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Abstract

Government or company decisions on whom to hire are mostly delegated to politicians, public sector officials or human resources and procurement managers. Due to anti-corruption laws, agents cannot sell contracts or positions that they are delegated to decide upon. Even if bribing is ruled out, those interested in the spoils may invest in a good relationship with the deciding agents in order to be remembered when the nomination is made. In this paper, we analyze such markets for connections in the presence of convex costs of networking.

JEL-Code: C790, D520, D720, D850, H570, L140.

Keywords: nominations, rent-seeking, networks, politicians, procurement.

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1 Introduction

Government or company decisions on whom to hire are mostly delegated to politicians, public sector officials or human resource and procurement managers, respectively. Politicians and public sector officials distribute government contracts and fill public sector jobs. Managers, acting for the part of their companies, are delegated to choose one among the best candidates for the job when recruiting personnel, accountants, consultants, or subcontractors. Such agents are in a position to distribute lucrative contracts or nominations. Yet, anti-corruption laws and best business practices forbid decision makers, civil servants or private sector managers alike, from privately selling the contracts or nominations and reaping gains for themselves. Although interested rent-seekers can not buy a position, it pays off to be on good terms with the decision maker: favors are passed to acquaintances only and therefore the rent-seeker needs a close connection with the decision maker to have a chance of being favored.

Keeping in touch is costly, however. It takes not only the rent-seeker's time and effort but also that of the decision maker. Why should the decision maker bother spending time with a rent-seeker? He or she must be compensated for doing so. The rent-seekers spend time with the decision makers by offering lunches and entertainment, and, in politics, by taking part in campaigns and fund-raising events to be remembered when rents are distributed. Each individual rent-seeker gains by rubbing shoulders with several decision makers. As will be illustrated formally in the paper, this results in rent dissipation due to time-consuming network formation. Thus, the fact that nominations and projects generating rents cannot be legally sold can have excessive network formation as a side effect.

We present a stylized and static model of rent-seeking when rent-seekers do not pay directly for nominations or projects, but where rent-seekers pay for the access to the decision makers to be remembered at the time when the spoil is distributed. Intuitively, we show that such rent-seeking may result in excessive networking and thus generates inefficiencies in the unique (up to permutations) stable network when any search rationale of networking is ruled out. We show how the surplus accruing to decision makers and to rent-seekers depends on the value of rents and on the relative size of the two groups. We also study how the stable network is structured.

Since the seminal contribution of Jackson and Wolinsky (1996) on strategic networks by mutual consent, a whole literature studying networks as outcomes of economic decisions has emerged. We follow this literature by assuming that for a rent-seeker to receive a spoil from a decision maker, a connection must be established between the two. We allow for transfers being paid from rent-seekers to decision makers as a remuneration for keeping up the connection.[1] We assume that parties don't observe the link quantities of the agents they are connected to but they have correct conjectures on the link quantities in the stable network. There may be price discrimination ex-ante so that transfers paid for the links may differ across rent-seekers. Our main methodological contribution to the literature on networks by mutual consent is to show that, in our specific rent-seeking environment, the unique pair-wise stable network with transfers coincides with a network established by a Walrasian auctioneer who announces a unique and uniform market price for links. Thus, there is no price discrimination in the stable network and a Walrasian approach can be used to simplify the analysis to a great extent. To our knowledge, the current paper is the first to consider economic but non-strategic (Walrasian) network formation, which yet has a game-theoretic foundation. Crucial for this result is also that agents on each side of the market are identical, and that there are strictly increasing marginal linking costs.

A related paper on bipartite networks is Kranton and Minehart (2001) who model economic interaction on an established network explicitly. They analyze strategic network formation followed by strategic trading on the thereby established platforms. They find that efficient networks are formed when highest valuation buyers pay the social opportunity cost for the good. In our paper, inefficiencies are due to the feature that decision makers who distribute the spoils are prevented from charging the social opportunity cost due to the anti-corruption laws which prevent selling the good. The implied high rents for the rent-seekers invite inefficiently large scale of networking. Moreover, in the model of Kranton and Minehart (2001), there is a constant cost of networking per each link whereas we assume convex linking costs to allow for increasing marginal opportunity cost of networking.

In a classical paper, Aumann (1964) pointed out that the core and the Walrasian equilibrium coincide when there are infinitely many agents on the market.[2] The core requires stability[3] with respect to deviations by any coalition of agents, including those where more than two agents jointly deviate. In our model, we have

[1]The extent of the transfer cannot influence the probability of receiving the spoil since anti-corruption laws are binding.

[2]See also Shubik (1959) and Debreu and Scarf (1963) who point out that the core converges to the Walrasian equilibrium when the number of agents tends to infinity. McKenzie (1955) and Arrow and Hahn (1971) establish the existence of the Walrasian equilbirium in an economy with externalities.

[3]An allocation is stable if no deviating coalition can reach a higher utility.

an equivalence of the Walrasian equilibrium and the pair-wise stable network even if we only have finitely many agents.[4] Of course, the setups are rather different. The classical equivalence of core and the Walrasian equilibrium is established in a pure exchange economy without trading costs, with no limitations on trade and no externalities. In our framework, instead, link formation is costly and anti-corruption laws forbid selling nominations, linking has negative externalities on rent-seekers, and decision makers cannot commit to sell a given number of links. This latter is the crucial difference with respect to Kranton and Minehart (2001) where networks are efficient. In our setup agents end up dissipating rent due to excessive network formation.[5]

Our analysis has common features with also another strand of literature, that on rent-seeking and lobbying contests (Tullock 1967, 1980; Bernheim and Whinston, 1986; Grossman and Helpman, 1994) which gains important insights into how lobbying may affect policy making. These models are similar to our model in that rent-seekers actively influence the decision makers' decisions on how to distribute rents. Yet, there are some differences. In our model, links are endogenous, requiring mutual consent. Moreover, the links are costly not only for the lobbying side but also for the decision makers. Payments are made in exchange for establishing links. In the rent-seeking and lobbying literature, the links are given at the outset, and payments are viewed as bids for rents to be distributed. The only previous contribution that endogenizes the relationship between decision makers and lobbyists is Felli and Merlo (2006). Our approach is complementary to theirs. Whereas they analyze ideological lobbying, we analyze lobbying on non-ideological spoils. Furthermore, Felli and Merlo (2006) assume that the links are costless whereas we assume that creating and maintaining links is costly.

Throughout the analysis, we assume that anti-corruption laws work and thus the spoils that decision makers distribute cannot be auctioned or traded even implicitly.[6] Previous literature on contests has already extensively analyzed the case where the anti-corruption laws can be circumvented.[7]

[4]The equivalence holds also for pairwise Nash stability. See Bloch and Jackson (2006).

[5]Due to the symmetry assumption, we are abstracting from the motivation in finding best matching group of agents on the other side, only the number of agents and the price of a match matters.

[6]The inability of decision makers to sell or to auction off nominations or projects when these arise could result from outside monitoring or from there being a fraction of honest citizens who would report asking or offering bribes, provided that punishments for corruption are sufficiently high.

[7]Therefore, we have especially in mind a modern democracy with a relatively low level of corruption. Well-fitting examples are EU 15 and especially Nordic countries. See the Transparency

The paper is structured as follows. Section 2 presents the model. Section 3 solves for the equilibria using a Walrasian approach. Section 4 shows that there is an equivalence between the non-strategic Walrasian approach and the strategic network through pairwise stability with transfers (Bloch and Jackson, 2006). Section 5 relates our results to existing literature and section 6 concludes.

2 The model

We analyze decisions to form connections in order to influence the distribution of spoils which cannot be legally sold. There are two types of agents. Type A is called a decision maker. She has a chance to distribute a valuable spoil, which could be either a nomination or a contract, with a positive probability. Type B is someone interested in spoil, a rent-seeker. Each decision maker distributes a spoil with probability p. A nominated rent-seeker receives surplus s where s is strictly positive. We define the expected rent as $\psi = ps$. The term spoil should be interpreted widely. It could refer to a politically filled position, a lucrative private or public sector job, or a contract to provide a certain type of service such that the contractee receives a rent. We assume that ψ is of intermediate size: it is sufficiently large so that all rent-seekers have at least one link, but sufficiently small so that no rent-seeker is linked to all decision makers.

There are n_A decision makers and n_B rent-seekers. The decision makers are indexed with $i = 1, ..., n_A$ and the rent-seekers with $j = 1, ..., n_B$. There are γ times more rent-seekers than decision makers, $n_B = \gamma n_A$, where γ is an integer strictly greater than one.[8]

Whether decision maker i is connected with rent-seeker j is captured by $m_{i,j}$. If i is connected with j then $m_{i,j} = 1$, if not then $m_{i,j} = 0$. A connection is established between a decision maker and a rent-seeker if both are willing to do so. Decision maker i's connections are described by $\mathbf{m}_i^A = (m_{i,1}, ..., m_{i,n_B})$ and rent-seeker j's connections are described by $\mathbf{m}_j^B = (m_{1,j}, ..., m_{n_A,j})$. Thus the network is

International Corruption Perceptions Index (2006).

[8]This simplification allows us to solve the model explicitly.

characterized by the matrix

$$
\begin{aligned}
M &= (\mathbf{m}_1^B, ..., \mathbf{m}_{n_B}^B) \\
&= \begin{pmatrix}
m_{1,1} & m_{1,2} & \cdots & \cdots & m_{1,n_B} \\
m_{2,1} & m_{2,2} & \cdots & \cdots & m_{2,n_B} \\
\vdots & \vdots & & & \vdots \\
\vdots & \vdots & & & \vdots \\
m_{n_A,1} & m_{n_A,2} & \cdots & \cdots & m_{n_A,n_B}
\end{pmatrix}. \\
&= (\mathbf{m}_1^A, ..., \mathbf{m}_{n_A}^A)'
\end{aligned}
$$

Notice that $m_{i,j} = m_{j,i}$ since i cannot be linked to j if j is not linked to i. Let the number of connections that rent-seeker j has to decision makers be denoted by $m_{jA} = \sum_{i=1}^{n_A} m_{i,j}$. The number of connections of decision maker i is denoted by $m_{iB} = \sum_{j=1}^{n_B} m_{i,j}$.

Maintaining a connection is costly. A decreasing marginal productivity in other activities, or a decreasing marginal utility of leisure, implies that the marginal cost of time spent on networking is strictly increasing. Thus, we adopt a strictly convex and increasing cost function for decision maker i of having a total number m_{iB} of connections, $c(m_{iB})$. Similarly rent-seeker i's cost of networking is $c(m_{iA})$. Both must contribute time and effort to keep up the relationship. Thus any given connection induces costs for both.

We assume that, ex ante, each decision maker is indifferent as to whom to nominate and each rent-seeker is indifferent as to who nominates him. However, we assume that in order for decision maker i to be able to nominate rent-seeker j, there has to be a *direct* connection between them, $m_{i,j} = 1$, as opposed to an *indirect* connection where i knows a third agent who knows j. This assumption rules out favors being passed to an acquaintance of an acquaintance and implies that the network will parallel a market place with connections between sellers and buyers, but without any intermediaries. While such indirect connections may indeed have some value, it is likely that, when the decision maker is ex ante indifferent as to whom to pass the favor, she is likely to favor a close rather than a distant acquaintance. The restriction that only close acquaintances can receive the favor serves as a simplifying assumption. Analogous assumptions are made in Kranton and Minehart (2001) and Kakade et al (2004). A decision maker nominates each rent-seeker connected to her with an equal probability.[9] Moreover, we suppose that a rent-seeker can accept several nominations. This is a simplifying assumption.[10]

Our model of network formation is based on economic decisions as opposed to taking the network as given or as an outcome of an exogenous process.[11] Typically, in models that consider networks as an outcome of economic decisions, networks arise as game theoretic solutions reflecting rational choice given what others choose.[12] This should be contrasted with non-strategic economic models where parties ignore or abstract from the impact of their own choices on others and on the market. In particular, this applies to general equilibrium models where parties take prices as given and do not consider the influence of their own demand or supply decision on the price.

In this paper, we present a model in which a non-strategic Walrasian approach and a game-theoretic approach of linking by mutual consent lead into an identical network and identical prices (up to permutations). When solving for a stable network under strategic network formation, we assume that decision-makers make offers to rent-seekers and verify that the links are stable against any pair-wise deviation by a pair of subjects. The approach is closely related to the concept of pairwise stability with transfers (Bloch and Jackson, 2006).[13] The approach thus allows for price discrimination ex ante. It turns out however, that in the unique pairwise stable network with tranfers, there is no price discrimination, under our assumption of lack of commitment by the decision makers and agents' incomplete information about the extent of the others' networks. We show that both the Walrasian and the pairwise stable network, as well as the associated prices for connections, are unique

[9] The decision maker may not in all instances dole out spoils uniformly but it is sufficient that rent-seekers believe that they do (or don't have prior knowledge so as to who will have higher chance).

[10] Assuming alternatively that each lobbyist can only receive one nomination would have two effects. First, the probability of being offered a nomination would depend positively on the number of connections that other lobbyists (linked to the decision maker) have to other decision makers. Second, the gain from an additional connection would not be constant but rather decreasing as with more connections to decision makers, the probability that only one nomination is offered is decreasing. The decision maker's incentives are unaffected by the alternative assumption, however, since she only cares about connections and rewards.

[11] See Jackson (2006) for a classification of network formation models.

[12] Bloch and Jackson (2006) distinguish between pair-wise stability and its derivatives on the one hand, and non-cooperative networking games where people simultaneously announce which links they would like to form, on the other.

[13] For our specific needs, we augment their definition to allow us to consider the stability of the rewards charged by the politicians in addition to the stability of the connections formed.

up to permutations. Our analysis provides a setting that can be applied to a variety of economic problems, including politics and networking for job opportunities.

3 Walrasian network

3.1 Modelling approach

Let us first proceed with the simpler Walrasian approach. The Walrasian auctioneer first announces a uniform market reward that any rent-seeker having a connection to a decision maker must pay to the latter. Each decision maker simultaneously announces a vector of integer numbers of connections that he is willing to have with that reward. Typically, when there is a unique optimum, there is a single scalar offer. But occasionally, there might be several optimal offers and the decision maker can inform the Walrasian auctioneer of all the optimal offers he is indifferent between. Similarly each rent-seeker announces a vector of demands. The Walrasian auctioneer then clears the market by assigning each potential connection (between any potential decision maker i and rent-seeker j) a probability that this connection is formed. The probability distribution over connections is such that in any resulting network, each agent has some number of connections that she demanded for. We will discuss the interpretation of the randomizations and provide examples later in this section. Note that the randomization can take place only between the connection numbers between which the market participant is indifferent.

A rent-seeker pays a decision maker a reward, r, for maintaining a connection. A stable market reward[14] r equates the supply of connections (by the decision makers) and the demand (by the rent-seekers). While there may be a continuum of rewards which clear the market, it turns out that the stable network offers and demands are the same in all these equilibria. Therefore, the social surplus is unaffected by the choice of market clearing reward. To simplify and to reflect the relative market power of the decision makers, we choose the stable network reward which maximizes the decision makers' profits.

The payoff of decision maker i when network M prevails with reward r reads

$$\pi_i(M, r) = m_{iB}r - c(m_{iB}), \tag{1}$$

[14]In this section *stable network* refers to the Walrasian equilibrium in the market for connections. In Section 4, we derive a corresponding pairwise stable network with transfers (Bloch and Jackson, 2006). Whether a Walrasian equilibrium network or a pair-wise stable network with transfers, we call such a network generally *stable*.

where $c(m)$ is a strictly increasing and strictly convex function with $c(0) = c'(0) = 0$ and $lim_{m \longrightarrow \infty} c'(m) = \infty$. The expected payoff when there is uncertainty about the links is defined in the obvious manner.

Decision maker i's maximization problem[15] is

$$\max_{m_{iB}} \{m_{iB} r - c(m_{iB})\}.$$

Due to the strict concavity of the payoffs in the number of connections, there can be at most two optimal connection quantities for each agent and these must be consecutive. It turns out that the optima are the same for all agents of a given type. Thus, we simplify and denote the total number of connections of a decision maker (to rent-seekers) in a stable network by m^*_{AB} and by m^*_{BA} the corresponding total number of connections of a rent-seeker (to decision makers). If decision makers have two optima, there may exist numbers m^*_{AB} and $m^*_{AB} + 1$ such that each decision maker has m^*_{AB} connections for sure and some have an additional $m^*_{AB} + 1$:th connection with a probability smaller than one (respectively numbers m^*_{BA} and $m^*_{BA} + 1$ such that each rent-seeker has m^*_{BA} connections for sure and some have an additional $m^*_{BA} + 1$:th connection with a probability smaller than one).[16] Furthermore, we denote by r^* the Walrasian stable network reward.

In a stable network, given rewards, increasing or decreasing the number of connections must not strictly pay off. Thus, we have the following condition for decision makers

$$c(m^*_{AB}) - c(m^*_{AB} - 1) \leq r^* \leq c(m^*_{AB} + 1) - c(m^*_{AB}). \tag{2}$$

For a rent-seeker, the probability that a connection to decision maker i results in a nomination, p_i, depends negatively on the number of connections that the decision maker has to other rent-seekers: the more connections to other rent-seekers, the less likely it is that the decision maker nominates the rent-seeker. Decision makers cannot commit not to sell additional connections. To reflect the fact that the rent-seekers cannot monitor the decision makers, we assume that the rent-seekers cannot observe how many other connections each decision maker is providing, not even ex post. Thus, the decision makers appear to the rent-seekers as ex ante identical. Yet,

[15] All the results could be generalized to allow decision makers to receive some direct benefit from networking with rent-seekers, as long as the time cost exceeds the benefit for decision makers at the margin. Furthermore, all the results would remain the same if the decision maker would also receive a surplus when allocating a rent to a rent-seeker.

[16] These equilibrium probabilities are coordinated by the Walrasian auctioneer.

the rent-seekers correctly anticipate the distribution of decision makers' number of connections in a stable network. Due to this uncertainty and due to the fact that the auctioneer potentially creates a connection only with a positive probability smaller than one, the number of links of the decision maker is potentially perceived as a random variable by the rent-seekers. Let the probability that decision maker has k connections (or the fraction of decision makers with k connections) be denoted by q_k^A. Since link quantities are not observable, the rent seeker's perceived ex-ante probability that a connection to a decision maker results in a nomination is the same across decision makers. Thus, for all i, $p_i = p_A \equiv \sum_k q_k^A \frac{p}{k}$. As noticed above, due to strictly convex costs, decision-makers randomize over at most two consecutive numbers. Thus without loss of generality we can write $p_A = q_k^A \frac{p}{k} + q_{k+1}^A \frac{p}{k+1}$ for suitably chosen positive integer k. Notice that if decision makers all demand k or $k+1$, then probability q_{k+1}^A can be interpreted as follows : If $q_{k+1} = \frac{n}{n_A}$ for some $n \in \{1, ..., n_A - 1\}$ then this means that the auctioneer allocates $k+1$ offers to n of the decision makers and k offers to $n_A - n$ decision makers. If $q_{k+1} \in (\frac{n}{n_A}, \frac{n+1}{n_A})$ for some $n \in \{0, ..., n_A - 1\}$ then the auctioneer allocates $k+1$ offers to n of the decision makers and moreover with probability $q_{k+1} - \frac{n}{n_A}$, $k+1$ links will be allocated to one of the remaining $n_A - n$ decision makers. This implies that the rent seeker's perceived ex-ante probability of k links is q_k and that of $k+1$ links is q_{k+1}.

Similarly, if rent-seekers have various link quantities or they are allocated a probability distribution of their demands by the auctioneer, the link number of a rent-seeker is a random variable and the probability that rent seeker has k connections will be denoted by q_k^B.

Since the rent-seekers perceive the decision makers as ex ante identical, the expected payoff of rent-seeker j when network M prevails with reward r can be written as[17]

$$\mathbb{E}\pi_j(M, r) = m_{jA} p_A s - m_{jA} r - c(m_{jA}). \tag{3}$$

Each rent-seeker takes as given the reward, r, and correctly anticipates the expected probability of being nominated, p_A. The rent-seeker maximizes

$$\max_{m_{jA}} \{m_{jA} p_A s - m_{jA} r - c(m_{jA})\}.$$

[17] This formulation relates to Tullock (1980). Yet, here we consider a dichotomic decision whether to connect with a decision maker or not and all lobbyists who are connected have an equal probability of being nominated. Moreover, we differ from Tullock in that the cost of networking is not linear but convex in the number of connections.

Again, in stable network, all rent-seekers behave identically and, given rewards, increasing or decreasing the number of connections must not strictly pay off:

$$p_A s + c(m^*_{BA}) - c(m^*_{BA} + 1) \leq r^* \leq p_A s + c(m^*_{BA} - 1) - c(m^*_{BA}). \tag{4}$$

On the left-hand side of (4), we have the expected rent from the last connection ($p_A s$), minus the networking cost if adding one more link, $(c(m^*_{BA} + 1) - c(m^*_{BA}))$. When this is smaller than or equal to $r*$, it does not pay off to add one more link. On the right-hand of (4), we have the expected rent from the last link, minus the networking cost of maintaining the last link, $(c(m^*_{BA}) - c(m^*_{BA} - 1))$. When this is larger than or equal to $r*$, it does not pay off to eliminate the last link.

In stable network, the auctioneer sets the highest reward that the rent-seekers are willing to pay given the decision maker's (expected) connections. Thus, one of the upper bounds of r^* in (2) and in (4) must be binding. A Walrasian stable network exists if there exists such a market clearing price.

Note that r is a gross price, and it has to compensate the decision maker for her marginal cost of linking. We show in Proposition 1 that a market clearing price always exists. Competition between decision makers on the supply side and between rent-seekers on the demand side determines a unique (decision makers' profit maximizing) stable network reward that is approximately equal to the marginal linking costs and that equilibrates decision makers' supply and rent-seekers' demand for connections.

This unique stable network is symmetric in the sense that all decision makers make the same offers and all the rent seekers make the same demands.[18] However, in case of indifference, the number of connections of two decision makers, for instance, need not be the same ex-post due to allocation decisions by the auctioneer.[19] Consequently, more than one value of q^B_k may be strictly positive.

3.2 Stable network regimes

As anticipated in the previous section, the number of connections in a stable network is unique. However depending on the parameters, there are four structures of the

[18]Symmetry is a property of any equilibrium. It is not exogenously assumed. The equilibrium would be symmetric even if we chose a market clearing r which does not maximize the decision makers' profits.

[19]In Bloch and Jackson (2006) model where the pairwise stability with transfers is introduced, the links are observable. In our model, however, links are private information. Thus, lobbyists are not willing to change the number of links once the network is formed.

Walrasian stable network. These four regimes are described below.

(i) All rent-seekers have an identical number of connections and all decision makers have an identical number of connections. The reward keeps the rent-seekers indifferent between their stable network connections and having one connection less. The stable network reward is increasing in ψ.

(ii) All rent-seekers have an identical number of connections and all decision makers have an identical number of connections. The reward keeps the decision makers indifferent between their stable network connections and having one connection more. The reward does not adjust to small changes in ψ.

(iii) Some rent-seekers have a connection more than others and some decision makers have a connection more than others. The reward does not adjust to small changes in ψ.

(iv) Some rent-seekers have a connection more than others and all decision makers have an equal number of connections. The stable network reward is increasing in ψ.

The network structure and the stable network reward are driven by the incentive constraints (2) and (4). The decision makers supply connections at the highest reward that the rent-seekers are willing to pay. However, the rent-seekers correctly anticipate how many connections the decision makers have in stable network, which affects their demand and willingness to pay. In regime (i), the network structure does not change as ψ increases and, thus, the rent-seekers' willingness to pay for each connection increases. Therefore, the decision makers are able to capitalize on increases in ψ in the market value of connections, r^*.

Eventually the reward, r^*, becomes so high that the decision makers do not mind supplying an additional connection and, if it is raised further, the decision makers would strictly prefer to provide an additional connection. This would lead to an oversupply of connections. Moreover, the rent-seekers would anticipate that if the decision makers sold more connections, each individual connection would be associated with a lower expected probability of nomination. This would further imbalance the supply and the demand. Thus, in regime (ii) we say that *a reward cap* binds. As ψ increases, the reward cap regime persists until the connections become sufficiently more attractive so that the rent-seekers do not mind demanding an additional connection and the stable network shifts to regime (iii) where both are indifferent between two consecutive connection quantities.

In regime (i) and in regime (ii), rent-seekers have m^*_{BA} connections and decision makers have $m^*_{AB} = \gamma m^*_{BA}$ connections. As ψ increases, the stable network shifts to regime (iii) where there are decision makers with γm^*_{BA} connections and others with $\gamma m^*_{BA} + 1$ connections, and there are rent-seekers with m^*_{BA} connections and others with $m^*_{BA} + 1$ connections. From regime (iii), we enter regime (iv) when there is sufficient demand for all decision makers to provide eventually $\gamma m^*_{BA} + 1$ connections whereas the rent-seekers keep on having either m^*_{BA} or $m^*_{BA} + 1$ connections. As ψ increases even further, we enter regime (iii) again, but now while some rent-seekers have m^*_{BA} connections and others have $m^*_{BA} + 1$ connections, some decision makers have $\gamma m^*_{BA} + 1$ and others have $\gamma m^*_{BA} + 2$ connections. Regimes of types (iii) and (iv) alternate until eventually all rent-seekers strictly prefer to demand an additional connection and we move from regime (iii) to regime (i), now with $m^*_{BA} + 1$ and $\gamma(m^*_{BA} + 1)$ connections for rent-seekers and decision makers, respectively.

We formally derive the order of stable network regimes in the appendix. Figure 1 illustrates this. We fix $\gamma = 2$ and let the expected rent of the nomination increase when moving downwards. The number of connections increases and we move from one regime to another as shown in Figure 1.

We denote the expected stable network payoffs by π^*_A, π^*_B. The costs of networking are defined as $TC^* = n_A(q^A_{m^*_{AB}} c(m^*_{AB}) + (1 - q^A_{m^*_{AB}}) c(m^*_{AB} + 1)) + n_B(q^B_{m^*_{BA}} c(m^*_{BA}) + (1 - q^B_{m^*_{BA}}) c(m^*_{BA} + 1))$ where $q^A_{m^*_{AB}}$ and $q^B_{m^*_{BA}}$ are the stable network probabilities that a decision maker has m^*_{AB} connections and a rent-seeker has m^*_{BA} connections, respectively. The sum of payoffs is defined by $W^* = n_A \pi^*_A + n_B \pi^*_B$. The main results of this section can be summarized as follows:

Proposition 1

- There is a unique stable network, provided that ψ is sufficiently large. It is symmetric.
- The stable network numbers of connections, $m^*_{AB}(\psi, \gamma)$ and $m^*_{BA}(\psi, \gamma)$, are increasing in ψ (and thus in p and s) and decreasing in γ.
- The stable network payoffs, π^*_A, π^*_B, the costs of networking, TC^*, and the sum of payoffs, W^*, are continuous and increasing in ψ but not strictly increasing.

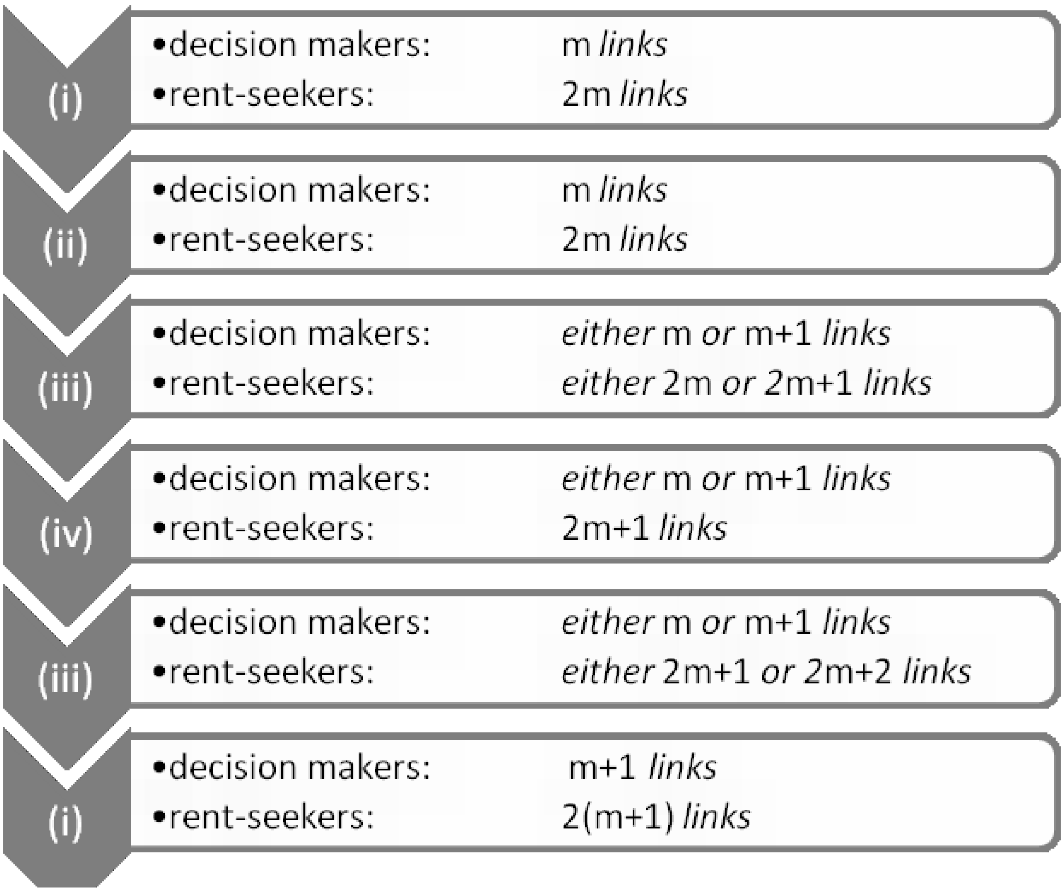

Figure 1: Equilibrium regimes, $\gamma = 2$.

For all parameter values, either the payoffs of the rent-seekers remain constant or the payoffs of the decision makers remain constant as ψ increases or both.

The payoffs are all continuous and increasing in the expected value of the nomination. Intuitively, the rent-seekers are willing to demand more connections when the rents are higher. The decision makers can charge higher rewards not only for these added connections but also for the inframarginal connections in all regimes, except for regime (ii) where the reward cap binds and the rent-seekers reap the gains from small increases in the rents, ψ. The higher rewards charged by the decision makers are offset by higher networking costs in regime (iii) and neither the decision makers nor the rent-seekers gain. When both decision makers and rent-seekers randomize between two consecutive sold and bought link quantities, an increase in the rents results in an increase in the probability weight of the higher of the link quantities that are allocated to the decision makers and rent seekers. Neither the expected payoff from a link to the rent-seeker nor the price of the link can increase, as otherwise at least one of the agents would no longer be indifferent between the two quantities. Therefore, the whole increase in the value of rents is dissipated in wasteful network formation as long as both agents continue to attach a weight larger than zero also to the smaller quantity. In regimes (i) and (iv), the gains of the higher rents accrue to the decision makers.

The payoff functions are illustrated in Figure 2 for the special case $n_A = 2$, $\gamma = 2$ and $c = \frac{1}{2}m^2$. As a function of ψ, the rent-seeker's stable network payoff is the curve on the bottom, the decision maker's payoff is the second curve from the bottom and the aggregate surplus for two decision makers and four rent-seekers is the third curve from the bottom.

The total expected value of nominations is $n_A\psi$. In Figure 2, this is illustrated by the line starting from the origin with a slope equal to two. Notice that the sum of payoffs falls short of this total expected value and the distance between these two increases in ψ. The distance coincides with the total costs of networking.

4 Pair-wise stability with transfers

The approach in section 3 implicitly assumes a Walrasian auctioneer who sets the price and coordinates the demand and the supply of connections. In this section, we adopt a game theoretical approach to networks requiring pairwise stability and

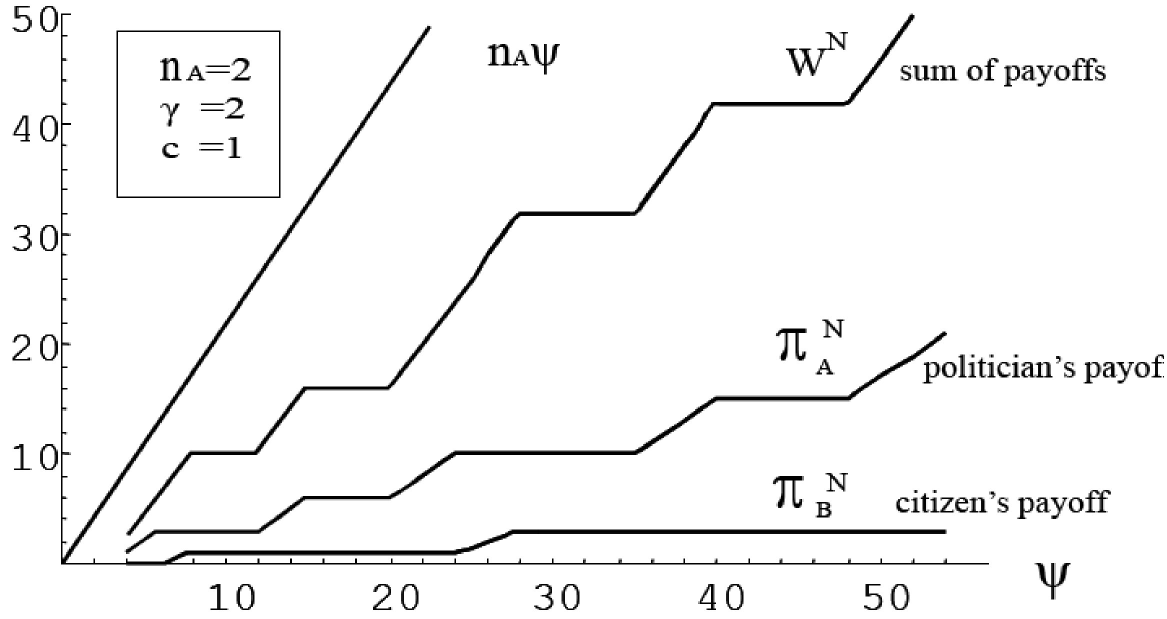

Figure 2: Equilibrium payoffs.

allowing transfers between agents (Bloch and Jackson, 2006) leading to the same conclusion as the Walrasian approach.[20]

Remember that a network is characterized by a matrix M of zeros and ones on which the restriction $m_{i,j} = m_{j,i}$ is imposed. Network $M - ij$ is one where the connection between i and j present in M is abolished. Network $M + ij$ is one where the connection ij not present in M is created. Network $M \pm ij$ is any such network where at least one connection in M is abolished and a new connection is created between i and j. Since this is a set of networks we denote by $\overline{\pi}_i(M \pm ij, \mathbf{R}^o)$ the maximal payoff for i in this set, i.e. the one where the lowest return link from i to $k \neq j$ is abolished.

We build on the concept of *pair-wise stability with transfers* developed by Bloch and Jackson (2006). This concept allows for monetary transfers being paid between the connecting parties which in our setup translate into payments from rent-seekers to decision makers in remuneration for their investment in keeping up the connection. A priori there may be price discrimination, different payments may be charged across rent-seekers and connections. The reward that decision maker i charges from rent-seeker j is denoted $r_{i,j}$ and her profile of rewards is denoted by $\mathbf{r}_i = (r_{i,1}, ..., r_{i,n_B})$.[21] The matrix of reward profiles is denoted by $\mathbf{R} = (\mathbf{r}_1, ..., \mathbf{r}_{n_A})'$. A network is stable if, no decision maker or rent-seeker would gain by abolishing any of the specified connections or by adding or replacing a connection where an arbitrary transfer can be charged for that connection. The formal statement is given in the definition below. Define $q_{i,j}$ as the probability that there is a connection between i and j perceived from rent-seekers' perspective. For a given $\mathbf{R}$ let $\mathbf{R}^o$ satisfy $r_i^o = (r_{i,1}, ..., r_{i,j-1}, r_{i,j}^o, r_{i,j+1}, ..., r_{i,n_B})$ where $r_{i,j}^o \neq r_{i,j}$ and $r_k^o = r_k$ for $k \neq i$. (In $\mathbf{R}^o$ there is only one decision maker whose reward scheme is different from that in $\mathbf{R}$ and only one reward offer of that decision maker is different from the offers in $\mathbf{R}$.)

Definition 1 *The network M is pair-wise stable with transfers, provided that for all $\mathbf{R}^o$*

i. *if $q_{i,j} > 0$, then $\pi_i(M, \mathbf{R}) \geq \pi_i(M - ij, \mathbf{R})$ and $E\pi_j(M, \mathbf{R}) \geq E\pi_j(M - ij, \mathbf{R})$ and that*

[20]Pair-wise stability with transfers is defined in Bloch and Jackson (2006). Unlike in Bloch and Jackson, however, we keep on assuming that there is incomplete information about the number of connections but offered rewards are observable and can be used to make inferences about the number of links others have. In the stable network expectations are correct of course.

[21]These would appear in equations (3) and (1) in section 3 and r would be replaced by $r_{i,j}$.

ii. *if* $q_{i,j} = 0$, *then*

1. $\pi_i(M, \mathbf{R}) < \overline{\pi}_i(M + ij, \mathbf{R}^o)$ *implies* $\mathbb{E}\pi_j(M, \mathbf{R}) > \mathbb{E}\overline{\pi}_j(M + ij, \mathbf{R}^o)$;

2. *for every* $M \pm ij$, $\pi_i(M, \mathbf{R}) < \overline{\pi}_i(M \pm ij, \mathbf{R}^o)$ *implies* $\mathbb{E}\pi_j(M, \mathbf{R}) > \mathbb{E}\overline{\pi}_j(M \pm ij, \mathbf{R}^o)$;

3. $\mathbb{E}\pi_j(M, \mathbf{R}) < \mathbb{E}\pi_j(M + ij, \mathbf{R}^o)$ *implies* $\pi_i(M, \mathbf{R}) > \pi_i(M + ij, \mathbf{R}^o)$;

4. *for every* $M \pm ij$, $\mathbb{E}\pi_j(M, \mathbf{R}) < \mathbb{E}\overline{\pi}_j(M \pm ij, \mathbf{R}^o)$ *implies* $\pi_i(M, \mathbf{R}) > \overline{\pi}_i(M \pm ij, \mathbf{R}^o)$.

Notice first that in part (ii) of the definition, one could equally well write $\mathbf{R}^o$ instead of $\mathbf{R}$ since $q_{i,j} = 0$ implies that originally no transfers are passed between i and j. The definition lists essentially the *pair-wise stability with transfers* conditions of Bloch and Jackson (2006) where transfers between parties are allowed for.[22] Our approach differs from Bloch and Jackson (2006) in one aspect: we allow for pair-wise deviations where two players form a connection and at the same time each of them abandons one of their connections rather than just deviations where either a new link is formed or an existing link is abandoned but not both.

Condition (i) states that any connection which is formed with a positive probability in a stable state benefits both parties and benefits strictly at least one of the two. The first pair of inequalities of condition (ii) of definition 1 implies that if i strictly prefers to deviate and form a connection with j whereas j is indifferent, then the connection between them will be formed with a positive probability. The second pair of inequalities of condition (ii) includes the case where replacing some connection of decision maker i and some connection of rent-seeker j by a connection between i and j would benefit decision maker i but harm rent-seeker j. The third and the fourth pairs of inequalities have the corresponding cases where the rent-seeker would gain and the decision maker would lose. Notice that the pair-wise stability conditions (i) and (ii) do not say whether there is a positive or a zero probability of forming a connection if both are indifferent.

[22]Bloch and Jackson (2006) discuss the relationship between Nash-like solutions concept in of a non-cooperative link formation game and pair-wise stability. Our uniqueness and equivalence result holds also for pair-wise Nash stability. Notice that given our assumption on incomplete information about the decision makers' number of links, we could more precisely employ the conjectural pair-wise stability concept designed for incomplete information analysis (McBride, 2006). Translating his approach to the current setting would imply that each decision maker's $\mathbf{R}$ provides a signal for the rent-seekers and thus allows the rent-seekers to have correct conjectures about the number of links of the decision makers.

This pairwise stable network approach leads to exactly the same outcome as the Walrasian approach of section 3. As in the Walrasian approach, there may exist several pair-wise stable reward profiles with equal rewards for all decision makers which sustain the same network constellation. We choose to consider the one where decision makers' payoffs are the highest. These stable networks and reward profiles coincide with the Walrasian equilibria where decision maker's payoffs are highest. Every Walrasian and pair-wise stable network is identical (up to permutations).

Proposition 2 *There is a unique pairwise stable network (up to permutations) associated with a uniform market price for a connection. The market price and the network coincide with the Walrasian stable network (up to permutations).*

Proof. The last subsection in the appendix shows that the unique (decision-maker-payoff maximizing) pair-wise stable network with transfers is unique and coincides with the Walrasian stable network. ■

The intuition behind this result is the following. The numbers of connections of the decision makers have to be the same (in regimes (i) and (ii)) or one of two consecutive integers (regimes (iii) and (iv)). Otherwise, a rent-seeker connected to a decision maker with more connections could pair-wise deviate with a decision maker with at least two connections less. These two establish a new connection and the rent-seeker abolishes the connection with the decision maker with more connections. It is easy to see that there are rewards such that the rent-seeker strictly benefits and the decision maker is indifferent. In a pairwise stable network, rewards must be the same since if they are not, then a rent-seeker connected with a decision maker with a higher reward can pair-wise deviate and establish a link with a decision maker with a lower reward and abolish the one with high reward decision maker. It is easy to see that there are rewards such that both gain. However, if the stable quantities of connections of agents of the same type are equal and the rewards are equal to the marginal costs or benefits of all rent-seekers or all decision makers, then no pair-wise deviation will strictly benefit one party without harming the other.

There are three novel features to our approach. First, there is incomplete information about the number of links any other agent has. Second and relatedly, uncertainty reigns about which links are actually formed. Third, we allow for pair-wise deviations where i and j create a link while each abolishes one of their links. The randomization assumption together with that on incomplete information, although unusual in network studies, greatly simplify analysis in regimes (iii) and (iv). They can also be motivated on the grounds that without those assumptions, each decision maker could exclusively sell just one link and charge the value of the spoil from the rent-seeker she connects with. When one can sell the spoil directly in this manner, networking becomes uninteresting. Notice also, that the combination of random links and incomplete information generates payoff symmetry and continuity in the stable network that simplifies analysis in the inherently discrete world of networks. We exploit these features in the proofs of Lemma 1 and Proposition 2. Yet incomplete information and population interpretation of the randomization are fairly intuitive and plausible.[23]

5 Related literature

Our model differs from contest and all-pay auction models of rent-seeking in four ways. First, *connections* rather than monetary bribes or *bids* determine winning probabilities and connections either exist or not whereas in contests and auctions bids are typically continuously adjustable. Second, in our model connections generate strictly convex opportunity costs both to rent-seekers and to decision makers whereas in contests only rent-seekers bear costs and these are typically linear. Third in our model, rent-seekers must, in addition to bearing the opportunity costs, reward decision makers for keeping up the connection. The rewards are set by the decision makers (or a Walrasian auctioneer) and can be continuously adjusted. Fourth, there are multiple decision makers in our model and thus there is competition on both sides. We are not aware of corresponding research on all-pay auctions where there is competition between contests.[24]

[23]To give a simple example, consider a case where each decision maker has at least m links for sure and there is an equilibrium probability of q (as perceived by each rent-seeker) that each decision maker has an additional link. Then clearly there is $0 \leq k < n_A$ such that $q \in [\frac{k}{n_A}, \frac{k+1}{n_A})$. Thus such an equilibrium can be interpreted as k of the n_A decision makers having $m+1$ links and one of the remaining ones establishing a link with probability $q - \frac{k}{n_A}$ while the remaining $n_A - k - 1$ decision makers have m links for sure. This means that each of the rent-seekers, of whom there are γ times more, will have at least $\overline{m}/\gamma$ links where $\overline{m}$ is the largest integer smaller than m that is divisible by γ and the $n_A(m - \overline{m}) + k$ will have an additional $\overline{m}/\gamma + 1$:th link. Finally of the remaining $n_B - n_A(m - \overline{m}) - k$, one has a probability $q - \frac{k}{n_A}$ of establishing an additional $\overline{m}/\gamma + 1$:th link and the rest have $\overline{m}/\gamma$ links. Each agent's probability of ending up to each of these three alternatives is proportional to the respective frequencies of the alternatives. It is easy to verify that this compound lottery results in the rent seekers having the needed (correct) equilibrium beliefs that decision makers, for instance, have q_m and q_{m+1} links, respectively and the probability of a link between i and j matches with the probability of there being a link between j and i. Finally, supply $n_A m + k + q - \frac{k}{n_A}$ equals demand $\gamma n_A \frac{\overline{m}}{\gamma} + n_A(m - \overline{m}) + k + q - \frac{k}{n_A} = n_A m + k + q - \frac{k}{n_A}$.

[24]Perhaps since replicating a single contest, does not influence the prediction in the contest when preferences are quasi-linear and thus there are no income effects. To the contrary due to convex linking costs, competition between decision makers has non-trivial impact on the outcome in our

Also Prat and Rustichini (2003) consider a multi-agent common agency setting. Their paper is related to ours in that a non-cooperative network model (Bala and Goyal, 2000) where rent-reekers first offer rewards for links and these are thereafter non-cooperatively chosen and sponsored by the decision makers would be a special case of their setup. Yet, we consider a network model where links are formed by mutual consent, both sides incur costs of networking, and it is the decision makers on the short-side of the market who set the prices for connecting to rent-seekers. Each agent only observes the prices offered to him and thus the decision maker is unable to commit not to sell additional links to other rent-seekers nor to condition the reward on the number of links sold. Thus, unlike in Prat and Rustichini (2003) in our setup equilibria are generally inefficient. Finally, Groseclose and Snyder (1996) and Diermeier and Myerson (1999) analyze competition between two lobbyists who may target multiple decision-makers, assuming that the second-moving lobbyist observes offers by the first mover. We assume, instead, that rent-seekers cannot observe each other's actions, and that decision-makers cannot commit not to establish new links.

6 Conclusion

In this paper, we present a stylized framework to analyze network formation between decision makers and rent-seekers when decision makers distribute valuable nominations and are not allowed to sell these. We show that with a given number of decision makers and rent-seekers, there is a unique Walrasian stable network, in terms of the numbers of connections that each agent has (up to permutations), for any expected value of nominations. The stable network is symmetric and coincides with the unique pair-wise stable network with transfers. This finding simplifies future theoretical work, by allowing other researchers to use only the easier-to-solve Walrasian approach also in settings of decentralized rent-seeking if indirect links do not play a role.

Our unique stable network may include uncertainty about the formation of some of the links. As an implication, a given agent's number of links may take one of two consecutive values. Furthermore in the stable network, the payoffs of decision makers and rent-seekers are both non-decreasing in the value of nominations, as are also the costs of networking. However, both agents' payoffs are not strictly increasing: for all parameter values, either the payoffs of the rent-seekers remain

model.

constant or the payoffs of the decision makers remain constant as the value of rents increases or both. Typically it is one of the sides of the market that reaps all gains from marginal increases in the value of perks.

There are two competing hypotheses against which our predictions can be tested. First, if one thinks that lobbying does not reflect the value of perks distributed, then the value of nominations and projects should be uncorrelated with the amount of time decision makers spend networking, and with the amount of campaign contributions that decision makers receive. Second test relates to the relevance of our model of wasteful network formation against the view of lobbying as an auction in which rent-seekers are engaged either in a contest or an all-pay auction with decision makers giving the projects to those who pay them most (see section on related literature). Our model predicts that an increase in the value of nominations and projects should be associated with an increase both in the money changing hands and in time spent in networking by both decision makers and rent-seekers. Previous models predict that the amount of time that decision makers spend should not increase, while agreeing in that the amount of money changing hands increases. The relative suitability of the two approaches is likely to depend on institutional settings and on how well anti-corruption laws function. We expect that our model has highest explanatory power in societies with a relatively low level of corruption.

Our framework also invites a number of theoretical extensions. First, we could allow for rent-seekers to differ in their skills and preferences thus bringing the framework closer to a typical matching market setup. In that case, if information about skills and preferences are only transmitted through links, decision makers would have an incentive to be connected with the rent-seekers both in order to search for a competent one and to cash in the rent-seekers' desire for nomination. Second, we could introduce an additional stage to the game in which the agents who are connected to the decision-maker would lobby the decision-maker with additional transfers if the decision-maker has a rent to distribute. We would have to solve first this second-stage allocation mechanism, and then introduce the outcome to the network formation game at the first stage. Third, we could endogenize the identity of the decision makers in the citizen-candidate tradition pioneered by Osborne and Slivinski (1996) and Besley and Coate (1997). Finally concerning political applications, we abstract from the role of ideological considerations. In a richer model applicable to politics, the rent-seekers and decision makers would differ in their ideology. In that case, the nominating politician could face a choice between the ideologically more appealing candidates and those willing to pay more for gaining access. Such trade-offs and heterogeneity in the ideological importance of positions could help to explain why some positions are typically filled by ideologically close candidates, while others are used as rewards for contributors. For example in the United States, Presidents have nominated campaign contributors as Ambassadors while the Justices of the Supreme Court are chosen according to other criteria.

7 Appendix A

7.1 Lemma 1

Lemma 1 lists the parameter values for which each stable network regime exists given numbers of connections in stable network.

Lemma 1 *Given* c *and* γ *and that* $\psi \geq c\gamma^2$*, one and only one of the regimes prevails for each* ψ*.*

- *stable network regime (i) where rent-seekers are connected with* m^*_{BA} *decision makers and decision makers are connected with* γm^*_{BA} *rent-seekers prevails iff*

$$\begin{aligned} & \gamma m^*_{BA}[c(\gamma m^*_{BA}) - c(\gamma m^*_{BA} - 1) + c(m^*_{BA}) - c(m^*_{BA} - 1)] \\ \leq \quad & \psi \leq \gamma m^*_{BA}[c(\gamma m^*_{BA} + 1) - c(\gamma m^*_{BA}) + c(m^*_{BA}) - c(m^*_{BA} - 1)]. \end{aligned}$$

  If ψ *is increased above the upper bound, one enters an interval belonging to regime (ii) with each rent-seeker having* m^*_{BA} *connections.*

- *stable network regime (ii) where rent-seekers are connected with* m^*_{BA} *decision makers and decision makers are connected with* γm^*_{BA} *rent-seekers prevails iff*

$$\begin{aligned} & \gamma m^*_{BA}[c(\gamma m^*_{BA} + 1) - c(\gamma m^*_{BA}) + c(m^*_{BA}) - c(m^*_{BA} - 1)] \\ \leq \quad & \psi \leq \gamma m^*_{BA}[c(\gamma m^*_{BA} + 1) - c(\gamma m^*_{BA}) + c(m^*_{BA} + 1) - c(m^*_{BA})]. \end{aligned}$$

  If ψ *is increased above the upper bound, one enters an interval belonging to regime (iii) with each rent-seeker having* m^*_{BA} *or* $m^*_{BA} + 1$ *connections.*

- *stable network regime (iii) where rent-seekers have* m^N_{BA} *or* $m^N_{BA}+1$ *connections whereas decision makers have* m^*_{AB} *or* $m^*_{AB} + 1$ *connections where* $\gamma m^*_{BA} \leq$

$m^*_{AB} < \gamma(m^*_{BA}+1)$ *prevails iff*

$$\begin{aligned} & m^*_{AB}[c(m^*_{AB}+1) - c(m^*_{AB}) + c(m^*_{BA}+1) - c(m^*_{BA})] \\ < \ & \psi < (m^*_{AB}+1)[c(m^*_{AB}+1) - c(m^*_{AB}) + c(m^*_{BA}+1) - c(m^*_{BA})]. \end{aligned}$$

If ψ *is increased above the upper bound and*

- *if* $m^*_{AB} < \gamma(m^*_{BA}+1) - 1$,
  one enters an interval belonging to regime (iv) with $m^*_{AB}+1$ *connections for decision makers.*
- *if* $m^*_{AB} = \gamma(m^*_{BA}+1) - 1$,
  one enters an interval belonging to regime (i) with $m^*_{BA}+1$ *connections for rent-seekers.*

- *Stable network regime (iv) where rent-seekers have* m^*_{BA} *or* $m^*_{BA}+1$ *connections whereas decision makers have* m^*_{AB} *connections where* $\gamma m^*_{BA}+1 \leq m^*_{AB} < \gamma(m^*_{BA}+1)$ *prevails iff*

$$\begin{aligned} & m^*_{AB}[c(m^*_{AB}) - c(m^*_{AB}-1) + c(m^*_{BA}+1) - c(m^*_{BA})] \\ \leq \ & \psi \leq m^*_{AB}[c(m^*_{AB}+1) - c(m^*_{AB}) + c(m^*_{BA}+1) - c(m^*_{BA})]. \end{aligned}$$

If ψ *is increased above the upper bound, one enters an interval belonging to regime (iii) with decision makers mixing between* m^*_{AB} *and* $m^*_{AB}+1$ *connections.*

7.2 Lemma 2

Lemma 2 *Stable network payoffs and the sum of payoffs are non-negative and given by*

$$\begin{aligned} \pi^*_A &= \psi + \gamma m^*_{BA}[c(m^*_{BA}-1) - c(m^*_{BA})] - c(\gamma m^*_{BA}) \\ \pi^*_B &= m^*_{BA}[c(m^*_{BA}) - c(m^*_{BA}-1)] - c(m^*_{BA}) \\ W^* &= n_A[\psi - c(\gamma m^*_{BA}) - \gamma c(m^*_{BA})] \end{aligned}$$

in regime (i);

$$\begin{aligned}
\pi_A^* &= \gamma m_{BA}^*[c(\gamma m_{BA}^*+1)-c(\gamma m_{BA}^*)]-c(\gamma m_{BA}^*)\\
\pi_B^* &= \frac{\psi}{\gamma}-m_{BA}^*[c(\gamma m_{BA}^*+1)-c(\gamma m_{BA}^*)]-c(m_{BA}^*)\\
W^* &= [\psi-\gamma c(m_{BA}^*)-c(\gamma m_{BA}^*)]n_A
\end{aligned}$$

in regime (ii);

$$\begin{aligned}
\pi_A^* &= m_{AB}^*[c(m_{AB}^*+1)-c(m_{AB}^*)]-c(m_{AB}^*)\\
\pi_B^* &= m_{BA}^*[c(m_{BA}^*+1)-c(m_{BA}^*)]-c(m_{AB}^*)\\
W^* &= n_A[m_{AB}^*[c(m_{AB}^*+1)-c(m_{AB}^*)]-c(m_{AB}^*)\\
&\quad +\gamma m_{BA}^*[c(m_{BA}^*+1)-c(m_{BA}^*)]-\gamma c(m_{AB}^*)]
\end{aligned}$$

in regime (iii); and

$$\begin{aligned}
\pi_A^* &= \psi+m_{AB}^*[c(m_{BA}^*)-c(m_{BA}^*+1)]-c(m_{AB}^*)\\
\pi_B^* &= m_{BA}^*[c(m_{BA}^*+1)-c(m_{BA}^*)]-c(m_{BA}^*)\\
W^* &= n_A[\psi-c(m_{AB}^*)-\gamma c(m_{BA}^*)]
\end{aligned}$$

in regime (iv).

7.3 Proof of Lemma 1

Proof. We will first show that each stable network regime exists in each of its intervals of ψ in the claim. For each regime, the proof proceeds regime by regime using a market clearing condition and the two optimality conditions (4) and (2) where either one or the other must be equal to one of its bounds. The market clearing condition is given by $\sum_{m_{AB}} q^A_{m_{AB}} m_{AB} = \sum_{m_{BA}} q^B_{m_{BA}} m_{BA}$ where $q^A_{m_{AB}}$ and $q^B_{m_{BA}}$ are the probabilities that a decision maker has m_{AB} connections and a rent-seeker has m_{BA} connections, respectively. Below, we will illustrate how the bounds are derived for regime (i). Supplementary material provides an extended version of the proof including the details of the proof for each regime.

Bounds of regime (i). In stable network, the supply of connections by decision makers has to equal the demand by rent-seekers and thus $m_{AB}^* = \gamma m_{BA}^*$. We consider the stable network reward which maximizes the decision makers' payoffs. Thus, the latter inequality of (4) is satisfied as an equality, and solving for r^* gives

$$r^* = \frac{\psi}{\gamma m^*_{BA}} + c(m^*_{BA} - 1) - c(m^*_{BA}). \tag{A1}$$

Now (2) must be satisfied yielding

$$\begin{aligned} & \gamma m^*_{BA}[c(\gamma m^*_{BA}) - c(\gamma m^*_{BA} - 1) + c(m^*_{BA}) - c(m^*_{BA} - 1)] \\ \leq \quad & \psi \leq \gamma m^*_{BA}[c(\gamma m^*_{BA} + 1) - c(\gamma m^*_{BA}) + c(m^*_{BA}) - c(m^*_{BA} - 1)]. \end{aligned}$$

Thus, if and only if these conditions hold, we have a regime (i) stable network with rent-seekers having m^*_{BA} connections.

Regime (ii). In regime (ii), the upper bound of (3) is satisfied as an equality and not the upper bound of (4) as in regime (i)

$$r^* = c(m^*_{AB} + 1) - c(m^*_{AB}). \tag{A2}$$

If the reward was above this reward cap, decision-makers would supply an additional link each and supply would exceed demand. Now (3) must be satisfied. Plugging (A2) into (3) and solving the two inequalities for ψ yields

$$\begin{aligned} & \gamma m_{BA}[c(\gamma m_{BA} + 1) - c(\gamma m_{BA}) + c(m_{BA}) - c(m_{BA} - 1)] \\ \leq \quad & \psi \leq \gamma m_{BA}[c(\gamma m_{BA} + 1) - c(\gamma m_{BA}) + c(m_{BA} + 1) - c(m_{BA})] \end{aligned}$$

Thus regime (ii) with rent-seekers having m^*_{BA} links prevails if and only if these two inequalities hold.

Regime (iii). All decision-makers are indifferent between selling m^*_{AB} or $m^*_{AB} + 1$ links where $\gamma m^*_{BA} \leq m^*_{AB}, m^*_{AB} + 1 < \gamma(m^*_{BA} + 1)$, and all rent-seekers are indifferent between buying m^*_{BA} or $m^*_{BA} + 1$ links in stable network. Due to the indifference, the second inequality in (3) and the first inequality in (4) respectively are satisfied as equalities implying

$$r^* = c(m^*_{AB} + 1) - c(m^*_{AB}) \tag{A3}$$

$$r^* = p^*_A s + c(m_{BA}) - c(m_{BA} + 1). \tag{A4}$$

rent-seekers have correct expectations on how many links decision-makers are going to sell, on average. Denote the share of decision-makers selling $m^*_{AB} + 1$ links by $q^*_{m^*_{AB}+1}$.

For a rent-seeker, the probability of receiving a spoil from buying a link is then

$$p_A^* = \frac{q^*_{m^*_{AB}+1} p}{m^*_{AB}+1} + \frac{(1-q^*_{m^*_{AB}+1})p}{m^*_{AB}}.$$

Plugging this into (A4) yields

$$r^* = \frac{q^*_{m^*_{AB}+1} \psi}{m^*_{AB}+1} + \frac{(1-q^*_{m^*_{AB}+1})\psi}{m^*_{AB}} + c(m_{BA}) - c(m_{BA}+1).$$

We plug in r^* from (A3) and solve the resulting equation for $q^*_{m^*_{AB}+1}$ yielding

$$c(m^*_{AB}+1) - c(m^*_{AB}) = \frac{q^*_{m^*_{AB}+1} \psi}{m^*_{AB}+1} + \frac{(1-q^*_{m^*_{AB}+1})\psi}{m^*_{AB}} + c(m_{BA}) - c(m_{BA}+1).$$

$$q^*_{m^*_{AB}+1} = (m^*_{AB}+1) - \frac{m^*_{AB}(m^*_{AB}+1)}{\psi}[c(m^*_{AB}+1) - c(m^*_{AB}) + c(m_{BA}+1) - c(m_{BA})].$$

To guarantee that $q^*_{m^*_{AB}+1}$ is between 0 and 1 we need

$$\begin{aligned} 0 &< (m^*_{AB}+1) - \frac{m^*_{AB}(m^*_{AB}+1)}{\psi}[c(m^*_{AB}+1) - c(m^*_{AB}) + c(m_{BA}+1) - c(m_{BA})] < 1 \\ &\Leftrightarrow 0 < 1 - \frac{m^*_{AB}}{\psi}[c(m^*_{AB}+1) - c(m^*_{AB}) + c(m_{BA}+1) - c(m_{BA})] < 1/(m^*_{AB}+1) \end{aligned}$$

or

$$\psi > m^*_{AB}[c(m^*_{AB}+1) - c(m^*_{AB}) + c(m_{BA}+1) - c(m_{BA})]$$

and

$$\psi < (m^*_{AB}+1)[c(m^*_{AB}+1) - c(m^*_{AB}) + c(m_{BA}+1) - c(m_{BA})]$$

This gives us the bounds of regime (iii) which exists for given linking quantities if and only if

$$\begin{aligned} & m^*_{AB}[c(m^*_{AB}+1) - c(m^*_{AB}) + c(m_{BA}+1) - c(m_{BA})] \\ < \; & \psi < (m^*_{AB}+1)[c(m^*_{AB}+1) - c(m^*_{AB}) + c(m_{BA}+1) - c(m_{BA})] \end{aligned}$$

where $\gamma m^*_{BA} \leq m^*_{AB} < \gamma(m^*_{BA}+1)$.

Regime (iv). In regime (iv), each decision-maker has $\gamma m^*_{BA} < m^*_{AB} < \gamma(m^*_{BA}+1)$ links. The rent-seeker is indifferent between m^*_{BA} or $m^*_{BA}+1$ links. Let fraction $q^*_{m^*_{BA}+1}$ of rent-seekers have $m^*_{BA}+1$ links in stable network. Due to indifference, the first inequality of (4) is satisfied as an equality yielding (A4). The supply of links equals the demand for links:

$$n_A m^*_{AB} = n_B(q^*_{m^*_{BA}+1}(m^*_{BA}+1) + (1 - q^*_{m^*_{BA}+1})m^*_{BA}).$$

Equivalently

$$r^* = \frac{\psi}{m^*_{AB}} + c(m_{BA}) - c(m_{BA}+1)$$
$$m^*_{AB} = \gamma(m^*_{BA} + q^*_{m^*_{BA}+1})$$

where

$$p^*_A = \frac{p}{m^*_{AB}} = \frac{p}{\gamma(m^*_{BA} + q^*_{m^*_{BA}+1})}$$

is the probability that rent-seeker receives a spoil from buying a link in the stable network.

Now (3) must be satisfied yielding

$$c(m^*_{AB}) - c(m^*_{AB} - 1) \le \frac{\psi}{m^*_{AB}} + c(m_{BA}) - c(m_{BA}+1) \le c(m^*_{AB}+1) - c(m^*_{AB})$$

where $\psi = ps$ by definition. On the other hand, solving $m^*_{AB} = \gamma(m^*_{BA} + q^*_{m^*_{BA}+1})$ for $q^*_{m^*_{BA}+1}$ we see that $0 < \frac{m^*_{AB}}{\gamma} - m^*_{BA} < 1$ since $q^*_{m^*_{BA}+1}$ must be between 0 and 1. Thus for each m^*_{BA} and m^*_{AB} there exists a stable network of regime (iv) with fraction $q^*_{m^*_{BA}+1}$ of rent-seekers having $m^*_{BA}+1$ links and fraction $1 - q^*_{m^*_{BA}+1}$ of rent-seekers having m^*_{BA} links if and only if

$$\gamma(1 + m^*_{BA}) > m^*_{AB} > \gamma m^*_{BA}$$

and

$$\begin{aligned} & m^*_{AB}[c(m^*_{AB}) - c(m^*_{AB}-1) + c(m_{BA}+1) - c(m_{BA})] \\ \le \quad & \psi \le m^*_{AB}[c(m^*_{AB}+1) - c(m^*_{AB}) + c(m_{BA}+1) - c(m_{BA}). \end{aligned}$$

It is easy to verify that, for each pair c and γ, the regime intervals are ordered as in the statement of Lemma 1. When $m^*_{BA} = 0 = m^*_{AB}$, for example, the lower bound of (iii) equals 0. The uniqueness and the existence and the order of regime intervals follow since the intervals form a partition of $(0, \infty)$. ■

7.4 Proof of Lemma 2

Proof. From the proof of Lemma 1, we obtain the stable network payoffs and welfare in various regimes by substituting in the expected connection quantities and rewards in the stable network. Using the boundaries of the existence condition of the regime in Lemma 1, it is easy to verify that the stable network payoffs of the rent-seeker and the decision maker in regimes (i), (ii), (iii) and (iv) respectively are non-negative. ■

7.5 Proof of Proposition 1

Proof. We first show that there can be no other equilibria but those which belong to regimes (i)-(iv). The claim then follows from Lemma 1.

Given reward, the payoff of the decision maker is strictly concave in the number of connections, $rm_{AB} - c(m_{AB})$. Thus there can be at most two integer amounts that constitute optimal demands for the decision maker and they are consecutive. Moreover, since rent-seekers can only infer this but don't know the actual number of links, the rent-seeker's expected return from each link to decision makers (at most one to each) is the same. Thus, also the rent-seeker's expected payoff is strictly concave in the number of connections. A strictly concave function has at most two maximizers which are moreover consecutive.

There is no symmetric stable network where all rent-seekers have an equal number of connections but decision makers are indifferent and fraction $q^A_{m_{AB}}$ of them have m_{AB} and fraction $1 - q^A_{m_{AB}}$ have $m_{AB} + 1$ connections since then demand for connections would not equal the supply. Moreover, given that the decision makers can be indifferent between connection quantities that differ at most by one, market would not clear if the reward r is such that the quantity demanded by each decision maker, m_{AB} is not in the set $\{\gamma m^*_{BA}, ..., \gamma(m^*_{BA} + 1)\}$ (where m^*_{BA} is the smallest stable network connection quantity of the rent-seekers).[25]

We have ruled out any other type of stable network regime but (i)-(iv). By Lemma 1, one and only one of these regimes prevails and, by the transformation rule of the regimes, the (minimum) stable network quantities, $m^*_{AB}(\psi, \gamma)$ and $m^*_{BA}(\psi, \gamma)$ are increasing in ψ. Moreover by Lemma 1, the bounds of each regime with given stable network quantities are increasing in γ. Thus, by the regime transformation rule, the stable network quantities are decreasing in γ.

[25] By the same arguments, there can be no two agents of the same type whose payoff maximizing link quantities differ by more than one.

The last bullet of Proposition 1 follows from noticing that the stable network payoff functions and the sum of payoffs in Lemma 2 are continuous increasing functions in ψ. To see this, notice that by Lemma 1, the regimes constitute a partition of $(0, \infty)$ and it is easy to check that the stable network payoff functions and the sum of payoffs in Lemma 2 are continuous at the regime shift values of ψ. That TC^* is increasing in ψ follows from the fact that $m^*_{AB}(\psi, \gamma)$ and $m^*_{BA}(\psi, \gamma)$ are increasing in ψ. ■

7.6 Proof of Proposition 2 (networks approach)

Proof. Before proceeding to the proof of Proposition 2 itself, we first need to reconsider the proof of Lemma 1. In all regimes, the Walrasian proof above verifies that the supply of connections equals their demand. It also verifies that in all regimes, by (2) and (4), neither rent-seekers nor decision makers are willing to increase or reduce their number of connections. But if, first, there are equal rewards for all rent-seekers, and second, if decision makers and rent-seekers have connection quantities which satisfy (2) and (4), then conditions (1) and (2) in Definition 1 (the stability of the network) are satisfied. Thus given $\mathbf{R}$, the network is stable. In regime (i), decreasing any $r_{i,j}$ would not increase decision maker's profits, since the decision maker is not willing to be connected with more rent-seekers due to the fact that reward is below the marginal cost of an added connection. Increasing any $r_{i,j}$ would render rent-seeker j willing to replace his connection with that decision maker with a connection to another decision maker and this latter would be indifferent between replacing and not replacing the connection. Thus, such an $\mathbf{R}'$ is not stable. In regime (ii), it does not pay off for the decision maker to reduce her reward, since this would reduce her payoff for each current customer and the reward would be lower than the marginal networking cost to an added rent-seeker. If a decision maker charges a reward higher than the reward cap, there exists a pair-wise replacement deviation where one of her customers replaces the connection with the decision maker with a connection with a decision maker whose reward equals the reward cap. Thus, the rewards are stable. In regime (iii), it does not pay off for the decision maker to reduce her reward, since this would reduce her payoff for each current customer and the reward would be lower than the marginal networking cost to an added rent-seeker. If a decision maker charges a higher reward, there exists a pair-wise replacement deviation where one of her customers replaces the connection with the decision maker with a connection with a decision maker whose reward equals the original reward. Thus, the rewards are stable. In regime (iv), by the same arguments as in the previous case the network is stable given $\mathbf{R}$ and, on the other hand, $\mathbf{R}$ is stable.

Now we move to the core of the proof of the proposition. In proving the uniqueness (under the restriction that the rewards are the most favorable for decision makers given a network structure, and up to permutations) in the network approach, in addition to what is done in the Walrasian proof, we need to verify that there is no price discrimination across rent-seekers in stable network or that the rewards of two decision makers cannot differ.

If there are two decision makers whose expected supplied quantities differ say $m'_{AB} < m''_{AB}$ then the marginal networking costs (MC) satisfy $MC' < MC''$ and $p'_A > p''_A$. Moreover, $MC' < MC'' \leq r''$. However, now the one with less connections can slightly undercut r'' and provide an additional connection to the rent-seeker who is offered r'' and this rent-seeker is willing to take the offer since $m'_{AB} < m''_{AB}$ and thus the probability of being nominated when connecting to this other decision maker is at least p''_A with the original decision maker. Thus all decision makers must have an equal number of connections.

Suppose now that all decision makers have an equal number of connections and there are two decision makers whose rewards at two implemented connections differ (notice that any network where a single decision maker price discriminates against her rent-seekers implies this). Now obviously, the decision maker with a lower offer can abolish this low reward connection and slightly undercut the offer made to the rent-seeker to which the decision maker with the higher offer is connected. This pays off to both the lower offer decision maker and the higher offer rent-seeker. Thus, all decision makers must be connected to an equal number of rent-seekers in expected terms and the offers must be equal. ■

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