CESifo Working Papers

www.cesifo.org/wp

Are User Fees Really Regressive?

George Economides
Apostolis Philippopoulos

CESIFO WORKING PAPER NO. 3875
CATEGORY 6: FISCAL POLICY, MACROECONOMICS AND GROWTH
JULY 2012

An electronic version of the paper may be downloaded
- *from the SSRN website: www.SSRN.com*
- *from the RePEc website: www.RePEc.org*
- *from the CESifo website: www.CESifo-group.org/wp*

CESifo *Working Paper No. 3875*

Are User Fees Really Regressive?

Abstract

This paper studies the aggregate and distributional implications of introducing user fees for publicly provided excludable public goods into a model with consumption and income taxes. The setup is a neoclassical growth model where agents differ in earnings and second-best policy is chosen by a Ramsey government. Our main result is that the adoption of user fees by the Ramsey government not only increases aggregate efficiency, but it also decreases inequality. This result is in contrast to common view and policy practice.

JEL-Code: H400, H200, D600.

Keywords: user fees, Ramsey taxation, efficiency, inequality.

George Economides
Athens University of Economics and Business
76 Patission street
Greece – Athens 10434
gecon@aueb.gr

Apostolis Philippopoulos
Athens University of Economics and Business
76 Patission street
Greece – Athens 10434
aphil@aueb.gr

July 2, 2012
We thank Konstantinos Angelopoulos, Jim Malley, Hyun Park and Vanghelis Vassilatos for discussions and comments. We also thank Dimitris Papageorgiou for help with the data. Any errors are ours.

Please do not quote without the authors’ permission.

1. Introduction

There are publicly provided goods and services where exclusion is not possible (e.g. national security and defence) or non-desirable from a social point of view (e.g. services provided by the police and the court system). But there are also publicly provided goods and services from which exclusion is possible and hence user fees, or user prices, could be charged for their use.[1] The most well-known examples are education, health care and transportation systems (see Atkinson and Stiglitz, 1980, chapter 16, and Hillman, 2009, chapter 3). Such goods may have some of the properties of public goods (because of externalities) but, at the same time, they are also like private goods since the benefit received from them is mainly personal. However, in most countries, user fees are often zero, or partial, with the government subsidizing the public provision of such goods from other taxes, like income and consumption taxes.[2]

The most commonly expressed objection to user fees is the view that they are unjust, in the sense that they would increase inequality shifting the burden of taxation from the rich onto the poor. But, is it so? Does the introduction of user fees worsen inequality?

To answer the above questions, this paper studies the aggregate and distributional implications of introducing user fees as an additional public financing policy instrument. We work in a dynamic general equilibrium setup with Ramsey second-best optimal policy. Our main result is that, ceteris paribus, the adoption of optimally chosen user fees not only increases aggregate efficiency, but it also decreases inequality. Thus, the common view seems to be a fallacy. This holds especially when the use of the publicly provided good by one agent creates external, public-good benefits for other agents (namely, the publicly provided good is not private).

We deliberately work within a simple and recognizable setup. We build on the neoclassical growth model. Demand for the publicly provided excludable public good, as a

[1] See Hillman (2009, chapter 3) for a discussion of excludable public goods and user fees. User fees do not make sense if the public good is pure. Recall that if a public good is excludable or rival, it is not pure. Here we focus on excludability.

[2] For instance, in the euro area, as well as in EU-27, "Payments for non-market output as share of GDP" are less than 1% as share of GDP. The data source is OECD: National Accounts Statistics, General Government Accounts, Main Aggregates, 2011. This item represents services provided by the government "at prices economically non-significant" and includes fees paid by students for public universities and colleges, tickets for museums, etc. It is interesting to note, however, that in many countries the reliance on user fees has been increasing over the years. The US is a good example (see Huber and Runkel, 2009, for details). Also, in Germany, the above ratio is relatively high (2.16%). By contrast, it is less than 0.5% in France and South European countries.

negative function of the user fee, comes from individual agents' utility maximization problem (see Ott and Turnovsky, 2006). To study distributional implications, we obviously use a model with heterogeneous agents with a potential conflict of interests. We choose to work with Judd's (1985) model, in which agents are divided into two groups, called capitalists and workers, where workers do not participate in capital markets. This has been one of the most commonly used models with heterogeneity in the literature on optimal taxation.[3] The government is allowed to finance the provision of the excludable public good by a mix of income taxes, consumption taxes and user fees, all of which are proportional to their own tax base.

We work in two steps. We first solve a Ramsey-type second-best optimal policy problem where the government chooses all the above tax-spending policy instruments optimally. In turn, in the second step, we study what happens when the same amount of public goods, as found in the first step, is financed by income and consumption taxes only, chosen again by the Ramsey government. In other words, in the second step, we set the total amount of the public good, as well as the amount enjoyed by each social group, as in the first step, where these amounts were chosen by utility-maximizing private agents, and also assume away the use of user fees. By working in this way, the two regimes (with and without user fees) are directly comparable.

Following most of the Ramsey literature, we focus on the long run. That is, we compare the long-run solutions of Ramsey policy/allocation with and without user fees. Our main results are as follows.

First, when we compare an economy with income and consumption taxes only to an economy that also makes use of user fees, the latter is more efficient. Both groups, capitalists and workers, get better in terms of net income and welfare. This happens because user fees are less distorting than consumption taxes, which, in turn, are less distorting than income taxes. A more efficient economy benefits all groups.

Second, net income inequality is also reduced by the introduction of user fees. In particular, although the gross income of capitalists rises by more than the gross income of workers, at the same time, their tax burden also gets heavier than that on workers. The tax burden effect more than offsets the gross income effect, so the introduction of user fees makes workers better relative to capitalists. In particular, the effective tax rate on each

[3] In addition to Judd (1985), see also Lansing (1999), Krusell (2002), Fowler and Young (2006), Angelopoulos et al. (2011) and many others. As Fowler and Young (2006) argue, the assumptions of this model are not unreasonable in terms of wealth concentration.

capitalist rises, while the effective rate on each worker falls,[4] as we switch to the economy with user fees. Therefore, the introduction of user fees seems to work as a progressive tax that hurts those who have benefited more from the switch to a more efficient economy and this reduces net income inequality. These results are robust to the presence of external, public-good effects from the publicly provided good. Actually, the presence of such effects implies that the introduction of user fees decreases not only net income but also welfare inequality.

Although user prices have always been a debated issue in economic policy circles, the academic literature is relatively limited. Exceptions include Gertler et al. (1987), Fraser (1996), Ott and Turnovsky (2006), Swope and Janeba (2006), Fuest and Kolmar (2007) and Huber and Runkel (2009).[5] The paper closer to ours is that of Ott and Turnovsky (2006), who also use a dynamic general equilibrium model with endogenously determined tax bases. But, when they study the determination of income taxes and user fees, they focus on the implementation of the first best; thus, they do not solve for Ramsey policy. More importantly, they work with a representative agent model so they study efficiency issues only.

Thus, to the best of our knowledge, our paper is the first that evaluates the aggregate and distributional implications of introducing user fees into a general equilibrium growth model with income and consumption taxes when all policy instruments are optimally chosen.

The rest of the paper is as follows. Section 2 begins with a situation where agents are identical. Section 3 models the more general case where agents differ. Section 4 closes the paper.

2. A model with identical agents

We start with a situation where agents are identical. Although this cannot address issues of equality, it helps us to introduce the more general model in section 3 where agents differ. For simplicity the model is deterministic. Time is discrete and the horizon is infinite.

[4] The effective (average) tax rate is defined as the tax burden as a ratio of gross income.

[5] Gertler et al. (1987) study the introduction of user fees in the health care system in Peru. They find that user fees hurt the poor. Fraser (1996) focuses on the provision of public goods under different public financing schemes including user fees. Swope and Janeba (2006) analyze how populations with different preferences choose different public financing schemes. Fuest and Kolmar (2007) focus on the use of user fees under cross-border externalities. Huber and Runkel (2009) also focus on the use of user fees under tax competition; they also provide useful empirical evidence for the use of user fees in the US.

The setup is the standard neoclassical growth model extended to include demand for the publicly provided excludable public good coming from individuals' utility maximization problem (see also Ott and Turnovsky, 2006). Subject to this setup, the Ramsey government maximizes the welfare of the representative individual by choosing income taxes, consumption taxes and user fees, as well as the associated amount of the excludable public good. Policy is chosen once-and-for-all.

Households

There are $i = 1, 2, ..., N$ identical households. Each i maximizes lifetime utility:

$$\sum_{t=0}^{\infty} \beta^t u(c_{i,t}, l_{i,t}, \tilde{g}_{i,t}) \tag{1}$$

where $c_{i,t}$ is i's private consumption, $l_{i,t}$ is i's work hours, $\tilde{g}_{i,t}$ is the public good from the point of view of each i, and $0 < \beta < 1$ is the time preference rate.[6]

We find it convenient to follow e.g. Alesina and Wacziarg (1999) and define $\tilde{g}_{i,t}$ as:

$$\tilde{g}_{i,t} \equiv g_{i,t} + \gamma \sum_{j \neq i}^{N} g_{j,t} \qquad \text{where } 0 \leq \gamma < 1 \tag{2}$$

That is, γ measures the strength of external effects from other agents. If $\gamma = 0$, the publicly provided good is private.[7]

In our numerical solutions, we will use a simple period utility function of the form:

$$u_{i,t} = \mu_1 \log(c_{i,t}) + \mu_2 \log(1 - l_{i,t}) + \mu_3 \log(\tilde{g}_{i,t}) \tag{3}$$

where the parameters $\mu_1, \mu_2, \mu_3 > 0$ are preference weights.

The period budget constraint of each i is:

[6] For notational simplicity, we do not also include a pure public good (see Ott and Turnovsky, 2006, for both pure and excludable public goods). Our results are not affected by this.

[7] If $\gamma = 1$, the good is a pure public good. In this case, we do not get a well-defined solution. As Ott and Turnovsky (2006) and Hillman (2009) also explain, user fees can apply to impure public goods, namely, public goods that are rival and excludable. Here we focus on excludable public goods.

$$(1+\tau_t^c)c_{i,t} + k_{i,t+1} - (1-\delta)k_{i,t} + p_t g_{i,t} = (1-\tau_t^y)(r_t k_{i,t} + w_t l_{i,t}) \tag{4}$$

where $k_{i,t+1}$ is end-of-period capital, r_t is the return to beginning-of-period capital $k_{i,t}$, w_t is the return to labor $l_{i,t}$, p_t is the user fee paid by i for the service provided to him/her, $g_{i,t}$, $0<\tau_t^c, \tau_t^y < 1$ are consumption and income tax rates respectively, and the parameter $0<\delta<1$ is the capital depreciation rate.

Individuals act competitively. The first-order conditions for $c_{i,t}, l_{i,t}, k_{i,t+1}, g_{i,t}$ include the budget constraint above and:

$$\frac{1}{(1+\tau_t^c)c_{i,t}} = \frac{\beta[1-\delta+(1-\tau_{t+1}^y)r_{t+1}]}{(1+\tau_{t+1}^c)c_{i,t+1}} \tag{5a}$$

$$\frac{\mu_1(1-\tau_t^y)w_t}{(1+\tau_t^c)c_{i,t}} = \frac{\mu_2}{1-l_{i,t}} \tag{5b}$$

$$\frac{\mu_1 p_t}{(1+\tau_t^c)c_{i,t}} = \frac{\mu_3}{[1+\gamma(N-1)]g_{i,t}} \tag{5c}$$

where (5a) is a standard Euler equation for capital, (5b) is a standard labor supply condition, while (5c) is the first-order condition for $g_{i,t}$ and gives the demand for the excludable public good (recall that N is the number of identical households).

Firms

There are $f=1,2,...,N$ identical firms owned by households. Firms are modeled in the standard way. Each f maximizes profits:

$$\pi_{f,t} = y_{f,t} - r_t\, k_{f,t} - w_t l_{f,t} \tag{6}$$

subject to:

$$y_{f,t} = A(k_{f,t})^{\alpha}(l_{f,t})^{1-\alpha} \tag{7}$$

where $k_{f,t}$ and $l_{f,t}$ are capital and labor inputs respectively, while $A>0$ and $0<\alpha<1$ are usual technology parameters.

Firms act competitively. The usual first-order conditions for the two inputs are:

$$r_t = \frac{\alpha y_{f,t}}{k_{f,t}} \tag{8a}$$

$$w_t = \frac{(1-\alpha) y_{f,t}}{l_{f,t}} \tag{8b}$$

so that profits are zero.

Government budget constraint

The period budget constraint of the government is (in aggregate terms):

$$G_t = N[\tau_t^y (r_t k_{i,t} + w_t l_{i,t}) + \tau_t^c c_{i,t} + p_t g_{i,t}] \tag{9}$$

where G_t denotes the total provision of the excludable public good.[8]

Decentralized competitive equilibrium (for any feasible policy)

In the decentralized competitive equilibrium (DCE), households maximize utility, firms maximize profits, all constraints are satisfied and all markets clear (see Appendix A.1 for the market-clearing condition for the excludable public good and the associated equilibrium value of the user fee).[9]

Since agents are identical, we can drop subscripts. The DCE is summarized by the following five equations (quantities are in per capita terms):

[8] We use a single income tax, rather than separate taxes on capital income and labour income, because, if a Ramsey government has access to capital income, labor income and consumption taxes, it can implement the first-best (see e.g. Correia, 2010). We get similar results (available upon request) in our model. We also do not include public debt because as is known, in a Ramsey equilibrium, long-run debt cannot be pinned down by long-run conditions only (see Chamley, 1986, and Guo and Lansing, 1999), except if we add some friction. Since the presence of public debt is not expected to matter to our main results, we assume it away to avoid such complexities.

[9] We assume that the government sets the quantity of the excludable public good, g_t, while the price of the latter, p_t, follows endogenously. Results are the same if we treat p_t as a policy instrument and allow g_t to follow endogenously. This also applies to the next section.

$$\frac{1}{(1+\tau_t^c)c_t}=\frac{\beta[1-\delta+(1-\tau_{t+1}^y)r_{t+1}]}{(1+\tau_{t+1}^c)c_{t+1}} \tag{10a}$$

$$\frac{\mu_1(1-\tau_t^y)w_t}{(1+\tau_t^c)c_t}=\frac{\mu_2}{1-l_t} \tag{10b}$$

$$c_t+k_{t+1}-(1-\delta)k_t+g_t=y_t \tag{10c}$$

$$g_t=\tau_t^y y_t+\tau_t^c c_t+p_t g_t \tag{10d}$$

$$p_t=\frac{\mu_3(1+\tau_t^c)c_t}{\mu_1 g_t[1+\gamma(N-1)]} \tag{10e}$$

where we use $y_t = A(k_t)^{\alpha}(l_t)^{1-\alpha}$, $r_t = \frac{\alpha y_t}{k_t}$ and $w_t = \frac{(1-\alpha)y_t}{l_t}$.

We thus have five equations in $\{c_t, l_t, k_{t+1}, p_t\}_{t=0}^{\infty}$ and one of the policy instruments, $\{\tau_t^c, \tau_t^y, g_t\}_{t=0}^{\infty}$, which adjusts to satisfy the government budget constraint. The DCE is for any feasible policy. We now turn to optimal policy.

Ramsey policy and allocation with user fees

To solve for Ramsey policy and the associated Ramsey equilibrium, we follow the so-called dual approach.[10] The objective of the government is to maximize the representative household's utility function. To do so, it chooses $\{\tau_t^c, \tau_t^y, g_t, c_t, l_t, k_{t+1}, p_t\}_{t=0}^{\infty}$ subject to the DCE equations above. This policy problem is presented in detail in Appendix A.2.

Since the resulting equilibrium system cannot be solved analytically,[11] we present numerical solutions using common parameter values (see the notes in Table 1 for parameter values used). We report that our results are robust to changes in parameter values. Following most of the literature on Ramsey policy, we focus on the long-run solution. A numerical long-run solution is presented in Table 1, Regime A. The two columns of Regime A present the solution with, and without, externalities respectively, where in the former case we set $\gamma = 0.3$ which can be thought as a mild degree of external effects. Before we comment on results, we solve the same economy without user fees.

[10] For the dynamic Ramsey policy problem, see the rich review in Ljungqvist and Sargent (2004, chapter 15).

[11] We report that we can get analytical results for the long run for some variables. For instance, we can show that the income tax rate is zero in the long run. This is a reminiscent of zero capital income taxes in Chamley (1986) and Judd (1985). This also applies to the next section. See Guo and Lansing (1999) and Economides and Philippopoulos (2008) for non-zero long-run capital income taxes in the presence of imperfections.

Table 1 around here

Ramsey policy and allocation without user fees

We now examine what happens when the government provides the same amount of public goods as found above without charging user fees. That is, now we set $\{g_t\}_{t=0}^{\infty}$ as found above and assume that the government's public financing instruments are consumption and income taxes only. This makes the comparison to the previous regime with user fees meaningful.

In particular, the government chooses $\{\tau_t^c, \tau_t^y, c_t, l_t, k_{t+1}\}_{t=0}^{\infty}$ to maximize the same objective function taking $\{g_t\}_{t=0}^{\infty}$ as given subject to the new DCE equations. The latter are as in (10a-d) where now the user fee is set at zero and households treat the amount of the public good as given. This policy problem is presented in detail in Appendix A.3. Notice that this is very similar to the standard Ramsey problem in the literature.

Using the same parameter values as above, a numerical solution for the long run of this economy is presented in Table 1, Regime B. The two columns of Regime B present the solution with, and without, externalities respectively.

A comparison of regimes A and B in Table 1 reveals that the economy with user fees is more efficient, both in terms of per capita income and welfare. This is because user fees do not distort the decision to work or save, while consumption and income taxes do (see equations (10a)-(10b)). The interesting question here is whether the gain in efficiency happens at the cost of a more unequal society as is usually believed. We address this question in the next section where we add two different groups of agents.

Before we move on to the next section, it is worth pointing out two properties of Ramsey tax policy in Table 1. First, in all cases, the optimal long-run income tax rate is zero. This is because there are less distorting public financing instruments available to use, like consumption taxes and user fees. Second, in Regime A, where there is a policy choice between consumption taxes and user fees, both of them should be positive when there are externalities, $0 < \gamma < 1$; by contrast, when the publicly provided good is exclusively private, $\gamma = 0$, it is optimal to use user fees only. The non-zero value of the consumption tax in the case with externalities is rationalized by the corrective role of tax policy. Namely, the government uses a relatively distorting policy instrument (as said, consumption taxes are more distorting than user prices) to correct for a market failure, here in the form of externalities.

3. A model with heterogeneous agents

We now add heterogeneity among individuals. Agents differ in capital ownership as in Judd (1985). Capital is in the hands of a small group called capitalists, while workers cannot save or borrow. This is the only difference from the model in the previous section. Since we follow the same steps and use the same notation as above, we avoid details when unnecessary.

Capitalists

There are $k = 1, 2, ..., N^k$ identical capitalists. Each k maximizes lifetime utility:

$$\sum_{t=0}^{\infty} \beta^t u(c_{k,t}, l_{k,t}, \tilde{g}_{k,t}) \tag{11}$$

where, as in the previous section, we have respectively for the period utility function, the specification of the excludable public good and the period budget constraint:

$$u(c_{k,t}, l_{k,t}, \tilde{g}_{k,t}) = \mu_1 \log(c_{k,t}) + \mu_2 \log(1 - l_{k,t}) + \mu_3 \log(\tilde{g}_{k,t}) \tag{12}$$

$$\tilde{g}_{k,t} \equiv g_{k,t} + \gamma \sum_{j \neq i}^{N} g_{k,t} \qquad \text{where } 0 \leq \gamma < 1 \tag{13}$$

$$(1 + \tau_t^c) c_{k,t} + k_{k,t+1} - (1 - \delta) k_{k,t} + p_t g_{k,t} = (1 - \tau_t^y)(r_t k_{k,t} + w_t l_{k,t}) \tag{14}$$

The first-order conditions for $c_{k,t}, l_{k,t}, k_{k,t+1}, g_{k,t}$ include the budget constraint and:

$$\frac{1}{(1 + \tau_t^c) c_{k,t}} = \frac{\beta [1 - \delta + (1 - \tau_{t+1}^y) r_{t+1}]}{(1 + \tau_{t+1}^c) c_{k,t+1}} \tag{15a}$$

$$\frac{\mu_1 (1 - \tau_t^y) w_t}{(1 + \tau_t^c) c_{k,t}} = \frac{\mu_2}{1 - l_{k,t}} \tag{15b}$$

$$\frac{\mu_1 p_t}{(1 + \tau_t^c) c_{k,t}} = \frac{\mu_3}{[1 + \gamma (N^k - 1)] g_{k,t} + \gamma N^w g_{w,t}} \tag{15c}$$

which are like equations (5a-c) above. That is, (15c) is the first-order condition for $g_{k,t}$ and gives the capitalist's demand function for the excludable public good (where N^k is the number of identical capitalists and $N^w = N - N^k$ is the number of identical workers).

Workers

There are $w = 1, 2, ..., N^w = N - N^k$ identical workers. Each w maximizes lifetime utility:

$$\sum_{t=0}^{\infty} \beta^t u(c_{w,t}, l_{w,t}, \tilde{g}_{w,t}) \tag{16}$$

where, as above, we have:

$$u(c_{w,t}, l_{w,t}, \tilde{g}_{w,t}) = \mu_1 \log(c_{w,t}) + \mu_2 \log(1 - l_{w,t}) + \mu_3 \log(\tilde{g}_{w,t}) \tag{17}$$

$$\tilde{g}_{w,t} \equiv g_{w,t} + \gamma \sum_{j \neq i}^{N} g_{w,t} \qquad \text{where } 0 \leq \gamma < 1 \tag{18}$$

$$(1 + \tau_t^c) c_{w,t} + p_t g_{w,t} = (1 - \tau_t^y) w_t l_{w,t} \tag{19}$$

This is a static problem. The first-order conditions for $c_{w,t}, l_{w,t}, g_{w,t}$ include the budget constraint and:

$$\frac{\mu_1 (1 - \tau_t^y) w_t}{(1 + \tau_t^c) c_{w,t}} = \frac{\mu_2}{1 - l_{w,t}} \tag{20a}$$

$$\frac{\mu_1 p_t}{(1 + \tau_t^c) c_{w,t}} = \frac{\mu_3}{[1 + \gamma (N^w - 1)] g_{w,t} + \gamma N^k g_{k,t}} \tag{20b}$$

which are analogous to (15b-c) above.

Firms

There are $f = 1, 2, ..., N^k$ firms owned by capitalists. Thus, each capitalist owns one firm. The behavior of the firm remains as in the previous section.

Government budget constraint

The period budget constraint of the government is now (in aggregate terms):

$$G_t = N^k[\tau_t^y(r_t k_{k,t} + w_t l_{k,t}) + \tau_t^c c_{k,t} + p_t g_{k,t}] + N^w[\tau_t^y w_t l_{w,t} + \tau_t^c c_{w,t} + p_t g_{w,t}] \tag{21}$$

Decentralized competitive equilibrium (for any feasible policy)

In the decentralized competitive equilibrium (DCE), capitalists and workers maximize utility, firms maximize profits, all constraints are satisfied and all markets clear (see Appendix B.1 for the market-clearing condition for the excludable public good and the associated equilibrium value of the user fee).[12]

It is convenient to define the population shares of the two groups, $n^k \equiv N^k / N$ and $n^w \equiv N^w / N = 1 - n^k$. Then, the DCE is summarized by the following nine equations (quantities are in per capita terms):

$$\frac{1}{(1+\tau_t^c)c_{k,t}} = \frac{\beta[1-\delta+(1-\tau_{t+1}^y)r_{t+1}]}{(1+\tau_{t+1}^c)c_{k,t+1}} \tag{22a}$$

$$\frac{\mu_1(1-\tau_t^y)w_t}{(1+\tau_t^c)c_{k,t}} = \frac{\mu_2}{1-l_{k,t}} \tag{22b}$$

$$\frac{\mu_1(1-\tau_t^y)w_t}{(1+\tau_t^c)c_{w,t}} = \frac{\mu_2}{1-l_{w,t}} \tag{22c}$$

$$(1+\tau_t^c)c_{w,t} + \frac{\{[1+\gamma(N^k-1)]c_{w,t} - \gamma N^k c_{k,t}\}g_t p_t}{\left[n^k\{[1+\gamma(N^w-1)]c_{k,t} - \gamma N^w c_{w,t}\} + n^w\{[1+\gamma(N^k-1)]c_{w,t} - \gamma N^k c_{k,t}\}\right]} = (1-\tau_t^y)w_t l_{w,t} \tag{22d}$$

$$n^k c_{k,t} + n^k[k_{k,t+1} - (1-\delta)k_{k,t}] + n^w c_{w,t} + g_t = n^k y_{f,t} \tag{22e}$$

$$g_t = \tau_t^y n^k y_{f,t} + \tau_t^c(n^k c_{k,t} + n^w c_{w,t}) + p_t g_t \tag{22f}$$

$$p_t = \frac{\mu_3(1+\tau_t^c)\left[n^k\{[1+\gamma(N^w-1)]c_{k,t} - \gamma N^w c_{w,t}\} + n^w\{[1+\gamma(N^k-1)]c_{w,t} - \gamma N^k c_{k,t}\}\right]}{\mu_1 g_t\{[1+\gamma(N^k-1)][1+\gamma(N^w-1)] - \gamma^2 N^k N^w\}} \tag{22g}$$

and, in turn, the amount of the excludable public good used by each type follows from:

[12] The market-clearing conditions in the labour and capital markets are respectively $N^f l_{f,t} = N^k l_{k,t} + N^w l_{w,t}$ and $N^f k_{f,t} = N^k k_{k,t}$. Recall that $N^f = N^k$; namely, the number of capitalists equals the number of firms.

$$g_{k,t} = \frac{\{[1+\gamma(N^w - 1)]c_{k,t} - \gamma N^w c_{w,t}\}}{\left[n^k\{[1+\gamma(N^w-1)]c_{k,t} - \gamma N^w c_{w,t}\} + n^w\{[1+\gamma(N^k-1)]c_{w,t} - \gamma N^k c_{k,t}\}\right]} g_t \quad (22h)$$

$$g_{w,t} = \frac{\{[1+\gamma(N^k - 1)]c_{w,t} - \gamma N^k c_{k,t}\}}{\left[n^k\{[1+\gamma(N^w-1)]c_{k,t} - \gamma N^w c_{w,t}\} + n^w\{[1+\gamma(N^k-1)]c_{w,t} - \gamma N^k c_{k,t}\}\right]} g_t \quad (22i)$$

where we use $n^k y_{f,t} = A(n^k k_{k,t})^{\alpha}(n^k l_{k,t} + n^w l_{w,t})^{1-\alpha}$, $r_t = \frac{\alpha y_{f,t}}{k_{k,t}}$ and $w_t = \frac{(1-\alpha)n^k y_{f,t}}{(n^k l_{k,t} + n^w l_{w,t})}$.

We thus have nine equations in $\{c_{k,t}, l_{k,t}, k_{k,t+1}, g_{k,t}, c_{w,t}, l_{w,t}, g_{w,t}, p_t\}_{t=0}^{\infty}$ and one of the policy instruments, $\{\tau_t^c, \tau_t^y, g_t\}_{t=0}^{\infty}$, which adjusts to satisfy the government budget constraint. This is for any feasible policy. We now turn to optimal policy.

Ramsey policy and allocation with user fees

The objective of the government is to maximize a weighted average of capitalists' and workers' utility concerning private goods (i.e. consumption and leisure) plus the utility enjoyed by both groups (i.e. capitalists and workers) from the provision of the public good. To do so, the government chooses $\{\tau_t^c, \tau_t^y, g_t, c_{k,t}, l_{k,t}, k_{k,t+1}, g_{k,t}, c_{w,t}, l_{w,t}, g_{w,t}, p_t\}_{t=0}^{\infty}$ subject to the DCE equations above. This policy problem is presented in detail in Appendix B.2.

We again solve the model numerically (see the notes in Table 2 for parameter values used, which are as in Table 1 plus values for the population shares of the two groups). We report that our results are robust to changes in parameter values. A numerical long-run solution is presented in Table 2, Regime A. The four columns in Regime A present the solution for four different values of the externality parameter $\gamma \geq 0$. In particular, we set $\gamma = 0.4$, $\gamma = 0.3$, $\gamma = 0.2$ and $\gamma = 0$, where the last case means that the publicly provided good is private. As in the previous section, before we discuss results, we solve the same economy without user fees.

Table 2 around here

Ramsey policy and allocation without user fees

We now examine what happens when the government provides the same amount of public goods as found above without charging user fees. That is, now we set $\{g_t, g_{k,t}, g_{w,t}\}_{t=0}^{\infty}$ as found above and assume that the government's public financing instruments are consumption and income taxes only.

In particular, the government chooses $\{\tau_t^c, \tau_t^y, c_{k,t}, l_{k,t}, k_{k,t+1}, c_{w,t}, l_{w,t}\}_{t=0}^{\infty}$ to maximize the same objective function taking $\{g_t, g_{k,t}, g_{w,t}\}_{t=0}^{\infty}$ as given subject to the new DCE equations. The latter are as in (22a-f) above where now the user fee is set at zero and private agents treat the amount of the excludable public good as given. This policy problem is presented in detail in Appendix B.3.

Using the same parameter values as above, a numerical solution for the long run of this economy is presented in Table 2, Regime B. A comparison of Regimes A and B in Table 2 reveals that the economy with user fees is not only more efficient (in terms of per capita output and welfare) but is also more just in terms of net income. In particular, the net income of capitalists relative to workers, denoted as y_k / y_w,[13] falls as we move from Regime B (case without user fees) to regime A (case with user fees). This happens both when the publicly provided good is private, $\gamma = 0$, and when it creates public good externalities, $\gamma > 0$.

To understand this redistribution result, recall that, by definition, changes in net income are driven by changes in gross income and/or tax payments. Our solution implies that, as we move from Regime B to Regime A, the gross income of both agents rises, since we move to a more efficient economy. Actually, the gross income of capitalists rises by more than the gross income of workers, so it cannot be changes in gross income that drive the redistribution result. In turn, inspection of tax payments implies that the tax burden of capitalists rises, while the tax burden of workers falls, as we move to Regime B to Regime A. Combining effects, tax payments as a ratio of gross income - namely, the average effective tax rate - rises for each capitalist and falls for each worker, as we move from Regime B to Regime A.[14] Therefore, the tax burden effect more than offsets the gross income effect and drives the net redistribution result. Notice that this is the case both with, and without, externalities, although the redistribution in favor of workers is stronger in the more general case with externalities, $\gamma > 0$.

---

[13] Thus, $y_k \equiv (1-\tau^y)(rk_k + wl_k) - \tau^c c_k - pg_k$ and $y_w \equiv (1-\tau^y)wl_w - \tau^c c_w - pg_w$.

[14] In particular, for the capitalist, $\frac{\tau^c c_k + pg_k}{rk_k + wl_k}$ in Regime A is higher than $\frac{\tau^c c_k}{rk_k + wl_k}$ in Regime B. By contrast, for the worker, $\frac{\tau^c c_w + pg_w}{wl_w}$ in Regime A is lower than $\frac{\tau^c c_w}{wl_w}$ in Regime B. Recall that, in the long run, the government finds it optimal to choose $\tau^y = 0$ in both regimes.

Depending on the value of the externality parameter, $\gamma \geq 0$, workers can get better off in terms of welfare too. This happens when γ is sufficiently high. Under our parameterization, when $\gamma \geq 0.3$, u_k / u_w rises as we move from Regime B to Regime A,[15] meaning that workers get relatively better when we introduce user fees (recall that $u_k, u_w < 0$, so an increase in absolute value denotes lower welfare). In general, as Table 2 shows, the stronger the external effects from the use of the public good, the stronger the fall in (both income and welfare) inequality.

4. Concluding remarks and extensions

We have studied the aggregate and distributional implications of introducing user prices for a publicly provided excludable public good. We believe that, although we have not provided a theory, we have used a standard model to show that user fees not only increase efficiency but can also reduce inequality.

Our paper belongs to a group of papers that question the validity of some widely perceived views in public policy (see Mankiw et al., 2009, for the gap between taxation in theory and practice). In the same spirit, Correia (2010) has shown that an exogenous policy reform that replaces the current US tax system with a flat consumption tax rate, accompanied by a lump-sum transfer that increases the progressivity of the tax system, can increase efficiency and reduce inequality in a model calibrated to the US economy. Similarly, in a companion paper (Economides and Philippopoulos, 2012), we show that, other things being equal, the introduction of consumption taxes to a model with income taxes only can reduce net income inequality between workers and capitalists when policy is chosen by a Ramsey government.

The paper can be enriched in several ways. For instance, we can also study transition effects. That is, we can study the aggregate and distributional implications when we depart from the long run of the economy without user fees and travel towards the long run of the same economy with user fees. Besides, we have set aside a lot of interesting issues in public economics (see the discussion in Hillman, 2009, chapter 3). For example, we have not examined the issue of asymmetric information. As is known, user prices provide useful information about personal benefits from public goods. Also, we have focused on the case in

[15] It is easy to find parameterizations where the fall in inequality starts taking place at lower critical values of

which the excludable public good is publicly provided. There are nevertheless cases where such goods are privately produced (by the so-called private providers) subject to government regulation. We leave these extensions for future work.

$\gamma \geq 0$. For instance, this can happen when the valuation given to leisure, μ_2, decreases.

References

Alesina A. and R. Wacziarg (1999): Is Europe is going too far? *Carnegie-Rocherster Conference Papers on Public Policy*, 51, 1-42.

Angelopoulos K., J. Malley and A. Philippopoulos (2011): Time-consistent fiscal policy under heterogeneity: conflicting or common interests?, *CESifo Working Paper*, no. 3444, Munich.

Atkinson A. and J. Stiglitz (1980): *Lectures on Public Economics*. McGraw Hill, London.

Chamley C. (1986): Optimal taxation of capital income in general equilibrium with infinite lives, *Econometrica*, 54, 607-622.

Correia I. (2010): Consumption taxes and redistribution, *American Economic Review*, 100, 1673-1694.

Economides G. and A. Philippopoulos (2008): Growth enhancing policy is the means to sustain the environment, *Review of Economic Dynamics*, 11, 207-219.

Economides G. and A. Philippopoulos (2012): A case for consumption taxes, *mimeo*.

Fowler S. and E. Young (2006): Uncertainty and fiscal policy, mimeo.

Fraser D. C. (1996): On the provision of excludable public goods, *Journal of Public Economics*, 60, 111-130.

Fuest C. and M. Kolmar (2007): A theory of user-fee competition, *Journal of Public Economics*, 91, 497-509.

Gertler P., L. Locay and W. Sanderson (1987): Are user fees regressive? The welfare implications of health care financing proposals in Peru, *Journal of Econometrics*, 36, 67-88.

Guo J.T. and K. Lansing (1999): Optimal taxation of capital income with imperfectly competitive product markets, *Journal of Economic Dynamics and Control*, 23, 967-995.

Hillman A. (2009): *Public Finance and Public Policy*. Cambridge University Press, second edition, Cambridge.

Hubel B. and M. Runkel (2009): Tax competition, excludable public goods and user charges, *International Tax and Public Finance*, 16, 321-336.

Judd K. (1985): Redistributive taxation in a simple perfect foresight model, *Journal of Public Economics*, 28, 59-83.

Krusell P. (2002): Time-consistent redistribution, *European Economic Review*, 46, 755-769.

Lansing (1999): Optimal redistributive capital taxation in a neoclassical growth model, *Journal of Public Economics*, 73, *423-453.*

Ljungqvist L. and T. Sargent (2004): *Recursive Macroeconomic Theory*. The MIT Press, second edition, Cambridge, Mass.

Mankiw G., M. Weinzierl and D. Yagan (2009): Optimal taxation in theory and practice, *Journal of Economic Perspectives*, 23, 147-174.

OECD: National Accounts Statistics, General Government Accounts, Main Aggregates, 2011.

Ott I. and S. Turnovsky (2006): Excludable and non-excludable public inputs: Consequences for economic growth, *Economica*, 73, 725-748.

Park H. and A. Philippopoulos (2003): On the dynamics of growth and fiscal policy with redistributive transfers, *Journal of Public Economics*, 87, 515-538.

Swope J. K. and E. Janeba (2005): Taxes or fees? The political economy of providing excludable public goods, *Journal of Public Economic Theory*, 7, 405–426.

APPENDICES

Appendix A: Identical agents

A.1 Market-clearing in the market for the excludable public good

The market for the excludable public good clears when $G_t = Ng_{i,t}$. Using the household's optimality condition (5c), the user fee is:

$$p_t = \frac{\mu_3(1+\tau_t^c)c_t}{\mu_1 g_t[1+\gamma(N-1)]} \tag{A.1.1}$$

where $g_t \equiv \frac{G_t}{N}$ is a policy instrument. Equation (A.1.1), or (10e) in the text, is used to substitute out the user fee in the DCE equations. Ott and Turnovsky (2006) work similarly.

A.2 Ramsey problem with user fees

(a) The DCE rewritten in terms of net factor returns

Following common practice, we work with net factor returns. In particular, we define $W_t \equiv (1-\tau_t^y)w_t$. Thus, $(1-\tau_t^y)r_t = \frac{\alpha}{(1-\alpha)}\frac{W_t l_t}{k_t}$ and $(1-\tau_t^y)y_t = \frac{W_t l_t}{(1-\alpha)}$. Then, using also (A.1.1) or (10e) to substitute out p_t, we rewrite the DCE equations (10a-d) in the text as:

$$\frac{1}{(1+\tau_t^c)c_t} = \frac{\beta[1-\delta+\frac{\alpha}{(1-\alpha)}\frac{W_{t+1}l_{t+1}}{k_{t+1}}]}{(1+\tau_{t+1}^c)c_{t+1}} \tag{A.2.1}$$

$$\frac{\mu_1 W_t}{(1+\tau_t^c)c_t} = \frac{\mu_2}{1-l_t} \tag{A.2.2}$$

$$c_t + k_{t+1} - (1-\delta)k_t + g_t = y_t \tag{A.2.3}$$

$$g_t = y_t - \frac{W_t l_t}{(1-\alpha)} + \tau_t^c c_t + \frac{\mu_3(1+\tau_t^c)c_t}{\mu_1[1+\gamma(N-1)]} \tag{A.2.4}$$

where we use $y_t = A(k_t)^{\alpha}(l_t)^{1-\alpha}$. We thus have 4 equations in $\{c_t, l_t, k_{t+1}\}_{t=0}^{\infty}$ and one of the policy instruments, $\{\tau_t^c, W_t, g_t\}_{t=0}^{\infty}$.

(b) Ramsey problem

As said in the text, the government chooses $\{\tau_t^c, \tau_t^y, g_t, c_t, l_t, k_{t+1}, p_t\}_{t=0}^{\infty}$ to maximize:

$$\sum_{t=0}^{\infty}\beta^t[\mu_1\log(c_t)+\mu_2\log(1-l_t)+\mu_3\log(g_t)] \tag{A.2.5}$$

subject to (10a-e) in the text.

The above problem is equivalent to choosing $\{\tau_t^c, W_t, g_t, c_t, l_t, k_{t+1}\}_{t=0}^{\infty}$ to maximize (A.2.5) subject to (A.2.1)-(A.2.4). The Lagrangean is:

$$\sum_{t=0}^{\infty}\beta^t\left\{\begin{array}{l}\mu_1\log(c_t)+\mu_2\log(1-l_t)+\mu_3\log(g_t)+\lambda_{1t}\left[(1-\alpha)k_{t+1}(1+\tau_{t+1}^c)c_{t+1}-\beta(1+\tau_t^c)c_t(\alpha W_{t+1}l_{t+1}+(1-\delta)(1-\alpha)k_{t+1})\right]\\+\lambda_{2t}\left[\mu_2(1+\tau_t^c)c_t-\mu_1W_t(1-l_t)\right]+\lambda_{3t}\left[c_t+k_{t+1}-(1-\delta)k_t+g_t-Ak_t^{\alpha}l_t^{1-\alpha}\right]\\+\lambda_{4t}\left[g_t-Ak_t^{\alpha}l_t^{1-\alpha}+\dfrac{W_tl_t}{1-\alpha}-\tau_t^cc_t-\dfrac{\mu_3(1+\tau_t^c)c_t}{\mu_1[1+\gamma(N-1)]}\right]\end{array}\right\}$$

(A.2.6)

As is known, first-order conditions at $t=0$ and $t\geq 1$ differ. At $t\geq 1$, the first-order conditions with respect to $\tau_t^c, W_t, g_t, c_t, l_t, k_{t+1}$ are respectively:

$$\lambda_{1t-1}c_t(1-\alpha)k_t-\beta^2\lambda_{1t}c_t(\alpha W_{t+1}l_{t+1}+(1-\delta)(1-\alpha)k_{t+1})+\beta\lambda_{2t}\mu_2c_t-\beta\lambda_{4t}c_t-\beta\lambda_{4t}\frac{\mu_3c_t}{\mu_1[1+\gamma(N-1)]}=0 \tag{A.2.7.1}$$

$$-\alpha\lambda_{1t-1}(1+\tau_{t-1}^c)c_{t-1}l_t-\mu_1\lambda_{2t}(1-l_t)+\lambda_{4t}\frac{1}{1-\alpha}l_t=0 \tag{A.2.7.2}$$

$$\mu_3+(\lambda_{3t}+\lambda_{4t})g_t=0 \tag{A.2.7.3}$$

$$\begin{gathered}\beta\frac{\mu_1}{c_t}+\lambda_{1t-1}(1+\tau_t^c)(1-\alpha)k_t-\beta^2\lambda_{1t}(1+\tau_t^c)(\alpha W_{t+1}l_{t+1}+(1-\delta)(1-\alpha)k_{t+1})+\beta\lambda_{2t}\mu_2(1+\tau_t^c)+\beta\lambda_{3t}\\-\beta\lambda_{4t}\tau_t^c-\beta\lambda_{4t}\frac{\mu_3(1+\tau_t^c)}{\mu_1[1+\gamma(N-1)]}=0\end{gathered} \tag{A.2.7.4}$$

$$\begin{gathered}-\frac{\mu_2}{1-l_t}-\lambda_{1t-1}(1+\tau_{t-1}^c)c_{t-1}\alpha W_t+\lambda_{2t}\mu_1W_t-\lambda_{3t}(1-\alpha)Ak_t^{\alpha}l_t^{-\alpha}-\lambda_{4t}(1-\alpha)Ak_t^{\alpha}l_t^{-\alpha}\\+\lambda_{4t}\frac{W_t}{1-\alpha}=0\end{gathered} \tag{A.2.7.5}$$

$$\begin{gathered}\lambda_{1t}\Big((1+\tau_{t+1}^c)c_{t+1}(1-\alpha)-\beta(1+\tau_t^c)c_t(1-\alpha)(1-\delta)\Big)+\lambda_{3t}-\beta\lambda_{3t+1}(1-\delta+\alpha Ak_{t+1}^{\alpha-1}l_{t+1}^{1-\alpha})\\-\beta\lambda_{4t+1}\alpha Ak_{t+1}^{\alpha-1}l_{t+1}^{1-\alpha}=0\end{gathered} \tag{A.2.7.6}$$

Equations (A.2.7.1)-(A.2.7.6) and (A.2.1)-(A.2.4) constitute a ten-equation system in $\tau_t^c, W_t, g_t, c_t, l_t, k_{t+1}, \lambda_{1t}, \lambda_{2t}, \lambda_{3t}$ and λ_{4t}, where $\lambda_{1t}, \lambda_{2t}, \lambda_{3t}, \lambda_{4t}$ are dynamic multipliers. In turn, the long-run system follows from this system when variables do not change.

A.3 Ramsey problem without user fees

(a) The DCE rewritten in terms of net factor returns

Without user fees, equations (A.2.1)-(A.2.4) simplify to:

$$\frac{1}{(1+\tau_t^c)c_t} = \frac{\beta[1-\delta+\frac{\alpha}{(1-\alpha)}\frac{W_{t+1}l_{t+1}}{k_{t+1}}]}{(1+\tau_{t+1}^c)c_{t+1}} \tag{A.3.1}$$

$$\frac{\mu_1 W_t}{(1+\tau_t^c)c_t} = \frac{\mu_2}{1-l_t} \tag{A.3.2}$$

$$c_t + k_{t+1} - (1-\delta)k_t + g_t = y_t \tag{A.3.3}$$

$$g_t = y_t - \frac{W_t l_t}{(1-\alpha)} + \tau_t^c c_t \tag{A.3.4}$$

where we use $y_t = A(k_t)^\alpha (l_t)^{1-\alpha}$. We thus have 4 equations in $\{c_t, l_t, k_{t+1}\}_{t=0}^\infty$ and one of the policy instruments, $\{\tau_t^c, W_t\}_{t=0}^\infty$. Recall that now $\{g_t\}_{t=0}^\infty$ is set.

(b) Ramsey problem

Now the government chooses $\{\tau_t^c, W_t, c_t, l_t, k_{t+1}\}_{t=0}^\infty$ to maximize (A.2.5) subject to (A.3.1)-(A.3.4). The Lagrangean is:

$$\sum_{t=0}^{\infty}\beta^t \left\{ \begin{array}{l} \mu_1 \log(c_t) + \mu_2 \log(1-l_t) + \mu_3 \log(g_t) + \lambda_{1t}\left[(1-\alpha)k_{t+1}(1+\tau_{t+1}^c)c_{t+1} - \beta(1+\tau_t^c)c_t(\alpha W_{t+1}l_{t+1} + (1-\delta)(1-\alpha)k_{t+1})\right] \\ + \lambda_{2t}\left[\mu_2(1+\tau_t^c)c_t - \mu_1 W_t(1-l_t)\right] + \lambda_{3t}\left[c_t + k_{t+1} - (1-\delta)k_t + g_t - Ak_t^\alpha l_t^{1-\alpha}\right] + \lambda_{4t}\left[g_t - Ak_t^\alpha l_t^{1-\alpha} + \frac{W_t l_t}{1-\alpha} - \tau_t^c c_t\right] \end{array} \right\} \tag{A.3.5}$$

As is known, first-order conditions at $t=0$ and $t \geq 1$ differ. At $t \geq 1$, the first-order conditions with respect to $\tau_t^c, W_t, c_t, l_t, k_{t+1}$ are respectively:

$$\lambda_{1t-1}c_t(1-\alpha)k_t - \beta^2\lambda_{1t}c_t(\alpha W_{t+1}l_{t+1} + (1-\delta)(1-\alpha)k_{t+1}) + \beta\lambda_{2t}\mu_2 c_t - \beta\lambda_{4t}c_t = 0 \tag{A.3.6.1}$$

$$-\alpha\lambda_{1t-1}(1+\tau_{t-1}^c)c_{t-1}l_t - \mu_1\lambda_{2t}(1-l_t) + \lambda_{4t}\frac{1}{1-\alpha}l_t = 0 \tag{A.3.6.2}$$

$$\beta\frac{\mu_1}{c_t} + \lambda_{1t-1}(1+\tau_t^c)(1-\alpha)k_t - \beta^2\lambda_{1t}(1+\tau_t^c)(\alpha W_{t+1}l_{t+1} + (1-\delta)(1-\alpha)k_{t+1}) + \beta\lambda_{2t}\mu_2(1+\tau_t^c) + \beta\lambda_{3t} - \beta\lambda_{4t}\tau_t^c = 0 \tag{A.3.6.3}$$

$$-\frac{\mu_2}{1-l_t} - \lambda_{1t-1}(1+\tau_{t-1}^c)c_{t-1}\alpha W_t + \lambda_{2t}\mu_1 W_t - \lambda_{3t}(1-\alpha)Ak_t^\alpha l_t^{-\alpha} - \lambda_{4t}(1-\alpha)Ak_t^\alpha l_t^{-\alpha} + \lambda_{4t}\frac{W_t}{1-\alpha} = 0 \tag{A.3.6.4}$$

$$\lambda_{1t}\Big((1+\tau_{t+1}^{c})c_{t+1}(1-\alpha)-\beta(1+\tau_{t}^{c})c_{t}(1-\alpha)(1-\delta)\Big)+\lambda_{3t}-\beta\lambda_{3t+1}(1-\delta+\alpha Ak_{t+1}^{\alpha-1}l_{t+1}^{1-\alpha}) \\ -\beta\lambda_{4t+1}\alpha Ak_{t+1}^{\alpha-1}l_{t+1}^{1-\alpha}=0 \tag{A.3.6.5}$$

Equations (A.3.6.1)-(A.3.6.5) and (A.3.1)-(A.3.4) constitute a nine-equation system in $\tau_t^c, W_t, c_t, l_t, k_{t+1}, \lambda_{1t}, \lambda_{2t}, \lambda_{3t}$ and λ_{4t}, where $\lambda_{1t}, \lambda_{2t}, \lambda_{3t}, \lambda_{4t}$ are new dynamic multipliers. In turn, the long-run system follows from this system when variables do not change.

Appendix B: Heterogeneous agents

B.1 Market-clearing in the market for the excludable public good

Capitalists' and workers' optimality conditions for the public good, (15c) and (20b) respectively, imply:

$$g_{k,t}=\frac{\mu_3(1+\tau_t^c)}{\mu_1 p_t}\frac{\{[1+\gamma(N^w-1)]c_{k,t}-\gamma N^w c_{w,t}\}}{\{[1+\gamma(N^k-1)][1+\gamma(N^w-1)]-\gamma^2 N^k N^w\}} \tag{B.1.1}$$

$$g_{w,t}=\frac{\mu_3(1+\tau_t^c)}{\mu_1 p_t}\frac{\{[1+\gamma(N^k-1)]c_{w,t}-\gamma N^k c_{k,t}\}}{\{[1+\gamma(N^k-1)][1+\gamma(N^w-1)]-\gamma^2 N^k N^w\}} \tag{B.1.2}$$

The market for the excludable public good clears when $G_t = N^k g_{k,t} + N^w g_{w,t}$. Using (B.1.1)-(B.1.2) into this market-clearing condition, we get for the user fee:

$$p_t=\frac{\mu_3(1+\tau_t^c)\Big[n^k\{[1+\gamma(N^w-1)]c_{k,t}-\gamma N^w c_{w,t}\}+n^w\{[1+\gamma(N^k-1)]c_{w,t}-\gamma N^k c_{k,t}\}\Big]}{\mu_1 g_t\{[1+\gamma(N^k-1)][1+\gamma(N^w-1)]-\gamma^2 N^k N^w\}} \tag{B.1.3}$$

Using this expression for the user fee back into (B.1.1)-(B.1.2), we have:

$$g_{k,t}=\frac{\{[1+\gamma(N^w-1)]c_{k,t}-\gamma N^w c_{w,t}\}}{\Big[n^k\{[1+\gamma(N^w-1)]c_{k,t}-\gamma N^w c_{w,t}\}+n^w\{[1+\gamma(N^k-1)]c_{w,t}-\gamma N^k c_{k,t}\}\Big]}g_t \tag{B.1.4}$$

$$g_{w,t}=\frac{\{[1+\gamma(N^k-1)]c_{w,t}-\gamma N^k c_{k,t}\}}{\Big[n^k\{[1+\gamma(N^w-1)]c_{k,t}-\gamma N^w c_{w,t}\}+n^w\{[1+\gamma(N^k-1)]c_{w,t}-\gamma N^k c_{k,t}\}\Big]}g_t \tag{B.1.5}$$

where $g_t \equiv \dfrac{G_t}{N}$ is a policy instrument.

B.2 Ramsey problem with user fees

(a) The DCE rewritten in terms of net factor returns

We define $W_t \equiv (1-\tau_t^y)w_t$. Thus, $(1-\tau_t^y)r_t = \frac{\alpha W_t}{(1-\alpha)}\frac{(n^k l_{k,t}+n^w l_{w,t})}{n^k k_{k,t}}$ and $(1-\tau_t^y)y_{f,t} = \frac{W_t}{(1-\alpha)}\frac{(n^k l_{k,t}+n^w l_{w,t})}{n^k}$.

Then, using also (B.1.3) or (22g) to substitute out p_t, we rewrite the DCE equations (22a-f) in the text as:

$$\frac{1}{(1+\tau_t^c)c_{k,t}} = \frac{\beta\left[1-\delta+\frac{\alpha W_{t+1}}{(1-\alpha)}\frac{(n^k l_{k,t+1}+n^w l_{w,t+1})}{n^k k_{k,t+1}}\right]}{(1+\tau_{t+1}^c)c_{k,t+1}} \quad \text{(B.2.1)}$$

$$\frac{\mu_1 W_t}{(1+\tau_t^c)c_{k,t}} = \frac{\mu_2}{1-l_{k,t}} \quad \text{(B.2.2)}$$

$$\frac{\mu_1 W_t}{(1+\tau_t^c)c_{w,t}} = \frac{\mu_2}{1-l_{w,t}} \quad \text{(B.2.3)}$$

$$(1+\tau_t^c)c_{w,t} + \frac{\mu_3(1+\tau_t^c)}{\mu_1}\frac{\{[1+\gamma(N^k-1)]c_{w,t}-\gamma N^k c_{k,t}\}}{\{[1+\gamma(N^k-1)][1+\gamma(N^w-1)]-\gamma^2 N^k N^w\}} = W_t l_{w,t} \quad \text{(B.2.4)}$$

$$n^k c_{k,t} + n^k[k_{k,t+1}-(1-\delta)k_{k,t}] + n^w c_{w,t} + g_t = n^k y_{f,t} \quad \text{(B.2.5)}$$

$$g_t = n^k y_{f,t} - \frac{W_t(n^k l_{k,t}+n^w l_{w,t})}{(1-\alpha)} + \tau_t^c(n^k c_{k,t}+n^w c_{w,t}) + \quad \text{(B.2.6)}$$

$$+\frac{\mu_3(1+\tau_t^c)\left[n^k\{[1+\gamma(N^w-1)]c_{k,t}-\gamma N^w c_{w,t}\}+n^w\{[1+\gamma(N^k-1)]c_{w,t}-\gamma N^k c_{k,t}\}\right]}{\mu_1\{[1+\gamma(N^k-1)][1+\gamma(N^w-1)]-\gamma^2 N^k N^w\}}$$

where we use $n^k y_{f,t} = A(n^k k_{k,t})^\alpha (n^k l_{k,t}+n^w l_{w,t})^{1-\alpha}$. We thus have 6 equations in $\{c_{k,t}, l_{k,t}, k_{k,t+1}, c_{w,t}, l_{w,t}\}_{t=0}^\infty$ and one of the policy instruments, $\{\tau_t^c, W_t, g_t\}_{t=0}^\infty$. Notice that in turn $\{p_t, g_{k,t}, g_{w,t}\}_{t=0}^\infty$ can follow residually from (B.1.3), (B.1.4) and (B.1.5) respectively.

(b) Ramsey problem

As said in the text, the government chooses $\{\tau_t^c, \tau_t^y, g_t, c_{k,t}, l_{k,t}, k_{k,t+1}, g_{k,t}, c_{w,t}, l_{w,t}, g_{w,t}, p_t\}_{t=0}^\infty$ to maximize:

$$\sum_{t=0}^\infty \beta^t\{\lambda[\mu_1\log(c_{k,t})+\mu_2\log(1-l_{k,t})]+(1-\lambda)[\mu_1\log(c_{w,t})+\mu_2\log(1-l_{w,t})]+\mu_3\log(g_t)\} \quad \text{(B.2.7)}$$

where $0\le\lambda\le1$ is the weight given to capitalists (we will focus on the Benthamite case in which $\lambda\equiv n^k$ and so $1-\lambda\equiv n^w$). This maximization is subject to (22a)-(22g) in the text. As said in the text, in (B.2.7), the government's objective is a weighted average of capitalists' and workers' utility, the only difference is that, instead of having the weighted average of the utility perceived by capitalists, $g_{k,t}$, and the utility perceived by workers, $g_{w,t}$, the government maximizes the utility of the total provision of the public good, g_t.

The above problem is equivalent to choosing $\{\tau_t^c, W_t, g_t, c_{k,t}, l_{k,t}, k_{k,t+1}, c_{w,t}, l_{w,t}\}_{t=0}^{\infty}$ to maximize (B.2.7) subject to (B.2.1)-(B.2.6). The Lagrangean is:

$$\sum_{t=0}^{\infty}\beta^t \left\{ \begin{array}{l} \lambda\left[\mu_1\log(c_{k,t})+\mu_2\log(1-l_{k,t})\right]+(1-\lambda)\left[\mu_1\log(c_{w,t})+\mu_2\log(1-l_{w,t})\right]+\mu_3\log(g_t) \\ +\lambda_{1t}\left[(1-\alpha)n^k k_{k,t+1}(1+\tau_{t+1}^c)c_{k,t+1}-\beta(1+\tau_t^c)c_{k,t}(\alpha W_{t+1}(n^k l_{k,t+1}+n^w l_{w,t+1})+(1-\delta)(1-\alpha)n^k k_{k,t+1})\right] \\ +\lambda_{2t}\left[\mu_1 W_t(1-l_{k,t})-\mu_2(1+\tau_t^c)c_{k,t}\right]+\lambda_{3t}\left[\mu_1 W_t(1-l_{w,t})-\mu_2(1+\tau_t^c)c_{w,t}\right] \\ +\lambda_{4t}\left[(1+\tau_t^c)c_{w,t}+B(1+\tau_t^c)\{[1+\gamma(N^k-1)]c_{w,t}-\gamma N^k c_{k,t}\}-W_t l_{w,t}\right] \\ +\lambda_{5t}\left[\begin{array}{l} g_t-A(n^k k_{k,t})^{\alpha}(n^k l_{k,t}+n^w l_{w,t})^{1-\alpha}+\dfrac{W_t(n^k l_{k,t}+n^w l_{w,t})}{1-\alpha}-\tau_t^c(n^k c_{k,t}+n^w c_{w,t}) \\ -B(1+\tau_t^c)\left[n^k\{[1+\gamma(N^w-1)]c_{k,t}-\gamma N^w c_{w,t}\}+n^w\{[1+\gamma(N^k-1)]c_{w,t}-\gamma N^k c_{k,t}\}\right] \end{array}\right] \\ +\lambda_{6t}\left[n^k c_{k,t}+n^k k_{k,t+1}-n^k(1-\delta)k_{k,t}+n^w c_{w,t}+g_t-A(n^k k_{k,t})^{\alpha}(n^k l_{k,t}+n^w l_{w,t})^{1-\alpha}\right] \end{array} \right\}$$

(B.2.8)

where $B\equiv\dfrac{\mu_3}{\mu_1}\left\{[1+\gamma(N^k-1)][1+\gamma(N^w-1)]-\gamma N^k N^w\right\}^{-1}$.

As is known, first-order conditions at $t=0$ and $t\geq 1$ differ. At $t\geq 1$, the first-order conditions with respect to $\tau_t^c, W_t, g_t, c_{k,t}, l_{k,t}, k_{k,t+1}, c_{w,t}, l_{w,t}$ are respectively:

$$\begin{array}{l} \lambda_{1t-1}(1-\alpha)n^k k_{k,t}c_{k,t}-\beta^2\lambda_{1t}c_{k,t}\left[\alpha W_{t+1}(n^k l_{k,t}+n^w l_{w,t})+(1-\delta)(1-\alpha)n^k k_{k,t+1}\right]-\beta\lambda_{2t}\mu_2 c_{k,t}-\beta\lambda_{3t}\mu_2 c_{w,t} \\ +\beta\lambda_{4t}c_{w,t}+\beta\lambda_{4t}B\{[1+\gamma(N^k-1)]c_{w,t}-\gamma N^k c_{k,t}\}-\beta\lambda_{5t}(n^k c_{k,t}+n^w c_{w,t}) \\ -\beta\lambda_{5t}B\left[n^k\{[1+\gamma(N^w-1)]c_{k,t}-\gamma N^w c_{w,t}\}+n^w\{[1+\gamma(N^k-1)]c_{w,t}-\gamma N^k c_{k,t}\}\right]=0 \end{array}$$

(B.2.9.1)

$$\begin{array}{l} -\alpha\lambda_{1t-1}(1+\tau_{t-1}^c)c_{k,t-1}(n^k l_{k,t}+n^w l_{w,t})+\lambda_{2t}\mu_1(1-l_{k,t})+\lambda_{3t}\mu_1(1-l_{w,t})-\lambda_{4t}l_{w,t} \\ +\lambda_{5t}\dfrac{1}{1-\alpha}(n^k l_{k,t}+n^w l_{w,t})=0 \end{array} \quad \text{(B.2.9.2)}$$

$$\mu_3+(\lambda_{5t}+\lambda_{6t})g_t=0 \quad \text{(B.2.9.3)}$$

$$\begin{array}{l} \beta\lambda\mu_1+\lambda_{1t-1}(1-\alpha)n^k k_{k,t}(1+\tau_t^c)c_{k,t}-\beta^2\lambda_{1t}(1+\tau_t^c)c_{k,t}[\alpha W_{t+1}(n^k l_{k,t}+n^w l_{w,t})+(1-\delta)(1-\alpha)n^k k_{k,t+1}] \\ -\beta\lambda_{2t}\mu_2(1+\tau_t^c)c_{k,t}-\beta\lambda_{4t}B(1+\tau_t^c)c_{k,t}\gamma N^k-\beta\lambda_{5t}\tau_t^c c_{k,t}n^k-\beta\lambda_{5t}B(1+\tau_t^c)c_{k,t}\left\{n^k[1+\gamma(N^w-1)]-n^w\gamma N^k\right\} \\ +\beta\lambda_{6t}n^k c_{k,t}=0 \end{array}$$

(B.2.9.4)

$$-\lambda\mu_2(n^k l_{k,t}+n^w l_{w,t})^\alpha-\lambda_{1t-1}(1+\tau_{t-1}^c)c_{k,t-1}\alpha W_t n^k(1-l_{k,t})(n^k l_{k,t}+n^w l_{w,t})^\alpha$$
$$-\lambda_{2t}\mu_1 W_t(1-l_{k,t})(n^k l_{k,t}+n^w l_{w,t})^\alpha-\lambda_{5t}(1-\alpha)A(n^k k_{k,t})^\alpha n^k(1-l_{k,t}) \quad \text{(B.2.9.5)}$$
$$+\lambda_{5t}\frac{n^k}{1-\alpha}W_t(1-l_{k,t})(n^k l_{k,t}+n^w l_{w,t})^\alpha-\lambda_{6t}(1-\alpha)A(n^k k_{k,t})^\alpha n^k(1-l_{k,t})=0$$

$$\lambda_{1t}k_{k,t+1}\left((1-\alpha)n^k(1+\tau_{t+1}^c)c_{k,t+1}-\beta(1+\tau_t^c)(1-\delta)(1-\alpha)n^k c_{k,t}\right)$$
$$-\beta\lambda_{5t+1}\alpha A(n^k k_{k,t+1})^\alpha(n^k l_{k,t+1}+n^w l_{w,t+1})^{1-\alpha}+\lambda_{6t}n^k k_{k,t+1} \quad \text{(B.2.9.6)}$$
$$-\beta\lambda_{6t+1}\left((1-\delta)n^k k_{k,t+1}+\alpha A(n^k k_{k,t+1})^\alpha(n^k l_{k,t+1}+n^w l_{w,t+1})^{1-\alpha}\right)=0$$

$$(1-\lambda)\mu_1-\lambda_{3t}\mu_2(1+\tau_t^c)c_{w,t}+\lambda_{4t}(1+\tau_t^c+B(1+\tau_t^c)[1+\gamma(N^k-1)])c_{w,t}-\lambda_{5t}\tau_t^c c_{w,t}n^w$$
$$-\lambda_{5t}B(1+\tau_t^c)c_{w,t}\left\{n^w[1+\gamma(N^k-1)]-n^k\gamma N^w\right\}+\lambda_{6t}n^w c_{w,t}=0 \quad \text{(B.2.9.7)}$$

$$-(1-\lambda)\mu_2(n^k l_{k,t}+n^w l_{w,t})^\alpha-\lambda_{1t-1}(1+\tau_{t-1}^c)c_{k,t-1}\alpha W_t n^w(1-l_{k,t})(n^k l_{k,t}+n^w l_{w,t})^\alpha$$
$$-\lambda_{3t}\mu_1 W_t(1-l_{w,t})(n^k l_{k,t}+n^w l_{w,t})^\alpha-\lambda_{4t}W_t(1-l_{w,t})(n^k l_{k,t}+n^w l_{w,t})^\alpha$$
$$-\lambda_{5t}(1-\alpha)A(n^k k_{k,t})^\alpha n^w(1-l_{k,t})+\lambda_{5t}\frac{n^w}{1-\alpha}W_t(1-l_{k,t})(n^k l_{k,t}+n^w l_{w,t})^\alpha \quad \text{(B.2.9.8)}$$
$$-\lambda_{6t}(1-\alpha)A(n^k k_{k,t})^\alpha n^w(1-l_{k,t})=0$$

Equations (B.2.9.1)-(B.2.9.8) and (B.2.1)-(B.2.6) constitute a fourteen-equation system in $\tau_t^c, W_t, g_t, c_{k,t}, l_{k,t}, k_{k,t+1}, c_{w,t}, l_{w,t}, \lambda_{1t}, \lambda_{2t}, \lambda_{3t}, \lambda_{4t}, \lambda_{5t}$ and λ_{6t}, where $\lambda_{1t}, \lambda_{2t}, \lambda_{3t}, \lambda_{4t}, \lambda_{5t}, \lambda_{6t}$ are new dynamic multipliers. In turn, the long-run system follows from this system when variables do not change.

B.3 Ramsey problem without user fees

(a) The DCE rewritten in terms of net factor returns

Without user fees, equations (B.2.1)-(B.2.6) simplify to:

$$\frac{1}{(1+\tau_t^c)c_{k,t}}=\frac{\beta\left[1-\delta+\dfrac{\alpha W_{t+1}}{(1-\alpha)}\dfrac{(n^k l_{k,t+1}+n^w l_{w,t+1})}{n^k k_{k,t+1}}\right]}{(1+\tau_{t+1}^c)c_{k,t+1}} \quad \text{(B.3.1)}$$

$$\frac{\mu_1 W_t}{(1+\tau_t^c)c_{k,t}}=\frac{\mu_2}{1-l_{k,t}} \quad \text{(B.3.2)}$$

$$\frac{\mu_1 W_t}{(1+\tau_t^c)c_{w,t}}=\frac{\mu_2}{1-l_{w,t}} \quad \text{(B.3.3)}$$

$$(1+\tau_t^c)c_{w,t}=W_t l_{w,t} \quad \text{(B.3.4)}$$

$$n^k c_{k,t}+n^k[k_{k,t+1}-(1-\delta)k_{k,t}]+n^w c_{w,t}+g_t=n^k y_{f,t} \quad \text{(B.3.5)}$$

$$g_t = n^k y_{f,t} - \frac{W_t(n^k l_{k,t} + n^w l_{w,t})}{(1-\alpha)} + \tau_t^c (n^k c_{k,t} + n^w c_{w,t}) \tag{B.3.6}$$

where we use $n^k y_{f,t} = A(n^k k_{k,t})^{\alpha} (n^k l_{k,t} + n^w l_{w,t})^{1-\alpha}$. We have 6 equations in $\{c_{k,t}, l_{k,t}, k_{k,t+1}, c_{w,t}, l_{w,t}\}_{t=0}^{\infty}$ and one of the policy instruments, $\{\tau_t^c, W_t\}_{t=0}^{\infty}$. Recall that now $\{g_t, g_{k,t}, g_{w,t}\}_{t=0}^{\infty}$ are set.

(b) Ramsey problem

Now the government chooses $\{\tau_t^c, W_t, c_{k,t}, l_{k,t}, k_{k,t+1}, c_{w,t}, l_{w,t}\}_{t=0}^{\infty}$ to maximize (B.2.7) subject to equations (B.3.1)-(B.3.6). The Lagrangean is:

$$\sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \lambda\left[\mu_1 \log(c_{k,t}) + \mu_2 \log(1-l_{k,t})\right] + (1-\lambda)\left[\mu_1 \log(c_{w,t}) + \mu_2 \log(1-l_{w,t})\right] + \mu_3 \log(g_t) \\ + \lambda_{1t}\left[(1-\alpha) n^k k_{k,t+1}(1+\tau_{t+1}^c) c_{k,t+1} - \beta(1+\tau_t^c) c_{k,t}(\alpha W_{t+1}(n^k l_{k,t+1} + n^w l_{w,t+1}) + (1-\delta)(1-\alpha) n^k k_{k,t+1})\right] \\ + \lambda_{2t}\left[\mu_1 W_t (1-l_{k,t}) - \mu_2 (1+\tau_t^c) c_{k,t}\right] + \lambda_{3t}\left[\mu_1 W_t (1-l_{w,t}) - \mu_2 (1+\tau_t^c) c_{w,t}\right] \\ + \lambda_{4t}\left[(1+\tau_t^c) c_{w,t} - W_t l_{w,t}\right] \\ + \lambda_{5t}\left[g_t - A(n^k k_{k,t})^{\alpha}(n^k l_{k,t} + n^w l_{w,t})^{1-\alpha} + \dfrac{W_t(n^k l_{k,t} + n^w l_{w,t})}{1-\alpha} - \tau_t^c (n^k c_{k,t} + n^w c_{w,t})\right] \\ + \lambda_{6t}\left[n^k c_{k,t} + n^k k_{k,t+1} - n^k(1-\delta) k_{k,t} + n^w c_{w,t} + g_t - A(n^k k_{k,t})^{\alpha}(n^k l_{k,t} + n^w l_{w,t})^{1-\alpha}\right] \end{array} \right\}$$

(B.3.7)

As is known, first-order conditions at t=0 and $t \geq 1$ differ. At $t \geq 1$, the first-order conditions with respect to $\tau_t^c, W_t, c_{k,t}, l_{k,t}, k_{k,t+1}, c_{w,t}, l_{w,t}$ are respectively:

$$\begin{array}{l} \lambda_{1t-1}(1-\alpha) n^k k_{k,t} c_{k,t} - \beta^2 \lambda_{1t} c_{k,t}\left[\alpha W_{t+1}(n^k l_{k,t} + n^w l_{w,t}) + (1-\delta)(1-\alpha) n^k k_{k,t+1}\right] - \beta\lambda_{2t}\mu_2 c_{k,t} - \beta\lambda_{3t}\mu_2 c_{w,t} \\ + \beta\lambda_{4t} c_{w,t} - \beta\lambda_{5t}(n^k c_{k,t} + n^w c_{w,t}) = 0 \end{array}$$

(B.3.8.1)

$$\begin{array}{l} -\alpha\lambda_{1t-1}(1+\tau_{t-1}^c) c_{k,t-1}(n^k l_{k,t} + n^w l_{w,t}) + \lambda_{2t}\mu_1(1-l_{k,t}) + \lambda_{3t}\mu_1(1-l_{w,t}) - \lambda_{4t} l_{w,t} \\ + \lambda_{5t}\dfrac{1}{1-\alpha}(n^k l_{k,t} + n^w l_{w,t}) = 0 \end{array} \tag{B.3.8.2}$$

$$\begin{array}{l} \beta\lambda\mu_1 + \lambda_{1t-1}(1-\alpha) n^k k_{k,t}(1+\tau_t^c) c_{k,t} - \beta^2 \lambda_{1t}(1+\tau_t^c) c_{k,t}[\alpha W_{t+1}(n^k l_{k,t} + n^w l_{w,t}) + (1-\delta)(1-\alpha) n^k k_{k,t+1}] \\ - \beta\lambda_{2t}\mu_2(1+\tau_t^c) c_{k,t} - \beta\lambda_{5t}\tau_t^c c_{k,t} n^k + \beta\lambda_{6t} n^k c_{k,t} = 0 \end{array}$$

(B.3.8.3)

$$\begin{array}{l} -\lambda\mu_2(n^k l_{k,t} + n^w l_{w,t})^{\alpha} - \lambda_{1t-1}(1+\tau_{t-1}^c) c_{k,t-1}\alpha W_t n^k (1-l_{k,t})(n^k l_{k,t} + n^w l_{w,t})^{\alpha} \\ - \lambda_{2t}\mu_1 W_t (1-l_{k,t})(n^k l_{k,t} + n^w l_{w,t})^{\alpha} - \lambda_{5t}(1-\alpha) A(n^k k_{k,t})^{\alpha} n^k (1-l_{k,t}) \\ + \lambda_{5t}\dfrac{n^k}{1-\alpha} W_t (1-l_{k,t})(n^k l_{k,t} + n^w l_{w,t})^{\alpha} - \lambda_{6t}(1-\alpha) A(n^k k_{k,t})^{\alpha} n^k (1-l_{k,t}) = 0 \end{array} \tag{B.3.8.4}$$

$$\lambda_{1t}k_{k,t+1}\Big((1-\alpha)n^k(1+\tau_{t+1}^c)c_{k,t+1}-\beta(1+\tau_t^c)(1-\delta)(1-\alpha)n^k c_{k,t}\Big)$$
$$-\beta\lambda_{5t+1}\alpha A(n^k k_{k,t+1})^{\alpha}(n^k l_{k,t+1}+n^w l_{w,t+1})^{1-\alpha}+\lambda_{6t}n^k k_{k,t+1} \quad \text{(B.3.8.5)}$$
$$-\beta\lambda_{6t+1}\Big((1-\delta)n^k k_{k,t+1}+\alpha A(n^k k_{k,t+1})^{\alpha}(n^k l_{k,t+1}+n^w l_{w,t+1})^{1-\alpha}\Big)=0$$

$$(1-\lambda)\mu_1-\lambda_{3t}\mu_2(1+\tau_t^c)c_{w,t}+\lambda_{4t}(1+\tau_t^c)c_{w,t}-\lambda_{5t}\tau_t^c c_{w,t}n^w+\lambda_{6t}n^w c_{w,t}=0 \quad \text{(B.3.8.6)}$$

$$-(1-\lambda)\mu_2(n^k l_{k,t}+n^w l_{w,t})^{\alpha}-\lambda_{1t-1}(1+\tau_{t-1}^c)c_{k,t-1}\alpha W_t n^w(1-l_{k,t})(n^k l_{k,t}+n^w l_{w,t})^{\alpha}$$
$$-\lambda_{3t}\mu_1 W_t(1-l_{w,t})(n^k l_{k,t}+n^w l_{w,t})^{\alpha}-\lambda_{4t}W_t(1-l_{w,t})(n^k l_{k,t}+n^w l_{w,t})^{\alpha}$$
$$-\lambda_{5t}(1-\alpha)A(n^k k_{k,t})^{\alpha}n^w(1-l_{k,t})+\lambda_{5t}\frac{n^w}{1-\alpha}W_t(1-l_{k,t})(n^k l_{k,t}+n^w l_{w,t})^{\alpha} \quad \text{(B.3.8.7)}$$
$$-\lambda_{6t}(1-\alpha)A(n^k k_{k,t})^{\alpha}n^w(1-l_{k,t})=0$$

Equations (B.3.8.1)-(B.3.8.7) and (B.3.1)-(B.3.6) constitute a thirteen-equation system in $\tau_t^c, W_t, c_{k,t}, l_{k,t}, k_{k,t+1}, c_{w,t}, l_{w,t}, \lambda_{1t}, \lambda_{2t}, \lambda_{3t}, \lambda_{4t}, \lambda_{5t}$ and λ_{6t}, where $\lambda_{1t}, \lambda_{2t}, \lambda_{3t}, \lambda_{4t}, \lambda_{5t}, \lambda_{6t}$ are new dynamic multipliers. In turn, the long-run system follows from this system when variables do not change.

Table 1: Long-run solution with identical agents

Endogenous variables	Regime A: with user fees		Regime B: without user fees	
	$\gamma = 0.3$	$\gamma = 0$	$\gamma = 0.3$	$\gamma = 0$
c	0.3305	0.3778	0.3049	0.2892
l	0.3137	0.3586	0.2954	0.2954
k	1.7085	1.9530	1.6090	1.6090
g	0.1102	0.1259	0.1102 (set)	0.1259 (set)
y	0.5774	0.6600	0.5438	0.5438
τ^y	0	0	0	0
τ^c	0.2231	0	0.3615	0.4354
p	0.3306	1	-	-
u	-0.6477	-0.7656	-0.6562	-0.7895

Notes: $\alpha = 0.36$, $\beta = 0.96$, $A = 1$, $\delta = 0.08$, $\mu_1 = 0.3$, $\mu_2 = 0.6$, $\mu_3 = 0.1$, $N = 10$.

Table 2: Long-run solution with heterogeneous agents

Endogenous variables	Regime A: with user fees				Regime B: without user fees			
	$\gamma = 0.4$	$\gamma = 0.3$	$\gamma = 0.2$	$\gamma = 0$	$\gamma = 0.4$	$\gamma = 0.3$	$\gamma = 0.2$	$\gamma = 0$
c_k	0.3661	0.3712	0.3793	0.4348	0.3466	0.3448	0.3421	0.3255
l_k	0.2317	0.2325	0.2353	0.2619	0.2069	0.2069	0.2069	0.2069
k_k	5.6313	5.6949	5.7978	6.5100	5.3632	5.3632	5.3632	5.3632
g_k	0.2005	0.1767	0.1591	0.1449	0.2005 (set)	0.1767 (set)	0.1591 (set)	0.1449 (set)
c_w	0.3127	0.3152	0.3197	0.3534	0.2914	0.2899	0.2876	0.2736
l_w	0.3438	0.3484	0.3553	0.4000	0.3333	0.3333	0.3333	0.3333
g_w	0.0671	0.0796	0.0905	0.1178	0.0671 (set)	0.0796 (set)	0.0905 (set)	0.1178 (set)
y	1.9032	1.9247	1.9595	2.2001	1.8126	1.8126	1.8126	1.8126
g	0.1071	0.1087	0.1111	0.1259	0.1071 (set)	0.1087 (set)	0.1111 (set)	0.1259 (set)
τ^y	0	0	0	0	0	0	0	0
τ^c	0.2363	0.2178	0.1877	0	0.3478	0.3548	0.3655	0.4354
p	0.2749	0.3350	0.4296	1	-	-	-	-
u_k	-0.5197	-0.5360	-0.5570	-0.6252	-0.5169	-0.5383	-0.5660	-0.6690
u_w	-0.6773	-0.6997	-0.7277	-0.8324	-0.6890	-0.7110	-0.7394	-0.8460
u	-0.6300	-0.6506	-0.6765	-0.7702	-0.6374	-0.6592	-0.6874	-0.7929
y_k / y_w	2.6115	2.6234	2.6369	2.7038	2.6622	2.6699	2.6816	2.7580
c_k / c_w	1.1707	1.1778	1.1862	1.2302	1.1897	1.1897	1.1897	1.1897
u_k / u_w	0.7673	0.7660	0.7654	0.7511	0.7502	0.7571	0.7655	0.7908

Notes: $\alpha = 0.36, \beta = 0.96, A = 1, \delta = 0.08, \mu_1 = 0.3, \mu_2 = 0.6, \mu_3 = 0.1, N^k = 3, N^w = 7, v^k = 0.3, v^w = 0.7, v = 0.3$.