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Family Taxation, Fertility, and Horizontal Equity: A Political Economy Perspective

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Abstract

This paper intends to make a two-fold contribution to the literature. First, it studies a political economy model of family taxation using a household economics approach to behaviour; the nature of the winning policy is found to depend on whether i) the parents control their fertility or not, ii) they value their children or not. Second, it investigates the question whether the winning policy is capable to achieve horizontal equity (i.e. the requirement that all agents who are in all "relevant" senses identical should be treated identically); it turns out that under endogenous fertility, any winning policy trivially satisfies horizontal equity, but if fertility is exogenous for some of (or all) the parents, horizontal equity is virtually impossible to satisfy. The assessment on whether a given family taxation scheme attains horizontal equity objectives cannot therefore be independent from the assessment on the nature of fertility behaviour.

JEL-Code: D130, D720, H310, J130.

Keywords: family taxation, horizontal equity, fertility, political economy, median voter, family size.

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1 Introduction

Should parents be compensated for the cost of bringing up their children? In the real word, fiscal systems often include subsidisation schemes for large families (e.g. UK and France); where these are lacking or inadequate (e.g. Italy and Spain), their introduction or expansion is being advocated, for a variety of reasons. For example, it is often argued that the financial stability and viability of the pay-as-you-go pension systems would be improved, in the long run, if the fertility rates increased, so as to obtain a more favourable demographic structure. While these and other reasons are obviously important, they fall outside the scope of the present paper, that focuses instead on a very specific motivation, based on the horizontal equity (HE) principle.

HE requires that the agents who are in all "relevant" senses identical should be treated identically. This formulation seems incontrovertible: still, it is often argued that actual tax systems ignore HE requirements (see e.g. Gravelle and Gravelle, 2006). At first sight, this appears difficult to explain. However, it has long been recognised that the actual implementation of the principle is far from straightforward (Auerbach and Hassett, 2002 provide a relatively recent treatment of the issue). In fact, one way of viewing the present contribution is to say that it offers yet another example of such difficulties. Let us see how.

The problem lies with the meaning of the adjective "relevant". In some instances, it is obvious: tax systems should not discriminate on the basis of, say, religion, skin colour, etc. In other cases, instead, the meaning is less clear. For example, in our context, if the number of children is taken as given, and is not seen as an admissible distinguishing characteristic (i.e. as "relevant"), it follows that subsidies should be used to restore HE in case families who are identical, except for their fertility level, enjoy different after-tax utilities. This argument rests however on the implicit assumption that children are like disabling accidents or chronic illnesses, unavoidable negative events against which families should be insured. Now, while the case for subsidising large families on HE grounds is intuitively obvious if parents do not control their fertility and do not value their children, it becomes much less obvious if it is believed otherwise. And there is no compelling reason to refuse a priori the idea that the parents control, to some extent anyway, their fertility or that they value their children, either because they care for them or because they expect a return from them (Dasgupta 1993). In fact, the reasons mentioned above for family size-related subsidies, such as favouring the viability of social security, are necessarily based on a view of fertility as an endogenous phenomenon.

The economic literature on fertility-related fiscal policies goes back at least to Pollak and Wales (1979); other important past contributions are Cigno (1986) and Atkinson and Bourguignon (1989), while, more recently, similar ground has been covered by e.g. Balestrino et al. (2002)

and Cremer et al. (2003). However, these authors argue against or in favour of compensations for large families on the basis of conventional social welfare maximisation arguments. There is also a branch of the literature that looks at the interesting question of the extent to which social welfare maximisation and HE conflict with each other (early contributions on this are King 1983 and Balcer and Sadka 1986; for a more recent take on the subject, see Jordahl and Micheletto, 2005).[1] In the present contribution, however, we ignore social welfare maximisation objectives and the possible ensuing conflict with HE.

What the present paper does, instead, is to evaluate in HE terms the policy that emerges at a political equilibrium, i.e. the policies that are actually feasible as opposed to those that would ideally maximise social welfare. This allows us to make also a contribution to the political economy of family tax policy, which is not a particularly well-researched area. Previous attempts in this direction (such as Bergstrom and Blomquist 1996) have looked at day care, that may be construed as an in-kind transfer to the families with children, but we not aware, to the best of our knowledge, of voting models in which the policy considered is specifically a family tax/transfer scheme.

After setting the stage for the analysis in Section 2, we will develop in Section 3 a median voter model for two alternative scenarios: exogenous and endogenous fertility. In the first scenario, we will face the additional difficulty that the agents differ along two dimensions, i.e. the wage rate and the number of children, a circumstance that will require an adjustment to the standard mode of analysis; in the second, we will be able to apply the model using the normal procedure. We will also consider a mixed case in which some agents control their fertility and other don't. We will compare the results across all these alternatives scenarios, and we will also verify for each of them whether the winning policy satisfies HE or not. We will see that the requirements of HE change radically in different environments: it makes a huge difference whether we believe that fertility is exogenous or endogenous, and also the extent to which the parents value their children matters. A policy that is very far from satisfying HE under exogenous fertility may be able to satisfy it under endogenous fertility. It is therefore crucial to understand which of the various scenarios, if any, is more apt to describe the situation of real-world economies: otherwise, it becomes impossible to make meaningful theoretical statements and issue relevant policy recommendations. This is the task we take up in Section 4, that serves also as the conclusion of the analysis.

[1] This conflict is also touched upon in Balestrino (2000), from which the present paper borrows the model of household choice presented in the next Section.

2 Background: policy instruments and family choices

We construct two versions, each further subdivided into three variants, of a finite-horizon model where only parents and children exist (Nerlove *et al.* 1987; Cigno, 1991). Let us begin with the common background. There is only one parent, who consumes c units of a consumption good, supplies l units of time, and has n children, whose quality (of life) q is the same for all (Willis 1973) and is a function of a children good e and home-time s; we also set a subsistence threshold for quality $\overline{q}$. The price of c equals unity, while that of e is denoted by p. The wage is denoted by w and the time constraint is $ns + l = t$, where t is the time endowment (there is no leisure).

We consider a second-best economy in which the number of children and the income of each household are public information, while the wage rate and the time endowment are taken to be private information.[2] Given this informational structure, it is clear that when we will consider the political process, we will have to assume that each candidate can credibly commit only to linear policies. There will be two policy instruments: a per-child subsidy or demogrant G, whose total amount Gn is clearly conditional on the number of children, and a proportional income tax, whose rate is denoted by θ.

We divide the parent's problem in two stages (Cigno 1991). In the first, she minimises the cost of rearing her n identical children, all of quality q, that is she chooses e and s for each child so as to

$$\min \; pe + (1-\theta)\, ws \text{ s.t. } q\,(e,d) \geq \overline{q}, \;\; (\sigma)\,; \tag{1}$$

the Lagrange multiplier is in parentheses. The first order conditions are

$$p = \sigma \frac{\partial q}{\partial e}; \;\; (1-\theta)\, w = \sigma \frac{\partial q}{\partial e}. \tag{2}$$

Let the minimum value function (expenditure function) descending from this problem be

$$\pi = \pi(\overline{q}; p, (1-\theta)\, w); \tag{3}$$

using the envelope theorem, we find that

$$\sigma\,(p, (1-\theta)\, w) = \frac{\partial \pi}{\partial \overline{q}}. \tag{4}$$

Then, σ can be interpreted as the "price of quality"; if we assume constant returns to scale, it

[2] In tax models, it is customary to assume that neither the wage rate nor the *effort level* (labour supply) are observable. Here, we have no leisure and therefore effort could be identifiable by the government, e.g. by observing families with no children, if the time endowment were publicly known (for in that case, the labour supply would actually equal the time endowment).

can be shown that it is independent of the level of q:[3]

$$\frac{\partial\sigma}{\partial\overline{q}} = 0. \tag{5}$$

We can also define the minimum value function (3) as the "price of quantity"; again using the envelope theorem, we find that

$$\frac{\partial\pi}{\partial p} > 0;\ \frac{\partial\pi}{\partial w} > 0;\ \frac{\partial\pi}{\partial\overline{q}} = 0. \tag{6}$$

This is indeed an intuitively appealing, and at the same time manageable (it avoids non-linearities in the household's problem) framework: the prices of quality and quantity differ across households as long as the wage rates differ, but they are given for any household.

We are now ready to consider the two alternative scenarios, one with endogenous and the other with exogenous fertility, each specialised in three variants depending on whether and how the parent values her children: she might value them as consumption goods, or as investment goods, or not value them at all.

The literature on fiscal compensations for family size tends to focus on either the "exogenous fertility/valueless children" variant or on the "endogenous fertility/valuable children" one: for example, equivalence scales models (Deaton and Muellbauer 1980, ch. 8) fall within the first category, while most of the works following the household economics approach belong to the second category – some taking children as consumption goods (Becker 1991), others as investment goods (Cigno 1991). For completeness, we cover also the other possible combinations, and postpone a discussion of which is the most plausible set of assumptions until the last Section.

2.1 Exogenous fertility

If children are valued as consumption goods, parental utility is $u(c, q; n)$, maximised subject to:

$$c + n\pi(\cdot) = (1-\theta)\,wt + Gn; \tag{7}$$

$$q \geq \overline{q}. \tag{8}$$

where β is the marginal utility of income. In general (8) will not be binding. The first order conditions (FOCs) are

$$\frac{\partial u}{\partial c} = \beta;\ \frac{\partial u}{\partial q} = \beta n\sigma. \tag{9}$$

[3] The proof can be based on the fact that, as a consequence, the marginal productivities are homogeneous of degree zero. Using the same straightforward comparative statics exercise, it also possible to show that technological complementarity ($\partial q/\partial e\partial s \geq 0$) is sufficient (not necessary) to guarantee that $q(\cdot)$ depends positively on P and w.

If children are valued as investment goods (perhaps because of old-age support), the benefit accruing to their parent must be reflected in the budget constraint, e.g. via a function $a(n)$; then, the parent maximises $u(c;n)$ subject to:

$$c + n\pi(\cdot) = (1-\theta)\,wt + Gn + a(n) \tag{10}$$

and to (8), which will now be binding. The parents will therefore set $q = \overline{q}$ and $c = (1-\theta)\,wt + Gn + a(n) - n\pi(\overline{q}; p, (1-\theta)\,w)$.

If children are valueless, parental utility is $u(c)$, to be maximised subject to (7) and (8), which will again be binding. The parent sets $q = \overline{q}$ and $c = (1-\theta)\,wt + n\,(G - \pi(\overline{q}; p, (1-\theta)\,w))$.

In all three cases, indirect utility is identified by substituting the demand functions back into the utility function and is written as

$$v = v\,(p, (1-\theta)\,w, G; n). \tag{11}$$

For future use, remembering that $l = t - ns$, we note that

$$\frac{\partial v}{\partial \theta} = -\beta wl;\ \frac{\partial v}{\partial G} = \beta n, \tag{12}$$

2.2 Endogenous fertility

In this scenario, the parent chooses also n (with certainty)[4]; utility is $u(c,q,n)$ when children are consumption goods or $u(c,n)$ when they are investment goods, and the relevant constraints are (7) or (10) and (8), plus a a physiological ceiling on the number of children, $n \leq m$. This ceiling might or might not bite: if it does, we would return to the exogenous fertility scenario, because we could simply substitute the binding constraint back into the parent's maximisation problem. We could describe the household equilibrium with children as consumption goods by means of the following FOCs:

$$\frac{\partial u}{\partial c} = \beta;\ \frac{\partial u}{\partial q} = \beta\sigma;\ \frac{\partial u}{\partial n} = \beta\,(\pi\,(q; p, (1-\theta)\,w) - G). \tag{13}$$

If children are instead investment goods, we would have $q = \overline{q}$ and

$$\frac{\partial u}{\partial c} = \beta;\ a' = \pi\,(\overline{q}; p, (1-\theta)\,w) - G. \tag{14}$$

[4] The assumption that n is chosen with certainty is not to be taken literally: rather, it is an approximation to the fact that the parents, in the real world, can affect the probability distribution of births by an appropriate choice of instruments for the control of fertility. Models with uncertainty about the survival of children are rare; Sah (1991) and Cigno (1998) provide two examples. We discuss briefly in the last Section of this paper the consequences of relaxing this assumption by allowing for the possibility that agents make mistakes in their attempts at controlling fertility.

Finally, with valueless children, utility is $u(c)$ and the parent faces constraints (7) and (8), plus the fertility ceiling. As long as the price of quantity is positive, we have $n = q = 0$ and $c = wt$.

The indirect utility function under endogenous fertility is derived in the same way as above: however, it will *not* have the number of children among its arguments:

$$v = v(p, (1-\theta) w, G). \tag{15}$$

Correspondingly, the expressions for the derivatives w.r.t. the policy instruments look the same as in (12), but n is now *not* given: it is the demand for children.

3 Horizontal equity at the political equilibrium

The question we want now to answer is how family policy will look like at the political equilibrium. We will also ask whether HE is satisfied; to this end, we need to state the basic requirement of HE as follows:

Definition 1 *Public interventions are said to satisfy HE if those families whose pre-intervention utilities are equal, excluding the irrelevant features, have equal after-intervention utilities, including the irrelevant features.*

To proceed, we must now distinguish the case in which fertility is assumed to be exogenous from that in which is assumed to be endogenous.

3.1 Exogenous fertility

In this scenario, the families differ in two characteristics: their wage rates (say that there are $h = 1...H$ wage groups), and their fertility levels (let us take $i = 1...I$ possible fertility levels). We take fertility levels as "non-relevant" from the perspective of HE, that is we do not want the families to be treated differently according to their size. Then, Definition 1 requires that all households within a given wage group should have the same after-tax utility.

3.1.1 The political equilibrium

Let us begin with identifying the winning policy. In the present setup, there are two policy instruments that can be linked through a budget constraint (see below), so that the policy choice is actually unidimensional; we can further assume that the candidates are office-motivated and that they can credibly pre-commit to policies. This paves the way for a straightforward usage of the median voter theorem: unfortunately, though, the usual procedure for ensuring that the majority voting equilibrium exists is not applicable here. Such a procedure consists of checking whether the marginal rate of substitution between policy tools in the agents' indirect utility functions is monotonic in type or, equivalently, that the indifference curves in the policy space exhibit single-crossing (Gans and Smart 1996). But this presupposes that types are defined along one dimension, typically income or wage rate. Here, we have a differentiation according to both wage rate and the number of children.

In order to appreciate the difficulty more clearly, and to verify if there is way around it, let us now identify the above-mentioned marginal rate of substitution. Using (12), we see that no matter whether children are valueless or valuable, we have that

$$MRS_{\theta G} = -\frac{\partial v/\partial \theta}{\partial v/\partial G} = \frac{wl\left(\theta\right)}{n} > 0. \tag{16}$$

The indifference curves in the policy space are thus positively sloped; the MRS coincides with the gross income per child. For any given w, the slope varies with the number of children; more precisely, the curves become flatter the more children one has, since the denominator increases and the numerator decreases (recall that $l = t - ns$, where s is independent of n). In the standard case in which total gross income wl increases with the wage rate (i.e. $\partial l/\partial w > 0$, increasing labour supply curve), we can also claim that, for any given number of children, the curves become steeper the higher is the wage. Notice also that all the curves for any given agent have the same slope at any given value of θ. More precisely, using again that $\partial l/\partial w > 0$ and therefore $\partial l/\partial \theta < 0$, we have that

$$\frac{\partial MRS_{\theta G}}{\partial \theta} = \frac{w}{n}\frac{\partial l}{\partial \theta} < 0 : \tag{17}$$

the indifference curves become flatter as we move towards higher value of θ – they are concave in the (θ, G)-space.

If we now define the public budget constraint, we can depict the agent's policy problem – her choice of preferred policy. Let there be μ^h parents for each wage group and let $N = \Sigma_h \Sigma_i n^{hi}$ denote the total number of children; then the government's budget is:

$$\theta \Sigma_h \mu^h w^h \Sigma_i l^{hi} = NG, \tag{18}$$

from which we can easily obtain an expression for G:

$$G\left(\theta\right) = \frac{\theta \Sigma_h \mu^h w^h \Sigma_i l^{hi}}{N}. \tag{19}$$

In order to simplify the notation, we let $Y\left(\theta\right) = \Sigma_h \mu^h w^h \Sigma_i l^{hi}$ denote the national income. In the (θ, G)-space, the revenue curve is assumed to have a standard concave inverted-U shape. We have

$$G'\left(\theta\right) = \frac{Y\left(\theta\right)}{N} + \theta\frac{Y'\left(\theta\right)}{N}; \tag{20}$$

hence, the sign of G' depends on that of the revenue elasticity w.r.t. the tax rate, which is commonly taken to be positive up to a point and decreasing afterwards; for future use, let us denote the revenue-maximising value of the tax rate as $\theta^{\max}$. The sign of G'' is assumed to be negative.

We can now identify two groups of agents depending on the relationship between *the indifference curve through the origin* and the revenue curve. There are two possibilities: i) the indifference curve always stays above the revenue curve; ii) the indifference curve cuts the revenue curve from below. It is then clear that the first group prefers no policy, while the second group comprises all those who prefer an active policy of varying intensity. To see this in more detail, compare the slopes of the indifference curves and of the revenue curve at the origin:

$$MRS_{\theta G}|_{\theta=0} = wl\left(0\right)/n;\; G'\left(0\right) = Y\left(0\right)/N. \tag{21}$$

Some agents have a *laissez-faire* gross income per child ($wl\left(0\right)/n$) that is above the economy-wide average *laissez-faire* gross income per child ($Y\left(0\right)/N$); in other words, they have, in the free market, a relatively large income and/or relatively few children. For these agents, the indifference curve that goes through the origin is steeper than the revenue curve. As we move away from the origin, the revenue curve becomes flatter; the indifference curve also becomes flatter, but does so at a slower pace – and in any case it remains positively sloped while the revenue has an inverted-U shape. Then the preferred policy for these agents is $G = \theta = 0$. Among them, we certainly have to include those who have no children, as their indifference curves are straight lines orthogonal to the abscissa in the (θ, G)-space.

Other agents have a relatively low income and/or relatively few children; for them, the indifference curve is flatter than the revenue curve at the origin, but then becomes relatively steeper (i.e., less flat than the revenue curve) and thus cuts through the latter, either to the left or to the right of $\theta^{\max}$. Still exploiting the fact that all indifference curves for any given agent are parallel to each other, it is easy to see in this case that the preferred tax rate is somewhere between 0 and the value of θ for which the indifference curve through the origin cuts the revenue curve. The lower is $wl\left(0\right)/n$, the larger is the preferred tax rate. If the value of θ for which the curves cross is sufficiently to the right of $\theta^{\max}$, the preferred policy is $(\theta^{\max}, G\left(\theta^{\max}\right))$; in fact there will be a critical value of $wl\left(0\right)/n$ below which all parents are bunched at the revenue-maximising policy.

From this discussion, it emerges that we take the free market gross income per child, $wl\left(0\right)/n$, as a synthetic index of the agent's type, we can order the agents themselves from the one with the lowest income per child to the highest, and find that their preferred tax level is monotonic in the index: to each $wl\left(0\right)/n$ corresponds a different slope of the indifference curves – specifically, higher-index agents have steeper indifference curves – and therefore, the higher the index, the lower the preferred tax rate. Assuming that there is an odd number of types, the median voter model then applies in this modified context: a simple majority voting procedure can guarantee the existence of an equilibrium. The tax level preferred by the agent who has the median free market gross income per child wins against any other preferred tax level.

If the median agent happens to belong to the first group, the one that includes all those who prefer no policy, we have no support for families with children at the political equilibrium; if she belongs to the second group, we have an active policy – and in the extreme case in which her income per child is very low, we could even have the most generous policy that is actually feasible.

3.1.2 Horizontal equity

How do we check whether the winning policy satisfies HE? One way would be to identify, for each variant of the household choice model, how total utility changes as the number of children varies. If a marginal child, within each wage group, induces neither utility losses or gains and does so because of the action of policy, then we can say that such a policy satisfies HE. Conversely, if a marginal child does induce losses or gains, policy notwithstanding, we can see how these are distributed among the households.

With valuable children, the utility change associated with an increase in fertility is

$$\frac{\mathrm{d}v^{hi}}{\mathrm{d}n^{hi}} = \beta^{hi}\left(\frac{\partial u^{hi}}{\partial n^{hi}} - \left(\pi^{h}\left(\theta\right) - G\right)\right) \text{ or } \frac{\mathrm{d}v^{hi}}{\mathrm{d}n^{hi}} = \beta^{hi}\left(a' - \left(\pi^{h}\left(\theta\right) - G\right)\right), \tag{22}$$

depending on whether the children are consumption or investment goods.

The sign of these expressions depends on the relative magnitude of the marginal benefit of a child (the willingness to pay for quantity or the marginal rate of return from children) *vis-à-vis* the "consumer" price of a child. In our setup, the marginal benefit is decreasing is n, while π is independent from n. In the free market then, given $\pi > 0$, it is possible to identify a constrained optimum, i.e. the level of n for which the marginal benefit equals the price of quantity ($\partial u/\partial n - \pi = 0$ or $a' - \pi = 0$, both evaluated at $\theta = G = 0$). However, the actual number of children, being exogenously determined, might happen to be less than the constrained optimum or more than that: only by chance a parent might happen to have exactly the optimal number of children.

Under these circumstances, HE requires to equalise utilities within each wage group at the level enjoyed by the parents whose n coincides with the constrained optimum (quite independently from whether they exist or not). Notice that we have emphasised in (22) how the price of quantity depends on θ; since π^h is increasing in the wage rate by (6), it is by the same token decreasing in the income tax rate. Now, the introduction of G presupposes that of θ to finance it (alternatively, if the demogrant is in fact a tax, it might finance a negative rate of income tax). We should therefore characterise the HE-optimal policy by taking into account the two parameters at the same time: they both impact on the after-tax price of quantity.

HE would require then a (θ, G) pair such that $\pi(\theta) - G = 0$ for all the children of all the families. If the actual n falls short of the constrained optimum, we have $\partial u/\partial n - \pi > 0$ or $a' - \pi > 0$ in *laissez-faire*; HE would require $G < 0$, a tax on children that could finance a income subsidy, $\theta < 0$, that in turn would also work in the direction of increasing the price of quantity. If the actual n exceeds the constrained optimum, we have either $\partial u/\partial n - \pi < 0$ or $a' - \pi < 0$, and therefore we should have a per-child subsidy $G > 0$, and an income tax $\theta > 0$: the combined effect of the two policy instruments would be that of reducing the price of quantity. Notice that this subcase covers also the valueless children scenario: when the optimal number of children is zero, any positive number of children is larger than the optimal one.

This means that, in order to satisfy HE, some families should in principle be *compensated* for having *too many* children, while others should be *taxed* for having *too few*. It is clear that the actual policy emerging at the political equilibrium *cannot* satisfy HE. The informational requirements are too demanding. The government should know the wage rate of each household so as to compute the price of quantity π^h, and we ruled that out at the outset. More generally, the logic of the coalitions supporting the winning policy is inconsistent with that of HE, because these coalitions do not necessarily include all those who have the same wage rate. For example, the agents who have no children support a no-policy political equilibrium no matter what their wage rate is; a parent whose wage rate is less than or the same as that of some other parent may however have less children and perhaps favour a less generous policy towards the families, and so on and so forth.

A further question that we may ask is, which family size is favoured at the tax equilibrium? To begin with, notice that $G < 0$ cannot arise at the equilibrium. This circumstance is reflected in the common practice of requesting that large families are compensated, not that small one are taxed: it must be recognised, however, that it further reduces the scope of policy in HE terms. Let then $G^E \geq 0$ be the equilibrium demogrant. With a no-policy equilibrium, $\pi^h - G^E \equiv \pi^h > 0$ for all; HE fails for all wage groups. With an active policy, and given that $\partial \pi/\partial w > 0$, there will be a cut-off value of h above which $\pi^h > G^E$ and below which $\pi^h < G^E$. At the cut-off, $\pi^h - G^E$ will be close to zero, and, by pure chance, it might happen to be actually zero. Therefore, HE is achieved, approximately or exactly, only for the threshold wage group: within that group, no matter how many children you have, your after-tax utility is the same. In all the others groups, parents will be over- or undercompensated for having a non-optimal number of children. For example, if you belong to a low-wage group such that $\pi^h < G^E$, the more children you happen to have, the better (overcompensation); if you belong to a high-wage group, and $\pi^h > G^E$, then any additional child you happen to have represents a cost (undercompensation). If we believe that fertility is exogenous, we must therefore conclude that any policy generates a distinction between some parents whose wage is low enough to make a large family advantageous in fiscal terms and other parents whose wage is too high for that.

3.2 Endogenous fertility

Under endogenous fertility, wage rate differences survive, but the number of children is a choice variable and therefore cannot be taken as a characteristic of the household. Households might have different fertility ceilings, and these could in principle be taken as an additional distinguishing characteristic. Unlike the *actual* number of children, however, the *potential* number of children is not publicly observable. We cannot therefore imagine that the candidates can credibly commit to policies based on whether fertility ceilings are hit or not. It would also be difficult to base HE policies on this. Claims that the fertility ceiling has been hit cannot be costlessly verified, and therefore compensations could not be awarded, due to incentive-compatibility problems. Perhaps, it might be argued that couples in which it is ascertained that the fertility ceiling is zero are necessarily constrained. Still, incentive-compatibility issues cannot be ruled out, because the couple could have desired to have no children anyway; if it were known that zero fertility couples are entitled to a compensation, these could *pretend* to have desired a child.

It would still be true, however, that the impact of policy may vary depending on whether the fertility ceilings are operative or not. Thus, we do have to consider that, in practice, three subcases might arise: i) none of the fertility constraint is binding; ii) some are binding and some are not; iii) they are all binding. Since with valueless children the optimal amount of children is zero, subcases ii) and iii) do not arise for that variant. Also, subcase iii) with valuable children is in fact equivalent to the exogenous fertility scenario, because by substituting the binding constraint back into the parent's optimisation problem we can re-frame it as one of exogenous fertility: the only difference is that families with too many children do no exist when fertility is controlled by the parent with certainty. This leaves us with only two situations to discuss: that in which fertility is fully endogenous, and that in which children are valuable and fertility is endogenous for some of the families and exogenous for others.

3.2.1 No fertility constraint is binding

In this case, only w varies, and therefore it may constitute a valid index for ordering the agents from bottom to top; our setup is simple enough to permit a direct application of the standard median voter model (provided that the number of types H is odd). To see this, consider that the

MRS between policy tools is

$$MRS_{\theta G} = -\frac{\partial v/\partial \theta}{\partial v/\partial G} = \frac{wl(\theta)}{n(\theta)} > 0, \tag{23}$$

and that

$$\frac{\partial MRS_{\theta G}}{\partial w} = \frac{\left(l + w\frac{\partial l}{\partial w}\right)n - \frac{\partial n}{\partial \pi}\frac{\partial \pi}{\partial w}wl}{n^2} > 0, \tag{24}$$

where the sign follows from $\partial l/\partial w > 0$ (increasing labour supply curve) and $\partial n/\partial \pi < 0$ (children as normal goods); we know from (6) that $\partial \pi/\partial w > 0$. The MRS is thus monotonic in type, and the median voter theorem applies.

We will not discuss the details of the policy emerging at the at the political equilibrium. We can notice that also in this case the indifference curve is concave:

$$\frac{\partial MRS_{\theta G}}{\partial \theta} = \frac{w\frac{\partial l}{\partial \theta}n - \frac{\partial n}{\partial \pi}\frac{\partial \pi}{\partial \theta}wl}{n^2} < 0; \tag{25}$$

the sign follows because $\partial l/\partial \theta < 0$, $\partial n/\partial \pi < 0$ and $\partial \pi/\partial \theta < 0$. The situation is thus similar to the one depicted in the exogenous fertility scenario, with the difference that the agents are ordered on the basis of their wage rate w and not on their income per child as above.

From our point of view, the interesting question is whether the equilibrium policy satisfies HE or not. To ascertain this, we need to know very little. There are two possibilities:

i) if children are valueless, no-one will have them. So, the median voter will favour the same policy as everybody else, namely no policy. Now, all the families within each wage group choose the same number of children, i.e. zero, and no compensation is needed to achieve HE. In other words, HE requires $G = \theta = 0$, which is what actually emerges at the political equilibrium.

ii) if children are valuable, typically, the median voter will favour an active policy. But, in this case, the HE-optimal policy is indeterminate: all parents sharing the same wage rate have the same number of children, so no matter how θ and G are fixed, their pre- and post-tax utilities remain equal within the group. Remarkably, then, any policy, including trivially the winning one, satisfies HE.

This outcome is as distant from the one emerging in the exogenous fertility scenario as it can be. There, it was impossible to satisfy HE; here, it is always and effortlessly satisfied. As we warned in the Introduction, the assessment of the capability of any policy to achieve HE varies dramatically with changing assumptions on the nature of fertility.

3.2.2 Some fertility constraints are binding and children are valuable

The analysis becomes more involved when children are valuable, and some families are constrained while others aren't. The complication arises from the fact that we are in a mixed situation: the unconstrained families continue to be distinguished along the wage dimension, but the constrained ones, being in fact in a condition of exogenous fertility, must now be ordered along two dimensions, the wage rate *and* the number of children (see above). Since we took H to be an odd number, in order to keep the number of possible types odd, we will study only the case in which the number of groups that have constrained families is even. For example, there might be three wage groups: in that case, we would have only two of them with constrained families, so that we would in fact have five types. In order to simplify the analysis, we will in fact refer to this example in what follows: the generalisation to unspecified odd and even numbers of, respectively, possible and constrained types is immediate and left to the reader.

In order to understand what happens in this situation, it is important to notice that constrained families have *steeper* indifference curves relative to unconstrained families with the same wage rate. This can be ascertained from a comparison between (17) and (25), and has the following intuitive explanation. Recall that constrained families are just an instance of families whose fertility is exogenous. Now, when fertility is *endo*genous, if θ rises, work becomes less convenient but parenthood becomes cheaper (π goes down): more taxes may reduce your income, but you may partly compensate by increasing the quantity of children. Instead, when fertility is *exo*genous, there is no compensation available from that angle when taxes rise. Therefore, for any given increase in θ, it takes a larger increase in G to keep the agent indifferent (in other words, the indifference curve must be steeper) when fertility is exogenous that when it is endogenous.

The fact that in the mixed case there are types whose indifference curves are steeper suggest that the policy emerging at the political equilibrium might have the same characteristic as in the scenario with no constrained families, or might be less active, in the sense of prescribing a lower income tax rate – and correspondingly a lower demogrant. It is easy to confirm this in our three/five types example. We have three cases to consider:

1. let us compare the political equilibrium without constrained families with that in which the constrained families are those at the two extremes of the wage distribution, i.e. 1 and 3 (from the lowest to the highest). In the fully endogenous fertility scenario, the median type is represented by the parents in the wage group 2; their preferred policy, let us say an active one, is the winner. In the scenario with constrained families, there are five groups: the two extra-types, call them $\widehat{1}$ and $\widehat{3}$, have steeper indifference curves than 1 and 3. Type $\widehat{3}$ poses no problem: she will prefer a less active policy that type 3, and will be the new extreme on that side. Type $\widehat{1}$ is more interesting: she prefers a less active policy than 1, and her indifference curve might in principle be steeper that that of 2, i.e. she might become the new median type. If that is the case, the political equilibrium will exhibit a less active policy than in the fully endogenous scenario; otherwise, the winning policy will be the same;

2. now, let us compare the political equilibrium without constrained families with that in which the constrained families are 1 and 2. It is easy to see that in this case, the outcome is as above;

3. finally, let us compare the political equilibrium without constrained families with that in which the constrained families are 2 and 3. As before, $\widehat{3}$ poses no problem. As for $\widehat{2}$, she will become the new median type, or, in case her indifference curve is especially steep, will leave that role to 3. In either case, the equilibrium policy with constrained families will be less active then when there no constrained families.

What about HE? Within the same wage group, all unconstrained families have the same (optimal) number of children, while the constrained ones have less children (less than the other families, and less than the optimal number). For the former, the marginal benefit for quantity equals its price; for the latter, the marginal benefit exceeds the price. Hence, the former should receive no compensation; the latter should be taxed for having too few children. Instead, they all receive a compensation, actually the same per-child subsidy, no matter whether they are constrained or not. HE is therefore violated. Now, it is true that the actual implementation of the policy required to satisfy HE is virtually impossible: since the fertility ceiling may bite at different level for different families, within as well as across wage rate groups, the taxes should be tailored to each single case, and this is impossible because not only the wage rate, but also the fertility ceiling is unobservable. However, it is remarkable that the policy actually implemented is not even an approximation of what would be required for HE purposes: rather than imposing taxes when needed, it concedes subsidies to everybody.

4 Concluding remarks

It is commonly argued that families with many children should receive fiscal compensation, and the argument is often based on horizontal equity considerations. Our analysis demonstrates that the correct reasoning is much more nuanced than that. Under one set of specific assumptions, namely exogenous fertility and valueless children, compensations for large families are clearly desirable on the basis of HE considerations; if instead children are valuable, HE calls for subsidies to large families and taxes on small families. On the other hand, if fertility is endogenous and children are valueless HE is actually achieved without intervention; but, if children are valuable, any policy satisfies HE. Finally, in the mixed case in which children are valuable and some families can control their fertility while others are constrained, HE allows to make a case for taxing the constrained families (because they are too small). Deciding which policy satisfies HE is no easy matter.

An additional problem with HE-oriented family policies is that they are too informationally demanding. Since the cost of raising children depend chiefly on the shadow-price of time, a correct compensation framework should differentiate the subsidies on the basis of the wage rate – which is instead typically unobservable. If fertility is believed to be endogenous and, realistically, the possibility that some families hit their fertility ceiling is allowed for, the informational requirements become even more difficult to satisfy, as the policy-maker should also be able to distinguish, within a wage group, the constrained families from the unconstrained ones. In the example of the family with no children, the government should separate those who actually have chosen not to procreate from those who have unwittingly failed to do so, something that clearly cannot be done with any satisfactory degree of accuracy.

In fact, we argued that the feasible policies, the ones that might emerge from a voting process at the political equilibrium, have to be based on realistic assumptions about what is publicly observable and what is not; as a consequence, they will in general *not* satisfy HE conditions under exogenous fertility or in the mixed case. They will instead satisfy such conditions trivially in the fully endogenous fertility case with valuable children, but only because in that specific case any family tax policy is equivalent from an HE perspective.

A first conclusion that emerges from the above analysis is that it is impossible to evaluate a family taxation scheme in HE terms without making explicit assumptions on the nature of fertility. We noticed above, at the very end of Section 2, that economics seems to have a schizophrenic attitude in this respect. The literature on equivalence scales is based on the idea that fertility is exogenous and children are valueless, although these assumptions are rarely made explicit: the household economics literature, instead, usually takes for granted that fertility is endogenous and that children are valuable (the debate is mostly on the reason why they are valuable). The authors working along these two lines of research seem to ignore each other: a side-effect, perhaps, of the increased specialisation within the discipline. The question to be settled is therefore: which set of assumptions is more plausible?

As far as the value of children is concerned, the idea that human beings do not value them really seems somewhat implausible. A species whose individuals do not care for their offspring would have disappeared long ago from Earth, annihilated by the natural selection mechanisms. Or, to put it from the opposite perspective, our success as a species in this world must mean that, on average, the individuals care for their offspring. *Why* they care is less important for our present argument: whether the biological impulses convert themselves into selfish and strategic considerations or pure love is not really relevant for our purposes here, although it might matter for other economic analyses. What we require, in fact, is simply that children generate benefits as well as costs, so that it is in principle possible to identify a non-zero optimal number of them.

As far as fertility is concerned, an argument in favour of endogeneity that appears most convincing in its simplicity has been proposed, among others, by Dasgupta (2000): since the total fertility rates that we actually observe never reach the maximum possible rate, it must be the case the households control, to some extent anyway, the number of children they have. Casual observation confirms this, and so do several empirical studies (such as those cited in Jones and Tertilt 2008). Contraceptive methods, including abortion as an *extrema ratio*, have always been available over the ages, indeed since pre-historic times. These methods vary drammatically in effectiveness, but even the most ineffectual and unsafe at the individual level end up exterting some influence on fertility outcomes at the level of the population as a whole.

It seems then reasonable to accept endogenous fertility with valuable children as our working hypothesis. It is however probably excessive to assume that the all the couples are always able to control their fertility completely. In the model, we recognised that some couples might have less children than planned. This seems like a very concrete possibility in some situations. For example, it is well known that, in the Western countries, nowadays couples tend to have their first child later than in the past. Biologically, women reach their fertility peak around the age of 20: our bodies are still, in evolutionary terms, those of our short-lived ancestors, the hunter/gatherers and cave-dwellers of the past.[5] After that, fertility declines steadily: when a couple in which the woman is over 35 attempts a pregnancy, the chances of failure are not negligible. The ever increasing recourse to artificial fertilisation techniques is an indirect proof of this.

It is of course also possible that the couples make mistakes in the opposite direction, to wit that they have more children than planned.[6] We did not consider this case in the model, but it is clear that it can be treated symmetrically. On the one hand, even if we considered the possibility that parents have an extra-child by mistake, this would not alter the set of feasible policies, as these could not be based on the unverifiable claim that the last child was unwanted; on the other hand, there really are couples who have a *larger*-than-optimal number of children, and this should be accounted for, both in the identification of the winning policy and in the evaluation of that policy in HE terms.

In order to proceed in that direction, we then take the following scenario as the most plausible:

[5] This phenomenon, known as "time lag", is recognised in evolutionary biology as one of the commonest constraint on adaptation: since environments change rapidly, while biological evolution proceeds at an immensely slower pace, we, as other animals, are "very probably out of date, built under the influences of genes that were selected in some earlier era when conditions were different" (Dawkins 1982, ch. 3).

[6] There might also be timing errors (i.e. the couple might have a child sooner than planned), but this is irrelevant in our static environment.

the majority of couples control their fertility, but some make mistakes and end up with at least one extra-child, while others cannot achieve their desired number of children Adapting the results in Section 3 above, we can argue immediately that the couples who have more children than planned, if they exist, are the only ones who are treated at the political equilibrium in the way in which HE would recommend. Since we took children to be valuable, the winning policy will presumably be active and therefore all families will receive a compensation for each child: on HE grounds, however, only those who have too many children would be entitled to such a compensation, because only for them the cost of a child at the margin exceeds the benefit (the amount of the compensation will of course be presumably incorrect).

What do we conclude from this? A possible way of reading the analysis so far is to state that the pursuing of horizontal equity in family tax policy is a hopeless task. The policies than can credibly be proposed by the candidates and, as consequence, those that will emerge at the political equilibrium as the winning ones, are informationally constrained in such a way that they cannot be employed for horizontal equity purposes. The analysis, if correct, also says that we should not be surprised to find that actual family taxation schemes fail to achieve HE: indeed, they can't. The extent, and the gravity, of this failure is however something that can only be ascertained empirically. If it were possible to find out exactly how many families end up having the "wrong" number of children in any given society, we could then gauge the degree to which HE hasn't been achieved. In fact, it is the presence of these families that causes the problem: in an ideal world, where all couples have exactly as many children as they wanted, HE would not be an issue. The most promising avenue for pursuing it, therefore, would seem not to tinker with tax instruments, but to implement effective social and health policies that help family planning: on the one hand, it would be useful to minimise unwanted pregnancies, and ideally reduce them to zero, and on the other hand, it would be also useful to remove all the obstacles towards a desired pregnancy. Evidently, all these questions fall well outside the scope of this paper, and of tax analysis in general. It is also obvious that the objectives indicated are highly desirable in themselves, and not merely as a way of helping the achievement of HE. For once, it is however heartening to discover that what would appear as commendable to everybody on the basis of common sense and common decency coincides with an economically sound recommendation. Perhaps, we are not too dismal after all.

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