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Abstract

We propose a stylized monopolistic competition model of international trade where firms differ with respect to the expected economic lifetime of their innovations. Upon entry, they receive a commonly observed signal which is updated over time. Jointly with partial irreversibility of investment, this generates heterogeneity in effective discount rates and, thus, in the cost of finance. In line with evidence, the model predicts a negative correlation between firms' financing costs and their age. Over a firm's life-cycle, per period net profits and the export participation probability grow. Exporters are less likely to exit than purely domestic firms. Belief updating entails excessive financing of incumbents relative to entrants and too much exporting. Asymptotically, trade liberalization reduces overall general equilibrium exit rates, but it does not necessarily increase welfare. With multiple asymmetric export markets, firms gradually expand their market coverage and total sales. A confidence crisis modeled by belief reversion causes an over-proportional decrease in exports, thereby offering a novel interpretation of the recent trade slump.

JEL-Code: F120, F360.

Keywords: international trade, monopolistic competition, heterogeneous firms, heterogeneous fixed costs, Bayesian updating.

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1 Introduction

Recent theoretical work pioneered by Melitz (2003) has shed light on the role of productivity heterogeneity for the effect of international trade on firm behavior and aggregate outcomes. Given the presence of fixed costs, only more productive firms sort into exporting, and a reduction of trade costs increases aggregate productivity. Similar selection effects can be derived from firm-level differences in perceived product quality (Baldwin & Harrigan, 2011) or the degree of tradability of output (Bergin & Glick, 2009). The core prediction of these models, namely that more competitive firms are more likely to be exporters, enjoys massive empirical support (Bernard et al., 2007). A smaller strand of theoretical work introduces heterogeneity regarding fixed market access costs into the Krugman (1980) framework while keeping marginal revenues constant across firms (Schmitt & Yu, 2001; Jorgenson & Schröder, 2008).

Unrecognized in the recent trade literature, firms also differ with respect to their exit probabilities, at least as *perceived* by financial markets.[1] Ashcraft & Santos (2009) study data on credit default swaps and document a remarkable degree of heterogeneity amongst firms with respect to their perceived risk of business discontinuation. The Melitz (2003) model does not capture this stylized fact, since at each period, all firms are equally likely to be hit by a death shock. Plant death is important for aggregate statistics: Bernard and Jensen (2007) show that plant deaths account for more than half of gross job destruction in U.S. manufacturing. In our model, we continue to view business discontinuation as a discrete exogenous shock.[2] But we allow firms to differ with respect to the probability of such death shocks. Upon innovating a new product, firms trigger uncertain, publicly observable signals about the viability of their new product (i.e., their type), yielding beliefs that are correct in expectation and that are updated according to Bayes' law in case of firm survival. In the

[1] Pflüger and Russek (2011) are the only exception known to us: they use a two-sector Melitz (2003) model where exit probabilities are assumed to be inversely related to firm-level productivity.

[2] We are silent about the exact source of the shock. It may due to the the sudden disappearance of demand due to the emergence of a cheaper perfect substitute of the firm's variety or due to a technology shock causing the immediate depreciation of the firm's assets.

presence of partial irreversibility of investment,this assumption implies firm-level differences with respect to their cost of finance.[3]

As in Melitz (2003), in our framework, firms are identical *ex ante*. The financial markets are risk neutral and perfectly competitive. However, the 'true' life expectancy of a firm is unknown to all agents (i.e., to producers, financial markets, consumers). At the beginning of each period, producers must invest a fixed cost which cannot be recovered at any stage and which depreciates at the end of the period. Assuming, without loss of generality, that funds are available at a zero baseline interest rate, a firm's effective financing cost is equal to its per-period exit probability. If a firm survives, at the end of the period, market participants update their believed exit rates downwards. So, as time elapses, the funding of fixed costs activities (such as exporting) becomes gradually cheaper.

Firms' marginal revenues remain constant over time, so that the model enjoys the tractability of Schmitt & Yu (2001). However, despite its simplicity, the setup generates additional insights that are not available in the Melitz (2003) framework. As only firms with sufficiently low exit hazards enter foreign markets, exporters are on average longer-lived than domestic firms. Trade liberalization allows those formerly domestic firms with lowest effective interest rates to take up exporting while domestic firms, facing high interest rates, are forced to exit. So, trade liberalization lowers the expected average survival time of exporters but increases that of domestic firms. Due to a composition effect, in the overall economy, expected average survival increases. Hence, liberalization leads to higher ex post stability of firms, but effects differ between exporters and domestic firms.

The model also yields insights about firm and firm-generation dynamics. Recent literature studies the dynamic behavior of firms in open economies. The common objective is to explain the obvious stylized fact that firms are not typically born as exporters but evolve into exporting, and possibly out of it, over time. Dynamics may arise from the evo-

[3] Impullitti, Irarrazabal & Oppromolla (2012) use a Melitz (2003) model with a stochastic evolution of productivity and irreversibility of investment. They provide a rich discussion of the empirical importance of sunk costs in trade related applications.

lution of firm types. Impulliti, Irarrazabal & Oppromolla (2011) work with productivity shocks and irreversible investment in an otherwise standard Melitz (2003) model. Fajgelbaum (2011) stresses labor market frictions. Burstein & Melitz (2011) analyze the role of innovation. Alternatively, dynamics may also arise from learning about foreign markets or foreign customers. Nguyen (2012) studies the role of uncertainty about foreign market demand; Albornoz et al. (2012) offer a model of sequential exporting where firms gradually learn about foreign market profitability; Araujo et al. (2012) investigate the build up of trust between a producer and the foreign client in the absence of complete contracts.[4] In our model, uncertainty concerns the type of the producer or, equivalently, characteristics of the product, the 'true' economic life expectancy of a firm or product being unknown to all market participants. Dynamics are driven by two very simple mechanisms; the cleansing mechanism: *inferior firms are more likely to default*, and the updating mechanism: *trust in firms increases in firm age.*

The cleansing mechanism yields firm generation dynamics. As firms with high exit probability default more likely, the type distribution of firm generations evolves over time. Average exit probabilities of firm generations decrease with respect to their age, yielding decreasing average discount rates, increasing average net profits and an increasing fraction of exporters. The updating mechanism is driven by type uncertainty and the resulting Bayesian updating, yielding similar firm specific dynamics as the cleansing mechanism implies for firm generations. The older a firm, the lower the discount rate it is being assigned, yielding lower costs of finance, increasing net profits and increasing probability of exporting. Besides, as firms anticipate these life cycle patterns, there are some firms that enter the domestic market realizing negative profits initially.[5] In contrast, on the export market such early entries do not occur as active firms can wait until belief updating pushes their discount rate below the threshold ensuring positive profits.

[4] Aeberhardt et al. (2011) also study learning in the context of contract incompleteness.

[5] Belief updating requires that the firm is active, i.e., producing, and therefore observed by market participants.

Even though belief updating is rational on the individual level of the firm, the joint analysis of the cleansing and updating mechanisms reveals that updating leads to misvaluation of firm generation averages. While the evolution of true average exit probabilities is solely driven by the cleansing mechanism, the evolution of perceived average exit probabilities is driven by the cleansing *and* the updating mechanism. Thus, the older a firm generation gets, the further perceived and true magnitudes drift apart. Average discount rates of incumbents are inefficiently small, yielding excessive financing of incumbents relative to entrants (innovators). As incumbents and entrants compete for workforce, this yields insufficient entry of new firms. A corollary of this is that belief updating implies excessive exporting: If a firm enters the export market by a misjudgment of its type, it will, in expectation, default before accumulated profits balance exporting fixed costs, yielding a negative welfare effect.

The predictions of our model are consistent with a number of empirical stylized facts: (i) firm survival and export status are positively correlated (Greenaway et al., 2009), the link between the two running through access to finance (Goerg and Spaliara, 2009); (ii) over longer horizons of time, about 40% of total export growth occurs at the extensive margin (Bernard and Jensen, 2004); (iii) over time, firms gradually expand the number of export markets that they serve (Lawless, 2009); (iv) export activities are heavily persistent due to the existence of sunk costs (Das et al., 2007).

We use the model to study a crisis of confidence, in which market participants revise their beliefs, i.e., they delete a portion of the updating history. Since type beliefs of exporters are on average farther away from true types, this revision leads to a stronger decline in exports and, by trade balance, of imports relative to domestic sales. Credit conditions of large old firms (exporters) deteriorate more strongly than of small young ones. These observations are in line with the effect of the Lehman Brothers crash on September 15, 2008. This shock led to a tightening of credit restrictions, in particular of large firms, and to a collapse of trade. The differential effect on small relative to large firms is in line with firm-level evidence from

Germany, Italy, and France.[6]

The remainder of this paper is structured as follows. Section 2 describes the basic framework; Section 3 derives our core results under the simplifying assumption that firms' expected life times are known with certainty after entry; Section 4 extends the analysis to the more realistic case of uncertain default probabilities; Section 5 concludes.

2 Setup

Households: We consider $n+1$ symmetric countries. We relax symmetry in Section 4.3 below. Each country is populated by a representative household of size L, who supplies labor inelastically, and who cares about the quantity of a final good C according to a linear utility function. Hence, per capita utility is $u = C/L$.

Production: In each country, there is a mass M of monopolistically competitive producers of differentiated intermediate inputs, indexed by ω. These inputs are assembled by a perfectly competitive final goods sector into the final good Y according to the CES production function:

$$Y = \Big(\int q(\omega)^{\rho} d\omega \Big)^{1/\rho} = C + I, \rho \in (0,1). \tag{1}$$

The final good $Y = C + I$ can be either consumed by households or used as investment by firms. While the final good is freely tradable, differentiated inputs are subject to standard iceberg trade costs $\tau \geq 1$. Standard manipulation yields optimal input demands of final goods producers and associated expenditures:

$$q(\omega) = Q\Big(\frac{p(\omega)}{P}\Big)^{-\sigma} \text{ and } r(\omega) = R\Big(\frac{p(\omega)}{P}\Big)^{1-\sigma}, \tag{2}$$

[6] Our model is too stylized to be used for a full quantitative analysis of the crisis. Rather, we wish to highlight a novel theoretical mechanism that may have played a role along more standard determinants such as the strong decline in demand.

with $\sigma = 1/(1-\rho) > 1$ and Dixit Stiglitz aggregates P is the associated price of the final output good, normalized to unity by choice of numeraire, Q is an ideal quantity index, and $R = PQ = Y$. Input goods are produced via a one-to-one technology, $q = l$, with labor l being the only factor of production. As firms do not differ in productivity they charge identical prices, p_d on the domestic and p_x on the export markets:

$$p_d = \frac{w}{\rho}, \text{ and } p_x = \tau p_d, \tag{3}$$

where w denotes the wage rate. Thus, domestic per period *operating* profits and revenues are identical for all firms and are given by:

$$\pi_d = (p_d - w)q_d = \left(\frac{wq_d}{\sigma - 1}\right), \text{ and } r_d = p_d q_d = \sigma \pi_d, \tag{4}$$

with analogous expressions for exporters.

Heterogeneity: Firm heterogeneity is introduced via firm specific per period exit probabilities $\delta \in [0,1]$ distributed with pdf $g(\delta)$ and cdf $G(\delta)$. In Chapter 3 we assume that start-up investments reveal true types δ of firms, thereby deactivating the updating mechanism and isolating the dynamics generated by the cleansing mechanism. Then, from Chapter 4 onwards, we drop this assumption, introducing *perceived* types $\hat{\delta}$ that evolve according to Bayes' law and analyze the full dynamics triggered by the cleansing and the updating mechanism.

Financial Market: We consider a risk neutral, perfectly competitive financial market and normalize the interest rate required by households to zero. Thus, in case of revealed types, a firm δ is charged a per period rate of $\delta\alpha$ for a loan with nominal α, yielding zero expected profits for creditors.[7] Analogously, in case of type uncertainty, a firm of perceived

[7] Here we restrict our analysis to sunk fixed costs, that can not be recovered subsequent to firm default.

type $\hat{\delta}$ is charged $\hat{\delta}\alpha$.

Timing: We analyze an infinitely repeated game of symmetric information. Each period $t \in \mathbb{N}$ consists of three stages: $s = 1$: Inactive firms may turn active by sinking f_I units of the final output good into research and development. This effort yields a new variety of the differentiated input for sure, but the viability of the innovation δ is drawn from $g(\delta)$ and differs across firms. The market receives signals that reveal true firm types δ (Chapter 3), or that yield certain beliefs of firm types $\hat{\delta}$ (Chapter 4). $s = 2$: Active firms consider to either turn inactive or to sell on the domestic market (which requires fixed domestic market access of f_d), or to additionally engage in exporting (which requires fixed export market access costs of f_x). Both f_d and f_x are measured in units of the final output good. $s = 3$: Active firms may be forced to exit the market by idiosyncratic shocks, that arrive according to their per period exit probability δ, and turn inactive. Survivors remain active, generate profits and conduct loan rate repayments. In case of type uncertainty (Chapter 4), beliefs are updated contingent on firm survival.

Aggregation: A long-run equilibrium is characterized by a mass M and a type distribution $h(\delta)$ of active firms and a mass M_x and a type distribution $h_x(\delta)$ of exporters in every country. As all active firms charge the same domestic price p_d and all exporters charge the same price p_x for their exports:

$$1 = P = \Big(\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \Big)^{1/(1-\sigma)} = \Big(\int_0^1 p_d^{1-\sigma} M h(\delta) d\delta + n \int_0^1 p_x^{1-\sigma} M_x h_x(\delta) d\delta \Big)^{1/(1-\sigma)}$$

$$= (M p_d^{1-\sigma} + n M_x p_x^{1-\sigma})^{1/(1-\sigma)}, \tag{5}$$

by choice of numeraire. Analogously we get $Q = (M q_d^{\rho} + n M_x q_x^{\rho})^{1/\rho}$ and $R = M r_d + n M_x r_x$.

One could additionally introduce a component that is not sunk. As additional insights are small – if more units of final good are needed for investment, aggregate consumption decreases, but idiosyncratic interest rates of firms are not affected – we simply assume sunkness of fixed costs for the purpose of technical simplicity.

3 The Cleansing Mechanism

In this section we focus on the *cleansing* mechanism and its impact on firm generation dynamics. The *updating* mechanism is switched off by assuming perfect observability of firm types. Additionally, we assume $g(0) = 0$, i.e. no firm shall be able to survive all possible shocks. We denote expected values with respect to a certain distribution χ by $E_\chi(\cdot)$ and impose the technical assumption $E_g(1/\delta) \in (1, \infty)$.[8]

Zero Cut-Off Profit Conditions: Market access costs f_d and f_x are modeled as flow fixed costs which occur at the beginning of each period and which are sunk. So, in case of firm default they are lost and in case of firm survival firms repay them at the end of the period, and apply for new loans at the beginning of the next period. As the financial market is risk neutral and perfectly competitive an active firm of type δ faces per period loan rates of $\delta P f_d = \delta f_d$, plus $n\delta f_x$ in case of exporting. Thus, domestic entry occurs only if per period operating profits π_d dominate per period loan rates δf_d, yielding $\pi_d = \delta_d^* f_d$, with δ_d^* denoting the domestic cut-off type. Analogously we get $\pi_x = \delta_x^* f_x$, with the exporting cut-off type δ_x^*. As per period operating profits earned at each market do not depend on firm type we have:

$$\pi_d = \delta_d^* f_d \quad \text{and} \quad \pi_x = \delta_x^* f_x, \tag{6}$$

for all firms. Importantly, per period *net* profits do depend on firm types as loan rate repayments δf_d for domestic market entry and δf_x for foreign market entry are type-dependent. Thus, a firm of type $\delta \leq \delta_d^*$ realizes per period net profits of:

$$\pi^n(\delta) = \begin{cases} \pi_d^n(\delta) = \pi_d - \delta f_d = (\delta_d^* - \delta) f_d & \text{if } \delta \in (\delta_x^*, \delta_d^*], \\ \pi_d^n(\delta) + n\pi_x^n(\delta) = (\delta_d^* - \delta) f_d + n(\delta_x^* - \delta) f_x & \text{if } \delta \in (0, \delta_x^*]. \end{cases} \tag{7}$$

[8] The restriction $E_g(1/\delta) < \infty$ is equivalent to requiring that the density $g(\delta)$ converges faster than linearly towards zero as its argument δ converges against the boundary $\delta \to 0$. The restriction $E_g(1/\delta) > 1$ precludes convergence towards the degenerate density that assigns all probability to the outcome $\delta = 1$.

Dividing domestic and exporting per period profits and applying (2) and (3), we get a one-to-one correspondence between cut-off types δ_x^* and δ_d^*:

$$\frac{\delta_x^* f_x}{\delta_d^* f_d} = \frac{\pi_x}{\pi_d} = \tau^{1-\sigma} \Rightarrow \delta_x^* = \tau^{1-\sigma}\frac{f_d}{f_x}\delta_d^*. \tag{8}$$

To ensure that all active firms serve their domestic market and only a subset of domestically active firms engages in exporting, we assume $f_x \geq f_d$.[9]

Free Entry Condition: As firm types are unobservable ex ante, firms are not able to offer banks the repayment of a fixed nominal in order to be granted the loan needed for carrying out the start-up investment f_I. If, for example, the firm turns out to be of the domestic cut-off type δ_d^*, it will realize zero per period net profits and hence will not be able to deduct any positive rate payments. Therefore, firms offer the repayment of a type dependent nominal $\alpha(\delta)$ that has to be less than their expected total net profits $\alpha(\delta) \leq \Sigma_{t=0}^{\infty}(1-\delta)^t \pi^n(\delta) = \pi^n(\delta)/\delta$. Banks accept only if they do not incur losses in expectation. As start-up investment costs f_I are sunk and as only a fraction $G(\delta_d^*)$ of new firms is able to enter the market, this yields $E_g(\alpha(\delta)|\delta \leq \delta_d^*) \geq f_I/G(\delta_d^*)$. As banks face perfect competition, this inequality is binding. Free entry of firms drives down profits until nominal and expected total net profits coincide $\alpha(\delta) = \pi^n(\delta)/\delta$, leaving firms with with zero profits and yielding:

$$E_g(\pi^n(\delta)/\delta|\delta \leq \delta_d^*) = f_I/G(\delta_d^*). \tag{9}$$

In the Appendix we prove that cut-off values δ_d^* and δ_x^* exist and are uniquely determined by (7), (8) and (9). Moreover, we also prove the following Proposition:

Proposition 1 (Trade Liberalization and Firm Churning) *A reduction in variable trade costs τ lowers δ_d^* but increases δ_x^*. Trade liberalization yields lower average firm churning, while churning of exporters increases.*

[9] A similar condition ensure the empirically relevant sorting pattern in the Melitz (2003) model.

Long Run Distribution of Active Firms: In expectation, low-δ-firms drop out from the market later than high-δ-firms. Thus, the long-run distribution of active firms $h(\delta)$ differs from the distribution of entering firms $g(\delta)$. Every period a certain measure M_e of g-distributed firms tries to enter the market (henceforth denoted as firm generation), yielding a certain measure $M_e g(\delta)$ of entrants per type δ. Let $i(\delta)$ denote the measure of incumbents of type δ, then firms of type δ accumulate until the measure of entrants $M_e g(\delta)$ coincides with the measure of defaulting firms $\delta i(\delta)$, yielding $i(\delta) = M_e g(\delta)/\delta$. Thus, the type distribution of active firms is given by

$$h(\delta) = \begin{cases} \frac{g(\delta)/\delta}{\int_0^{\delta_d^*} g(\delta)/\delta \, d\delta} & \text{if } \delta \in (0, \delta_d^*], \\ 0 & \text{otherwise.} \end{cases} \tag{10}$$

Correspondingly the type distribution of exporters follows $h(\delta|\delta \leq \delta_x^*)$. As $h(\delta)$ shifts mass towards low values of δ, average turnover of firms entering the market $E_g(\delta|\delta \leq \delta_d^*)$ is higher than average market turnover $E_h(\delta)$. Summarizing, we obtain the next proposition.

Proposition 2 (Cleansing Mechanism) *The older a firm generation, the lower its average exit probability.*

As loan rates, size of net profits and entry into exporting are determined by firms exit probabilities, we can directly infer

Proposition 3 (Firm Generation Effects) *The older a firm generation, the lower its average loan rate, the higher its average net profit and the higher the fraction of exporters among its members.*

With $\delta_d^*, \delta_x^*, h(\delta)$ characterized, now we close the model by determining firm masses and per period consumption.

Firm Masses: In steady state, firm entry balances firm exit, yielding $M_e = E_h(\delta)M/G(\delta_d^*)$. Using labor market clearing $L = Mq_d + nM_x\tau q_x$ and the relative mass of exporting firms $M_x = H(\delta_x^*)M$, with $H(\delta)$ denoting the cumulative density function of the long-run distribution of firm types $h(\delta)$, we get:[10]

$$M = wL/[(\sigma-1)(f_d\delta_d^* + f_x n H(\delta_x^*)\delta_x^*)], \tag{11}$$

which is a first relation linking the two remaining unknown endogenous variables M, w.

Consumption: We can determine the equilibrium wage rate w from $P = 1$, obtaining aggregate per period consumption:

$$C = Lw/P = Lw = L\rho(M + nH(\delta_x^*)M\tau^{1-\sigma})^{1/(\sigma-1)}, \tag{12}$$

a second relation linking M, w. As utility is linear in consumption, (12) constitutes a measure of welfare. A detailed derivation of (12) is provided in the Appendix. From the measure of entering firms and the fixed costs they have to bear, we can directly determine the quantity of the final product spent for start-up investments and market entries every period:

$$I = (f_I + f_d G(\delta_d^*) + nf_x G(\delta_x^*))M_e. \tag{13}$$

From (2) and (3) we get $\tau p_x = \tau^{1-\sigma} q_d < q_d$. Thus, trade liberalization increases the number of available varieties in every country. Moreover, trade liberalization increases average productivity: Proposition 1 establishes that trade liberalization forces firms with low net profits out of the market ($\hat{\delta}_d^* \downarrow$) shifting production towards more efficient firms. As per period net profits constitute the difference of per period profits (that are independent of firm type) and per period fixed costs (that decrease in length of firm life), trade liberalization raises $Y - I = C$ and we get:

Proposition 4 (Trade Liberalization and Welfare) *Trade liberalization increases wel-*

[10] See the Appendix for a step-by-step derivation.

fare.

4 Uncertain Firm Types and Updating

In this section, we discuss variations and applications of our simple baseline model from above. First, we introduce type uncertainty, leaving everything else unchanged (subsection 4.1), then we discuss consequences of a confidence crisis (subsection 4.2) and conclude with the analysis of the asymmetric country case (subsection 4.3). Henceforth start-up investments trigger uncertain signals, yielding perceived types $\hat{\delta}_0 \in [0, 1]$. When referring to the cross section of firms we drop the age indicating subscript and denote perceived types with $\hat{\delta}$. Perceived types $\hat{\delta}$ constitute expected values of their corresponding belief $\delta \sim b_{\hat{\delta}}(\delta)$, i.e. $E_{b_{\hat{\delta}}}(\delta) = \hat{\delta}$. Initial perceived types $\hat{\delta}_0$ are correct in expectation. Thus, perceived and true types are both distributed with the true type pdf g introduced in Chapter 3 initially. Again, we impose the technical assumption $E_{b_{\hat{\delta}}}(1/\delta) \in (1, \infty)$ for all $\hat{\delta}$.[11] Turning to the perceived type evolution of individual firms, we can frame a very simple mechanism. Every period a firm survives, its perceived type is being updated according to Bayes' law until it is hit by a shock and forced to exit the market. As updating is only triggered by good news (firm survival), we get $\hat{\delta}_0 > \hat{\delta}_1 > \cdots > \hat{\delta}_t > \ldots$ for all periods a firm survives, with $\hat{\delta}_t$ denoting its perceived type in its t^{th} period subsequent foundation.

Proposition 5 (Updating Mechanism) *The older a firm, the lower its perceived exit probability.*

4.1 Symmetric Countries

Except from the type uncertainty introduced above, the setup from Chapter 3 remains unchanged.

[11] The restriction $E_{b_{\hat{\delta}}}(1/\delta) < \infty$ is equivalent to requiring that the density $b_{\hat{\delta}}(\delta)$ converges faster than linearly towards zero as its argument δ converges against the boundary $\delta \to 0$. The restriction $E_{b_{\hat{\delta}}}(1/\delta) > 1$ precludes convergence towards the degenerate density that assigns all probability to the outcome $\delta = 1$.

Zero Cut-Off Profit Conditions: As loans for market access costs are negotiated on a per period basis, firms face rate payments $\hat{\delta}_t f_d$ (plus $n\hat{\delta}_t f_x$ in case of exporting) that always reflect current firm status $\hat{\delta}_t$. Thus, the older a firm the lower its rate payments. As firms anticipate this life cycle pattern, the entry decision arises from comparing present value of expected future profits with present value of expected future costs. Hence, some firms enter even though they are facing negative per period net profits initially. Consider a firm with initial perceived type $\hat{\delta}_0$, then present value of expected future profits from domestic activity equals $E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\pi(\hat{\delta}_t)) = E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\pi_d + \Sigma_{t=t(\hat{\delta}_0)}^{\infty}(1-\delta)^t n\pi_x) = E_{b_{\hat{\delta}_0}}(1/\delta)\pi_d + E_{b_{\hat{\delta}_0}}((1-\delta)^{t(\hat{\delta}_0)}/\delta)n\pi_x$, with $t(\hat{\delta}_0)$ denoting the period of entry into exporting in case of survival. Let $\psi(\hat{\delta}_0)$ denote the weighted probability of survival until entry into exporting. It is defined by the condition satisfying $E_{b_{\hat{\delta}_0}}(\psi(\hat{\delta}_0)/\delta) = E_{b_{\hat{\delta}_0}}((1-\delta)^{t(\hat{\delta}_0)}/\delta)$. Then, the present value of expected future profits can be rewritten as $E_{b_{\hat{\delta}_0}}(1/\delta)(\pi_d + \psi(\hat{\delta}_0)n\pi_x)$. The present value of expected future costs from domestic entry equals $E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\hat{\delta}_t f_d + \Sigma_{t=t(\hat{\delta}_0)}^{\infty}(1-\delta)^t n\hat{\delta}_t f_x) = E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\bar{\delta}(\hat{\delta}_0)f_d + \Sigma_{t=t(\hat{\delta}_0)}^{\infty}(1-\delta)^t n\bar{\delta}(\hat{\delta}_{t(\hat{\delta}_0)})f_x) = E_{b_{\hat{\delta}_0}}(1/\delta)(\bar{\delta}(\hat{\delta}_0)f_d + \psi(\hat{\delta}_0)n\bar{\delta}(\hat{\delta}_{t(\hat{\delta}_0)})f_x)$, with $\bar{\delta}(\hat{\delta})$ denoting the expected future average perceived type of a firm with perceived type $\hat{\delta}$.[12] For the cut-off value $\hat{\delta}_d^*$, present value of expected future profits and present value of expected future costs coincide, yielding $\pi_d + \psi(\hat{\delta}_d^*)n\pi_x = \bar{\delta}(\hat{\delta}_d^*)f_d + \psi(\hat{\delta}_d^*)n\bar{\delta}(\hat{\delta}_{t(\hat{\delta}_d^*)})f_x$. Differently, in case of exporting, firms wait until their perceived type is low enough to realize positive per period net profits from exporting. As domestic and exporting per period operating profits do not depend on firm type we get:

$$\pi_d = \bar{\delta}(\hat{\delta}_d^*)f_d - \psi(\hat{\delta}_d^*)(\hat{\delta}_x^* - \bar{\delta}(\hat{\delta}_{t(\hat{\delta}_d^*)}))nf_x \quad \text{and} \quad \pi_x = \hat{\delta}_x^* f_x. \tag{14}$$

Even if the probability of exporting was zero $\psi(\hat{\delta}_d^*) = 0$, firms $\hat{\delta}_0 \in (\bar{\delta}(\hat{\delta}_d^*), \hat{\delta}_d^*]$ would realize

[12] As $\hat{\delta}_t$ decreases monotonically in t, the expected amount of cleared entry costs $E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\hat{\delta}_t f_d) < E_{b_{\hat{\delta}_0}}(1/\delta)\hat{\delta}_0 f_d < \infty$ is finite by assumption $E_{b_{\hat{\delta}}}(1/\delta) \in (1,\infty)$. Thus, there exists a unique $\bar{\delta}(\hat{\delta}_0) \in (0, \hat{\delta}_0)$ fulfilling $E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\hat{\delta}_t f_d) = E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\bar{\delta}(\hat{\delta}_0)f_d)$. Existence and uniqueness of $\bar{\delta}(\hat{\delta}_{t(\hat{\delta}_0)})$ holds analogously.

negative net profits $(\bar{\delta}(\hat{\delta}_d^*) - \hat{\delta}_0) f_d < 0$ initially, speculating on positive net profits $(\bar{\delta}(\hat{\delta}_d^*) - \hat{\delta}_t) f_d > 0$ in future periods. The prospect of positive exporting profits lowers initial profits even further. A firm of age t and perceived type $\hat{\delta}_t$ realizes a per period net profit of:

$$\pi^n(\hat{\delta}_t) = \begin{cases} \pi_d^n(\hat{\delta}_t) = \pi_d - \hat{\delta}_t f_d & \text{if } \hat{\delta}_t \in (\hat{\delta}_x^*, \hat{\delta}_d^*], \\ \pi_d^n(\hat{\delta}_t) + n\pi_x^n(\hat{\delta}_t) = \pi_d - \hat{\delta}_t f_d + n(\pi_x - \hat{\delta}_t f_x) & \text{if } \hat{\delta}_t \in (0, \hat{\delta}_x^*]. \end{cases} \tag{15}$$

Dividing domestic and exporting per period operating profits and applying (2) and (3), we get a one-to-one correspondence between $\hat{\delta}_x^*$ and $\hat{\delta}_d^*$:

$$\begin{aligned} \frac{\hat{\delta}_x^* f_x}{\bar{\delta}(\hat{\delta}_d^*) f_d - \psi(\hat{\delta}_d^*)(\hat{\delta}_x^* - \bar{\delta}(\hat{\delta}_{t(\hat{\delta}_d^*)}))n f_x} &= \frac{\pi_x}{\pi_d} = \frac{q_x}{q_d}\left(\frac{p_x - \tau w}{p_d - w}\right) = \tau^{1-\sigma} \\ &\Rightarrow \hat{\delta}_x^* = \tau^{1-\sigma} \frac{\bar{\delta}(\hat{\delta}_d^*) f_d + \psi(\hat{\delta}_d^*)\bar{\delta}(\hat{\delta}_{t(\hat{\delta}_d^*)}) n f_x}{(1 + \tau^{1-\sigma}\psi(\hat{\delta}_d^*) n) f_x}. \end{aligned} \tag{16}$$

Summarizing, the updating mechanism from proposition 5 yields:

Proposition 6 (Firm Specific Effects) *Net profits of firms and ex ante probability of exporting increase in firm age. Some firms face negative per period net profits from domestic activity initially, while entry into export occurs only in case of positive per period net profits.*

Free Entry Condition: In line with the known firm type case, firms offer the repayment of their signal dependent expected total net profits $E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t \pi^n(\hat{\delta}_t))$ and risk neutral, perfectly competitive banks grant loans until expected profits coincide with expected costs:

$$E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t \pi^n(\hat{\delta}_t)) | \hat{\delta}_0 \leq \hat{\delta}_d^*) = f_I / G(\hat{\delta}_d^*). \tag{17}$$

In the Appendix we prove that cut-off values $\hat{\delta}_d^*$ and $\hat{\delta}_x^*$ exist and are uniquely determined by (14), (15), (16) and (17). Moreover, we prove that lower iceberg trade costs τ lowers $\hat{\delta}_d^*$ but increases $\hat{\delta}_x^*$, yielding identical trade liberalization effects on firm churning as in the known firm type case (Proposition 1).

Long Run Distributions of Active Firms: First we determine the steady state distribution of true types and then, second, the steady state distribution of perceived types. Every period a certain measure M_e of firms with g-distributed true types tries to enter the market. As true types are not observable, even high-δ-firms may enter if their start-up signal is sufficiently good, i.e. if $\hat{\delta}_0 \leq \hat{\delta}^*_d$, yielding the modified distribution of entrants $j(\delta) = \int_0^{\hat{\delta}^*_d} b_{\hat{\delta}_0}(\delta) g(\hat{\delta}_0) d\hat{\delta}_0$. Let $i(\delta)$ denote the aggregate mass of incumbents of type δ. Then firms of true type δ accumulate until the measure of entrants, $M_e j(\delta)$, coincides with the measure of defaulting firms $\delta i(\delta)$ yielding $i(\delta) = M_e j(\delta)/\delta$. Thus, we get the long-run true type distributions of active firms:

$$h(\delta) = \frac{j(\delta)/\delta}{\int_0^1 j(\delta)/\delta \, d\delta}. \tag{18}$$

Perceived types of entrants are distributed with $g(\hat{\delta}_0 | \hat{\delta}_0 \leq \hat{\delta}^*_d)$ and evolve according to the Bayesian updating process subsequently. Thus, if we fix a perceived type $\hat{\delta}$ and want to determine the density of incumbents for this perceived type, we have to consider two components: new entrants with perceived type $\hat{\delta}_0 = \hat{\delta}$ and older firms that started with a start-up perceived type $\hat{\delta}'_0 > \hat{\delta}$ and happen to be assigned a current perceived type $\hat{\delta}$ by Bayesian updating. Let $\hat{\delta}_{-t} > \hat{\delta}$ denote the start-up perceived type that coincides with $\hat{\delta}$ after t periods of Bayesian updating. Then, the entry density of perceived type $\hat{\delta}_{-t}$ equals $M_e g(\hat{\delta}_{-t})$ and the probability that firms of this perceived type survive for t periods is given by $E_{b_{\hat{\delta}_{-t}}}((1-\delta)^t)$ yielding the perceived type density of incumbents $\hat{j}(\hat{\delta}) = \Sigma_{t=0}^{T(\hat{\delta})} E_{b_{\hat{\delta}_{-t}}}((1-\delta)^t) M_e g(\hat{\delta}_{-t})$.[13] Thus, we get long-run perceived type distributions of domestic firms firm:

$$\hat{h}(\hat{\delta}) = \begin{cases} \frac{\hat{j}(\hat{\delta})}{\int_0^{\hat{\delta}^*_d} \hat{j}(\hat{\delta}) \, d\hat{\delta}} & \text{if } \hat{\delta} \in (0, \hat{\delta}^*_d], \\ 0 & \text{otherwise.} \end{cases} \tag{19}$$

Misvaluation of Active Firms: Even though belief updating is rational on the firm level,

[13] As $g(0) = 0$, only perceived types $\hat{\delta} > 0$ are possible. And as $\lim_{t \to \infty} (\hat{\delta}^*_d)_t = 0$, there always exists a finite t s.t. $(\hat{\delta}^*_d)_t < \hat{\delta}$. Thus, $T(\hat{\delta})$ is finite for all $\hat{\delta} > 0$.

it leads to misvaluation in aggregate terms. As start-up signals $\hat{\delta}_0$ for individual firms are correct in expectation and as only firms $\hat{\delta}_0 \leq \hat{\delta}_d^*$ turn active, perceived average exit probabilities of new born firm generations are smaller than their true average exit probability. Both decline with respect to generation age by excess exit of high-δ-types according to the cleansing mechanism (Proposition 2). But as the decline of aggregate perceived types is amplified by the updating mechanism (Proposition 5), perceived types are increasingly biased downwards the older a firm generation gets. By this misjudgment, incumbents face interest rates that are too small in expectation, and too many firms become exporters.[14] Active and inactive firms compete for workers in order to produce, to enter new markets, or to conduct the start-up investments. The relative advantage of incumbents yields too little entry of new firms. Summarizing, the joint impact of the cleansing mechanism from Proposition 2 and the updating mechanism from Proposition 5 implies:

Proposition 7 (Firm Generation Effects) *The older a firm generation, the further perceived and true average exit probabilities deviate, yielding inefficiently low interest rates for incumbents. Thus, the steady state exhibits excessive exporting and insufficient start-up investment.*

Firm type generation effects from the known type case (Proposition 3) carry over to the uncertain firm type case.

Firm Masses: In steady state, firm entry balances firm exit, yielding $M_e = E_h(\delta)M/G(\hat{\delta}_d^*)$. Using labor market clearing $L = Mq_d + nM_x\tau q_x$ and the mass of exporting firms $M_x = \hat{H}(\hat{\delta}_x^*)M$, with $\hat{H}(\hat{\delta})$ denoting the cumulative density function of the long-run distribution of perceived firm types $\hat{h}(\hat{\delta})$, we get:[15]

$$M = wL/[(\sigma-1)(f_d\bar{\delta}(\hat{\delta}_d^*) + nf_x(\hat{H}(\hat{\delta}_x^*) - \psi(\hat{\delta}_d^*))\hat{\delta}_x^*))]. \tag{20}$$

[14]If a firm with true type $\delta > \hat{\delta}_x^*$ enters the export market by a misjudgment of its type $\hat{\delta}_t \leq \hat{\delta}_x^*$, it will (in expectation) default before sunk entry costs nf_x are balanced by accumulated per period net profits.

[15]See the Appendix for details.

Consumption: We determine the equilibrium wage rate w from $P = 1$ and obtain aggregate per period consumption:

$$C = Lw/P = Lw = L\rho(M + n\hat{H}(\hat{\delta}_x^*)M\tau^{1-\sigma})^{1/(\sigma-1)}. \tag{21}$$

As utility is linear in consumption, (21) constitutes a measure of welfare. A detailed derivation of (21) is provided in the Appendix. Apart from consumption, the final product is spent for start-up investments, market entry of new firms and for foreign market entry of incumbents that turn exporters by Bayesian updating. Let $(\hat{\delta}_x^*)_{-t}$ denote the start-up perceived type that coincides with $\hat{\delta}_x^*$ after t periods of updating, then (conditional on survival) all firms with start-up perceived types $\hat{\delta}_0 \in ((\hat{\delta}_x^*)_{-(t-1)}, (\hat{\delta}_x^*)_{-t}]$ will turn exporters in their t^{th} period. As the entry density of a perceived type $\hat{\delta}_0$ equals $M_e g(\hat{\delta}_0)$ and the probability that firms of this perceived type survive for t periods is given by $E_{b_{\hat{\delta}_0}}((1-\delta)^t)$, the measure of firms of age t that turn exporters by Bayesian updating every period equals $\int_{(\hat{\delta}_x^*)_{-(t-1)}}^{(\hat{\delta}_x^*)_{-t}} E_{b_{\hat{\delta}_0}}((1-\delta)^t)M_e g(\hat{\delta}_0)d\hat{\delta}_0$. Adding all possible ages $t = 1, 2, \ldots, T(\hat{\delta}_x^*) < \infty$ we obtain:[16]

$$\begin{aligned} I &= (f_I + f_d G(\hat{\delta}_d^*) + n f_x G(\hat{\delta}_x^*))M_e \\ &\quad + n f_x \Sigma_{t=1}^{T(\hat{\delta}_x^*)} \int_{(\hat{\delta}_x^*)_{-(t-1)}}^{(\hat{\delta}_x^*)_{-t}} E_{b_{\hat{\delta}_0}}((1-\delta)^t)M_e g(\hat{\delta}_0)d\hat{\delta}_0. \end{aligned} \tag{22}$$

Similar to the known type case trade liberalization forces firms with low net profits out of the market shifting production towards firms with higher net profits. But as loan rates (that depend on perceived types) and real per period fixed costs (that depend on real types) differ systematically, this shift does not always improve average efficiency of the economy. As we prove in the Appendix the welfare result from Proposition 4 does not carry over to uncertain

[16] $T(\hat{\delta}_x^*)$ denotes the number of periods of Bayesian updating a firm with highest possible start-up perceived exit probability $\hat{\delta}_d^*$ needs to turn exporter. As $\lim_{t\to\infty} \hat{\delta}_t = 0$ for all $\hat{\delta}$, $T(\hat{\delta}_x^*)$ has to be finite.

firm types.

Proposition 8 (Trade Liberalization and Welfare) *In case of uncertain firm types, trade liberalization can have a negative welfare effect.*

The intuition for this result lies in the fact that belief-updating leads to excessive exporting, as explained above. Lower variable trade costs can exacerbate this inefficiency, which can lead to welfare losses from trade liberalization.

4.2 Crisis of Confidence

For many observers, the world-wide recession of 2008/09 has been particularly severe and pervasive because it involved a massive reversal of beliefs on the stability of the financial system (Bacchetta et al., 2010). The relationship between output drop, falling demand, and the banking crisis epitomized by the collapse of the investment bank Lehman Brothers in September 15, 2008, is still a matter of academic debate. Our model is, of course, much too stylized to give a quantitative assessment of the crisis. However, it allows to shed light on the different effects of a belief revision on small as compared to large firms. It captures, admittedly in a a very stylized way, the facts that (i) exports dropped much more than GDP in most countries and in the world (see Behrens et al., 2010, for a discussion) and that (ii) large firms saw their financing conditions deteriorate more strongly than small ones. This second fact has been documented using firm-level data for Germany by Rottmann and Wolmershaüser (2010), Costa et al. (2011) for Italy, and Kremp and Sevestre (2011) for France.[17] Costa also shows that exporting firms have been more severely affected than non-exporters.

Belief Revision: We consider a shock that triggers all agents to return to former beliefs,

[17] This finding relates to the change in the costs of funding; large firms still obtain credit at lower cost than small ones.

i.e. some firm survival information is deleted.[18] There are several natural ways to model a belief revision. A belief revision could prompt all agents to return to their beliefs a certain number of periods ago, it could prompt all agents to delete a certain fraction of firm survival histories, or in the extreme case prompt all agents to return to start-up perceived types of firms. These scenarios have in common that agents become suddenly less optimistic as to the survival of firms. As start-up beliefs constitute lower bounds for belief revisions, the shock does not force any firms to exit domestic markets. However, some firms stop exporting.

Proposition 9 (Crisis of Confidence) *Implementing a crisis of confidence by a belief revision, it forces some firms to exit foreign markets while leaving the number of domestic firms unaltered.*

4.3 Asymmetric Countries

By incorporating country heterogeneity with respect to fixed market entry costs, we generate multi-level growth into exporting. The older a firm the more export destinations it will serve. To avoid technical complications, we consider a continuum of countries $\iota \in [0, 1]$, each being of zero measure. A foreign firm faces fixed costs f_ι upon market entry in country ι. Countries are ordered according to the size of their entry costs, i.e. $\iota < \kappa \Rightarrow f_\iota < f_\kappa$, each denominated in terms of the final good. To circumvent the special case of all firms only serving the market of country $\iota = 0$, which arises due to our simplifying assumption of free tradability of final goods, we introduce an additional stage of production. The final goods produced by countries shall henceforth be referred to as country good. Those country goods are then used to produce the "new" final good without requiring other inputs according to the standard CES-production function. Both, country goods and final goods, are traded freely. This setup extension nests all previous results, as all countries produce identical amounts of country goods in the symmetric country case. Under this additional stage of

[18]Entry or exit information is excluded from the revision, as neither defaulted firms can be reanimated, nor new born firms can be eliminated by a change in belief.

production the (normalized) price index of the final good is given by:

$$1 = P = \Big(\int_0^1 P_\iota^{1-\sigma} d\iota\Big)^{1/(1-\sigma)}, \tag{23}$$

with

$$P_\iota = \Big(\int_0^1 \Big(\int_{\omega_{\kappa,\iota}\in\Omega_{\kappa,\iota}} p(\omega_{\kappa,\iota})^{1-\sigma} d\omega_{\kappa,\iota}\Big) d\kappa\Big)^{1/(1-\sigma)}, \tag{24}$$

denoting the price index of country ι, where $\Omega_{\kappa,\iota}$ denotes the set of intermediate goods imported from country κ. All payments, such as wage payments, loan rates or fixed costs, are still measured in units of final good. Whenever results are independent of country type, we suppress the country indicating subscript.

Uniform Wage Rate: Since individual countries are of zero measure, costs or profits a firm faces within one country are infinitesimal and hence negligible. Only costs or profits a firm faces within a positive measure of countries will influence its actions. Thus, all firms that conduct the start-up investment will enter domestic markets, as this entrance at infinitesimal entry costs entails a positive probability of entry into a positive measure of foreign countries, yielding positive expected profits. Hence, true and perceived types of entrants are distributed with probability density function g in all countries. Besides domestic entry fees, also domestic profits are infinitesimal and hence negligible. Thus, firm actions (the choice of export destinations and export prices) solely depend on perceived firm type and are independent of firm location. As neither the distribution, nor the action of firms depend on their location, the aggregate production of intermediate inputs by firms located in one country, is identical for all countries. Thus, by trade balance, all countries are compensated with identical amounts of final good yielding identical wages in all countries.

Zero Cut-Off Profit Conditions: Firms enter a foreign market ι as soon as per period profit π_ι dominates per period costs $\hat{\delta}_t f_\iota$, yielding the first zero cut-off profit condition

$\pi_\iota = \hat{\delta}^*_\iota f_\iota$, with $\hat{\delta}^*_\iota$ denoting the cut-off type for entry into market ι. Dividing per period profits, we get the second zero cut-off profit condition $\hat{\delta}^*_\iota = (f_\kappa/f_\iota)\hat{\delta}^*_\kappa$ for all $\iota, \kappa \in [0,1]$. Thus, $f_\iota < f_\kappa$ yields $\hat{\delta}^*_\iota > \hat{\delta}^*_\kappa$, i.e. the higher the market entry costs the smaller the set of perceived firm types that enter. Let $\kappa(\hat{\delta}_t)$ denote the "last" country a firm of perceived type $\hat{\delta}_t$ exports to, i.e. the country with cut-off value $\hat{\delta}^*_\kappa = \hat{\delta}_t$. Then, a firm of perceived type $\hat{\delta}_t$ will export to all countries $\iota \in [0, \kappa(\hat{\delta}_t)]$. The lower the firms' perceived exit probability $\hat{\delta}_t$ the greater its measure of export destinations, until, for $\hat{\delta}_t \leq \hat{\delta}^*_{\iota=1}$ it exports to all countries.

Free Entry Condition: Free entry of firms ensures that expected future profits $E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t \int_0^{\kappa(\hat{\delta}_t)}(\pi_\kappa - \hat{\delta}_t f_\kappa)d\kappa))$ coincide with costs for the start-up investment f_I. As we prove in the Appendix, zero cutoff and free entry conditions determine cut-off values $\hat{\delta}^*_\iota$ uniquely. From the ordering of cut-off values ($\iota < \kappa \Rightarrow \hat{\delta}^*_\iota > \hat{\delta}^*_\kappa$) and the updating mechanism (Proposition 5) we find that firms enter more and more markets as they grow in age.

Proposition 10 (Firm Specific Effects) *The measure of export destinations increases in firm age. In a crisis of confidence, firms exit markets with highest fixed costs or market presence first.*

Long Run Distributions of Active Firms: As firms of all types enter, the true type distribution of incumbents equals $h(\delta) = (g(\delta)/\delta)/(\int_0^1 (g(\delta)/\delta)d\delta)$ and the perceived type distribution equals $\hat{h}(\hat{\delta}) = \hat{j}(\hat{\delta})/\int_0^1 \hat{j}(\hat{\delta})d\hat{\delta}$, with $\hat{j}(\hat{\delta}) = \Sigma_{t=0}^{T(\hat{\delta})} E_{b_{\hat{\delta}_{-t}}}((1-\delta)^t)M_e g(\hat{\delta}_{-t})$.

Firm Masses: All firms that conduct the start-up investment enter and firm exit occurs with respect to the true type distribution. Thus, the steady state correspondence of firm masses of entrants and incumbents equals $M_e = E_h(\delta)M$. Firm export status depends on perceived firm type. Thus, the mass of firms within a certain country that export to country κ equals $M_\kappa = \hat{H}(\hat{\delta}^*_\kappa)M$. Additionally, taking into account the labor market clearing

$L = \int_0^1 M_\kappa \tau q_\kappa d\kappa$, we obtain:[19]

$$M = wL/[(\sigma-1)\int_0^1 \hat{H}(\hat{\delta}^*_\kappa)\hat{\delta}^*_\kappa f_\kappa d\kappa]. \tag{25}$$

Consumption: Determining the equilibrium wage rate w from $P = 1$, we receive aggregate per period consumption:[20]

$$C = Lw/P = Lw = L(\rho/\tau)\Big(M \int_0^1 \hat{H}(\hat{\delta}^*_\iota)d\iota\Big)^{1/(\sigma-1)}. \tag{26}$$

In line with Section 4.1 aggregate per period investment consists of the units of final product needed for start-up investment, $f_I M_e$, the units needed for direct market entry, $\int_0^1 G(\hat{\delta}^*_\iota)f_\iota d\iota M_e$, and the units needed for entry into market ι by Bayesian updating, $f_\iota \Sigma_{t=1}^{T(\hat{\delta}^*_\iota)} \int_{(\hat{\delta}^*_\iota)_{-(t-1)}}^{(\hat{\delta}^*_\iota)_{-t}} E_{b_{\hat{\delta}_0}}((1-\delta)^t)M_e g(\hat{\delta}_0)d\hat{\delta}_0$. As the last term arises for all markets, we get:

$$\begin{aligned} I &= f_I M_e + \int_0^1 G(\hat{\delta}^*_\iota)f_\iota d\iota M_e \\ &\quad + \int_0^1 \Big(f_\iota \Sigma_{t=1}^{T(\hat{\delta}^*_\iota)} \int_{(\hat{\delta}^*_\iota)_{-(t-1)}}^{(\hat{\delta}^*_\iota)_{-t}} E_{b_{\hat{\delta}_0}}((1-\delta)^t)M_e g(\hat{\delta}_0)d\hat{\delta}_0\Big)d\iota, \end{aligned} \tag{27}$$

which completes the characterization of the general equilibrium under type uncertainty in an asymmetric country setting.

5 Conclusion

Newly created firms are uncertain as to the viability of their new product. Market expectations about the lifetime of an innovation determine the effective costs of finance for firms. So, if some fraction of firms' investment needs are irreversible, firms differing with respect to the perceived probability of death shocks face different financing possibilities.

[19]Details of the derivation are in the Appendix.
[20]Again, see the Appendix for detailed derivations.

International trade interacts with this heterogeneity: firms with lower perceived default probabilities are more likely to be exporters, lower trade costs make the expected survival rates of domestic firms smaller but those of exporters larger; firm survival is longer in open compared to closed economies. All this facts are well supported by empirical evidence.

In contrast to firm-level heterogeneity in productivity or product quality, a firm's life expectancy cannot be easily inferred from its production process or its sales statistics. Rather, it is more likely that market participants only receive a noisy signal about the true type of a firm. Conditional on survival of the firm, market participants update their beliefs. This process has important further implications for firm behavior and aggregate outcomes. First, it implies that the financial conditions faced by firms improve over time. Second, due to this, firms will be gradually growing as they enter more and more markets. Third, the updating process leads to an excessive expansion of large incumbents to the expense of startups, so that the number of existing firms tends to be too small. Fourth, a sudden reversal of beliefs leads to reduction in economic activity, but the collapse of trade flows is larger than that of total income. Again, these facts square well with empirical facts.

The main advantage of the framework is its simplicity and generality. As long as firms are homogeneous with respect to variable components of revenue, aggregation is very simple. This allows an analytical characterization of firm dynamics without making assumptions on the form of distribution functions. It also makes further extensions of the model possible. One interesting avenue for further research would be to add a more complete description of financial frictions to the model or to allow for a second source of heterogeneity, possibly of the form used in Melitz (2003).

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Appendices

Appendix A: Baseline Model

Existence and Uniqueness of Cut-Off Values: Starting with (9) and applying (7) in the third step of the calculation, we get:

$$
\begin{aligned}
f_I/G(\delta_d^*) &= E_g(\pi^n(\delta)/\delta|\delta \leq \delta_d^*) \\
&= E_g(\pi_d^n(\delta)/\delta|\delta \leq \delta_d^*) + (G(\delta_x^*)/G(\delta_d^*))E_g(n\pi_x^n(\delta)/\delta|\delta \leq \delta_x^*) \\
&= E_g((\delta_d^* - \delta)f_d/\delta|\delta \leq \delta_d^*) + (G(\delta_x^*)/G(\delta_d^*))E_g(n(\delta_x^* - \delta)f_x/\delta|\delta \leq \delta_x^*) \\
&= f_d(\delta_d^* E_g(1/\delta|\delta \leq \delta_d^*) - 1) + nf_x(G(\delta_x^*)/G(\delta_d^*))(\delta_x^* E_g(1/\delta|\delta \leq \delta_x^*) - 1),
\end{aligned}
$$

yielding:

$$
f_d G(\delta_d^*)(\delta_d^* E_g(1/\delta|\delta \leq \delta_d^*) - 1) + nf_x G(\delta_x^*)(\delta_x^* E_g(1/\delta|\delta \leq \delta_x^*) - 1) = f_I. \tag{28}
$$

Replacing δ_x^* by δ_d^* via (8), the left hand side of (28) is a continuous function of δ_d^* that equals 0 for $\delta_d^* = 0$ and is strictly positive for $\delta_d^* = 1$ as $E(1/\delta) > 1$. Thus, it is always possible to choose $f_I > 0$ sufficiently small in order to ensure the existence of a solution of (28). Uniqueness follows from proof by contradiction: Assume there are at least two different domestic cut-off values $\delta_d^\natural < \delta_d^\flat$ solving (28). Then net per period profits of firms $\delta \in (\delta_d^\natural, \delta_d^\flat)$ have to be less or equal to net per period profits of firm $\delta^\flat$, which yields a contradiction as net per period profits strictly decrease in δ.

Proof of Proposition 1: Equation (8) exhibits a direct effect $\tau \downarrow \Rightarrow \delta_x^* \uparrow$. As $\delta_x^* \uparrow$ yields $\delta_x^* \downarrow$ via (28) and as there is no direct effect of τ on (28). The statements contained in the Proposition follow.

Derivation of (11): Labor market clearing $L = Mq_d + nM_x\tau q_x$ yields $wL = M(r_d - \pi_d) + nM_x(r_x - \pi_x) = M((r_d - \pi_d) + nH(\delta_x^*)(r_x - \pi_x))$. Transforming r_d and r_x according to (4) and (6) and replacing π_d and π_x via (6) we get:

$$\begin{aligned} wL &= M((r_d - \pi_d) + nH(\delta_x^*)(r_x - \pi_x)) = M(\sigma\pi_d - \pi_d + nH(\delta_x^*)(\sigma\pi_x - \pi_x)) \\ &= M((\sigma - 1)\delta_d^* f_d + nH(\delta_x^*)(\sigma - 1)\delta_x^* f_x) = M(\sigma - 1)(\delta_d^* f_d + nH(\delta_x^*)\delta_x^* f_x) \end{aligned}$$

yielding (11).

Derivation of (12): From $1 = P = (Mp_d^{1-\sigma} + nM_x p_x^{1-\sigma})^{\frac{1}{1-\sigma}}$ we get:

$$\begin{aligned} w &= w/P = w(Mp_d^{1-\sigma} + nM_x p_x^{1-\sigma})^{\frac{1}{\sigma-1}} = w(Mp_d^{1-\sigma} + nH(\delta_x^*)M(\tau p_d)^{1-\sigma})^{\frac{1}{\sigma-1}} \\ &= (w/p_d)(1 + nH(\delta_x^*)\tau^{1-\sigma})^{\frac{1}{\sigma-1}} M^{\frac{1}{\sigma-1}} = \rho(1 + nH(\delta_x^*)\tau^{1-\sigma})^{\frac{1}{\sigma-1}} M^{\frac{1}{\sigma-1}}. \end{aligned}$$

Together with $C = Lw/P$, this implies (12).

Appendix B: Uncertain Firm Types (Symmetric Countries)

Existence and Uniqueness of Cut-Off Values: Starting with (17) and applying (15) in the fourth and (14) in the fifth step of the calculation, we get:

$$
\begin{aligned}
f_I/G(\hat{\delta}_d^*) &= E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\pi^n(\hat{\delta}_t))|\hat{\delta}_0 \le \hat{\delta}_d^*) \\
&= E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\pi_d^n(\hat{\delta}_t) + \Sigma_{t=t(\hat{\delta}_0)}^{\infty}(1-\delta)^t n\pi_x^n(\hat{\delta}_t))|\hat{\delta}_0 \le \hat{\delta}_d^*) \\
&= E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\pi_d^n(\hat{\delta}_t)) \\
&\quad +E_{b_{\hat{\delta}_0}}((1-\delta)^{t(\hat{\delta}_0)}\Sigma_{t=0}^{\infty}(1-\delta)^t n\pi_x^n(\hat{\delta}_{t(\hat{\delta}_0)+t}))|\hat{\delta}_0 \le \hat{\delta}_d^*) \\
&= E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t(\pi_d - \hat{\delta}_t f_d) \\
&\quad +E_{b_{\hat{\delta}_0}}((1-\delta)^{t(\hat{\delta}_0)}\Sigma_{t=0}^{\infty}(1-\delta)^t n(\pi_x - \hat{\delta}_{t(\hat{\delta}_0)+t} f_x))|\hat{\delta}_0 \le \hat{\delta}_d^*) \\
&= E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t((\bar{\delta}(\hat{\delta}_d^*) - \hat{\delta}_t)f_d - \psi(\hat{\delta}_d^*)(\hat{\delta}_x^* - \bar{\delta}(\hat{\delta}_{t(\hat{\delta}_d^*)}))nf_x) \\
&\quad +E_{b_{\hat{\delta}_0}}((1-\delta)^{t(\hat{\delta}_0)}\Sigma_{t=0}^{\infty}(1-\delta)^t n(\hat{\delta}_x^* - \hat{\delta}_{t(\hat{\delta}_0)+t})f_x)|\hat{\delta}_0 \le \hat{\delta}_d^*) \\
&= E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t((\bar{\delta}(\hat{\delta}_d^*) - \bar{\delta}(\hat{\delta}_0))f_d - \psi(\hat{\delta}_d^*)(\hat{\delta}_x^* - \bar{\delta}(\hat{\delta}_{t(\hat{\delta}_d^*)}))nf_x) \\
&\quad +E_{b_{\hat{\delta}_0}}((1-\delta)^{t(\hat{\delta}_0)}\Sigma_{t=0}^{\infty}(1-\delta)^t(\hat{\delta}_x^* - \bar{\delta}(\hat{\delta}_{t(\hat{\delta}_0)}))nf_x)|\hat{\delta}_0 \le \hat{\delta}_d^*) \\
&= E_g(((\bar{\delta}(\hat{\delta}_d^*) - \bar{\delta}(\hat{\delta}_0))f_d - \psi(\hat{\delta}_d^*)(\hat{\delta}_x^* - \bar{\delta}(\hat{\delta}_{t(\hat{\delta}_d^*)}))nf_x)E_{b_{\hat{\delta}_0}}(1/\delta) \\
&\quad +(\hat{\delta}_x^* - \bar{\delta}(\hat{\delta}_{t(\hat{\delta}_0)}))nf_x E_{b_{\hat{\delta}_0}}((1-\delta)^{t(\hat{\delta}_0)}/\delta)|\hat{\delta}_0 \le \hat{\delta}_d^*) \\
&= E_g(((\bar{\delta}(\hat{\delta}_d^*) - \bar{\delta}(\hat{\delta}_0))f_d - \psi(\hat{\delta}_d^*)(\hat{\delta}_x^* - \bar{\delta}(\hat{\delta}_{t(\hat{\delta}_d^*)}))nf_x \\
&\quad +\psi(\hat{\delta}_0)(\hat{\delta}_x^* - \bar{\delta}(\hat{\delta}_{t(\hat{\delta}_0)}))nf_x)E_{b_{\hat{\delta}_0}}(1/\delta)|\hat{\delta}_0 \le \hat{\delta}_d^*)
\end{aligned}
$$

yielding: $f_I =$

$$
G(\hat{\delta}_d^*)E_g[[(\bar{\delta}(\hat{\delta}_d^*)-\bar{\delta}(\hat{\delta}_0))f_d-\psi(\hat{\delta}_d^*)(\hat{\delta}_x^*-\bar{\delta}(\hat{\delta}_{t(\hat{\delta}_d^*)}))nf_x+\psi(\hat{\delta}_0)(\hat{\delta}_x^*-\bar{\delta}(\hat{\delta}_{t(\hat{\delta}_0)}))nf_x]E_{b_{\hat{\delta}_0}}(1/\delta)|\hat{\delta}_0 \le \hat{\delta}_d^*], \tag{29}
$$

with $t(\hat{\delta}_0)$ denoting the period in which a firm of start-up perceived type $\hat{\delta}_0$ would enter the export market in case of survival. Existence and uniqueness of the solution $\hat{\delta}_d^*$ follows from analog arguments as in the known firm type case.

Effects of trade liberalization: From (16) we get $\partial\hat{\delta}_x^*/\partial\tau < 0$, yielding $\tau\downarrow\Rightarrow\hat{\delta}_x^*\uparrow$. As $\psi(\hat{\delta}_d^*) \leq \psi(\hat{\delta}_0)$ yields the correspondence $\hat{\delta}_x^*\uparrow\Rightarrow\hat{\delta}_x^*\downarrow$ via (29), and as there is no direct effect of τ on (29), we obtain the claims made in the text.

Derivation of (20): Labor market clearing $L = Mq_d + nM_x\tau q_x$ yields $wL = M(r_d - \pi_d) + nM_x(r_x - \pi_x) = M(r_d - \pi_d + n\hat{H}(\hat{\delta}_x^*)(r_x - \pi_x))$. Transforming r_d and r_x via (4) and (14) and replacing π_d and π_x according to (14), yields:

$$\begin{aligned}
wL &= M(r_d - \pi_d + n\hat{H}(\hat{\delta}_x^*)(r_x - \pi_x)) \\
&= M(\sigma\pi_d - \pi_d + n\hat{H}(\hat{\delta}_x^*)(\sigma\pi_x - \pi_x)) \\
&= M((\sigma-1)(\bar{\delta}(\hat{\delta}_d^*)f_d - \psi(\hat{\delta}_d^*)\hat{\delta}_x^* n f_x) + n\hat{H}(\hat{\delta}_x^*)((\sigma-1)\hat{\delta}_x^* f_x) \\
&= M(\sigma-1)(f_d\bar{\delta}(\hat{\delta}_d^*) + nf_x(\hat{H}(\hat{\delta}_x^*) - \psi(\hat{\delta}_d^*))\hat{\delta}_x^*),
\end{aligned}$$

yielding (20).

Derivation of (21): From $1 = P = (Mp_d^{1-\sigma} + nM_xp_x^{1-\sigma})^{\frac{1}{1-\sigma}}$ we get:

$$\begin{aligned}
w &= w/P \\
&= w(Mp_d^{1-\sigma} + nM_xp_x^{1-\sigma})^{\frac{1}{\sigma-1}} \\
&= w(Mp_d^{1-\sigma} + n\hat{H}(\hat{\delta}_x^*)M(\tau p_d)^{1-\sigma})^{\frac{1}{\sigma-1}} \\
&= (w/p_d)(1 + n\hat{H}(\hat{\delta}_x^*)\tau^{1-\sigma})^{\frac{1}{\sigma-1}} M^{\frac{1}{\sigma-1}} \\
&= \rho(1 + n\hat{H}(\hat{\delta}_x^*)\tau^{1-\sigma})^{\frac{1}{\sigma-1}} M^{\frac{1}{\sigma-1}}.
\end{aligned}$$

Together with $C = Lw/P$, this implies (21).

Proof of Proposition 8: If a firm starts exporting by a misjudgment of its true type, expected profits from exporting $\Sigma_{t=0}^{\infty}(1-\delta)^t\pi_x$ are dominated by costs f_x. In this case the firm uses up more units of final good of a country than it produces, yielding a negative welfare effect. By constructing a very special ex ante distribution g' of true and perceived firm types, we can increase the fraction of firms that enter by misjudgment of their type almost to 1. Let $(\hat{\delta}_x^*)_{-1}$ denote the value of the start-up perceived type that coincides with the exporting cut-off value after one period of updating and let $R = \int_{\hat{\delta}_x^*}^{(\hat{\delta}_x^*)_{-1}} g(\hat{\delta}_0)d\hat{\delta}_0$ denote the fraction of start-up firms $\hat{\delta}_0$ within $(\hat{\delta}_x^*, (\hat{\delta}_x^*)_{-1})$. Then, those start-up firms will enter foreign markets in their second period of operation, yielding a negative aggregate welfare effect, as start-up perceived firm types are correct in expectation. By shifting probability density towards a value $\hat{\delta}_0'$ within the open interval $(\hat{\delta}_x^*, (\hat{\delta}_x^*)_{-1})$, we can push R arbitrarily close towards 1.[21] For some value of R' (close enough to 1) the negative welfare effect from export entry of firms belonging to this fraction will outweigh the possibly positive welfare effect from export entry of the residual $1-R'$. Under such an ex ante distribution g' of true and perceived firm types a change to prohibitive variable trade costs $\tau \to \infty$ or to $n \to 0$ accessible foreign markets increases welfare. Hence, by the mean value theorem of differential calculus, there exists a τ' and a n' at which liberalizing trade yields negative welfare effects.

Appendix C: Uncertain Firm Types (Asymmetric Countries)

Existence and Uniqueness of Cut-Off Values: Using the zero cut-off profit conditions $\pi_\iota = \hat{\delta}_\iota^* f_\iota$ and $\hat{\delta}_\iota^* = (f_\kappa/f_\iota)\hat{\delta}_\kappa^*$ we can transform the free entry condition into an equation with

[21]As this shifting of probability density draws $\hat{\delta}_x^*$ and $(\hat{\delta}_x^*)_{-1}$ closer together, $\hat{\delta}_0'$ has to belong to the interval subsequent the shifting of probability density. As $\hat{\delta}_x^* < (\hat{\delta}_x^*)_{-1}$ for all non-degenerate distributions g, such a $\hat{\delta}_0'$ always exists.

only one unknown $\hat{\delta}_0^*$:

$$
\begin{aligned}
f_I &= E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t \int_0^{\kappa(\hat{\delta}_t)} (\pi_\kappa - \hat{\delta}_t f_\kappa) d\kappa)) \\
&= E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t \int_0^{\kappa(\hat{\delta}_t)} ((\hat{\delta}_\kappa^* - \hat{\delta}_t) f_\kappa) d\kappa)) \\
&= E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t \int_0^{\kappa(\hat{\delta}_t)} ((f_0/f_\kappa)\hat{\delta}_0^* - \hat{\delta}_t) f_\kappa d\kappa)) \qquad (30)
\end{aligned}
$$

The right hand side of (30) is a continuous monotonically increasing function of $\hat{\delta}_0^*$. It equals zero for $\hat{\delta}_0^* = 0$, as in this case all cut-off values vanish $\hat{\delta}_\kappa^* = (f_0/f_\kappa)\hat{\delta}_0^* = 0$ and thus no firm will enter into exporting. If $\hat{\delta}_0^* = 1$ all firms will export to country $\iota = 0$ and to countries with similarly low market entry costs $\iota = 0 + \epsilon$.[22] Hence $\int_0^{\kappa(\hat{\delta}_0)}((f_0/f_\kappa)\hat{\delta}_0^* - \hat{\delta}_0) f_\kappa d\kappa > 0$ for all $\hat{\delta}_0$, yielding a strictly positive right hand side of (30). Thus, for all sufficiently small $f_I > 0$, there exists a unique solution $\hat{\delta}_0^*$ of (30).

Derivation of (25): From the labor market clearing condition $L = \int_0^1 M_\kappa \tau q_\kappa d\kappa$, we get:

$$
\begin{aligned}
wL &= \int_0^1 M_\kappa w\tau q_\kappa d\kappa = \int_0^1 M_\kappa (p_\kappa q_\kappa - (p_\kappa - w\tau) q_\kappa) d\kappa \\
&= \int_0^1 M_\kappa (r_\kappa - \pi_\kappa) d\kappa = \int_0^1 M_\kappa (\sigma - 1)\pi_\kappa d\kappa \\
&= \int_0^1 M_\kappa (\sigma - 1)\hat{\delta}_\kappa^* f_\kappa d\kappa = \int_0^1 \hat{H}(\hat{\delta}_\kappa^*) M(\sigma - 1)\hat{\delta}_\kappa^* f_\kappa d\kappa \\
&= M(\sigma - 1)\int_0^1 \hat{H}(\hat{\delta}_\kappa^*)\hat{\delta}_\kappa^* f_\kappa d\kappa,
\end{aligned}
$$

yielding (25).

[22] We assume that market entry costs f_ι increase continuously in ι.

Derivation of (26): Determining the country index

$$
\begin{aligned}
P_\iota &= \Big(\int_0^1 \Big(\int_{\Omega_{\kappa,\iota}} p(\omega_{\kappa,\iota})^{1-\sigma} d\omega_{\kappa,\iota}\Big) d\kappa\Big)^{1/(1-\sigma)} \\
&= \Big(\int_0^1 \Big(\int_{\Omega_{\kappa,\iota}} p_\iota^{1-\sigma} d\omega_{\kappa,\iota}\Big) d\kappa\Big)^{1/(1-\sigma)} = \Big(\int_0^1 p_\iota^{1-\sigma} M_\iota d\kappa\Big)^{1/(1-\sigma)} \\
&= p_\iota M_\iota^{1/(1-\sigma)} = w(\tau/\rho)(\hat{H}(\hat{\delta}_\iota^*)M)^{1/(1-\sigma)}
\end{aligned}
$$

and plugging it into $P = \Big(\int_0^1 P_\iota^{1-\sigma} d\iota\Big)^{1/(1-\sigma)} = w(\tau/\rho)\Big(M \int_0^1 \hat{H}(\hat{\delta}_\iota^*) d\iota\Big)^{1/(1-\sigma)}$ we receive $C = Lw/P = L(\rho/\tau)\Big(M \int_0^1 \hat{H}(\hat{\delta}_\iota^*) d\iota\Big)^{1/(\sigma-1)}$.