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Abstract

What has been the overall global welfare impact of the accession to the World Trade Organization of a large country like China, or the global welfare impact of the completion of the Uruguay round of GATT negotiations? Can we come up with a simple user-friendly formula to calculate the global welfare impact of the simultaneous trade liberalization of a number of countries? How sensitive is the answer to the assumption of the trade model? We find a striking answer to these questions. We find that, for a very broad class of models and settings, the global welfare impact of trade liberalization in a country, or a simultaneous liberalization of a number of countries, is given by the same simple formula. We find that the global welfare impact of the simultaneous trade liberalization of different countries only depends on two sets of statistics: (i) the ratio of the value of bilateral trade between each and every pair of trading partners and global income; and (ii) the change in exporting cost for each and every pair of trading partners. Most interestingly, the formula applies to a very broad class of models and settings, which include the general Ricardian model (including, for example, Anderson, 1979, and Eaton and Kortum, 2002), the models of Krugman (1980), Melitz (2003) and its extensions, and the extensions of these models to the multi-sectoral case, multi-factor production technology, multi-stage production, the existence of tradable intermediate goods and the existence of a large outside good sector in each country. We find that global welfare would have been 0.05% lower in the year 2008 if China had not gained accession to the WTO in 2001.

JEL-Code: F100, F120, F130.

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1 Introduction

What has been the overall global welfare impact of the accession to the World Trade Organization of a large country like China, or the global welfare impact of the tariff reductions resulting from the completion of the Uruguay round of GATT negotiations? Can we come up with a simple user-friendly formula to calculate the global welfare impact of the simultaneous trade liberalization of a number of countries? How sensitive is the answer to this question to the assumption of the trade model? This paper tries to answer these questions with the help of recent researches in the properties of the new trade models.

Economists are interested in estimating the gains from trade or trade liberalization for good reasons. Policy makers need to know the magnitudes of the impacts of their policies, such as trade liberalization, establishing trade agreements or accession to the World Trade Organization (WTO). In order to estimate gains from trade or trade liberalization, we need to assume a trade model. Recently, Arkolakis, Costinot & Rodriguez-Clare (AER, forthcoming) (hereinafter referred to as ACR) show that a country's gains from trade (and trade liberalization) conditional on the import penetration ratio and the trade elasticity is the same regardless of what model is used as long as they belong to the same class of trade models, which include Krugman (1980), Melitz (2003), Anderson (1979), and Eaton and Kortum (2002), among others. Specifically, a country's gains from trade are always given by the same formula and is determined by only two sufficient statistics: (i) the share of expenditure on domestic goods, which is equal to one minus the import penetration ratio; and (ii) the elasticity of imports with respect to the variable trade cost. This is no doubt a significant result. However, it no longer holds when one extends these models to multi-sectoral settings. Consequently, it may limit the empirical applicability of the result.[1]

The message that the details of firms' responses are of secondary importance to the estimation of the welfare impact of trade liberalization is also elaborated by Atkeson & Burstein (2010), who find that though changes in trade costs can have a substantial impact on heterogeneous firms' exit, export, and process innovation decisions, the impact of these changes on a country's welfare largely offset each other. In other words, the welfare impact of trade liberalization calculated from the trade models based on monopolistic competition such as Krugman (1980) or Melitz (2003) mainly stems from the direct effect of the reduction in trade costs.

Besides evaluating the gains from trade or trade liberalization of an individual country, we often are interested in estimating the gains from trade liberalization for the world as a whole, i.e. in the global gains from trade liberalization. For example, one might be interested in evaluating the global welfare impact of the accession to the WTO of a large country like China, or the global welfare impact of the completion of the Uruguay round of GATT negotiations. Can we come up with a simple user-friendly formula to calculate the global welfare impact of the simultaneous trade

[1] The fact that the result is not robust to the extension to the multi-sectoral setting is also pointed out by Balistreri, Hillberry and Rutherford (2010, 2011)

liberalization of a number of countries? How sensitive is the answer to the assumption of the trade model? We find a striking answer to these questions. We find that, in a very broad class of models and settings, the global welfare impact of trade liberalization in a country, or the simultaneous liberalization of a number of countries, is given by the same simple formula. Specifically, we find that the change in global welfare resulting from simultaneous small changes in trade costs in different countries is simply given by the sales-share-weighted average of percentage changes in exporting costs for all pairs of trading partners, namely, given by the expression $-\sum_{j=1}^{n}\sum_{i=1}^{n}\frac{X_{ij}}{\sum_{k=1}^{n}Y_k}\widehat{\tau}_{ij}$, where X_{ij} is the value of exports from country i to country j, Y_k is the GDP of country k, and $\widehat{\tau}_{ij}$ is the percentage change in the cost of exporting from i to j. Hence, the global welfare impact of the simultaneous trade liberalization in mulitple countries only depends on two sets of statistics: (i) the ratio of the value of bilateral trade between each and every pair of trading partners and global income; and (ii) the change in exporting cost between each and every pair of trading partners. Note that this expression does not involve the changes in prices in any country, or changes in the extensive margins of any country. In other words, the general equilibrium adjustments of relative prices of factors or goods between countries, or adjustments of extensive margins in the countries have no effect on global welfare. Put differently, the global gains from trade liberalization solely arises from the direct effect of the reductions in trade costs (import price reductions); the indirect effects totally offset each other. Most interestingly, the formula applies to a very broad class of models and settings, which include the general Ricardian model (including, for example, Anderson, 1979, and Eaton and Kortum, 2002), the models of Krugman (1980), Melitz (2003) and its extensions, and the extensions of these models to the multi-sectoral case, multi-factor production technology, and multi-stage production. Finally, it continues to hold in the setting when there are tradable intermediate goods (as in Alvarez and Lucas, 2007) or when relative wages are exogenously determined by the existence of a large outside good sector in every country. Our work is inspired by that of ACR. Yet, our result applies to a considerably broader set of models and settings than theirs.

The structure of this paper is as follow. In section 2, we lay down the framework of the model. In section 3, under a set of assumptions R1, R2 and R3 as given by ACR, we prove that the change in global welfare is given by $-\sum_{j=1}^{n}\sum_{i=1}^{n}\frac{X_{ij}}{\sum_{k=1}^{n}Y_k}\widehat{\tau}_{ij}$. This is satisfied by (i) the general Ricardian model with complete specialization; and (ii) Melitz's (2003) model with Pareto firm productivity distribution. In other words, the bilateral trade volumes between all pairs of trading partners and the changes in trade costs for all pairs of trading partners are two sets of sufficient statistics for calculating the change in global welfare induced by small changes in trade costs. In section 4, we prove that the above equation continues to hold in a number of "multi-X" extensions, e.g. multi-sector models, multi-factor models, and multi-stage Ricardian model proposed by Yi (2003). In section 5, we prove that the equation continues to hold even when there are tradable intermediate goods (as in Alvarez and Lucas, 2007) or when there is a large outside good sector in each country that determines the relative wage there exogenously. Section 6 reports an empirical application of the model to China's accession to the WTO. We find that global welfare would have been 0.05%

lower in the year 2008 if China had not gained accession to the WTO in 2001. The last section concludes.

2 The Basic Model

Before delving into the main result, we first introduce the basic elements of the model, which is by and large adopted from ACR. We consider a world economy consisting of n countries indexed by $i = 1, ..., n$, with a single factor of input, labor, which is inelastically supplied and immobile across countries. There is a continuum of goods indexed by $\omega \in [0, N]$. We use L_i and w_i to denote the total endowment of labor and the wage level in country i respectively.

2.1 Preference, Technology and Market Structure

Preferences: In each country i, the representative agent with Dixit-Stiglitz preferences chooses her consumption bundle to maximize her utility subject to her budget constraint. Her utility is given by

$$U = \left[\int_{\omega\in\Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}} \tag{1}$$

where $q(\omega)$ represents the consumption of variety ω where $\omega \in \Omega = [0, N]$ and N is the exogenous measure of total quantity of varieties. Consequently, we have

$$P_i = \left[\int_{\omega\in\Omega} p_i(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}$$

where P_i is the exact price index in country i and $p_i(\omega)$ is the price of variety ω there.

Technology: For every good $\omega \in [0, N]$, there is a blueprint that can be acquired by one or many firms depending on the market structure (to be described below). For any exporting country i and any importing country j, the blueprint associated with good ω contains a set of destination-specific techniques $t \in \left[\underline{t}, \overline{t}\right]$ that can be used to produce the good in country i to be sold to country j. If a firm from country i uses technology $\mathbf{t} \equiv \{t_j\}$ to produce $\mathbf{q} \equiv \{q_{ij}(\omega)\}$ units of good ω to be sold to country j, its cost function is given by

$$C_i(\mathbf{w}, \mathbf{q}, \mathbf{t}, \omega) = \sum_{j=1}^{n} \left[c_{ij}(w_i, t_j, \omega) q_{ij}(\omega) + f_{ij}(w_i, w_j, t_j, \omega) \mathbf{1}(q_{ij}(\omega) > 0)\right]$$

with

$$c_{ij}(w_i, t_j, \omega) \equiv \tau_{ij} \cdot w_i \cdot \alpha_{ij}(\omega) \cdot t_j^{\frac{1}{1-\sigma}}$$
$$f_{ij}(w_i, w_j, t_j, \omega) \equiv \xi_{ij} \cdot h_{ij}(w_i, w_j) \cdot \phi_{ij}(\omega) \cdot m_{ij}(t_j)$$

where $\mathbf{w} \equiv \{w_j\}$ is the vector of wages, and $c_{ij}(w_i, t_j, \omega)$ represents the constant marginal cost inclusive of trade cost, and $f_{ij}(w_i, w_j, t_j, \omega)$ represents the fixed exporting cost. In the marginal cost and fixed cost functions, $\tau_{ij} \geq 1$ and $\xi_{ij} > 0$ are exogenously given for all blueprints and their changes capture the changes in variable and fixed trade costs. For simplicity, we define $h_{ij}(w_i, w_j) = w_i^{\mu} w_j^{1-\mu}$, which we draw from Arkolakis (2010). The constant μ captures the share of the fixed exporting cost borne by the source country, and $1-\mu$ the share borne by the destination country. The parameters $\alpha_{ij}(\omega)$ and $\phi_{ij}(\omega)$ reflect the heterogeneity across blueprints, and $m_{ij}(t_j)$ reflects endogenous destination-specific technological decision.

Market Structure: We consider two market structures: (i) perfect competition and (ii) monopolistic competition (with either restricted or free entry). Under both market structures, there is a large number of firms, and all goods-markets and labor markets clear. Under perfect competition, firms have free access to all blueprints, and there are no fixed exporting costs.

Under monopolistic competition with restricted entry, we assume that an exogenous number of firms $N_i < N$ each freely obtains monopoly power by freely acquiring a blueprint. The assignment of blueprints to firms is random. Under monopolistic competition with free entry, by contrast, a firm from country i needs to hire F_i units of labor to develop a blueprint, which gives it monopoly power. The measure of goods N_i that can be produced in country i is endogenously determined by the zero profit condition. In equilibrium, the entry cost, $w_i F_i$, is equal to the expected profit of each firm.

2.2 Restrictions

Besides the above structure, we also require the model to satisfy two macro-level restrictions for each country: (i) trade is balanced in the aggregate; (ii) aggregate profit flow in each period is a constant share of GDP. For the monopolistic competition models, the third restriction of "import demand is CES" is also required. In the following, we describe these restrictions in detail.

1. **Trade is balanced.** Let X_{ij} denote the total value of exports from country i to country j. The first macro-level restriction is that the value of imports must be equal to the value of exports:

R1 *For any country* j, $\sum_{i=1}^{n} X_{ij} = \sum_{i=1}^{n} X_{ji}$.

2. **Aggregate profit flow is a constant share of total revenue in each period.** Let Π_j denote country j's aggregate profit flow gross of entry cost. Therefore, total revenue $R_j = \sum_{i=1}^{n} X_{ji}$. The second macro-level restriction is given by

R2 *For any country* j, Π_j / R_j *is constant.*

Under perfect competition, R2 trivially holds since aggregate profits are equal to zero. Under monopolistic competition with and without free entry and constant markup, R2 necessarily holds.

3. The third restriction is

R3 *The import demand system is such that for any importer* j *and any pair of exporters* $i \neq j$

and $i' \neq j$, $\varepsilon_j^{ii'} \equiv \frac{\partial \ln(X_{ij}/X_{jj})}{\partial \ln \tau_{i'j}} = \varepsilon < 0$ *if* $i = i'$ *and zero otherwise.*

Each elasticity $\varepsilon_j^{ii'}$ captures the percentage change in the relative imports from country i in country j associated with a small change in the marginal trade cost between country i' and j, holding wages and the measure of goods that can be produced in each country fixed. Under perfect competition, R3 implies complete specialization in the sense that for all i and $i' \neq i$, and j, the measure of goods in $\Omega_{ij} \cap \Omega_{i'j}$ must be equal to zero, where Ω_{ij} is the set of goods exported from i to j. However, under perfect competition, we do not need **R3** to obtain our result. We only need complete specialization. Under monopolistic competition, R3 implies that the measure of firms that are indifferent about selling in a particular market must be equal to zero.

3 Global Welfare Impact of Trade Liberalization

The welfare of country j is given by $U_j = Y_j/P_j$, where P_j denotes the exact price index faced by the representative consumer in country j. We will show below that the change in global welfare (expressed as the GDP-weighted sum of the percentage change in welfare of individual countries) resulting from small changes in trade costs is a sales-share-weighted average of changes in trade costs:

$$\sum_{j=1}^{n} s_j \widehat{U}_j = -\sum_{j=1}^{n} \sum_{i=1}^{n} \frac{X_{ij}}{\sum_{k=1}^{n} Y_k} \widehat{\tau}_{ij} \tag{2}$$

where $s_j = \frac{Y_j}{\sum_{k=1}^{n} Y_k}$ is country j's share of global income. In the rest of this section, we only outline the proofs of this result, while relegating the detailed proofs to the appendix.

3.1 One-sector model under perfect competition

In this subsection, we analyze the change in global welfare under perfect competition. In contrast to ACR, for models with perfect competition, we only need to assume complete specialization instead of R3. Complete specialization means that, from the point of view of any representative consumer in any country, the intersection of the sets of goods that originate from different countries is a set with zero measure. In the appendix, we prove the following proposition.

Proposition 1 *Suppose that the market structure is perfect competition. Under trade balance and complete specialization, the change in global welfare associated with small changes in trade costs is given by equation (2).*

Under complete specialization, as each good consumed in a country is solely provided by a single country (domestic or foreign), the change in its price just depends on the change in the unit cost (wage of the producing country) and the trade cost. Other than that, the "specialization effect", which reflects a change in the pattern of specialization, does not affect the price index.[2] Hence, we can express the percentage change in a consumer's real income as:

$$\widehat{w}_j - \widehat{P}_j = -\textstyle\sum_{i=1}^{n} \lambda_{ij} \left(\widehat{\tau}_{ij} + \widehat{w}_i - \widehat{w}_j \right) \tag{3}$$

where $\lambda_{ij} \equiv X_{ij}/Y_j$ is country j's share of expenditure on goods originating from country i and consumed in country j. The first term on the right hand side (RHS), $\sum_{i=1}^{n} \lambda_{ij} \widehat{\tau}_{ij}$, captures the direct effect of changes in trade costs on welfare in country j, which we refer to as "direct price effect". The second term, $\sum_{i=1}^{n} \lambda_{ij} \left(\widehat{w}_i - \widehat{w}_j \right)$, captures the change in purchasing power induced by the general equilibrium adjustments in relative wages, which we refer to as "relative wage effect".

From the perspective of global welfare, the relative wage effects have a zero sum character, as a consequence of trade balance. The change in the wage level of a country can affect other countries' welfare through its exports to these countries. However, from the global welfare point of view, these effects will be offset by the change in this country's welfare caused by changes in other countries' wages, through imports. As a result, we have $\sum_{j=1}^{n} \sum_{i=1}^{n} s_j \lambda_{ij} \left(\widehat{w}_i - \widehat{w}_j \right) = 0$. This result plays a crucial role in obtaining equation (2). Here is a brief description of the proof:

Step 1: *The change in real income of a country is given by:*

$$\widehat{U}_j = \widehat{w}_j - \widehat{P}_j$$

Step 2: Because of complete specialization, *the change in the consumer price index is given by:*

$$\widehat{P}_j = \textstyle\sum_{i=1}^{n} \lambda_{ij} \widehat{c}_{ij} = \sum_{i=1}^{n} \lambda_{ij} \left(\widehat{w}_i + \widehat{\tau}_{ij} \right)$$

As shown in ACR, this can be easily shown to be satisfied by, for example, the Eaton-Kortum (2002) model.

Step 3: From the equations in steps 1 and 2, we can show that the *change in global welfare is given by equation (2).*

The key in the proof of this step is that trade balance $\sum_{i=1}^{n} X_{ij} = \sum_{i=1}^{n} X_{ji}$ (R1), together with the facts that $s_j = \frac{Y_j}{\sum_{k=1}^{n} Y_k}$ and $\lambda_{ij} = \frac{X_{ij}}{Y_j}$, imply that

$$\textstyle\sum_{j=1}^{n} \sum_{i=1}^{n} s_j \lambda_{ij} \left(\widehat{w}_i - \widehat{w}_j \right) = 0 \quad \text{i.e. indirect effects sum to zero.}$$

[2] The specialization effect works through a change in the pattern of specialization. This term is attributed to Wilson (1980). For example, the set of goods produced by Home only (or Foreign only) will change during trade liberalization, leading to welfare effect on each country. (This happens in, say, Dornbusch-Fisher-Samuelson (1977)). Complete specialization can avoid discontinuity in the effect of specialization on welfare under a small change in trade cost.

Therefore,

$$\sum_{j=1}^{n} s_j \widehat{U}_j = -\sum_{j=1}^{n}\sum_{i=1}^{n} s_j \lambda_{ij} \widehat{\tau}_{ij} = -\sum_{j=1}^{n}\sum_{i=1}^{n} \frac{X_{ij}}{\sum_{k=1}^{n} Y_k} \widehat{\tau}_{ij}$$

As a result, we have Proposition 1. The complete offset of the relative wage effects from different countries is a result of balanced trade, which plays an important role in this paper.

Examples of models to which this proposition applies: Dornbusch-Fisher-Samuelson (1977); Anderson (1979), and Eaton and Kortum (2002).

3.2 One-sector model under monopolistic competition

In this subsection, we analyze the change in global welfare under monopolistic competition resulting from simultaneous changes in exporting costs for a number of pairs of trading partners. In the appendix, we prove the following proposition.

Proposition 2 *Suppose that the market structure is monopolistic competition. Under R1, R2 and R3, the change in global welfare associated with small changes in trade costs is given by equation (2).*

Under R1, R2 and R3, we can write country j's welfare as:

$$\widehat{w}_j - \widehat{P}_j = -\sum_{i=1}^{n} \lambda_{ij} \left[\widehat{\tau}_{ij} + \left(\frac{\mu}{\sigma - 1} + \frac{\mu}{\varepsilon} + 1 \right) (\widehat{w}_i - \widehat{w}_j) + \frac{\widehat{N}_i}{\varepsilon} \right] \tag{4}$$

where ε is the elasticity of trade as defined in R3. The first term inside the squared brackets of (4) captures the direct price effect. The second term there captures the relative wage effect, while the third term captures the firm number effect, which will be explained below. Comparing this expression with equation (3), we have one more indirect effect, namely that of changes in firm numbers, which are induced by entry and exit of firms due to changes in trade costs. Hereinafter, we shall refer to this effect as the "firm number effect". The number of firms in each country is constant either because it is exogenously determined under restrictive entry or it is unchanged in equilibrium under free entry with a fixed overhead cost in each period. Therefore, in equilibrium, $\widehat{N}_i = 0$; so the "firm number effect" for an individual country is zero. Therefore, only the "direct price effect" and "relative wage effect" affect the welfare of an individual country.[3], [4] From the global welfare point of view, however, the relative wage effects from different countries have a zero-sum character as a result of trade balance. Therefore, only the direct price effect remains to be relevant for determining the global welfare impact of trade liberalization. Hence, we have Proposition 2. The following is an outline of the proof.

Step 1: *The change in real income in country j is given by $\widehat{U}_j = \widehat{w}_j - \widehat{P}_j$.*

Step 2: Totally differentiating the price index equation given in subsection 2.1 yields an expression for *the change in consumer price index, given by:*

$$\widehat{P}_j = \sum_{i=1}^{n} \frac{\lambda_{ij}}{1-\sigma-\gamma_j} \left\{ \left[1-\sigma-\left(\frac{\mu}{\sigma-1}+1\right)\gamma_{ij}\right]\widehat{w}_i + \left(\frac{\mu}{\sigma-1}\right)\gamma_{ij}\widehat{w}_j + (1-\sigma-\gamma_{ij})\,\widehat{\tau}_{ij} + \widehat{N_i} \right\}$$

where γ_{ij} is the elasticity of extensive margin with respect to the variable cost, and $\gamma_j = \sum_{i=1}^{n} \lambda_{ij}\gamma_{ij}$.

Step 3: Since R3 implies that $\gamma_{ij} = 1-\sigma-\varepsilon$ (which is a constant) for all i, *the change in the consumer price index can be simplified to equation (4).* The fact that γ_{ij} is a constant is important in obtaining the final result.

Step 4: R1 and R2 implies that *the change in the number of goods that can be produced is given by* $\widehat{N_i} = 0$ for both restricted entry and free entry. This directly follows from ACR.

Step 5: Like in perfect competition, trade balance (R1) implies that $\sum_{j=1}^{n}\sum_{i=1}^{n} s_j\lambda_{ij}\left(\widehat{w}_i - \widehat{w}_j\right) = 0$; therefore *the change in global welfare is given by (2).* In other words, the relative wage effect is equal to zero in the aggregate.

Examples of models to which this proposition applies are Krugman (1980), Melitz (2003) and Arkolakis (2010).

The proofs for the perfect competition model and monopolistic competition model form the basis of the proofs in the extensions discussed in sections 4 and 5 below. Because of the tedious nature of the proofs, We only outline them below while relegating the detailed proofs to the appendix.

[3] In Krugman's (1980) monopolistic competition model, $\varepsilon = 1-\sigma$, equation (4) is reduced to equation (3). Based on Melitz's (2003) model with Pareto distribution of firm productivity, $\varepsilon \neq 1-\sigma$, yet the changes in the magnitudes of the relative wage effects, compared with the perfect competition models, are also small. This is because that the trade elasticity $\varepsilon = -\gamma$, where γ is is the shape parameter of Pareto distribution. According to Axtell (2001), $\frac{-\varepsilon}{\sigma-1}$ is close to one. Hence, the difference in the relative wage effect between perfect competition and monopolistic competition, given by $\left(\frac{\mu}{\sigma-1}+\frac{\mu}{\varepsilon}\right)(\widehat{w}_i - \widehat{w}_j)$, is close to zero. This result also implies that, consistent with Atkeson & Burstein (2010), small changes in trade barriers usually have similar aggregate effects on welfare for each individual country in models with and without firm heterogeneity.

[4] However, the firm number effect for an individual country will not be zero under the multi-sector extension, as we shall see in section 4.1.

4 Global Welfare Impact: "Multi-X" Extensions

In this section, we consider a number of extensions of the basic models in the last section by introducing "multi-X" extensions in the model: the cases with multiple sectors and multiple factors, and the Ricardian model with multiple stages of production. We conclude that Propositions 1 and 2 continue to hold under these settings. In other words, the structure of the trade model does not affect the property that the change in global welfare only depends on the direct effect of the reduction of trade costs. The proofs in these cases are fundamentally similar to those explained in the last section. Trade balance is crucial in the complete offset of the relative wage effects from different countries with each other.

4.1 Multiple Sectors

Suppose that goods $\omega \in \Omega$ are separated into $s = 1, ..., S$ groups of goods, each of which is denoted by the set Ω^s, which we refer to as sector s. Consumers in country j have their preferences represented by the following utility function:

$$U_j = \prod_{s=1}^{S} [U_j(s)]^{\eta^s}$$
$$\text{where} \quad U_j(s) = \left[\sum_{i=1}^{n} \int_{\omega \in \Omega_i} q_{ij}^s(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}},$$

$0 \leq \eta^s \leq 1$ denotes the (constant) share of expenditure on goods in sector s in any country, $q_{ij}^s(\omega)$ denotes the consumption of variety ω in sector s in country j of goods originating from country i. Now, we assume that our macro-level restrictions apply to the sectoral level variables, namely, total profit of sector s of country j, Π_j^s; total revenue of sector s of country j, R_j^s; total value of sector s's goods exported from country i to country j, X_{ij}^s. Like ACR, R2 and R3 are now modified to:

RM2: *For any country* j and any sector s, Π_j^s / R_j^s *is constant.*

RM3: The bilateral value of exports from country i to country j in sector s, X_{ij}^s, satisfies:

$$\frac{\partial \ln(X_{ij}^s / X_{jj}^s)}{\partial \ln \tau_{i'j}^{s'}} = \varepsilon \text{ if } s' = s \text{ and } i' = i$$
$$\frac{\partial \ln(X_{ij}^s / X_{jj}^s)}{\partial \ln \tau_{i'j}^{s'}} = 0 \text{ otherwise}$$

Furthermore, when we refer to restricted entry under monopolistic competition in country j, we assume that there is an exogenous number of firms, N_j^s, each of which freely receives monopoly power as a result of freely acquiring ownership of a blueprint in sector s. Under monopolistic competition with free entry, by contrast, a firm in any sector in country i needs to employ F_i units of labor to develop a blueprint, which gives it monopoly power. Thus, the entry cost is $w_i F_i$.[5] We prove the following proposition in the appendix.

[5] Our results continue to hold if we assume the entry costs are different across sectors.

Proposition 3 *If R1 holds, and* (i) *there is complete specialization for models under perfect competition, or* (ii) *RM2 and RM3 hold for models under monopolistic competition, the change in global welfare associated with small changes in trade costs across countries is given by*

$$\sum_{j=1}^{n} s_j \widehat{U}_j = -\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{s=1}^{S} \frac{X_{ij}^s}{\sum_{k=1}^{n} Y_k} \widehat{\tau_{ij}^s}.$$

Therefore, if we assume that $\tau_{ij}^s = \tau_{ij}$ for any $s = 1, ..., S$, the change in global welfare again reduces to (2).

Here we outline the intuition leading to Proposition 3 by way of comparison with the one-sector models. Under perfect competition, the proof of Proposition 3 is almost identical to that of the proof of Proposition 1, and so will not be elaborated here. Under monopolistic competition, the change in welfare in sector s is given by

$$\sum_{s=1}^{S} \eta^s \left[\widehat{w}_j - \widehat{P}_j(s)\right] = -\sum_{s=1}^{S} \eta^s \sum_{i=1}^{n} \lambda_{ij}^s \left[\widehat{\tau}_{ij}^s + \left(\frac{\mu}{\sigma-1} + \frac{\mu}{\varepsilon} + 1\right)(\widehat{w}_i - \widehat{w}_j) + \frac{\widehat{N_i^s}}{\varepsilon}\right]$$

It is instructive to compare this equation with (4) in the one-sector monopolistic competition model. One important difference between the two equations is that the firm number effect on individual country's welfare captured by $\widehat{N_i^s}$ here is not zero anymore under free entry with a fixed overhead cost in each period.[6] In other words, $\widehat{N_i^s} \neq 0$ under free entry, as resources reallocate across sectors as trade costs change. In general, the change in firm number in a sector may have a positive or negative welfare effect on a country. As shown in Fan, Lai & Qi (2011), a small *reduction* in trade cost in a country *may* lead to a *decrease* in national welfare in a multi-sectoral framework when the inter-sectoral resource reallocation effect (or firm number effect) dominates the import price reduction effect (or direct price effect). From the global welfare point of view, however, in addition to the complete offset of the relative wage effects from different countries, the firm number effects of different countries also completely offset each other. This leaves the direct price effect as the only relevant effect on global welfare. The mechanism is the following. An increase in firm number in a country increases the consumer welfare in other countries as the variety of goods available for the latter increases. Quantitatively, the global welfare impact of an increase in firm number in sector s of country i is equal to $\left(R_i^s \cdot \widehat{N_i^s}\right) / \sum_{k=1}^{n} Y_k$. Therefore, the total global welfare effect of changes in firm numbers in all sectors of country i is equal to $\sum_{s=1}^{S}\left(R_i^s \cdot \widehat{N_i^s}\right) / \sum_{k=1}^{n} Y_k$, which is equal to $\sum_{s=1}^{S} R_i^s \left(\widehat{R_i^s} - \widehat{w}_i\right) / \sum_{k=1}^{n} Y_k$, as $\widehat{R_i^s} = \widehat{N_i^s} + \widehat{w}_i$ due to RM2 and the free entry condition.[7] The trade balance condition, which is equivalent to total expenditure $w_i L_i$ equals total revenue $\sum_{s=1}^{S} R_i^s$, implies that $\widehat{w}_i = \left(\sum_{s=1}^{S} R_i^s \cdot \widehat{R_i^s}\right) / \sum_{s=1}^{S} R_i^s$, which in turn implies that $\sum_{s=1}^{S} R_i^s \left(\widehat{R_i^s} - \widehat{w}_i\right) = 0$.[8] Hence, the firm number effects across sectors within country i offset each other.

[6] Under restrictive entry, N_i^s is exogenous; therefore the firm number effect on individual country's welfare is zero.

[7] RM2 and free entry condition implies that $\varsigma R_i^s = \Pi_i^s = N_i^s w_i F_i$, which in turn implies that $\widehat{R_i^s} = \widehat{N_i^s} + \widehat{w}_i$.

[8] This means that the global welfare impact of the change in firm number in each individual country induced by trade liberalization in multiple countries is equal to zero.

Examples of models to which this equation applies are: multi-sector Melitz, Hsieh and Ossa (2011), Okubo (2008), multi-sector Eaton-Kortum, Chor (2010), Donaldson (2010), Costinot, Donaldson and Komunjer (2010).

4.2 Multiple Factors and Sectors

In the previous sections, the only factor input is labor. Here we consider multiple factor inputs. We assume that the factor endowment in country i is given by L_{ki}, where $k = 1, ..., K$ denote indexes of factors. Suppose that goods $\omega \in \Omega$ are divided into $s = 1, ..., S$ groups of goods, each of which is denoted by Ω^s, which we refer to as sector s. The representative consumer in country j has preferences given by the following utility function:

$$U_j = \prod_{s=1}^{S} [U_j(s)]^{\eta^s}$$
$$\text{where} \quad U_j(s) = \left[\sum_{i=1}^{n} \int_{\omega \in \Omega_i} q_{ij}^s(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}},$$

$0 \leq \eta^s \leq 1$ denotes the (constant) share of consumption expenditure in any country on goods in sector s; $q_{ij}^s(\omega)$ denotes the consumption of variety ω in sector s exported from country i to country j. Note that $\sum_{s=1}^{S} \eta^s = 1$. The production function of each good in sector s exported from country i to country j is given by $f^s(l_1, ..., l_K) = A_{ij}(\omega) \prod_{k=1}^{K} (l_k)^{\beta_k(s)}$, where $\beta_k(s)$ is the cost share of factor k in sector s so that $\sum_{k=1}^{K} \beta_k(s) = 1$ for all s; l_k is the input of factor k; $A_{ij}(\omega)$ denotes the TFP corresponding to country pair $\{i, j\}$. In order to be consistent with the one-sector model, the cost function for producing quantities $\mathbf{q} \equiv \{q_{ij}^s(\omega)\}$ is given by

$$C_i^s(\mathbf{w}, \mathbf{q}, \mathbf{t}, \omega) = \sum_{j=1}^{n} \left[c_{ij}^s(v_i^s, t_j, \omega) \cdot q_{ij}^s(\omega) + f_{ij}(v_i^s, t_j, \omega) \cdot \mathbf{1}\left(q_{ij}^s(\omega) > 0\right)\right]$$

where the marginal cost $c_{ij}^s(v_i^s, t_j, \omega)$ and the fixed exporting cost $f_{ij}(v_i^s, t_j, \omega)$ satisfy:

$$c_{ij}^s(v_i^s, t_j, \omega) = \tau_{ij}^s v_i^s \alpha_{ij}(\omega) t_j^{\frac{1}{1-\sigma}}$$
$$f_{ij}(v_i^s, t_j, \omega) = \xi_{ij} v_i^s \phi_{ij}(\omega) m_{ij}(t_j)$$

where $v_i^s = \prod_{k=1}^{K} \left(\frac{w_{ki}}{\beta_k(s)}\right)^{\beta_k(s)}$ is the marginal factor cost in sector s of country i. As far as market structure is concerned, the results for perfect competition and for monopolistic competition with restricted entry are the same as in the previous models. Under monopolistic competition with free entry, each firm from country i needs to pay $v_i^s F_i$ to acquire a blueprint. We prove the following proposition in the appendix.

Proposition 4 *If R1 holds, and (i) there is complete specialization for models under perfect competition, or (ii) RM2 & RM3 hold for models under monopolistic competition, then the change in global welfare associated with small changes in trade costs is given by equation (2).*

Compared with the single-factor models, each country's welfare is related to the relative marginal factor cost for producing each good v_i^s instead of relative wage w_i. Like the relative wage effects, these relative marginal factor cost effects offset each other as a result of trade balance.

Examples of models to which this equation applies are: Bernard, Redding and Schott (2007), Burstein and Vogel (2010), Heckscher-Ohlin models such as the 2x2x2 model under perfect competition and DFS (1980), which features complete specialization by both countries.[9]

4.3 Multiple-stage Production under Perfect Competition

In the previous sections, all goods are for final consumption. In the present extension we allow some goods to be used only as intermediate inputs into the production of other goods. Each country produces a single nontraded final good. The final good is produced in three sequential stages. The final good is assembled from a continuum of stage-2 outputs $\omega \in \Omega$. The production of stage-2 output ω requires the input of stage-1 output ω and labor. The production of stage-1 output requires labor. All stage-1 and stage-2 outputs are tradable, and all countries possess the technologies of production for the first two stages. The production function in the first stage is given by

$$y_{1i}(\omega) = A_{1i}(\omega)\, l_{1i}(\omega)$$

where $y_{1i}(\omega)$ is the quantity of stage-1 output ω, $A_{1i}(\omega)$ is country i's TFP associated with the production of stage-1 output ω, and $l_{1i}(\omega)$ is country i's labor used in producing the stage-1 output ω. The stage-1 output ω is used as an intermediate input in the production of the stage-2 output ω; a continuum of the stage-2 outputs $\omega \in \Omega$ are assembled into the non-traded final good. The stage-1 output ω and labor are combined in a nested Cobb-Douglas production function at the second stage to produce the stage-2 output ω:

$$y_{2i}(\omega) = x_{1i}(\omega)^{\theta}\left[A_{2i}(\omega)\, l_{2i}(\omega)\right]^{1-\theta} \quad \text{where } \omega \in \Omega$$

where $y_{2i}(\omega)$ is the quantity of stage-2 output ω, $A_{2i}(\omega)$ is country i's TFP associated with the production of stage-2 output ω, $l_{2i}(\omega)$ is country i's labor input in the production of stage-2 output ω, $x_{1i}(\omega)$ is country i's employment of the stage-1 output ω, and the constant θ represents the cost share of the intermediate input. In the final stage, stage-2 outputs are assembled into the nontraded final good using a constant elasticity of substitution (CES) technology:

$$Y_i = \left[\int_{\omega\in\Omega} x_{2i}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}},$$

where $x_{2i}(\omega)$ is country i's use of the stage-2 output ω and σ is the elasticity of substitution between the inputs in this stage. The consumer price index in country j, which can be derived from the

[9] Note that in the standard 2x2x2 model, the elasticity of extensive margin with respect to variable trade cost is equal to zero, as both countries produce both goods before and after trade liberalization.

utility function, is therefore given by

$$P_j = \left[\sum_{i=1}^{n} \int_{\omega \in \Omega_{ij}} p_{ij} (\omega)^{1-\sigma} \, d\omega \right]^{\frac{1}{1-\sigma}}$$

We prove the following proposition in the appendix.

Proposition 5 *Under R1, complete specialization and multi-stage production under perfect competition, the change of global welfare associated with small changes in trade costs is given by equation (2).*

When there are many stages of production, the effect of a small change in trade cost in each stage will accumulate to affect the consumer price index, and this direct effect in each stage is only related to the trade value and trade cost in that stage. Again, balance of trade implies that the relative wage effects offset each other from the global welfare point of view. As a result, the change in global welfare resulting from small changes in trade costs is only affected by the cumulative trade value in all stages between each pair of trading partners and changes in trade costs in each and every country. It is noteworthy that this result applies to a model with more than three stages, though we only discuss the three-stage example here for simplicity of exposition.

An example of model to which this result applies is Yi (2003).

5 Global Welfare Impact: Other Extensions

In the previous sections, we show that Propositions 1 and 2 hold in the extensions to the "multi-X" cases. In this section, we analyze whether Propositions 1 and 2 continue to hold when there are tradable intermediate goods or when there is a large outside-good sector in each country that exogenously fixes the relative wages.

5.1 Tradable Intermediate Goods

In this subsection, we assume that the good $\omega \in \Omega$ can either be consumed as final good or used as input in the production of other goods in Ω, which can in turn be either consumed or used as input in the production of other goods in Ω. As in Eaton and Kortum (2002), Alvarez and Lucas (2007), Atkeson and Burstein (2010), we assume that all goods $\omega \in \Omega$ can either be used to produce a unique non-tradeable final good or used as intermediate inputs. Thus, P_j now represents both the consumer price index in country j and the price index of each intermediate good in this country. There is only one factor input, labor. Accordingly, the marginal cost for each good is given by

$$c_{ij} (w_i, P_i, t_j, \omega) \equiv \tau_{ij} w_i^{\beta} P_i^{1-\beta} \alpha_{ij} (\omega) \, t_j^{\frac{1}{1-\sigma}}$$

where it is assumed that the production of the final good is Cobb-Douglas with labor cost share β and intermediate good share $1-\beta$. To be consistent with section 2, we keep all other assumptions unchanged. In this environment, Propositions 1 and 2 continue to hold. We prove the following proposition in the appendix.

Proposition 6 *If R1 holds, and* (i) *there is complete specialization for models under perfect competition, or* (ii) *R2 and R3 holds for models under monopolistic competition, the change in global welfare associated with small changes in trade costs is given by equation (2).*

The intuition is as follow. The change in the real income in country j is given by

$$\widehat{U_j} = \widehat{w}_j - \widehat{P}_j = -\sum_{i=1}^{n} \lambda_{ij} \left[\kappa \cdot (\widehat{w}_i - \widehat{w}_j) - (1-\beta) \left(\widehat{w}_i - \widehat{P}_i \right) + \widehat{\tau}_{ij} \right]$$

where $\kappa = 1$ under perfect competition and $\kappa = \frac{\mu}{\sigma-1} + \frac{\mu}{\varepsilon} + 1$ under monopolistic competition. Therefore, the change in global welfare is given by

$$\begin{aligned} \sum_{j=1}^{n} s_j \widehat{U}_j &= -\sum_{j=1}^{n} \sum_{i=1}^{n} s_j \frac{\lambda_{ij}}{\beta} \left[\kappa \cdot (\widehat{w}_i - \widehat{w}_j) + \widehat{\tau}_{ij} \right] \\ &= -\sum_{j=1}^{n} \sum_{i=1}^{n} \frac{s_j}{\beta} \lambda_{ij} \widehat{\tau}_{ij} \quad \text{as } \sum_{j=1}^{n} \sum_{i=1}^{n} s_j \lambda_{ij} \left(\widehat{w}_i - \widehat{w}_j \right) \text{ due to trade balance} \\ &= -\sum_{j=1}^{n} \sum_{i=1}^{n} \frac{X_{ij}}{\sum_{k=1}^{n} Y_k} \widehat{\tau}_{ij} \end{aligned}$$

From these expressions, we can see that the change in real income in country j increases by $1/\beta$ times, compared with the model without tradable intermediate goods. At the same time, the trade volumes also increase by $1/\beta$ times. Hence, the same equation (2) captures the total global welfare impact of trade liberalization.

5.2 Model with a large outside-good sector

In this subsection, we assume that there are two sectors: sector H (the "outside good sector"), which can be a homogeneous-good sector or non-traded-good sector, and sector D, which encompasses a set of differentiated products. There is perfect competition in sector H, where production is characterized by constant returns to scale. If we interpret sector H as a homogeneous good sector, then we assume that all countries produce this good in equilibrium, and that the trade cost for this good is zero. [10] In the differentiated-good sector, there are two possible market structures: perfect competition and monopolistic competition. Under monopolistic competition with restricted entry, we assume that there is an exogenous number of firms, N_{Di}, in the differentiated good sector of

[10] When the sector H is interpreted as a homogeneous-good sector, its existence in all countries will be satisfied if the demand for the homogeneous good in all countries are sufficiently high, i.e. b (the share of expenditure on the homogeneous good) is sufficiently large.

country i. Under monopolistic competition with free entry, the assumptions are the same as in the basic model. The utility function for the representative consumer in country j is given by

$$U_j = H_j^b D_j^{1-b}$$
$$D_j = \left[\sum_{i=1}^n \int_{\omega \in \Omega_i} q_{ij}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}}$$

where $1-b$ is the constant share of expenditure on differentiated goods, $q_{ij}(\omega)$ is as defined in the basic model. Now, we assume that RM2 and RM3 hold in the differentiated-good sector. In other words, the following two assumptions will replace RM2 and RM3:

RH2: *For any country j, Π_{Dj}/R_{Dj} is constant in the differentiated-good sector.*

RH3: *In the differentiated-good sector, the import demand system is such that for any importer j and any pair of exporters $i \neq j$ and $i' \neq j$, $\varepsilon_{sj}^{ii'} = \frac{\partial \ln(X_{Dij}/X_{Djj})}{\partial \ln \tau_{i'j}} = \varepsilon < 0$ if $i = i'$ and zero otherwise.*

If sector H is interpreted as a homogeneous-good sector, then the change in global welfare associated with small changes in trade costs is only dependent on the changes in trade volumes and trade costs in the differentiated-good sector, as the trade cost for the homogeneous-good is already zero as assumed. It can be easily shown that the same qualitative outcome will be obtained if the outside-good sector is a differentiated-good sector too, as long as trade cost for this sector is zero. However, the global welfare impact of a small change in each country's firm number is not zero. This is because the resource allocation between the homogeneous-good sector and the differentiated-good sector induces a firm number effect that does not exist in other models presented earlier in this paper. However, the total global welfare impact of small changes in firm numbers in all countries is still zero.[11]

If sector H is interpreted as a non-traded-good sector, the change in global welfare associated with small changes in trade costs is also only dependent on the changes in trade volumes and trade costs in the differentiated-good sector. This case is equivalent to one where the outside-good sector is a differentiated-good sector where trade cost is infinite.

We can prove the following proposition.[12]

Proposition 7 *If R1 holds, and (i) there is complete specialization for models under perfect competition, or (ii) RH2 & RH3 hold for models under monopolistic competition, the change in global*

[11] The total expenditure share on the differentiated goods in the world is $1-b$, i.e., $\sum_{j=1}^n R_j = (1-b)\sum_{j=1}^n w_j L_j$. This implies that $\sum_{j=1}^n \frac{R_i}{Y_i}\left(\widehat{R_i} - \widehat{w}_i\right) = 0$. Therefore, the total global welfare effect of the changes in firm numbers in all countries in the differented-good sector, $\sum_{i=1}^n \frac{R_i}{\sum_{k=1}^n Y_k}\widehat{N_i}$, is equal to zero since a small change in the revenue in the differentiated-good sector is equal to the sum of the small changes in wage and firm mass in this sector by RM2 and the free entry condition, i.e. $\widehat{R_i} = \widehat{N_i} + \widehat{w}_i$. Hence, the global welfare impact of the firm number effects in different countries offset each other.

[12] The proof is available from the authors upon request.

welfare associated with small changes in trade costs is given by equation (2), where X_{ij} denotes the total value of differentiated goods exported from country i to country j.

Examples of models to which this equation applies are Demidova (2008), Fan, Lai and Qi (2011), Eaton and Kortum (2002) and Alvarez and Lucas (2007).

6 Empirical Application

In this section, we apply our theory to estimate the impact of China's accession to the WTO on global welfare. China acceded to the WTO in December 2001. Since then there was a series of tariff reductions for a number of years. We use the changes in the average ad valorem duties imposed by China on each country to measure trade liberalization. It is well-documented that the conditions of WTO accession require China to reduce its import barriers with very little corresponding changes in its trade partners' import barriers against China. Therefore, we only need to focus on the changes in China's import tariffs. Trade volumes and MFN-applied tariff data for the period 2001-2008 are obtained from the CEPII and WTO, respectively.[13], [14] The world is assumed to consist of 16 countries or regions: China, Unite States, Argentina, Brazil, Canada, France, Germany, India, Italy, Japan, Mexico, United Kingdom, Africa, Other Asia, Other Europe, and Other Latin America. Based on our theory, we use the following equation to estimate the global welfare effect of China's accession to the WTO:

$$-\sum_{t=2001}^{2007}\sum_{i=1}^{n}\frac{X_{ict}}{\sum_{k=1}^{n}Y_{kt}}\widehat{\tau}_{ict}$$

where the subscript c, t refer to China and year respectively; X_{ict} is total volume of exports from country i to China in the year t; $\widehat{\tau}_{ict}$ is the change in tariff from year t to year $t+1$; Y_{kt} is the GDP of country k in year t obtained from the World Bank-WDI database.[15]

Table 1 summarizes the estimated global welfare impact of China's accession to the WTO. The second column gives the estimated change in global welfare in year $t+1$. As can be seen, world welfare is estimated to have increased by a cumulative 0.0498 percent in 2008 as the import tariff reduced from 14.83% in 2001 to 4.93% in 2008. In other words, global welfare would have been 0.0498 percent lower in 2008 if China had not gained accession to the WTO in 2001.

[13] Since applied MFN tariff is available at the HS eight-digit disaggregated level but trade value is available at the HS six-digit disaggregated level, we calculate the mean of tariff at six-digit disaggregated level in order to make them consistent.

[14] The full name of CEPII is Centre d'Etudes Prospectives et d'Informations Internationales. Its website is http://www.cepii.fr/anglaisgraph/bdd/bdd.htm.

[15] τ_{ict} is the average ad valorem duties imposed by China for goods from country i.

Table 1: Welfare Gain from China's entering WTO

Year t	Welfare Gains of World
2001	0.0367%
2002	0.0057%
2003	0.0041%
2004	0.0020%
2005	0.0007%
2006	0.0004%
2007	0.0002%
2001-2007 (cumulative)	0.0498%

7 Conclusion

Our theoretical analysis conveys two messages. First, ACR find that the gains from trade for a country based on a broad class of trade models conditional on the import penetration ratio and trade elasticity are the same regardless of the trade model used. The class of models include the Armington model, Eaton and Kortum (2002), Krugman (1980), Melitz (2003). However, when one extends these models to multi-sector settings, the result no longer holds, as pointed out by Balistreri, Hillberry and Rutherford (2010). In fact, it can be shown that the gains from trade based on a multi-sector model can be very different from those based on a one-sector model, even when the observed import penetration ratio and trade elasticity are the same.[16] In this paper, we prove that a much broader class of models yield the same global gains from trade liberalization conditional on the same set of trade flows and changes in trade costs. The class of models includes the general Ricardian model (including, for example, Anderson, 1979, and Eaton and Kortum, 2002), the models of Krugman (1980), Melitz (2003) and its extensions, and the extensions of these models to the multi-sectoral case, multi-factor production technology, multi-stage production, the existence of tradable intermediate goods (as in Alvarez and Lucas, 2007), and the existence of a large outside good sector in each country. In other words, the formula for calculating the gains from trade liberalization for the world as a whole is very robust to the trade model used. This is a significant result, as the set of trade models and settings to which the equation is applicable is considerably broader than that of ACR.

The second message is that the global welfare impact of trade liberalization only depends on two sets of sufficient statistics: (i) the ratio of the bilateral trade volume between each and every pair of trading partners and global income; and (ii) the change in exporting cost for each and every

[16] See, for example, Fan, Lai and Qi (2011).

pair of trading partners. The fact that the equation is independent of changes in wages and factor prices or firm numbers is a consequence of the fact that all indirect effects offset each other as a result of trade balance so that in the end only the direct effect remains.

However, the gains from trade liberalization of an individual country conditional on the same set of trade flows and changes in trade costs is in general not robust to the trade model used. It would be interesting to calculate how the global gains from some important episode of trade liberalization (e.g. Uruguay Round of GATT) are divided among different countries using different trade models, and see how different models yield different results. This is left for future research.

Appendix

For any importing country j, we denote by $G_j(\alpha_1, ..., \alpha_n, \phi_1, ..., \phi_n)$ the share of goods $\omega \in [0, N]$ such that $\alpha_{ij}(\omega) \leq \alpha_i$ and $\phi_{ij}(\omega) \leq \phi_i$ for all i, and by $g_j(\alpha_1, ..., \alpha_n, \phi_1, ..., \phi_n)$ the corresponding density function.

A Proof of Proposition 1 (One-sector perfect competition)

Under perfect competition, $\underline{t} = \overline{t} = 1$ and $\alpha_{ij}(\omega) \equiv \alpha_i(\omega)$. Following ACR (AER forthcoming), we use the following notations:

$$c_{ij} = w_i \tau_{ij} \tag{5}$$

$$g_{ij}(\alpha_i, c_{1j}, ..., c_{nj}) = \int_{\alpha_1 > \frac{\alpha_i c_{ij}}{c_{1j}}} ... \int_{\alpha_{i-1} > \frac{\alpha_i c_{ij}}{c_{i-1j}}} \int_{\alpha_{i+1} > \frac{\alpha_i c_{ij}}{c_{i+1j}}} ... \int_{\alpha_n > \frac{\alpha_i c_{ij}}{c_{nj}}} g_j(\boldsymbol{\alpha}, \mathbf{0})\, d\boldsymbol{\alpha}_{-i} \tag{6}$$

where $\boldsymbol{\alpha} \equiv (\alpha_1, ..., \alpha_n)$, $\mathbf{0} = (0, ..., 0)$ and $\boldsymbol{\alpha}_{-i}$ denotes the vector $\boldsymbol{\alpha}$ with the i-th component removed. In the following, we proceed to prove Proposition 1 in three steps.

Step 1: *The change in real income satisfies:*

$$\widehat{U}_j = \widehat{w}_j - \widehat{P}_j \tag{7}$$

Proof: By definition, we know that $\widehat{U}_j = \widehat{Y}_j - \widehat{P}_j$. Since the total profit of firms $\Pi_j = 0$ under perfect competition, we have $Y_j = w_j L_j$, which implies $\widehat{Y}_j = \widehat{w}_j$. These two observations imply Equation (7)

Step 2: *The change in the consumer price index satisfies:*

$$\widehat{P}_j = \textstyle\sum_{i=1}^{n} \lambda_{ij} \widehat{c}_{ij} = \sum_{i=1}^{n} \lambda_{ij} (\widehat{w}_i + \widehat{\tau}_{ij}) \tag{8}$$

Proof: By Lemma 1 in ACR, we know that, due to complete specialization, the consumer price index can be written as:

$$P_j = \left[\textstyle\sum_{i=1}^{n} \displaystyle\int_0^{\infty} (c_{ij}\alpha_i)^{1-\sigma} N g_{ij}(\alpha_i, c_{1j}, ..., c_{nj})\, d\alpha_i \right]^{\frac{1}{1-\sigma}} \tag{9}$$

Totally differentiating this expression, we obtain:

$$\widehat{P}_j = \frac{1}{1-\sigma} \textstyle\sum_{i=1}^{n} \lambda_{ij} \left[(1-\sigma)\, \widehat{c}_{ij} + \sum_{i'=1}^{n} \gamma_{ij}^{i'} \widehat{c}_{i'j} \right] \tag{10}$$

where

$$\lambda_{ij} = \frac{\int_0^\infty (c_{ij}\alpha_i)^{1-\sigma} g_{ij}(\alpha_i, c_{1j}, ..., c_{nj}) d\alpha_i}{\sum_{i'=1}^n \int_0^\infty \left(c_{i'j}\alpha_{i'}\right)^{1-\sigma} g_{i'j}(\alpha_{i'}, c_{1j}, ..., c_{nj}) d\alpha_{i'}}$$
$$\gamma_{ij}^{i'} = \frac{\partial \ln\left(\int_0^\infty \alpha_i^{1-\sigma} g_{ij}(\alpha_i, c_{1j}, ..., c_{nj}) d\alpha_i\right)}{\partial \ln c_{i'j}}$$

where λ_{ij} is the penetration ratio of country i's goods in country j, $\gamma_{ij}^{i'}$ is the extensive margin elasticity. By Equation (6), the extensive margin elasticities satisfy $\gamma_{ij}^{i} = -\sum_{i'\neq i}\gamma_{ij}^{i'}$. Thus, we can rewrite Equation (10) as:

$$\widehat{P}_j = \frac{1}{1-\sigma}\sum_{i=1}^n \lambda_{ij}\left[(1-\sigma)\widehat{c}_{ij} + \sum_{i'\neq i}\gamma_{ij}^{i'}\left(\widehat{c}_{i'j} - \widehat{c}_{ij}\right)\right]$$

The extensive margin elasticities also satisfy $\lambda_{ij}\gamma_{ij}^{i'} = \lambda_{i'j}\gamma_{i'j}^{i}$ according to Equation (6), which implies:

$$\sum_{i=1}^n\sum_{i'\neq i}\lambda_{ij}\gamma_{ij}^{i'}\left(\widehat{c}_{i'j} - \widehat{c}_{ij}\right) = 0$$

Combining the previous two equations, we obtain the first equality in Equation (8).

Step 3: *Change in global welfare satisfies (2):*

Proof: By Equations (7) and (8), the change in global welfare is given by:

$$\sum_{j=1}^n s_j\widehat{U}_j = -\sum_{j=1}^n\sum_{i=1}^n s_j\lambda_{ij}\left(\widehat{w}_i - \widehat{w}_j + \widehat{\tau}_{ij}\right) \tag{11}$$

as $\sum_{i=1}^n\lambda_{ij} = 1$. Trade balance condition $\sum_{i=1}^n X_{ij} = \sum_{i=1}^n X_{ji}$ (R1) implies:

$$\sum_{i=1}^n s_j\lambda_{ij} = \sum_{i=1}^n s_i\lambda_{ji}$$
$$\Leftrightarrow \quad \sum_{j=1}^n\sum_{i=1}^n s_j\lambda_{ij}\widehat{w_i} = \sum_{j=1}^n\sum_{i=1}^n s_i\lambda_{ji}\widehat{w_i}$$
$$\Leftrightarrow \quad \sum_{j=1}^n\sum_{i=1}^n s_j\lambda_{ij}\widehat{w_i} = \sum_{j=1}^n\sum_{i=1}^n s_j\lambda_{ij}\widehat{w_j}$$
$$\Leftrightarrow \quad \sum_{j=1}^n\sum_{i=1}^n s_j\lambda_{ij}\left(\widehat{w}_i - \widehat{w}_j\right) = 0 \tag{12}$$

where the RHS of the third equation is obtained by switching i to j and j to i on the RHS of the second equation. Combining equations (11), (12) and the facts that $s_j = \frac{Y_j}{\sum_{k=1}^n Y_k}$ and $\lambda_{ij} = \frac{X_{ij}}{Y_j}$, we obtain

$$\sum_{j=1}^n s_j\widehat{U}_j = -\sum_{j=1}^n\sum_{i=1}^n s_j\lambda_{ij}\widehat{\tau}_{ij} = -\sum_{j=1}^n\sum_{i=1}^n\frac{X_{ij}}{\sum_{k=1}^n Y_k}\widehat{\tau}_{ij}$$

which is (2).

B Proof of Proposition 2 (One-sector monopolistic competition)

Following ACR, we use the following additional notation:

$$\alpha_{ij}^{*} \equiv \sigma^{\frac{\sigma}{1-\sigma}} (\sigma - 1) \left(\frac{P_j}{w_i \tau_{ij}}\right) \left(\frac{\xi_{ij} w_i^{\mu} w_j^{1-\mu}}{Y_j}\right)^{\frac{1}{1-\sigma}} \tag{13}$$

$$t_{ij} \left(\alpha, \phi, \alpha_{ij}^{*}\right) \equiv \underset{t \in \left[\underline{t}, \overline{t}\right]}{\arg\max} \left[t \left(\frac{\alpha}{\alpha_{ij}^{*}}\right)^{1-\sigma} - m_{ij}(t) \phi \right] \tag{14}$$

$$c_{ij} \left(\alpha, \phi, \alpha_{ij}^{*}\right) \equiv w_i \tau_{ij} \left(\frac{\sigma}{\sigma - 1}\right) \left[t_{ij} \left(\alpha, \phi, \alpha_{ij}^{*}\right)\right]^{\frac{1}{1-\sigma}} \alpha \tag{15}$$

$$\overline{\alpha}_{ij}(\phi) \equiv \sup \left\{ \alpha > 0 \left| t_{ij}(\alpha, \phi, 1) \alpha^{1-\sigma} - m_{ij} \left[t_{ij}(\alpha, \phi, 1)\right] \phi > 0 \right. \right\} \tag{16}$$

$$g_{ij}(\alpha, \phi) \equiv \textstyle\int_0^{+\infty} g_j \left(\alpha_{-i}, \alpha_i = \alpha, \phi_{-i}, \phi_i = \phi\right) d\alpha_{-i} d\phi_{-i} \tag{17}$$

Noter that α_{ij}^{*} is equivalent to the productivity cutoff for exporting in Melitz (2003). In the following, we proceed to prove Proposition 2 in five steps.

Step 1: *The change in real income satisfies (7):*

Proof: By definition, we know that $\widehat{U}_j = \widehat{Y}_j - \widehat{P}_j$. Suppose first that there is free entry in country j. In this case, $Y_j = w_j L_j$ since $\Pi_j = N_j w_j F_j$. This immediately implies Equation (7). Suppose instead that there is restricted entry in country j. In this case, $Y_j = w_j L_j + \Pi_j$. By R1 and R2, we must also have $\Pi_j = \zeta R_j = \zeta Y_j$ with $0 < \zeta < 1$. Combining the two previous equations, we get $\widehat{Y}_j = \widehat{w}_j$, which again implies Equation (7).

Step 2: *The change in the consumer price index satisfies:*

$$\widehat{P}_j = \textstyle\sum_{i=1}^{n} \frac{\lambda_{ij}}{1 - \sigma - \gamma_j} \left\{ \left[1 - \sigma - \left(\frac{\mu}{\sigma - 1} + 1\right) \gamma_{ij}\right] \widehat{w}_i + \left(\frac{\mu}{\sigma - 1}\right) \gamma_{ij} \widehat{w}_j + (1 - \sigma - \gamma_{ij}) \widehat{\tau}_{ij} + \widehat{N_i} \right\} \tag{18}$$

where γ_{ij} is given by Equation (21) below, and $\gamma_j = \sum_{i=1}^{n} \lambda_{ij} \gamma_{ij}$.

Proof: By Lemma 2 in ACR, we know that, under assumption R3, the consumer price index can be written as:

$$P_j = \left[\textstyle\sum_{i=1}^{n} N_i \int_0^{\infty} \int_0^{\overline{\alpha}_{ij}(\phi) \alpha_{ij}^{*}} c_{ij}^{1-\sigma} \left(\alpha, \phi, \alpha_{ij}^{*}\right) g_{ij}(\alpha, \phi) \, d\alpha d\phi \right]^{\frac{1}{1-\sigma}} \tag{19}$$

Totally differentiating this expression, we obtain

$$\widehat{P}_j = \textstyle\sum_{i=1}^{n} \frac{\lambda_{ij}}{1 - \sigma} \left[(1 - \sigma) \left(\widehat{w}_i + \widehat{\tau}_{ij}\right) + \widehat{N_i} + \gamma_{ij} \widehat{\alpha}_{ij}^{*} \right] \tag{20}$$

where

$$\gamma_{ij} = \frac{\int_{\underline{\phi}_{ij}}^{\infty} c_{ij}^{1-\sigma}\left(\overline{\alpha}_{ij}(\phi)\,\alpha_{ij}^{*},\phi,\alpha_{ij}^{*}\right)\overline{\alpha}_{ij}(\phi)\,\alpha_{ij}^{*} g_{ij}\left(\overline{\alpha}_{ij}(\phi)\,\alpha_{ij}^{*},\phi\right)d\phi}{\int_{0}^{\infty}\int_{0}^{\overline{\alpha}_{ij}(\phi)\alpha_{ij}^{*}} c_{ij}^{1-\sigma}\left(\alpha,\phi,\alpha_{ij}^{*}\right) g_{ij}(\alpha,\phi)\,d\alpha d\phi} + \frac{\int_{0}^{\infty}\int_{0}^{\overline{\alpha}_{ij}(\phi)\alpha_{ij}^{*}} c_{ij}^{1-\sigma}\left(\alpha,\phi,\alpha_{ij}^{*}\right)\left[\frac{\partial\ln t_{ij}\left(\alpha,\phi,\alpha_{ij}^{*}\right)}{\partial\ln\alpha_{ij}^{*}}\right] g_{ij}(\alpha,\phi)\,d\alpha d\phi}{\int_{0}^{\infty}\int_{0}^{\overline{\alpha}_{ij}(\phi)\alpha_{ij}^{*}} c_{ij}^{1-\sigma}\left(\alpha,\phi,\alpha_{ij}^{*}\right) g_{ij}(\alpha,\phi)\,d\alpha d\phi} \tag{21}$$

where γ_{ij} is the counterpart of the extensive margin elasticities under perfect competition, with $\underline{\phi}_{ij} = \inf\{\phi > 0 \mid \overline{\alpha}_{ij}(\phi) < +\infty\}$. By definition of α_{ij}^{*}, we know that

$$\widehat{\alpha}_{ij}^{*} = \widehat{P}_j - \widehat{\tau}_{ij} - \widehat{w}_i - \frac{\mu}{\sigma-1}(\widehat{w}_i - \widehat{w}_j) \tag{22}$$

Combining Equations (20) and (22), we get Equation (18).

Step 3: *The change in the consumer price index satisfies:*

$$\widehat{P}_j = \textstyle\sum_{i=1}^{n}\lambda_{ij}\left[\widehat{w}_i + \left(\frac{\mu}{\sigma-1} + \frac{\mu}{\varepsilon}\right)(\widehat{w}_i - \widehat{w}_j) + \widehat{\tau}_{ij} + \frac{\widehat{N_i}}{\varepsilon}\right] \tag{23}$$

Proof: By Lemma 2 in ACR, the total value of exports from country i to country j equals

$$X_{ij} = \frac{N_i\int_{0}^{\infty}\int_{0}^{\overline{\alpha}_{ij}(\phi)\alpha_{ij}^{*}} c_{ij}^{1-\sigma}\left(\alpha,\phi,\alpha_{ij}^{*}\right) g_{ij}(\alpha,\phi)\,d\alpha d\phi}{\sum_{i'=1}^{n}N_{i'}\int_{0}^{\infty}\int_{0}^{\overline{\alpha}_{i'j}(\phi)\alpha_{i'j}^{*}} c_{i'j}^{1-\sigma}\left(\alpha,\phi,\alpha_{i'j}^{*}\right) g_{i'j}(\alpha,\phi)\,d\alpha d\phi} Y_j$$

Totally differentiating the above expression yields:

$$\widehat{X}_{ij} - \widehat{X}_{jj} = (1-\sigma)(\widehat{w}_i - \widehat{w}_j + \widehat{\tau}_{ij}) + \gamma_{ij}\widehat{\alpha}_{ij}^{*} - \gamma_{jj}\widehat{\alpha}_{jj}^{*} + \widehat{N_i} - \widehat{N}_j$$

By Equation (22), we also have

$$\widehat{\alpha}_{ij}^{*} = \widehat{\alpha}_{jj}^{*} - \widehat{\tau}_{ij} - \left(\frac{\mu}{\sigma-1} + 1\right)(\widehat{w}_i - \widehat{w}_j)$$

The last two equations imply:

$$\widehat{X}_{ij} - \widehat{X}_{jj} = \left[1 - \sigma - \left(\frac{\mu}{\sigma-1} + 1\right)\gamma_{ij}\right](\widehat{w}_i - \widehat{w}_j) + (1-\sigma-\gamma_{ij})\,\widehat{\tau}_{ij} + (\gamma_{ij} - \gamma_{jj})\,\widehat{\alpha}_{jj}^{*} + \widehat{N_i} - \widehat{N}_j$$

This expression implies, for any $i, i' \neq j$,

$$\frac{\partial\ln(X_{ij}/X_{jj})}{\partial\ln\tau_{i'j}} = \varepsilon_j^{ii'} = \begin{cases} 1 - \sigma - \gamma_{ij} + (\gamma_{ij} - \gamma_{jj})\frac{\partial\ln\alpha_{jj}^{*}}{\partial\ln\tau_{ij}} \text{ for } i' = i \\ (\gamma_{ij} - \gamma_{jj})\frac{\partial\ln\alpha_{jj}^{*}}{\partial\ln\tau_{i'j}}, \text{ for } i' \neq i \end{cases}$$

Since $\frac{\partial\ln\alpha_{jj}^{*}}{\partial\ln\tau_{i'j}} > 0$, the above equation together with R3 implies that $\gamma_{ij} = 1 - \sigma - \varepsilon$ for all i. Substituting $\gamma_{ij} = 1 - \sigma - \varepsilon$ into Equation (18), we get Equation (23).

Step 4: *The change in the number of goods that can be produced satisfies* $\widehat{N_i} = 0$.

Proof: Suppose first that entry is restricted in country j. In this case, we immediately get $\widehat{N_i} = 0$. Now suppose that we have free entry instead. In this case, zero profit implies that $\Pi_j = N_j w_j F_j$. Since $\Pi_j = \zeta Y_j = \zeta w_j L_j$ by R1 and R2, we therefore have $\widehat{\Pi}_j = \widehat{w}_j$, which implies $\widehat{N}_j = 0$ for all j from the previous equation.

Step 5: *The change in global welfare satisfies (2):*

Proof: Step 4, together with Equations (7), (23) and the fact that $\sum_{i=1}^{n} \lambda_{ij} = 1$, implies that change in global welfare satisfies:

$$\sum_{j=1}^{n} s_j \widehat{U}_j = -\sum_{j=1}^{n} \sum_{i=1}^{n} s_j \lambda_{ij} \left[\left(\frac{\mu}{\sigma - 1} + \frac{\mu}{\varepsilon} + 1 \right) (\widehat{w}_i - \widehat{w}_j) + \widehat{\tau}_{ij} \right]$$

As shown in step 3 of the proof of proposition 1, trade balance condition implies (12). Combining (12) with the last equation, we obtain Equation (2).

C Proof of Proposition 3 (Multi-sector)

We proceed to prove in two steps. Now, the consumer price index becomes

$$P_j = \prod_{s=1}^{S} [P_j(s)]^{\eta^s} \tag{24}$$

where $P_j(s)$ denotes the aggregate price index for sector s.

Step 1: The *change in global welfare satisfies:*

$$\sum_{j=1}^{n} s_j \widehat{U}_j = \sum_{j=1}^{n} \sum_{s=1}^{S} s_j \eta^s \left[\widehat{w}_j - \widehat{P}_j(s) \right] \tag{25}$$

Proof: Totally differentiating the consumer price index (24), we get

$$\widehat{P}_j = \sum_{s=1}^{S} \eta^s \widehat{P}_j(s)$$

Similar to step 1 of the proofs in Propositions 1 and 2, by R1 and RM2, we have $\widehat{Y}_j = \widehat{w}_j$ under both perfect competition and monopolistic competition. Then, combining the fact that $\sum_{s=1}^{S} \eta^s = 1$ and the last equation, we have

$$\widehat{U}_j = \widehat{w}_j - \widehat{P}_j = \sum_{s=1}^{S} \eta^s \left(\widehat{w}_j - \widehat{P}_j(s) \right)$$

which implies Equation (25)

Step 2: The *change in global welfare satisfies:*

$$\sum_{j=1}^{n} s_j \widehat{U}_j = -\sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{s=1}^{S} s_j \eta^s \lambda_{ij}^s \widehat{\tau}_{ij}^s = -\sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{s=1}^{S} \frac{X_{ij}^s}{\sum_{k=1}^{n} Y_k} \widehat{\tau}_{ij}^s \tag{26}$$

Proof: Here, we will show that Equation (26) holds under both perfect competition and monopolistic competition.

Under Perfect competition:

Based on the same derivation as in the proof of Proposition 1, Equation (8) continues to hold for each of the sectors. In other words, the change in the aggregate price index $P_j(s)$ in sector s is given by

$$\widehat{P}_j(s) = \sum_{i=1}^{n} \lambda_{ij}^s \left(\widehat{w}_i + \widehat{\tau}_{ij}^s\right)$$

Substituting this expression into Equation (25), we obtain

$$\sum_{j=1}^{n} s_j \widehat{U}_j = -\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{s=1}^{S} s_j \eta^s \lambda_{ij}^s \left(\widehat{w}_i - \widehat{w}_j + \widehat{\tau}_{ij}^s\right) \tag{27}$$

as $\sum_{i=1}^{n}\lambda_{ij}^s = 1$. Trade balance condition, $\sum_{i=1}^{n} X_{ij} = \sum_{i=1}^{n} X_{ji}$, which is equivalent to $\sum_{i=1}^{n}\sum_{s=1}^{S} X_{ij}^s = \sum_{i=1}^{n}\sum_{s=1}^{S} X_{ji}^s$, implies:

$$\sum_{i=1}^{n}\sum_{s=1}^{S} s_j \eta^s \lambda_{ij}^s = \sum_{i=1}^{n}\sum_{s=1}^{S} s_i \eta^s \lambda_{ji}^s$$

Hence, the indirect effect from the change in relative wage offset each other from the perspective of global real income. That is to say,

$$\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{s=1}^{S} s_j \eta^s \lambda_{ij}^s \left(\widehat{w}_i - \widehat{w}_j\right) = 0 \tag{28}$$

Combining Equations (27) and (28), we get Equation (26).

Under Monopolistic competition:

Based on same derivation in Proof of Proposition 2, Equation (23) continues to hold from the perspective of sector under assumption RM3. Hence, the aggregate price index in sector s, satisfies:

$$\widehat{P}_j(s) = \sum_{i=1}^{n} \lambda_{ij}^s \left[\widehat{w}_i + \left(\frac{\mu}{\sigma-1} + \frac{\mu}{\varepsilon}\right)\left(\widehat{w}_i - \widehat{w}_j\right) + \widehat{\tau}_{ij}^s + \frac{\widehat{N_i^s}}{\varepsilon}\right]$$

where N_i^s is expected firm mass in sector s in country i. Substituting this expression into Equation (25), we have that the change in global welfare is given by:

$$\sum_{j=1}^{n} s_j \widehat{U}_j = -\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{s=1}^{S} s_j \eta^s \lambda_{ij}^s \left[\left(\frac{\mu}{\sigma-1} + \frac{\mu}{\varepsilon} + 1\right)\left(\widehat{w}_i - \widehat{w}_j\right) + \widehat{\tau}_{ij}^s + \frac{\widehat{N_i^s}}{\varepsilon}\right] \tag{29}$$

Invoking the same logic as step 3 of the proof of Proposition 1, trade balance implies (28).

In addition, under monopolistic competition with restricted entry, N_i^s is exogenous, i.e., $\widehat{N_i^s} = 0$, which, together with Equations (29) and (28), imply (26). Suppose that we have free entry instead. In this case, $\zeta R_j^s = \Pi_j^s = N_j^s w_j F_j$ for any s, j by RM2 and free entry condition. Therefore, $\widehat{R_j^s} = \widehat{N_j^s} + \widehat{w}_j$. The trade balance condition is equivalent to $w_j L_j = \sum_{s=1}^{S} R_j^s$, which implies that $\widehat{w}_j = \sum_{s=1}^{S} \frac{R_j^s}{\sum_{s=1}^{S} R_j^s}\widehat{R_j^s} = \sum_{s=1}^{S} \frac{R_j^s}{\sum_{s=1}^{S} R_j^s}\left(\widehat{N_j^s} + \widehat{w}_j\right)$, which in turn implies that $\sum_{s=1}^{S} R_i^s \widehat{N_i^s} = 0$.

Hence, in both the unrestricted entry and restricted entry cases, we have $\sum_{s=1}^{S} R_i^s \widehat{N}_i^s = 0$. This means that

$$\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{s=1}^{S} s_j \eta^s \lambda_{ij}^s \widehat{N}_i^s = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{s=1}^{S} X_{ij}^s \widehat{N}_i^s}{\sum_{k=1}^{n} Y_k} = \frac{\sum_{i=1}^{n}\sum_{s=1}^{S} R_i^s \widehat{N}_i^s}{\sum_{k=1}^{n} Y_k} = 0$$

Combining the above expression with equations (28) and (29), we obtain equation (26).

D Proof of Proposition 4 (Multi-factor with multiple sectors)

Endowment of factors in country i is L_{ki}, $k = 1, ..., K$. Now, the production function in sector s is $f^s(l_1, ..., l_K) = A_{ij}(\omega) \prod_{k=1}^{K} l_k^{\beta_k(s)}$ with $\sum_{k=1}^{K} \beta_k(s) = 1$. To be consistent with ACR, now the marginal cost is $c_{ij}^s(v_i^s, t_j, \omega) = \tau_{ij} v_i^s \alpha_{ij}(\omega) t_j^{\frac{1}{1-\sigma}}$. The exporting cost is assumed to be $\xi_{ij} v_i^s \phi_{ij}(\omega) m_{ij}(t_j)$, and entry cost under monopolistic competition with free entry is $v_i^s F_i$. In addition, $v_i^s = \prod_{k=1}^{K} \left(\frac{w_{ki}}{\beta_k(s)}\right)^{\beta_k(s)}$ implies $\widehat{v}_i^s = \sum_{k=1}^{K} \beta_k(s) \widehat{w}_{ki}$. Now, the consumer price index in country j becomes

$$P_j = \prod_{s=1}^{S} [P_j(s)]^{\eta^s} \tag{30}$$

In the following, we proceed in three steps.

Step 1: *The change in GDP satisfies*

$$\widehat{Y}_j = \frac{\sum_{i=1}^{n}\sum_{s=1}^{S} X_{ji}^s \widehat{v}_j^s}{Y_j} \tag{31}$$

Proof: The production function, the exporting cost function, and the entry cost function in each sector are all Cobb-Douglas functions. As a result, the total expenditure on factor k in sector s is the ratio $\beta_k(s)$ of the total expenditure on all factors in sector s. Under perfect competition and monopolistic competition with free entry, the total expenditure on all factors in country j in sector s is the revenue of country j in this sector $R_j^s = \sum_{i=1}^{n} X_{ji}^s$. Hence the total expenditure on factor k in sector s in country j is $\beta_k(s) \sum_{i=1}^{n} X_{ji}^s$. Then the total expenditure on factor k in country j is $\sum_{s=1}^{S} \beta_k(s) \sum_{i=1}^{n} X_{ji}^s$, which equals to $w_{kj} L_{kj}$ by factor market clearing condition. Under monopolistic competition with restricted entry, the total expenditure on all factors in country j is a fraction $1-\zeta$ of the total revenue of country j in this sector, since the profits of firms are a fraction ζ of the total revenue in this sector by assumption RM2. Hence, the total expenditure on factor k in sector s in country j is $(1-\zeta) \beta_k(s) \sum_{i=1}^{n} X_{ji}^s$. Then, the total expenditure on factor k in country j is $(1-\zeta) \sum_{s=1}^{S} \beta_k(s) \sum_{i=1}^{n} X_{ji}^s$, which equals $w_{kj} L_{kj}$ by the factor market clearing condition.

Then, under perfect competition and monopolistic competition with free entry, we have $Y_j =$

$\sum_{k=1}^{K} w_{kj} L_{kj}$, which implies:

$$\begin{aligned}\widehat{Y}_j &= \sum_{k=1}^{K} \frac{w_{kj} L_{kj}}{\sum_{k=1}^{K} w_{kj} L_{kj}} \widehat{w}_{kj} = \sum_{k=1}^{K} \frac{\sum_{s=1}^{S} \beta_k (s) \sum_{i=1}^{n} X_{ji}^{s}}{Y_j} \widehat{w}_{kj} \\ &= \frac{\sum_{i=1}^{n} \sum_{s=1}^{S} \sum_{k=1}^{K} \beta_k (s) X_{ji}^{s} \widehat{w}_{kj}}{Y_j} = \frac{\sum_{i=1}^{n} \sum_{s=1}^{S} X_{ji}^{s} \left(\sum_{k=1}^{K} \beta_k (s) \widehat{w}_{kj} \right)}{Y_j} \\ &= \frac{\sum_{i=1}^{n} \sum_{s=1}^{S} X_{ji}^{s} \widehat{v}_j^{s}}{Y_j}\end{aligned}$$

where the last equality follows from $\widehat{v}_i^s = \sum_{k=1}^{K} \beta_k (s) \widehat{w}_{ki}$.

Similarly, under monopolistic competition with restricted entry, we have $Y_j = \frac{1}{1-\zeta} \sum_{k=1}^{K} w_{kj} L_{kj}$ by assumption RM2. Totally differentiating this equation yields:

$$\begin{aligned}\widehat{Y}_j &= \sum_{k=1}^{K} \frac{w_{kj} L_{kj}}{\sum_{k=1}^{K} w_{kj} L_{kj}} \widehat{w}_{kj} = \sum_{k=1}^{K} \frac{(1-\zeta) \sum_{s=1}^{S} \beta_k (s) \sum_{i=1}^{n} X_{ji}^{s}}{(1-\zeta) Y_j} \widehat{w}_{kj} \\ &= \frac{\sum_{i=1}^{n} \sum_{s=1}^{S} \sum_{k=1}^{K} \beta_k (s) X_{ji}^{s} \widehat{w}_{kj}}{Y_j} = \frac{\sum_{i=1}^{n} \sum_{s=1}^{S} X_{ji}^{s} \widehat{v}_j^{s}}{Y_j}\end{aligned}$$

Step 2: *The change in global welfare satisfies:*

$$\sum_{j=1}^{n} s_j \widehat{U}_j = \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{s=1}^{S} s_j \eta^s \lambda_{ji}^{s} \widehat{v}_j^{s} - \sum_{j=1}^{n} \sum_{s=1}^{S} s_j \eta^s \widehat{P}_j (s)$$

Proof: Totally differentiating (30) yields

$$\widehat{P}_j = \sum_{s=1}^{S} \eta^s \widehat{P}_j (s) \tag{32}$$

Equation (31), together with $s_j = \frac{Y_j}{\sum_{k=1}^{n} Y_k}$ and $X_{ji}^s = \eta^s Y_i \lambda_{ji}^s$, implies:

$$\sum_{j=1}^{n} s_j \widehat{Y}_j = \sum_{j=1}^{n} s_j \frac{\sum_{i=1}^{n} \sum_{s=1}^{S} X_{ji}^{s} \widehat{v}_j^{s}}{Y_j} = \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{s=1}^{S} s_i \eta^s \lambda_{ji}^{s} \widehat{v}_j^{s} \tag{33}$$

Then, based on Equations (32) and (33), the change in global welfare satisfies

$$\sum_{j=1}^{n} s_j \widehat{U}_j = \sum_{j=1}^{n} s_j \left(\widehat{Y}_j - \widehat{P}_j \right) = \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{s=1}^{S} s_i \eta^s \lambda_{ji}^{s} \widehat{v}_j^{s} - \sum_{j=1}^{n} s_j \sum_{s=1}^{S} \eta^s \widehat{P}_j (s)$$

Step 3: The change in global welfare *satisfies (26).*

Proof: We will show that this step holds under perfect competition and under monopolistic competition, separately

Under Perfect Competition:

Based on same derivation as in Proof of Proposition 1, Equation (8) continues to hold from the perspective of a single sector. In other words, the change in the aggregate price index $P_j (s)$ in sector s is given by:

$$\widehat{P}_j (s) = \sum_{i=1}^{n} \lambda_{ij}^{s} \left(\widehat{v}_i^{s} + \widehat{\tau}_{ij}^{s} \right)$$

Combining this with the previous expression proved in Step 2, we obtain

$$\begin{aligned}
\textstyle\sum_{j=1}^{n} s_j \widehat{U}_j &= \textstyle\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{s=1}^{S} s_i \eta^s \lambda_{ji}^s \widehat{v}_j^s - \sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{s=1}^{S} s_j \eta^s \lambda_{ij}^s \left(\widehat{v}_i^s + \widehat{\tau}_{ij}^s\right) \\
&= \textstyle\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{s=1}^{S} s_j \eta^s \lambda_{ij}^s \widehat{v}_i^s - \sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{s=1}^{S} s_j \eta^s \lambda_{ij}^s \left(\widehat{v}_i^s + \widehat{\tau}_{ij}^s\right) \\
&= -\textstyle\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{s=1}^{S} s_j \eta^s \lambda_{ij}^s \widehat{\tau}_{ij}^s = -\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{s=1}^{S} \dfrac{X_{ij}^s}{\sum_{k=1}^{n} Y_k} \widehat{\tau}_{ij}^s
\end{aligned}$$

where the second equality stems from switching i to j and j to i in the first term; while the last equality follows from $s_j = \frac{Y_j}{\sum_{k=1}^{n} Y_k}$ and $\lambda_{ij}^s = \frac{X_{ij}^s}{\eta^s Y_j}$. The last line of the above equations is (26).

Under Monopolistic Competition:

Based on the similar derivation as the first three steps in the Proof of Proposition 2, except that $\widehat{Y}_j$ is not equal to $\widehat{w}_j$, we obtain the aggregate price index in sector s:

$$\widehat{P}_j(s) = \textstyle\sum_{i=1}^{n} \lambda_{ij}^s \left[\widehat{v}_i^s + \left(\dfrac{1}{\sigma-1} + \dfrac{1}{\varepsilon}\right)\left(\widehat{v}_i^s - \widehat{Y}_j\right) + \widehat{\tau}_{ij}^s + \dfrac{\widehat{N_i^s}}{\varepsilon}\right] \tag{34}$$

$$= \textstyle\sum_{i=1}^{n} \lambda_{ij}^s \left[\left(\dfrac{1}{\sigma-1} + \dfrac{1}{\varepsilon} + 1\right)\widehat{v}_i^s + \widehat{\tau}_{ij}^s + \dfrac{\widehat{N_i^s}}{\varepsilon}\right] - \left(\dfrac{1}{\sigma-1} + \dfrac{1}{\varepsilon}\right)\widehat{Y}_j \tag{35}$$

where (34) is analogous to (23), with $\widehat{v}_i^s$ replacing $\widehat{w}_i$ and μ set equal to one. The last equality is due to $\sum_{i=1}^{n} \lambda_{ij}^s = 1$. Hence, the change in global welfare is given by:

$$\begin{aligned}
\textstyle\sum_{j=1}^{n} s_j \widehat{U}_j &= \textstyle\sum_{j=1}^{n} s_j \left(\widehat{Y}_j - \widehat{P}_j\right) = \sum_{j=1}^{n} s_j \left[\widehat{Y}_j - \sum_{s=1}^{S} \eta^s \widehat{P}_j(s)\right] \\
&= \left(\dfrac{1}{\sigma-1} + \dfrac{1}{\varepsilon} + 1\right)\textstyle\sum_{j=1}^{n} s_j \widehat{Y}_j - \sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{s=1}^{S} s_j \eta^s \lambda_{ij}^s \left[\left(\dfrac{1}{\sigma-1} + \dfrac{1}{\varepsilon} + 1\right)\widehat{v}_i^s + \widehat{\tau}_{ij}^s + \dfrac{\widehat{N_i^s}}{\varepsilon}\right] \\
&= \left(\dfrac{1}{\sigma-1} + \dfrac{1}{\varepsilon} + 1\right)\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{s=1}^{S} s_i \eta^s \lambda_{ji}^s \widehat{v}_j^s - \sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{s=1}^{S} s_j \eta^s \lambda_{ij}^s \left[\left(\dfrac{1}{\sigma-1} + \dfrac{1}{\varepsilon} + 1\right)\widehat{v}_i^s + \widehat{\tau}_{ij}^s + \dfrac{\widehat{N_i^s}}{\varepsilon}\right] \\
&= \left(\dfrac{1}{\sigma-1} + \dfrac{1}{\varepsilon} + 1\right)\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{s=1}^{S} s_j \eta^s \lambda_{ij}^s \widehat{v}_i^s - \sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{s=1}^{S} s_j \eta^s \lambda_{ij}^s \left[\left(\dfrac{1}{\sigma-1} + \dfrac{1}{\varepsilon} + 1\right)\widehat{v}_i^s + \widehat{\tau}_{ij}^s + \dfrac{\widehat{N_i^s}}{\varepsilon}\right] \\
&= -\textstyle\sum_{j=1}^{n}\sum_{s=1}^{S}\sum_{i=1}^{n} s_j \eta^s \lambda_{ij}^s \left(\widehat{\tau}_{ij}^s + \dfrac{\widehat{N_i^s}}{\varepsilon}\right) \qquad (36)
\end{aligned}$$

where the second equality follows from Equation (32); the third equality stems from (35) and $\sum_{s=1}^{S} \eta^s = 1$; the fourth equality stems from (33); the fifth equality comes from switching i to j and j to i in the first term. Under monopolistic competition with restricted entry, N_i^s is exogenous, i.e., $\widehat{N_i^s} = 0$, which, together with Equation (36), immediately implies Equation (26). Suppose that there is free entry instead. By R1 and RM2, we have $\sum_{s=1}^{S} \Pi_j^s = \zeta \sum_{s=1}^{S} R_j^s = \zeta Y_j$. Then,

$$\textstyle\sum_{s=1}^{S} \dfrac{R_j^s}{Y_j}\left(\widehat{N}_j^s + \widehat{v}_j^s\right) = d\ln\left(\sum_{s=1}^{S} \Pi_j^s\right) = d\ln\left(\zeta Y_j\right) = \sum_{s=1}^{S} \dfrac{R_j^s}{Y_j} \widehat{v}_j^s \tag{37}$$

where the first equality stems from the zero profit condition $\Pi_j^s = N_j^s v_j^s F_j$;[17] the second equality stems from $\sum_{s=1}^{S} \Pi_j^s = \zeta Y_j$, and the last equality stems from step 1, which shows that $\widehat{Y}_j =$

[17] $d\ln\left(\sum_{s=1}^{S} \Pi_j^s\right) = \frac{d\left(\sum_{s=1}^{S} \Pi_j^s\right)}{\sum_{s=1}^{S} \Pi_j^s} = \frac{\sum_{s=1}^{S} d\Pi_j^s}{\sum_{s=1}^{S} \Pi_j^s} = \frac{\sum_{s=1}^{S} \widehat{\Pi}_j^s \cdot \Pi_j^s}{\sum_{s=1}^{S} \Pi_j^s} = \frac{\sum_{s=1}^{S}\left(\widehat{N}_j^s + \widehat{v}_j^s\right)\cdot \varsigma R_j^s}{\varsigma Y_j} = \sum_{s=1}^{S} \frac{R_j^s}{Y_j}\left(\widehat{N}_j^s + \widehat{v}_j^s\right)$

$\frac{\sum_{i=1}^{n}\sum_{s=1}^{S}X_{ji}^{s}\widehat{v}_{j}^{s}}{Y_j} = \sum_{s=1}^{S}\frac{R_j^s}{Y_j}\widehat{v}_j^s$, since $R_j^s = \sum_{i=1}^{n}X_{ji}^s$. From Equation (37), we have

$$\sum_{s=1}^{S}R_j^s\widehat{N}_j^s = 0$$

Therefore,

$$\sum_{j=1}^{n}\sum_{s=1}^{S}\sum_{i=1}^{n}s_j\eta^s\lambda_{ij}^s\widehat{N_i^s} = \frac{\sum_{j=1}^{n}\sum_{s=1}^{S}\sum_{i=1}^{n}X_{ij}^s\widehat{N_i^s}}{\sum_{k=1}^{n}Y_k} = \frac{\sum_{i=1}^{n}\sum_{s=1}^{S}R_i^s\widehat{N_i^s}}{\sum_{k=1}^{n}Y_k} = 0 \tag{38}$$

where the third equality stems from $\sum_{s=1}^{S}R_i^s\widehat{N_i^s} = 0$ above. Combining (36) and (38), we obtain Equation (26) under monopolistic competition with free entry.

E Proof of Proposition 5 (Multi-stage Production under Perfect Competition)

We proceed to prove this proposition in two steps.

Step 1: *The change in consumer price index satisfies*

$$\widehat{U}_j = -\sum_{i=1}^{n}\sum_{m=1}^{n}\lambda_{mij}\left[\theta\left(\widehat{w}_m - \widehat{w}_i\right) + \widehat{w}_i - \widehat{w}_j + \theta\widehat{\tau}_{mi} + \widehat{\tau}_{ij}\right] \tag{39}$$

Proof: Based on similar derivation as in the proof of Proposition 1, we can show that equation (8) continues to hold. However, the marginal cost now is related to where the first-stage production happens. Totally differentiating the aggregate price index given in section 4.3, we have

$$\widehat{P}_j = \sum_{i=1}^{n}\sum_{m=1}^{n}\lambda_{mij}\widehat{c}_{mij} = \sum_{i=1}^{n}\sum_{m=1}^{n}\lambda_{mij}\left[\theta\widehat{w}_m + \theta\widehat{\tau}_{mi} + (1-\theta)\widehat{w}_i + \widehat{\tau}_{ij}\right] \tag{40}$$

where λ_{mij} denotes the expenditure share of country j on imported stage-2 good from country i, whose production use the stage 1 good from country m, $c_{mij} = \left(\frac{w_m\tau_{mi}}{\theta}\right)^{\theta}\left(\frac{w_i}{1-\theta}\right)^{1-\theta}\tau_{ij}$ denotes the marginal cost for goods imported into country j from country i at stage 2, whose production use the stage 1 good from country m. Under perfect competition, $Y_j = w_jL_j$, which implies $\widehat{U}_j = \widehat{Y}_j - \widehat{P}_j = \widehat{w}_j - \widehat{P}_j$. Substituting (40) into this equation, we get Equation (39).

Step 2: *The change in global welfare satisfies (2).*

Proof: Let λ_{ij} denote the expenditure share of goods imported from country i to country j. It contains two parts: (i) imports of stage-1 goods from country i to country j for producing stage 2 good. This value equals $\sum_{m=1}^{n}\theta\lambda_{ijm}Y_m$ since for each dollar of country m's imports of stage-2 goods from country j, only θ dollar of stage-1 goods is imported from country i to country j. (ii) imports of stage-2 goods from i to j for producing final good; this value equal to $\sum_{m=1}^{n}\lambda_{mij}Y_j$. Hence, expenditure share λ_{ij} satisfies:

$$\begin{aligned}\lambda_{ij}w_jL_j &= \sum_{m=1}^{n}\lambda_{mij}Y_j + \theta\sum_{m=1}^{n}\lambda_{ijm}Y_m \\ \Leftrightarrow s_j\lambda_{ij} &= \sum_{m=1}^{n}s_j\lambda_{mij} + \theta\sum_{m=1}^{n}s_m\lambda_{ijm}\end{aligned} \tag{41}$$

Then, from (39), we can derive the change in global welfare as:

$$\begin{aligned}
\textstyle\sum_{j=1}^{n} s_j \widehat{U}_j &= -\textstyle\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{m=1}^{n} s_j \lambda_{mij} \left[\theta\left(\widehat{w}_m - \widehat{w}_i\right) + \widehat{w}_i - \widehat{w}_j + \theta\widehat{\tau}_{mi} + \widehat{\tau}_{ij}\right] \\
&= -\theta\textstyle\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{m=1}^{n} s_j \lambda_{mij}\left(\widehat{w}_m - \widehat{w}_i + \widehat{\tau}_{mi}\right) - \sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{m=1}^{n} s_j \lambda_{mij}\left(\widehat{w}_i - \widehat{w}_j + \widehat{\tau}_{ij}\right) \\
&= -\textstyle\sum_{j=1}^{n}\sum_{i=1}^{n}\left(\theta\sum_{m=1}^{n} s_m \lambda_{ijm} + \sum_{m=1}^{n} s_j \lambda_{mij}\right)\left(\widehat{w}_i - \widehat{w}_j + \widehat{\tau}_{ij}\right) \\
&= -\textstyle\sum_{j=1}^{n}\sum_{i=1}^{n} s_j \lambda_{ij}\left(\widehat{w}_i - \widehat{w}_j + \widehat{\tau}_{ij}\right) \\
&= -\frac{\sum_{j=1}^{n}\sum_{i=1}^{n} X_{ij}\widehat{\tau}_{ij}}{\sum_{k=1}^{n} Y_k}, \text{ which is (2),}
\end{aligned}$$

where the third equality arises from switching m to i, i to j, j to m for the first term; the fourth equality follows from Equation (41); the last equality is from the trade balance condition as shown in (12)

F Proof of Proposition 6 (Tradable Intermediate Goods)

We present the proofs under perfect competition and monopolistic competition separately.

Under Perfect competition:

Based on same derivation in Proof of Proposition 1, Equation (8) continues to hold, i.e.,

$$\widehat{P}_j = \textstyle\sum_{i=1}^{n} \lambda_{ij}\left[\beta\widehat{w}_i + (1-\beta)\widehat{P}_i + \widehat{\tau}_{ij}\right]$$

where $\beta\widehat{w}_i + (1-\beta)\widehat{P}_i$ replaces $\widehat{w}_i$ in the equation for the one-sectoral model without intermediate goods. Then, the change in global real income is given:

$$\widehat{U}_j = \widehat{w}_j - \widehat{P}_j = -\textstyle\sum_{i=1}^{n}\lambda_{ij}\left[\widehat{w}_i - \widehat{w}_j - (1-\beta)\left(\widehat{w}_i - \widehat{P}_i\right) + \widehat{\tau}_{ij}\right]$$

Hence, the change in global welfare satisfies:

$$\textstyle\sum_{j=1}^{n} s_j \widehat{U}_j = -\sum_{j=1}^{n} s_j \sum_{i=1}^{n}\lambda_{ij}\left[\widehat{w}_i - \widehat{w}_j - (1-\beta)\widehat{U}_i + \widehat{\tau}_{ij}\right] \qquad (42)$$

Trade balance condition, $\sum_{i=1}^{n} X_{ij} = \sum_{i=1}^{n} X_{ji}$, implies:

$$\textstyle\sum_{j=1}^{n} s_j \sum_{i=1}^{n}\lambda_{ij}\left(\widehat{w}_i - \widehat{w}_j\right) = 0$$

Hence, Equation (42) can be rewritten as

$$\begin{aligned}
\textstyle\sum_{j=1}^{n} s_j \widehat{U}_j &= -\textstyle\sum_{j=1}^{n}\sum_{i=1}^{n} s_j\lambda_{ij}\left[-(1-\beta)\widehat{U}_i + \widehat{\tau}_{ij}\right] \\
&= (1-\beta)\textstyle\sum_{j=1}^{n}\sum_{i=1}^{n} s_j\lambda_{ij}\widehat{U}_i - \sum_{j=1}^{n}\sum_{i=1}^{n} s_j\lambda_{ij}\widehat{\tau}_{ij} \\
&= (1-\beta)\textstyle\sum_{j=1}^{n}\sum_{i=1}^{n} s_j\lambda_{ij}\widehat{U}_j - \sum_{j=1}^{n}\sum_{i=1}^{n} s_j\lambda_{ij}\widehat{\tau}_{ij} \\
&= (1-\beta)\textstyle\sum_{j=1}^{n} s_j\widehat{U}_j - \sum_{j=1}^{n}\sum_{i=1}^{n} s_j\lambda_{ij}\widehat{\tau}_{ij} \\
&= -\textstyle\sum_{j=1}^{n}\sum_{i=1}^{n}\frac{s_j}{\beta}\lambda_{ij}\widehat{\tau}_{ij} = -\sum_{j=1}^{n}\sum_{i=1}^{n}\frac{X_{ij}}{\sum_{k=1}^{n} Y_k}\widehat{\tau}_{ij}
\end{aligned}$$

where the third equality stems from the trade balance condition, which implies that $\sum_{j=1}^{n}\sum_{i=1}^{n} s_j \lambda_{ij} \widehat{U}_i = \sum_{j=1}^{n}\sum_{i=1}^{n} s_j \lambda_{ij} \widehat{U}_j$; the fourth equality is from $\sum_{i=1}^{n} \lambda_{ij} = 1$ and the last equality follows from $X_{ij} = \lambda_{ij} \frac{w_j L_j}{\beta} = \lambda_{ij} \frac{Y_j}{\beta}$ and $s_j = \frac{Y_j}{\sum_{k=1}^{n} Y_k}$. Note that when there exist tradable intermediate goods, the total expenditure is $\frac{w_j L_j}{\beta}$ instead of $w_j L_j$.

Under Monopolistic competition:

Based on the same derivation as in Proof of Proposition 2, Equation (23) continues to hold, i.e.,

$$\widehat{P}_j = \sum_{i=1}^{n} \lambda_{ij} \left[\beta \widehat{w}_i + (1-\beta) \widehat{P}_i + \widehat{\tau}_{ij} + \left(\frac{\mu}{\sigma - 1} + \frac{\mu}{\varepsilon} \right) (\widehat{w}_i - \widehat{w}_j) + \frac{\widehat{N}_i}{\varepsilon} \right]$$

where $\beta \widehat{w}_i + (1-\beta) \widehat{P}_i$ replaces $\widehat{w}_i$ in the equation for the one-sectoral model without intermediate goods. Hence, the change in global welfare, according to the same derivation as that for perfect competition, is given by:

$$\sum_{j=1}^{n} s_j \widehat{U}_j = -\sum_{j=1}^{n}\sum_{i=1}^{n} s_j \lambda_{ij} \left[-(1-\beta) \widehat{U}_i + \widehat{\tau}_{ij} + \left(\frac{\mu}{\sigma - 1} + \frac{\mu}{\varepsilon} + 1 \right) (\widehat{w}_i - \widehat{w}_j) + \frac{\widehat{N}_i}{\varepsilon} \right]$$

Under trade balance condition, the above expression can be rewritten as

$$\sum_{j=1}^{n} s_j \widehat{U}_j = -\sum_{j=1}^{n}\sum_{i=1}^{n} \frac{s_j}{\beta} \lambda_{ij} \left(\widehat{\tau}_{ij} + \frac{\widehat{N}_i}{\varepsilon} \right) \tag{43}$$

In addition, under monopolistic competition with restricted entry, N_i is exogenous, i.e., $\widehat{N}_i = 0$. If we consider free entry instead, then the total profit $\Pi_j = N_j w_j F_j$. By R1 and R2, we have $N_j w_j F_j = \Pi_j = \zeta w_j L_j$, which implies $\widehat{N}_i = 0$. Hence, which implies

$$\sum_{j=1}^{n} s_j \widehat{U}_j = -\sum_{j=1}^{n}\sum_{i=1}^{n} \frac{s_j}{\beta} \lambda_{ij} \left(\widehat{\tau}_{ij} + \frac{\widehat{N}_i}{\varepsilon} \right) = -\sum_{j=1}^{n}\sum_{i=1}^{n} \frac{s_j}{\beta} \lambda_{ij} \widehat{\tau}_{ij} = -\sum_{j=1}^{n}\sum_{i=1}^{n} \frac{X_{ij}}{\sum_{k=1}^{n} Y_k} \widehat{\tau}_{ij}$$

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