CESifo Working Papers

www.cesifo.org/wp

Differential Prize Taxation and Structural Discrimination in Contests

Yosef Mealem

Shmuel Nitzan

CESIFO WORKING PAPER NO. 3831

CATEGORY 2: PUBLIC CHOICE

MAY 2012

An electronic version of the paper may be downloaded

- *from the SSRN website: www.SSRN.com*
- *from the RePEc website: www.RePEc.org*
- *from the CESifo website: www.CESifo-group.org/wp*

CESifo *Working Paper No. 3831*

Differential Prize Taxation and Structural Discrimination in Contests

Abstract

This paper evaluates differential prize taxation and structural discrimination as a means of increasing efforts in the most widely studied contests. We establish that a designer who maximizes efforts subject to a balanced-budget constraint prefers dual discrimination, namely, change of the contestants' prize valuations as well as bias of the impact of their efforts. Optimal twofold discrimination is often superior to any single mode of discrimination under any lottery. Surprisingly, in the general *N*-player contest game, under the prototypical simple lottery, it can yield the maximal possible efforts: the highest valuation of the contested prize. If a single mode of discrimination is allowed, then differential taxation is superior to structural discrimination.

JEL-Code: D700, D720, D740, D780.

Keywords: contest design, balanced-budget-constraint, differential prize taxation, structural discrimination, dual polarized discrimination, contest success function, lottery.

Yosef Mealem
The School of Banking & Finance
Netanya Academic College
Netanya / Israel
42365yossime1@bezeqint.net

Shmuel Nitzan
Department of Economics
Bar-Ilan University
Ramat Gan / Israel, 52900
nitzans@biu.ac.il

Financial support from the Adar Foundation of the Economics Department of Bar-Ilan University is gratefully acknowledged.

1. Introduction

In the vast contest literature that has numerous applications (internal labor market tournaments, promotional competitions, R&D races, rent-seeking, political and public policy competitions, litigation and sports), the lotteries proposed by Tullock (1980) are most commonly assumed as the contest success function (CSF), see Konrad (2009) and references therein. In two-player contests, these logit functions take the form

$$p_1(x_1, x_2) = \frac{x_1^\alpha}{x_1^\alpha + (\delta x_2)^\alpha} \tag{1}$$

Usually, $x_1 \geq 0$ and $x_2 \geq 0$ are interpreted as the efforts of the contestants 1 and 2 and p_1 as the winning probability of contestant 1 (if $x_1 = x_2 = 0$ then $p_1 = 0.5$). The winning probability of contestant 2 is equal to $p_2 = 1 - p_1$. The exponent α is a parameter that represents the effect of a real unit of investment on the winning probability of a contestant while the asymmetry between the impact of the contestants' efforts is captured by the parameter δ, $\delta \geq 0$. One reason for the popularity of this CSF is that it has appealing axiomatization, see Skaperdas (1996), Clark and Riis (1998), Jia (2008, 2010), Corchon and Dahm (2010), Hirshleifer and Riley (1992), Fullerton and McAfee (1999), Baye and Hoppe (2003)[1]. The special attention given to the simple lottery CSF, where $\alpha = 1$ and $\delta = 1$, can be justified, as recently argued by Franke et al. (2011), on the grounds that it lends itself to a very appealing competitive-market interpretation.

In our setting, the exponent α is viewed as a given parameter and it is assumed that $0 < \alpha \leq 2$, which guarantees, as is well known, see Konrad (2009) and references therein, that the contest game has a unique pure-strategy equilibrium. However, we do enable the contest designer to control the parameter δ, as first suggested in Lien (1986), (1990) and later by Clark and Riis (2000). This means that the designer can apply structural discrimination that affects the contestants' winning probabilities (the same efforts may yield different winning probabilities, depending on the value of this parameter). By (1), a reduction in δ increases the bias in favor of contestant 1, who is assumed to be, with no loss of generality, the more motivated contestant (the one with the higher prize valuation). Furthermore, $0 \leq \delta < 1$ implies a

[1] Munster (2009) has recently generalized the axiomatic approach to group CSFs.

bias in favor of contestant 1, with an extreme bias when $\delta = 0$ (contestant 1 is the certain winner, $p_1(x_1, x_2) = 1$). When $\delta = 1$ the contest is fair, there is no bias. When $\delta > 1$ the bias is in favor of contestant 2. The empirical relevance of such discrimination in contests with a logit lottery is thoroughly discussed in Epstein et al. (2011a), Franke (2012) and Franke et al. (2011).[2] Epstein et al. (2011b) have recently shown that structural discrimination is effective; it is useful as a means of increasing the contestants' efforts when applied independently.

The contest designer can also carry out another type of discrimination that affects the contestants' incentives, not by controlling the parameter δ (in which case $\delta = 1$), but by directly changing the contestants' prize valuations (their rewards in case of winning the contest), thereby increasing or decreasing the gap between these valuations. Such a policy is usually based on a "give and take" mechanism in case of winning, which is henceforth referred to "differential prize taxation". When the designer faces a balanced–budget constraint, the expected net resources transferred to the contestants applying this type of discrimination must be equal to zero. Note that the budget of the contest designer does not include the awarded prize and the exerted efforts of the contestants. The former assumption is plausible because the awarded prize is often not monetary, taking the form of some privilege, such as a monopoly permit. The latter assumption is plausible because the exerted efforts may not be monetary, they can be monetary, but not transfers to the designer and, finally, the efforts can be monetary transfers, but not part of the budget (illegal transfers). One simple form of a tax scheme involves transfers that are taken from the winner and given to the loser. The transferred amounts depend on the identity of the winner. Clearly, this tax scheme automatically satisfies the balanced-budget constraint, independent of the contestants' winning probabilities and even if the designer holds just one contest. Yet another possible tax scheme consists of two numbers (one negative and one positive) that are added to the contestants' initial prize valuations. These numbers need not be equal in their absolute value. However, they need to satisfy the requirement that the expected transfer to one contestant in case of his winning the contest must be equal to the expected "tax" paid by the other contestant

[2] Franke et al. (2012) have recently allowed $\delta \neq 1$, but still focusing on the simple lottery case ($\alpha = 1$). They have shown that in this setting, for *N* players the maximal efforts secured by the optimal APA are larger than those obtained by any lottery.

in case of his winning the contest. Hence, a tax scheme that satisfies the balanced-budget constraint now depends on the equilibrium winning probabilities of the contestants. We will show that this more subtle type of a tax scheme, which has to satisfy a more subtle form of the balanced-budget constraint, is more plausible because it is superior from the designer's point of view to the former simpler tax scheme. The balanced-budget constraint corresponding to this more plausible tax scheme is also realistic when the designer controls a series of identical contests that are held during a fixed period (typically weekly, monthly or quarterly contests that are held during the budget year). In such a case, the designer actually ensures that during the relevant period the net transfers between the contestants are cancelled out such that his budget is balanced. Discrimination via contingent taxation of the prize won in a contest can be applied in various public-economic contexts. In particular, it can be used to explain the expected change in the existing income inequality between interest groups (e.g., the “poor” and the “rich”) that compete on the prize (gain or loss of income) associated with a proposed reform in the tax system. Such interest groups are typically represented by lobbyists who are the actual contestants. Mealem and Nitzan (2012) have recently shown that discrimination implemented by the more subtle type of a tax scheme is also effective when applied independently. That is, when the designer resorts solely to this mode of discrimination, he can increase the contestants’ efforts. Note that Mealem and Nitzan (2012) focus on the application of differential prize taxation disregarding structural discrimination. Their main purpose is to show that in this setting the all-pay-auction induces more efforts than any lottery with $0 < \alpha < 2$. In contrast, in the current study, we allow the two modes of discrimination focusing on the maximal efforts the designer can induce in a contest based on a lottery.

In light of the separate effectiveness of the above two modes of discrimination, the main objective of this study is to examine whether both of these modes of discrimination are needed when they can be applied simultaneously and to study their effectiveness in generating efforts. Given that the gap between the contestants can be closed by discrimination, either by modifying the contestants’ prize valuations or by structurally changing the impact of their efforts, it seems that the designer can resort just to one type of discrimination. Interestingly, our preliminary claim establishes that, when $0 < \alpha < 2$, both types of discrimination are effective, not only when applied independently, but also when applied simultaneously. Furthermore, by the first main result, under any lottery exhibiting constant or increasing returns to scale and under some lotteries exhibiting decreasing returns to scale, the combined effects of the proposed dual discrimination increase the designer's revenue beyond the average value of the initial prize valuations, which is the maximal effort obtained by either mode of discrimination under *any* possible lottery[3]. In particular, when $\alpha = 1$, the combined effects of the two proposed modes of discrimination can yield efforts that are almost equal to the highest initial prize valuation. These efforts are exerted by the contestant who initially has the lower prize valuation. This contestant is offered an illusion of winning a very large prize. However, this attractively high prize is almost always unattainable, because the designer ensures that it is almost never won. Note that these efforts are the largest possible under any mechanism and, as is well known, they can be extracted under various versions of the 'take it or leave it' mechanism. The disadvantages of the 'take it or leave it' expropriating mechanism relative to the simple lottery with the dual mode of discrimination are spelled out in section 3B.

The extreme effectiveness of dual discrimination is robust to an increase in the number of the contestants. That is, if $\alpha = 1$, a designer who simultaneously applies the two modes of discrimination can induce the largest possible efforts in any *N*-player contest.[4] Our second main result reinforces the first one by establishing that the extreme, twofold polarized discrimination strategies presented in the first result are optimal. Surprisingly, this result implies that if the designer can control the two modes of discrimination as well as the exponent α of the CSF in (1), he can secure the largest possible efforts that are almost equal to n_1 by selecting the widely studied simple intermediate lottery where $\alpha = 1$.[5] The superiority of this constant-returns-to-scale-lottery is in marked contrast to its non-optimality when the designer is not

---

[3] See Epstein et al. (2011b) and Mealem and Nitzan (2012).

[4] For a thorough study of equilibrium efforts in contests based on a lottery with $\alpha \geq 2$, Alcalde and Dahm (2010) have shown that there exists an equilibrium in mixed strategies that is equivalent to the equilibrium of the APA. However, so far a characterization of the complete set of mixed-strategy equilibria is not available, even for the "simple" case without a balanced-budget constraint and without structural discrimination. Nevertheless, our result implies that, even when the designer can control the parameter α without being subjected to any constraint, $0 < \alpha \leq \infty$, the total maximal efforts are still obtained for $\alpha = 1$, because these efforts are the largest possible for any lottery.

[5] In the designer's maximization problem (10), the exponent α is a given parameter, $0 < \alpha \leq 2$. That is, the designer does not control α. Still , the solution of his problem for any $0 < \alpha \leq 2$ implies that the maximal contestants' efforts are obtained for $\alpha = 1$. In other words, the indirect effort function is maximized at $\alpha = 1$.So this value of the exponent would be the designer's preferred value if he could select the parameter α .

allowed to apply any mode of discrimination between the contestants, or when he is allowed to apply just one of these modes of discrimination.

The secondary objective of the paper is to compare the effectiveness of the two modes of discrimination that we study when applied independently. That is, our purpose is to answer the question which type of discrimination yields larger efforts when applied separately. We show that, when $0 < \alpha < 2$, differential prize taxation is superior to structural discrimination; it yields larger efforts. Differential prize taxation has another advantage beyond its superior effectiveness in inducing efforts; practically, it seems to be easier to implement because taxation is a common method of intervention, especially when insisting on satisfying the balanced-budget constraint. In contrast, structural discrimination might be normatively difficult to implement, especially when the legal system requires equal treatment of the contestants. Notice that by independently applying either mode of discrimination, the designer can maximize the intensity of the contest and equalize the contestants' winning probabilities. The question is whether this strategy also maximizes the contestants' efforts. It turns out that with structural discrimination this is indeed the case. In contrast, with differential prize taxation it is usually not the case with one exception. Specifically, when $0 < \alpha < 2$, prize taxation that equalizes the contestants' winning probabilities does not yield maximal efforts. Such equalization does, however, yield the maximal efforts under the extreme lottery where $\alpha = 2$.

2. The setting

In our contest there are two risk-neutral contestants, the high and low benefit contestants, 1 and 2. The initial prize valuations of the contestants are denoted by n_1 and n_2 and, with no loss of generality, we assume that $n_1 \geq n_2$ or $k = \frac{n_1}{n_2} \geq 1$ and that the contest designer has full knowledge of the contestants' prize valuations. Heterogeneity in the contestants' prize valuations is usually attributed to differences in preferences or to differences in the value of the awarded non-monetary privilege (monopoly permit). Given the contestants' fixed prize valuations and the CSF, the function that specifies the contestants' winning probability given their efforts, $p_i(x_1, x_2)$, the expected net payoff (surplus) of contestant *i* is:

(2) $$E(u_i) = p_i(x_1, x_2)n_i - x_i\,,\ \ (i\text{=}1,2)$$

where $p_i(x_1, x_2)$ is the lottery given by (1). In the optimal contest design setting, the objective function of the contest designer is:

(3) $$G = x_1 + x_2$$

Resorting just to structural discrimination means that the contest designer maximizes his objective function (3) by selecting δ (given any α that satisfies $0 < \alpha \leq 2$)[6]. Resorting solely to differential prize taxation means that the designer changes the contestants' prize valuations from n_1 and n_2 to $(n_1 + \varepsilon_1)$ and $(n_2 + \varepsilon_2)$ by selecting the (positive or negative) amounts ε_1 and ε_2 (given any α that satisfies $0 < \alpha \leq 2$). A contest designer who applies such discrimination must ensure that the transformed prize valuations are positive. Otherwise the contestants will not voluntarily take part in the contest and the designer's revenue will be equal to zero. If the contest designer faces a balanced-budget constraint, then ε_1 and ε_2 must also satisfy the equality $p_1\varepsilon_1 + p_2\varepsilon_2 = 0$. That is, the expected transfer to one contestant in case of his winning the contest must be equal to the expected "tax" paid by the other contestant in case of his winning the contest.[7] When the designer can apply both types of discrimination, he maximizes his objective function (3) by selecting δ, ε_1 and ε_2 (again, for any α that satisfies $0 < \alpha \leq 2$), given the anticipated Nash equilibrium efforts of the contestants. The particular choice of his preferred discrimination policy together with the corresponding efforts of the contestants, constitute the equilibrium of the game. The contest game that we study has therefore a two-stage structure:

---

[6] The analysis in this study is confined to lotteries with an exponent α such that $0 < \alpha \leq 2$. These lotteries include the constant and decreasing-returns-to-scale lotteries that are economically the most plausible ones. Note that, as is well known, the contest games based on these lotteries have a unique pure-strategy equilibrium, see Konrad (2009) and references therein.

[7] An alternative interpretation to the balanced-budget constraint is the following one. Suppose that the contested prize is divisible; in this case p_i can be interpreted, as in Corchon and Dahm (2010), as the share of the prize won by contestant i and ε_i can be interpreted as the tax/subsidy levied on (transferred to) contestant i assuming that he wins the whole prize. Hence, the constraint $p_1\varepsilon_1 + p_2\varepsilon_2 = 0$ means that the tax collected from one contestant is transferred to the other one although, clearly, it is possible that $\varepsilon_1 + \varepsilon_2 \neq 0$.

1. In the first stage the designer determines the discrimination policy, by selecting δ, ε_1 and ε_2 (for any α that satisfies $0 < \alpha \le 2$),
2. In the second stage the contestants simultaneously make decisions on their exerted efforts x_1 and x_2 taking as given the discrimination policy set by the designer.

The solution of this contest game is a sub-game-perfect Nash equilibrium.

Suppose that given a CSF of the logit form (1) where $0 < \alpha \le 2$, the designer can apply the two modes of discrimination, that is, select δ, ε_1 and ε_2. In this case the two contestants maximize their expected payoffs:

(4) $$E(u_1) = \frac{x_1^\alpha}{x_1^\alpha + (\delta x_2)^\alpha}(n_1 + \varepsilon_1) - x_1 \text{ and } E(u_2) = \frac{(\delta x_2)^\alpha}{x_1^\alpha + (\delta x_2)^\alpha}(n_2 + \varepsilon_2) - x_2$$

Let $a = \dfrac{n_1 + \varepsilon_1}{n_2 + \varepsilon_2}$ and $d = \left(\dfrac{a}{\delta}\right)^\alpha$. By the first order conditions,

(5) $$x_1^* = \frac{\alpha d(n_1 + \varepsilon_1)}{(d+1)^2} \text{ and } x_2^* = \frac{\alpha d(n_2 + \varepsilon_2)}{(d+1)^2}$$

and, therefore,

(6) $$G = x_1^* + x_2^* = \frac{\alpha d(n_1 + \varepsilon_1 + n_2 + \varepsilon_2)}{(d+1)^2}$$

(7) $$p_1 = \frac{d}{d+1} \text{ and } p_2 = \frac{1}{d+1}$$

and the balanced-budget constraint takes the form

(8) $$p_1\varepsilon_1 + p_2\varepsilon_2 = \frac{d}{d+1}\varepsilon_1 + \frac{1}{d+1}\varepsilon_2 = 0$$

or

(9) $$d\varepsilon_1 + \varepsilon_2 = 0$$

The designer's problem is therefore:

(10)
$$
\begin{aligned}
&\underset{\varepsilon_1,\varepsilon_2,\delta}{Max}\ \left(x_1^* + x_2^*\right) = \frac{\alpha d\left(n_1 + \varepsilon_1 + n_2 + \varepsilon_2\right)}{\left(d+1\right)^2} \\
&s.t. \\
&1.\ 1 - \alpha + d \geq 0 \\
&2.\ \left(1-\alpha\right)d + 1 \geq 0 \\
&3.\ \delta \geq 0 \\
&4.\ d\varepsilon_1 + \varepsilon_2 = 0 \\
&5.\ n_1 + \varepsilon_1 > 0 \\
&6.\ n_2 + \varepsilon_2 > 0
\end{aligned}
$$

In Appendix A we present the justification for constraints 1 and 2. It will be shown that these constraints guarantee that the contestants' utilities are not negative as well as the fulfillment of the second-order conditions in the contestants' maximization problems.[8]

3. Two-mode discrimination

3.A Results

Let us start by clarifying the effectiveness of discrimination when it can take the form of both differential prize taxation and structural discrimination. More precisely, our preliminary claim is the following one:

Claim 1:

For $k > 1$ and $0 < \alpha < 2$, dual discrimination yields larger efforts than those obtained just by differential prize taxation.[9]

The proof of the preliminary claim (see Appendix B) uses the following idea: the designer increases the polarization between the contestants' stakes by reducing the

[8] The justification in Appendix A of constraints 1 and 2 in problem (10) is similar to that presented in Appendix B in Epstein et al. (2011b).

[9] In section 4 we show that differential prize taxation is more effective than structural discrimination. This implies that dual discrimination yields larger efforts than those obtained just by structural discrimination.

stake of contestant 1 and increasing the stake of contestant 2. The increase in polarization is associated with an increase in the sum of the contestants' prize valuations. But to enable the increased polarization, the balanced-budget constraint requires creating a structural bias in favor of contestant 1, the contestant whose stake has been reduced, by selecting δ which is smaller than 1. In the proof, the required reduction in δ results in the preservation of the contestants' winning probabilities while increasing the sum of their stakes, and this causes the increase in their exerted efforts (see (6)). This idea raises the following question: what happens to the total efforts if the designer "maximizes" the extent of polarization between the contestants' stakes by reducing the stake of contestant 1 almost to zero ($\varepsilon_1 \to -n_1^+$) and by increasing the stake of contestant 2 to a "very large" level. Clearly, to make sure that the balanced-budget constraint is satisfied, the designer must create an appropriate bias in favor of contestant 1 by selecting a very small δ, such that the balanced-budget constraint (9) is satisfied:

(11) $$\delta = \left(\frac{n_1 + \varepsilon_1}{n_2 + \varepsilon_2}\right)\left(-\frac{\varepsilon_1}{\varepsilon_2}\right)^{\frac{1}{\alpha}}$$

The two types of discrimination exerted in this case are somewhat different than those described in the proof of claim 1, because the designer does not confine himself to preserving the winning probabilities of the contestants. It turns out that, for $\alpha = 1$, this dual discrimination with maximal polarization, that is, $\varepsilon_1 \to -n_1^+$ and $\varepsilon_2 \to \infty$, is an optimal strategy yielding efforts that are almost equal to n_1 , the initial higher prize valuation of contestant 1. For example, for $\alpha = 1$, $n_1 = 100$ and $n_2 = 2$, if the designer considerably increases the polarization between the contestants' stakes by selecting $(\varepsilon_1, \varepsilon_2) = (-99.9, 100000)$ and δ according to (11), $\delta \cong 9.9898 \cdot 10^{-10}$, the total efforts will be equal to $G = 99.7$. This example illustrates the more general finding obtained in the first part of the first proposition presented below. That is, when $0 < \alpha \leq 1$, dual discrimination with maximal stake polarization and selection of δ that satisfies (11) yields efforts that are almost equal to αn_1. When $\alpha = 1$ these efforts are equal to those obtained under the "take it or leave it" mechanism. However, in our setting n_1 is almost obtained using a standard simple lottery that allows structural bias between the contestants, without setting a minimal effort for contestant 1, without disregard for the balanced-budget constraint and without deterring the participation of one of the contestants.

Undertaking the extreme dual discrimination with maximal polarization, $\varepsilon_1 \to -n_1^+$ and $\varepsilon_2 \to \infty$, while choosing δ according to (11), such that the balanced-budget constraint (9) is satisfied, is possible for $0 < \alpha \le 1$. But it is not possible for $1 < \alpha \le 2$. The reason is that the designer's selection of $(\varepsilon_1, \varepsilon_2, \delta)$ must ensure that the utility of the contestants is not negative, to prevent their abandonment from the competition and, in turn, the decline of the contestants' efforts to zero. In other words, constraints 1 and 2 in problem (10) that ensure the existence of competition, as well as the second order conditions for utility maximization, must be satisfied. It can be verified that when $0 < \alpha \le 1$, for any degree of polarization between the contestants, (any positive value of *d*), these two constraints are satisfied. However, an increase of α beyond 1 does not enable any degree of polarization (any value of *d*) in which contestant 1's stake is reduced and contestant 2's stake is increased, as implied by constraint 2 in problem (10) that ensures the entry of contestant 2 to the competition. In this case ($1 < \alpha \le 2$) the designer can set a maximal value for *d* which is equal to $d_{\max} = \dfrac{1}{\alpha - 1}$.[10] Combining this equality with the condition for the existence of the balanced-budget constraint, $d = -\dfrac{\varepsilon_2}{\varepsilon_1}$, gives the maximal value of ε_2 (given $\varepsilon_1 < 0$) that can be set by the designer, $\varepsilon_2 = -\dfrac{\varepsilon_1}{\alpha - 1}$. This means that the maximal polarization in this case is obtained when $\varepsilon_1 \to -n_1^+$ and $\varepsilon_2 \to \dfrac{n_1}{\alpha - 1}$. Can this maximal degree of polarization together with the value of δ determined by (11) yield total efforts that converge to n_1? By the second part of Proposition 1, the answer is negative.

[10] Notice that if the designer chooses $d = d_{\max} = \dfrac{1}{\alpha - 1}$, then constraint 1 in problem (10) is also satisfied.

Proposition 1:[11]

1. For $k > 1$ and $0 < \alpha \leq 1$, when $\varepsilon_1 \to -n_1^+$, $\varepsilon_2 \to \infty$ and δ is set according to (11), the winning probability of contestant 1 converges to 1, but his effort converges to zero and the winning probability of contestant 2 converges to zero, but his effort converges to αn_1. Total efforts therefore converge to αn_1, $E(u_1) \to 0$ and $E(u_2) \to (1-\alpha)n_1$.

2. For $k > 1$ and $1 < \alpha \leq 2$, when $\varepsilon_1 \to -n_1^+$, $\varepsilon_2 = \dfrac{\varepsilon_1}{1-\alpha}$ and δ is set according to (11), the winning probability of contestant 1 is $\dfrac{1}{\alpha}$, but his effort converges to zero and the winning probability of contestant 2 is $\left(1 - \dfrac{1}{\alpha}\right)$ and his effort converges to $\dfrac{1}{\alpha} n_1 + \left(1 - \dfrac{1}{\alpha}\right) n_2$. Total efforts therefore converge to a value that is smaller than n_1 and $E(u_1) = E(u_2) \to 0$.[12]

The special appeal of the dual polarized discrimination strategies presented in Proposition 1 is highlighted by the following result.

Proposition 2: For any $0 < \alpha \leq 2$, the dual polarized discrimination strategies applied in Proposition 1 yield total efforts that converge to the lowest upper bound of the possible equilibrium efforts of the contestants.

The relationship between the exponent α of a lottery and the maximal attainable efforts G is presented in Figure 1. By Proposition 1, under any lottery exhibiting constant or increasing returns to scale, $1 \leq \alpha < 2$, and under lotteries exhibiting decreasing returns to scale, such that $0.5 + \dfrac{1}{2k} < \alpha < 1$, the combined effects of the extreme polar modes of discrimination increase the designer's revenue beyond the

---

[11] The proofs of this and the next propositions appear in the Appendix.

[12] For $k = 1$, that is, when $n_1 = n_2 = n$, in the range $0 < \alpha < 1$, we would get that $G \to \alpha n$ and in the range $1 \leq \alpha \leq 2$, we would get that $G \to n$.

average value of the initial prize valuations, $0.5(n_1 + n_2)$, which is the maximal effort obtained by either mode of discrimination under *any* possible lottery.

Proposition 2 implies that when the designer applies the two modes of discrimination, each type has a positive "added value" that enhances the exertion of efforts relative to the situation where the designer resorts to just one mode of discrimination. That is, the two modes of discrimination are supportive or "complementing" - their combination yields larger efforts than those obtained by separate application of one of these modes of discrimination for almost any given level of α ($0 < \alpha < 2$). Furthermore, under lotteries with increasing or constant returns to scale, as well as under some lotteries with decreasing returns to scale, such dual discrimination yields efforts that are larger than the average prize valuation (see ABC in Figure 1), which is the largest possible total effort under separate application of these modes of discrimination. The advantage of combining these two types of discrimination relative to the use of a single mode of discrimination is due to the distinctive features of the contribution of each of these modes of discrimination to the exerted efforts as described below.

(i) Differential prize taxation increases as much as possible the initially lower prize valuation while reducing the initially higher prize valuation almost to zero. This increases the sum of the contestants' prize valuations to infinity and makes the 'income effect' (associated with a scheme that increases the sum of the final stakes from $(n_1 + n_2)$ to $(n_1 + \varepsilon_1 + n_2 + \varepsilon_2))$ of this mode of discrimination the dominant effect.[13]

(ii) The maximal possible increase in the sum of the contestants' prize valuations is not the result of differential prize taxation alone. It is rendered possible by structural discrimination that makes sure that the balanced-budget constraint is satisfied. Specifically, stuctural discrimination counterbalances the above 'income effect' by almost completely favorably discriminating contestant 1, ensuring that his winning probability converges to zero.

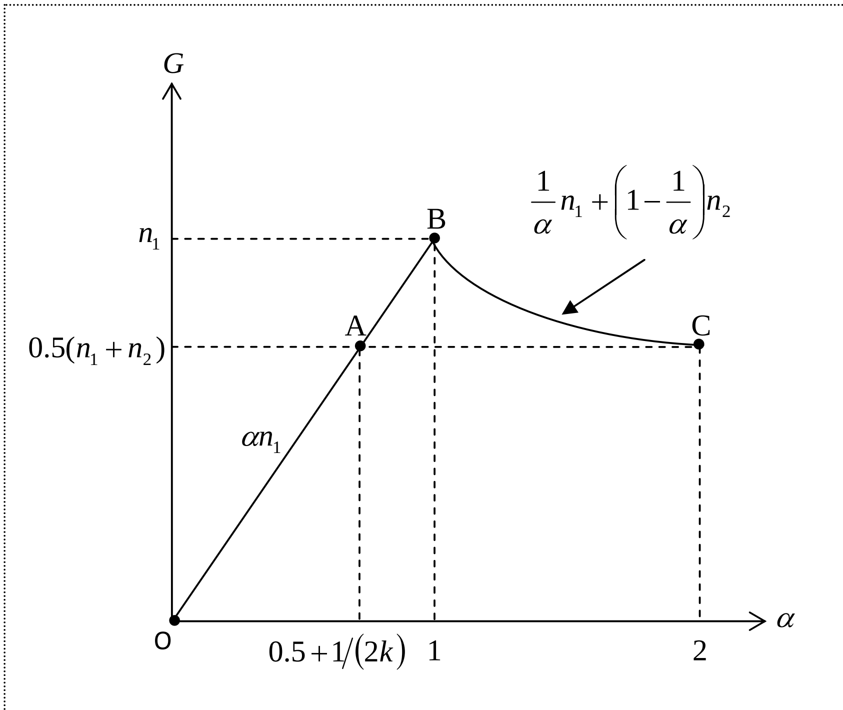
The moderating effect described in (**ii**) is necessary to attain the maximal efforts. While structural discrimination has a 'second order' effect on efforts that moderates the income effect of differential prize taxation, it also enables the

[13] For a clarification of the meaning of the income effect associated with differential prize taxation, see the discussion following Proposition 2 in Mealem and Nitzan (2012).

dominance of this 'first order' income effect on efforts described in (**i**), namely, the increase in efforts due to the increase in the sum of the contestants' prize valuations. The dominance of the effect of differential prize taxation means that the more extreme this mode of discrimination, the higher the total efforts and this requires the extremity of structural discrimination.

Proposition 2 also implies that if the designer can control δ, ε_1, ε_2 as well as α, he can secure almost the largest possible efforts n_1 by selecting $\alpha = 1$ (recall that we have already proved in part 2 of Proposition 1 that the efforts exerted when

Figure 1*: The relationship between the exponent α and the maximal attainable efforts *G



$1 < \alpha \le 2$ converge to a value that is smaller than n_1). Any lottery with $\alpha \ne 1$ is therefore inferior to a simple lottery where $\alpha = 1$, when in both cases the designer applies the optimal discrimination strategy, viz, the dual polarized discrimination strategy. The superiority of $\alpha = 1$ is in marked contrast to its non-optimality when the designer is not allowed to discriminate between the contestants or when the designer is allowed to discriminate between the contestants, but apply just one mode of discrimination.

Finally, let us turn to the justification of the particular form of differential prize taxation we have assumed by comparing it to an alternative simpler form.

The alternative simpler differential prize taxation: Suppose that, before the contest is held, the designer announces that in case of winning contestant *i* ($i=1,2$) is subjected to a tax equal to β_i, which is transferred to his rival (according to Appendix B, in equilibrium the requirement is that $0 \le \beta_1 + \beta_2 < n_i$).

It turns out that, from the designer's point of view, this simpler form of a tax scheme that automatically satisfies the balanced-budget constraint is inferior to the assumed mode of differential prize taxation that applies the pair $(\varepsilon_1, \varepsilon_2)$. That is, for any given lottery, where $0 < \alpha < 2$, the maximal efforts obtained under the optimal dual discrimination based on the simpler alternative differential prize taxation are smaller than the maximal efforts obtained in our model under the optimal dual discrimination. When $\alpha = 2$, the two models yield the same total efforts $0.5(n_1 + n_2)$. Specifically,

Proposition 3: For any $0 < \alpha < 2$, the optimal dual polarized discrimination strategy in our model yields total efforts that are larger than $0.25\alpha(n_1 + n_2)$, the maximal efforts obtained under the optimal dual discrimination strategy based on the alternative simpler mode of differential prize taxation. When $\alpha = 2$, the two models yield the same total efforts $0.5(n_1 + n_2)$.

The proof of this proposition is obtained by showing that, under the simpler alternative of differential prize taxation, the optimal dual discrimination requires that $\beta_1 = \beta_2 = 0$ and $\delta = k$. In this case, $\beta_1 = \beta_2 = 0$ because setting positive β_i's reduces the prize of contestant *i* in case of winning and increases his payoff in case of losing. Both of these effects reduce his incentive to exert effort. Hence, the designer has no incentive to apply differential prize taxation and he applies just structural discrimination setting $\delta = k$, which maximizes the intensity of the competition. Note that an increase in the stake of contestant 2, which is realized when he loses the contest, induces him to reduce his effort. In our setting, however, an increase of ε_2 in contestant 2's stake induces him to increase his effort.

3.B Discussion

By part 1 of Proposition 1, the application of dual polarized discrimination can yield the maximal efforts. This extreme result is particularly interesting because it is obtained under a variant of the simple and most commonly studied CSF in the contest literature; the simple lottery where $\alpha = 1$ that allows structural bias between the contestants. When the designer does not face a balanced-budget constraint (and any surplus is allowed), it can be easily shown that with complete information on the contestants' prize valuations, he can apply a "take it or leave it" mechanism that expropriates almost the largest possible efforts (n_1). In such a case he can basically confiscate (arbitrarily close to) the complete higher value of the prize (n_1) by taxing all the lower value of the prize (n_2) and by taxing almost all of n_1, ensuring the winning of contestant 1 with the higher prize valuation. There are three crucial differences, however, between the effort-maximizing mechanisms applied by the designer in this case and in our setting that highlight the advantage of a simple lottery accompanied by an optimal dual discrimination strategy.

1. In the current study, the designer can apply a strategy of double discrimination, but the set of allowed strategies is confined by the balanced-budget constraint. Despite this restriction, he can induce the contestants to exert the maximal efforts. In the alternative "take it or leave it" setting, the designer has more flexibility because he can select any strategy of differential taxation of the prize (including strategies that result in a surplus). But such excessive flexibility might be questionable and perhaps infeasible for lack of a balanced budget. Note that in the alternative setting the designer's flexibility is reduced because he cannot apply structural discrimination. He can still attain the maximal revenue.
2. In the current paper, the designer's utility is defined as the contestants' efforts, as commonly assumed in the contest literature. In the alternative "take it or leave it" setting a less common definition is used. Since the designer does not face a balanced-budget constraint, the objective function of the designer consists of the contestants' efforts plus the collected tax (the budget surplus).

3. In the alternative "take it or leave it" setting the designer extracts from contestant 1 almost all his higher prize valuation. In contrast, in the current study, the situation is reversed; although initially contestant 1's stake is the higher one, after the implementation of the dual polarized discrimination strategy, the stake of contestant 1 is reduced (almost) to zero, so practically virtually no efforts can be extracted from him. However, contestant 2's stake is very much increased and his winning probability is reduced substantially (almost to zero), such that his effort becomes almost equal to the initial stake of contestant 1 and his utility converges to zero. This result is due to the "illusion" offered to contestant 2; the possibility of his winning a very large prize. However, as explained above, this attractively high prize is almost always unattainable, because the designer ensures that it is almost never won.[14] Still contestant 2 (not contestant 1) is induced to exert efforts that are almost equal to n_1.

Two important conclusions can be drawn from the results. First, for $k > 1$, when the designer applies the (optimal) dual polarized discrimination strategy (the strategy that maximizes the contestants' efforts), an increase in α from $\alpha = 1$ to $\alpha = 2$ reduces efforts. Second, given any number of contestants *N*, such that $n_1 \geq n_2, ..., \geq n_N$, when $\alpha = 1$ and the designer applies dual polarized discrimination strategy, he can attain the almost n_1. In the more general multi-player contest, the designer has to reduce the stakes of $N - 2$ contestants to zero, making sure that contestant 1 with the highest stake is not included among them. Applying the dual polarized discrimination strategy with respect to the two remaining contestants, the designer can induce efforts that are almost equal to n_1.

The last question we wish to address is whether the designer can induce the maximal efforts that are almost equal to n_1 by applying dual polarized discrimination strategy, but by reversing the roles of the contestants (reducing the stake of contestant 2 almost to zero and substantially increasing the stake of contestant 1). The answer to this question is negative. For $k > 1$ and $0 < \alpha \leq 1$, if the designer chooses $\varepsilon_1 \to \infty$,

[14] The designer also offers an illusion to contestant 2 whenever $1 < \alpha \leq 2$, although in these cases the exerted efforts by contestant 2 converge to a value smaller than n_1.

$\varepsilon_2 \to -n_2^+$ and sets δ according to (11), then it can be easily shown that total efforts converge to αn_2. For $k > 1$ and $1 < \alpha \le 2$, it can be shown that reversal of the roles of the two contestants results in total efforts that converge to a value that is smaller than n_1.

4. The preferred single mode of discrimination

Suppose that the designer can select just one mode of discrimination. When he chooses structural discrimination, $(\varepsilon_1, \varepsilon_2) = (0,0)$, the contestants' payoffs, efforts, winning probabilities and the problem of the designer are obtained by substituting $(\varepsilon_1, \varepsilon_2) = (0,0)$ in (4), (5), (6), (7) and (10).[15] In this case, the designer selects the optimal δ, $\delta = k$, and the corresponding efforts for $0 < \alpha \le 2$ are equal to $G = 0.25\alpha(n_1 + n_2)$, see Epstein et al. (2011b). When the designer resorts solely to differential prize taxation, $\delta = 1$, the contestants' payoffs, efforts, winning probabilities and the problem of the designer are now obtained by substituting $\delta = 1$ in (4), (5), (6), (7) and (10). In this case the designer can select $(\varepsilon_1, \varepsilon_2) = (-0.5(n_1 - n_2), 0.5(n_1 - n_2))$, which satisfies all the constraints in problem (10) and induce efforts that are equal to $G = 0.25\alpha(n_1 + n_2)$. By Proposition 2 in Mealem and Nitzan (2012), when $0 < \alpha < 2$, the designer does not equalize the stakes of the contestants. He prefers a different strategy $(\varepsilon_1, \varepsilon_2) \neq (-0.5(n_1 - n_2), 0.5(n_1 - n_2))$, such that $\varepsilon_1 + \varepsilon_2 > 0$ which yields efforts that are larger than $G = 0.25\alpha(n_1 + n_2)$, the efforts obtained under structural discrimination.[16] We therefore get that for $0 < \alpha < 2$, differential prize taxation yields efforts that are larger than those obtained under structural discrimination. When

[15] Notice that in this case (8) and (9) are always satisfied.

[16] The reason that the designer prefers a differential prize taxation scheme that does not equalize the contestants' prize valuations is spelled out in length in Mealem and Nitzan (2012). The equalitarian taxation scheme represented by $(\varepsilon_1, \varepsilon_2) = (-0.5(n_1 - n_2), 0.5(n_1 - n_2))$ enables the designer to neutralize the initial difference in the contestants' stakes and thus increase the intensity of competition and, in turn, the contestants' efforts relative to the initial situation represented by $(\varepsilon_1, \varepsilon_2) = (0,0)$. The move from point $(\varepsilon_1, \varepsilon_2) = (-0.5(n_1 - n_2), 0.5(n_1 - n_2))$ to the equilibrium point (that satisfies $\varepsilon_1 + \varepsilon_2 > 0$) enables the designer to further increase the contestants' efforts by fully taking advantage of the potential "income effect" associated with a scheme that increases the sum of the final stakes from ($n_1 + n_2$) to ($n_1 + \varepsilon_1 + n_2 + \varepsilon_2$). This positive income effect dominates the negative effect on total efforts due to the reduced competition associated with the creation of a gap between the contestants' final stakes.

$\alpha = 2$, the two modes of discrimination are equivalent. That is, they induce the same efforts.[17]

5. Conclusion

5.A A brief summary of the main contribution

Under common knowledge of the contestants' prize valuations and any Tullock-type lottery associated with a pure-strategy equilibrium, optimal contest design can be implemented by applying structural discrimination that biases the effect of the contestants' exerted efforts, Epstein et al. (2011a), (2011b). Alternatively, such design can be carried out by affecting the contestants' prize valuations via differential taxation of the prize, subject to a balanced-budget constraint. Our results establish that:

(i) Both modes of discrimination are effective and, therefore will be used by the designer, when they can be applied simultaneously; Furthermore, under lotteries exhibiting constant or increasing returns to scale (with the exception of the case $\alpha = 2$) and under certain lotteries exhibiting decreasing returns to scale, $0.5 + \frac{1}{2k} < \alpha < 1$, the combined effects of these modes of discrimination can increase the designer's revenue beyond the average value of the initial prize valuations, which is the maximal effort obtained by either mode of discrimination under *any* possible lottery, Epstein et al. (2011b), Mealem and Nitzan (2012);

(ii) The dual polarized discrimination strategies corresponding to lotteries where $0 < \alpha \leq 2$ are optimal;

(iii) When $\alpha = 1$, a variant of the prototypical simple and most commonly studied lottery that applies extreme structural discrimination, together with a polar differential prize taxation is the designer's optimal strategy that yields the

[17] **Proof:** When $\alpha = 2$, optimal structural discrimination requires that $\delta = k$ and the corresponding efforts are equal to $G = 0.5(n_1 + n_2)$. With differential prize taxation, when $\alpha = 2$, the designer must set $(\delta, \varepsilon_1, \varepsilon_2)$ such that constraints 1 and 2 in his problem (10) are satisfied. Since by constraint 1, $d \geq 1$ and, by constraint 2, $d \leq 1$, the designer's optimal strategy requires that $d = 1$. Substituting $d = 1$ in the balanced-budget constraint, we get that $\varepsilon_1 + \varepsilon_2 = 0$ which implies, by (6), that the efforts of the contestants are equal to $G = 0.5(n_1 + n_2)$.

largest possible efforts (efforts that are almost equal to the initially higher prize valuation);

(iv) If the contest designer can use just a single mode of discrimination, then differential prize taxation is superior to structural discrimination; When the lottery is extreme, $\alpha = 2$, the two modes of discrimination are equivalent, yielding the same revenue.

(v) The plausibility of the particular form of differential prize taxation (and the corresponding balanced-budget constraint) we have assumed is due to its superior effectiveness as a means of generating revenue relative to an alternative simpler form of differential prize taxation the designer could apply.

5.B Extraction of the maximal revenue

In an unconstrained environment that allows budget surplus and just differential prize taxation, the designer can expropriate almost all the initially higher prize valuation, applying a "take it or leave it" mechanism. In this case, both of the contestants are left without any surplus in case of winning and, since the contestant with the lower stake is deterred from taking part in the contest, contestant 1, who takes part in the contest and wins, transfers almost all his stake to the designer. In our balanced-budget constrained setting, the designer also captures almost all the initially higher stake when $\alpha = 1$, due to the simultaneous effective application of the two modes of discrimination. But in this case the designer gets hold of the actual efforts exerted by the contestants and not of the taxes collected from the winners (the expected value of these taxes is equal to zero). We have pointed out to substantial differences between the two situations that clarify the advantage of applying a standard lottery rather than a "give it or take it” mechanism.

5.C Generalization to *N*-player contests

A potential interesting extension of our study is the analysis of the multiple-player case. Only few studies dealt with *N*-player contests assuming lotteries with asymmetric contestants. Stein (2002), Fang (2002), Franke (2007) and Franke et al. (2011, 2012) assumed, for *N*-player, that $\alpha = 1$, and Cornes and Hartley (2005) allowed any α. Stein (2002) extended the two-player contest to *N*-player contest and examined how changes in the contestants’ prize valuations and in the measure of their prior relative chance of winning affect the equilibrium efforts. Fang (2002) compared the lottery model where $\alpha = 1$ and $\delta = 1$ to the All-Pay Auction and examined the conditions under which the total efforts corresponding to a lottery are larger than those corresponding to the All-Pay Auction. Franke (2007) compared these efforts under Affirmative Action (AA), where the designer affects the winning probabilities of the contestants (in our case, via the selection of δ) to the efforts obtained under Equal Treatment (ET). For two contestants, he extended his analysis to the case where $\alpha \leq 1$, but for *N* players he confined the analysis to $\alpha = 1$. For *N*-player contests, Franke et al. (2011, 2012) have recently allowed structural discrimination ($\delta \neq 1$), but still focusing on the simple lottery case ($\alpha = 1$). Franke et al. (2011) have shown that in this setting the designer will level the playing field by encouraging weak contestants, but he will not equalize the contestants' chances of winning the contest. Franke et al. (2012) have shown that the maximal efforts secured by the optimal APA are larger than those obtained by any lottery.

For *N* players, Cornes and Hartley proposed an elegant way to examine the existence of equilibrium for any α. Among other things, they have shown that, for $\alpha = 1$, there exists a unique equilibrium in pure strategies. But, for $\alpha > 1$, there is no explicit presentation of equilibrium and, in fact, multiple equilibria are possible, which precludes the possibility of conducting comparative statics, see footnote 24 in Franke (2007). This implies that, to attain consistency of the results, we can choose $\alpha > 1$, for two players or $\alpha = 1$, for any number of players. In our study the focus is on two-player contests and, therefore, we can compare the two modes of discrimination also for lotteries with $\alpha \leq 2$, despite our inability to compute explicitly the equilibrium outcome under differential prize taxation. This case is also more general than the one examined by Franke (2007), since he assumed for 2 players that $\alpha \leq 1$. The challenging question what happens when we move to an *N*-player contest for any α, $0 < \alpha \leq 2$, (note that in our study we have dealt only with the case $\alpha = 1$) seems an especially demanding challenge and is left for future research.

References

Alcalde, J. and Dahm, M., (2010), "Rent Seeking and Rent Dissipation: A Neutrality Result", *Journal of Public Economics*, 94, 1-7.,

Baye, Michael R. and Heidrun Hoppe, (2003), "The Strategic Equivalence of Rent-Seeking, Innovation, and Patent-Race Games", *Games and Economic Behavior*, 44(2), 217-226.

Clark, D.J., Riis, C., (1998), "Contest Success Functions: An Extension", *Economic Theory*, 11, 201-204.

Clark, D.J., Riis, C., (2000), "Allocation Efficiency in a Competitive Bribery Game", *Journal of Economic Behavior and Organization*, 42, 109-124.

Corchón Luis and Matthias Dahm (2010), "Foundations for Contest Success Functions", *Economic Theory*, 43, 81–98.

Cornes. R and R. Hartley., (2005), Asymmetric contests with general technologies, *Economic Theory*, 26(4), 923-946.

Epstein G.S., Mealem Y. and Nitzan S., (2011a), "Political Culture and Discrimination in Contests", *Journal of Public Economics*, 95(1-2), 88-93.

Epstein G.S., Mealem Y. and Nitzan S., (2011b), "Lotteries vs. All-Pay Auctions in Fair and Biased Contests", Discussion Paper No. 29, Department of Economics, Bar Ilan University.

Fang, H., (2002), "Lottery versus All-Pay Auction Models of Lobbying", *Public Choice*, 112, 351-371.

Franke, J., (2012), "Affirmative Action in Contest Games", *European Journal of Political Economy*, 28(1), 115-118.

Franke J. , Kanzow C. Leininger W. and Schwartz A. , (2011), "Effort Maximization in Asymmetric Contest Games with Heterogeneous Contestants", *Economic Theory*, forthcoming.

Franke, Jorg; Kanzow, Christian; Leininger, Wolfgang and Schwartz, Alexandra., (2012), "Lottery versus All-Pay Auction Contests - A Revenue Dominance Theorem" Ruhr Economic Papers #315, TU Dortmund.

Fullerton, Richard L. and R. Preston McAfee, (1999), "Auctioning Entry into Tournaments", *Journal of Political Economy*, 107(3), 573-605.

Hirshleier, Jack and John G. Riley, (1992), "The Analytics of Uncertainty and Information", Cambridge University Press, Cambridge UK.

Jia, H., (2008), “A Stochastic Derivation of the Ratio Form of Contest Success Functions”, *Public Choice*, 135, 125–130.

Jia, H., (2010), "On a Class of Contest Success Functions", *The B.E. Journal of Theoretical Economics*, 10(1).

Konrad, K., (2009), Strategy and Dynamics in Contests (London School of Economic Perspectives in Economic Analysis), Oxford University Press, USA.

Lien, D., (1986), “A Note on Competitive Bribery Games”, *Economics Letters*, 22, 337-341.

Lien, D., (1990), “Corruption and Allocation Efficiency”, *Journal of Development Economics*, 33, 153-164.

Mealem, Yosef and Nitzan, Shmuel., (2012), “Equity and Effectiveness of Optimal Taxation in Contests under an All-Pay Auction", Working Paper No 3712, CESifo.

Munster, J., (2009), "Group Contest Success Functions", *Economic Theory*, 41(2), 345-357.

Nti, K.O., (2004), "Maximum Efforts in Contests with Asymmetric Valuations", *European Journal of Political Economy*, 20(4), 1059-1066.

Skaperdas, S., (1996), "Contest success functions", *Economic Theory*, 7, 283-290.

Stein, W., (2002), "Asymmetric rent-seeking with more than two contestants", *Public Choice* 113, 325–336.

Tullock, G., (1980), Efficient Rent-Seeking. In: Buchanan J.M, Tollison R.D, Tullock G., (ed) Toward a Theory of the Rent-Seeking Society, College Station, TX, Texas A. and M., University Press, 97-112.

Appendix A

In problem (10), the designer controls the parameters δ, ε_1 and ε_2. From the expected payoff of the contestants, equations (4), we get the first order conditions:

$$\frac{\partial E(u_1)}{\partial x_1} = \frac{\alpha x_1^{\alpha-1}(\delta x_2)^{\alpha}(n_1+\varepsilon_1)}{\left[x_1^{\alpha}+(\delta x_2)^{\alpha}\right]^2} - 1 = 0$$

(A1) and

$$\frac{\partial E(u_2)}{\partial x_2} = \frac{\alpha \delta^{\alpha} x_2^{\alpha-1} x_1^{\alpha}(n_2+\varepsilon_2)}{\left[x_1^{\alpha}+(\delta x_2)^{\alpha}\right]^2} - 1 = 0$$

and, after rearranging, we get the contestants' equilibrium efforts, see (5). Substituting these efforts in the contestants' expected payoffs, see (4), we get:

(A2) $$E(u_1^*) = \frac{d(n_1+\varepsilon_1)(1-\alpha+d)}{(d+1)^2} \text{ and } E(u_2^*) = \frac{(n_2+\varepsilon_2)[(1-\alpha)d+1]}{(d+1)^2}$$

The second order equilibrium conditions (SOC) are:

$$\frac{\partial^2 E(u_1)}{\partial x_1^2} = \frac{\alpha(n_1+\varepsilon_1)(\delta x_2)^{2\alpha} x_1^{\alpha-2}\left[\alpha-1-(\alpha+1)\left(\frac{x_1/x_2}{\delta}\right)^{\alpha}\right]}{\left[x_1^{\alpha}+(\delta x_2)^{\alpha}\right]^3} \le 0$$

and

$$\frac{\partial^2 E(u_2)}{\partial x_2^2} = \frac{\alpha(n_2+\varepsilon_2)\delta^{2\alpha} x_1^{\alpha} x_2^{2\alpha-2}\left[(\alpha-1)\left(\frac{x_1/x_2}{\delta}\right)^{\alpha}-(\alpha+1)\right]}{\left[x_1^{\alpha}+(\delta x_2)^{\alpha}\right]^3} \le 0$$

In equilibrium, we obtain that $\frac{x_1^*}{x_2^*} = a$. Since $d = \left(\frac{a}{\delta}\right)^{\alpha}$, the SOC can be written as:

(A3) $$1-\alpha+(\alpha+1)d \ge 0 \text{ and } (1-\alpha)d+\alpha+1 \ge 0$$

Also, in equilibrium, the expected contestants' payoffs must be non-negative, that is, $E(u_1^*) \ge 0$ and $E(u_2^*) \ge 0$, which requires (see (A2)) that:

(A4) $$1-\alpha+d \ge 0 \text{ and } (1-\alpha)d+1 \ge 0$$

Note that the conditions in (A4), namely, constraints 1 and 2 in problem (10), ensure that the SOC in (A3) are also satisfied.

Q.E.D

Appendix B

Claim 1:

For $k>1$ and $0<\alpha<2$, dual discrimination yields larger efforts than those obtained just by differential prize taxation.

Proof: Resorting solely to differential prize taxation means that the designer chooses $(\varepsilon_1,\varepsilon_2)$, given that $\delta=1$. That is, in problem (10), $d=a^\alpha$. In this case, for $0<\alpha<2$ and $k>1$, $\varepsilon_1<0<\varepsilon_2$.[18] Denote by $(\varepsilon_1^E,\varepsilon_2^E)$ the equilibrium pair of $(\varepsilon_1,\varepsilon_2)$ and by d_E the corresponding d, $d_E=\left(\frac{n_1+\varepsilon_1^E}{n_2+\varepsilon_2^E}\right)^\alpha$.Since, in equilibrium, $n_1+\varepsilon_1^E>0$ and $\varepsilon_1^E<0$, we get that $-\frac{n_1}{\varepsilon_1^E}>1$ and, by the balanced-budget constraint (9), in equilibrium $d_E=-\frac{\varepsilon_2^E}{\varepsilon_1^E}$. Hence, by (6), in equilibrium total efforts are equal to:

$$\text{(B1)} \qquad G_E=\alpha\frac{d_E}{(d_E+1)^2}\left(n_1+\varepsilon_1^E+n_2+\varepsilon_2^E\right)$$

Let us show that allowing the designer to change δ relative to $\delta=1$ as well as $(\varepsilon_1,\varepsilon_2)$ increases the contestants' efforts. The idea of the proof is based on enabling changes in the contestants' stakes, such that their winning probabilities are preserved (*d* remains constant, see (7)), but the sum of their modified prize valuations is increased. This certainly increases their efforts as implied by (6). Consider then the following two changes made by the designer.

1. Multiplying the equilibrium $(\varepsilon_1^E,\varepsilon_2^E)$ by λ. The new mode of discrimination therefore becomes $(\lambda\varepsilon_1^E,\lambda\varepsilon_2^E)$, such that λ satisfies the requirement $1<\lambda<-\frac{n_1}{\varepsilon_1^E}$.
2. Setting a new δ (the structural discrimination parameter that will differ from 1), such that the balanced-budget constraint is satisfied.

[18] This has been established in the proof of Proposition 1 (Part 1) in Mealem and Nitzan (2012).

The reason for the requirement $\lambda < -\frac{n_1}{\varepsilon_1^E}$ in the first change is that the new stake of contestant 1 must be positive, otherwise he leaves the contest and the contestants' efforts are equal to zero. This requirement ensures therefore that the new prize valuations of the contestants $\left(n_1 + \lambda\varepsilon_1^E, n_2 + \lambda\varepsilon_2^E\right)$ are positive. The new value of a is $a_N = \frac{n_1 + \lambda\varepsilon_1^E}{n_2 + \lambda\varepsilon_2^E}$ and the corresponding new d is equal to the original one, $d_N = -\frac{\lambda\varepsilon_2^E}{\lambda\varepsilon_1^E} = -\frac{\varepsilon_2^E}{\varepsilon_1^E} = d_E$. This means that the winning probabilities of the contestants are unaltered. By the balanced-budget constraint, the feasibility of the first change requires that the new δ satisfies $d_N\lambda\varepsilon_1^E + \lambda\varepsilon_2^E = 0$. Since $d_N = \left(\frac{\frac{n_1 + \lambda\varepsilon_1^E}{n_2 + \lambda\varepsilon_2^E}}{\delta}\right)^\alpha$, δ must satisfy $\left(\frac{\frac{n_1 + \lambda\varepsilon_1^E}{n_2 + \lambda\varepsilon_2^E}}{\delta}\right)^\alpha \lambda\varepsilon_1^E + \lambda\varepsilon_2^E = 0$ or:[19]

(B2) $$\delta = \left(\frac{n_1 + \lambda\varepsilon_1^E}{n_2 + \lambda\varepsilon_2^E}\right)\left(-\frac{\varepsilon_1^E}{\varepsilon_2^E}\right)^{\frac{1}{\alpha}} (> 0)$$

Given that $d_N = d_E$, the new efforts are:

(B3) $$G_N = \alpha \frac{d_E}{(d_E + 1)^2}\left(n_1 + \lambda\varepsilon_1^E + n_2 + \lambda\varepsilon_2^E\right)$$

[19] This choice requires that $0 < \delta < 1$ because, by the balanced-budget constraint, in the original equilibrium $\left(\frac{n_1 + \varepsilon_1^E}{n_2 + \varepsilon_2^E}\right)\left(-\frac{\varepsilon_1^E}{\varepsilon_2^E}\right)^{\frac{1}{\alpha}} = 1$. Since $\varepsilon_1^E < 0 < \varepsilon_2^E$ and $1 < \lambda < -\frac{n_1}{\varepsilon_1^E}$, $\frac{n_1 + \varepsilon_1^E}{n_2 + \varepsilon_2^E} > \frac{n_1 + \lambda\varepsilon_1^E}{n_2 + \lambda\varepsilon_2^E}$ and therefore, $1 = \left(\frac{n_1 + \varepsilon_1^E}{n_2 + \varepsilon_2^E}\right)\left(-\frac{\varepsilon_1^E}{\varepsilon_2^E}\right)^{\frac{1}{\alpha}} > \left(\frac{n_1 + \lambda\varepsilon_1^E}{n_2 + \lambda\varepsilon_2^E}\right)\left(-\frac{\varepsilon_1^E}{\varepsilon_2^E}\right)^{\frac{1}{\alpha}} = \delta$.

Since $\lambda > 1$, we get that $\lambda\varepsilon_1^E + \lambda\varepsilon_2^E = \lambda\left(\varepsilon_1^E + \varepsilon_2^E\right) > \varepsilon_1^E + \varepsilon_2^E (> 0)$ and, therefore, the total efforts in (B3) are larger than the total efforts in (B1), $G_N > G_E$. This means that the selection of δ that differs from 1 together with the larger $(\varepsilon_1, \varepsilon_2)$, which is not necessarily the optimal selection, increases the total efforts of the contestants.

Q.E.D.

Proposition 1:[20]

1. For $k > 1$ and $0 < \alpha \le 1$, when $\varepsilon_1 \to -n_1^+$, $\varepsilon_2 \to \infty$ and δ is set according to (11), the winning probability of contestant 1 converges to 1, but his effort converges to zero and the winning probability of contestant 2 converges to zero, but his effort converges to αn_1. Total efforts therefore converge to αn_1, $E(u_1) \to 0$ and $E(u_2) \to (1-\alpha)n_1$.
2. For $k > 1$ and $1 < \alpha \le 2$, when $\varepsilon_1 \to -n_1^+$, $\varepsilon_2 = \dfrac{\varepsilon_1}{1-\alpha}$ and δ is set according to (11), the winning probability of contestant 1 is $\dfrac{1}{\alpha}$, but his effort converges to zero and the winning probability of contestant 2 is $\left(1-\dfrac{1}{\alpha}\right)$ and his effort converges to $\dfrac{1}{\alpha}n_1 + \left(1-\dfrac{1}{\alpha}\right)n_2$. Total efforts therefore converge to a value that is smaller than n_1 and $E(u_1) = E(u_2) \to 0$.

Proof:

Part 1. By the balanced-budget constraint (9), $d = -\dfrac{\varepsilon_2}{\varepsilon_1}$ and, therefore, when $0 < \alpha \le 1$, constraints 1 and 2 in problem (10) are always satisfied. Substituting $d = -\dfrac{\varepsilon_2}{\varepsilon_1}$ in (6) we get that $G = \dfrac{\alpha\left(-\dfrac{\varepsilon_2}{\varepsilon_1}\right)(n_1 + \varepsilon_1 + n_2 + \varepsilon_2)}{\left(-\dfrac{\varepsilon_2}{\varepsilon_1} + 1\right)^2}$. Multiplying the

[20] Note the proof is indirect not using the Kuhn-Tucker conditions. The reason is that the constraints in problem (10) imply that the feasible set of the control variables is not compact. In particular, note that constraints 5 and 6 have the form of strict inequalities. In addition, note that the objective function is not continuous at $\varepsilon_i = -n_i$. The standard Kuhn-Tucker conditions cannot therefore be used.

nominator and denominator of the above expression by $\left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2$ we get that

$G = -\frac{\alpha\varepsilon_1\left(\frac{n_1}{\varepsilon_2}+\frac{\varepsilon_1}{\varepsilon_2}+\frac{n_2}{\varepsilon_2}+1\right)}{\left(-1+\frac{\varepsilon_1}{\varepsilon_2}\right)^2}$. Now, for $0<\alpha\leq 1$, where $\varepsilon_1 \to -n_1^+$, $\varepsilon_2 \to \infty$ and δ

is determined according to (11), we get that

$$G \to \frac{\alpha n_1\left(\frac{n_1}{\varepsilon_2}-\frac{n_1}{\varepsilon_2}+\frac{n_2}{\varepsilon_2}+1\right)}{\left(-1-\frac{n_1}{\varepsilon_2}\right)^2} = \frac{\alpha n_1(0+1)}{(-1-0)^2} = \alpha n_1$$

Since $d \to \infty$, the winning probability of contestant 1 converges to 1, because

$$p_1 = \frac{d}{d+1} = \frac{1}{1+\frac{1}{d}} \to 1$$

so $p_2 \to 0$. By (5), the exerted effort of contestant 1 is:

$$x_1^* = \frac{\alpha d(n_1+\varepsilon_1)}{(d+1)^2} = \alpha(n_1+\varepsilon_1)\frac{d}{(d+1)}\frac{1}{(d+1)}$$

Since $\varepsilon_1 \to -n_1^+$, $d \to \infty$ and $p_1 = \frac{d}{d+1} \to 1$, $x_1^* \to 0$. The exerted effort of contestant 2 is equal to:

$$x_2^* = \frac{\alpha d(n_2+\varepsilon_2)}{(d+1)^2} = \alpha\left(\frac{n_2+\varepsilon_2}{d+1}\right)\frac{d}{(d+1)}$$

Substituting $d = -\frac{\varepsilon_2}{\varepsilon_1}$ in the second term of the above expression, we get that

$$x_2^* = \alpha\left(\frac{n_2+\varepsilon_2}{-\frac{\varepsilon_2}{\varepsilon_1}+1}\right)\frac{d}{(d+1)}$$

Multiplying the nominator and the denominator in the second term by $-\frac{\varepsilon_1}{\varepsilon_2}$, we get that

$$x_2^* = \alpha\left(\frac{-\frac{\varepsilon_1 n_2}{\varepsilon_2} - \varepsilon_1}{1 - \frac{\varepsilon_1}{\varepsilon_2}}\right)\frac{d}{(d+1)}$$

Since $\varepsilon_1 \to -n_1^+$, $\frac{d}{d+1} \to 1$ and $\varepsilon_2 \to \infty$, we get that $x_2^* \to \alpha n_1$,

$$x_2^* = \alpha\left(\frac{-\frac{\varepsilon_1 n_2}{\varepsilon_2} - \varepsilon_1}{1 - \frac{\varepsilon_1}{\varepsilon_2}}\right)\frac{d}{(d+1)} \to \alpha\left(\frac{\frac{n_1 n_2}{\infty} + n_1}{1 - \frac{n_1}{\infty}}\right)\cdot 1 = \alpha n_1$$

The utility of contestant 1 is $E(u_1) = p_1(n_1 + \varepsilon_1) - x_1$. Since $p_1 \to 1$, $n_1 + \varepsilon_1 \to 0^+$ (because $\varepsilon_1 \to -n_1^+$) and $x_1^* \to 0$ hence, $E(u_1) \to 0$. The utility of contestant 2 is $E(u_2) = p_2(n_2 + \varepsilon_2) - x_2$. By the balanced-budget constraint (9), $\varepsilon_2 = -d\varepsilon_1 \to dn_1$ and therefore $n_2 + \varepsilon_2 \to n_2 + dn_1$. Since $n_2 + \varepsilon_2 \to n_2 + dn_1$, $p_2 = \frac{1}{d+1}$ and $x_2^* \to \alpha n_1$, $E(u_2) \to \frac{1}{d+1}(n_2 + dn_1) - \alpha n_1$. When $d \to \infty$, we get that $E(u_2) \to (1-\alpha)n_1$.

Part 2. As already noted in the discussion before Proposition 1, for $1 < \alpha \leq 2$, maximal polarization requires that $\varepsilon_1 \to -n_1^+$ and $\varepsilon_2 \to \frac{n_1}{\alpha - 1}$, where δ is determined by (11). To find out the limit of the corresponding efforts, let us substitute in (6), $\varepsilon_1 = -n_1$, $\varepsilon_2 = \frac{n_1}{\alpha - 1}$ and $d = \frac{1}{\alpha - 1}$ to obtain:

$$G \to \frac{\alpha\frac{1}{\alpha-1}\left(n_1 - n_1 + n_2 + \frac{n_1}{\alpha - 1}\right)}{\left(\frac{1}{\alpha-1} + 1\right)^2} = \frac{1}{\alpha}n_1 + \left(1 - \frac{1}{\alpha}\right)n_2 < n_1$$

(it can be readily verified that the last inequality holds because, by assumption, $n_2 < n_1$). Since $d = \frac{1}{\alpha - 1}$, the winning probability of contestant 1 is $p_1 = \frac{d}{d+1} = \frac{1}{\alpha}$ so $p_2 = 1 - \frac{1}{\alpha}$. By (5), the exerted effort of contestant 1 is equal to $x_1^* = \frac{\alpha d(n_1 + \varepsilon_1)}{(d+1)^2}$. Since $\varepsilon_1 \to -n_1^+$, $x_1^* \to 0$. The exerted effort of contestant 2 is equal to

$x_2^* = \frac{\alpha d(n_2+\varepsilon_2)}{(d+1)^2}$. Substituting $d = \frac{1}{\alpha-1}$ and $\varepsilon_2 \to \frac{n_1}{\alpha-1}$, we get that $x_2^* \to \frac{1}{\alpha} n_1 + \left(1-\frac{1}{\alpha}\right) n_2$. Since $\varepsilon_1 \to -n_1^+$, $n_1+\varepsilon_1 \to 0^+$ and, therefore, $E(u_1) \to 0$.

The utility of contestant 2 is:

$$E(u_2) = p_2(n_2+\varepsilon_2) - x_2 \to \left(1-\frac{1}{\alpha}\right)\left(n_2+\frac{n_1}{\alpha-1}\right) - \left[\frac{1}{\alpha}n_1 + \left(1-\frac{1}{\alpha}\right)n_2\right] = 0$$

Q.E.D.

Proposition 2: For any $0 < \alpha \le 2$, the dual polarized discrimination strategies applied in Proposition 1 yield total efforts that converge to the lowest upper bound of the possible equilibrium efforts of the contestants.

Proof: Let us divide the proof to two parts dealing with $0 < \alpha \le 1$ and then with $1 < \alpha \le 2$.

Part 1. If $0 < \alpha \le 1$, then by part 1 of Proposition 1 the polarized discrimination strategy yields efforts that are equal to αn_1. We therefore have to show that the total efforts given by (6) do not exceed αn_1. That is, $\frac{\alpha d(n_1+\varepsilon_1+n_2+\varepsilon_2)}{(d+1)^2} \le \alpha n_1$ or, after some simplifications, $0 \le d^2 n_1 - d\varepsilon_2 + dn_1 - dn_2 + n_1 - d\varepsilon_1$. By the balanced-budget constraint, $\varepsilon_2 = -d\varepsilon_1$. Substituting this term (twice) in the last inequality and then adding and subtracting n_2, the inequality takes the form:

$$0 \le d^2 n_1 - d(-d\varepsilon_1) + dn_1 - dn_2 + n_1 - n_2 + n_2 + \varepsilon_2,$$

which, after simplification becomes:

$$0 \le d^2(n_1+\varepsilon_1) + (d+1)(n_1-n_2) + n_2 + \varepsilon_2$$

Since, $d > 0$, $n_1+\varepsilon_1 > 0$, $n_1 \ge n_2$ and $n_2+\varepsilon_2 > 0$, the above inequality holds.

Part 2. If $1 < \alpha \le 2$, then by part 2 of Proposition 1 the polarized discrimination strategy yields efforts that converge to $\frac{1}{\alpha}n_1 + \left(1-\frac{1}{\alpha}\right)n_2$. We therefore have to show that the total efforts given by (6) do not exceed this level. Let us first show that the contestants' equilibrium efforts do not exceed $\frac{dn_1 + n_2}{d+1}$. For that purpose, let us substitute the equilibrium efforts of (5) in (4), to obtain the equilibrium utilities:

$$E(u_1^*) = \frac{(n_1 + \varepsilon_1)d(1 - \alpha + d)}{(d+1)^2} \text{ and } E(u_2^*) = \frac{(n_2 + \varepsilon_2)[(1-\alpha)d + 1]}{(d+1)^2}$$

In equilibrium, the utility of a contestant is not negative so the sum of these utilities is not negative. That is,

$$E(u_1^*) + E(u_2^*) = \frac{(n_1 + \varepsilon_1)d(1 - \alpha + d)}{(d+1)^2} + \frac{(n_2 + \varepsilon_2)[(1-\alpha)d + 1]}{(d+1)^2} \geq 0$$

or, after some simplification,

$$dn_1 + n_2 + d\varepsilon_1 + \varepsilon_2 \geq \frac{\alpha d(n_1 + \varepsilon_1 + n_2 + \varepsilon_2)}{(d+1)}$$

Since, by the balanced-budget constraint $d\varepsilon_1 + \varepsilon_2 = 0$, the above inequality takes the form:

$$dn_1 + n_2 \geq \frac{\alpha d(n_1 + \varepsilon_1 + n_2 + \varepsilon_2)}{(d+1)}$$

or, dividing both sides of the inequality by $(d+1)$,

$$\frac{dn_1 + n_2}{d+1} \geq \frac{\alpha d(n_1 + \varepsilon_1 + n_2 + \varepsilon_2)}{(d+1)^2}$$

To complete the proof, let us show that

$$\frac{1}{\alpha} n_1 + \left(1 - \frac{1}{\alpha}\right) n_2 \geq \frac{dn_1 + n_2}{d+1}$$

or

$$(d+1)n_1 + (\alpha - 1)(d+1)n_2 \geq \alpha d n_1 + \alpha n_2$$

or

$$[(1-\alpha)d + 1]n_1 \geq [(1-\alpha)d + 1]n_2$$

Since the utility of contestant 2 is not negative, by constraint (2) in Problem (10), $(1-\alpha)d + 1 \geq 0$. Therefore, if $(1-\alpha)d + 1 = 0$, the latter condition is satisfied as equality and if $(1-\alpha)d + 1 > 0$, the latter condition takes the form $n_1 \geq n_2$, which is also satisfied.

Q.E.D.

Proposition 3: For any $0 < \alpha < 2$, the optimal dual polarized discrimination strategy in our model yields total efforts that are larger than $0.25\alpha(n_1 + n_2)$ - the maximal efforts obtained under the optimal dual discrimination strategy based on the alternative simpler form of differential prize taxation. When $\alpha = 2$, the two models yield the same total efforts $0.5(n_1 + n_2)$.

Proof: Let us prove that the optimal dual discrimination strategy under the simpler mode of differential prize taxation yields efforts that are equal to $0.25\alpha(n_1 + n_2)$. This will complete the proof since, for $0 < \alpha \le 1$, $0.25\alpha(n_1 + n_2) < \alpha n_1$ and for $0 < \alpha < 2$, $0.25\alpha(n_1 + n_2) < \frac{1}{\alpha} n_1 + \left(1 - \frac{1}{\alpha}\right) n_2$.

Suppose that given a CSF of the logit form (1) where $0 < \alpha \le 2$, the designer can apply the two modes of discrimination, that is, select δ, β_1 and β_2. In this case the two contestants maximize their expected payoffs:

$$E(u_1) = \frac{x_1^\alpha}{x_1^\alpha + (\delta x_2)^\alpha}(n_1 - \beta_1) + \frac{(\delta x_2)^\alpha}{x_1^\alpha + (\delta x_2)^\alpha}\beta_2 - x_1$$

and

$$E(u_2) = \frac{(\delta x_2)^\alpha}{x_1^\alpha + (\delta x_2)^\alpha}(n_2 - \beta_2) + \frac{x_1^\alpha}{x_1^\alpha + (\delta x_2)^\alpha}\beta_1 - x_2$$

Let $b = \frac{n_1 - \beta_1 - \beta_2}{n_2 - \beta_1 - \beta_2}$ and $e = \left(\frac{b}{\delta}\right)^\alpha$. By the first order conditions,

$$x_1^* = \frac{\alpha e(n_1 - \beta_1 - \beta_2)}{(e+1)^2} \text{ and } x_2^* = \frac{\alpha e(n_2 - \beta_1 - \beta_2)}{(e+1)^2}$$

Hence, in equilibrium, $n_2 > \beta_1 + \beta_2$, because otherwise total efforts converge to zero (if this inequality is not satisfied, contestant 2 does not take part in the contest and contest 1 exerts a negligible effort, which ensures his winning). Total efforts are therefore equal to

$$G = x_1^* + x_2^* = \frac{\alpha e(n_1 + n_2 - 2\beta_1 - 2\beta_2)}{(e+1)^2} = \frac{e}{(e+1)^2}\alpha(n_1 + n_2 - 2\beta_1 - 2\beta_2)$$

Note that $\alpha(n_1 + n_2 - 2\beta_1 - 2\beta_2)$ is maximal and equal to $\alpha(n_1 + n_2)$ when $\beta_1 = \beta_2 = 0$, independent of $\frac{e}{(e+1)^2}$. This latter term is maximal and equal to 0.25 when $e = 1$. The maximization of $\frac{e}{(e+1)^2}\alpha(n_1 + n_2 - 2\beta_1 - 2\beta_2)$ is therefore obtained when $\beta_1 = \beta_2 = 0$ and $\delta = k$ (which ensures that $e = 1$) because the second order conditions for maximization are satisfied and the utility of every contestant in not negative. The maximal total efforts are therefore equal to $0.25\alpha(n_1 + n_2)$.

Q.E.D.