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Optimal Participation Taxes

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Abstract

Addressing extensive labour supply responses, the literature has established a case for *larger* transfers to the working poor than to even poorer non-working people. This paper casts new light on this result. I argue that the result hinges crucially on the labour supply responses of people in income brackets above that of the working poor. Then distinguishing between more and less poor working people, I show that if a single working type faces a negative tax, it is the poorer one. I extend the analysis by endogenising wages and show that key conditions will be of the same form as with exogenous wages.

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1. Introduction

The literature on optimal labour income taxation is concerned with labour supply responses and distortions at the intensive margin (changes in labour supply among those working) and the extensive margin (the decision to participate in the labour market or not). This paper will revisit some key aspects of extensive response models in the spirit of Diamond (1980) and Saez (2002) with a somewhat different focus by highlighting behavioural responses. The question is how one would like to set income taxes that affect individuals' decisions to work or not to work when the government is also concerned with distributional effects of taxes. As I abstract from other margins of labour supply than participation in the labour market, I will refer to the taxes as participation taxes. The basic assumption is that conditional on individual characteristics ("ability") people can obtain different earnings if working. An income tax will then appropriate some of the remuneration for participating in the labour market, or subsidise it, if the tax is negative.

Under what has been considered to be reasonable assumptions, three key results have been highlighted in the literature on participation taxes (see Saez, 2002). Firstly, and unsurprisingly, there is an argument for a transfer to non-working, and hence poor, people, Secondly, there is also a case for a transfer to the working poor (low-income working people). Thirdly, and more surprisingly, there is a case for *larger* transfers to the working poor than to the even poorer non-working people, which implies that there is a negative marginal tax rate at the lower end of the income distribution. The findings of the participation tax model have been very influential, also in contexts where it is recognised that such a pure participation model takes a too narrow view of labour supply. For instance, there can be little doubt that the Mirrlees Review (Mirrlees et al. 2010) is one of the applications of tax theory that is heavily influenced by the case for in-work benefits. (See chapter 3 on the taxation of labour earnings and Brewer et al. (2010).)

The first aim of the current paper is to shed further light on the case for transfers to low-wage working agents (a working or earned income tax credit) and to achieve a deeper understanding of the conditions required for this case to be valid. This is accomplished by shifting the focus more to behavioural responses compared to previous studies. I will argue that the result hinges crucially on the labour supply responses of people in income brackets above that of the working poor. The next question to be addressed is whether it is always at the very bottom of the income distribution that a negative marginal tax, if any, will occur. An affirmative answer is given. Starting out from the conventional assumption of exogenous wages, I then extend the analysis by endogenising wages and show that key conditions will be of the same form as before. The entire analysis considers homogeneous decision makers and abstracts from the choices of couples between one or both spouses participating in the labour market.

The analysis will start out from the simplest possible model with only two types of working agents and exogenous wages. The basic analysis is presented in section 2. The extension to several (three) working types is considered in section 3, and endogenous wages are introduced in section 4. Section 5 concludes.

2. The basic participation model

Suppose that there are three types of individuals: some who are unable to work (type 0) , some who can work in occupation 1 (type 1) and some who can work in occupation 2 (type 2). The numbers of the respective types are n_0, n_1 and n_2. The exogenous gross income levels of the respective occupations (including unemployment) are $w_0 = 0, w_1$ and $w_2 > w_1$, and the corresponding disposable incomes are c_0, c_1 and c_2. Workers in occupation 1 and occupation 2 face the respective income taxes $T_1 = w_1 - c_1$ and $T_2 = w_2 - c_2$, one of which may conceivably be negative. The disposable income of the non-working is due to a transfer from the government. (Alternative models of labour market participation are obtained by assuming that people can choose not to work because they are endowed with (inherited) wealth, can work informally[2] or can share the income of a spouse; see e.g. Immervoll et al. (2011).)

Benefit payments to the non-working can be of different kinds. One could have a negative income tax with a uniform transfer to everyone who cannot work or choose not to work, for whatever reason. We would then interpret c_0 as the guaranteed income of the tax-transfer scheme. Alternatively, the transfer may be intended for those uable to work, but types are non-observable (or at least non-verifiable) which implies that a type of individual able to work in an occupation may choose not to work and receive the same transfer as those unable to work.

I focus on labour force participation (the extensive margin) and neglect the possibility that an individual chooses to be work-active in a lower-paid occupation than the one he is qualified for. As it is the participation decision I want to shed further light on, a pure model is more suitable than one that mixes several types of decisions even if the latter would be more realistic. Saez (2002) considered an

[2] As mentioned in Brewer et al. (2011).

arbitrary number of types (equal to I+1). Limiting the number to three is a pure simplification which does not confine the generality of our major insights. However, I will extend the model to four types below in order to go into some further details.

Each group consisting of a type of worker (apart from the disabled) is indeed assumed to be heterogeneous as each individual of type 1 is also characterised by a parameter α which is interpretered as a measure of his unwillingness to work, which can be due to a (monetary) cost of working or disutiltiy of working. Specifying this "taste parameter", will allow us to discuss in further detail the determinants of optimal taxes[3]. Analogously, each individual of type 2 is characterised by a similar parameter β. Supports are given by $a \le \alpha \le A$ and $b \le \beta \le B$, and density functions are denoted by $f(\alpha)$ and $g(\beta)$, respectively. Denote the corresponding cumulative distribution functions by $F(\alpha)$ and $G(\beta)$.

If *a* (or *b*) is negative α (or β) can take on negative values, and some individuals would derive benefits from working and would be willing to work even if incurring a loss of income by working. This case is explicitly assumed away by Saez (2002, p.1048), but could be permitted without changing the analysis, where we assume that the marginal worker must have a positive compensation for working.

In most welfare schemes, agents are not free to choose living on social benefits, as assumed above. One will have to pass eligibility tests, or the receipt of benefits may require certain activities, e.g. searching for a job if unemployed[4]. Those who are not eligible for benefits may find it costly to mimic those who are unable to work or to find a job because they will be subjected to eligibility tests and have to pretend to qualify for benefits. The parameter α (β) can therefore be interpreted as an indicator of the net cost of working after subtracting the cost of faking. A high value of the parameter may therefore reflect a low cost of mimicking, for instance because the agent is close to being eligible and may be able to appear credible without excessive efforts.

The respective utility levels are then functions of consumption levels and the parameters α, β. Assuming that some able agents choose not to work, there exist cut-off points $\bar{\alpha}$ and $\bar{\beta}$ such that those with lower parameter values choose to work and those with larger parameter values choose not to work. Denote by $u(c_0)$ utility of type 0 agents. Utility levels for type 1 agents are given by

[3] Saez (2002) assumes that individuals have heterogeneous tastes without specifying explicitly a "taste parameter".

[4] A typical scheme will combine various targeted transfers to specific, eligible groups (those eligible for unemployment benefits, disabled, old people, etc.) with, usually less generous, welfare payments to others without any means.

$u_{11}(c_1,\alpha)$ where $\alpha \le \bar{\alpha}$ and $u_{10}(c_0)$ where $\alpha > \bar{\alpha}$. Utility levels for type 2 agents are given by $u_{22}(c_2,\beta)$ where $\beta \le \bar{\beta}$ and $u_{20}(c_0)$ where $\beta > \bar{\beta}$. Cut-off points are determined by the equations $u_{11}(c_1,\bar{\alpha}) = u_{10}(c_0)$ and $u_{22}(c_2,\bar{\beta}) = u_{20}(c_0)$, respectively[5]. Denote by h_{10}, h_{11} the numbers of type 1 individuals not being work active and working in occupation 1, respectively. In formal expressions, $h_{10} = n_1 \int_{\bar{\alpha}}^{A} f(\alpha)d\alpha = n_1(1-F(\bar{\alpha}))$ and $h_{11} = \int_{a}^{\bar{\alpha}} n_1 f(\alpha)d\alpha = n_1 F(\bar{\alpha})$. The numbers h_{20}, h_{22} are defined analogously. The equations $u_{11}(c_1,\bar{\alpha}) = u_{10}(c_0)$ and $u_{22}(c_2,\bar{\beta}) = u_{20}(c_0)$ define $\bar{\alpha}$ as a function of c_0, c_1 and define $\bar{\beta}$ as a function of c_0, c_2. In turn, it follows that we can write $h_{10}(c_0,c_1), h_{11}(c_0,c_1)$ and $h_{20}(c_0,c_2), h_{22}(c_0,c_2)$.

Assuming purely redistributive policy, the government's budget constraint is

(1) $$h_0 c_0 = h_{11}T_1 + h_{22}T_2$$

where $h_0 = n_0 + h_{10} + h_{20}$.

Consider the following welfare function

(2) $$\begin{aligned} W &= n_0 u(c_0) + n_1 \int_{a}^{\bar{\alpha}} u_{11}(w_1 - T_1, \alpha) f(\alpha) d\alpha + n_1 u_{10}(c_0)(1-F(\bar{\alpha})) \\ &+ n_2 \int_{b}^{\bar{\beta}} u_{22}(w_2 - T_2, \beta) f(\beta) d\beta + n_2 u_{20}(c_0)(1-G(\bar{\beta})) \end{aligned}$$

In order to characterise the welfare-maximising policy subject to the government's budget constraint, we formulate the Lagrange function

(3) $$\begin{aligned} \Lambda &= W + \lambda\left[T_1 n_1 F(\bar{\alpha}) + T_2 n_2 G(\bar{\beta}) - n_0 c_0 - n_1 c_0 (1-F(\bar{\alpha})) - n_2 c_0 (1-G(\bar{\beta}))\right] \\ &= W + \lambda\left[T_1 h_{11} + T_2 h_{22} - n_0 c_0 - h_{10} c_0 - h_{20} c_0\right] \end{aligned}$$

Marginal utilities of income are denoted by u_0', u_{i0}', u_{ii}' (i=1,2).

Maximising wrt T_1 we obtain the first order condition

[5] It is not important whether u , u_{10} and u_{20} are different or identical functions.

(4) $$-n_1\int_{a}^{\bar{\alpha}} u'_{11}(w_1 - T_1, \alpha) f(\alpha) d\alpha + \lambda h_{11} + \lambda T_1\left(-\frac{\partial h_{11}}{\partial c_1}\right) + \lambda c_0\left(-\frac{\partial h_{11}}{\partial c_1}\right) = 0,$$

recalling that $h_{10} = n_1 - h_{11}$.

Denoting by $\overline{u}'_{11}$ the average marginal utility of income among working type 1 agents, we can rewrite (4) as

(5) $$-\frac{\overline{u}'_{11}}{\lambda} + 1 - (T_1 + c_0)\frac{\partial h_{11}}{\partial c_1} / h_{11} = 0$$

$\frac{\overline{u}'_{11}}{\lambda}$ is the marginal utility of (private) income denominated in terms of public revenue. In the sequel, most references to marginal utility of income will be to this measure.

Analogously, we obtain the following condition when maximising wrt T_2.

(6) $$-\frac{\overline{u}'_{22}}{\lambda} + 1 - (T_2 + c_0)\frac{\partial h_{22}}{\partial c_2} / h_{22} = 0$$

Finally, we derive the first order condition wrt c_0.

(7) $$n_0 u'_0 + h_{10}u'_{10} + h_{20}u'_{20} - \lambda(n_0 + h_{10} + h_{20}) + \lambda(T_1 + c_0)\frac{\partial h_{11}}{\partial c_0} + \lambda(T_2 + c_0)\frac{\partial h_{22}}{\partial c_0} = 0$$

Denoting by $\overline{u}'_0$ the marginal utility of income among non-working agents, we can rewrite (7) as

(8) $$\frac{\overline{u}'_0}{\lambda} - 1 + (T_1 + c_0)\frac{\partial h_{11}}{\partial c_0} / h_0 + (T_2 + c_0)\frac{\partial h_{22}}{\partial c_0} / h_0 = 0$$

Now normalise the size of the population to unity:

(9) $$h_0 + h_{11} + h_{22} = 1.$$

Then consider the special case where

(10) $$\frac{\partial h_{11}}{\partial c_0} = -\frac{\partial h_{11}}{\partial c_1} \text{ and } \frac{\partial h_{22}}{\partial c_0} = -\frac{\partial h_{22}}{\partial c_2}$$

This is equivalent to h_{11} being a function solely of $c_1 - c_0$, which is the assumption made by Saez (2002, p. 1048)). In my model the assumption is tantamount to $\bar{\alpha}$ being a function solely of $c_1 - c_0$. (Similarly, h_{22} and $\bar{\beta}$ are functions of $c_2 - c_0$.) This assumption has important bearing on the utility functions in our analysis. The cut-off value $\bar{\alpha}$ is then determined by setting $u(c_0) = u_{11}(c_1, \bar{\alpha})$. By the assumption above $\bar{\alpha} = \bar{\alpha}(c_1 - c_0) = H(c_1 - c_0)$ where $\bar{\alpha}'(c_1 - c_0) = H'(c_1 - c_0) > 0$. Taking the inverse of H we have $c_1 - c_0 = H^{-1}(\bar{\alpha}) = h(\bar{\alpha})$, and $c_0 = c_1 - h(\bar{\alpha})$. Plugging this equation into $u(c_0)$, we get $u(c_0) = u(c_1 - h(\bar{\alpha})) = u_{11}(c_1, \bar{\alpha})$, and in general we have $u_{11}(c_1, \alpha) = u(c_1 - h(\alpha))$. In an analogous way, we get $u_{22}(c_2, \beta) = u(c_2 - m(\beta))$ since α and β represent the same kind of parameter. When working becomes more burdensome utility decreases so that $h'(\alpha) > 0$, $m'(\beta) > 0$.

Obviously, the agent will make the same choice for any cardinalisation of u(), i.e. any utility function can be replaced by an increasing monotonic transformation when analysing consumer behaviour. The concavity of u() will determine the relative social marginal utility of income assigned to different income levels c^* and $\hat{c}$: $u'(c^*)/u'(\hat{c})$. I assume that the chosen cardinalisation is the one that reflects the distributional preferences of the government.

Multiply by $-h_{11}$ on both sides of (5), by $-h_{22}$ on both sides of (6) and by h_0 on both sides of (8). Summing the resulting equations and invoking (9), we obtain

$$(11) \quad \frac{\bar{u}'_{11}}{\lambda} h_{11} + (T_1 + c_0)\frac{\partial h_{11}}{\partial c_1} + \frac{\bar{u}'_{22}}{\lambda} h_{22} + (T_2 + c_0)\frac{\partial h_{22}}{\partial c_2} + \frac{\bar{u}'_0}{\lambda} h_0 + (T_1 + c_0)\frac{\partial h_{11}}{\partial c_0} + (T_2 + c_0)\frac{\partial h_{22}}{\partial c_0} = 1$$

Then assuming the special case (10)

$$(12) \qquad \frac{\bar{u}'_{11}}{\lambda} h_{11} + \frac{\bar{u}'_{22}}{\lambda} h_{22} + \frac{\bar{u}'_0}{\lambda} h_0 = 1$$

The average marginal utility of income in the population is one. This is equivalent to equation (2) in Saez (2002, p. 1047).

Now simplify the notation by means of

$$(13) \qquad v_0 = \frac{\bar{u}'_0}{\lambda}, v_1 = \frac{\bar{u}'_{11}}{\lambda}, v_2 = \frac{\bar{u}'_{22}}{\lambda}$$

(5) and (6) can be reformulated as inverse elasticity rules (as in Saez, 2002).

(14)
$$\frac{T_1+c_0}{c_1}=\frac{1-v_1}{\dfrac{\partial h_{11}}{\partial c_1}\dfrac{c_1}{h_{11}}}=\frac{1-v_1}{\sigma_{11}}$$

(15)
$$\frac{T_2+c_0}{c_2}=\frac{1-v_2}{\dfrac{\partial h_{22}}{\partial c_2}\dfrac{c_2}{h_{22}}}=\frac{1-v_2}{\sigma_{22}}$$

where σ_{11} and σ_{22} are labour supply elasticities as implicitly defined by the latter equations in (14) and (15)[6].

We can interpret T_1+c_0 as the effective tax (participation tax) on workers in occupation 1. By working they will forego the transfer obtained if not working, c_0, and pay a tax T_1. It's a straightforward conclusion that if the average marginal utility of income for type 1 $\left(\overline{u}'_{11}/\lambda\right)$ exceeds one, ie, the population mean for the marginal utility of income, then the participation tax is negative. Saez argues that, with an arbitrary number of types of individuals, there may be types *i>0* for which the marginal utility of income is above the mean, i.e. above 1. In our three-type model this would correspond to type one having a marginal utility of income above 1. The assumption that low-wage working agents may have an above average marginal utility of income has been taken to be a (possible) case for an earned income tax credit. A negative participation tax on type 1 means that low-income working agents receive a larger transfer than the non-working individuals. I will shed a different light on this result by highlighting behavioural responses rather than focussing directly on welfare weights.

It may seem appealing to stimulate work effort by rewarding those who choose to work. However, by itself, subsidising work effort is no less distortionary than taxing it and a further rationale will have to be established for such a policy. Moreover, we recall that agents characterised by $\alpha>\overline{\alpha}$ will choose c_0. Individuals with $\alpha<\overline{\alpha}$ will choose c_1. Where $\alpha<\overline{\alpha}$, obviously $c_1-h(\alpha)>c_0$ and $u'_0=u'(c_0)>u'(c_1-h(\overline{\alpha}))=u'_{11}$. The marginal utility of income of those working is smaller than the marginal utility of income of those not working so it is not immediately clear that a larger transfer to the former is justified. One might think that those who incur a large disbenefit by toiling in the labour market would have a large marginal utility of income, but those who would be in that situation have already opted out and chosen c_0.

[6] Saez uses a slightly different formulation by defining labour supply elasticites w.r.t. c_1-c_0.

The crucial question is how plausible it is to assume that low-wage working agents have an above average marginal utility of income and should be favoured by a large transfer. The assumption is hard to assess in and by itself. The marginal utility of income is endogenous and is determined both by distributional preferences and behavioural responses. This is easily seen by considering the sum of (5) and (8).

(16) $$v_0 - v_1 + (T_1 + c_0)\left(\frac{\partial h_{11}}{\partial c_0}/h_0 - \frac{\partial h_{11}}{\partial c_1}/h_{11}\right) + (T_2 + c_0)\frac{\partial h_{22}}{\partial c_0}/h_0 = 0$$

which reflects effects of a transfer from workers in the low-paid occupation to the non-working agents.

We note that the following signs apply.

$\frac{\partial h_{11}}{\partial c_0}/h_0 - \frac{\partial h_{11}}{\partial c_1}/h_{11} < 0$, $\frac{\partial h_{22}}{\partial c_0}/h_0 < 0$. A higher transfer to the non-working will discourage participation, while a larger disposable income for the work-active will encourage participation. As we have seen above $u_0' > u_{11}'$, and the first difference on the left hand side of (16) is positive. In order to have $T_1 + c_0 < 0$, the last term on the left hand side must be sufficiently negative. In other words, people in the high-wage occupation must pay a sufficiently high tax, $T_2 + c_0$, and respond sufficiently strongly to an increase in the income received if not working. We see that what is crucial for the sign of $T_1 + c_0$ is the labour supply response of those with higher income rather than the labour supply response of the low-wage working themselves. It is often argued that people with higher income have a lower demand response. If so, it will weaken the case for an earned income tax credit. We see that if the high-skilled have a negligible propensity to opt out of the labour market the last tem of (16) is negligible and it follows immediately that $T_1 + c_0 > 0$. There is no earned income tax credit.

In order to recognise the case for introducing an earned income tax credit, let us take as our point of departure $T_1 + c_0 = 0$. A transfer from type 2 to type 1 can be less costly in terms of social efficiency than a transfer from type 2 to type 0. In both cases a larger T_2 will induce type 2 people to quit the labour force. If the transfer is given to type 0 there will be a further inducement to leave the labour force which is avoided if the transfer goes to type 1. Hence there may be a gain from lowering T_1 and hence lowering $T_1 + c_0$ below zero. Thus it is the high income agents' propensity to leave the labour force which is crucial for the sign of $T_1 + c_0$. A transfer from type 0 to type 1 will be a transfer from poorer to richer. However, there is an efficiency gain because one weakens the incentive for type 2 people to quit working. The undesirable redistribution is offset by a smaller loss of social efficiency.

To summarise, the purpose is not in itself to give a particulary large transfer to the working poor. The explanation for the transfer levels is rather that there is a case for keeping the transfer to the non-working poor low as otherwise too many able people would be induced to opt for the transfer. As a result of this concern, the transfer to the non-working poor may fall short of the transfer to the working poor.

3. Extension to three able types

While the model above is sufficient to obtain certain key insights, further details about the tax structure require an extension to more types. The smallest possible extension to three types of able individuals will do for this purpose. We then add n_3 individuals of type three with income w_3 now being the largest income in society. The taste parameter of type three individuals is denoted η with density function $k(\eta)$ and cumulative distribution $K(\eta)$. In other respects the notation is analogous to the one above, for instance with $\bar{\eta}$ denoting the cut-off level between those working and those choosing not to work. The welfare function can then be written as

$$
\begin{aligned}
W &= n_0 u(c_0) + n_1 \int_a^{\bar{\alpha}} u_{11}(w_1 - T_1, \alpha) f(\alpha) d\alpha + n_1 u_{10}(c_0)(1 - F(\bar{\alpha})) \\
&+ n_2 \int_b^{\bar{\beta}} u_{22}(w_2 - T_2, \beta) g(\beta) d\beta + n_2 u_{20}(c_0)(1 - G(\bar{\beta})) \\
&+ n_3 \int_d^{\bar{\delta}} u_{33}(w_3 - T_3, \eta) k(\eta) d\eta + n_3 u_{30}(c_0)(1 - K(\bar{\eta}))
\end{aligned} \tag{17}
$$

and the government budget constraint becomes

$$T_1 h_{11} + T_2 h_{22} + T_3 h_{33} - n_0 c_0 - h_{10} c_0 - h_{20} c_0 - h_{30} c_0 = 0 \tag{18}$$

Maximising welfare wrt T_1, T_2 and T_3, respectively, s.t. the budget constraint , we obtain the same type of first order conditions as above.

$$-v_1 + 1 - (T_1 + c_0) \frac{\partial h_{11}}{\partial c_1} / h_{11} = 0 \tag{19}$$

$$-v_2 + 1 - (T_2 + c_0) \frac{\partial h_{22}}{\partial c_2} / h_{22} = 0 \tag{20}$$

(21) $$-v_3 + 1 - (T_3 + c_0)\frac{\partial h_{33}}{\partial c_3} / h_{33} = 0$$

The welfare maximising policy subject to the government's budget constraint is characterised in further detail in appendix A. According to (A11) and (A12) of the appendix the following conditions must hold.

(22) $$v_0 - v_1 + (T_1 + c_0)\left(\frac{\partial h_{11}}{\partial c_0} / h_0 - \frac{\partial h_{11}}{\partial c_1} / h_{11}\right) + (T_2 + c_0)\frac{\partial h_{22}}{\partial c_0} / h_0 + (T_3 + c_0)\frac{\partial h_{33}}{\partial c_0} / h_0 = 0$$

(23) $$v_0 - v_2 + (T_2 + c_0)\left(\frac{\partial h_{22}}{\partial c_0} / h_0 - \frac{\partial h_{22}}{\partial c_2} / h_{22}\right) + (T_1 + c_0)\frac{\partial h_{11}}{\partial c_0} / h_0 + (T_3 + c_0)\frac{\partial h_{33}}{\partial c_0} / h_0 = 0$$

By the same reasoning as in Section 2 we can conclude that $v_0 > v_1$ and $v_0 > v_2$. In the same manner as above, we can also conclude that if those with income above w_1 have a negligible labour supply response, $\frac{\partial h_{22}}{\partial c_0}$ and $\frac{\partial h_{33}}{\partial c_0}$ are negligible, $T_1 + c_0 > 0$, while stronger responsiveness is compatible with a tax structure where $T_1 + c_0 < 0, T_2 + c_0 > 0$ and $T_3 + c_0 > 0$. This is a trivial generalisation as the previous high income group has just been split in two high-income groups. Likewise, sufficiently large $-(T_3 + c_0)\frac{\partial h_{33}}{\partial c_0} / h_0$, is compatible with $T_1 + c_0 < 0, T_2 + c_0 < 0$ and $T_3 + c_0 > 0$, which we can interpret as the previous low-income group having been divided into two low-income groups.

A more interesting question at this stage is whether it is possible that type 1 may face a positive participation tax while type 2 faces a negative participation tax. According to (A13) derived in the appendix ,

(24) $$v_1 - v_2 + (T_1 + c_0)\frac{\partial h_{11}}{\partial c_1} / h_{11} - (T_2 + c_0)\frac{\partial h_{22}}{\partial c_2} / h_{22} = 0$$

We immediately realise that under the assumption that type one has a larger marginal utility of income than type two, $v_1 - v_2$ >0, the combination $T_1 + c_0 > 0, T_2 + c_0 < 0$ is ruled out. *If a single working type faces a negative tax it is the least skilful working type who does.*

One can argue that $v_1 - v_2 > 0$ is a plausible assumption. The tax structure $T_1 + c_0 > 0$ and

$T_2 + c_0 < 0$ implies that $T_2 < -c_0 < T_1$. It would follow that $c_1 = w_1 - T_1 < c_2 = w_2 - T_2$. Type 2 has a larger disposable income. It is also plausible that more able people have a larger proportion of people with low cost of working. Part of the reason for this may be that the more able will incur a higher cost of mimicking those who are unable to work.

To get some intuition for the result above, consider the hypothetical case where $T_1 + c_0 > 0$ and $T_2 + c_0 < 0$. Then making a transfer from working type 2 agents to working type 1agents will have the following effects. There will be a beneficial distributional effect since a larger welfare weight is assigned to income for the latter. Type 1 will be encouraged to work more since a higher disposable income is obtained by working. This will be a beneficial efficiency effect since there is initially a downward distortion of labour supply. The work effort of type 2 people will be discouraged as a lower disposable income is received when working. This is a beneficial efficiency effect as there is initially an upward distortion of labour supply due to the subsidy. As the transfer has only beneficial effects one is obviously not at a social optimum. At the optimum, $T_1 + c_0$ and $T_2 + c_0$ must have the same sign or $T_1 + c_0 < 0$ and $T_2 + c_0 > 0$.

From a practical policy perspective, the discrete number of income levels of the model can be conceived of as an approximation to an almost continuous income distribution. In practice, income tax schedules are piece-wise linear with a small number of tax brackets[7], which in the participation model above will be represented by the same small number of discrete income levels. If the extensive labour supply response is largest at the very bottom of the income distribution and declines rapidly with income, the bracket(s) with a negative marginal tax may constitute a small interval and be of limited practical significance. Alternatively, small brackets will be avoided, and those in tax brackets

[7] For further arguments, see Rees et al. (2009).

beyond the lowest one will reach income levels where the extensive response is small. Then it follows from the analysis above that there is no case for a negative marginal tax.

4. Endogenous incomes

Let me now go back to the model with two able types to consider a different extension. I will endogenise incomes by assuming that total production (income) is determined by the macro production function $x = f(h_{11}, h_{22})$ with constant returns to scale, which is the endogenous income model closest to the previous model[8]. The production function is assumed to have standard properties. The wage of each type is determined by the marginal product of each type of labour.

(25) $$w_i = \frac{\partial f(h_{11}, h_{22})}{\partial h_{ii}} = w_i(h_{11}, h_{22})\text{; i=1,2.}$$

As above, the labour supply is determined by disposable incomes when working and not working, respectively. We express the labour supply functions as

(26) $$h_{11} = \ell_{11}(c_0, w_1(h_{11}, h_{22}) - T_1)$$

(27) $$h_{22} = \ell_{22}(c_0, w_2(h_{11}, h_{22}) - T_2)$$

which implicitly define h_{11} and h_{22} as functions of the policy instruments. It follows that w_1 and w_2 are also functions of the policy parameters. As before, by definition $c_1 = w_1 - T_1$ $c_2 = w_2 - T_2$. It follows that $\partial c_1 / \partial T_1 = \partial w_1 / \partial T_1 - 1, \partial c_1 / \partial T_2 = \partial w_1 / \partial T_2$, $\partial c_2 / \partial T_2 = \partial w_2 / \partial T_2 - 1$, $\partial c_2 / \partial T_1 = \partial w_2 / \partial T_1$.

We can express the unit cost function as $\overline{c}(w_1, w_2) = 1$ in equilibrium. It follows from the Envelope Theorem that $\frac{\partial \overline{c}}{\partial w_1}\frac{\partial w_1}{\partial T_i} + \frac{\partial \overline{c}}{\partial w_2}\frac{\partial w_2}{\partial T_i} = \frac{h_{11}}{x}\frac{\partial w_1}{\partial T_i} + \frac{h_{22}}{x}\frac{\partial w_2}{\partial T_i} = 0$.

(28) $$\frac{\partial w_2}{\partial T_1} = -\frac{h_{11}}{h_{22}}\frac{\partial w_1}{\partial T_1}$$

[8] It is close to the original model as inputs are the same and no profit will arise.

(29) $$\frac{\partial w_2}{\partial T_2} = -\frac{h_{11}}{h_{22}}\frac{\partial w_1}{\partial T_2}$$

(30) $$\frac{\partial w_1}{\partial T_2} = -\frac{\partial w_2}{\partial T_2}\frac{h_{22}}{h_{11}}$$

An alternative assumption is that there is a third production factor, say capital, *k*, which is infinitely elastically supplied at a price *r*, conceivably in the world market. Then $x = f(h_{11}, h_{22}, k)$, which is assumed to exhibit constant returns to scale. Then the unit cost function is $\overline{c}(w_1, w_2, r) = 1$. It follows that $\frac{\partial \overline{c}}{\partial w_1}\frac{\partial w_1}{\partial T_i} + \frac{\partial \overline{c}}{\partial w_2}\frac{\partial w_2}{\partial T_i} = \frac{h_{11}}{x}\frac{\partial w_1}{\partial T_i} + \frac{h_{22}}{x}\frac{\partial w_2}{\partial T_i} = 0$.

In either case it also holds that $\frac{\partial \overline{c}}{\partial w_1}\frac{\partial w_1}{\partial c_0} + \frac{\partial \overline{c}}{\partial w_2}\frac{\partial w_2}{\partial c_0} = \frac{h_{11}}{x}\frac{\partial w_1}{\partial c_0} + \frac{h_{22}}{x}\frac{\partial w_2}{\partial c_0} = 0$

As before, total welfare is

$$W = n_0 u(c_0) + n_1 \int_a^{\overline{\alpha}} u_{11}(w_1 - T_1, \alpha) f(\alpha) d\alpha + n_1 u(c_0)(1 - F(\overline{\alpha}))$$
$$+ n_2 \int_b^{\overline{\beta}} u_{22}(w_2 - T_2, \beta) f(\beta) d\beta + n_2 u(c_0)(1 - G(\overline{\beta}))$$

where the cut-off values $\overline{\alpha}, \overline{\beta}$ are determined by $u_{11}(c_1, \overline{\alpha}) = u_{10}(c_0)$ and $u_{22}(c_2, \overline{\beta}) = u_{20}(c_0)$, which means that they are ultimately functions of the policy parameters. Social welfare is maximised subject to the same budget constraint as before. Optimality conditions are derived in Appendix B.

Solving for the relative tax rates, we get.

(31) $$\theta_1 = \frac{T_1 + c_0}{c_1} = \frac{1 - v_1}{\sigma_{11}}$$

(32) $$\theta_2 = \frac{T_2 + c_0}{c_2} = \frac{1 - v_2}{\sigma_{22}}$$

as shown in Appendix B, eqs. (B8) and (B9).

Recalling (14) and (15), we note that the relative tax rates are determined by the same inverse elasticity rules irrespective of whether wages are exogenous or endogenous.

This is different from the analysis of the intensive margin in the Mirrlees model where the characterisation of the marginal tax rate changes when assuming endogenous wage rates even if this has only been studied within a two-type model (see Stiglitz, 1982).

Even though special assumptions have been made, and we cannot expect the result to hold in general, it is interesting that the characterisation of optimal taxation does not necessarily change when wages are endogenised.

In order to discuss the sign of $T_1 + c_0$ let us consider the following condition due to (B10) of the appendix.

$$(33) \quad v_0 - v_1 + (c_0 + T_1)\left(\frac{\partial h_{11}}{\partial c_0}\frac{1}{h_0} - \frac{\partial h_{11}}{\partial c_1}\frac{1}{h_{11}}\right) + (c_0 + T_2)\frac{\partial h_{22}}{\partial c_0}\frac{1}{h_0} = 0$$

which takes us back to exactly the same form as (16).

The criterion for determining the sign of the effective tax on the working poor is the same whether wages are endogenous or exogenous.

5. Concluding remarks

How taxes affect labour market participation has recently atttracted considerable attention as it has been recognised that labour supply is more elastic at the extensive margin than at the intensive margin. There is a broad range of participation decisions depending on what is the source of consumption if opting out of the labour market. This paper has only considered the case where a non-working person receives a transfer from the government. Similar to Saez (2002), workers are of different types, each of whom either works in a specific occupation or doesn't work.

The paper has made several contributions.

It has elaborated on the tax-setting at the lower end of the income distribution. In particular, it has discussed the conditions under which there is a negative marginal tax at the lower end of the income distribution – a result that has recently received considerable attention. It is argued that the result hinges on sufficiently large elastic labour supply responses among workers in the income brackets above that of the working poor rather than the labour supply elasticity of the working poor themselves.

Considering two groups of more or less poor workers with different skills, it is shown that if a single working type faces a negative tax it is the least skilful one who does.

Taking as point of departure the case of exogenous wages, the analysis has been extended to the case of endogenous incomes determined by labour supply depending on disposable wages and labour demand derived from a linearly homogenous macro production function. It is shown that the condition for a negative marginal income tax at the lower end of the income distribution as well as the characterisation of optimal taxes are of the same form as in the case of exogenous wages. To get some intuition for this result, one should note that tax shifting has a bearing on both the (dis)incentive effect and the distributional effect that are traded off at the optimum.

Appendix A

The case of three able types. In order to characterise the welfare-maximising policy subject to the government's budget constraint, we formulate the Lagrange function

(A.1)
$$\Lambda = W + \lambda[T_1 n_1 F(\overline{\alpha}) + T_2 n_2 G(\overline{\beta}) + T_3 n_3 K(\overline{\eta}) - n_0 c_0 - n_1 c_0 (1 - F(\overline{\alpha})) - n_2 c_0 (1 - G(\overline{\beta})) \\ - n_3 c_0 (1 - K(\overline{\eta}))] = W + \lambda[T_1 h_{11} + T_2 h_{22} + T_3 h_{33} - n_0 c_0 - h_{10} c_0 - h_{20} c_0 - h_{30} c_0]$$

Maximising welfare wrt T_1, T_2 and T_3, respectively, s.t. the budget constraint , we obtain the same type of first order conditions as above.

(A.2)
$$-\frac{\overline{u}'_{11}}{\lambda} + 1 - (T_1 + c_0)\frac{\partial h_{11}}{\partial c_1} / h_{11} = 0$$

(A.3)
$$-\frac{\overline{u}'_{22}}{\lambda} + 1 - (T_2 + c_0)\frac{\partial h_{22}}{\partial c_2} / h_{22} = 0$$

(A.4)
$$-\frac{\overline{u}'_{33}}{\lambda} + 1 - (T_3 + c_0)\frac{\partial h_{33}}{\partial c_3} / h_{33} = 0$$

Finally, we derive the first order condition wrt c_0.

(A.5)

$$n_0 u'_0 + h_{10} u'_{10} + h_{20} u'_{20} + h_{30} u'_{30} - \lambda(n_0 + h_{10} + h_{20} + h_{30}) + \lambda(T_1 + c_0)\frac{\partial h_{11}}{\partial c_0} + \lambda(T_2 + c_0)\frac{\partial h_{22}}{\partial c_0} + \lambda(T_3 + c_0)\frac{\partial h_{33}}{\partial c_0} = 0$$

Denoting by $\overline{u}'_0$ the marginal utility of income among non-working agents, we can rewrite (A5) as

(A.6) $$\frac{\overline{u}_0'}{\lambda}-1+(T_1+c_0)\frac{\partial h_{11}}{\partial c_0}/h_0+(T_2+c_0)\frac{\partial h_{22}}{\partial c_0}/h_0+(T_3+c_0)\frac{\partial h_{33}}{\partial c_0}/h_0=0$$

Now normalise the size of the population to unity:

(A.7) $$h_0+h_{11}+h_{22}+h_{33}=1$$

Multiply both sides of (A2) by $-h_{11}$, both sides of (A3) by $-h_{22}$, both sides of (A4) $-h_{33}$ and multiply by h_0 on both sides of (A6). Summing the resulting equations and invoking (A7), we obtain

$$\text{(A8)}\ \frac{\overline{u}_{11}'}{\lambda}h_{11}+(T_1+c_0)\frac{\partial h_{11}}{\partial c_1}+\frac{\overline{u}_{22}'}{\lambda}h_{22}+(T_2+c_0)\frac{\partial h_{22}}{\partial c_2}+\frac{\overline{u}_{33}'}{\lambda}h_{33}+(T_3+c_0)\frac{\partial h_{33}}{\partial c_3}$$
$$+\frac{\overline{u}_0'}{\lambda}h_0+(T_1+c_0)\frac{\partial h_{11}}{\partial c_0}+(T_2+c_0)\frac{\partial h_{22}}{\partial c_0}+(T_3+c_0)\frac{\partial h_{33}}{\partial c_0}=1$$

Then assuming the special case where

(A.9) $$\frac{\partial h_{11}}{\partial c_0}=-\frac{\partial h_{11}}{\partial c_1},\quad \frac{\partial h_{22}}{\partial c_0}=-\frac{\partial h_{22}}{\partial c_2}\ \text{ and }\ \frac{\partial h_{33}}{\partial c_0}=-\frac{\partial h_{33}}{\partial c_3}$$

we obtain

(A.10) $$\frac{\overline{u}_{11}'}{\lambda}h_{11}+\frac{\overline{u}_{22}'}{\lambda}h_{22}+\frac{\overline{u}_{33}'}{\lambda}h_{33}+\frac{\overline{u}_0'}{\lambda}h_0=1$$

Summing (A2) and (A6), and using the simplified notation introduced in (13), we get

(A.11) $$v_0-v_1+(T_1+c_0)\left(\frac{\partial h_{11}}{\partial c_0}/h_0-\frac{\partial h_{11}}{\partial c_1}/h_{11}\right)+(T_2+c_0)\frac{\partial h_{22}}{\partial c_0}/h_0+(T_3+c_0)\frac{\partial h_{33}}{\partial c_0}/h_0=0$$

Summing (A3) and (A6) yields

(A.12) $$v_0-v_2+(T_2+c_0)\left(\frac{\partial h_{22}}{\partial c_0}/h_0-\frac{\partial h_{22}}{\partial c_2}/h_{22}\right)+(T_1+c_0)\frac{\partial h_{11}}{\partial c_0}/h_0+(T_3+c_0)\frac{\partial h_{33}}{\partial c_0}/h_0=0$$

To discuss further issues, it is helpful to subtract (A2) from (A3) to obtain

(A.13) $$v_1-v_2+(T_1+c_0)\frac{\partial h_{11}}{\partial c_1}/h_{11}-(T_2+c_0)\frac{\partial h_{22}}{\partial c_2}/h_{22}=0$$

Appendix B

The case of endogenous incomes. In order to characterise the welfare-maximising policy in Section 4 subject to the government's budget constraint, we formulate the Lagrange function

$$\Lambda = W + \lambda\left[T_1 n_1 F(\bar{\alpha}) + T_2 n_2 G(\bar{\beta}) - n_0 c_0 - n_1 c_0 (1 - F(\bar{\alpha})) - n_2 c_0 (1 - G(\bar{\beta}))\right]$$

(B.1)

$$= W + \lambda\left[T_1 h_{11} + T_2 h_{22} - n_0 c_0 - h_{10} c_0 - h_{20} c_0\right]$$

Differentiating with respect to T_1 and T_2, and recalling that $h_{i0} = n_i - h_{i0}$ (i=1,2), we can state the first order conditions analogous to (5) and (6) in the main text above.

(B.2) $$\frac{\bar{u}'_{11}}{\lambda}\left(\frac{\partial w_1}{\partial T_1} - 1\right) - \frac{\bar{u}'_{22}}{\lambda}\frac{\partial w_1}{\partial T_1} + 1 + (T_1 + c_0)\left(\frac{\partial w_1}{\partial T_1} - 1\right)\frac{\partial h_{11}}{\partial c_1} / h_{11} - (T_2 + c_0)\frac{\partial h_{22}}{\partial c_2}\frac{1}{h_{22}}\frac{\partial w_1}{\partial T_1} = 0$$

(B.3) $$\frac{\bar{u}'_{22}}{\lambda}\left(\frac{\partial w_2}{\partial T_2} - 1\right) - \frac{\bar{u}'_{11}}{\lambda}\frac{\partial w_2}{\partial T_2} + 1 + (T_2 + c_0)\left(\frac{\partial w_2}{\partial T_2} - 1\right)\frac{\partial h_{22}}{\partial c_2} / h_{22} - (T_1 + c_0)\frac{\partial h_{11}}{\partial c_1}\frac{1}{h_{11}}\frac{\partial w_2}{\partial T_2} = 0$$

Differentiating wrt c_0, we get

$$\begin{aligned}&\frac{\bar{u}'_0}{\lambda}h_0 + \frac{\bar{u}'_{11}}{\lambda}\frac{\partial w_1}{\partial c_0}h_{11} + \frac{\bar{u}'_{22}}{\lambda}\frac{\partial w_2}{\partial c_0}h_{22} - h_0 - \frac{\partial h_{10}}{\partial c_0}c_0 - \frac{\partial h_{20}}{\partial c_0}c_0 - \frac{\partial h_{10}}{\partial c_1}\frac{\partial w_1}{\partial c_0}c_0 - \frac{\partial h_{20}}{\partial c_2}\frac{\partial w_2}{\partial c_0}c_0 \\ &+ T_1\frac{\partial h_{11}}{\partial c_0} + T_2\frac{\partial h_{22}}{\partial c_0} + T_1\frac{\partial h_{11}}{\partial c_1}\frac{\partial w_1}{\partial c_0} + T_2\frac{\partial h_{22}}{\partial c_2}\frac{\partial w_2}{\partial c_0} = 0\end{aligned} \qquad \text{(B.4)}$$

A sequence of reformulations of (B2) and (B3) and making use of the simplified notation of (13), yield

$$(T_1 + c_0)\left(\frac{\partial w_1}{\partial T_1} - 1\right)\frac{\partial h_{11}}{\partial c_1} / h_{11} - (T_2 + c_0)\frac{\partial h_{22}}{\partial c_2}\frac{1}{h_{22}}\frac{\partial w_1}{\partial T_1} = v_2\frac{\partial w_1}{\partial T_1} - 1 - v_1\left(\frac{\partial w_1}{\partial T_1} - 1\right)$$

$$-(T_1 + c_0)\frac{\partial h_{11}}{\partial c_1}\frac{1}{h_{11}}\frac{\partial w_2}{\partial T_2} + (T_2 + c_0)\left(\frac{\partial w_2}{\partial T_2} - 1\right)\frac{\partial h_{22}}{\partial c_2} / h_{22} = v_1\frac{\partial w_2}{\partial T_2} - v_2\left(\frac{\partial w_2}{\partial T_2} - 1\right) - 1$$

$$\frac{(T_1 + c_0)}{c_1}\left(\frac{\partial w_1}{\partial T_1} - 1\right)\frac{\partial h_{11}}{\partial c_1}\frac{c_1}{h_{11}} - \frac{(T_2 + c_0)}{c_2}\frac{\partial h_{22}}{\partial c_2}\frac{c_2}{h_{22}}\frac{\partial w_1}{\partial T_1} = v_2\frac{\partial w_1}{\partial T_1} - 1 - v_1\left(\frac{\partial w_1}{\partial T_1} - 1\right)$$

$$-\frac{(T_1 + c_0)}{c_1}\frac{\partial h_{11}}{\partial c_1}\frac{c_1}{h_{11}}\frac{\partial w_2}{\partial T_2} + \frac{(T_2 + c_0)}{c_2}\left(\frac{\partial w_2}{\partial T_2} - 1\right)\frac{\partial h_{22}}{\partial c_2}\frac{c_2}{h_{22}} = v_1\frac{\partial w_2}{\partial T_2} - v_2\left(\frac{\partial w_2}{\partial T_2} - 1\right) - 1$$

$$\frac{(T_1 + c_0)}{c_1}\left(\frac{\partial w_1}{\partial T_1} - 1\right)\sigma_{11} - \frac{(T_2 + c_0)}{c_2}\sigma_{22}\frac{\partial w_1}{\partial T_1} = v_2\frac{\partial w_1}{\partial T_1} - 1 - v_1\left(\frac{\partial w_1}{\partial T_1} - 1\right)$$

$$-\frac{(T_1 + c_0)}{c_1}\frac{\partial w_2}{\partial T_2}\sigma_{11} + \frac{(T_2 + c_0)}{c_2}\sigma_{22}\left(\frac{\partial w_2}{\partial T_2} - 1\right) = v_1\frac{\partial w_2}{\partial T_2} - v_2\left(\frac{\partial w_2}{\partial T_2} - 1\right) - 1$$

Then simplify the notation by defining the relative tax rates

(B.5) $$\theta_1 = \frac{T_1 + c_0}{c_1},\ \theta_2 = \frac{T_2 + c_0}{c_2}$$

Further reformulations then yield

$$\theta_1\left(\frac{\partial w_1}{\partial T_1} - 1\right)\sigma_{11} - \theta_2\sigma_{22}\frac{\partial w_1}{\partial T_1} = v_2\frac{\partial w_1}{\partial T_1} - 1 - v_1\left(\frac{\partial w_1}{\partial T_1} - 1\right)$$

(B.6)

$$-\theta_1\frac{\partial w_2}{\partial T_2}\sigma_{11} + \theta_2\sigma_{22}\left(\frac{\partial w_2}{\partial T_2} - 1\right) = v_1\frac{\partial w_2}{\partial T_2} - v_2\left(\frac{\partial w_2}{\partial T_2} - 1\right) - 1$$

Define

(B.7) $$D = \left(\frac{\partial w_1}{\partial T_1} - 1\right)\left(\frac{\partial w_2}{\partial T_2} - 1\right)\sigma_{11}\sigma_{22} - \sigma_{11}\sigma_{22}\frac{\partial w_1}{\partial T_1}\frac{\partial w_2}{\partial T_2} = \sigma_{11}\sigma_{22}\left(1 - \frac{\partial w_1}{\partial T_1} - \frac{\partial w_2}{\partial T_2}\right)$$

Solving for the relative tax rates, we get

(B.8) $$\theta_1 = (1 - v_1)\left(1 - \frac{\partial w_1}{\partial T_1} - \frac{\partial w_2}{\partial T_2}\right)\sigma_{22} / D = \frac{1 - v_1}{\sigma_{11}}$$

(B.9) $$\theta_2 = (1 - v_2)\left(1 - \frac{\partial w_2}{\partial T_2} - \frac{\partial w_1}{\partial T_1}\right)\sigma_{11} / D = \frac{1 - v_2}{\sigma_{22}}$$

In order to discuss the sign of $T_1 + c_0$, let us return to (B4).

$$\frac{\overline{u}_0'}{\lambda}h_0 + \frac{\overline{u}_{11}'}{\lambda}\frac{\partial w_1}{\partial c_0}h_{11} + \frac{\overline{u}_{22}'}{\lambda}\frac{\partial w_2}{\partial c_0}h_{22} - h_0 - \frac{\partial h_{10}}{\partial c_0}c_0 - \frac{\partial h_{20}}{\partial c_0}c_0 - \frac{\partial h_{10}}{\partial c_1}\frac{\partial w_1}{\partial c_0}c_0 - \frac{\partial h_{20}}{\partial c_2}\frac{\partial w_2}{\partial c_0}c_0$$
$$+T_1\frac{\partial h_{11}}{\partial c_0} + T_2\frac{\partial h_{22}}{\partial c_0} + T_1\frac{\partial h_{11}}{\partial c_1}\frac{\partial w_1}{\partial c_0} + T_2\frac{\partial h_{22}}{\partial c_2}\frac{\partial w_2}{\partial c_0} = 0$$

Since $\frac{\partial h_{i0}}{\partial c_0} = -\frac{\partial h_{ii}}{\partial c_0}$ for i=1,2, we can reformulate (B4) as

$$v_0h_0 + v_1\frac{\partial w_1}{\partial c_0}h_{11} + v_2\frac{\partial w_2}{\partial c_0}h_{22} - h_0 + (c_0 + T_1)\frac{\partial h_{11}}{\partial c_0} + (c_0 + T_2)\frac{\partial h_{22}}{\partial c_0} + (c_0 + T_1)\frac{\partial h_{11}}{\partial c_1}\frac{\partial w_1}{\partial c_0} + (c_0 + T_2)\frac{\partial w_2}{\partial c_0}\frac{\partial h_{22}}{\partial c_2} = 0$$

and since $\frac{\partial w_2}{\partial c_0} = -\frac{\partial w_1}{\partial c_0}\frac{h_{11}}{h_{22}}$

$$v_0 - 1 + v_1 \frac{\partial w_1}{\partial c_0}\frac{h_{11}}{h_0} + v_2\left(-\frac{\partial w_1}{\partial c_0}\right)\frac{h_{11}}{h_0} + \left(c_0 + T_1\right)\frac{\partial h_{11}}{\partial c_0}\frac{1}{h_0} + \left(c_0 + T_2\right)\frac{\partial h_{22}}{\partial c_0}\frac{1}{h_0} + \left(c_0 + T_1\right)\frac{\partial h_{11}}{\partial c_1}\frac{\partial w_1}{\partial c_0}\frac{1}{h_0}$$
$$+\left(c_0 + T_2\right)\left(-\frac{h_{11}}{h_{22}}\frac{\partial w_1}{\partial c_0}\right)\frac{\partial h_{22}}{\partial c_2}\frac{1}{h_0} = 0$$

Now make partial substitution by means of $(1-v_1)h_{11} = \left(T_1 + c_0\right)\frac{\partial h_{11}}{\partial c_1}$ and $(1-v_2)h_{22} = \left(T_2 + c_0\right)\frac{\partial h_{22}}{\partial c_2}$.

$$v_0 - 1 + v_1 \frac{\partial w_1}{\partial c_0}\frac{h_{11}}{h_0} + v_2\left(-\frac{\partial w_1}{\partial c_0}\right)\frac{h_{11}}{h_0} + \left(c_0 + T_1\right)\frac{\partial h_{11}}{\partial c_0}\frac{1}{h_0} + \left(c_0 + T_2\right)\frac{\partial h_{22}}{\partial c_0}\frac{1}{h_0} + \left(1 - v_1\right)\frac{h_{11}}{h_0}\frac{\partial w_1}{\partial c_0} - \left(1 - v_2\right)\frac{h_{11}}{h_0}\frac{\partial w_1}{\partial c_0} = 0$$

$$v_0 - 1 + \left(c_0 + T_1\right)\frac{\partial h_{11}}{\partial c_0}\frac{1}{h_0} + \left(c_0 + T_2\right)\frac{\partial h_{22}}{\partial c_0}\frac{1}{h_0} = 0$$

Also using

$$1 - v_1 - \left(T_1 + c_0\right)\frac{\partial h_{11}}{\partial c_1}\frac{1}{h_{11}} = 0$$

Adding these two expressions,

$$\text{(B.10)} \quad v_0 - v_1 + \left(c_0 + T_1\right)\left(\frac{\partial h_{11}}{\partial c_0}\frac{1}{h_0} - \frac{\partial h_{11}}{\partial c_1}\frac{1}{h_{11}}\right) + \left(c_0 + T_2\right)\frac{\partial h_{22}}{\partial c_0}\frac{1}{h_0} = 0$$

which takes us back to (16). The criterion for determining the sign of the effective tax is the same as before.

Let us examine the effect of T_1 on wages. The labour input ratio will be determined by relative wages and labour input equals labour supply as expressed by (26) and (27).

$$\varphi\left(\frac{\ell_{11}(w_1 - T_1 - c_0)}{\ell_{22}(w_2 - T_2 - c_0)}\right) = \frac{w_1}{w_2}$$

$$\varphi' \frac{1}{h_{22}^2}\left(\ell_{22}\frac{\partial \ell_{11}}{\partial c_1}\left(\frac{\partial w_1}{\partial T_1} - 1\right) - \ell_{11}\frac{\partial \ell_{22}}{\partial c_2}\frac{\partial w_2}{\partial T_1}\right) = \frac{1}{w_2^2}\left(w_2\frac{\partial w_1}{\partial T_1} - w_1\frac{\partial w_2}{\partial T_1}\right)$$

$$\varphi' \frac{1}{h_{22}^2}\left(\ell_{22}\frac{\partial \ell_{11}}{\partial c_1}\left(\frac{\partial w_1}{\partial T_1} - 1\right) - \ell_{11}\frac{\partial \ell_{22}}{\partial c_2}\left(-\frac{h_{11}}{h_{22}}\frac{\partial w_1}{\partial T_1}\right)\right) = \frac{1}{w_2^2}\left(w_2\frac{\partial w_1}{\partial T_1} - w_1\left(-\frac{h_{11}}{h_{22}}\frac{\partial w_1}{\partial T_1}\right)\right)$$

$$\varphi' \frac{1}{h_{22}^2}\left(\ell_{22}\frac{\partial \ell_{11}}{\partial c_1} + \frac{h_{11}}{h_{22}}\ell_{11}\frac{\partial \ell_{22}}{\partial c_2}\right)\frac{\partial w_1}{\partial T_1} - \frac{1}{w_2^2}\left(w_2 + \frac{h_{11}}{h_{22}}w_1\right)\frac{\partial w_1}{\partial T_1} = \varphi' \frac{1}{h_{22}^2}\ell_{22}\frac{\partial \ell_{11}}{\partial c_1}$$

The right hand side is negative, and the left hand side is negative if $\frac{\partial w_1}{\partial T_1} > 0$, which then must hold.

$$\varphi' \frac{1}{h_{22}^2}\left(\ell_{22}\frac{\partial \ell_{11}}{\partial c_1}\frac{\partial w_1}{\partial T_2} - \ell_{11}\frac{\partial \ell_{22}}{\partial c_2}\left(\frac{\partial w_2}{\partial T_2} - 1\right)\right) = \frac{1}{w_2^2}\left(w_2\frac{\partial w_1}{\partial T_2} - w_1\frac{\partial w_2}{\partial T_2}\right)$$

$$\varphi' \frac{1}{h_{22}^2}\left(\ell_{22}\frac{\partial \ell_{11}}{\partial c_1}\frac{\partial w_1}{\partial T_2} - \ell_{11}\frac{\partial \ell_{22}}{\partial c_2}\left(-\frac{h_{11}}{h_{22}}\frac{\partial w_1}{\partial T_2} - 1\right)\right) = \frac{1}{w_2^2}\left(w_2\frac{\partial w_1}{\partial T_2} - w_1\left(-\frac{h_{11}}{h_{22}}\frac{\partial w_1}{\partial T_2}\right)\right)$$

$$\varphi' \frac{1}{h_{22}^2}\left(\ell_{22}\frac{\partial \ell_{11}}{\partial c_1} + \frac{h_{11}}{h_{22}}\ell_{11}\frac{\partial \ell_{22}}{\partial c_2}\right)\frac{\partial w_1}{\partial T_2} - \frac{1}{w_2^2}\left(w_2 + \frac{h_{11}}{h_{22}}w_1\right)\frac{\partial w_1}{\partial T_2} = -\varphi' \frac{1}{h_{22}^2}\ell_{11}\frac{\partial \ell_{22}}{\partial c_2}$$

It follows that $\frac{\partial w_1}{\partial T_2} < 0$.

$$\varphi' \frac{1}{h_{22}^2}\left(\ell_{22}\frac{\partial \ell_{11}}{\partial c_1}\left(\frac{\partial w_1}{\partial c_0} - 1\right) - \ell_{11}\frac{\partial \ell_{22}}{\partial c_2}\left(\frac{\partial w_2}{\partial c_0} - 1\right)\right) = \frac{1}{w_2^2}\left(w_2\frac{\partial w_1}{\partial c_0} - w_1\frac{\partial w_2}{\partial c_{01}}\right)$$

$$\varphi' \frac{1}{h_{22}^2}\left(\ell_{22}\frac{\partial \ell_{11}}{\partial c_1}\left(\frac{\partial w_1}{\partial c_0} - 1\right) - \ell_{11}\frac{\partial \ell_{22}}{\partial c_2}\left(-\frac{h_{11}}{h_{22}}\frac{\partial w_1}{\partial c_0} - 1\right)\right) = \frac{1}{w_2^2}\left(w_2 + \frac{h_{11}}{h_{22}}w_1\right)\frac{\partial w_1}{\partial c_0}$$

$$\varphi' \frac{1}{h_{22}^2}\left(\ell_{22}\frac{\partial \ell_{11}}{\partial c_1} + \frac{h_{11}}{h_{22}}\ell_{11}\frac{\partial \ell_{22}}{\partial c_2}\right)\frac{\partial w_1}{\partial c_0} - \frac{1}{w_2^2}\left(w_2 + \frac{h_{11}}{h_{22}}w_1\right)\frac{\partial w_1}{\partial c_0} = \varphi' \frac{1}{h_{22}^2}\left(\ell_{22}\frac{\partial \ell_{11}}{\partial c_1} - \ell_{11}\frac{\partial \ell_{22}}{\partial c_2}\right)$$

$$\varphi' \frac{1}{h_{22}^2}\left(\ell_{22}\frac{\partial \ell_{11}}{\partial c_1} + \frac{h_{11}}{h_{22}}\ell_{11}\frac{\partial \ell_{22}}{\partial c_2}\right)\frac{\partial w_1}{\partial c_0} - \frac{1}{w_2^2}\left(w_2 + \frac{h_{11}}{h_{22}}w_1\right)\frac{\partial w_1}{\partial c_0} = \varphi' \frac{1}{h_{22}^2}\frac{\ell_{11}\ell_{22}}{c_0}\left(\frac{\partial \ell_{22}}{\partial c_0}\frac{c_0}{\ell_{22}} - \frac{\partial \ell_{11}}{\partial c_0}\frac{c_0}{\ell_{11}}\right)$$

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