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Abstract

We investigate externalities in higher education enrollment over the course of development in a two-sector model. Each sector works with only one type of labor, skilled or unskilled, and individuals are differentiated according to their cost of acquiring human capital. Both sectors exhibit productivity externalities in the size of the skill-specific labor and in the average human capital of workers. When skill-biased technological change prevails, it may well be the case that intermediate stages of development witness underenrollment in higher education, while highly developed economies experience overenrollment.

JEL-Code: D620, H230, I210, J240, O100.

Keywords: higher education, enrollment, externalities, two-sector model.

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1 Introduction

Is the number of college graduates in the population too high or too low? Over the last decades, the number of workers who hold an academic degree has increased tremendously. Nowadays, around 40 per cent of a birth cohort graduate from a theory-based program of tertiary education in OECD countries, ranging from graduation rates around 20% in Mexico and Turkey to rates exceeding 50% in Poland, Iceland and the Slovak Republic (OECD, 2011). High and even increasing skill premia in terms of lifetime income for people holding academic degrees in recent decades (Mitchell, 2005), even in many developing countries (Ripoll, 2005), underpin this trend. On the other hand, there are worries about future shortages of semi-skilled workers like nurses and technicians, raising doubts about whether or not the current share of students in higher education is already detrimental for growth or welfare. Indeed, hinting to the skill premium is not convincing, as only the productivity increase of the marginal, least talented, individual is of importance, corrected for possible externalities. The phenomenon of overeducation has been discussed in the empirical literature, though not in a conclusive way (Sicherman, 1991; Büchel, 2003; Chevalier, 2003). Being employed in an occupation that does not require the actual formal qualification of the worker at some given point in time does not necessarily indicate overeducation. It may easily go along with a substantial positive return to human capital investment in higher education in a lifetime perspective.

Our paper addresses the question of which pattern of overenrollment and underenrollment can be expected over the course of development. More specifically, the stylized facts suggest a move from underenrollment in some intermediate stage of industrialization to overenrollment in advanced economies. We try to explain this pattern within a simple structure of production externalities that are stationary in terms of parameters describing the externality, where the stage of development is described by the strength of the skill-bias of the technology used in the production process. When dealing with the issue of whether undereducation or overeducation prevails, our focus lies on externalities of the enrollment decision. We discuss why market forces lead to overinvestment or underinvestment in higher education, justifying government intervention in that sector. Though only a minority of the population enrolls in tertiary education, it is subsidized to a large extent in many countries, where policies toward tuition fees are far from uniform. While tuition fees are either negligible or even absent in many continental European countries, they can take values clearly exceeding average cost at several US universities. In the absence of market failures, standard considerations state that individual decisions to study will not be distorted with a proportional income tax when the subsidy rate of the direct cost coincides with the income tax rate (Trostel, 1993; Nielsen and Sorensen, 1997). This is true as the income tax reduces the returns to education and its opportunity cost by the same factor.

Our contribution focuses on externalities suggested by endogenous growth theory. Individuals are differentiated according to ability, which translates into differences in the cost of acquiring a university degree. Such a heterogeneity can be attributed to direct costs, e.g. need for additional tuition, opportunity costs, e.g. need to repeat some exams, or even psychic costs, as learning with lower ability will be harder. Although our formulation describes such psychic costs, generalizations would be straightforward.

We embed the endogenous enrollment decision in a simple model of a production economy with two sectors employing one type of labor - either skilled or unskilled - together with skill-specific technologies. We abstract from neoclassical scarcity effects from diminishing returns as they will typically not be a source of an externality. We also ignore the argument that when it comes to bargaining at the individual level, workers will only get a share of the productivity gain by education or training, thus pointing to underinvestment in human capital (Acemoglu, 1996; Acemoglu and Pischke, 1999).

Two main sources of market failure are considered, (i) an average human capital externality, and (ii) a size externality. Productivity in each sector depends on average human capital of the workers in the spirit of Lucas (1988). When the marginal individual decides to go to college, he disregards that average human capital will go down in each sector. This average human capital externality is clearly a source of overeducation from the point of view of a social planner. A similar overenrollment phenomenon would occur in a matching framework where lower average human capital levels would reduce investment of firms (Charlot and Decreuse, 2005).

Productivity of a sector also depends on the size of the sector, which may reflect learning by doing or productivity gains through improved division of labor. When enrollment in higher education increases, the skilled sector becomes larger and the unskilled sector becomes smaller. Hence, there is a negative externality on the unskilled sector and a positive externality on the skilled sector. Consequently, the net effect of a change in enrollment depends on the interplay between these externalities.

Over the course of development, the size of the skilled sector tends to grow, for example due to skilled-biased technological change. It may well be the case that the net effect of the enrollment on the aggregate welfare is negative in poor economies, positive in some medium range, and again negative in rich economies. This structure of externalities may give rise to a pattern of overinvestment in education in early and late stages of development and underenrollment in between.

The remainder of the paper is organized as follows. Section 2 introduces the model, and Section 3 deals with its equilibria and comparative statics. Optimal enrollment is discussed in Section 4. The final Section 5 concludes and indicates directions for future research.

2 The Model

2.1 Individuals and wages

Each individual lives for one period. Upon learning her ability type, she chooses whether or not to enroll in higher education. All university students graduate and work in the skilled sector, the other individuals work in the unskilled sector. Individuals are heterogeneous in ability a. For simplicity, let ability a be uniformly distributed on $[0, 1]$. Wages reflect productivity differences proportionally. In the unskilled sector, the income of an individual of ability level a is given by $y_u(a) = w_u a$, where w_u is a standard wage in the unskilled sector that would be paid to an individual with the highest ability $a = 1$. In the skilled sector, a worker of ability a earns $y_s(a) = w_s a$.

To keep the analysis tractable, utility is assumed to be logarithmic in income, $U(y) = \log(y)$. Acquiring skills is associated with a utility cost $C(a) = \log(1/a)$. Thus, individuals with the highest ability have utility cost of zero, and individuals with the lowest ability level will face an infinite cost. This ensures that the endogenous ability threshold that separates the skilled workers from the unskilled is interior whenever $w_s > w_u$. Individuals possess perfect foresight with respect to their prospective wage. An individual of ability a enrolls in education when net utility from doing so exceeds utility from remaining unskilled, that is, if $\log(w_s a) - \log(1/a) > \log(w_u a)$ holds. This implies that an agent will enroll if ability a exceeds the threshold level a^*, with

$$a^* = \frac{w_u}{w_s}. \tag{1}$$

2.2 Production

The economy under consideration consists of two sectors. For simplicity, each sector exclusively uses one type of labor, which is either skilled and unskilled. Both sectors are assumed to work under linear production functions: $Y_j = A_j H_j$, with $j \in \{s, u\}$, where H_j is the aggregate sector specific human capital. The coefficient A_j represents the level of technology. Since firms in each sector behave competitively, there is no residual income. One unit of human capital, corresponding to the highest ability level, is paid according to marginal productivity:

$$w_j = A_j. \tag{2}$$

The technology in each sector is determined by

$$A_j = \overline{A}_j \left(\widetilde{h}_j\right)^{\phi_j} (1 + N_j)^{\delta_j}, \tag{3}$$

with $\phi_j, \delta_j \in (0, 1)$. The term $\overline{A}_j$ expresses the exogenous productivity level of sector j in the period under consideration. Although it seems plausible that the current technology depends on the level of the previous period, or historical enrollment levels, we ignore such intertemporal spillovers. The term $\left(\widetilde{h}_j\right)^{\phi_j}$ displays an average human capital externality - the higher the average quality of workers in that sector, the more productive any unit of human capital is. Such an externality may occur if production takes place in teams, where a higher team quality in terms of human capital increases output of each worker in the team. Finally, $(1 + N_j)^{\delta_j}$ describes the size externality, expressing that productivity of each worker increases in the size of the sector N_j.[1]

[1] For microfoundations of such a productivity function see, for example, Schiopu (2010).

At given enrollment threshold a^*, the average human capital levels $\widetilde{h}_u$ and $\widetilde{h}_s$ in the unskilled and skilled sectors are:

$$\widetilde{h}_u = \frac{a^*}{2} \tag{4}$$

$$\widetilde{h}_s = \frac{a^*+1}{2} \tag{5}$$

As the population size is normalized to unity, we have sector sizes $N_u = a^*$ and $N_s = 1 - a^*$. We impose $\overline{A}_s > 2^{(\delta_u - \phi_u)}\overline{A}_u$. This condition ensures that $w_s > w_u$ holds at $a^* = 1$. Thus, if everybody plans to work in the unskilled sector, there is an incentive for the most talented type to enroll in university education.

3 Equilibrium and comparative statics

An interior equilibrium is defined by a market enrollment threshold $a_m = a^*$ satisfying equations (1)-(5). Notice that an additional equilibrium exists at $a^* = 0$. This trivial equilibrium is however unstable, as demonstrated below. Proposition 1 shows sufficient conditions under which a unique interior market enrollment rate always exists.

Proposition 1 *A market enrollment threshold $a_m \in (0,1)$ always exists. The market enrollment threshold is unique if $\phi_u + \delta_s + \frac{\delta_u - \phi_s}{2} \leq 1$ when $\delta_u - \phi_s > 0$, or $\phi_u + \delta_s < 1$ when $\delta_u - \phi_s \leq 0$.*

Proof. See Appendix A. □

In order to derive a stability condition for comparative static analysis, we consider the related differential equation

$$\dot{a}^* \equiv \frac{da^*}{dt} = f\left(\frac{w_u}{w_s} - a^*\right) = f(Z - a^*) \tag{6}$$

where $f(0) = 0$ and $f' > 0$. Hence, if the marginal individual would lose from enrolling, the enrollment threshold will go up, and vice versa. An equilibrium a_m will be stable if and only if $\frac{d\dot{a}^*}{da^*} \leq 0$. The sufficient stability condition requires that the strict inequality has to hold, which is equivalent here to $\left.\frac{\partial Z}{\partial a^*}\right|_{a^*=a_m} < 1$. This condition is fulfilled if the uniqueness condition from Proposition 1 holds. Since the latter is always met if the coefficients describing the externalitiy are sufficiently small, it is not particularly restrictive. In the following we assume that the condition of Proposition 1 is met, ensuring uniqueness and stability of the interior market equilibrium. Notice that the equilibrium at $a^* = 0$ is always unstable due to $\lim_{a^* \to 0} \frac{\partial Z}{\partial a^*} = \infty$ and can therefore be neglected. The comparative static properties of the interior market equilibrium a_m with respect to the technology parameters $\overline{A}_u$ and $\overline{A}_s$ can be derived in a straightforward fashion.

Proposition 2 *A higher* $\overline{A}_u/\overline{A}_s$ *increases the market enrollment threshold* a_m.

Proof. The market enrollment threshold is determined by $Z - a^* = 0$ with Z being defined as above. According to the implicit function theorem, $\frac{\partial a_m}{\partial \left(\overline{A}_u/\overline{A}_s\right)} = -\frac{\partial Z/\partial \left(\overline{A}_u/\overline{A}_s\right)}{\frac{\partial Z}{\partial a^*} - 1}$. Since the stability condition $\left.\frac{\partial Z}{\partial a^*}\right|_{a^*=a_m} < 1$ has to be met for meaningful comparative statics, it follows that

$$sgn\left[\frac{\partial a_m}{\partial \left(\overline{A}_u/\overline{A}_s\right)}\right] = sgn\left[\frac{\partial Z}{\partial \left(\overline{A}_u/\overline{A}_s\right)}\right] = sgn\left[Z/\left(\overline{A}_u/\overline{A}_s\right)\right] > 0. \tag{7}$$

□

The comparative static properties are easily understood. A lower $\overline{A}_u/\overline{A}_s$ indicates a stronger relative technological advantage of the skilled sector. As this will be translated into a higher skill premium at any enrollment level, the enrollment incentives are increased. This in turn leads to a lower enrollment threshold a_m and a higher enrollment rate $1 - a_m$.

4 Optimal enrollment

Welfare W is represented by a Benthamite utilitarian welfare function, aggregating utility from wage income minus utility losses due to acquiring human capital:

$$W = \int_0^{a^*} \log(w_u a) f(a) da + \int_{a^*}^{1} \log(w_s a) f(a) da - \int_{a^*}^{1} \log(1/a) f(a) da \tag{8}$$

Thus, aggregate welfare is derived by adding utility from income of the unskilled, $\int_0^{a^*} \log(w_u a) f(a) da$, to utility from income of the skilled, $\int_{a^*}^1 \log(w_s a) f(a) da$, net of the aggregate utility cost of higher education, $\int_{a^*}^1 \log(1/a) f(a) da$.

We are interested in how the net externality at the market enrollment evolves over the course of development. The development process is described by skilled-biased technological change, where $\overline{A}_u/\overline{A}_s$, the relative standard productivity of the unskilled sector, is falling over time. Accordingly, market enrollment rates are increasing and threshold abilities are decreasing. We consider market enrollment rates shrinking from almost unity to almost zero. Proposition 3 summarizes the results.

Proposition 3 *(i) If $a_m \to 1$, there is overenrollment, that is, $\frac{\partial W}{\partial a^*}(a_m) > 0$. (ii) If $a_m \to 0$, there is overenrollment, that is, $\frac{\partial W}{\partial a^*}(a_m) > 0$ provided that $\phi_s + \phi_u - \frac{1}{2}\delta_s > 0$. (iii) If there is overenrollment for $a_m \to 0$, underenrollment may occur for some $a_m \in \{0, 1\}$.*

Proof. Rewriting the welfare function yields

$$
\begin{aligned}
W &= \int_0^{a^*} \log(w_u a) f(a) da + \int_{a^*}^1 \log(w_s a) f(a) da - \int_{a^*}^1 \log(1/a) f(a) da \qquad (9) \\
&= \int_0^{a^*} \log(w_u) f(a) da + \int_0^{a^*} \log(a) f(a) da + \int_{a^*}^1 \log(w_s) f(a) da + 2\int_{a^*}^1 \log(a) f(a) da \\
&= a^* \log(w_u) + (1 - a^*) \log(w_s) + \int_{a^*}^1 \log(a) f(a) da + \int_0^1 \log(a) f(a) da \\
&= a^* \log\left[\overline{A}_u \left(a^*/2\right)^{\phi_u} \left(1 + a^*\right)^{\delta_u}\right] + (1 - a^*) \log\left[\overline{A}_s \left((a^* + 1)/2\right)^{\phi_s} \left(2 - a^*\right)^{\delta_s}\right] \\
&\quad + \int_{a^*}^1 \log(a) f(a) da + \int_0^1 \log(a) f(a) da
\end{aligned}
$$

Increasing the cutoff ability, thus decreasing enrollment, affects welfare as follows:

$$\begin{aligned}\frac{\partial W}{\partial a^*} &= \log(w_u) + \frac{a^*}{w_u}\frac{\partial w_u}{\partial a^*} - \log(w_s) + \frac{1-a^*}{w_s}\frac{\partial w_s}{\partial a^*} - \log(a^*) \\ &= \log\frac{w_u}{a^* w_s} + \frac{a^*}{w_u}\left[\frac{\phi_u}{a^*}w_u + \frac{\delta_u}{1+a^*}w_u\right] + \frac{1-a^*}{w_s}\left[\frac{\phi_s}{1+a^*}w_s - \frac{\delta_s}{2-a^*}w_s\right] \\ &= \log\frac{w_u}{a^* w_s} + \phi_u + \delta_u\frac{a^*}{1+a^*} + \phi_s\frac{1-a^*}{a^*+1} - \delta_s\frac{1-a^*}{2-a^*}.\end{aligned} \tag{10}$$

When evaluating $\frac{\partial W}{\partial a^*}$ at the market solution a_m, the first term becomes zero, according to (1):

$$\frac{\partial W}{\partial a^*}(a_m) = \phi_u + \phi_s\frac{1-a_m}{1+a_m} + \delta_u\frac{a_m}{1+a_m} - \delta_s\frac{1-a_m}{2-a_m}. \tag{11}$$

Claim (i) then is immediate from $\lim_{a^m \to 1}\frac{\partial W}{\partial a^*}(a_m) = \phi_u + \frac{1}{2}\delta_u > 0$. Considering $\lim_{a^m \to 0}\frac{\partial W}{\partial a^*}(a_m) = \phi_u + \phi_s - \frac{1}{2}\delta_s$ yields claim (ii). The final claim (iii) can be proved, for example, by considering the situation in which half of the population opts for working in the skilled sector, $\frac{\partial W}{\partial a^*}(\frac{1}{2}) = \phi_u + \frac{1}{3}\delta_u + \frac{1}{3}\phi_s - \frac{1}{3}\delta_s$. Should $\phi_s = \frac{1}{2}\delta_s$, while at the same time ϕ_u and δ_u are comparatively small, we arrive at a scenario with underenrollment in intermediate stages of development, switching again to overenrollment late. □

Decomposing the total change in welfare when there is a marginal positive change in a_m, that is, lower enrollment, yields the result shown in equation (11). The first term ϕ_u shows the positive effect on the average ability of the unskilled workers, while the second term $\phi_s\frac{1-a_m}{1+a_m}$ captures the positive impact on the average ability of the skilled workers. The size externalities have counteracting signs. While $\delta_u\frac{a_m}{1+a_m}$ expresses the positive impact on the market size of the unskilled technologies, the final term $\delta_s\frac{1-a_m}{2-a_m}$ shows a negative effect on the market size of the skilled technologies.

In the early stages of development, a_m is close to unity, where the impacts on the unskilled technology dominate. As enrollment decisions are made ignoring the negative size and quality effects on the unskilled sector, we arrive at overeducation. In late stages, when enrollment is pretty high, the size effect on the unskilled sector vanishes. In such a situation, the negative quality effect of enrollment on the skilled technology is already strong. This may be different in some intermediate range, as the quality externality in the skilled sector is more sensitive to the enrollment level than the size externality. In the example, the externalities in the skilled sector tend to offset each other in late stages of development, whereas the size externality dominates in the intermediate range. At the same time, the externalities in the unskilled sector remain comparatively small. In that event, we arrive at a scenario with underenrollment in intermediate stages of development, switching again to overenrollment late.

A possible objection is that the underenrollment outcome in intermediate stages requires an unrealistic high coefficient δ_s relative to ϕ_s. But recalling that we are still ignoring the underinvestment argument from the matching literature, stressing that productivity gains will not be fully reflected in the wage growth at the individual level, a scenario becomes likely with moving from an overenrollment situation in a developing country stage to underenrollment under intermediate industrialization, and back to overenrollment again in advanced countries.

Next, we study conditions under which the enrollment threshold that maximizes the aggregate welfare is interior and unique. Proposition 4 displays a sufficient existence condition on the miminal superiority of the skilled technology over the unskilled technology. If the coefficients describing the externalities are small, it is only slightly more demanding than set of the conditions ensuring the existence and uniqueness of market equilibria.

Proposition 4 *If* $\overline{A}_s/\overline{A}_u > 2^{\delta_u - \phi_u} \exp\left(\phi_u + \frac{\delta_u}{2}\right)$, *then there exists an interior enrollment threshold that maximizes welfare. This threshold is unique if the externality coefficients* $\delta_u, \phi_u, \delta_s, \phi_s$ *are sufficiently small.*

Proof. See Appendix B.

When inspecting the proof of Proposition 4, it transpires that ensuring uniqueness of the welfare maximum may entail conditions being somewhat more restrictive than related conditions on the uniqueness of market equilibria. Since it is reasonable to focus on externality coefficients that are small, uniqueness of the interior welfare optimum is guaranteed for any realistic specification. In fact, it is rather difficult to construct examples in which there are multiple solutions to the first-order condition characterizing an interior welfare optimum, though not impossible.

It is obvious that the government can improve welfare by introducing a subsidy to higher education in the case of underenrollment or a tax in the case of overenrollment, where tax revenue may be returned to all citizens in a lump-sum fashion. By appropriate choice of the level of the subsidy or tax, it can also induce any enrollment threshold $\widetilde{a}$ that maximizes welfare W.

Proposition 5 *If a unique welfare-maximizing enrollment threshold* $\widetilde{a} \in (0,1)$ *exists, it can be implemented by setting a tuition fee* $\theta w_s(a^*)$ *with*

$$\theta = 1 - \exp\left\{-\left[\phi_u + \delta_u \frac{\widetilde{a}}{1+\widetilde{a}} + \phi_s \frac{1-\widetilde{a}}{1+\widetilde{a}} - \delta_s \frac{1-\widetilde{a}}{2-\widetilde{a}}\right]\right\}.$$

Proof. The welfare-maximizing enrollment threshold $\widetilde{a}$ satisfies the first-order condition

$$\frac{\partial W}{\partial a^*}(\widetilde{a}) = \log \frac{w_u}{a^* w_s} + \phi_u + \delta_u \frac{a^*}{1+a^*} + \phi_s \frac{1-a^*}{a^*+1} - \delta_s \frac{1-a^*}{2-a^*} = 0. \tag{12}$$

With a tuition fee of $\theta w_s(a^*)$, the marginal individual with ability a^* will be characterized by

$$\log\left(w_s(a^*) - \theta w_s(a^*)\right) - \log(1/a^*) = \log\left(w_u(a^*)\right) \tag{13}$$

It remains to be shown that this tuition fee indeed induces $a^* = \widetilde{a}$. Rearranging (13) yields

$$\log(1-\theta) = \log \frac{w_u}{w_s a^*} \tag{14}$$

Using the definition of θ and equation (14) shows that the induced enrollment threshold indeed satisfies (12), that is, $a^* = \widetilde{a}$. □

The optimal tuition fee is related to the resulting market equilibrium standard skilled wage $w_s(a^*)$, where the "tax rate" θ is designed so as to internalize the externalities at the welfare-maximizing enrollment level. Taking into account the externalities, the decentralized market solution then coincides with the social optimum. When the net externalities of the enrollment decision at the social optimum

$\widetilde{a}$ are negative, $\phi_u + \delta_u \frac{\widetilde{a}}{1+\widetilde{a}} + \phi_s \frac{1-\widetilde{a}}{1+\widetilde{a}} - \delta_s \frac{1-\widetilde{a}}{2-\widetilde{a}} > 0$, the Pigouvian tuition fee is positive, $\theta > 0$, while a positive net externality at the welfare optimum requires a subsidy, $\theta < 0$.

5 Conclusions

We have analyzed a simple two-sector model with two types of technological externalities where the decision to acquire skill is endogenous. We show that an economy described in this fashion may well switch from overenrollment to underenrollment and back to overenrollment over the course of development without any government intervention. In such a situation, the optimal policy to discourage enrollment or encourage people to take higher education depends on the state of technological development.

We are still ignoring some underinvestment arguments from the literature, in particular that the wage premium of the worker will fall short of the individual productivity gain due to bargaining in an imperfect labor market. Moreover, underinvestment may occur in an intertemporal perspective if technological progress depends on the number of skilled workers in earlier periods.

On the other hand, we also neglect the overinvestment tendencies due to imperfect labor markets when payment by degree prevails. Depending on the extent of collectivity in wage setting institutions, firms tend to pay their workers by their formal qualification level - and not according to the personal human capital endowment of the worker. If this happens, the skill premium reflects the difference in marginal productivities for average workers in the respective sectors. As this skill premium is far larger than the productivity differential for the marginal individual, the incentive to enroll in university is too strong. Hence, payment by degree is a source of overinvestment in human capital. It clearly works in the same direction as the average human capital externality, but may induce a far stronger distortion.

Summing up, it is quite plausible that in an extended model being enriched by these arguments for overinvestment and underinvestment in university education a similar outcome of moving from undereducation to overeducation occurs when reaching late stages of economic development.

Appendix

A: Proof of Proposition 1

As firms behave competitively, the sector-specific wage levels are determined by the zero profit condition:

$$w_u = \overline{A}_u \left(\frac{a^*}{2}\right)^{\phi_u} (1+a^*)^{\delta_u}\,, \tag{15}$$

$$w_s = \overline{A}_s \left(\frac{a^*+1}{2}\right)^{\phi_s} (2-a^*)^{\delta_s}\,. \tag{16}$$

Using (1), we obtain the following expression in the market enrollment threshold:

$$a^* = \frac{\overline{A}_u \left(\frac{a^*}{2}\right)^{\phi_u} (1+a^*)^{\delta_u}}{\overline{A}_s \left(\frac{a^*+1}{2}\right)^{\phi_s} (2-a^*)^{\delta_s}} \tag{17}$$

Denote the RHS of equation (17) as Z. We can see that $\lim_{a^*\to 0} Z = 0$ and

$$\lim_{a^*\to 1} Z = \frac{\overline{A}_u\,(2)^{\delta_u-\phi_u}}{\overline{A}_s}, \tag{18}$$

being smaller than unity by our assumption $\overline{A}_s > 2^{(\delta_u-\phi_u)}\overline{A}_u$. Further,

$$\frac{\partial Z}{\partial a^*} = Z\left[\frac{\phi_u}{a^*} + \frac{\delta_u-\phi_s}{1+a^*} + \frac{\delta_s}{2-a^*}\right]. \tag{19}$$

Since $0 < \phi_u < 1$, it follows that $\lim_{a^*\to 0} \frac{\partial Z}{\partial a^*} = \infty$, ensuring that equation (17) has a solution $a^* \in (0,1)$. Denote the solution of this equation a_m. Uniqueness of this solution turns out if $\frac{\partial Z}{\partial a^*} < 1$ at any candidate solution a^*. Note that at any candidate solution $Z = a^*$, implying

$$\left.\frac{\partial Z}{\partial a^*}\right|_{a^*=a_m} = \phi_u + \frac{(\delta_u-\phi_s)\,a^*}{1+a^*} + \frac{\delta_s a^*}{2-a^*}. \tag{20}$$

Since both $a^*/(1+a^*)$ and $a^*/(2-a^*)$ are strictly increasing in the interval $[0,1]$, the RHS of (20) assumes its maximum at $a^*=1$ if $\delta_u-\phi_s>0$. Therefore, $\left.\frac{\partial Z}{\partial a^*}\right|_{a^*=a_m} < \phi_u+\delta_s+\frac{\delta_u-\phi_s}{2}$ when $\delta_u-\phi_s>0$, and $\left.\frac{\partial Z}{\partial a^*}\right|_{a^*=a_m} < \phi_u+\delta_s$ when $\delta_u-\phi_s \le 0$. This suffices to establish uniqueness of a_m.

B: Proof of Proposition 4

Given the threshold ability level a^*, welfare can be written as

$$W(a^*) = a^* \log\left[\overline{A}_u (a^*/2)^{\phi_u} (1+a^*)^{\delta_u}\right] \tag{21}$$
$$+(1-a^*)\log\left[\overline{A}_s ((a^*+1)/2)^{\phi_s} (2-a^*)^{\delta_s}\right] - 2 - a^*(\log a^* - 1).$$

The first derivative of (21) with respect to a^* is

$$\frac{\partial W}{\partial a^*} = \log\frac{w_u}{a^* w_s} + \phi_u + \delta_u \frac{a^*}{1+a^*} + \phi_s \frac{1-a^*}{a^*+1} - \delta_s \frac{1-a^*}{2-a^*} \tag{22}$$
$$= \log\frac{2^{\phi_s-\phi_u}\overline{A}_u}{\overline{A}_s} + (\phi_u - 1)\log a^* + (\delta_u - \phi_s)\log(1+a^*)$$
$$-\delta_s \log(2-a^*) + \phi_u + \delta_u \frac{a^*}{1+a^*} + \phi_s \frac{1-a^*}{a^*+1} - \delta_s \frac{1-a^*}{2-a^*}.$$

Note that $lim_{a^* \longrightarrow 0} \frac{\partial W}{\partial a^*}(a^*) = \infty$. Then we compute

$$\lim_{a^* \to 1} \frac{\partial W}{\partial a^*}(a^*) = \log\frac{2^{\phi_s-\phi_u}\overline{A}_u}{\overline{A}_s} + \phi_u + \delta_u \frac{1}{2} + \delta_u \log 2 - \phi_s \log 2 \tag{23}$$
$$= \log\frac{\overline{A}_u 2^{\delta_u - \phi_u}}{\overline{A}_s} + \phi_u + \frac{\delta_u}{2}$$
$$= -\log\frac{\overline{A}_s}{\overline{A}_u 2^{\delta_u-\phi_u}} + \phi_u + \frac{\delta_u}{2}.$$

Thus, $\lim_{a^* \to 1} \frac{\partial W}{\partial a^*}(a^*) < 0$ iff

$$\log\frac{\overline{A}_s}{\overline{A}_u 2^{\delta_u-\phi_u}} > \phi_u + \frac{\delta_u}{2}. \tag{24}$$

Since $W(a^*)$ is differentiable on the interval $[0,1]$, the properties $\lim_{a^* \to 0} \frac{\partial W}{\partial a^*}(a^*) > \infty$ and $\lim_{a^* \to 1} \frac{\partial W}{\partial a^*}(a^*) < 0$ ensure the existence of an interior welfare maximum. The maximum is unique if $\frac{\partial^2 W}{(\partial a^*)^2} < 0$ holds at any a^* for which $\frac{\partial W}{\partial a^*}(a^*) = 0$.

Straightforward derivation shows

$$\frac{\partial^2 W}{(\partial a^*)^2} = \frac{1}{a^*}\left[\phi_u + \frac{(\delta_u - \phi_s)\, a^*}{1+a^*} + \frac{\delta_s a^*}{2-a^*} - 1\right] \tag{25}$$
$$+.\frac{\delta_u - 2\phi_s}{(1+a^*)^2} + \frac{\delta_s}{(2-a^*)^2},$$

which is negative provided that the externality coefficients $\delta_u, \phi_u, \delta_s, \phi_s$ are sufficiently small.

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