her benefit is $\alpha_{\tau(i)}w_1$ if her preferred party wins and 0 otherwise, we get the above expression. The expression of $\Pi(p)$ is as given in Börgers (2004), who also shows that this is a differentiable and decreasing function for all $p \in (0, 1)$.

We denote by $B_{\tau(i)}(c_H, c_L)$ the total expected benefit of voting of an individual i with type $\tau(i)$ as a function of the thresholds c_H and c_L. An equilibrium is given by thresholds c_H^* and c_L^* such that $B_{\tau(i)}(c_H^*, c_L^*) \geq c_i$ for all i who vote and $B_{\tau(i)}(c_H^*, c_L^*) \leq c_i$ for all i who abstain.

For the four types of second period equilibria described in Lemma 1, we have the following total expected benefits of voting, where p, λ_H, λ_L are functions of c_H and c_L:

1. If agents choose to interact with both voters and non-voters,

$$B_{\tau(i)}(c_H, c_L) = \alpha_{\tau(i)} \left\{ \frac{w_1}{2}\Pi(p) \right\} \tag{7}$$

2. If agents choose to interact with only voters,

$$B_{\tau(i)}(c_H, c_L) = \alpha_{\tau(i)} \left\{ \frac{w_1}{2}\Pi(p) + w_2 p(n-1)[\lambda_H \alpha_H + (1-\lambda_H)\alpha_L - d] \right\} \tag{8}$$

3. If agents choose to interact with only non-voters,

$$B_{\tau(i)}(c_H, c_L) = \alpha_{\tau(i)} \left\{ \frac{w_1}{2}\Pi(p) - w_2(1-p)(n-1)[(1-\lambda_L)\alpha_H + \lambda_L \alpha_L - d] \right\} \tag{9}$$

4. If agents choose to interact with neither voters nor non-voters,

$$B_{\tau(i)}(c_H, c_L) = \alpha_{\tau(i)} \left\{ \frac{w_1}{2}\Pi(p) \right\} \tag{10}$$

In all the four types of the second period equilibria, the expected electoral benefit is the same. However, the expected signaling benefit differs. In the second type, only voters match among each other. Hence, if an individual votes, she matches with all voters in her neighborhood. Her expected payoff from a single match is $w_2\alpha_{\tau(i)}[\lambda_H\alpha_H + (1-\lambda_H)\alpha_L - d]$ and the expected number of voters (and so of matches) in her neighborhood but her is $p(n-1)$. Hence, if she votes, this gives her an expected payoff of $w_2\alpha_{\tau(i)}p(n-1)[\lambda_H\alpha_H + (1-\lambda_H)\alpha_L - d]$ in the second period. If she does not vote, she does not match with anyone, so her payoff is 0 in the second period. The payoff difference between the two cases where she votes or does not vote gives the expected signaling benefit of voting.

In the first and fourth types, everyone matches with each other or no one matches irrespective of the voting behavior. Hence, the expected signaling benefit is nil. In the third type, only non-voters match among each other. Hence, if an individual votes, she does not match with anyone, so her payoff is 0 in the second period. If she does not vote, she matches with all non-voters in her neighborhood. Her expected payoff from a single match is $w_2\alpha_{\tau(i)}[(1-\lambda_L)\alpha_H + \lambda_L\alpha_L - d]$ and the expected number of non-voters (and so of matches) in her neighborhood but her is $(1-p)(n-1)$. Hence, if she does not vote, this gives her an expected payoff of $w_2\alpha_{\tau(i)}(1-p)(n-1)[(1-\lambda_L)\alpha_H + \lambda_L\alpha_L - d]$ in the second period. Hence, there is a negative expected signaling benefit. However, as we show next, this case cannot occur in an equilibrium with $0 < p < 1$.

We observe that $B_L = \mu B_H$ in any case where $\mu = \alpha_L/\alpha_H$. Using this, we can show the following[9]:

Lemma 2. *In any equilibrium with* $0 < p < 1$*, we have* $\lambda_H > 1 - \lambda_L$.

As shown in Lemma 1, the third type of second period equilibria (where only non-voters match among each other) can happen only if $\lambda_H\alpha_H + (1-\lambda_H)\alpha_L \leq d$ and $(1-\lambda_L)\alpha_H + \lambda_L\alpha_L \geq d$. But these inequalities cannot hold together when $\lambda_H > 1-\lambda_L$. Hence, by Lemma 2, the third type of second period equilibria cannot occur if $0 < p < 1$.[10]

Given that $B_H > B_L$, we have six possible types of first period equilibria:

(i) Everyone votes: $B_H(c_{max}, c_{max}), B_L(c_{max}, c_{max}) \geq c_{max}$.

[9]The proof is included in the appendix.

[10]In fact, it can occur if $p = 0$ or $p = 1$, but only when we take advantage of arbitrariness of beliefs out of equilibrium path and assign "unrealistic" beliefs.

(ii) Nobody votes: $B_H(c_{min}, c_{min}), B_L(c_{min}, c_{min}) \leq c_{min}$.

(iii) All high type individuals vote and none of low type individuals votes: $B_H(c_{max}, c_{min}) \geq c_{max}$ and $B_L(c_{max}, c_{min}) \leq c_{min}$.

(iv) All high type individuals vote and some of low type individuals vote: $B_H(c_{max}, c_L^*) \geq c_{max}$ and $B_L(c_{max}, c_L^*) = c_L^*$ where $c_{min} < c_L^* < c_{max}$.

(v) Some of high type individuals vote and none of low type individuals votes: $B_H(c_H^*, c_{min}) = c_H^*$ and $B_L(c_H^*, c_{min}) \leq c_{min}$ where $c_{min} < c_H^* < c_{max}$.

(vi) Some of high type individuals and some of low type individuals vote: $B_H(c_H^*, c_L^*) = c_H^*$ and $B_L(c_H^*, c_L^*) = c_L^*$ where $c_{min} < c_L^*, c_H^* < c_{max}$.

We focus on the last type of equilibria where some of each type of individuals vote. We call these equilibria as *interior equilibria*.

An interior equilibrium implies $c_L^* = \mu c_H^*$ since $B_L = \mu B_H$, and satisfies $0 < p < 1$. So, by Lemma 2, an interior equilibrium is not consistent with the third type of second period equilibria where only non-voters match among each other. In addition, an equilibrium including the first or fourth type of second period equilibria involves no signaling. These equilibria are not very interesting, in the sense that they generate the same final outcomes as Börgers (2004)[11] and, therefore, provide no new insight into voters' behavior than the existing literature. Our main interest being the signaling aspect of voting, we focus on interior equilibria with the second type of second period equilibria where only voters match among each other. We call these equilibria as *interior equilibria with signaling*.

Since $c_L^* = \mu c_H^*$, we can summarize the condition for an interior equilibrium as follows:

$$B_H(c_H^*, \mu c_H^*) = c_H^* \tag{11}$$

where $c_{min}/\mu < c_H^* < c_{max}$. Since then, $B_L(c_H^*, c_L^*) = c_L^*$ and $c_{min} < c_L^* < c_{max}$ follow immediately. Such an equilibrium is stable if after a slight increase (decrease) in c_H, and the corresponding increase (decrease) in $c_L = \mu c_H$, the benefit from voting falls short of (exceeds) the cost so that the share of voters falls (rises) back to the equilibrium. Formally, defining $\mathcal{B}_H(c_H) \equiv B_H(c_H, \mu c_H)$ for all c_H, we we can write the

[11]The only difference being that the analysis in Börgers (2004) should be applied both to high type and low type individuals.

expected benefit for type H as a function of only the cutoff c_H. With this definition, the equilibrium is stable if $\frac{\partial \mathcal{B}_H}{\partial c_H} - 1 < 0$.

In order to show that the subsequent analysis of stable interior equilibria is well founded, we complete this section by proving that such an equilibrium exists for some parameter values of the model. For this purpose, consider the following inequalities:

$$B_H(c_{min}/\mu, c_{min}) = \alpha_H \Big\{ \frac{w_1}{2}\Pi(qF(c_{min}/\mu)) + w_2 qF(c_{min}/\mu)(n-1)[\alpha_H - d] \Big\} > c_{min}/\mu \tag{12}$$

$$B_H(c_{max}, \mu c_{max}) = \alpha_H \Big\{ \frac{w_1}{2}\Pi(q + (1-q)F(\mu c_{max})) + w_2(n-1)[q(\alpha_H - d) - (1-q)F(\mu c_{max})(d - \alpha_L)] \Big\} < c_{max} \tag{13}$$

$$\frac{(1-q)F(\mu c_H)}{qF(c_H)} \leq \frac{\alpha_H - d}{d - \alpha_L} \leq \frac{(1-q)(1-F(\mu c_H))}{q(1-F(c_H))} \tag{14}$$

Note that inequality (14) is equivalent to the inequalities, $\lambda_H \alpha_H + (1-\lambda_H)\alpha_L \geq d$ and $(1-\lambda_L)\alpha_H + \lambda_L \alpha_L \leq d$, which ensure that only voters match among each other in the second period (see Lemma 1, case 2). Inequalities (12) and (13) are boundary conditions requiring that the benefit of voting exceeds (falls short of) the cost of voting if the turnout is very low (very high).

Proposition 1.

(i) If inequality (14) holds for all $c_H \in [c_{min}/\mu, c_{max}]$, and inequalities (12) and (13) hold, then a stable interior equilibrium with signaling exists.

(ii) There exist parameter values of the model which satisfy simultaneously the above inequalities.

4 Comparative Statics

In this section, we provide the main comparative statics of stable interior equilibria with signaling, which have been shown to exist in the previous section. In the first subsection 4.1, we derive some direct effects of the model's parameters on equilibrium turnout. In subsection 4.2, we turn to the interaction between signaling and the incentives to vote, which is the main focus of our analysis.

4.1 Direct effects

By substituting the posterior beliefs (5) and (6) in equation (8) and by linking the cut-off value of low types to the cut-off value of high types via $c_L = \mu c_H$, the equilibrium condition (11) can be formulated as:

$$\begin{aligned}\mathcal{B}_H(c_H) &\equiv B_H(c_H, \mu c_H) \\ &= \alpha_H \Big\{ \frac{w_1}{2} \Pi \left[qF(c_H) + (1-q)F(\mu c_H) \right] \\ &\quad + w_2(n-1)[qF(c_H)(\alpha_H - d) + (1-q)F(\mu c_H)(\alpha_L - d)] \Big\} = c_H \end{aligned} \tag{15}$$

By using the implicit function theorem one can compute the effect of a change of a parameter, say x, of the model to the equilibrium cutoff c_H^*:

$$\frac{dc_H^*}{dx} = -\frac{\frac{\partial \mathcal{B}_H}{\partial x}}{\frac{\partial \mathcal{B}_H}{\partial c_H} - 1} \tag{16}$$

Since we are considering a stable equilibrium of the game, we know that $\partial \mathcal{B}_H / \partial c_H < 1$, so that the denominator of the above expression is negative. Therefore, the change of the equilibrium cutoff c_H^* has the same sign as the change of the total expected utility ($\mathcal{B}_H$) with respect to the parameter x. Moreover, the change of the equilibrium turnout $p^* = qF(c_H^*) + (1-q)F(\mu c_H^*)$ has also the same sign unless the parameter x is q, α_H, or α_L. As a consequence, we have the following comparative statics of the model:

(i) $\frac{dp^*}{dd} < 0$: An increase in the cost of the second stage interaction decreases the value of signaling and equilibrium turnout.

(ii) $\frac{dp^*}{dw_1} > 0$ and $\frac{dp^*}{dw_2} > 0$: Directly increasing the significance that voters put in the election or the significance of signaling increases equilibrium turnout.

(iii) $\frac{dp^*}{dN} < 0$ but $\frac{dp^*}{dn} > 0$: Increasing the size of the electorate reduces the probability of being pivotal and hence the electoral benefit and thus equilibrium turnout decrease. This is, of course, a direct implication of the $\Pi(p)$ function, which is the same as in Börgers (2004). However, notice that, even if N is arbitrarily large, the value of signaling remains strictly positive in the set of equilibria that we examine and the equilibrium turnout does not fall to zero. To put it differently, even if we examine arbitrarily large societies, we can find values for the remaining parameters such that an interior equilibrium with strictly positive turnout exists and the **paradox of not voting** does not take place. This is because, even though agents can not affect the outcome of the election, they receive strictly positive utility by signaling their type to other agents. On the other hand, an increase in the number of neighbors increases the value of signaling and equilibrium turnout.

The comparative statics above have a straightforward interpretation, which comes directly from the model: any change that increases the value of the electoral outcome or the value of signaling or both, increases the willingness of the marginal voter to vote and, therefore, it increases the equilibrium turnout. As the following result shows, the model however also generates some effects which are more involved.

(iv) $\frac{dp^*}{d\alpha_H}$ has an ambiguous sign[12]: On the one hand, as expected, turnout ratio of high type agents increases unambiguously if they value the benefit of voting higher. On the other hand, turnout ratio of low type agents may decrease or increase due to two opposing effects: the decrease in voting benefit due to lower pivotal probability induced by the higher turnout of high type agents, and the increase in signaling benefit through the increased benefit of a match with a high type agent and the increase of the numbers of matches with high type agents (again due to the higher turnout of high type agents). If turnout ratio of low type agents increases, then overall turnout ratio clearly increases. Otherwise, the result is ambiguous.

4.2 Interaction of signaling and voting incentives

After discussing these comparative static effects, which are direct consequences of introducing signaling into the model, we turn to the more subtle, and possibly even more interesting, indirect effects. Specifically, we ask: How do the benefits of voting and of signaling interact? Does the presence of signaling increase the sensitivity of turnout to the importance of the election outcome for voters? In other words, we would like to investigate the conditions under which the presence of signaling in a voting game **reinforces** or **dampens** the sensitivity of turnout to the electoral incentives. This is interesting both in terms of empirical implications (are countries with better connected

[12]The computations and more detailed explanations are in the appendix. The other comparative static analyses $\frac{dp^*}{d\alpha_L}$ and $\frac{dp^*}{dq}$ will be skipped to save space, since they have similar flavor to $\frac{dp^*}{d\alpha_H}$.

communities expected to have more volatile turnout?) and in terms of policy implications (should governments adopt community friendly policies to increase the sensitivity of voters to political issues?). For brevity, whenever the sensitivity of the turnout to electoral incentives increases with signaling we say that we have a **reinforcing signaling** effect, while whenever the sensitivity of the turnout to electoral incentives decreases with signaling we say that we have a **dampening signaling** effect.

Moreover, we investigate whether there can be a **bandwagon effect**, i.e. whether a voter is more likely to vote when there is higher turnout. Note that in the absence of signaling, this is impossible, since higher turnout decreases the pivotal probability of a voter, who is then less likely to vote. In addition, we ask whether an increase in turnout ratio can be substantial in case of a small increase of the election's significance (w_1). Note again that this cannot be the case in the absence of signaling, since the effect of w_1 is downgraded by small pivotal probabilities.

4.2.1 Reinforcing or dampening signaling effects

In terms of formal analysis, we study whether signaling is reinforcing or dampening by examining how the change of c_H^* due to an increase in the significance of the elections is affected by an increase in the value of signaling. Therefore, if $\frac{d^2 c_H^*}{dw_1 dw_2} > 0$ we have reinforcing signaling and if $\frac{d^2 c_H^*}{dw_1 dw_2} < 0$ we have dampening signaling. Since an increase in the equilibrium cut-off value c_H^* always increases the equilibrium turnout p^* for given values of q, α_H and α_L, examining the effect on c_H^* also gives us the impact on p^*. By setting $x = w_1$ and by taking the derivative of (16) with respect to w_2 we find:

$$\frac{d^2 c_H^*}{dw_1 dw_2} = \frac{\frac{\partial^2 \mathcal{B}_H}{\partial c_H \partial w_2} \frac{\partial \mathcal{B}_H}{\partial w_1}}{\left(\frac{\partial \mathcal{B}_H}{\partial c_H} - 1\right)^2}$$

Since the denominator and $\frac{\partial \mathcal{B}_H}{\partial w_1}$ are both positive, the sign of the expression above has the same sign as $\frac{\partial^2 \mathcal{B}_H}{\partial c_H \partial w_2}$. By computing the latter cross-derivative and rearranging we find that we have reinforcing signaling if and only if (recall that $\mu = \alpha_L/\alpha_H$):

$$(\alpha_H - d) q f(c_H) + (\alpha_L - d)(1-q)\mu f(\mu c_H) > 0 \tag{17}$$

Inequality (17) illustrates the interaction of voting and signaling incentives. When the election importance (w_1) increases, there are $qf(c_H)$ additional individuals of high type and $(1-q)\mu f(\mu c_H)$ additional individuals of low type who decide to vote. Inequality

(17) states that the expected payoff of matching with these additional voters is positive. In this case, the expected signaling benefit of voting increases, which reinforces the increase in turnout due to the higher importance of the election.

Solving inequality (17) for the parameter d we find a critical threshold value (let us call it $\tilde{\alpha}$), such that if d is below this threshold, then we have reinforcing signaling, while if d is above this threshold we have dampening signaling. We summarize this result in the following proposition, which is directly derived from the analysis so far:

Proposition 2. *In any stable interior equilibrium with signaling (i.e. agents interact only with voters) we have a reinforcing signaling effect whenever the cost of matching d is below the threshold value $\tilde{\alpha}$ and dampening signaling otherwise, with*

$$\tilde{\alpha} \equiv \frac{\alpha_H q f(c_H) + \alpha_L (1-q)\mu f(\mu c_H)}{q f(c_H) + (1-q)\mu f(\mu c_H)} \tag{18}$$

Note that, if we define $w_2 p(n-1)[\lambda_H \alpha_H + (1-\lambda_H)\alpha_L - d]$ in equation (8) as the signaling benefit of voting, then it is easy to show that:

$$\frac{\partial^2 \mathcal{B}_H}{\partial c_H \partial w_2} = \frac{1}{w_2} \frac{\partial(\text{signaling benefit})}{\partial c_H}$$

Hence, if an increase in the total turnout has a positive effect on the value of signaling, then this implies that signaling has a reinforcing effect on voting. The interpretation is that if the significance of the elections increases (w_1 increases) then turnout will increase because the overall expected benefit for voters increases. But whether this effect is larger or smaller than in a society where the signaling benefit is absent (i.e. Börgers (2004)) or where communities are less important (lower value of w_2), depends on the impact of the increased turnout on the signaling benefit. If turnout has a positive impact on signaling then the increase in turnout will be greater in the society with stronger community ties ($\frac{d^2 c_H^*}{dw_1 dw_2} > 0$), because the initial increase in the value of voting is further reinforced by the fact that voting is also more beneficial for signaling one's type to her "neighbors". Of course, the opposite is true if the signaling benefit is negatively affected by higher turnout.

Proposition 2 makes clear that in a society where the cost of social interactions is low ($d < \tilde{\alpha}$), for instance due to inadequate substitutes to social interactions or because of well-established communication channels, signaling has a reinforcing effect, while the opposite is true for a society with high cost of social interactions. Hence, we expect the turnout ratio to be more sensitive to the importance of the electoral outcome in societies with low cost of social interactions.

Beyond this general result, it is worthwhile to investigate in more detail whether, and in what circumstances, the condition $d < \tilde{\alpha}$ is likely to be satisfied in an equilibrium with signaling. To answer this question, we relate $\tilde{\alpha}$ to the inequalities laid down in case 2 of Lemma 1. These inequalities implicitly define an interval $[d_L, d_H]$, within which the cost d of the match must lie for an equilibrium with signaling to obtain.

If $\tilde{\alpha}$ is greater than the upper bound of the interval $[d_L, d_H]$, i.e. $\tilde{\alpha} > d_H$, then signaling has a reinforcing effect on voting irrespectively of the other parameters of the model. If $\tilde{\alpha}$ is lower than the lower bound of the interval, i.e. $\tilde{\alpha} < d_L$, then signaling has a dampening effect on voting, irrespectively of the other parameters of the model, and if $\tilde{\alpha}$ is in the interior of the interval, the effect of signaling is either reinforcing or dampening, depending on the other parameters of the model. The following proposition relates these cases to the distribution of voting costs[13]:

Proposition 3. *Consider an interior equilibrium with signaling and cutoff value c_H^* for the high types. Then:*

(i) If $\frac{f(c_H^*)}{F(c_H^*)} > \frac{\mu f(\mu c_H^*)}{F(\mu c_H^*)}$, then the effect of signaling is reinforcing.

(ii) If $\frac{f(c_H^*)}{1-F(c_H^*)} > \frac{\mu f(\mu c_H^*)}{1-F(\mu c_H^*)}$, the effect of signaling is reinforcing for some parameter values and dampening for the rest.

(iii) If $\frac{f(c_H^*)}{1-F(c_H^*)} < \frac{\mu f(\mu c_H^*)}{1-F(\mu c_H^*)}$, then the effect of signaling is dampening.

Note that the condition of part (ii) in Proposition 3 is a weaker version of the increasing hazard rate, which is commonly used in the literature. This means that, if the distribution of voting costs satisfies the increasing hazard rate property, then whether signaling has a reinforcing or dampening effect depends on the cost of social interactions, d, as given in Proposition 2. On the other hand, ensuring that all the stable interior equilibria of the model for any set of parameter values exhibit reinforcing signaling requires the condition of part (i). This condition, which is akin to the "reverse" hazard rate, is stronger than condition (ii). The most commonly used distributions in the literature, such as the uniform, the normal and the exponential distribution, do not satisfy the condition of part (i) but satisfy the condition of part (ii) globally. Thus, it is

[13]The proof is included in the appendix.

reasonable to expect case (ii) to occur, which means that the cost of social interaction is indeed crucial for signaling to have a reinforcing effect on voting incentives.

4.2.2 Bandwagon Effect

Next, we investigate whether there can be a bandwagon effect in our model. Mathematically, a bandwagon effect exists if and only if[14] $\frac{\partial \mathcal{B}_H}{\partial c_H} > 0$, i.e. higher turnout increases the voting benefit. This derivative is given by:

$$\begin{aligned} \frac{\partial \mathcal{B}_H}{\partial c_H} =& \alpha_H \Big\{ \frac{w_1}{2} \Pi'(p) \left[q f(c_H) + (1-q)\mu f(\mu c_H) \right] \\ & + w_2 (n-1) [q f(c_H)(\alpha_H - d) + (1-q)\mu f(\mu c_H)(\alpha_L - d)] \Big\} \end{aligned} \tag{19}$$

Since $\Pi'(p)$ is negative, the first term in the curly brackets is negative. This term shows that electoral benefit decreases with higher turnout. The second term, which corresponds to the change of signaling benefit, is positive if and only if signaling is reinforcing, i.e. inequality (17) holds. Hence, a necessary condition for a bandwagon effect ($\frac{\partial \mathcal{B}_H}{\partial c_H} > 0$) is reinforcing signaling. Given that signaling is reinforcing, a bandwagon effect exists as long as the second term is higher in absolute value than the first term, for instance, for a high enough value for the importance of social interactions (w_2).

The intuition is as follows: With a higher turnout, electoral benefit of a voter decreases due to a smaller pivotal probability. However, if signaling benefit increases with a higher turnout, or equivalently if signaling is reinforcing, then the bandwagon effect may arise. The bandwagon effect exists when the increase in signaling benefit is higher in magnitude than the decrease in electoral benefit.

4.2.3 Magnitude of $\frac{dc_H^*}{dw_1}$

Until here, we were interested in the sign of various effects. Finally, we analyze the magnitude of the increase of turnout ratio due to a small increase of the election's significance (w_1). Note that in a model of voting which does not include signaling benefit, the response of turnout to changes of w_1 is small due to low pivotal probabilities for voters. Therefore, it is important to see whether the inclusion of the signaling benefit can change this result.

[14] Expressing this condition in terms of the voting benefit of a high type agent is sufficient, since the voting benefit of a low type agent is proportional.

As we showed earlier, the election's significance becomes more important for turnout ratio when signaling is reinforcing. Indeed, if this reinforcement is strong enough so that there exists an important bandwagon effect, a small change in the importance of the election may have a large impact on equilibrium turnout. Mathematically, replacing x by w_1 in equation (16) gives:

$$\frac{dc_H^*}{dw_1} = -\frac{\frac{\partial \mathcal{B}_H}{\partial w_1}}{\frac{\partial \mathcal{B}_H}{\partial c_H} - 1} \tag{20}$$

where $\frac{\partial \mathcal{B}_H}{\partial c_H}$ is given in equation (19) and $\frac{\partial \mathcal{B}_H}{\partial w_1}$ is given by

$$\frac{\partial \mathcal{B}_H}{\partial w_1} = \frac{\alpha_H}{2}\Pi(p)$$

Since $\Pi(p)$ is relatively small, the numerator in equation (20) is expected to be small. In the absence of signaling benefit ($w_2 = 0$), the denominator in absolute value is higher than 1, since $\frac{\partial \mathcal{B}_H}{\partial c_H}$ is negative. This leads to a low magnitude of $\frac{dc_H^*}{dw_1}$. However, in the presence of signaling, if signaling is reinforcing, $\frac{\partial \mathcal{B}_H}{\partial c_H}$ can be arbitrarily close to 1 (a stable equilibrium implies that $\frac{\partial \mathcal{B}_H}{\partial c_H} < 1$) for sufficiently high values of w_2. Then, this leads to a small denominator in absolute value and therefore to an important magnitude of $\frac{dc_H^*}{dw_1}$.

The intuition behind this result is that, if social interactions are very important for voters (high w_2), then even a small increase in the importance of the election may generate a large increase in turnout, because of the importance of signaling effects. In other words, since voters expect other voters to turn out in higher numbers, their own incentive to vote increases significantly due to signaling purposes and this may generate a substantial increase on total turnout. This is an important result of our paper, because it depends crucially on the existence of signaling benefits and can not be generated by the existing literature on rational voting.

5 Conclusion

The paper presents a formal model of voting as signaling device. By observing the voting behavior of others in their social circle, voters receive a signal about their 'neighbor's' value in social interactions. This generates an additional incentive to vote, apart from affecting the outcome of the election, as the early rational voting theory predicts. This additional incentive can account for the paradox of not voting in large societies and the role of social pressures in electoral turnouts. Moreover, the model generates several predictions which are consistent with empirical findings.

We believe that the model can be extended in order to shed light on the interaction between voting incentives and the role of political parties. In our model, party preferences are assumed to be independently distributed in each neighborhood. Also the benefit of social interaction is assumed to be independent of voters' preferences over political parties. However, one would reasonably assume that cooperation among individuals of similar ideological position is more beneficial than if they have very dissimilar views. Relaxing these assumptions may lead to understand better political parties' strategic use of advertising and the role of party activists, depending on the characteristics of neighborhoods.

Overall, we believe that this is a very fruitful avenue for further research and we intend to extend our model in the near future in these directions.

Appendix

Second Stage Equilibrium

In order to facilitate the proof of Lemma 1, we first present two results, which we summarize in the form of Claim 1. For the first result, note that, in principle, the choice of, say, i to interact with j or not may depend on all elements of her information set in the second stage game: her identity (i), her type ($\tau(i)$), the signal she has produced in the first stage (s_i) and the signal of her neighbor (s_j). Therefore, in principle we can write her choice as $I_i(\tau(i), s_i, s_j)$.

Claim 1. *In any equilibrium of the second stage game we have:*
(i) $I_i(\tau(i), s_i, s_j) = I(s_j)$
(ii)
$$\begin{aligned}
I(s_j = 1) = 1 &\quad \textit{if} \;\; \lambda_H\alpha_H + (1-\lambda_H)\alpha_L - d \; > \; 0 \\
I(s_j = 1) = 0 &\quad \textit{if} \;\; \lambda_H\alpha_H + (1-\lambda_H)\alpha_L - d \; < \; 0 \\
I(s_j = 0) = 1 &\quad \textit{if} \;\; (1-\lambda_L)\alpha_H + \lambda_L\alpha_L - d \; > \; 0 \\
I(s_j = 0) = 0 &\quad \textit{if} \;\; (1-\lambda_L)\alpha_H + \lambda_L\alpha_L - d \; < \; 0.
\end{aligned}$$

The first result shows that one's decision to interact is independent of one's own identity, type, and voting behavior. That is, an agent's choice to interact with a neighbor or not depends only on the neighbor's signal in the first stage. The second result shows how this choice changes according to the posterior beliefs of an agent regarding the neighbor's type. The results hold for pure strategy equilibria, which is the only type of equilibria we examine in this paper.

Proof of part (i): The most general formulation of i's expected second stage payoff (equation (2)) is given by the following expression:

$$EP_{ij} = w_2\alpha_{\tau(i)}\left[\lambda(s_j)(\alpha_H - d)I_j(H, s_j, s_i) + (1-\lambda(s_j))(\alpha_L - d)I_j(L, s_j, s_i)\right] I_i(\tau(i), s_i, s_j)$$

However, as one can infer from a quick look at the equation above, the best-response of i is independent of her type and her identity. This is because we have assumed that $\alpha_H > \alpha_L > 0$ and therefore the best-response of i is to interact with j if the term in the square brackets is positive and not to interact if the term is negative, irrespectively of her type. In terms of notation, $I_i(H, s_i, s_j) = I_i(L, s_i, s_j) = I(s_i, s_j)$. This implies that also j's best-response to interact with i is independent of j's type and hence we can rewrite i's payoff as:

$$EP_{ij} = w_2 \alpha_{\tau(i)} \left[\lambda(s_j)(\alpha_H - d) + (1 - \lambda(s_j))(\alpha_L - d) \right] I_i(s_i, s_j) I_j(s_j, s_i) \tag{A.1}$$

The subscript on $I_i(s_i, s_j)$ in the expression above is used only to denote who is making the decision, but the strategy itself is independent of one's identity. We keep it simply to facilitate exposition and drop it later on.

In the expression above, if $I_j(s_j, s_i) = 0$, the payoff of i is zero, as j does not want to interact with i, and hence any value of $I_i(s_i, s_j)$ is a best-response for i. This gives rise to a coordination failure equilibrium where, if j does not interact with i, it is a best response for i not to interact with j and vice versa. As commented in the main text, we do not consider this case further.

If $I_j(s_j, s_i) = 1$, then i's best-response depends on the sign of the term in the brackets of equation (A.1), which is independent of i's signal. Therefore, whether i voted or not in the first-stage does not alter her best-response. In terms of notation $I_i(0, s_j) = I_i(1, s_j) = I(s_j)$, which shows part (i) of our claim.

Proof of part (ii): The second part of the claim follows immediately. If the bracket in equation (A.1) is positive and $I(s_i) = 1$, then the best-response of i is to interact and if it is negative then the best-response is not to interact. Hence $I(s_j) = 1$ if $\lambda_H(\alpha_H - d) + (1 - \lambda_H)(\alpha_L - d) > 0$ or if $(1 - \lambda_L)(\alpha_H - d) + \lambda_L(\alpha_L - d) > 0$. $I(s_j) = 0$ if $\lambda_H(\alpha_H - d) + (1 - \lambda_H)(\alpha_L - d) < 0$ or if $(1 - \lambda_L)(\alpha_H - d) + \lambda_L(\alpha_L - d) < 0$ (recall that $\lambda_H = \lambda(s_j = 1)$ and $\lambda_L = 1 - \lambda(s_j = 0)$).

As commented in the first part of the claim, if $I(s_i) = 0$, then i's payoff in independent of her action and she may choose any as a best-response. However, if one uses the concept of trembling hand (which means that each player has an arbitrarily small chance of making a mistake and choosing the other action), i's indifference to her action breaks down and we reach the same conclusion as above (when $I(s_i) = 1$). ■

Proof of Lemma 1

The implication of the second part of the claim 1 is that, when the expected benefit of interaction is different from zero ($[\lambda(s_j)(\alpha_H - d) + (1 - \lambda(s_j))(\alpha_L - d)] \neq 0$), then there are four different equilibria of the second stage and they are summarized in section 3.1. Each one of them is directly derived by a combination of inequalities from the second part of claim 1. For example, if $\lambda_H \alpha_H + (1 - \lambda_H)\alpha_L > 0$ and $(1 - \lambda_L)\alpha_H + \lambda_L \alpha_L < 0$

then all agents choose to interact with voters and choose not to interact with non-voters, which results to case 2 in page 10.

When $[\lambda(s_j)(\alpha_H - d) + (1-\lambda(s_j))(\alpha_L - d)] = 0$, other equilibria may arise. This is because i is indifferent between interacting with j or not. Again, the best-response of i is any choice of $I_i(s_j)$. As a result, there can be asymmetric equilibria of the second stage, where some neighbors of j choose to interact with her and others do not interact. Since we are examining only symmetric equilibria in this paper, we ignore these asymmetric equilibria. Furthermore, the set of symmetric equilibria remains unchanged from the set defined in section 3.1, even when $[\lambda(s_j)(\alpha_H - d) + (1-\lambda(s_j))(\alpha_L - d)] = 0$. For example, if $\lambda_H\alpha_H + (1-\lambda_H)\alpha_L > 0$ and $(1-\lambda_L)\alpha_H + \lambda_L\alpha_L = 0$, there is an equilibrium of the second stage where agents interact with both voters and non-voters (case 1 in page 10), and another equilibrium, where agents interact only with voters (case 2 in page 10). Overall, when we focus in symmetric equilibria with no coordination failures, then the second stage produces the four equilibria we describe in section 3.1. ■

First-Stage Equilibrium

Proof of Lemma 2

Since $B_L = \mu B_H$, B_L and B_H have the same sign. When $0 < p < 1$, they are both positive since otherwise $p = 0$ from $c_{min} \geq 0$. Hence, $B_H > B_L$ since $\mu < 1$. Therefore, $c_H^* \geq c_L^*$. The case $c_H^* = c_L^*$ can occur only if $c_H^* \leq c_{min}$ so that $c_H^* = c_L^* = c_{min}$ or $c_L^* \geq c_{max}$ so that $c_H^* = c_L^* = c_{max}$. But, $c_H^* > c_{min}$ since otherwise $p = 0$. Also $c_L^* < c_{max}$, since otherwise $p = 1$. Hence, we conclude that $c_H^* > c_L^*$.

For $0 < p < 1$, λ_H and λ_L are given by equations (5) and (6) respectively. By simple algebraic manipulations, we get

$$\lambda_H - (1-\lambda_L) = \frac{q(1-q)}{p(1-p)}(F(c_H^*) - F(c_L^*))$$

where $p = qF(c_H^*) + (1-q)F(c_L^*)$. This difference is positive since $c_H^* > c_L^*$ and f has a positive density over all of the support. ■

Proof of Proposition 1

(i) When we plot $\mathcal{B}_H(c_H)$ on $c_{min}/\mu < c_H < c_{max}$, the intersection c_H^* with the 45° line would be an interior equilibrium satisfying (11). By the continuity of $\mathcal{B}_H(c_H)$ on the interval $[c_{min}/\mu, c_{max}]$, if $\mathcal{B}_H(c_{min}/\mu) > c_{min}/\mu$ (i.e. the starting point is above the 45° line) and $\mathcal{B}_H(c_{max}) < c_{max}$ (i.e. the ending point is below the 45° line), then at least one such intersection exists. Moreover, since at least one intersection is such that $\mathcal{B}_H(c_H)$ cuts the 45° line from above, a stable interior equilibrium exists if these two conditions are satisfied. From Lemma 1, case 2, in an interior equilibrium with signaling, the second period benefit is $w_2 p(n-1)[\lambda_H \alpha_H + (1-\lambda_H)\alpha_L - d]$. With the cutoff points $c_H = c_{min}/\mu$ and $c_L = c_{min}$, p is equal to $p = qF(c_{min}/\mu)$ and λ_H is equal to $\lambda_H = \frac{qF(c_{min}/\mu)}{qF(c_{min}/\mu)} = 1$. With the cutoff points $c_H = c_{max}$ and $c_L = \mu c_{max}$, p is equal to $p = q + (1-q)F(\mu c_{max})$ and λ_H is equal to $\lambda_H = \frac{q}{q+(1-q)F(\mu c_{max})}$. Replacing p and λ_H in (8) and rearranging, one finds that $\mathcal{B}_H(c_{min}/\mu) > c_{min}/\mu$ and $\mathcal{B}_H(c_{max}) < c_{max}$ are equivalent to inequalities (12) and (13).

In addition, we have to make sure that this intersection c_H^* gives an equilibrium with signaling. This is the case if the two conditions $\lambda_H \alpha_H + (1-\lambda_H)\alpha_L \geq d$ and $(1-\lambda_L)\alpha_H + \lambda_L \alpha_L \leq d$ hold for all $c_H \in [c_{min}/\mu, c_{max}]$ (i.e. for all possible intersection points). These two conditions are equivalent to inequality (14) holding for all $c_H \in [c_{min}/\mu, c_{max}]$.

(ii) The lhs of inequality (12) is always positive. Hence, this inequality is satisfied for for low enough c_{min}. The lhs of inequality (13) is bounded above by $\alpha_H\big\{\frac{w_1}{2} + w_2(n-1)q(\alpha_H - d)\big\}$. Hence, this inequality is satisfied for high enough c_{max}.

The lhs of inequality (14) is lower than $(1-q)/q$ since $F(\mu c_H) < F(c_H)$ for all $c_H \in [c_{min}/\mu, c_{max}]$. Similarly, the rhs of inequality (14) is greater than $(1-q)/q$ since $1 - F(\mu c_H) > 1 - F(c_H)$ for all $c_H \in [c_{min}/\mu, c_{max}]$. Then, for instance, if d is such that $\frac{\alpha_H - d}{d - \alpha_L} = \frac{1-q}{q}$ (equivalently $q\alpha_H + (1-q)\alpha_L = d$), both conditions are satisfied. Hence, there is a neighborhood of values of d around $q\alpha_H + (1-q)\alpha_L$ in which both conditions are satisfied. Note that this neighborhood for d is consistent with the fact that inequalities (12) and (13) hold for some parameter values, since the latter inequalities are satisfied by appropriate choice of c_{min} and c_{max}, irrespective of d.
■

Comparative Statics

Claim: $\frac{dp^*}{d\alpha_H}$ *has an ambiguous sign.*

Proof: $\frac{\partial \mathcal{B}_H}{\partial \alpha_H}$ is given by

$$\begin{aligned}\frac{\partial \mathcal{B}_H}{\partial \alpha_H} =& \Big\{\frac{w_1}{2}\Pi(p) + w_2(n-1)[qF(c_H)(\alpha_H - d) + (1-q)F(\mu c_H)(\alpha_L - d)]\Big\} + \\ & \alpha_H\Big\{\frac{w_1}{2}\Pi'(p)(1-q)f(\mu c_H)(-\frac{\alpha_L}{\alpha_H^2})c_H \\ & + w_2(n-1)\big[qF(c_H) + (1-q)f(\mu c_H)(\alpha_L - d)(-\frac{\alpha_L}{\alpha_H^2})c_H\big]\Big\} \end{aligned} \tag{A.2}$$

The term in the first bracket is clearly positive and corresponds to the direct effect of the increased benefit of voting and signaling. The first and the second terms in the second bracket are also positive and correspond respectively to the increase in voting benefit through higher pivotal probability (for a given c_H, $c_L = \mu c_H$ is lower since an increase in α_H decreases μ, which leads to less turnout of low type agents), and the increase in signaling benefit through the increased benefit of a match with a high type agent and the decrease of the numbers of matches with low type agents (again due to the lower turnout of low type agents). Hence, turnout ratio of high type agents increases unambiguously.

Similar to $\mathcal{B}_H(c_H)$, we define $\mathcal{B}_L(c_L)$ as

$$\begin{aligned}\mathcal{B}_L(c_L) &\equiv B_L(c_L/\mu, c_L) \\ &= \alpha_L\Big\{\frac{w_1}{2}\Pi\left[qF(c_L/\mu) + (1-q)F(c_L)\right] \\ &+ w_2(n-1)\big[qF(c_L/\mu)(\alpha_H - d) + (1-q)F(c_L)(\alpha_L - d)\big]\Big\} = c_L \end{aligned} \tag{A.3}$$

By the same argument as in the text, $dc_L/d\alpha_H$ has the same sign as $\partial\mathcal{B}_L/\partial\alpha_H$ which is given by

$$\frac{\partial \mathcal{B}_L}{\partial \alpha_H} = \alpha_L\Big\{\frac{w_1}{2}\Pi'(p)qf(c_L/\mu)\frac{c_L}{\alpha_L} + w_2(n-1)\big[qF(c_L/\mu) + qf(c_L/\mu)(\alpha_H - d)\frac{c_L}{\alpha_L}\big]\Big\} \tag{A.4}$$

The first term in the bracket is negative and corresponds to the first effect mentioned in the text, whereas the second term is positive and corresponds to the second effect. Hence, the sign of the change of low type agents' turnout ratio is ambiguous. Therefore, overall turnout ratio can increase or decrease, depending on parameter values. ■

Proof of Proposition 3

First we derive the thresholds d_H and d_L from case 2 of Lemma 1, by substituting the relevant values for λ_H and λ_L:

$$\begin{aligned}
d_H &= \lambda_H \alpha_H + (1-\lambda_H)\alpha_L \\
\Rightarrow d_H &= \frac{qF(c_H)}{qF(c_H)+(1-q)F(\mu c_H)}\alpha_H + \left(1 - \frac{qF(c_H)}{qF(c_H)+(1-q)F(\mu c_H)}\right)\alpha_L \\
\Rightarrow d_H &= \frac{\alpha_H qF(c_H) + \alpha_L(1-q)F(\mu c_H)}{qF(c_H)+(1-q)F(\mu c_H)}
\end{aligned}$$

Similarly:

$$d_L = (1-\lambda_L)\alpha_H + \lambda_L \alpha_L \;\Rightarrow\; d_L = \frac{\alpha_H q(1-F(c_H)) + \alpha_L(1-q)(1-F(\mu c_H))}{q(1-F(c_H)) + (1-q)(1-F(\mu c_H))}$$

For part (i), suppose that $f(c_H)/F(c_H) \geq \mu f(\mu c_H)/F(\mu c_H)$. One has

$$\frac{f(c_H)}{F(c_H)} > (=) \frac{\mu f(\mu c_H)}{F(\mu c_H)} \Leftrightarrow (\alpha_H - \alpha_L) f(c_H) F(\mu c_H) > (=) (\alpha_H - \alpha_L)\mu f(\mu c_H) F(c_H)$$

$$\Leftrightarrow \alpha_H f(c_H)F(\mu c_H) + \alpha_L \mu f(\mu c_H) F(c_H) > (=) \alpha_H \mu f(\mu c_H) F(c_H) + \alpha_L f(c_H) F(\mu c_H)$$

Multiplying both sides by $q(1-q)$ and adding $\alpha_H q^2 f(c_H)F(c_H)$ and $\alpha_L(1-q)^2 \mu f(\mu c_H) F(\mu c_H)$ on both sides yields:

$$\begin{aligned}
&\alpha_H q^2 f(c_H)F(c_H) + \alpha_H q(1-q) f(c_H) F(\mu c_H) \\
&\qquad + \alpha_L q(1-q)\mu f(\mu c_H) F(c_H) + \alpha_L (1-q)^2 \mu f(\mu c_H) F(\mu c_H) \\
&\quad > (=) \;\; \alpha_H q^2 f(c_H) F(c_H) + \alpha_H q(1-q)\mu f(\mu c_H) F(c_H) \\
&\qquad + \alpha_L q(1-q) f(c_H) F(\mu c_H) + \alpha_L(1-q)^2 \mu f(\mu c_H) F(\mu c_H) \\
&\Leftrightarrow \big[\alpha_H q f(c_H) + \alpha_L(1-q)\mu f(\mu c_H)\big]\big[qF(c_H) + (1-q)F(\mu c_H)\big] \\
&\quad > (=) \;\; \big[\alpha_H q F(c_H) + \alpha_L (1-q) F(\mu c_H)\big]\big[q f(c_H) + (1-q)\mu f(\mu c_H)\big] \\
&\Leftrightarrow \frac{\alpha_H q f(c_H) + \alpha_L(1-q)\mu f(\mu c_H)}{q f(c_H) + (1-q)\mu f(\mu c_H)} \;\; > (=) \;\; \frac{\alpha_H q F(c_H) + \alpha_L(1-q)F(\mu c_H)}{qF(c_H) + (1-q)F(\mu c_H)} \\
&\Leftrightarrow \tilde{\alpha} \; > (=) \; d_H
\end{aligned}$$

From the lines above, we conclude more specifically that

$$\frac{f(c_H)}{F(c_H)} \;\; > (=) \;\; \frac{\mu f(\mu c_H)}{F(\mu c_H)} \Leftrightarrow \tilde{\alpha} \; > (=) \; d_H$$

When $\tilde{\alpha}$ is greater than (resp. equal to) d_H, this implies that any value of d that satisfies the equilibrium conditions also satisfies $d < \tilde{\alpha}$ (resp. $d \leq \tilde{\alpha}$) . Hence $\frac{d^2 c_H^*}{dw_1 dw_2} > 0$ (resp. ≥ 0).

For part (ii), substitute in the proof above the terms $1 - F(c_H)$ and $1 - F(\mu c_H)$ for the terms $F(c_H)$ and $F(\mu c_H)$ respectively and iterate the same steps. Then we obtain:

$$\begin{aligned}
\frac{f(c_H)}{1 - F(c_H)} &> (=) \quad \frac{\mu f(\mu c_H)}{1 - F(\mu c_H)} \\
\Leftrightarrow \frac{\alpha_H q f(c_H) + \alpha_L (1-q) \mu f(\mu c_H)}{q f(c_H) + (1-q) \mu f(\mu c_H)} &> (=) \quad \frac{\alpha_H q (1 - F(c_H)) + \alpha_L (1-q)(1 - F(\mu c_H))}{q(1 - F(c_H)) + (1-q)(1 - F(\mu c_H))} \\
\Leftrightarrow \tilde{\alpha} &> (=) \quad d_L
\end{aligned}$$

When $\tilde{\alpha}$ is greater than d_L, the cost of the match d may satisfy $d < \tilde{\alpha}$ or not. This depends on the other parameters of the model. Hence, either $\frac{d^2 c_H^*}{dw_1 dw_2} \geq 0$ or $\frac{d^2 c_H^*}{dw_1 dw_2} < 0$. When $\tilde{\alpha}$ is equal to d_L, $d \geq \tilde{\alpha}$. Hence, $\frac{d^2 c_H^*}{dw_1 dw_2} \leq 0$.

Finally, part (iii) follows from part (ii). This is because when the initial condition of part (ii) does not hold, then $\tilde{\alpha} < d_L$, which implies that the condition $d \leq \tilde{\alpha}$ is mutually exclusive with the equilibrium conditions. ■

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