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Abstract

This paper contributes to resolving the puzzle that in practice most countries use ad valorem (corporate income) taxation, while a large part of the tax competition literature views business taxes as unit (wealth) taxation. We point to the dual role corporate taxation plays in attracting mobile capital, on the one hand, and in absorbing economic rents, on the other hand. In contrast to the previous literature, we show (i) that detrimental tax competition may be less severe in a system of ad valorem taxes than in a system of unit taxes and (ii) that ad valorem taxation may be the equilibrium outcome in a decentralized world where countries decide themselves on the tax system. Interestingly, the decentralized choice of the ad valorem system may be a prisoner's dilemma since the countries' welfare may be higher if they choose unit taxes.

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1 Introduction

One of the main unresolved puzzles in the analysis of tax competition is that in practice almost all countries tax companies by ad valorem (corporate income) taxes, while the literature on tax competition mainly assumes that corporate taxes can be modelled as unit (wealth) taxes on capital.[1] It is even argued that unit taxes are superior to ad valorem taxes in the sense of less detrimental tax competition (Lockwood, 2004). The present paper contributes to resolving this puzzle. We point to the dual role that corporate taxation plays in attracting mobile capital, on the one hand, and in absorbing economic rents, on the other hand. In contrast to the previous literature, we show (i) that detrimental tax competition may be less severe in a system of ad valorem taxes than in a system of unit taxes and (ii) that ad valorem taxation may be the equilibrium outcome in a decentralized world where countries decide themselves on the tax system. Interestingly, the decentralized choice of the ad valorem system may be a prisoner's dilemma since the countries' welfare may be higher if they choose unit taxes.

We bring forward these results in a multi-country model of capital tax competition. This model is basically of the same type as the one used in the seminal papers of Zodrow and Mieszkowski (1986) and Wilson (1986). Each country is populated by a representative household, which consumes a private good and a local public good. The household receives income from supplying (mobile) capital on the world capital market and (immobile) labor on the local labor market. It also receives profit income from owing the representative firm in its country. The firm produces an output good with the help of capital and labor. It has to pay a corporate tax to the government of its country. We depart from the standard Zodrow-Mieszkowski-Wilson model in two important respects. First, we allow for pure economic rents by assuming that the production function exhibits non-increasing returns to scale. In case of constant returns to scale, economic rents are zero, while they become strictly positive for decreasing returns. Second, we consider both unit taxation as well as ad valorem taxation. While the tax base under unit taxation equals the amount of capital employed, taxable corporate income under ad valorem taxation is defined as the revenues from selling the output reduced by the total labor costs and the share of deductible capital costs.

Within this framework, corporate taxation plays two important roles. The first

[1] This modelling approach dates back to the initial analyses of Zodrow and Mieszkowski (1986) and Wilson (1986), and since then has been used in the largest part of the literature. Surveys on the tax competition literature can be found in, for example, Wilson (1999) and Wilson and Wildasin (2004).

role is that of an instrument in tax competition. By lowering the corporate tax rate, the countries may attract mobile capital and thereby improve their tax bases and tax revenues. As all countries behave in this way, however, they end up in a situation with inefficiently low taxation, which is the standard race to the bottom argument. With respect to this role of corporate taxation in detrimental tax competition, unit taxation dominates ad valorem taxation. The reason becomes clear if we consider the effective tax rate on capital. Under unit taxation, the effective tax rate is identical to the statutory tax rate. In contrast, under ad valorem taxation the effective tax rate depends on the marginal return to capital. Hence, a reduction in the statutory tax rate leads to a smaller reduction of the effective tax rate under unit taxation than under ad valorem taxation, since under the latter the induced increase in capital demand lowers the marginal return to capital and thereby has an additional effect on the effective tax rate. It follows that the tax rate elasticity of capital demand and thus the degree of detrimental tax competition is lower under unit taxation than ad valorem taxation.

The second role of corporate taxation comes into play when firms earn pure economic rents, which in our framework are generated by decreasing returns to scale in production.[2] In the presence of economic rents, governments can try to use corporate taxation as a means to absorb such rents and to shift resources from the private to the public sector. However, this works only under ad valorem taxation and not under unit taxation. The reason is the tax base definition under the two tax systems. Under unit taxation, the tax base equals the amount of capital employed and thus is not directly related to economic rents. Unit taxation can therefore not been used for rent absorption. In contrast, the tax base under ad valorem taxation equals taxable income which is positively related or even identical (when not only labor costs but also capital costs are fully tax deductible) to pure economic rents. Ad valorem taxation is thus an effective instrument to absorb economic rents. Hence, regarding the role of corporate taxes in rent absorption ad valorem taxation dominates unit taxation.

We derive our main results based on the two opposing roles of corporate taxation described above. We first characterize the Nash equilibrium of the countries' tax competition game under ad valorem taxation and under unit taxation, and then ask under which system tax competition is less detrimental. This comparison can be interpreted as a centralized choice of the tax system since we can think of a social planner who chooses the more efficient tax system before the countries engage in tax competition.

[2]Intuitively, our basic insights should also hold when economic rents are caused by other factors such as, for example, imperfect competition.

As a benchmark we first show that in case of constant returns to scale unit taxation causes less tax competition and thereby is superior to ad valorem taxation. The reason for that is the fact that for constant returns to scale there are no economic rents, and corporate taxation takes only the role of an instrument in tax competition. As explained above, with respect to this role unit taxation dominates ad valorem taxation. Under decreasing returns to scale, in contrast, the ranking of the tax systems may be reversed. In this case, economic rents arise and corporate taxation can also be used to absorb such rents. Hence, the effects of corporate taxation with respect to tax competition have to be traded-off against the effects of corporate taxation with respect to rent absorption. More specific, we show that under decreasing returns to scale ad valorem taxation leads to less tax competition and thus is superior to unit taxation, if the share of deductible capital costs is above a threshold value. The reason for this is that for high values of deductible capital costs the ad valorem tax is not too distortive in tax competition, so its disadvantage with respect to tax competition is relatively small and outweighed by its advantage with respect to rent absorption.

Beside this centralized choice of the tax system, we also consider a decentralized setting where countries not only engage in tax competition, but also choose the tax system in a non-cooperative way. Even though tractability reasons force us to restrict ourselves to numerical simulations of the two-country specification of our model, a clear pattern of the equilibrium choices can be identified. We show that under decreasing returns to scale the countries choose ad valorem taxation in equilibrium if the share of deductible capital costs is above a certain threshold value. This result as well as the intuition for it is basically the same as in the centralized setting mentioned above. However, in the decentralized world an important further effect occurs. In order to find their preferred tax system, countries not only have to consider the symmetric cases where both countries have the same tax system, but also asymmetric cases where one country chooses ad valorem taxation while the other chooses unit taxation. In such asymmetric cases terms of trade effects come into play and make tax competition profitable for the country facing the larger tax elasticity of capital demand.[3] As the tax elasticity is larger for the country with ad valorem taxation than for the country with unit taxation, the decentralized choice of the tax system is more biased in the direction of ad valorem taxation. It follows that the countries may end up in a prisoner's

[3]These terms of trade effects are similar to those identified in the asymmetric tax competition models of Bucovetsky (1991) and Wilson (1991). There, asymmetries are caused by differences in population sizes, while in our framework asymmetries correspond to the tax systems used.

dilemma situation where they both choose ad valorem taxation, but would do better if they decided to implement unit taxation.

Our results have important policy implications. From a positive point of view, our analysis may help to explain the widespread use of ad valorem (corporate income) taxation in practice. This is true for corporate income taxation in federal countries like, e.g., local business taxation in Germany or Switzerland where a central authority chooses the tax system. It is also true for corporate income taxation in economies where the tax system is determined locally. State corporate income taxation in the U.S. and corporate income taxation of the Canadian provinces are good examples. And also corporate income taxation on the international level fits our decentralized setting quite well. The interesting point from a normative point of view is that under certain circumstances the widespread use of ad valorem taxation in practice is inefficient. Under such circumstances it may be welfare-enhancing if countries agreed on a policy reform that implements more elements of unit taxation into the corporate tax system. Of course, in the end it is an empirical question whether the circumstances for such a profitable reform really hold and whether the effects identified in our paper are more important than other effects of corporate income taxation.

Recent years witnessed a growing literature on the comparison between unit and ad valorem taxation in capital tax competition. Our paper is most closely related to the central paper of Lockwood (2004). He focuses on the centralized case and demonstrates that with constant returns to scale and without any deductibility of capital costs, unit taxation dominates ad valorem taxation. Our benchmark result in case of constant returns to scale generalizes his result since we also allow for a strictly positive deductibility of capital costs. Lockwood (2004) does not consider decreasing returns to scale and therefore cannot obtain our rationale for ad valorem taxation. The same is true for the recent paper of Akai et al. (2011). They consider a setting with constant returns to scale and decentralized choice of the tax system. In their model it turns out that unit taxation is the Pareto-optimal tax system and that countries indeed choose unit taxation. Hence, they also cannot explain the widespread use of ad valorem taxation and, in contrast to our numerical results under decentralization, they do not derive a prisoner's dilemma in the choice of tax systems. To the best of our knowledge, the only paper that may help to explain the widespread use of ad valorem taxes in a tax competition framework is Ogawa (2011). He shows that for sufficiently large asymmetries between countries there exists an equilibrium where some countries choose ad valorem taxation. But in this equilibrium there are also countries which prefer unit taxation, different to our results. Moreover, Ogawa (2011) also focuses on constant returns to scale and zero deductibility of capital costs and thus cannot derive our rationale for ad valorem taxes.[4]

The paper is organized as follows. Section 2 presents the basic framework. Sections 3 and 4 characterize the tax competition equilibrium in the ad valorem and unit tax scenario, respectively. Section 5 considers the centralized choice of tax systems, while the decentralized setting is analyzed in Section 6. Section 7 concludes.

2 Basic Framework

We build on the Zodrow-Mieszkowski-Wilson tax competition model in the variant first investigated by Hoyt (1991). The economy consists of $n \geq 1$ countries, each hosting a representative firm. The firm in country $i \in \{1, \ldots, n\}$ uses k_i units of mobile capital and ℓ_i units of an immobile factor like labor in order to produce $F(k_i, \ell_i)$ units of a homogenous output good whose price is normalized to one. The production function F satisfies the usual assumptions. It exhibits positive and decreasing returns to each production factor, i.e. $F_k, F_\ell > 0$ and $F_{kk}, F_{\ell\ell} < 0$. Capital and labor are assumed to be complements in the sense that $F_{k\ell} > 0$. Moreover, we introduce

Assumption 1. *The production function F is homogenous of degree $\mu \in]0, 1]$ such that $F(k_i, \ell_i) = \ell_i^\mu H(k_i/\ell_i)$ with $\mu \in]0, 1]$ and $H(k_i/\ell_i) := F(k_i/\ell_i, 1)$.*

Lockwood (2004) focuses on constant returns to scale which are obtained as special case of Assumption 1 if $\mu = 1$. In contrast, we also allow values $\mu \in]0, 1[$ thereby taking into account that the production process may be characterized by decreasing returns to scale. In this case, there is a fixed third production factor such as for example, land or entrepreneurial service that gives rise to pure economic rents. For constant returns to scale ($\mu = 1$) these economic rents are zero.

[4] It is worth mentioning that a large body of literature deals with the difference between unit and ad valorem taxes in various market structures. Suits and Musgrave (1953) proved that any unit tax in a monopoly market can be substituted by an ad valorem tax which raises the same revenue but yields a lower market price. An even stronger result has been obtained by Skeath and Trandel (1994) who show that for any unit tax there exists an ad valorem tax which Pareto dominates it in the monopoly case (see also Blackorby and Murtya, 2007). Moreover, the findings of Suits and Musgrave (1953) are generalizable beyond the monopoly market structure, as has been shown by Delipalla and Keen (1992) for the oligopoly case (see also Anderson et al. 2001). However, all these studies analyse exclusively matters of tax incidence in closed economies, thus leaving the tax competition issue aside.

Each country is populated by a representative household. Country i's household inelastically supplies $\bar{\ell}$ units of labor on the local labor market at the wage rate w_i and $\bar{k}$ units of capital on the world capital market at the interest rate r. Moreover, it is the owner of the firm in country i and therefore obtains profit income π_i which equals the firm's after-tax profit. Country i's household uses its total income in order to purchase c_i units of a private consumption good. Its budget constraint therefore reads

$$c_i = r\bar{k} + w_i\bar{\ell} + \pi_i. \tag{1}$$

Additionally, the household in country i consumes g_i units of a local public good provided by the government of country i. Utility of country i's household can therefore be written as $U(c_i, g_i)$. The utility function U satisfies the standard assumptions. It is increasing in private and public consumption, i.e. $U_c, U_g > 0$, and strictly quasi-concave. Moreover, we follow Lockwood (2004) and impose

Assumption 2. *Both the private good and the public good are normal, and*

$$\frac{U_g[F(\bar{k},\bar{\ell}),0]}{U_c[F(\bar{k},\bar{\ell}),0]} > 1, \quad \frac{U_g[\bar{\ell}F_\ell(\bar{k},\bar{\ell}), F(\bar{k},\bar{\ell}) - \bar{\ell}F_\ell(\bar{k},\bar{\ell})]}{U_c[\bar{\ell}F_\ell(\bar{k},\bar{\ell}), F(\bar{k},\bar{\ell}) - \bar{\ell}F_\ell(\bar{k},\bar{\ell})]} < 1. \tag{2}$$

As will be seen in more detail below when we introduce corporate taxation, equation (2) states that the marginal rate of substitution between public and private consumption is larger (smaller) than one if corporate taxes are set at their minimum (maximum).

The local labor market in country i balances labor demand of country i's firm and labor supply of country i's household. The equilibrium condition therefore reads

$$\ell_i = \bar{\ell}. \tag{3}$$

This condition ensures that the wage rate in country i is endogenously determined. On the world capital market, capital demand of all firms meets total capital supply of the households. Hence, we obtain the equilibrium condition

$$\sum_{i=1}^{n} k_i = n\bar{k}. \tag{4}$$

Condition (4) endogenously determines the world interest rate r.

3 Ad Valorem Taxation

Market equilibrium. Under ad valorem taxation, the tax base equals output less deductible production costs. We follow the previous literature and assume that labor costs are perfectly deductible. With respect to capital costs, previous studies proceed on the implicit assumption that such costs cannot deducted from the tax base at all. We generalize this assumption and suppose that the firms are allowed to deduct a share $\rho \in [0,1]$ of the capital costs. As motivation, notice that most real world corporate tax systems contain depreciation allowances and allow to deduct the costs of debt financing, implying that ρ is strictly positive.

Denoting the ad valorem tax rate of country i by $t_i \in [0,1]$, the after-tax profit of the firm located in country i can be written as

$$\pi_i = (1-t_i)[F(k_i,\ell_i) - w_i\ell_i] - (1-\rho t_i)rk_i. \tag{5}$$

The firm chooses capital and labor input in order to maximize after-tax profit (5). The first-order conditions read

$$(1-t_i)F_k(k_i,\ell_i) - (1-\rho t_i)r = 0, \tag{6}$$

$$F_\ell(k_i,\ell_i) - w_i = 0. \tag{7}$$

Condition (6) and (7) state that for each production factor the (after-tax) marginal return equals the (after-tax) factor costs.

Together with (3) and (4), equation (6) and (7) determine the market equilibrium of the economy. Formally, we obtain the equilibrium factor allocation $\{k_i,\ell_i\}_{i=1}^n$ and the equilibrium factor prices $\{r,w_i\}_{i=1}^n$ as functions of the tax rates $\{t_i\}_{i=1}^n$. In the subsequent analysis, we focus on symmetric situations with $t_i = t$. From (3), (4), (6) and (7), it then follows $\ell_i = \bar{\ell}$ and $k_i = \bar{k}$. Conducting a comparative static analysis of (3),(4), (6) and (7) and then applying the symmetry property, Appendix A shows

$$\frac{\partial r}{\partial t_i} = -\frac{(1-\rho)F_k}{n(1-\rho t)^2} \leq 0, \tag{8}$$

$$\frac{\partial k_i}{\partial t_i} = -(n-1)\frac{\partial k_j}{\partial t_i} = \frac{n-1}{n}\frac{(1-\rho)F_k}{(1-t)(1-\rho t)F_{kk}} \leq 0, \tag{9}$$

$$\frac{\partial w_i}{\partial t_i} = -(n-1)\frac{\partial w_j}{\partial t_i} = \frac{n-1}{n}\frac{(1-\rho)F_kF_{k\ell}}{(1-t)(1-\rho t)F_{kk}} \leq 0. \tag{10}$$

As stated by (9), if capital costs are not perfectly deductible ($\rho \in [0,1[$), an increase in country i's tax rate induces a relocation of capital from country i to the other countries. According to (8), this relocation is brought about by a fall in the world interest rate. Moreover, equation (10) shows that, because of the complementarity of the production factors, labor demand in country i falls, implying a reduction in this country's wage rate. In contrast, for perfect deductibility of capital costs ($\rho = 1$), country i's tax rate influences neither the factor allocation nor the factor prices. Hence, ad valorem taxes distort the market equilibrium only if capital costs are not fully deductible.

Tax competition. The government of country i uses the revenue from corporate taxation in order to finance its provision of the local public good. Under ad valorem taxation, the public budget constraint of country i can be written as

$$g_i = t_i[F(k_i, \ell_i) - w_i \ell_i - \rho r k_i]. \tag{11}$$

Country i's government sets its tax rate in a welfare-maximizing way. It chooses t_i in order to to maximize $U(c_i, g_i)$ taking into account the private and public budget constraints in (1) and (11) as well as the comparative static effects in (8)–(10) and $\ell_i = \bar{\ell}$. Moreover, the government of country i takes as given the tax rates chosen by the governments of the other countries, so we obtain a Nash tax competition game.

The focus is on the symmetric equilibrium of this game with $t_i = t^a$, $k_i = \bar{k}$ and $\ell_i = \bar{\ell}$. In Appendix B it is shown that such an equilibrium exists and is unique if at least one of the two parameters μ and ρ is different from unity, which we will assume throughout.[5] In order to characterize the equilibrium, we introduce $\eta := \bar{k}F_k(\bar{k}, \bar{\ell})/F(\bar{k}, \bar{\ell}) > 0$ as the production elasticity of capital and $\theta := \bar{k}F_{kk}(\bar{k}, \bar{\ell})/F_k(\bar{k}, \bar{\ell}) < 0$ as the capital elasticity of the marginal product of capital. Both elasticities are evaluated at the symmetric equilibrium and, therefore, represent given parameters. Appendix B shows that the Nash equilibrium under ad valorem taxation is characterized by

$$\frac{U_g[F(\bar{k}, \bar{\ell}) - G(t^a, \rho), G(t^a, \rho)]}{U_c[F(\bar{k}, \bar{\ell}) - G(t^a, \rho), G(t^a, \rho)]} = \frac{1}{1 + \frac{\frac{n-1}{n}\frac{t^a(1-\rho)^2}{(1-t^a)(1-\rho t^a)^2}\frac{1}{\theta}}{\frac{1-\mu}{\eta} + \frac{1-\rho}{1-\rho t^a} + \frac{\rho t^a(1-\rho)}{n(1-\rho t^a)^2} - \frac{n-1}{n}\frac{t^a(1-\rho)}{(1-t^a)(1-\rho t^a)}\left(\frac{\mu-1}{\theta} - 1\right)}}, \tag{12}$$

with

$$G(t^a, \rho) = t^a(1-\mu)F(\bar{k}, \bar{\ell}) + \frac{t^a(1-\rho)}{1-\rho t^a}\bar{k}F_k(\bar{k}, \bar{\ell}) \tag{13}$$

denoting public expenditure as function of the tax rate and the share of deductible capital costs. Equation (12) states that in the Nash equilibrium of the tax competition

[5] If $\mu = \rho = 1$, constant returns to scale together with perfect deductibility of capital cost imply that the tax base of the ad valorem tax is zero. Hence, tax revenue and public good provision are always zero, which cannot be an equilibrium due to Assumption 2.

game under ad valorem taxation the marginal rate of substitution between public and private consumption (LHS) equals the marginal costs of public funds (RHS).

An important question is how the equilibrium tax rate t^a is influenced by the deductibility share ρ and how it relates to the efficient (cooperative) tax rate t^*. The efficient tax rate maximizes joint welfare of all countries, i.e. $t^* = \arg\max_{t_i} \sum_{i=1}^{n} U(c_i, g_i)$. It is straightforward to show that t^* satisfies the Samuelson condition $U_g(\cdot)/U_c(\cdot) = 1$ stating that the marginal rate of substitution equals the marginal rate of transformation between public and private consumption. In Appendix B we prove

Proposition 1. *Suppose the n countries compete in ad valorem taxes.*
(ii) For $\mu = 1$, t^a is strictly increasing in ρ for all $\rho \in [0, 1[$ but always smaller than t^*.
(ii) For $\mu \in [0, 1[$, t^a is strictly increasing in ρ for all $\rho \in [0, 1]$, and $t^a = t^*$ if $\rho = 1$.

According to Proposition 1 (i), under constant returns to scale the equilibrium ad valorem tax is increasing in the share of deductible capital costs, but it is always inefficiently low implying underprovision of public goods. The rationale of this result goes back to the distortionary effect of the ad valorem tax identified in the comparative static results (8)–(10) for $\rho \in [0, 1[$. For less than full deductibility, an increase in the ad valorem tax of one country raises the user costs of capital thereby reducing the incentive to invest in this country. The consequence is an outflow of capital to other countries. This distortionary effect renders the marginal costs of public funds on the RHS of (12) larger than one and thereby induces the countries to set tax rates too low. Undertaxation is the less pronounced, the larger is the share of deductible capital costs since an increase in ρ makes the ad valorem tax less distortive.

Proposition 1 (ii) shows that under decreasing returns to scale we qualitatively obtain the same results. For less than full deductibility of capital costs, the tax distorts the market equilibrium and thereby yields an incentive to set inefficiently low tax rates. However, under decreasing returns to scale an important exception from this undertaxation result occurs. Now there exists a Nash equilibrium also in the case of full deductibility of capital costs, and the tax rates as well as the provision of public goods are efficient in this equilibrium. The reason is that for full deductibility of capital costs, the ad valorem tax does not distort the market equilibrium as shown by (8)–(10) for $\rho = 1$. The ad valorem tax then turns into a non-distortionary pure profit tax which is capable to implement the efficient tax policy.

The insights from Proposition 1 highlight the two roles of corporate taxation which we mentioned earlier in the Introduction. On the one hand, countries can use corporate taxation as an instrument in tax competition. They ceteris paribus have an incentive to reduce corporate taxes in order to mitigate the distortive effects of taxation and to attract mobile capital. This role explains why for less than full deductibility of capital costs tax rates are inefficiently low. On the other hand, when capital costs are fully deductible, then the ad valorem tax is no longer distortive and cannot be used as a tax competition instrument. However, if production is characterized by decreasing returns to scale, then economic rents arise to the firms and the countries use corporate taxation in order to absorb these rents. This highlights the role of corporate taxation as a means to absorb economic rents and explains the efficiency result in case of decreasing returns to scale and full deductibility of capital costs.

4 Unit Taxation

Market equilibrium. Under unit taxation, the tax base of a firm equals this firm's capital input. Denoting country i's unit tax rate by τ_i, the after-tax profit of the firm located in country i can be written as

$$\pi_i = F(k_i, \ell_i) - w_i \ell_i - (r + \tau_i) k_i. \tag{14}$$

Profit maximization yields the first-order conditions

$$F_k(k_i, \ell_i) - r - \tau_i = 0, \tag{15}$$

$$F_\ell(k_i, \ell_i) - w_i = 0. \tag{16}$$

As under ad valorem taxation, these conditions equalize the (after-tax) marginal returns of the production factors and the factor prices.

By equations (3), (4), (15) and (16), the equilibrium factor allocation $\{k_i, \ell_i\}_{i=1}^n$ and equilibrium factor prices $\{r, w_i\}_{i=1}^n$ are now functions of the unit tax rates $\{\tau_i\}_{i=1}^n$. Totally differentiating and then applying the symmetry property, Appendix C proves

$$\frac{\partial r}{\partial \tau_i} = -\frac{1}{n} < 0, \tag{17}$$

$$\frac{\partial k_i}{\partial \tau_i} = -(n-1)\frac{\partial k_j}{\partial \tau_i} = \frac{n-1}{nF_{kk}} < 0, \tag{18}$$

$$\frac{\partial w_i}{\partial \tau_i} = -(n-1)\frac{\partial w_j}{\partial \tau_i} = \frac{(n-1)F_{k\ell}}{nF_{kk}} < 0. \tag{19}$$

These comparative static results are similar to those derived under ad valorem taxation. An increase in country i's unit tax rate lowers capital demand in this country and thereby reallocates capital to the other countries via a decline in the world interest rate, as formally shown by (17) and (18). Since capital and labor are complements, a decrease (increase) in capital leads to a decrease (increase) in labor demand, with the consequence that according to (19) the wage rate falls in country i and rises in the other countries. The important difference to the case of ad valorem taxation is that unit taxes always distort the market equilibrium, since unit taxes cannot be turned into pure profit taxes by adjusting the share of deductible capital costs.

Tax competition. With unit taxes, the public budget constraint in country i reads

$$g_i = \tau_i k_i. \tag{20}$$

We again consider a Nash tax competition game. The government of country i chooses τ_i such as to maximize its household's welfare $U(c_i, g_i)$ subject to (1) and (20), taking into account the comparative static effects in (17)–(19) and taking as given the unit taxes chosen by the governments of the other countries.

Appendix D proves existence and uniqueness of a symmetric equilibrium with $\tau_i = \tau^u$, $k_i = \bar{k}$ and $\ell_i = \bar{\ell}$. Denoting the capital demand elasticity in country i by $\varepsilon := (\partial k_i / \partial \tau_i)(\tau_i / k_i) < 0$, we derive in Appendix D the equilibrium condition

$$\frac{U_g[F(\bar{k}, \bar{\ell}) - \tau^u \bar{k}, \tau^u \bar{k}]}{U_c[F(\bar{k}, \bar{\ell}) - \tau^u \bar{k}, \tau^u \bar{k}]} = \frac{1}{1 + \varepsilon}. \tag{21}$$

This is the usual equilibrium condition known from previous studies. As under ad valorem taxation, it equates the marginal rate of substitution between public and private consumption (LHS) and the marginal costs of public funds (RHS). However, the marginal costs of public funds differ from those under ad valorem taxation. Due to $\varepsilon < 0$, they are greater than one and thus we always obtain inefficient undertaxation and underprovision of public goods, in contrast to ad valorem taxation. The reason is that unit taxes cannot be shaped such their effects on investment and wages vanishes. Hence, unit taxation always distorts the market equilibrium and the countries have an incentive to exploit this distortion in order to attract mobile capital.

This undertaxation result highlights the role which unit taxation plays in capital tax competition. For future purposes, however, it is important to note that unit taxation cannot be used as a means for rent absorption, in contrast to ad valorem taxation. The reason is that the tax base under unit taxation equals the amount of capital employed and not (taxable) corporate income, which is the tax base under ad valorem taxation.

Formally, a nice way to see the difference between the two tax systems with respect to rent taxation is to insert the firms' first-order conditions (6)–(7) and (15)–(16) into the profit functions (5) and (14), respectively. This yields the firm's maximized after-tax profits $\pi_i = F - k_i F_k - \ell_i F_\ell$ under unit taxation and $\pi_i = (1-t_i)(F - k_i F_k - \ell_i F_\ell)$ under ad valorem taxation. The expression $F - k_i F_k - \ell_i F_\ell$ represents the usual Euler term and thus stands for the firm's economic rents. These rents are zero for constant returns to scale, but strictly positive under decreasing returns to scale. Hence, country i's government can shift a part of the rents from the private to the public sector by means of ad valorem taxes, but not by means of unit taxes.

In order to further illustrate and to ease the comparison of unit and ad valorem taxation, notice that for each unit tax rate there is an equivalent ad valorem tax rate. Equivalence means that the equivalent ad valorem tax rate t^u with deductibility ρ generates the same welfare as the equilibrium unit tax rate τ^u. Welfare is the same if tax revenues are the same.[6] Hence, equivalence is given if $\tau^u = G(t^u, \rho)/\bar{k}$ with G defined in (13). Inserting into (21) and using (18) yields

$$\frac{U_g[F(\bar{k}, \bar{\ell}) - G(t^u, \rho), G(t^u, \rho)]}{U_c[F(\bar{k}, \bar{\ell}) - G(t^u, \rho), G(t^u, \rho)]} = \frac{1}{1 + \frac{n-1}{n} \frac{t^u}{\theta} \left(\frac{1-\mu}{\eta} + \frac{1-\rho}{1-\rho t^u} \right)}. \tag{22}$$

This condition implicitly determines the equivalent equilibrium ad valorem tax rate t^u as function of the deductibility share ρ. In Appendix D we prove

Proposition 2. *Suppose the n countries compete in unit taxes and denote the equivalent equilibrium ad valorem tax rate by t^u. Then, t^u is strictly increasing in ρ and smaller than t^* for all $\mu \in [0, 1]$ and $\rho \in [0, 1]$ with either $\mu \neq 1$ or $\rho \neq 1$.*

Proposition 2 confirms that under unit taxation we always have inefficient undertaxation, since the equivalent equilibrium ad valorem tax rate t^u falls short of the efficient tax rate t^*. Moreover, the equivalent equilibrium ad valorem tax rate t^u is strictly increasing in the deductibility share ρ. This relation will be helpful below, when we compare unit and ad valorem taxation. It follows from the equivalence condition $\tau^u = G(t^u, \rho)/\bar{k}$. For a given equilibrium unit tax rate τ^u, an increase in the deductibility share ρ narrows the tax base of the equivalent ad valorem tax. Hence, the tax rate of the equivalent ad valorem tax has to increase as well in order to ensure that the equilibrium tax revenue under unit taxation is met.

[6]Note that welfare in the equilibrium under unit taxation can be written as $U[F(\bar{k}, \bar{\ell}) - \tau^u \bar{k}, \tau^u \bar{k}]$. Thus, as far as tax revenues are equal to $\tau^u \bar{k}$ we always obtain the same welfare.

5 Centralized Choice of the Tax System

With the help of the insights derived in the previous sections, we are now in the position to compare ad valorem and unit taxation. In doing so, we investigate the question whether detrimental tax competition is more severe under unit taxation or ad valorem taxation. As already mentioned in the Introduction, this comparison can be viewed as centralized choice of the tax system, where on the first stage a social planner selects the more efficient tax system, taking into account the equilibrium of the countries' tax competition game on the second stage.

Comparing the two tax systems means that we have to consider the relation between the countries' welfare in the tax competition game under ad valorem taxation and the countries' welfare in the tax competition game under unit taxation. The easiest way to do is to compare the equilibrium ad valorem tax rate t^a determined by (12) with the equivalent equilibrium ad valorem tax rate t^u under unit taxation determined by (22). From Propositions 1 and 2 we know that t^a and t^u are never larger than the efficient tax rate t^*. Hence, for $t^a < t^u$ ($t^a > t^u$) undertaxation is more (less) severe under ad valorem taxation than under unit taxation. The same conclusion holds with respect to welfare: Since public consumption $G(t, \rho)$ defined by (13) is increasing in t and since welfare $U[F - G(\cdot), G(\cdot)]$ is increasing in $G(\cdot)$ whenever the marginal rate of substitution $U_g(\cdot)/U_c(\cdot)$ is larger than one, which is the case in the presence of undertaxation, welfare is lower (higher) under ad valorem taxation than under unit taxation if $t^a < t^u$ ($t^a > t^u$). For $t^a = t^u$ both tax system yields the same welfare.

Let us first consider constant returns to scale ($\mu = 1$). Lockwood (2004) also considers this case, but we generalize his analysis since we allow for deductibility of capital costs. Rewrite (12) and (22) for short as $A(t^a, \rho) = B^a(t^a, \rho)$ and $A(t^u, \rho) = B^u(t^u, \rho)$, where A represents the marginal rate of substitution on the respective LHS and B^a and B^u stand for the marginal costs of public funds on the respective RHS. Since $A(t, \rho)$ is decreasing in t according to $A_t < 0$ and $B^a(t, \rho)$ as well as $B^u(t, \rho)$ are non-decreasing in t according to $B^a_t \geq 0$ and $B^u_t \geq 0$ (see Appendix B and D), we obtain $t^a \lesseqqgtr t^u$ if $B^a(t, \rho) \gtreqqless B^u(t, \rho)$ for all feasible t. Now set $\mu = 1$ in $B^a(t, \rho)$ and $B^u(t, \rho)$. After some rearrangements, it is then straightforward to show that $B^a(t, \rho) > B^u(t, \rho)$ for all feasible t if $\rho \in [0, 1[$, which must be satisfied under constant returns to scale since otherwise no equilibrium exists. Hence, together with $t^u < t^*$ for all $\rho \in [0, 1[$, which is stated by Proposition 2, we have proven

Proposition 3. *Under constant returns to scale* ($\mu = 1$), $t^a < t^u < t^*$ *for all* $\rho \in [0, 1[$.

Proposition 3 shows that, under constant returns to scale, undertaxation is always less pronounced and welfare is higher under unit taxation than under ad valorem taxation, independent of the share of deductible capital costs. This result generalizes the insight derived by Lockwood (2004), who proves the superiority of unit taxation over ad valorem taxation for the special case of zero deductibility of capital costs.

The intuition provided by Lockwood (2004) extends to the general case with deductible capital costs. The key element is the effective tax rate on capital under the two tax systems. Under unit taxation, the effective tax rate in country i is equal to the statutory tax rate τ_i. Under ad valorem taxation, in contrast, the first-order condition (6) can be rewritten as $F_k(k_i, \bar{\ell}) = r + \tilde{\tau}_i$ where $\tilde{\tau}_i = t_i(1-\rho)F_k(k_i, \bar{\ell})/(1-\rho t_i)$ is the effective tax rate. Hence, under ad valorem taxation the effective tax rate in country i depends not only on the statutory tax rate t_i, but also on the marginal return on capital F_k which, in turn, is influenced by capital demand k_i. It follows that a reduction in the statutory tax rate causes a smaller fall in the effective tax rate under unit taxation than under ad valorem taxation, since under the latter the induced increase in capital demand exerts an additional negative effect on the effective tax rate via a fall in the marginal return to capital. As consequence, the tax rate elasticity of capital demand is smaller and thus detrimental tax competition is less severe under unit taxation than under ad valorem taxation. This intuition holds independent of the share of deductible capital costs, since for all $\rho \in [0, 1[$ the effective tax rate under ad valorem taxation depends on the marginal return to capital.

The superiority of unit taxation over ad valorem taxation identified in Proposition 3 solely rests on the role of corporate taxation as an instrument in tax competition. The role as a means of absorbing economic rents is not relevant in case of constant returns to scale, where economic rents are zero. However, rent absorption comes into play in case of decreasing returns to scale, with the consequence that the ranking of tax systems may be reversed. This can already be seen if we focus on the case without deductibility of capital costs, as considered by Lockwood (2004). Obviously, for $\rho = 0$ and $\mu \in [0, 1[$ the RHS of (12) may become smaller than the RHS of (22) if μ is sufficiently small. In such cases, the marginal costs of public funds are lower and welfare is higher under ad valorem taxation than under unit taxation. We can generalize this result when we allow for positive values of the deductibility share. More specific, from Propositions 1 and 2 we know that the equilibrium tax rate under ad valorem taxation, t^a, and the equivalent tax rate under unit taxation, t^u, are both increasing in ρ. Moreover, t^u is always smaller than the efficient tax rate t^*, while t^a is efficient if we have full deductibility $\rho = 1$. It immediately follows

Proposition 4. *Under decreasing returns to scale* ($\mu \in [0, 1[$) *there always exists a* $\bar{\rho} \in [0, 1[$ *such that* $t^u < t^a \leq t^*$ *if* $\rho \in [\bar{\rho}, 1]$.

Proposition 4 states that under decreasing returns to scale ad valorem taxation is more efficient than unit taxation whenever the share of deductible capital costs is larger than a threshold value, which is strictly smaller than 100 percent. The intuition of this result is the following. Under constant returns to scale, corporate taxation is only used as an instrument in capital tax competition, and with respect to this role unit taxation causes lower efficiency losses than ad valorem taxation. For decreasing returns to scale, economic rents accrue to the firms and corporate taxation can additionally be used to absorb such rents. This works, however, only with ad valorem taxation; unit taxation is not suitable for rent taxation as explained already above. Hence, the disadvantage of ad valorem taxation with respect to tax competition may now be overcompensated by the advantage of ad valorem taxation with respect to rent absorption. This is the case when the share of deductible capital costs is relatively large, since then the distortive effect of ad valorem taxation in tax competition is relatively low. Consequently, ad valorem taxes become more efficient than unit taxes.

6 Decentralized Choice of Tax System

So far we have assumed that the countries only set tax rates on the second stage of the game, but take as given the tax system chosen by a central authority on the first stage. This section considers the decentralized setting in which the countries also choose the tax system on the first stage. Therefore, we obtain a non-cooperative game between the countries on both stages. As the analysis of this extended game is too involved, we confine ourselves to the two-country case ($n = 2$) and conduct numerical simulations.

More specific, on the first stage each country may choose a strategy from the set $\{\mathcal{A}, \mathcal{U}\}$, where $\mathcal{A}$ stands for the ad valorem tax and $\mathcal{U}$ represents the unit tax. Let $u_i^{s_1 s_2}$ be the welfare of country $i \in \{1, 2\}$ if country 1 chooses $s_1 \in \{\mathcal{A}, \mathcal{U}\}$ and country 2 chooses $s_2 \in \{\mathcal{A}, \mathcal{U}\}$. We then obtain the payoff matrix displayed in Table 1. The welfare levels $u_i^{\mathcal{U}\mathcal{U}}$ and $u_i^{\mathcal{A}\mathcal{A}}$ are the outcomes of the second stage tax competition games which we have already analyzed in depth in Sections 3 and 4. In these games both

Country 2

	$\mathcal{U}$	$\mathcal{A}$
Country 1 $\mathcal{U}$	$u_1^{\mathcal{UU}}$, $u_2^{\mathcal{UU}}$	$u_1^{\mathcal{UA}}$, $u_2^{\mathcal{UA}}$
$\mathcal{A}$	$u_1^{\mathcal{AU}}$, $u_2^{\mathcal{AU}}$	$u_1^{\mathcal{AA}}$, $u_2^{\mathcal{AA}}$

Table 1: Payoff matrix on the first stage of the game

countries use the same tax system. Open are the welfare levels $u_i^{\mathcal{UA}}$ and $u_i^{\mathcal{AU}}$ in the second stage tax competition games where the countries choose different tax systems.

As these mixed games are characterized by an inherent asymmetry, analytical solutions cannot be obtained due to the complexity of our approach. We therefore focus on numerical simulations. For this, we use the Cobb-Douglas utility function

$$u_i(c_i, g_i) = c_i^{\eta}\, g_i^{1-\eta}, \tag{23}$$

with $\eta \in]0,1[$. Moreover, the production function is specified as

$$F(k_i, \ell_i) = \alpha\, k_i - \frac{\beta\, k_i^2}{2} + \gamma\, \ell_i - \frac{\delta\, \ell_i^2}{2} + \varepsilon\, k_i\, \ell_i, \tag{24}$$

with $\alpha, \beta, \gamma, \delta > 0$ and $\varepsilon \geq 0$. This quadratic production function is not homogenous, in contrast to our Assumption 1. However, for $\beta\,\delta > \varepsilon^2$ the scale elasticity $\psi := k_i F_k/F + \ell_i F_\ell/F$ is smaller than one for all $k_i, \ell_i \geq 0$, implying that (24) displays decreasing returns to scale, which is the main focus of the analysis in this section.

For our numerical simulations, we choose $\alpha = \gamma = 25$, $\varepsilon = \beta = 2$, $\delta = 3$, and $\bar{\ell} = \bar{k} = 1$. We then obtain the results displayed in Table 2. The first column of this table shows that we vary deductibility of capital costs from zero to one. The equilibrium tax rate and the equilibrium welfare when both countries choose ad valorem taxation are contained in the second and third column. The fourth column gives the equilibrium equivalent ad valorem tax rate under unit taxation (the equilibrium unit tax rate is $\tau^u = 3.886$), and the fifth column contains the associated welfare. The equilibrium tax rates and welfare levels in the asymmetric setting, where country i uses the ad valorem tax and country j the unit tax are given by the last four columns.

The Nash equilibrium choice of tax systems is as follows. For relatively low values of deductibility $\rho \in [0, 0.611[$ the Nash equilibrium is that both countries choose unit taxation. For intermediate values $\rho \in]0.611, 0.635[$ we obtain two mixed equilibria in which country i chooses the ad valorem tax and country j the unit tax. For $\rho \in]0.635, 1]$ the Nash equilibrium is that both countries choose ad valorem taxation. These

	$s_1 = \mathcal{A}, s_2 = \mathcal{A}$		$s_1 = \mathcal{U}, s_2 = \mathcal{U}$		$s_i = \mathcal{A}, s_j = \mathcal{U}$			
ρ	t^a	$u_1^{\mathcal{AA}} = u_2^{\mathcal{AA}}$	t^u	$u_1^{\mathcal{UU}} = u_2^{\mathcal{UU}}$	t_i^a	τ_j^u	$u_i^{\mathcal{AU}}$	$u_j^{\mathcal{AU}}$
0	**0.147**	**7.002**	**0.152**	**7.193**	**0.153**	**3.710**	**7.061**	**7.124**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
0.3	0.198	7.057	0.203	7.193	0.206	3.716	7.104	7.142
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
0.611	**0.310**	**7.165**	**0.311**	**7.193**	**0.320**	**3.726**	**7.193**	**7.174**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
0.62	0.314	7.169	0.316	7.193	0.325	3.726	7.197	7.175
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
0.635	**0.323**	**7.177**	**0.324**	**7.193**	**0.334**	**3.727**	**7.204**	**7.177**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
0.65	0.333	7.185	0.333	7.193	0.343	3.727	7.211	7.179
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
0.664	**0.342**	**7.193**	**0.342**	**7.193**	**0.353**	**3.728**	**7.219**	**7.181**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
0.7	0.368	7.215	0.367	7.193	0.379	3.729	7.238	7.186
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Table 2: Equilibrium tax rate and welfare contingent on tax systems and deductibility

numerical insights therefore suggest that in the decentralized setting we get a similar result as in the centralized setting: In the presence of decreasing returns to scale and thus economic rents, ad valorem taxation dominates unit taxation, if the share of deductible capital costs is sufficiently large. The intuition is also the same as under the centralized choice. For a large share of deductible capital costs, the tax competition distortion of ad valorem taxation is relatively low and thus overcompensated by the advantage of ad valorem taxation with respect to rent absorption.

However, we can derive an interesting further result if we look for the Pareto efficient choice of tax systems. As shown in Table 2, for $\rho \in [0, 0.664[$ the efficient tax system is that both countries use unit taxation, while for $\rho \in]0.664, 1]$ a pure ad valorem tax system is efficient. Combined with our above results with respect to the Nash equilibrium choice of tax systems, we can therefore conclude that there is an intermediate range of deductibility $\rho \in]0.635, 0.664[$ where the countries choose ad valorem taxation, but where the move to the pure unit tax system would imply a Pareto improvement. Hence, in this range the choice of ad valorem taxation represents a prisoner's dilemma. The intuition of this insight goes back to the terms of trade effect that emerges in the mixed policy setting with ad valorem and unit taxation. We already know that, from a tax competition perspective, ad valorem taxes imply a larger tax elasticity of capital demand than unit taxes. Hence, the country that uses the ad valorem tax engages more and benefits more from tax competition than the country using the unit tax. In the decentralized setting, the choice of the tax systems is therefore biased more to ad valorem taxation, opening the possibility of the above mentioned prisoner's dilemma.

7 Conclusion

In this paper we have derived a possible explanation for the widespread use of ad valorem (corporate income) taxation in practice. In our framework, corporate taxation may be used for two purposes, tax competition and rent absorption. While ad valorem taxes are less efficient than unit taxes regarding tax competition, they dominate unit taxes in terms of rent absorption. Hence, if the share of deductible capital costs is not too low, the tax competition argument is less important than the rent absorption argument, with the consequence that ad valorem taxation becomes superior to unit taxation. We have shown that this kind of arguments holds in centralized as well as decentralized settings. Interestingly, under a decentralized choice of the tax systems, a Nash equilibrium with ad valorem taxation may be Pareto inefficient.

There are many options to extent our analysis. Perhaps most important, while we assume that the share of deductible capital costs under ad valorem taxation is exogenously given, in practice this share is to a large extent determined by policy. Hence, a suitable extension might be to consider the deductibility share as a decision variable of policy makers. However, within the present framework, the result of such an extension is intuitively obvious. Both a central planner as well as the countries themselves would set the deductibility rate equal to 100%, since by doing so they have an undistortive pure profit tax with which they can absorb economic rents. Our argument in favor of ad valorem taxation is then strengthened, since it unambiguously holds for full deductibility of capital costs. The extension with an endogenous deductibility rate is less clear cut, if we include something which shifts the deductibility rate below 100%.

An example is profit shifting, which is well known to give countries the incentive for a tax-cut-cum-base-broadening policy (e.g. Haufler and Schjelderup, 2000). But even within this extension, we conjecture that there are model specifications for which the deductibility rate is still high enough such that ad valorem taxation still dominates unit taxation. A thorough analysis of this point is left for future research.

Appendix

A. Derivation of equation (8) – (10). From (3) we obtain $\mathrm{d}\ell_i = 0$. Moreover, totally differentiating (4), (6) and (7) and then applying the symmetry property gives

$$\sum_{i=1}^{n} \mathrm{d}k_i = 0, \tag{A1}$$

$$(1-t)F_{kk}\mathrm{d}k_i - (1-\rho t)\mathrm{d}r - \frac{(1-\rho)F_k}{1-\rho t}\mathrm{d}t_i = 0, \tag{A2}$$

$$F_{k\ell}\mathrm{d}k_i - \mathrm{d}w_i = 0, \tag{A3}$$

where in (A2) we have used $F_k - \rho r = (1-\rho)F_k/(1-\rho t)$ from (6). Summing (A2) over all $i = 1, \ldots, n$, using (A1) and setting all but one $\mathrm{d}t_i = 0$ proves (8). Using (8) in (A2) gives (9). Finally, inserting (9) in (A3) proves (10).

B. Nash equilibrium under ad valorem taxation. We first derive the equilibrium condition (12). The first-order condition of maximizing $U(c_i, g_i)$ with respect to t_i reads

$$\frac{U_g(c_i, g_i)}{U_c(c_i, g_i)} = -\frac{\mathrm{d}c_i/\mathrm{d}t_i}{\mathrm{d}g_i/\mathrm{d}t_i}. \tag{B1}$$

In order to specify the RHS of this expression note first that from (5)–(7) and the Euler Theorem $\mu F = \bar{k}F_k + \bar{\ell}F_\ell$, following from Assumption 1, we obtain

$$\frac{\mathrm{d}\pi_i}{\mathrm{d}t_i} = -(1-\mu)F - \frac{1-\rho}{1-\rho t^a}\bar{k}F_k - (1-t^a)\bar{\ell}\frac{\partial w_i}{\partial t_i} - (1-\rho t^a)\bar{k}\frac{\partial r}{\partial t_i}. \tag{B2}$$

The derivative of private consumption (1) can then be written as

$$\frac{\mathrm{d}c_i}{\mathrm{d}t_i} = -(1-\mu)F - \frac{1-\rho}{1-\rho t^a}\bar{k}F_k + t^a\bar{\ell}\frac{\partial w_i}{\partial t_i} + \rho t^a\bar{k}\frac{\partial r}{\partial t_i}. \tag{B3}$$

From deriving (11) we analogously obtain

$$\frac{\mathrm{d}g_i}{\mathrm{d}t_i} = (1-\mu)F + \frac{1-\rho}{1-\rho t^a}\bar{k}F_k - t^a\bar{\ell}\frac{\partial w_i}{\partial t_i} - \rho t^a\bar{k}\frac{\partial r}{\partial t_i} + \frac{t^a(1-\rho)F_k}{1-\rho t^a}\frac{\partial k_i}{\partial t_i}. \tag{B4}$$

The RHS of (12) can now be proven by inserting (B3) and (B4) into (B1) and taking into account (8)–(10) and $\bar{\ell}F_kF_{k\ell}/F_{kk} = \bar{k}F_k[(\mu-1)/\theta-1]$, which follows from Assumption 1. In order to show the LHS of (12), use the Euler Theorem, (6) and (7) to rewrite (1) and (11) as $c_i = F - G(t,\rho)$ and $g_i = G(t,\rho)$, with $G(t,\rho)$ defined in (13).

The next step is to investigate existence and uniqueness of the symmetric equilibrium. For $\mu = \rho = 1$, equations (12) and (13) imply $G(t^a,\rho) = 0$ and $U_g(F,0)/U_c(F,0) = 1$. But the latter expression contradicts Assumption 2, so no equilibrium exists if $\mu = \rho = 1$. For the case $\mu \neq 1$ and/or $\rho \neq 1$ denote the LHS and the RHS of (12) by $A(t^a,\rho)$ and $B^a(t^a,\rho)$, respectively, and let $D^a(t^a,\rho)$ be the denominator of $B^a(t^a,\rho)$. Then, $t^a \in [0,\bar{t}^a]$ with the upper bound $\bar{t}^a := \min\{\hat{t}^a, 1\}$ and $\hat{t}^a$ implicitly defined by $D^a(\hat{t}^a,\rho) = 0$. For given ρ, $A(t,\rho)$ is strictly decreasing in t for all $t \in [0,\bar{t}^a]$ since

$$A_t = \frac{G_t}{U_c}\left[U_{gg} - \frac{U_g}{U_c}U_{cg} + \frac{U_g}{U_c}\left(U_{cc} - \frac{U_c}{U_g}U_{cg}\right)\right] < 0, \tag{B5}$$

where the sign of (B5) follows from $G_t = (1-\mu)F + (1-\rho)\bar{k}F_k/(1-\rho t)^2 > 0$ and Assumption 2 that both goods are normal. After tedious computations, we can also show that $B^a(t,\rho)$ is non-decreasing in t for all $t \in [0,\bar{t}^a]$ since

$$B^a_t = \frac{(n-1)(1-\rho)^2}{\theta n^2(1-t)^2(1-\rho t)^2(D^a)^2}\left[-\frac{1-\rho}{1-\rho t}\right.$$
$$\left. -(1-\mu)\left(\frac{1-\rho t + 2\rho t(1-t)}{\eta} + \frac{n-1}{\theta n}\frac{\rho t^2(1-\rho)}{1-\rho t}\right)\right] \geq 0. \tag{B6}$$

The sign of (B6) is due to $\mu - 1 > \theta < 0$ which follows from $F_{k\ell} > 0$ and Assumption 1. It remains to investigate A and B^a at the corners. At the lower bound we obtain $A(0,\rho) = U_g(F,0)/U_c(F,0) > 1 = B^a(0,\rho)$ by Assumption 2. If the upper bound is $\bar{t}^a = 1$, then using the Euler Theorem, Assumption 2 and $\mu - 1 > \theta$ gives $A(1,\rho) = U_g(\bar{\ell}F_\ell, F-\bar{\ell}F_\ell)/U_c(\bar{\ell}F_\ell, F-\bar{\ell}F_\ell) < 1 < (\mu-1-\theta)/(\mu-2-\theta) = B^a(1,\rho)$. If $\bar{t}^a = \hat{t}^a$, then $A(\hat{t}^a,\rho) < \infty = B^a(\hat{t}^a,\rho)$. From all this information on A and B^a, it follows that for $\mu \neq 1$ and or $\rho \neq 1$ there is exactly one intersection of $A(t,\rho)$ and $B^a(t,\rho)$ in the relevant range and thus there exists a unique symmetric equilibrium $t^a \in]0,\bar{t}^a[$.

Finally, we prove Proposition 1. Note first that for A_ρ we obtain the same expression as in (B5) except of replacing G_t by $G_\rho = -t(1-t)\bar{k}F_k/(1-\rho t^2) < 0$. Thus, $A_\rho > 0$. Moreover, the derivative of B^a with respect to ρ can be written as

$$B^a_\rho = \frac{(n-1)t(1-\rho)}{\theta n(1-\rho t)^2(D^a)^2}\left[\frac{2(1-\mu)}{\eta(1-\rho t)} + \frac{1-\rho}{(1-\rho t)^2} + \frac{t(1-\rho)}{n(1-t)(1-\rho t)^2}\right.$$
$$\left. - \frac{n-1}{n}\frac{t(1-\rho)}{(1-t)(1-\rho t)^2}\left(\frac{\mu-1}{\theta} - 1\right)\right] \leq 0. \tag{B7}$$

An increase in ρ therefore shifts the intersection between A and B^a to the right and thus t^a is strictly increasing in ρ (except for $\mu = \rho = 1$ where no equilibrium exists). Since $\mu - 1 > \theta$, equation (12) implies that for $\mu \in [0,1]$ and $\rho \in [0,1[$, $A(t^a,\rho) = B^a(t^a,\rho) > 1$. Since t^* is determined by the Samuelson rule $A(t^*,\rho) = 1$, it follows $t^a < t^*$. For $\mu \in [0,1[$ and $\rho = 1$, equation (12) yields $A(t^a,1) = B^a(t^a,1) = 1 = A(t^*,1)$ and thus $t^a = t^*$, which completes the proof of Proposition 1.

C. Derivation of equation (17) – (19). Totally differentiating (3), (4), (15) and (16) and then applying the symmetry property yields $\mathrm{d}\ell_i = 0$ and

$$\sum_{i=1}^{n} \mathrm{d}k_i = 0, \tag{C1}$$

$$F_{kk}\mathrm{d}k_i - \mathrm{d}r - \mathrm{d}\tau_i = 0, \tag{C2}$$

$$F_{k\ell}\mathrm{d}k_i - \mathrm{d}w_i = 0. \tag{C3}$$

Summing (C2) over all $i = 1, \ldots, n$, using (C1) and setting all but one $\mathrm{d}t_i = 0$, we obtain (17). Using (17) in (C2) proves (18). From (18) and (C3) it follows (19).

D. Nash equilibrium under unit taxation. The first-order condition of maximizing $U(c_i, g_i)$ with respect to τ_i can be written as

$$\frac{U_g(c_i,g_i)}{U_c(c_i,g_i)} = -\frac{\mathrm{d}c_i/\mathrm{d}\tau_i}{\mathrm{d}g_i/\mathrm{d}\tau_i}. \tag{D1}$$

With the help of (1), (3), (14), (18) and the symmetry property we obtain $\mathrm{d}c_i/\mathrm{d}\tau_i = -\bar{k}$. From (20) and the symmetry property it follows $\mathrm{d}g_i/\mathrm{d}\tau_i = \bar{k} + \tau^u(\partial k_i/\partial \tau_i)$. Inserting into (D1) and using the definition of ε proves (21).

In order to prove existence and uniqueness of the equilibrium, we consider the equivalent equilibrium ad valorem tax t^u determined by (22). Analogous to ad valorem taxation, no equilibrium exists if $\mu = \rho = 1$. For the case that either $\mu \neq 1$ or $\rho \neq 1$, define the LHS of (22) as $A(t^u,\rho)$ and the RHS as $B^u(t^u,\rho)$. Let $D^u(t^u,\rho)$ be the denominator of $B^u(t^u,\rho)$. We then have $t^u \in [0,\bar{t}^u]$ with $\bar{t}^u := \min\{\hat{t}^u, 1\}$ and $\hat{t}^u$ implicitly defined by $D^u(\hat{t}^u,\rho) = 0$. From (B5) we know $A_t < 0$ for all $t^u \in [0,\bar{t}^u]$. Moreover, (22) implies

$$B_t^u = -\frac{n-1}{\theta n (D^u)^2}\left[\frac{1-\mu}{\eta} + \frac{1-\rho}{(1-\rho t)^2}\right] > 0. \tag{D2}$$

At the lower bound of the tax rate we have $A(0,\rho) = U_g(F,0)/U_c(F,0) > 1 = B^u(0,\rho)$. If $\bar{t}^u = 1$ is the upper bound, then

$$A(1,\rho) = \frac{U_g(\bar{\ell}F_\ell, F - \bar{\ell}F_\ell)}{U_c(\bar{\ell}F_\ell, F - \bar{\ell}F_\ell)} < 1 < \frac{1}{1 + \frac{n-1}{\theta n}\left(\frac{1-\mu}{\eta} + 1\right)} = B^u(1,\rho). \tag{D3}$$

If $\bar{t}^u = \hat{t}^u$ is the upper bound of the tax rate, then $A(\hat{t}^u,\rho) < \infty = B(\hat{t}^u,\rho)$. Overall, there always exists a unique $t^u \in]0,\bar{t}^u[$ such that $A(t^u,\rho) = B^u(t^u,\rho)$.

Finally, we prove Proposition 2. As for ad valorem taxation, under unit taxation it follows $A_\rho > 0$. Moreover, equation (22) implies $B^u_\rho = [(n-1)t(1-t)]/[\theta n(D^u)^2(1-\rho t)^2] < 0$. If ρ increases, the intersection of $A(t,\rho)$ and $B^u(t,\rho)$ is therefore shifted to the right, which proves that t^u is strictly increasing in ρ. Undertaxation is simply shown by writing (22) as $A(t^u,\rho) = B(t^u,\rho) > 1 = A(t^*,\rho)$.

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