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Interpreting How Others Interpret It: Social Value of Public Information

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Abstract

This paper studies the social value of public information in environments without common knowledge of data-generating process. We show that the stronger is the coordination motive behind agents behaviour, the more they would like to interpret private or public signals in the way that they suspect others are doing it. Consequently, the negative impact of public communication noted by Morris and Shin (2004) can be amplified if agents have doubts whether others take the public signal too literally and/or are too inattentive to their private signals. The social welfare increases when each agent evaluates the precision of public signal correctly but believes that others did not understand the public signal at all, which suggests that there is a scope for the central bank to "obliterate" its communication in a specific way, by making it, e.g., sophisticated and technical.

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Keywords: central bank communication, transparency, common p-belief, coordination game, higher-order uncertainty.

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1 Introduction

There has been several suggestions in the literature on the optimal degree of transparency that a central bank (or other governing agencies) should adopt in order to maximize social welfare. Agents in the economy use this information in order to update their expectations about the fundamental uncertainty which affects their future payoffs and hence today's actions. They also use this information in order to predict better the behaviour of the others and to better coordinate their actions. However there has been no agreement on what the optimal degree of disclosure should be.

Specifically, Morris and Shin (2004) suggested that if agents utility depends to a larger extent on coordination with each other (rather than on how their actions match fundamental economic values) the central bank should be transparent only if it is able to provide a very precise information, specifically more precise information than the information that agents possess themselves. Those predictions are strengthened when importance of coordination motives increases compared to importance of fundamentals. In turn, Hellwig (2005), in a richer setup, though with a more specific structure of fundamentals and welfare function, argued that increase of public information is always beneficial. Later Angeletos and Pavan (2007) have generalized the framework of Morris and Shin (2004) and within the cases of their analysis as in the setup in Morris and Shin (2004) they reconfirmed their prescriptions on the optimal degree of public information[1].

Those papers, while being different in their assumptions on payoffs and welfare functions, were similar in their assumption about beliefs that agents possess. Specifically, in those models, agents agree how to interpret the public and private signals and this agreement is common knowledge. The only source for difference in beliefs was privately known individual signals, i.e. agents disagree only because of differences in their private information, and had private signals been commonly exchanged there would be no more disagreement (and it would be always beneficial to increase precision of public information).

The assumption of common knowledge of a prior or of a shared common prior has been prevalent in the theoretical literature through decades. However, in the same time, there has been accumulated an abundant empirical literature which puts into question the

[1] Other papers which studied the same question are Cornand and Heinemann (2008), Morris and Shin (2007) and Myatt and Wallace (2009). All those papers came essentially to the same conclusion, the social welfare can be decreasing in the precision of the common signal, and hence the optimal policy should try to reduce the commonality of the signal as much as possible, but not the precision per se.

validity of this assumption within many strategic situations observed in the reality. More often than not agents are miscalibrated and commonly agree to disagree, i.e., are quite far from sharing a common prior.

One basic reason for this may be overconfidence with respect own knowledge about the model of the world and strong belief in superiority of own information. Among psychological studies of financial market participants overconfidence is one of the most consistent findings. A study by Cooper at al (1988) shows that 81% of start-up businessman believe they have at least 70% better chance to succeed than others and that only 39% of others businesses can succeed as well as they can do it. A study by Svenson (1981) shows how in a sample of U.S. students 82% of them are believing to be in the top 30% of the group who is driving most safely. In the similar vein, Ben-David, Graham, and Harvey (2010) show that top financial executives are a way too overconfident with respect to their understanding of the model of the world. In fact, they are severely miscalibrated when asked to predict future returns – the eventually realized market returns fall only 33% of times into their 80% confidence interval, i.e., financial executives are persistently failing in learning how to make correct inferences from the data.

Furthermore, agents publicly keep on following their convictions, which they are persuaded to be correct about. Even after having learned that they disagree they do not seem to make many attempts to revise and merge their opinions. E.g. Odean (1998) gives an example how Warren Buffet and the Feshbach brothers held, respectively, long and short positions worth hundreds of millions of dollars in Wells Fargo Bank and while the bank's weak loan portfolio was public information, investors had quite different models on the impact that this could have on its future value. This difference in opinion was common knowledge between them, yet both followed own beliefs (at the end Feshbachs lost $50 million when closing their positions). A similar, more recent example in this vein is the short position of James Chanos with regard to securities linked to the China economy, who accuses China of artificially creating growth statistics and overall bubbleness. He himself makes large bets on the future collapse of China's economy, despite of the fact that many other large financial market players are having a completely opposite opinion (including e.g. Warren Buffet, Jim Rogers and Wilbur Ross)[2].

As a large-scale and persistent example for difference in models of the world, or more specifically in interpretation of information, one can take abnormal trading volumes after quarterly announcements of corporate earnings, as it is found in Kandel and Pearson

[2] See the article in The New York Times *"Contrarian Investor Sees Economic Crash in China"* from January 7, 2010

(1995). The volumes of trade documented cannot be consistent with the model where all agents are interpreting the same public signal in the same way, even if they possess some private signals. Similarly, Lahiri and Sheng (2008) provide a large scale study on how professional forecasters of macro variables while, arguably, observing roughly the same statistical data, persistently disagree on the future rates of inflation, unemployment and GDP growth. As they revise and publish anew their forecasts regularly and, arguably, they have occasionally a look at what their peers have forecasted, if they shared the common prior their next forecasts would be the same at a faster rate that what is observed in the reality.

Worse than just agreeing to disagree about the model of the world, people can polarize and diverge in their opinions even more once they have observed the same (public) information. This cannot happen under any Bayesian model even with heterogeneous priors. For example, Lord, Ross, and Lepper (1979) document an experiment where subjects supporting and opposing death penalty were exposed to two purported studies, one seemingly confirming and one seemingly disconfirming their existing beliefs about the deterrent efficacy of the death penalty. Both proponents and opponents of capital punishment considered those results and procedures that confirmed their own beliefs to be the more convincing and probative ones, so they shifted beliefs even further to the extremes. Another example is Batson (1975) who show how religious believers disagree even more with non-believers after having been presented with a "study" that the New Testament is a fake.

In light of those observations, it seems, hence, valuable to evaluate the question of desirability of a transparent communication by the central bank in environments where agents do not share a common prior[3]. Naturally, there is a multitude of patterns how agents beliefs can fail to be consistent with the assumption that agents share a common prior and moreover this itself is commonly known and agreed upon. In this paper we take a relatively conservative way to relax this assumption. We consider a class of environments where doubts about others' interpretation of public and private signals take the form of a (common) p-belief. Each agent believes that it is only with probability p others have the same model of the joint stochastic evolution of fundamentals and private and public signals, and with probability $(1-p)$ they interpret a given private or public signal

[3] As far as communication by the central bank is concerned, Rosa and Verga (2008) show that financial market participants need around 3 years to learn (commonly) how to interpret and believe ECB announcements of its policy plan – something which is not allowed by the previous literature on communication by the central bank.

as being actually of a different precision. Furthermore, each agent believes that others have the same doubts as he has about everyone's interpretation only with probability p and with probability $(1-p)$ others are certain that everyone else is certain that precision of signals are different from what each agent believes at the first order. And so on. Thus, we essentially relax only the part that agents commonly know the common prior. We discuss in the Section 5 possible results under a stronger relaxation – when agents agree to disagree, i.e. agents commonly know that they do not share a common prior.

As the main results of the paper, we decorticate the effect of absence of common knowledge of the prior along of two layers. First we study effects of a more precise public information in environments where each agent believes that with probability $(1-p)$ others fail to have the right precision of public signals. In the second environment, each agent believes that others fail to have the right precision of their private signals.

We find that welfare effect of a more transparent communication depends on those fine details of each environment. It matters whether agents believe with probability $(1-p)$ that others miss to have the right precision by *overreacting* to public (or private) signals, i.e others take those signals too literally or by *underreacting* to it, i.e. others just miss seeing the signals. (Note, overconfidence per se can be about both, assuming that others are naive and take the signals at face values but it is also a belief that others' are not noticing the signals at all). For the welfare measure as in Morris and Shin (2004) our findings are rather intuitive: if agents believe that with probability $(1-p)$ everyone is overreacting to the public signal or everyone is underreacting to his private signals, and if $(1-p)$ is significantly high, the communication can be even more detrimental to the social welfare than it is in Morris and Shin (2004). On the other hand, if agents believe that with probability $(1-p)$ everyone is underreacting to the public signal or everyone is overreacting to his individual signals, the negative effect of increased transparency of Morris and Shin (2004) disappears (for p relatively small). The reason for those results is each agent, despite having the correct evaluation of the importance of signals, overreacts to public information or underreact to his private signals if he suspects others to do so. That is, paradoxically, the more agent is confident that he is correct whereas believes others to be incorrect, the more he forgoes his correct estimation and behaves like he believes others to be behaving. Thus, extending the result of Hellwig and Veldkamp (2009), agents not only would like to know what others know, but because of coordination motives, they also would like to follow the signals in a way that they believe others are interpreting them.

Furthermore, for communication to be detrimental to welfare it may be enough that precision of public signal is just a little bit below the precision of private signals. Thus the critique of Svensson (2006) of results of Morris and Shin that they are valid only if precision of public signal is 8 times smaller than the precision of private signals would fail in our environment whenever p is taken to be strictly below 1 and the potential overreaction to the public signal by others, as perceived by each agent, is sufficiently high.

Our main conclusion, as far as the precision of the public signal is concerned, is that the central bank should do its best to increase each agent's perception of the public signal, i.e. to increase the precision of public signal as it is perceived by each agent and it should do its best to make each agent believe that others did not understand as much from his communication as this agent himself did. Hence, if at the end of each meeting with the public the central bank adds something along the famous quote of Alan Greenspan, "...if I seem unduly clear to you, you must have misunderstood what I said", it may, actually, have a welfare increasing effect, according to our findings, provided the public is fully confident about own understanding abilities and only its beliefs about others' perception of the communication by the central bank are shakened by such type of statements.

As further extensions of the model, we also study the impact of greater transparency in the markets where agents commonly agree to disagree about the data generating process. The robust result in this case is a greater transparency essentially outweighs miscoordination due to ex ante disagreement. Thus, assuming that in the reality there is a mixture of overconfidence and disagreement in agents beliefs, the optimal communication policy should take into account carefully also this effect of increased transparency. In addition we also consider a different welfare measure, following the suggestion in Woodford (2005). Contrary to his result that under that different welfare measure a greater transparency is always valuable, under common p-belief about the prior the detrimental effect can be back again even. Moreover, it may well happen that an increase in the precision of private signals can reduce welfare.

2 The Model

The basic assumption about the utility function of agents is the same as in Morris and Shin (2004). There is a continuum of agents of mass 1, indexed by $i \in [0,1]$. Each agent's utility is

$$u_i(a,\theta) = -(1-r)(a_i-\theta)^2 - r(L_i - \int_0^1 L_j dj) \tag{1}$$

with r constant and $0 < r < 1$. The parameter θ stands for a fundamental. The term $L_i \equiv \int_0^1 (a_j - a_i)^2 dj$ is the beauty contest term, a penalty that agents incur for deviating from actions of others.

The value of θ is unknown and represents, from the perspective of agents, a random variable. Each agent has some probabilistic assessments of θ and also beliefs about others' assessments. We will introduce more details below.

Given the individual utilities, we can now state the social welfare function (which is for moment the same as in Morris and Shin (2004) and we will discuss in the concluding section how predictions of the analysis change if we allow for a different welfare function).

$$\begin{aligned} W(a,\theta) &= \frac{1}{1-r}\int_0^1 u_i(a,\theta)d_i \\ &= -\int_0^1 (a_i-\theta)^2 di \end{aligned} \tag{2}$$

Hence, from the ex post perspective it is desirable that each agent's action matches perfectly θ.

3 Some preliminary analysis

From the agent's maximization program we can derive agent's best reply function

$$a_i(t_i) = (1-r)E_i[\theta|t_i] + rE_i[\overline{a}|t_i] \tag{3}$$

where $\overline{a}$ is the average action of other agents.

It is useful to note for a later reference that if θ were commonly known, given the welfare objective in (2), the (unique) optimal action maximizing the social welfare would be

$$a_i^* = \theta \tag{4}$$

Thus under complete information there is no distortions due to miscoordination and the social welfare is at its maximum.

When agents do not possess perfect common knowledge of the value of θ, their action depends on their probabilistic beliefs about the distribution of θ; furthermore, to find an optimal best reply to others action they need to evaluate what other agents' believe about are. Similarly agents must possess third and higher orders of beliefs about everyone else's belief about... θ. Thus, each order of beliefs has an impact on the equilibrium strategy a_i^*. By iterative evaluation of the best reply function we obtain a solution to agents' maximization problem in terms of their higher order beliefs as follows[4]:

$$a_i^*(t_i) = (1-r)\sum_{k=1}^{\infty} r^k E_i(\overline{E}^k(\theta)) \tag{5}$$

where $E_i(\overline{E}^k(\theta))$ means the expectation of agent i of the average expectation of the average expectation, etc. k times (e.g. $E_i(\overline{E}^2(\theta)) = E_i[\overline{E}[\overline{E}[\theta]]]$). Thus, the optimal action in (5) depends only on the first moment of each order of expectations about θ.

4 Common p-Belief About Priors and Overconfidence

4.1 Interim Beliefs

Following the literature we let the (objective) distribution of the fundamental be Gaussian with parameters μ and α:

$$\theta \backsim \mathcal{N}(\mu, \frac{1}{\alpha})$$

Whereas agents commonly know and agree that the fundamental follows the Gaussian distribution, they may disagree or be uncertain which specific values parameters μ and α assume.

We assume that it is common knowledge that signals are linear in θ. The public signal is given by

$$y = \theta + \eta$$

where the distribution of η is from the family of normal distributions

$$\eta \backsim \mathcal{N}(0, \frac{1}{\beta})$$

and again agents may possess different and only privately known beliefs about the parameter β.

In addition to the public signal agents observe individual signals

[4]This has been also derived in Morris and Shin (expression (14) on p.1526)

$$x_i = \theta + \varepsilon_i$$

where the noise ε_i is distributed according to

$$\varepsilon_i \backsim \mathcal{N}(0, \frac{1}{\gamma}).$$

In Morris and Shin (2004) and subsequent related literature the value of parameters $\alpha, \beta, \gamma, \mu$ is assumed to be the same and commonly known across all agents. To depart from this assumption and introduce absence of common knowledge and/or disagreement about parameters $(\alpha, \beta, \gamma, \mu)$ of the (objective) distribution of the vector $(\theta, y, \boldsymbol{x})$ we follow the framework of Monderer and Samet (1989) and allow agents to possess common p-beliefs about stochastic relationship between fundamentals and signals.

The common p-belief about the prior joint distribution is defined here by the following sequence of iterative statements:

1. Every agent believes with probability one that the joint distribution of $(\theta, y, \boldsymbol{x})$ is given by $\mathcal{N}(\boldsymbol{\mu}, \Sigma(\alpha, \beta, \gamma))$

2. Every agent believes that [1] holds with probability p and that with probability $(1 - p)$ everyone believes that with probability 1 the joint distribution of $(\theta, y, \boldsymbol{x})$ is given by some $\mathcal{N}(\boldsymbol{\mu'}, \Sigma')$

3. Every agent believes that [2] holds with probability p and that with probability $(1 - p)$ that everyone believes that with probability 1 that everyone believes that with probability 1 the joint distribution of $(\theta, y, \boldsymbol{x})$ is given by $\mathcal{N}(\boldsymbol{\mu'}, \Sigma')$.

4. .....

Hence, at each order each agent suspects that with probability $(1 - p)$ everyone else believes $(\theta, y, \boldsymbol{x})$ be drawn from $\mathcal{N}(\boldsymbol{\mu'}, \Sigma')$ and moreover that everyone is certain that everyone else is *certain* that the distribution is $\mathcal{N}(\boldsymbol{\mu'}, \Sigma')$. Note that here as $p \to 1$ the environment above converges to the environment where $\mathcal{N}(\boldsymbol{\mu}, \Sigma(\alpha, \beta, \gamma))$ is commonly known. On the other hand, if we let $p \to 0$ agents agree to disagree (though we need to modify distribution of $\mathcal{N}(\boldsymbol{\mu}, \Sigma)$ and $\mathcal{N}(\boldsymbol{\mu'}, \Sigma')$ to be consistent across the population).

In the construction above, at the first order, each agent actually assesses correctly that the joint distribution of $(\theta, y, \boldsymbol{x})$ is given by $\mathcal{N}(\boldsymbol{\mu}, \Sigma(\alpha, \beta, \gamma))$. Instead, of course, one could have introduced doubts about $\mathcal{N}(\boldsymbol{\mu}, \Sigma(\alpha, \beta, \gamma))$ already at the first order. But as we would like first to distill out the effect of *absence of common knowledge* rather than the effect of *absence of knowledge*, i.e. that already at the first level each agent is uncertain what the prior is, we follow the route above.

It is also noticeable that there are several different ways to relax the common knowledge assumption in this framework. Rather than considering a two points uncertainty as here, we could have introduced a richer belief structure and let, for example, that each agent believes with probability $(1-p_1)$ everyone believes that the distribution $\mathcal{N}(\boldsymbol{\mu}', \Sigma')$ and with probability $(1-p_2)$ everyone believes that the distribution is some $\mathcal{N}(\boldsymbol{\mu}'', \Sigma'')$, with $p_1 + p_2 = p + 1$. The higher levels of uncertainty can be enriched further along the same lines. Whereas such extensions are interesting, as we will discuss later in more details, the results cannot be expected to be qualitatively much different from the current framework.

4.2 Equilibrium

To disentangle the different effects of the absence of common knowledge of details of the prior distribution we will consider separately the case where agents have doubts about others' assessment of the actual precision of the public signal, i.e. of β and the case where agents have doubts about others' interpretation of their private signals[5].

Absence of common knowledge of β.

We first consider the case when agents are not certain how others interpret the public signal. Specifically each agent beliefs are

1. Every agent believes with probability one that the precision of the public signal is β
2. Every agent believes that [1] holds with probability p and that with probability $(1-p)$ everyone believes that with probability 1 the precision of the public signal is β'.
3. Every agent believes that [2] holds with probability p and that with probability $(1-p)$ everyone believes that with probability 1 everyone believes that with probability 1 the precision is β'.
4. .....

[5]In the section 5 we consider the case where agents have doubts about others' ex-ante expectation of θ.

The usual technique to solve the equilibrium by the method of undetermined coefficients cannot be applied to our framework[6]. Hence, as the equilibrium strategy can be expressed as a weighted sum of iterated average expectations (see the expression (5)), to calculate it will construct the infinite sequence of iterated expectations resulting from the iterative p-beliefs reasoning above, substitute it into the expression (5) and. The first order is

$$E_i[\theta|y,x_i] = \frac{\alpha\mu+\beta y+\gamma x_i}{\alpha+\beta+\gamma}, \tag{6}$$

the second order expectation, i.e. expectation about the average of others' expectations is

$$E_i\bar{E}[\theta|y,x_i] = p(\frac{\alpha\mu+\beta y}{\alpha+\beta+\gamma}+\frac{\gamma}{\alpha+\beta+\gamma}(\frac{\alpha\mu+\beta y+\gamma x_i}{\alpha+\beta+\gamma}))+ \tag{7}$$

$$+(1-p)(\frac{\alpha\mu+\beta' y}{\alpha+\beta'+\gamma}+\frac{\gamma}{\alpha+\beta'+\gamma}(\frac{\alpha\mu+\beta y+\gamma x_i}{\alpha+\beta+\gamma}))$$

the third order expectations are then

$$E_i\bar{E}\bar{E}[\theta|y,x_i] = p[p(\frac{\alpha\mu+\beta y}{\alpha+\beta+\gamma}+\frac{\gamma}{\alpha+\beta+\gamma}(\frac{\alpha\mu+\beta y+\gamma x_i}{\alpha+\beta+\gamma}))+ \tag{8}$$

$$+(1-p)(\frac{\alpha\mu+\beta' y}{\alpha+\beta'+\gamma}+\frac{\gamma}{\alpha+\beta'+\gamma}(\frac{\alpha\mu+\beta y+\gamma x_i}{\alpha+\beta+\gamma}))]+$$

$$+(1-p)[\frac{\alpha\mu+\beta' y}{\alpha+\beta'+\gamma}+\frac{\gamma}{\alpha+\beta'+\gamma}(\frac{\alpha\mu+\beta' y}{\alpha+\beta'+\gamma}+\frac{\gamma}{\alpha+\beta'+\gamma}(\frac{\alpha\mu+\beta y+\gamma x_i}{\alpha+\beta+\gamma}))]$$

One can derive similarly the forth and subsequent orders of expectations. To simplify somewhat the exposition we introduce the following notation

$$A := \frac{\alpha\mu+\beta y}{\alpha+\beta+\gamma} \tag{9}$$

[6]This is because agents' second order beliefs here are totally independent of their first order beliefs. The method of undetermined coefficients takes only first order of expectations and uses implicitly the fact that all information about the belief hierarchy can be derived from $E_i[\theta|y,x_i]$ (and the fact that all variables are normally distributed).

$$B := \frac{\alpha\mu + \beta y + \gamma x_i}{\alpha + \beta + \gamma}$$

$$C := \frac{\alpha\mu + \beta' y}{\alpha + \beta' + \gamma}$$

$$k := \frac{\gamma}{\alpha + \beta + \gamma}$$

$$q := \frac{\gamma}{\alpha + \beta' + \gamma}$$

Using this notation and substituting $(E_i[\theta|y, x_i],\ E_i\bar{E}[\theta|y, x_i],...)$ into (5) we obtain the equilibrium strategy as a sum of series

$$a_i = (1-r)B + (1-r)r(p(A+kB) + (1-p)(C+qB))$$

$$+(1-r)(1-p)\sum_{s=2}^{\infty} r^s\{C[\sum_{m=0}^{s-1} q^m + \sum_{m=0}^{s-2} p^{m+1} \sum_{n=0}^{s-m-2} q^n]$$

$$+A[p^s \sum_{m=0}^{s-1} k + \sum_{m=0}^{s-2} p^{m+1} q^{s-m-1} \sum_{j=0}^{m} q^j]$$

$$+B[q^s + p^s k^s + \sum_{m=0}^{s-2} p^{s-m-1} q^{m+1} k^{s-m-1}]\}$$

Simplifying summands we obtain

$$a^* = \frac{(1-r)rp(1-pqr)A}{(1-pr)(1-qr)(1-kpr)} + \frac{(1-r)(1-pqr)B}{(1-kpr)(1-qr)} + \frac{r(1-p)C}{(1-pr)(1-qr)} \tag{10}$$

To make our results more directly comparable to Morris and Shin (2004) we let $\alpha = 0$ for the moment. Using this and (9) we simplify (10) to obtain

$$a_i^*(y, x_i) = \frac{\beta'\gamma r(1-p) + \beta(\beta' + \gamma(1-r))}{(\beta' + \gamma(1-r))(\beta + \gamma(1-pr))} y + \frac{\beta'\gamma(1-r) + \gamma^2(1-r)(1-pr))}{(\beta' + \gamma(1-r))(\beta + \gamma(1-pr))} x_i \tag{11}$$

When $p = 1$ our results coincide, naturally, with those of M&S and the optimal strategy is

$$a_i^*(y, x_i) = \frac{\beta y + (1-r)\gamma x_i}{\beta + (1-r)\gamma} \tag{12}$$

It is noticeable that without any strategic consideration, the optimal action of an agent is equal to his expectation of θ given his signals y and x_i, namely to

$$a_i^{**}(x_i, y) = \frac{\beta y + \gamma x_i}{\beta + \gamma}. \tag{13}$$

Comparing (12) and (13) one can see that because of strategic considerations, in the equilibrium of M&S, each agent puts inefficiently low weight to his private signal compared to the public signal. Whenever there is a higher-order strategic uncertainty, i.e. uncertainty about beliefs about the prior distribution, as we have introduced it, the weight assigned to private information is also inefficiently low (this is a robust result, it will be always too low, no matter what p and β' are, as long as $r \neq 0$). However, if $\beta > \beta'$ the weight assigned to the private information may be yet higher compared to the case without uncertainty about the prior distribution (like it is in M&S), i.e. when $p = 1$, namely

Proposition 1. *In environments with common p-belief about the prior distribution where agents have doubts about others interpretation of public signals, if $\beta > \beta'$ ($\beta < \beta'$) the weight each agent assigns to own private signal is higher (lower) for any $p \in (0, 1)$ as compared to the case with $p = 1$. Moreover, for $p \in (0, 1)$ if $\beta > \beta'$ the weight assigned to private signal is decreasing in p (and it is increasing for $\beta < \beta'$).*

Proof. First part follows from the direct comparison of $\frac{\beta'\gamma(1-r)+\gamma^2(1-r)(1-pr))}{(\beta'+\gamma(1-r))(\beta+\gamma(1-pr))}$ with $\frac{(1-r)\gamma}{\beta+(1-r)\gamma}$, the second from differentiation of $\frac{\beta'\gamma(1-r)+\gamma^2(1-r)(1-pr))}{(\beta'+\gamma(1-r))(\beta+\gamma(1-pr))}$ with respect to p. □

This proposition is our first finding – if each agent believes that others are assigning too high weight ($\beta < \beta'$) to the public signal, he forgoes even further his own private signal. As everyone would like to predict the best the action of the others, everyone is also interested in predicting the best, not only what others private information is, but also *how do they interpret it*.

Uncertainty about γ

Arguably agents may also be uncertain how others interpret their private signals. In this section we allow for common knowledge of the precision of the public information and instead consider what are the optimal actions when there is no common knowledge of how private signals are to be interpreted. Specifically each agent beliefs are

1. Every agent believes with probability one that the precision of private signals is γ

2. Every agent believes that [1] holds with probability p and that with probability $(1 - p)$ everyone believes that with probability 1 the precision of the private signals is γ'.

3. Every agent believes that [2] holds with probability p and that with probability $(1 - p)$ everyone believes that with probability 1 everyone believes that with probability 1 the precision is γ'.

4. .....

Proceeding as in the precious subsection we obtain the equilibrium strategy as follows[7]:

$$a_i^*(y, x_i) = \frac{\beta\gamma r(1-p) + \beta(\beta + \gamma'(1-r))}{(\beta + \gamma'(1-r))(\beta + \gamma(1-pr))} y + \frac{\beta\gamma(1-r) + \gamma\gamma'(1-r)(1-pr))}{(\beta + \gamma'(1-r))(\beta + \gamma(1-pr))} x_i \quad (14)$$

As before, the weight that each agent assigns to his own private signal is inefficiently low compared to what he should do given just informational constraints. But if each agent beliefs that others are likely to over-value their own private signals (i.e. that $\gamma < \gamma'$) each agent starts to assign a higher weight to his own private signal.

Proposition 2. *In environments with common p-belief about the prior distribution, where agents have doubts how others are interpreting their private signals, if $\gamma < \gamma'$ ($\gamma > \gamma'$) the weight each agent assigns to own private signal is higher (lower) under any $p \in (0, 1)$ as compared to the weight assigned under $p = 1$. Moreover, for $p \in (0, 1)$ the weight assigned to the private signal is increasing in p if $\gamma > \gamma'$ and decreasing otherwise.*

Proof. Similar to the proof of the previous proposition. □

4.3 Optimal Communication

We study next what is the optimal degree of transparency given the equilibrium strategies and given agents' uncertainty about others' interpretation of public and private signals.

First we consider the case where there is no common knowledge of the precision of the public signal. Given the equilibrium action in (11) the social welfare function is

[7]For this it is enough to unfold some first orders of iterated expectations, then one can see that in (9) only q and C change to become $q' := \frac{\gamma'}{\alpha+\beta+\gamma'}$ and $C' := \frac{\alpha\mu+\beta y}{\alpha+\beta+\gamma'}$. Then, substituting those expression together with k,B,A unchanged (and $\alpha = 0$) one obtains the equilibrium (14).

$$E[SW^p(.)] = -\frac{(\beta'\gamma(1-p) + \beta(\beta' + \gamma(1-r)))^2 + \gamma(1-r)^2\beta(\beta' + \gamma(1-pr))^2}{\beta(\beta' + \gamma(1-r))^2(\beta + \gamma(1-pr))^2}$$

Our first preliminary result is quite general and holds for arbitrary β' and β,: the social welfare increases from higher-order uncertainty, i.e. if p goes down from 1, only if $\beta' < \beta$ (other things being equal):

Proposition 3. *The increase of higher-order uncertainty, as measured by decrease in p, increases the expected social welfare only if $\beta' < \beta$.*

We did not specify much until now where the value of β' comes from. In principle β' can be related to β by some function $\beta' = f(\beta)$. The function $\beta' = f(\beta)$ can be an exogenous property of the environment, i.e. of agents' belief types, how a given public signal is translated into common p-beliefs. It can also be the property of the communication process of the central bank. Specifically, it is realistic that if the precision of public signal increases, agents would believe that others reacting to this and take the public signal as indeed more precise. Hence $f(\beta)$ is an increasing function β. In what follows, we will work with a linear function and define $\beta'(\beta) = s\beta$ or

$$s := \frac{\beta'}{\beta}$$

The parameter s can be fixed and exogenous, or it can be endogenous to the communication strategy of the central bank.

With $\beta'(\beta) = s\beta$, the equilibrium strategy (11) can be rewritten as

$$a_i^*(y, x_i) = \frac{\beta s\gamma r(1-p) + \beta^2 s + \beta\gamma(1-r))}{(\beta s + \gamma(1-r))(\beta + \gamma(1-pr))}y + \frac{\beta s\gamma(1-r) + \gamma^2(1-r)(1-pr))}{(\beta s + \gamma(1-r))(\beta + \gamma(1-pr))}x_i$$

Given this expression and welfare measure in (2) we can calculate the expected welfare at θ:

$$E[SW^p(.)] = -\frac{\beta(\beta s + \gamma(1-r+rs(1-p)))^2 + \gamma(1-r)^2(\beta s + \gamma(1-pr))^2}{(\beta s + \gamma(1-r))^2(\beta + \gamma(1-pr))^2}$$

Proposition 4. *Expected social welfare is always increasing in γ. There exists thresholds $\hat{\gamma}$ and $\bar{\beta}$ such that if $\gamma \in [0, \hat{\gamma})$ and $\beta \in [0, \bar{\beta})$ the social welfare is decreasing in β. For $s > 1$ the threshold $\bar{\beta}$ is increasing in s and decreasing in p.*

Proof. Follows from taking derivative of $E[SW^p(.)]$ and solving a system of inequalities. □

This means that whenever agents expect sufficiently high probability (i.e. with probability $(1-p)$) that others takes the public signal as too informative, i.e. if $s > 1$, the social welfare is decreasing in precision of public information over a larger set of parameters, and more specifically even for γ relatively high. Recall that the main policy implication of Morris and Shin (2004) was that if the private signals are sufficiently precise and if the central bank cannot improve precision beyond a threshold, it can be optimal to set $\beta = 0$, i.e. no information should be released. In our case, the result of the Proposition above means that if s is sufficiently large ($s > 1$) and $(1-p)$ is large, even for r relatively low it may be optimal to set $\beta = 0$. Recall again that in Morris and Shin (2004) the social welfare is *always* increasing in β for $r < \frac{1}{2}$. The higher order uncertainty unsettles this result:

Proposition 5. *There exists $\bar{s} >> 1$ and $\bar{p}$ such that if $s > \bar{s}$ and $p < \bar{p}$ even for $r < \frac{1}{2}$ the social welfare is decreasing in β.*

An intuition behind those two results is relatively simple: s, when $s > 1$, is a multiplier of a negative effect that the public signal may have, if β is relatively low and its marginal increase is welfare decreasing, s worsens the situation further: not only agents put too much weight on the (likely wrong) public signal but they also believe that others put too high weight on it and it provides them with further incentives to put an even more higher weight on the public signal.

Till now we have treated the function $\beta'(\beta) = s\beta$ as exogenous. However, arguably, the central bank may have influence on both β and, to a limited extent on s: it may be explaining its information in a way which not only improves on agents' understanding of the central bank's signal, but also influencing their understanding of how others' might have understood the signal. For example, providing very technical information increases precision of the public signal for each agent, but at the same time it may each agent think that others are not as good at understanding such technical information and hence they are likely to disregard the public signal. The following proposition says that it is *always* optimal for the central bank, if feasible, to set $s = 0$

Proposition 6. *For any finite β the social welfare is decreasing in s. Hence, for any finite β it is optimal to set $s = 0$ or to any feasible s_{min}.*

Thus, notwithstanding the case where $\beta = \infty$, i.e. the public signal is fully revealing about θ, it is always optimal for the central bank to muddle the information it reveals *as* *long as this allows to introduce doubts into agents beliefs about how others were able to understand the revealed information* and whether they potentially considered it as imprecise and hence potentially assigned a low weight. Putting differently, if β cannot be increased beyond a threshold, there is always a welfare improving hiding of information provided it affects agents' second (and higher) order beliefs in a desired way.

Next we provide the analysis for the case where agents are uncertain how other agents are interpreting their private signals. The situation turns out to be diametrically opposite to the case where agents are uncertain how others are interpreting public signals. Again we can define

$$t := \frac{\gamma'}{\gamma}$$

In this case the expected social welfare is

$$E[SW^p(.)] = -\frac{\beta(\beta+\gamma(t+r(1-p-t)))^2+\gamma(1-r)^2(\beta+\gamma t(1-pr))^2}{(\beta+t\gamma(1-r))^2(\beta+\gamma(1-pr))^2}$$

Similarly to the case with uncertainty about the precision of the public signal we have

Proposition 7. *The value of $E[SW^p(.)]$ is always increasing in γ and t. (2) For $0 \leq t < 1$ ($t > 1$) the value of $E[SW^p(.)]$ is increasing (decreasing) in p, uncertainty about others' interpretation of private signals is welfare beneficial if $t > 1$. There exists a value of precision of public information $\bar{\beta}$ such that for any $\beta \in [0, \bar{\beta})$ the social welfare is decreasing in β, with $\bar{\beta}$ decreasing in t.*

Hence, the more agents believe that others take their private signals as too informative, the less there is danger in increasing precision of public information even at β relatively low. The overreaction to public information is dampened by agents' believing that others are too concentrated on their private signals. However opposite is true if everyone believes that others are disregarding their private signals too much.

Similarly to the previous section, it can be shown that even for $r < 1/2$ the social welfare can be decreasing in β.

Proposition 8. *There exists $\bar{t}$ and $\bar{p}$ such that if $t < \bar{t}$ and $p < \bar{p}$, such that the social welfare is decreasing for $\beta \in [0, \bar{\beta})$* even if $r < \frac{1}{2}$.

This proposition is illustrated in the Figure 1.

To summarize the results in this section: to evaluate the entire impact of more transparent communication one should take into account not only agents' own understanding

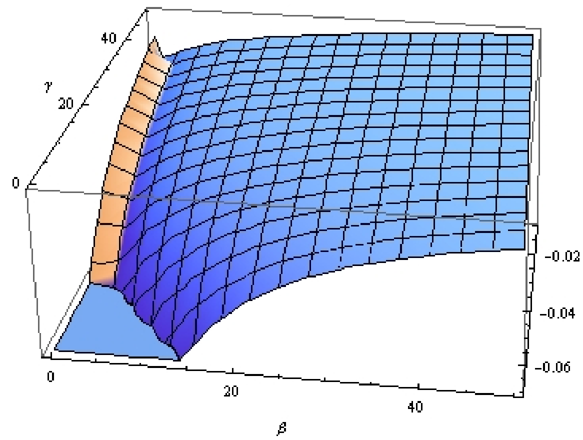

Figure 1: Social welfare with $r = \frac{1}{3}$, $p = \frac{1}{2}$, $t = \frac{1}{30}$

of public signals but also their perception of others' understanding of private and public signals. Fine details of those perceptions (whether agents perceive others being over or under reacting to the public and their private signals) matter for the optimal policy prescription. Whereas empirical research suggests that agents are overconfident about their abilities to interpret a stream of signals, it is left to clarify further how this overconfidence translates into their perception of others abilities.

5 Discussion and Extensions

5.1 Other Welfare Measures

Woodford (2005) has suggested that the welfare measure in Morris and Shin (2004) is very special, as it accounts only for discrepancy between agents actions and the fundamental, and instead the following measure is more appropriate:

$$E[SW^W(.)] = -\frac{1}{(1-r)}E[L_i] \tag{15}$$

where L_i is given by

$$L_i = (1-r)(\theta - a_i)^2 + rE_j(a_i - a_j)^2$$

i.e. the welfare measure should take into consideration also the fact that agents fail to coordinate with each other. In this case the social welfare for the model of Morris and

Shin (2004) is equal to

$$E[SW^W(.)] = -\frac{\beta + \gamma(1-r^2)}{(\beta + \gamma(1-r))^2}$$

and it is always an increasing function of β and γ, no matter what r is, i.e. more communication is always welfare improving.

Here we provide our analysis for this different welfare function given the absence of common knowledge about the prior distribution. It turns out that the results of Woodford does not hold any longer in this case. Substituting equilibrium actions in (15) with $\beta' = \beta s$ we obtain the following expression

$$E[SW^W(.)] = -\frac{\left(\gamma^2(1-r)(1-pr) + \beta\gamma s(1-s)\right)^2}{\gamma(\beta + \gamma - \gamma pr)^2(\gamma - \gamma r + \beta s)^2} -$$

$$-\frac{2r\left(\gamma^2(1-r)(1-pr) + \beta\gamma s - \beta\gamma rs\right)^2}{\gamma(1-r)(\beta + \gamma(1-pr))^2(\gamma - \gamma r + \beta s)^2} - \frac{\left(\beta\gamma(1-r) + \beta^2 s + \beta\gamma rs(1-s)\right)^2}{\beta(\beta + \gamma - \gamma pr)^2(\gamma - \gamma r + \beta s)^2}$$

The welfare maybe yet decreasing in β, even for $r < 1/2$, as figure 2 demonstrates.

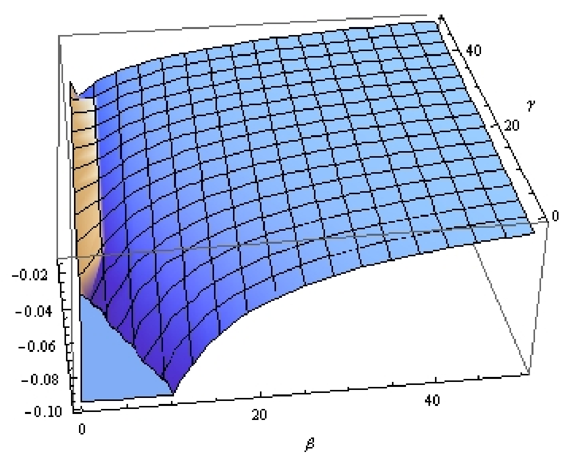

Figure 2: Social welfare under uncertainty about β, with $s = 30$, $p = 1/3$, $r = 1/3$.

The reason for this result the following. The new welfare measure accounts for two effects of increased precision of public information: one it is the part of M&S where for β low the welfare can be decreasing in β, another is the always positive gain in welfare under increased capacity to coordinate with each other. In the framework of M&S the second effect of increased β has been always larger than the first. Under common p-belief about β the first effect can be still dominating, if $\beta' = \beta s$ is sufficiently high due to high s.

By contrast, when there is uncertainty how others are interpreting their private signals, the social welfare is always increasing in β, however it may be *decreasing* in γ. Again substituting for $\gamma' = t\gamma$ we obtain the following expression for the expected social welfare

$$E[SW^W(.)] = -\frac{\left(\beta^2 - \beta\gamma(-1+p)r - \beta\gamma(-1+r)t\right)^2}{\beta(\beta+\gamma(1-pr))^2(\beta+\gamma s-\gamma rt)^2} -$$

$$-\frac{\left(\beta\gamma - \beta\gamma r + \gamma^2(1-r)(1-pr)t\right)^2}{\gamma(\beta+\gamma(1-pr))^2(\beta+\gamma t(1-r))^2} - \frac{2r\left(\beta\gamma - \beta\gamma r + g^2(1-r)(1-pr)t\right)^2}{\gamma(1-r)(\beta+\gamma(1-pr))^2(\beta+\gamma t(1-r))^2}$$

Figure 3 demonstrates the case with the decreasing welfare in γ under uncertainty about γ

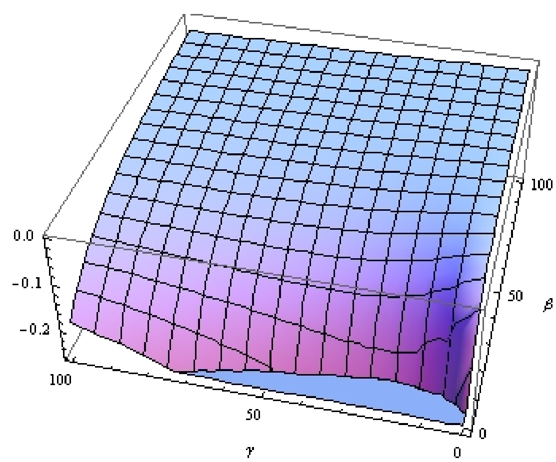

Figure 3: Social welfare under uncertainty about γ' with $t = 40$, $p = \frac{1}{2}$, $r = \frac{9}{10}$

The intuition for this result comes from the fact that now the welfare measure accounts for agents miscoordination among themselves. And if agents belief that others overreact to individual signals, but those signals themselves are quite noisy (γ is very low), each agent takes into account own private signal unjustifiably too much, which results in expected welfare losses.

To summarize, under common p-beliefs about the data generating process, the alternative welfare measure as in Woodford (2005) results in different predictions compared to the case where the prior is commonly known. Contrary to the results in Woodford (2005), too much of doubts about others abilities to interpret all information properly can add to welfare losses through the channel of missed mutual coordination.

5.2 Uncertainty About the Average Error

It is arguable that agents can be uncertain not only about how others interpret a given profile of signals. Each agent may bear doubts how others evaluate the expected error of the public signal. Or, similarly, how others evaluate the expected average of individual errors. In other words, each agent have the following hierarchy of common p-beliefs:

1. Every agent believes with probability one that the joint distribution of $(\theta, y, \boldsymbol{x})$ is given by $\mathcal{N}(\boldsymbol{\mu}, \Sigma(\alpha, \beta, \gamma))$
2. Every agent believes that [1] holds with probability p and that with probability $(1 - p)$ everyone believes that with probability 1 the joint distribution of $(\theta, y, \boldsymbol{x})$ is given by $\mathcal{N}(\boldsymbol{\mu}', \Sigma)$, with $\boldsymbol{\mu}' \neq \boldsymbol{\mu}$
3. Every agent believes that [2] holds with probability p and that with probability $(1 - p)$ that everyone believes that with probability 1 that everyone believes that with probability 1 the joint distribution of $(\theta, y, \boldsymbol{x})$ is given by $\mathcal{N}(\boldsymbol{\mu}', \Sigma)$.
4. .....

It turns out that the greater transparency is more likely to be welfare increasing:

Proposition 9. *Let $\boldsymbol{\mu}' \neq \boldsymbol{\mu}$, then an increase in β always increases the social welfare whenever p is sufficiently low.*

Details of the proof are left to the reader.

The intuition behind this proposition is quite simple – the doubts about others beliefs about the average of mistakes in the economy leads to a higher discrepancy between the average action and the value of the fundamental, which reduces the welfare. Yet, this discrepancy is important only proportionally to $\frac{1}{\beta}$ and as a consequence increase in β reduces it. A relatively low p makes the importance of the discrepancy higher in the social welfare, that is why when p is low, even if β itself is rather low but γ is high (i.e. we are in the region where under $p = 1$ increase of β would be welfare detrimental), the gains to welfare from the reduction in the discrepancy are higher compared to losses due to overreaction to the noisy public signal.

5.3 Other forms of higher order uncertainty

We have considered the case where the prior which $(1-p)$-believed is the same for each order of doubts (agents put essentially probability 1 on others believing with probability 1.... in β' or γ') and the same across agents. The question arises whether the conclusions of the paper would be different had we allowed for a richer framework, i.e. a richer belief structure and higher order doubts about the prior. It seems that while computationally the task of solving for the equilibrium becomes even more convoluted, the qualitative conclusions would not change much. The reason for this is inherent continuity of the equilibrium action in each order of expectations about others expectations. Moreover the impact that each subsequent order of doubts has on the equilibrium action is exponentially diminishing with each additional level of beliefs. This means that even if some higher order expectation at a level k does jump drastically, unless this jump outweighs significantly the weight r^k (recall that $r < 1$) assigned to that order of expectation, the total effect of the jump on the equilibrium strategy would not be too high. Thus, such results, with a richer structure of higher order expectations, can be of interest and policy importance only if they are driven by an empirical validation of a specific, rich hierarchy of beliefs.

6 Conclusions

In this paper we have not justified choice of a specific structure of higher order uncertainty and beliefs, instead we characterized equilibrium behaviour and the social welfare for a class possible belief hierarchies consistent with absence of common knowledge of a prior distribution.

Future research hence should be two-fold: necessarily one should make an empirical validation of different structures of higher order beliefs. Second, similarly to the paper by Myatt and Wallace (2011) it would be desirable to investigate under which conditions agents endogenously are failing to have common knowledge of the aggregate state, in our case the cross sectional distribution of prior beliefs (this agenda is pursued in Gizatulina (2011)).

Our model is general enough to allow for common knowledge of disagreement in a population of agents divided into two groups, where each agent considers that there is a fraction k of agents who believe that the precision of public signal is some β'' and another fraction $(1-k)$ who believe the precision to be β''' then, it is enough to substitute for $\beta' = k\beta'' + (1-k)\beta'''$ and let $p = 0$ in the above analysis to obtain the desired result. In this case results depend on how increase in β influences β'' and β'''. The main trade-off is that again increase in the precision of public information can be welfare detrimental due to too much noise and agents over-reaction to it, yet if both functions $\beta''(\beta)$ and $\beta'''(\beta)$ are monotonic in β, increase in β reduces simultaneously the discrepancy between $\beta''$and β''', which itself increases the welfare. The optimal policy, hence, should to be weighting off those two effects.

All those various extensions of the model being considered, the one-line conclusion from the main part of the paper is the central bank should do it best to increase each agent's perception of the public signal, i.e. to increase the precision of public signal as it is perceived by each agent and it should do its best to make each agent believe that others did not understand as much from his communication as this agent did.

References

ANGELETOS, G.-M., AND A. PAVAN (2007): "Efficient Use of Information and Social Value of Information," *Econometrica*, 75, 1103 – 1142.

BATSON, C. (1975): "Rational Processing or Rationalization? The Effect of Disconfirming Evidence on a Stated Religious Belief," *Journal of Personality and Social Psychology*, 32, 176–184.

BEN-DAVID, I., J. GRAHAM, AND C. HARVEY (2010): "Managerial Miscalibration," Working paper, NBER.

CORNAND, C., AND F. HEINEMANN (2008): "Optimal Degree of Public Information Dissemination," *The Economic Journal*, 118, 718 – 742.

GIZATULINA, A. (2011): "Endogenous Common Knowledge," work in progress, Max Planck Institute, Bonn.

HELLWIG, C. (2005): "Heterogeneous Information and the Welfare Effect of Public Information Disclosure," Working paper, UCLA.

HELLWIG, C., AND L. VELDKAMP (2009): "Knowing What Others Know: Coordination Motives in Information Acquisition," *Review of Economic Studies*, 76, 223 – 251.

KANDEL, E., AND N. PEARSON (1995): "Differential Interpretation of Public Signals and Trade in Speculative Markets," *Journal of Political Economy*, 103, 831 – 872.

LAHIRI, K., AND X. SHENG (2008): "Evolution of Forecast Disagreement in a Bayesian Learning Model," *Journal of Econometrics*, 144, 325 – 340.

LORD, C., L. ROSS, AND M. LEPPER (1979): "Biased Assimilation and Attitude Polarization: The effects of Prior Theories on Subsequently Considered Evidence," *Journal of Personality and Social Psychology*, 37, 2098 – 2109.

MONDERER, D., AND D. SAMET (1989): "Approximating Common Knowledge with Common Belief," *Games and Economic Behaviour*, 1, 170 – 190.

MORRIS, S., AND H. SHIN (2004): "Social Value of Public Information," *American Economic Review*, 92, 1521 – 1534.

MORRIS, S., AND H. SHIN (2007): "Optimal Communication," *Journal of European Economic Association*, 5, 594 – 602.

MYATT, D., AND C. WALLACE (2009): "On the Sources and Value of Information: Public Announcement and Macroeconomic Performance," Working paper, University of Oxford.

——— (2011): "Endogenous Information Acquisition in Coordination Games," *Review of Economic Studies*, pp. 1 – 38.

ODEAN, T. (1998): "Volume, Volatility, Price and Profit When all Traders are Above Average," *Journal of Finance*, 53, 1888 – 1934.

ROSA, C., AND G. VERGA (2008): "The Impact of Central Bank Announcements on Asset Prices in Real Time," *International Journal of Central Banking*, 4, 175–217.

SVENSON, O. (1981): "Are we all less risky and more skillful than our fellow drivers," *Acta Psychologica*, 47, 143 – 148.

SVENSSON, L. (2006): "Social Value of Public Information: Morris and Shin (2002) is Actually Pro-Transparency, not Con," *American Economic Review*, 96, 448 – 452.

WOODFORD, M. (2005): "Central Bank Communication and Policy Effectiveness," Working paper, NBER.