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Abstract

This paper explores the implications of gender-based income taxation in a non- cooperative model of a couple's time allocation between market work and providing a household public good. We find that the optimal structure of differential taxation by gender is solely determined by spouses' relative marginal rates of substitution between the public household good and private consumption. Breaking down this general rule into the primitives of the model, the spouse with a lower personal valuation of the public household good should be taxed at a higher rate. If these valuations are identical, a comparative advantage in home production relative to market work will imply a higher marginal tax rate. Using a realistic calibration, we show that these two results may combine to imply a higher optimal tax rate on female labor supply. This result stands in sharp contrast to previous models of gender-based taxation in which households select Pareto efficient allocations. Extending the model to include altruistic preferences, leisure, or human capital accumulation reduces optimal tax rates, while sequential labor supply decisions affect the optimal tax rate of the primary earner in an ambiguous direction.

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Keywords: gender-based taxation, non-cooperative family decision-making.

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1. Introduction

Should husband and wife be taxed differently? This question lies at the heart of the debate on how to tax family income. As constitutional rulings usually require governments to treat men and women alike, the space of political decision-making is largely limited to the choice between individual taxation and joint taxation. The latter formally implies that marginal tax rates of husband and wife are always identical, regardless of individual shares in generating total family income. However, if framed in terms of primary and secondary earner, a system of joint taxation with a progressive tax rate schedule disincentivizes secondary earners because the tax on their earned income starts at the highest marginal tax rate of the primary income. With this interpretation in mind, is is quite conceivable to construct a politically acceptable income tax schedule prescribing lower marginal taxes on secondary earners, appropriately defined. While such a specification may not be challenged as discriminatory, it would effectively allow for lower tax rates on female earnings.

Public finance theory has long acknowledged the importance of gender by highlighting the differences in the labor supply behavior of men and women and their implications for optimal income taxation. A common theme in most contributions is Ramsey's optimal taxation criterion whereby tax rates should be inversely proportional to the labor supply elasticity of the taxpayer. Since married women's labor supply is more elastic than that of men or single women (Pencavel, 1986; Evers et al., 2008), gender-based taxation with lower rates for women than for men—as advocated by Alesina et. al (2011, henceforth AIK)— and Apps and Rees (2011a) is desirable on grounds of economic efficiency. If the policy choice set is restricted to individual versus joint taxation, the logic of Ramsey taxation yields a preference in favor of the former method (Boskin and Sheshinski, 1983; Apps and Rees, 1999a; Meier and Wrede, 2013). The literature further argues that individual incomes should not be taxed independently (Brett, 2007), but probably in a fashion where marginal tax rates of the secondary earner fall in the income of the primary earner (Kleven et al., 2009). Several authors focus on the relevance of household production as an alternative to labor supply in the market, which remains untaxed in both its production and its trade component (Apps and Rees, 1999b) and may exhibit public good characteristics on the household level. Moreover, household production may involve time inputs of both husband and wife. Hence, not only the amount of production may be a source of inefficiency, but also the structure of time inputs. In this respect, Piggott and Whalley (1996) stress that joint taxation has the advantage to induce a symmetric distortion. Kleven and Kreiner (2007) argue that optimal marginal taxation of secondary earners may not fall short of taxation of primary earners if taxation of input goods that can be used in household production is taken into account.

Though existing studies of household taxation differ in many respects, they share the common characteristic of assuming that couples are able to determine their time allocations in an efficient manner. Justifications of this assumption point to cooperative bargaining or relational contracts within households (AIK, 2011; Apps and Rees, 2011b). Yet, in line with the theoretical argument that transaction costs may prevent couples from reaching cooperative outcomes (Pollak, 1985), recent research casts doubt on the systematic recourse to this efficiency assumption. For example, econometric evidence from time allocation models which allow for both efficient and inefficient intrahousehold behavior suggests that a sizeable proportion of couples behaves non-cooperatively (Del Boca and Flinn, 2012). Similarly, results from experiments that study family behavior in social dilemma games indicate that cooperation is not ubiquitous among maritally living couples (Cochard et al., 2009). We therefore find it important to examine the implications of gender-based taxation in a model that allows for inefficient family decision-making.[1]

To this end, we consider a non-cooperative framework of a family's time allocation between market work and providing a home-produced public good. We postulate a linear tax schedule and study the extent to which men and women should be taxed differently. Since labor supply is non-coordinated, couples fail to provide the optimal level of the family public good. In our basic setup, the main function of taxation is to counteract the externalities created by non-cooperative behavior. We show that the slope of the optimal gender-specific tax schedules is solely determined by spouses' *relative* marginal rates of substitution between the public household good and the private good. In particular, gender-based taxation with higher taxes on men is optimal when the marginal rate of substitution of men is smaller than the marginal rate of substitution of women, and *vice versa*.

Breaking down this general rule into the primitives of the model, we find that the optimal structure of differential taxation by gender depends on the deeper causes that sustain a gendered allocation of time. On the one hand, if women assume more household duties than men because they have higher valuations of home-produced public goods, then they should be taxed at a *lower* rate than men. However, if women perform more household production tasks than men because they have a comparative advantage in them, then gender-based taxation with *higher* marginal tax rates on women are optimal. In a calibration exercise, we show that, under reasonable parametric assumptions, these two results may combine to imply a higher optimal tax rate on female labor supply.

In order to check the limitations and the robustness of our predictions, we consider a few alternative model specifications. First, introducing altruistic "caring" preferences reduce optimal tax rates because the problem of underprovision of the family public good becomes less severe. Second, in a scenario where time allocation decisions are made sequentially and side payments between family members are feasible, the optimal tax treatment of the primary earner changes by a term reflecting whether inputs in household production function are complements or substitutes. Third, when accounting for ex-ante career choices, optimal tax rates are lower to reduce disincentives in human capital accumulation. Finally, introducing leisure as an alternative use of time again reduces optimal tax rates to some extent because the traditional labor supply distortion problem gains in importance. Each of these alternative specifications allows us to unearth an additional rationale for differentiated taxes by gender. However, none of the extensions changes our basic results substantially.

Our results stand in sharp contrast to those prevailing in the efficiency-based household taxation literature: AIK study gender-based taxation in a model in which labor supply elasticities emerge endogenously from a cooperatively bargained allocation of goods and time in the family. Their main conclusion is that a system of selective taxation with lower marginal tax rates for women is superior to an ungendered tax code, independently

[1] In this respect, our notion of non-cooperative behavior refers only to non-coordinated decisions across various dimensions, but never to some desire to do harm to the spouse.

of the deeper reasons that sustain gendered allocations of time. The main advantage of structuring tax schedules in favor of women in AIK's setup is that it minimizes the distortionary cost of taxation. In parallel, it offers the benefit of endogenously balancing the allocation of work across genders. In our setup, gender-based taxation with higher marginal tax rates on women may be necessary to counteract the externalities created by the non-coordinated labor supply behavior of couples.

From a policy perspective, our goal is not to argue that re-balancing the tax structure in favor of women is undesirable but to highlight a particular effect that strikes a cautionary note on thinking about its welfare consequences: efficiency models of the family and theories based on non-cooperative behavior suggest distinct and sometimes mutually exclusive optimal taxation criteria.

Apart from complementing the literature on the optimal taxation of couples, this study is also related to the literature on non-cooperative family decision-making. From an empirical viewpoint, the underlying motivation of our work stems from econometric estimates of Del Boca and Flinn (2012), which suggest roughly one-fourth of households behaves non-cooperatively. Relatedly, Jia (2005) empirically examines labor supply of retiring couples and concludes that more than one-half of households behaves according to a non-cooperative model of family decision-making. From a theoretical perspective, a close antecedent to our paper is Konrad and Lommerud (1995). They show that it is possible to influence non-cooperative household outcomes by lump-sum redistribution from one spouse to the other, and that such redistribution might lead to a Pareto-improvement. The non-cooperative approach has also been adopted by Anderberg (2007) analyzing the mix of government spending when family behavior is inefficient, and by Gugl (2009) who investigates the impact of tax regimes on inequality within the household. Finally, in our own work (Meier and Rainer, 2012) we show that joint taxation may be Pareto-superior to individual taxation under a Stackelberg equilibrium modeling assumption with a household public good.

The remainder of this paper is organized as follows. After introducing the basic model in Section 2, Section 3 exemplifies the optimal taxation of couples. The main result is disentangled into its components in Section 4. Having discussed some extensions in Section 5, Section 6 concludes and indicates possible directions for further research.

2. The Basic Model

2.1. Formal Structure

Consider a representative family consisting of two decision-makers, a (he) and b (she). Throughout we use the notation $i \in \{a, b\}$ to refer to either one of the two. Individual i's preferences are represented by a strictly increasing and strictly quasi-concave utility function defined over a private good, c_i, and a home-produced public good, q. Examples of the public good comprise the upbringing and education of children, and care for the elderly. Formally, the utility functions of a and b are

$$U^i(c_i, q) \quad \text{for } i = a, b. \tag{1}$$

We denote by $U_k^i(c_i, q)$ the first-order partial derivative of U^i with respect to its k-th argument ($k = c, q$). The second-order partial derivatives are represented by $U_{kl}^i(c_i, q)$ (or simply U_{kl}^i), where $k, l = c, q$.

Each partner has a unit of active time endowment, which can be allocated between working in the outside labor market ($1-\ell_i$) and working at home (ℓ_i), thereby contributing to the production of a household public good. The household production function f depends on time inputs $\ell \equiv (\ell_a, \ell_b)$:

$$q = f(\ell) \tag{2}$$

For each $i = a, b$, we denote by $f_i(\ell)$ and $f_{ii}(\ell)$ the first-order and second-order partial derivative of f with respect to ℓ_i. We assume that f is increasing and concave in its first two arguments: $f_i(\ell) > 0$ and $f_{ii}(\ell) < 0$.

Spouses may be differently productive in market work and home production. The productivity in the labor market is given by the gross market wage, w_i. The marginal productivity in household production is captured by $f_i(\ell)$.

The consumption levels of a and b are

$$c_i = w_i(1 - \tau_i)(1 - \ell_i) + \vartheta_i \quad \text{for } i = a, b \tag{3}$$

where $\tau_i \geq 0$ is the marginal tax rate and ϑ_i is a lump-sum transfer.

The sequence of events is as follows. First, the governments sets labor taxes $\tau = (\tau_a, \tau_b)$ and determines the lump-sum transfers. Second, the spouses non-cooperatively decide on how to allocate their time between market work and home production. To characterize equilibrium time allocations, we will consider both a simultaneous-move and a sequential-move game between the spouses. Labor taxes $\tau = (\tau_a, \tau_b)$ will be set to correct the externality from non-cooperative behavior. Moreover, we consider the case of lump-sum redistributed tax proceeds $\vartheta = (\vartheta_a, \vartheta_b)$ whereby each individual i receives a transfer ϑ_i that is equal to her labor income taxes.

$$\vartheta_i = \tau_i w_i(1 - \ell_i) \quad \text{for } i = a, b \tag{4}$$

Thus, all wage taxes paid are returned to family members as lump-sum benefits, implying a tax revenue requirement of zero. The specification of type-specific lump-sum transfers where taxes paid are returned in full allows for ruling out distributional goals of the government, ensuring that labor income taxes serve purely allocative purposes.

For convenience, we let

$$MRS_{qc}^i = \frac{U_q^i(c_i, q)}{U_c^i(c_i, q)}$$

denote i's marginal rate of substitution between the public and the private good.

2.2. First-Best Benchmark

To derive the first-best benchmark, we now replicate a Pareto efficient allocation without labor income taxes and lump-sum redistributed tax proceeds. Thus, we maximize one partner's utility subject to a given level of the other and the resource constraint. The

Lagrangian reads:

$$\mathcal{L} = U^a(c_a, q) + \lambda\Big[U^b(c_b, q) - \overline{U}^b\Big] + \mu\Big[\sum_{i\in\{a,b\}}\Big(w_i(1-\ell_i) - c_i\Big)\Big]$$

where λ and μ are Lagrange multipliers, and w_i is the gross market wage. In any interior solution, the first-order conditions are

$$U_c^a(c_a, q) - \mu = 0, \tag{5}$$

$$\lambda U_c^b(c_b, q) - \mu = 0, \tag{6}$$

$$\left[U_q^a(c_a, q) + \lambda U_q^b(c_b, q)\right] f_a(\ell) - \mu w_a = 0, \tag{7}$$

$$\left[U_q^a(c_a, q) + \lambda U_q^b(c_b, q)\right] f_b(\ell) - \mu w_b = 0. \tag{8}$$

These conditions can be simplified to express the Samuelson rule, stating that the sum of the marginal rates of substitution between the public and the private good must be equal to the marginal rate of transformation between these two goods.

$$MRS_{qc}^a + MRS_{qc}^b = \frac{w_i}{f_i(\ell)} \quad \text{for } i = a, b \tag{9}$$

Additionally, the (first-best) socially efficient allocation is also characterized by the marginal rates of transformation being equated across partners:

$$\frac{w_a}{f_a(\ell)} = \frac{w_b}{f_b(\ell)}. \tag{10}$$

This ensures efficiency in the production of the public good.

3. Optimal Labor Taxation of Non-Cooperative Couples

If there are no limits to cooperation, we would expect couples to achieve some first-best allocation as described above. We now follow the literature on non-cooperative family decision-making (see, e.g., Bergstrom, 1989; Lundberg and Pollak, 1993; Konrad and Lommerund, 1995; Chen and Woolley, 2001; Anderberg, 2007) in supposing that couples are not able to reach efficient outcomes. There are a number of ways of modeling non-cooperative family behavior. Our basic model focuses on an environment in which the individuals play a simultaneous move game and determine their time allocation independently. Later on, we will also extend our analysis to a sequential move game which allows for the possibility of side payments between spouses. Throughout, we will focus on interior private provision equilibria in which neither partner fully specializes in market work.

The two partners simultaneously and non-cooperatively choose how to divide their time endowments between market work and home production. We analyze the resulting time allocations that constitute a Nash equilibrium. An interior provision equilibrium can be characterized as follows. Each partner consumes quantities q^* and c_i^* of the public and the private good, respectively. Moreover, $q^* = f(\ell^*)$ and $c_i^* = w_i(1-\tau_i)(1-\ell_i^*) + \vartheta_i$

satisfy

$$\frac{MRS^i_{qc}}{1-\tau_i} = \frac{w_i}{f_i(\ell)} \quad \text{for } i = a, b \tag{11}$$

which depends on $\tau = (\tau_a, \tau_b)$ and $\vartheta = (\vartheta_a, \vartheta_b)$ as well as on the other model parameters. In equilibrium, each partner allocates her time between market work and home production such that her marginal rate of substitution between the public and the private good, multiplied by the tax wedge $\frac{1}{1-\tau_i}$ (i.e., the ratio of gross wage and net wage), equals the marginal rate of transformation between the two goods.

Compared to the first-best, non-cooperative behavior implies that there is no self-enforcing mechanism that induces the partners to internalize the impact of their choices on each other. As a consequence, each partner tends to supply an inefficiently high amount of time to the labor market, implying an inefficiently low provision of the household public good. In the presence of this inefficiency, wage taxes are no longer necessarily distortionary. Instead, they have a corrective element that may fully address the externality from non-cooperative behavior. We have:

Proposition 1. *The couple can be induced to attain the first-best allocation of time between market work and home production by implementing a set of corrective (Pigouvian) labor income taxes with lump-sum transfers to each individual that are equal to his or her labor income taxes. The first-best inducing labor income taxes $\tau^* = (\tau^*_a, \tau^*_b)$ satisfy:*

$$\tau_a = \frac{MRS^b_{qc}}{MRS^a_{qc} + MRS^b_{qc}} \quad \textit{and} \quad \tau_b = \frac{MRS^a_{qc}}{MRS^a_{qc} + MRS^b_{qc}} \tag{12}$$

Proof. In order to implement a socially efficient allocation, the partners' equilibrium choices have to satisfy both the Samuelson condition [eq. 9] and the home production efficiency requirement [eq. (10)]. Both conditions are simultaneously fulfilled if and only if

$$\frac{MRS^a_{qc}}{1-\tau_a} = MRS^a_{qc} + MRS^b_{qc} = \frac{MRS^b_{qc}}{1-\tau_b}.$$

Inserting (12) into the individual's first-order condition (11) shows that this condition will indeed hold. □

The first-best inducing marginal tax rates have the striking feature that they sum-up to one. Moreover, they solely depend on *gender differences in marginal rates of substitution*. In particular, gender-based taxation with higher taxes on men is optimal when the marginal rate of substitution of men is smaller than the marginal rate of substitution of women, and *vice versa*:

$$\tau^*_a \gtreqless \frac{1}{2} \gtreqless \tau^*_b \qquad \text{if and only if} \qquad MRS^a_{qc} \lesseqgtr MRS^b_{qc}$$

This result has a simple logic. It follows from the observation that the expression $\frac{MRS^i_{qc}}{MRS^a_{qc} + MRS^b_{qc}}$ $(i = a, b)$ captures the relative degree to which individual i "under-invests" into home production activities relative to the first-best. Indeed, the lower is partner i's marginal rate of substitution relative to the sum of marginal rates of substitution, the more severe is his or her underinvestment relative to that of the other spouse. The optimal gender-specific tax rates fully eliminates the inefficiency arising from non-cooperative behavior by imposing a tax rate on each individual based on the partner's relative degree of underinvestment. By requiring a higher marginal tax rate for the partner whose equilibrium choice more severely deviates from the socially efficient allocation, a *deviation from the first-best* principle is established.

4. Gendered Equilibria and Gender-Based Taxation

So far we have discussed differential taxation by gender in terms of marginal rates of substitution. However, the marginal rates of substitution cannot be taken as exogenous primitives, but depend endogenously on marginal tax rates, lump-sum transfers, wages, home productivities, and preference parameters. In other words, our main results so far do not provide an explicit solutions for the first-best inducing marginal tax rates, but characterize $\tau^* = (\tau_a^*, \tau_b^*)$ and $\vartheta^* = (\vartheta_a^*, \vartheta_b^*)$ as functions of the primitives of the model. Our analysis now proceeds as follows. First, we unearth the reasons that sustain a gendered allocation of time in our non-cooperative decision-making framework. Second, we ask how the optimal proportional tax rates on men relative to women depend on the reasons that sustain a gendered equilibrium.

We develop our results in a simpler framework where payoffs are additive. Let

$$U^i(c_i, q) = (1-\gamma_i)v(c_i) + \gamma_i z(q) \qquad \text{for } i = a, b, \tag{13}$$

where γ_i is a preference parameter which measures the relative importance of the household public good. We assume that $v(\cdot)$ and $z(\cdot)$ are well-behaved increasing and concave functions. To keep the analysis tractable, we additionally assume that the partners' time inputs into household production are "independent". Thus, we let the composition of the utility function $z(q)$ with the household production function $q = f(\ell)$ be given by

$$z(q) = z(f(\ell)) = \xi_a \mu(\ell_a) + \xi_b \mu(\ell_b), \tag{14}$$

where $\mu(\cdot)$ is a well-behaved increasing and concave function. The parameters ξ_a and ξ_b capture the partners' productivity in home production. For example, combining a Cobb-Douglas production function $q = (\ell_a)^{\xi_a}(\ell_b)^{\xi_b}$ with logarithmic utility, $z(q) = \ln q$, would yield $\mu(l_i) = \ln l_i$.

Under the above assumptions, the partners' time inputs into home production are neither complements nor substitutes. Therefore, each partner has a strictly dominant time allocation strategy. Indeed, an interior provision equilibrium now fulfills

$$\frac{MRS^i_{qc}}{1-\tau_i} = \frac{w_i}{\xi_i \mu'(\ell_i)} \quad \text{with} \quad MRS^i_{qc} = \frac{\gamma_i z'(q)}{(1-\gamma_i)v'(c_i)} \tag{15}$$

which only depends on ℓ_i. In general, the effect on i's own wage on ℓ_i is ambiguous due to conflicting income and substitution effects. Throughout the paper we will assume, however, that the substitution effect dominates so that individual i's time allocated to market work increases in the own wage. Formally, we impose $\varepsilon_{v'c} \equiv cv''(c)/v'(c) \in (-1, 0]$

and $\varepsilon_{z'q} \equiv qz''(q)/z'(q) \in (-1, 0]$. The latter ensures that when household productivity increases, the rising marginal utility of the respective time input is not offset by the diminishing marginal utility due to a higher level of the household good at given behavior.

We now discuss the extent to which the equilibrium gives rise to a gendered allocation of time. We have

Proposition 2. *Suppose that one of the three cases holds:*

(a) $\gamma_a < \gamma_b$ *with* $w_a = w_b$ *and* $\xi_a = \xi_b$.

(b) $w_a > w_b$ *with* $\xi_a = \xi_b$ *and* $\gamma_a = \gamma_b$.

(c) $\xi_a < \xi_b$ *with* $w_a = w_b$ *and* $\gamma_a = \gamma_b$.

Holding constant (τ_a, τ_b) *and* $(\vartheta_a, \vartheta_b)$ *at some arbitrary levels* $\bar{\tau}$ *and* $\bar{\vartheta}$ *respectively, men work more in the labor market than women and take less home duties than women* $(\ell_a^* < \ell_b^*)$.

Proof. See the Appendix. □

The proposition describes three cases which sustain a gendered equilibrium. In the first case, women have, for exogenous reasons, a higher preference for the household public good than men ($\gamma_a < \gamma_b$). Econometric studies of family behavior which identify preference parameters suggest that the average weight placed on home-produced public goods is indeed greater for women than for men (Del Boca and Flinn, 2012). In the second case, men receive exogenously (e.g., due to gender discrimination) a higher wage than women in the labor market ($w_a < w_b$). In the third case, women are exogenously (e.g., due to biological differences) more productive than men in performing home duties ($\xi_a < \xi_b$). Under any of these specifications, a gendered allocation of time arises in which women assume more responsibilities for home production.

What are the implications of these three cases for the optimal proportional tax rates on men relative to women? First, suppose that a gendered allocation of time stems from gender differences in preferences. We have:

Proposition 3. *If women value the household public good more than men* ($\gamma_a < \gamma_b$ *with* $w_a = w_b$ *and* $\xi_a = \xi_b$) *then gender-based taxation with higher marginal tax rates on men is optimal.*

Proof. See the Appendix. □

To illustrate one specific case, suppose the utility function of each individual is linear in consumption, i.e., let $v_i(c_i) = c_i$. In this case, the optimal marginal tax rates are given by

$$\tau_a^* = \frac{\kappa_b}{\kappa_a + \kappa_b} \quad \text{and} \quad \tau_b^* = \frac{\kappa_a}{\kappa_a + \kappa_b}, \quad \text{where} \quad \kappa_i = \frac{\gamma_i}{1 - \gamma_i}.$$

Each partner attaches a relative preference weight of κ_i to the public good when choosing how to allocate his or her time between work and home production. The socially efficient allocation, however, would require him or her to attach a relative weight of $\kappa_a + \kappa_b$ to the public good. Thus, if men value the public good less than women, their time allocation choice will deviate more from the first-best than that of women. Gender-based taxation with higher marginal tax rates on men address the relative severity of the underprovision problem among them, while at the same time guaranteeing that women also are incentivized to choose the first-best.

Next, consider the other two cases in which women assume more home duties than men because they have a comparative advantage in them:

Proposition 4. *If men receive a higher wage than women in the labor market ($w_a > w_b$ with $\xi_a = \xi_b$ and $\gamma_a = \gamma_b$), or if women are more productive than men in performing home duties ($\xi_a < \xi_b$ with $w_a = w_b$ and $\gamma_a = \gamma_b$), then gender-based taxation with higher marginal tax rates on women is optimal.*

Proof. See the Appendix. □

When women have a comparative advantage in home duties, then the marginal tax rate on women is higher at the optimum than that on men. The intuition behind this result is as follows. While even in our noncooperative setting there is a tendency towards specialization between partners in market work and the provision of household public goods, there is less specialization than in a cooperative model. Thus, if gender differences are assumed to originate from women's comparative advantage in home duties, then women's inputs into home production are more distorted than that of men from an allocative point of view. The optimal marginal tax rates ensure—by weighing relative input distortions—that the partners' joint contributions to the household public good correspond to the first-best.

A Cobb-Douglas Example. We now provide a simple Cobb-Douglas example which, in addition to illustrating the model, allows us to highlight a few more of its features. Let

$$U^i(c_i, q) = (1-\gamma_i)\ln c_i + \gamma_i \ln q \quad \text{and} \quad q = (\ell_a)^{\xi_a}(\ell_b)^{\xi_b} \tag{16}$$

In an interior equilibrium, ℓ_i^* and c_i^* are given by

$$\ell_i^*(\tau_i, \vartheta_i) = \frac{\gamma_i \xi_i [w_i(1-\tau_i) + \vartheta_i]}{w_i(1-\tau_i)(1-\gamma_i+\gamma_i\xi_i)} \quad 1 \text{ and } \quad c_i^*(\tau_i, \vartheta_i) = (1-\gamma_i)\left[\frac{w_i(1-\tau_i)+\vartheta_i}{1-\gamma_i+\gamma_i\xi_i}\right] \tag{17}$$

According to Proposition 1, the set of corrective taxes implementing the first best, together with lump-sum redistributed tax proceeds, simultaneously solve:

$$\begin{aligned} \tau_a &= \frac{\gamma_b c_b^*(\tau_b, \vartheta_b)(1-\gamma_a)}{\gamma_a c_a^*(\tau_a, \vartheta_a)(1-\gamma_b) + \gamma_b c_b^*(\tau_b, \vartheta_b)(1-\gamma_a)}, \\ \tau_b &= \frac{\gamma_a c_a^*(\tau_a, \vartheta_a)(1-\gamma_b)}{\gamma_a c_a^*(\tau_a, \vartheta_a)(1-\gamma_b) + \gamma_b c_b^*(\tau_b, \vartheta_b)(1-\gamma_a)} \\ \vartheta_a &= \tau_a w_a (1 - \ell_a^*(\tau_a, \vartheta_a)), \\ \vartheta_b &= \tau_b w_b (1 - \ell_b^*(\tau_b, \vartheta_b)), \end{aligned} \tag{18}$$

These conditions are used to prove:

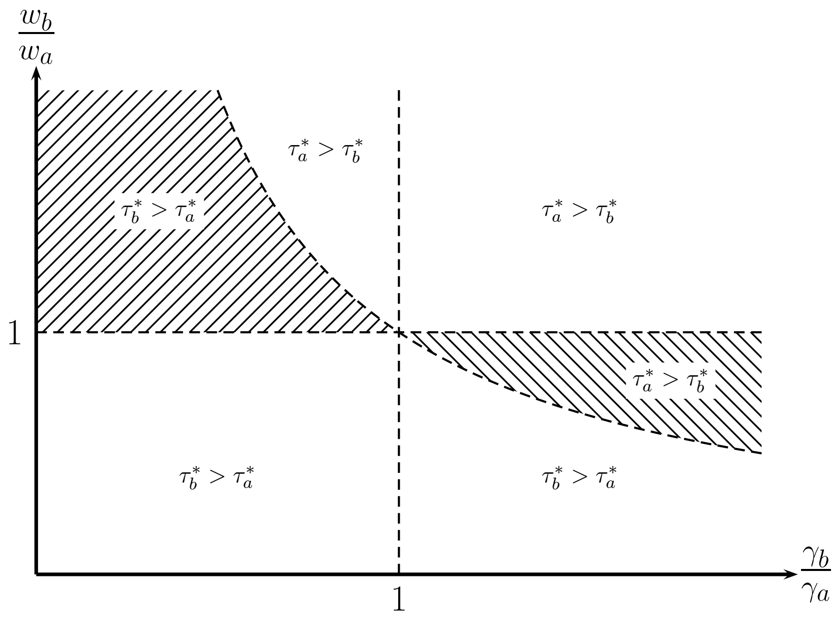

Figure 1: Optimal marginal tax rate regimes.

Proposition 5. *Let the partners' preferences and the home production technology be given by (16). The first best allocation of time between market work and home production can be implemented by a set of labor taxes $\tau^* = (\tau_a^*, \tau_b^*)$ satisfying:*

$$\tau_a^* = \frac{\gamma_b[w_b(1-\gamma_a+\gamma_a\xi_a) - w_a\gamma_a\xi_b]}{\gamma_a w_a(1-\gamma_b) + \gamma_b w_b(1-\gamma_a)} \quad \text{and} \quad \tau_b^* = \frac{\gamma_a[w_a(1-\gamma_b+\gamma_b\xi_b) - w_b\gamma_b\xi_a]}{\gamma_a w_a(1-\gamma_b) + \gamma_b w_b(1-\gamma_a)} \tag{19}$$

Lump-sum redistributed tax proceeds $\vartheta^* = (\vartheta_a^*, \vartheta_b^*)$ are:

$$\vartheta_i^* = \frac{(1-\gamma_i)[w_b(1-\gamma_a+\gamma_a\xi_a) - w_a\gamma_a\xi_b][w_a(1-\gamma_b+\gamma_b\xi_b) - w_b\gamma_b\xi_a]}{[(1-\gamma_a)(1-\gamma_b) + \gamma_a\xi_a(1-\gamma_b) + \gamma_b\xi_b(1-\gamma_a)][\gamma_a w_a(1-\gamma_b) + \gamma_b w_b(1-\gamma_a)]} \tag{20}$$

for $i = a, b$.

Proof. Inserting (17) into (18) and solving for (τ_a, τ_b) and $(\vartheta_a, \vartheta_b)$ yields the claims. □

Figure 1 illustrates these results.[2] It plots the ratio of wages, w_b/w_a, against the ratio of preferences, γ_b/γ_a, to illustrate the properties of the optimal gender-differentiated tax rates. From Proposition 3 we know that if we restrict our attention to gender differences

[2]From equation (19), it is easy to check that

$$\tau_a^* \gtrless \tau_b^* \qquad \text{if and only if} \qquad \frac{2\xi_b\left(\frac{w_b}{w_a}\frac{\xi_a}{\xi_b} - 1\right) + 1 - \frac{w_b}{w_a}}{2\frac{w_b}{w_a}} \gtrless \frac{1}{2\gamma_a}\left(\frac{1}{\frac{w_b}{w_a}\frac{\gamma_b}{\gamma_a}} - 1\right).$$

For illustrative purposes, we set $\xi_a = \xi_b = \frac{1}{2}$. It then follows immediately that

$$\tau_a^* \gtrless \tau_b^* \qquad \text{if and only if} \qquad \frac{w_b}{w_a} \gtrless \frac{1}{\frac{\gamma_b}{\gamma_a}}.$$

in preferences only, with women valuing the household public good more than men, then gender-based taxation with higher marginal tax rates on men would be optimal. However, if we allow differences in wages as well, even if $\gamma_b/\gamma_a > 1$, if the wages of men are sufficiently higher than the wages of women, differential taxation with higher marginal taxes on women would be optimal, and conversely, when $\gamma_b/\gamma_a < 1$.

To conclude the discussion of the main results of the model, we briefly highlight its implications. The main message here is as follows. When family members behave non-cooperatively, the optimal structure of differential taxation by gender depends on the deeper causes that sustain a gendered allocation of time. If men and women are almost identical in their market and home productivity but women value household public goods more than men, then women should be taxed at a lower rate than men. However, if men and women are almost identical in their preferences, while women assume more home duties than men because they have a comparative advantage in them, then gender-based taxation with higher marginal tax rates on women is optimal. Ultimately, the optimal gender-specific proportional tax rate on men relative to women depend in a non-trivial way on three sets of parameters – the partners' valuations of household public goods and market and home productivities.

Econometric analyses of household behavior may help to provide this information, but until recently there has been only very limited evidence on the relevant (unobserved) state variables in non-cooperative models of family decision-making. However, a recent paper by Del Boca and Flinn (2012) sheds some light on this issue. First, the study demonstrates empirically that not all households are able to operate on the Pareto frontier, implying that a non-cooperative model of family decision-making adequately captures the behavior of a non-negligible share of households. Second, the study also identifies gender-specific preference and home productivity parameters under a Nash equilibrium modeling assumption. In Del Boca and Flinn's (2012) model, as in ours, each partner cares about the consumption of a good produced in the household with time inputs of the family members. In order to estimate the model, the authors assume Cobb-Douglas preferences with a Cobb-Douglas household good production function. Using data from the 2005 wave of the PSID, they can then recover preference weights (γ_a, γ_b) and home productivities (ξ_a, ξ_b). Their nonparametric estimation of the distribution of state variables (see their Table 2, p. 12) reveals that the mean valuation of the home-produced public good of women ($\gamma_b = 0.698$) is roughly 11% higher than the mean valuation of men ($\gamma_a = 0.631$). The average productivity in home production of women ($\xi_b = 0.106$) is approximately 41% greater than the average productivity of men ($\xi_a = 0.075$). Finally, in terms of observed state variables of their analysis, the mean wage of men ($w_a = 22.009$) is roughly 39% greater than the mean wage of women ($w_b = 15.823$). Taken at face value, the above parameter estimates and observed mean wages would imply that, in order to implement the first-best, the marginal tax rate on women has to be higher than that of men [see equation (19)]:

$$\begin{pmatrix} \gamma_a \\ \gamma_b \\ \xi_a \\ \xi_b \\ w_a \\ w_b \end{pmatrix} = \begin{pmatrix} 0.631 \\ 0.698 \\ 0.075 \\ 0.106 \\ 22.01 \\ 15.82 \end{pmatrix} \qquad \Longrightarrow \qquad \begin{pmatrix} \tau_a^* \\ \tau_b^* \end{pmatrix} = \begin{pmatrix} 0.432 \\ 0.568 \end{pmatrix}$$

This raises the following—politically incorrect—question: is there a case in practice for differential taxation by gender with higher taxes on women than on men?

Our first issue is a robustness problem. Of course, our numerical example has to be interpreted with caution. One reason is that the average gender wage gap of 39% is rather large, due to wage outliers among men in the PSID (see Del Boca and Flinn, 2011, p.12). Indeed, cross-country evidence suggests that women are paid on average 17% less than men across developed countries (OECD, 2008). If we only use Del Boca and Flinn's (2012) estimated preference weights (γ_a, γ_b) and home productivities (ξ_a, ξ_b), our results on the optimal marginal tax rates [equation (19)] suggest that:

$$\tau_a^* \gtrless \tau_b^* \qquad \text{if and only if} \qquad \frac{w_b}{w_a} \gtrless 0.88$$

Thus, if the raw wage differential between men and women does not exceed 12%, gender-based taxation with lower marginal tax rates for women is optimal. However, if larger discrepancies in pay exist between men and women, the tax rate on women will be higher at the optimum than that of men. Over time, wage differentials worldwide have fallen substantially (Weichselbaumer and Winter-Ebmer, 2005). If wage differentials continue to fall toward zero, the case for gender-based taxation will rest on gender differences in preferences—as in our non-cooperative approach—and/or on uneven bargaining powers—as in Alesina et al.'s cooperative approach. In both cases, there are reasons to believe that gender-based taxation with lower marginal tax rates on women is superior to an ungendered tax rate.

The second important issue concerns the way economists think about household behavior. Alesina et al. (2011) view household time allocation decisions as being associated with a particular utility outcome on Pareto frontier. In their setting, gender-based taxation follows Ramsey's *inverse elasticity rule* (IER) and implies lower marginal tax rates for women, independently of the deeper reason that sustains a gendered equilibrium. We view household time allocation decisions as being associated with a non-cooperative Nash equilibrium point. In our setting, gender-based taxation follows the *deviation from the first-best principle* (DFP), which has the property that the optimal structure of differential taxation by gender depends on the reason that induces a gendered allocation of time. If women take on more home duties than men because they have a comparative advantage in them, the two rules have opposite implications for the structure of gender-based taxation: IER requires lower tax rates for women, while DFP implies lower marginal tax rates for men. Thus, it is conceivable that the optimal gender-based tax rates for non-cooperative and cooperative couples are at odds with each other. The structural estimates of Del Boca and Flinn (2011) suggest that one-fourth of households behave inefficiently. To fully account for both efficient and inefficient intrahousehold behavior, we would need to examine gender-based taxation in a model where couples endogenously sort into cooperative and non-cooperative time allocation regimes, which is beyond the scope of this paper.

5. Extensions

In this section we discuss the robustness of our basic results to some alternative specifications. First, we introduce the possibility of caring preferences. Second, we consider what would happen in the case where time allocation decisions are made sequentially and side payments between family members are feasible. Third, we consider the role of ex-ante career choices. Finally, we examine the role of leisure.

5.1. Concerns about the Distribution of Well-Being

So far we have considered only the possibility that couples' behavior is fully non-cooperative. Suppose instead that partners have a concern for each other's well-being, in the sense that each individual puts a weight of $\alpha_i \in (0, \frac{1}{2})$ on the private utility of the spouse and a larger weight $(1-\alpha_i)$ on own private utility. Thus, the partners' total preferences can be written as

$$V^i(c_a, c_b, q) = (1-\alpha_i)U^i(c_i, q) + \alpha_i U^j(c_j, q) \quad \text{for } i, j = a, b \tag{21}$$

where $q = f(\ell)$. Note that the caring parameters (α_a, α_b) effectively parameterize the degree of non-cooperation. Indeed, in the limit as $\alpha_a \to \frac{1}{2}$ and $\alpha_b \to \frac{1}{2}$, equilibrium behavior will come close to the related cooperative game.

It is easy to check that the socially efficient allocation of time is again governed by the Samuelson condition [equation (9)] and the home production efficiency requirement [equation (10)]. However, from the first-order condition

$$(1-\alpha_i)\left[U^i_c w_i (1-\tau) + U^i_q f_i\right] + \alpha_i U^j_q f_j = 0, \tag{22}$$

the equilibrium now satisfies

$$\frac{1}{1-\tau_i}\left[\frac{U^i_q}{U^i_c} + \frac{\alpha_i}{(1-\alpha_i)}\frac{U^j_q}{U^i_c}\right] = \frac{w_i}{f_i} \tag{23}$$

and thus

$$\frac{(1+\Phi_i)MRS^i_{qc}}{1-\tau_i} = \frac{w_i}{f_i(\ell)} \quad \text{with} \quad \Phi_i = \frac{\alpha_i}{1-\alpha_i}\frac{\gamma_j}{\gamma_i} = \frac{\alpha_i}{1-\alpha_i}\frac{U^j_q}{U^i_q} \tag{24}$$

for $i, j = a, b$. Since $\Phi_i > 0$, caring preferences reduce the degree to which the partners underinvest into home production relative to the first-best. Under the additively-separable specification of (13) and (14) we get $\Phi_i = \frac{\alpha_i}{1-\alpha_i}\frac{\gamma_j}{\gamma_i}$. Following the argument of Proposition 2, we obtain:

Proposition 6. *With caring preferences, the labor income taxes implementing the first-best are implicitly described by:*

$$\tau_a = \frac{MRS^b_{qc} - \Phi_a MRS^a_{qc}}{MRS^a_{qc} + MRS^b_{qc}} \quad \textit{and} \quad \tau_b = \frac{MRS^a_{qc} - \Phi_b MRS^b_{qc}}{MRS^a_{qc} + MRS^b_{qc}}.$$

Proof. See the Appendix □

Due to caring preferences, each partner partially internalizes the effect of her time allocation choice on the other. This in turn mitigates the inefficiencies created by fully non-cooperative behavior and shifts the couple closer to a first-best allocation of time. As a consequence, the first-best inducing marginal tax rates no longer sum up to one now. Indeed, while the optimal gender-specific tax schedule still requires a higher marginal tax rate for the partner whose equilibrium choice more severely deviates from the efficient allocation of time, it is now adjusted downwards by the degree to which concerns for the partner mitigate the inefficiencies described in the previous section.

Of course, the marginal rates of substitution now not only depend on wages, home productivities and preferences, but also on each partner's degree of caring. To illustrate a specific case, suppose that the partners' preferences and the home production technology are given by (16). Assuming additionally that the partners only differ in the degree of caring, but are otherwise identical, the optimal marginal tax rates are given by

$$\tau_a^* = \frac{1-2\alpha_a}{2(1-\alpha_a)} \quad \text{and} \quad \tau_b^* = \frac{1-2\alpha_b}{2(1-\alpha_b)}. \tag{25}$$

Efficiency now requires that the partner with the higher degree of caring is taxed at a lower rate than the other. Thus, gender differences in caring may provide an additional rationale for gender-based taxation.

5.2. Sequential-Move Game with Side Payments

Up to now, we have analyzed a simultaneous-move game. Studying the effect of changing the time structure of the game considered, we examine what happens when time allocations are made sequentially and side payments between the primary earner and the secondary earner are feasible. Thus, suppose that primary earner moves first by choosing (i) a money transfer $t \geq 0$ to be paid to his partner, and (ii) how to allocate his unit time endowment between market work and home production. The secondary earner observes these decisions, and then chooses her own time allocation.

Solving by backward induction, we deal with the choices of the two family members in reverse order. Thus, the secondary earner chooses ℓ_b to maximize $U^b(w_b(1-\tau_b)(1-\ell_b)+\vartheta_b+t, f(\ell))$, implying a condition for an interior solution analogous to (11):

$$\frac{MRS_{qc}^b}{1-\tau_b} = \frac{w_b}{f_b(\ell)}, \tag{26}$$

which describes her optimal time allocation given the primary earner's time allocation and transfer. The primary earner chooses his transfer and time allocation so as to maximize his utility, $U^a(w_a(1-\tau_a)(1-\ell_a)+\vartheta_a-t, f(\ell))$, subject to the way in which his partner will respond to his choices. The interior solution to this problem implies two conditions:

$$\frac{(1+\Delta)MRS_{qc}^a}{1-\tau_a} = \frac{w_a}{f_a(\ell)} \quad \text{where} \quad \Delta = \frac{f_b(\ell)}{f_a(\ell)}\frac{\partial \ell_b}{\partial \ell_a} \tag{27}$$

$$MRS^a_{qc} = \frac{1}{f_b(\ell)\frac{\partial \ell_b}{\partial t}}. \tag{28}$$

The first condition (27) reflects that the primary earner will take the reaction of the secondary earner to his time input into home production into account, as indicated by the term Δ. In an interior optimum, the primary earner sets the transfer to the point where his perceived price of the public good, determined by the partner's reaction $\frac{\partial \ell_b}{\partial t}$ and productivity $f_b(\ell)$, is equal to his marginal propensity to pay for the public good. Comparing these equilibrium conditions with those required for an efficient allocation, it is straightforward to establish:

Proposition 7. *In the sequential-move game with side payments, the labor income taxes implementing the first best are implicitly described by:*

$$\tau_a = \frac{MRS^b_{qc} - \Delta MRS^a_{qc}}{MRS^a_{qc} + MRS^b_{qc}} \quad \textit{and} \quad \tau_b = \frac{MRS^a_{qc}}{MRS^a_{qc} + MRS^b_{qc}}.$$

Proof. See the Appendix □

The sequential game structure changes the structure of the partners' optimal labor income taxes asymmetrically. As in the basic model, the tax rate of the secondary earner mirrors the relative marginal rates of substitution between the public household good and the private good. However, the tax rate of the primary earner not only reflects the relative marginal rates of substitution, but additionally contains the term Δ, whose sign depends on whether the partners' household production efforts are independent ($\Delta = 0$), strategic complements ($\Delta > 0$) or strategic substitutes ($\Delta < 0$). In the case of strategic independence, the home production effort of the primary earner has no effect on the effort of the secondary earner. Therefore, his tax rate mirrors the relative marginal rates of substitution, as in the simultaneous-move game. In the case of strategic complementarity, the primary earner's optimal tax rate is corrected downwards as an implicit reward for the fact that his own home production effort has a positive effect on his partners' effort, thereby reducing the inefficiency of the equilibrium. In the case of strategic substitutability, by contrast, the primary earner's optimal tax rate is adjusted upwards as an implicit punishment for the fact that his own home production efforts have a negative effect on his partners' efforts, which increases the inefficiency of the equilibrium. Notice that the optimal tax rates in Proposition 7 only refer to the time allocation of the primary earner while ignoring the transfer. Still, our extension underlines that an additional rationale for gender-based taxation may come from the strategic influence that one partner is able to exert on the other.

5.3. Career Choices

We now consider non-cooperative career investments, e_i, made *prior* to the intra-family allocation of time. As in Alesina et al. (2011), we interpret e_i as all those commitments that allow individuals to choose a market career with high wages. We let $\omega_i(e_i) = w_i e_i$ denote the effective wage that an individual gets as a result of career investments, and denote by $k(e_i) = \phi_i e_i$ career costs. To keep the analysis tractable, we assume that payoffs are additively separable, and that home production efforts are "independent": $U^i_{cq} = 0$ and $\partial^2 f(\ell_a, \ell_b)/\partial\ell_a\partial\ell_b = 0$.

We begin by observing that the conditions for a (first-best) socially efficient allocation are given by

$$MRS^a_{qc} + MRS^b_{qc} = \frac{w_i e_i}{f_i(\ell)} \tag{29}$$

$$U^i_c(c_i, q) w_i (1 - \ell_i) = \phi_i \tag{30}$$

for $i = a, b$. As in the basic model, the marginal rate of substitution between the public and the private good must be equal to marginal rate of transformation between the two goods. Additionally, the marginal return to career investment must be equal to its marginal cost.

In equilibrium, the stage-2 time allocation choices and stage-1 career investments are described by

$$\frac{MRS^i_{qc}}{1 - \tau_i} = \frac{w_i e_i}{f_i(\ell)} \tag{31}$$

$$U^i_c(c_i, q) w_i (1 - \ell_i) = \frac{\phi_i}{1 - \tau_i} \tag{32}$$

Given our assumptions, eqs. (20) and (21) characterizes ℓ_i and e_i as functions of (τ_i, ϑ_i), i.e., in equilibrium we have $\ell^*_i = \ell^*_i(\tau_i, \vartheta_i)$ and $e^*_i = e^*_i(\tau_i, \vartheta_i)$.

While satisfying the Samuelson condition still requires positive tax rates on labor supply, the first-best condition on career investment is satisfied only if labor remains untaxed. Consequently, it is no longer feasible to implement the unconstrained socially efficient allocation. Therefore, we now consider the second-best problem where a social planner chooses a gender-specific linear tax schedule—i.e., labor taxes $\tau = (\tau_a, \tau_b)$ and lump sum transfers $\vartheta = (\vartheta_a, \vartheta_b)$—to maximize the indirect utility of one partner subject to a given level of the other and the government budget constraint. In this and the following section, the social planner may also redistribute income between spouses by lump-sum transfers. The problem reads

$$\begin{aligned}\max_{\tau,\vartheta} \mathcal{L} = {} & U^a(w_a e^*_a(1 - \tau_a)(1 - \ell^*_a) + \vartheta_a, f(\ell^*_a, \ell^*_b)) - \phi_a e^*_a \\ & + \lambda_1 \left[U^b(w_b e^*_b(1 - \tau_b)(1 - \ell^*_b) + \vartheta_b, f(\ell^*_a, \ell^*_b)) - \phi_b e^*_b - \overline{U}^b \right] \\ & + \lambda_2 \left[\tau_a w_a e^*_a (1 - \ell^*_a) + \tau_b w_b e^*_b (1 - \ell^*_b) - \vartheta_a - \vartheta_b \right],\end{aligned} \tag{33}$$

where $e^*_i = e^*_i(\tau_i, \vartheta_i)$ and $\ell^*_i = \ell^*_i(\tau_i, \vartheta_i)$. We show that

Proposition 8. *With career investments, the labor income taxes implementing the second best are implicitly characterized by:*

$$\tau_a = \frac{MRS^b_{qc}}{(1 + \Psi_a) MRS^a_{qc} + MRS^b_{qc}} \quad \textit{and} \quad \tau_b = \frac{MRS^a_{qc}}{MRS^a_{qc} + (1 + \Psi_b) MRS^b_{qc}},$$

where

$$\Psi_i = 1 - (1 - \ell_i^*) \left[\frac{f_{ii}}{f_i} + f_i \frac{U_{qq}^i}{U_q^i} \right].$$

Proof. See the Appendix □

Since $\Psi_i > 0$, the main effect of introducing career investment is that it reduces the optimal tax rates described in the previous section. The intuition is simple and follows from the fact that the social planner now faces a trade-off between two targets: (a) ex-post time allocation choices which require labor income taxes that mirror relative marginal rates of substitution, and (b) ex-ante career investments which call for zero labor taxation. Intuitively, it is suboptimal for the social planner to focus on only one margin, or equivalently, to correct only one equilibrium condition. Instead, a social planner behaving optimally spreads distortions across both margins, thereby only partially eliminating the inefficiency arising from the non-cooperative allocation of time. As a consequence, both partners now face lower marginal tax rates compared to our baseline scenario. As far as gender-specific taxation is concerned, the downward adjustment of the tax rate is larger for the partner who eventually spends more time in market work.[3] Intuitively, the size of the career investment distortion is determined by the marginal return to career effort, which is larger for primary than for secondary earners. Counteracting the relative career investment distortions therefore implies that the optimal marginal taxation of primary earners may fall short of taxation of secondary earners.

5.4. The Role of Leisure

Our analysis so far has centered around the trade-off between market work and household production. We now extend our model to allow both partners to devote time to market work, household production, or leisure. Thus, suppose that partner i's utility function is given by $U^i(c_i, h_i, q)$, where h_i denotes leisure, $1 - h_i - l_i$ is labor supply, and $q = f(\ell_a, \ell_b)$. We assume that U^i is smooth, increasing and concave, and denote its second-order partial derivatives by U_{kl}^i, where $k, l = c, h, q$. We also keep the assumptions that payoffs are additively separable, $U_{ch}^i = U_{cq}^i = U_{hq}^i = 0$, and that home production efforts are independent, $\partial^2 f(\ell_a, \ell_b)/\partial \ell_a \partial \ell_b = 0$. The (first-best) socially efficient allocation of resources now satisfies the following two criteria:

$$MRS_{qc}^a + MRS_{qc}^b = \frac{w_i}{f_i(\ell)} \quad \text{and} \quad MRS_{hc}^i = w_i \tag{34}$$

For each $i = a, b$, the sum of the marginal rates of substitution between private consumption and the household public good must be equal to marginal rate of transformation between these two goods. Furthermore, the marginal rate of substitution between leisure and private consumption has to be equal to the marginal rate of transformation, given by the respective gross wage.

[3]It is straightforward to see that the adjustment factor Ψ_i strictly increases with own labor supply, $1 - \ell_i$.

In equilibrium, the partners' choices are characterized by:

$$\frac{MRS^a_{qc}}{1-\tau_i} = \frac{w_i}{f_i(\ell)} \quad \text{and} \quad \frac{MRS^a_{hc}}{1-\tau_i} = w_i \tag{35}$$

Given our assumptions, in equilibrium we have $h_i^* = h_i^*(\tau_i, \vartheta_i)$ and $\ell_i^* = \ell_i^*(\tau_i, \vartheta_i)$. In a tax-free world, individual choices regarding the consumption-leisure trade-off are socially efficient, but the Samuelson condition for the optimal provision of the household public good cannot be implemented. With labor income taxes, the appropriate choice of tax rates will implement the benchmark efficient level of the household public good, but this entails a distortion of individual consumption-leisure choices. Thus, implementation of an unconstrained socially efficient allocation is not feasible. Therefore, we now consider a (second-best) linear tax schedule which solves:

$$\begin{aligned}\max_{\tau,\vartheta} \mathcal{L} = {} & U^a(w_a(1-\tau_a)(1-\ell_a^*-h_a^*)+\vartheta_a, h_a^*, f(\ell_a^*, \ell_b^*)) \\ & + \lambda_1\Big[U^b(w_b(1-\tau_b)(1-\ell_b^*-h_b^*)+\vartheta_b, h_b^*, f(\ell_a^*, \ell_b^*)) - \overline{U}^b\Big] \\ & + \lambda_2\Big[\tau_a w_a(1-\ell_a^*-h_a^*)+\tau_b w_b(1-\ell_b^*-h_b^*)-\vartheta_a-\vartheta_b\Big],\end{aligned} \tag{36}$$

where $h_i^* = h_i^*(\tau_i, \vartheta_i)$ and $\ell_i^* = \ell_i^*(\tau_i, \vartheta_i)$. We show

Proposition 9. *In the model with three uses of time—i.e., market work, leisure, and household production—the labor income taxes implementing the second best are implicitly characterized by:*

$$\tau_a = \frac{\Theta_a MRS^b_{qc}}{MRS^a_{qc} + \Theta_a MRS^b_{qc}} \quad \textit{and} \quad \tau_b = \frac{\Theta_b MRS^a_{qc}}{\Theta_b MRS^a_{qc} + MRS^b_{qc}},$$

where

$$\Theta_i = \frac{U^i_{hh}}{U^i_q f_{ii} + U^i_{qq}(f_i)^2 + U^i_{hh}} \in (0,1)\,.$$

Proof. See the Appendix □

As far as the efficient provision of the home-produced public good is concerned, the social planner should implement labor income taxes according to relative marginal rates of substitution. On the other hand, for optimal individual choices about consumption and leisure, the labor income taxes should be zero. The social planner therefore chooses labor income tax rates by weighing these two considerations against each other. Unsurprisingly, the two competing concerns of the social planner result in lower marginal tax rates compared to our basic model without leisure.[4] For each partner, the size of the downward adjustment of the marginal tax rate depends negatively on the adjustment factor

[4]Indeed, since $\xi_i < 1$ $(i = a, b)$, it follows immediately that

$$\frac{\xi_a MRS^b_{qc}}{MRS^a_{qc} + \xi_a MRS^b_{qc}} < \frac{MRS^b_{qc}}{MRS^a_{qc} + MRS^b_{qc}} \quad \text{and} \quad \frac{\xi_b MRS^a_{qc}}{\xi_b MRS^a_{qc} + MRS^b_{qc}} < \frac{MRS^a_{qc}}{MRS^a_{qc} + MRS^b_{qc}}.$$

ξ_i, which in turn increases with the ratio of two second derivatives of the utility function, U^i_{hh}/U^i_{qq}. The weighting of the adjustment factor by the ratio of second derivatives is simply a result of the fact that optimal policy spreads distortions across the two relevant margins. Indeed, while an increase in labor income taxes leads to a distortion away from the first-best along the consumption-leisure margin, it implies a distortion towards the first-best along the household production margin. The positive effect in public good provision outweighs the negative consumption-leisure distortion if (ceteris paribus) either leisure exhibits large diminishing marginal utility or public good consumption exhibits small diminishing marginal utility. Put differently, the faster marginal utility of leisure is falling, the smaller the reaction of leisure to changing taxes and the more import it is to address the underprovision problem in household good production. The extent to which the law of diminishing marginal utility applies to two partners of opposite sex may, therefore, provide an additional rationale for gender-based taxation. In particular, all else equal, the optimal gender-specific linear tax schedule requires a higher marginal tax rate for the partner whose marginal utility of leisure diminishes at a higher rate.

6. Concluding Remarks

Economists have recently started to examine models of household behavior in which couples endogenously sort into efficient and inefficient time allocation regimes. Econometric estimates suggest that a model of inefficient family decision-making adequately captures the behavior of a substantial share of households. Motivated by this finding, this paper has explored the implications of gender-based taxation in a non-cooperative model of a couple's time allocation between market work and providing a home-produced public good. Our model allows for a rich set of specifications, giving rise to some clear-cut principles. Our approach highlights the function of wage taxation to tackle underprovision in household production, keeping in mind the problem of distorting labor supply. The key message of our paper is that there exist good reasons for a differentiated tax treatment of married couples by gender. While empirical regularities suggest lower marginal tax rates on female earnings due to women's higher valuation of household public goodes, this can be more than offset when women display a comparative advantage in home production. Several of our extensions point to lower levels of tax rates than in our baseline case, without substantially putting the optimality of gender-based taxation into question.

From a policy perspective, we do not argue that re-balancing the tax structure in favor of women is undesirable. However, our analysis suggests that optimal taxation criteria derived from non-cooperative models of household behavior can look very different from those in efficiency models. An interesting extension for future research would therefore be to allow for a more heterogeneous population and sorting of couples into regimes of non-cooperative and cooperative behavior. As it seems reasonable to expect that there are substantial shares of married couples of both types, the properties of an optimal income tax system that treats both groups alike will presumably depend on their relative sizes. While a high share of couples acting according to a cooperative game structure would push the balance in favor of lower taxes on female earnings, an opposite outcome cannot a priori be ruled out if non-coordinated labor supply behavior is a pervasive phenomenon.

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Appendix

Proof of Proposition 2. Given our assumptions, the first-order conditions for an interior provision equilibrium, ℓ_i^*, can be written as:

$$-(1-\gamma_i)v'(c_i)w_i(1-\tau_i) + \gamma_i\xi_i\mu'(\ell_i) = 0 \tag{A.1}$$

where $c_i = w_i(1-\tau_i)(1-\ell_i) + \vartheta_i$. Clearly, if the spouses are identical in every respect ($\gamma_a = \gamma_b$, $w_a = w_b$, $\xi_a = \xi_b$), they will make identical time allocation choices. The proposition then describes three cases which sustain a gendered allocation of time: (a) $\gamma_a < \gamma_b$ with $w_a = w_b$ and $\xi_a = \xi_b$; (b) $w_a > w_b$ with $\xi_a = \xi_b$ and $\gamma_a = \gamma_b$; and (c) $\xi_a < \xi_b$ with $w_a = w_b$ and $\gamma_a = \gamma_b$. These three cases follow immediately from noting that:

$$\frac{\partial \ell_i}{\partial \gamma_i} = -\frac{w_i(1-\tau_i)v'(c_i) + \xi_i\mu'(\ell_i)}{\chi_i} > 0 \tag{A.2}$$

$$\frac{\partial \ell_i}{\partial w_i} = \frac{(1-\gamma_i)(1-\tau_i)[v'(c_i) + w_i(1-\tau_i)(1-\ell_i)v''(c_i)]}{\chi_i} < 0 \tag{A.3}$$

$$\frac{\partial \ell_i}{\partial \xi_i} = -\frac{\gamma_i\mu'(\ell_i)}{\chi_i} > 0. \tag{A.4}$$

where $\chi_i \equiv (1-\gamma_i)w_i^2(1-\tau_i)^2v''(c_i) + \xi_i\gamma_i\mu''(\ell_i) < 0$ and (A.3) can be signed due to our assumption $\epsilon_{v'c} > -1$.

For future reference, it is also useful to have:

$$\frac{\partial \ell_i}{\partial \tau_i} = -\frac{(1-\gamma_i)w_i[v'(c_i) + w_i(1-\tau_i)(1-\ell_i)v''(c_i)]}{\chi_i} > 0 \tag{A.5}$$

$$\frac{\partial \ell_i}{\partial \vartheta_i} = \frac{(1-\gamma_i)w_i(1-\tau_i)v''(c_i)}{\chi_i} > 0 \tag{A.6}$$

□

Proof of Propositions 3 and 4. Expressing marginal rates of substitution in terms of primitives, the set of labor income taxes implementing the first best $\tau^* = (\tau_a^*, \tau_b^*)$, together with lump-sum redistributed tax proceeds $\vartheta^* = (\vartheta_a^*, \vartheta_b^*)$, are implicitly characterized by:

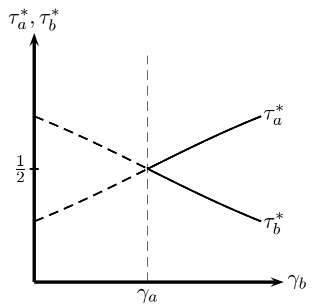
$$\tau_a^* + \tau_b^* = 1 \tag{A.7}$$

$$\frac{(1-\tau_a^*)\gamma_b}{(1-\gamma_b)v'(c_b^*)} = \frac{(1-\tau_b^*)\gamma_a}{(1-\gamma_a)v'(c_a^*)} \tag{A.8}$$

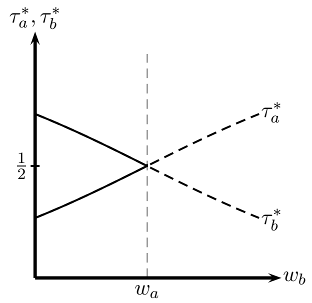
$$\vartheta_a = \tau_a^* w_a(1-\ell_a^*) \tag{A.9}$$

$$\vartheta_b = \tau_b^* w_b(1-\ell_b^*) \tag{A.10}$$

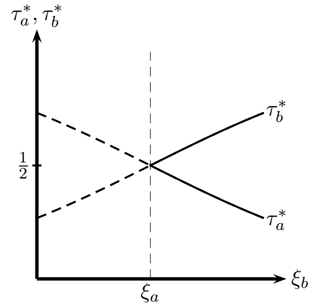
where $c_i^* = w_i(1-\tau_i^*)(1-\ell_i^*) + \vartheta_i^*$ with $\ell_i^* = \ell_i^*(\tau_i^*, \vartheta_i^*, w_i, \xi_i, \gamma_i)$. Clearly, $\tau_i^* = \tau_i^*(\gamma_a, \gamma_b, w_a, w_b, \xi_a, \xi_b)$ and $\vartheta_i^* = \vartheta_i^*(\gamma_a, \gamma_b, w_a, w_b, \xi_a, \xi_b)$. Now recall that, if the spouses are identical in every respect (i.e., $\gamma_a = \gamma_b$, $w_a = w_b$, $\xi_a = \xi_b$), they will make identical time allocation choices. In this case, it follows immediately from (A.7) and (A.8) that $\tau_a^* = \tau_b^* = \frac{1}{2}$. To establish Propositions 3 and 4, it is therefore sufficient to show that: (a) τ_b^* is strictly decreasing in γ_b, while τ_a^* is strictly increasing in γ_b; (b) τ_b^* is strictly decreasing in w_b, while τ_a^* is strictly increasing in w_b; and (c) τ_b^* is strictly increasing in ξ_b, while τ_a^* is strictly decreasing in ξ_b [see Figure 2].



(a) Preferences



(b) Wages



(c) Home Productivities

Figure 2: Proof of Propositions 3 and 4.

Totally differentiating eqs. (A.7) to (A.10), using eqs. (A.1) to (A.6) as well as eq. (A.8) to simplify, we obtain:

$$\frac{\partial \tau_a^*}{\partial \gamma_b} = -\frac{\partial \tau_b^*}{\partial \gamma_b} = \frac{\gamma_b \xi_b (1-\tau_a)\mu''(\ell_b^*)}{\Gamma \varrho_b (1-\gamma_b)^2 v'(c_b^*)} > 0$$

$$\frac{\partial \tau_a^*}{\partial w_b} = -\frac{\partial \tau_b^*}{\partial w_b} = \frac{\gamma_b (1-\tau_a)(1-\tau_b) w_b v''(c_b^*)}{\Gamma \varrho_b v'(c_b^*)} - \frac{\gamma_b^2 \xi_b (1-\tau_a)(1-\ell_b^*) v''(c_b^*)\mu''(\ell_b^*)}{\Gamma \varrho_b (1-\gamma_b)(v'(c_b^*))^2} - \frac{\gamma_a (1-\tau_a)(1-\tau_b) w_a^2 (v''(c_a^*))^2}{\Gamma \chi_a (v'(c_a^*))^2} > 0$$

$$\frac{\partial \tau_a^*}{\partial \xi_b} = -\frac{\partial \tau_b^*}{\partial \xi_b} = -\frac{\gamma_b^2 w_b (1-\tau_a)\mu'(\ell_b^*) v''(c_b^*)}{\Gamma \varrho_b (1-\gamma_b)(v'(c_b^*))^2} < 0$$

where $\chi_i \equiv (1-\gamma_i) w_i^2 (1-\tau_i)^2 v''(c_i) + \xi_i \gamma_i \mu''(\ell_i) < 0$, $\varrho_i \equiv (1-\gamma_i) w_i^2 (1-\tau_i) v''(c_i) + \xi_i \gamma_i \mu''(\ell_i) < 0$ and

$$\Gamma \equiv \frac{\gamma_a \gamma_b \xi_a \mu''(\ell_a^*)}{\varrho_a (1-\gamma_b) v'(c_b^*)} + \frac{\gamma_a \gamma_b \xi_b \mu''(\ell_b^*)}{\varrho_b (1-\gamma_a) v'(c_a^*)} > 0$$

Propositions 3 and 4 now follow immediately. □

Proof of Proposition 6. With social preferences, the equilibrium time allocation choices, ℓ_a^* and ℓ_b^*, simultaneously satisfy the Samuelson condition [eq. (9)] and the home production efficiency condition [eq. (10)] if and only if:

$$\frac{(1+\Phi_a) MRS_{qc}^a}{1-\tau_a} = MRS_{qc}^a + MRS_{qc}^b = \frac{(1+\Phi_b) MRS_{qc}^b}{1-\tau_b}, \tag{A.11}$$

where

$$\Phi_i = \frac{\alpha_i}{1-\alpha_i} \frac{\gamma_j}{\gamma_i} \quad \text{for } i,j = a,b \text{ and } i \neq j.$$

The proposition follows immediately after solving eq. (A.11) for τ_a and τ_b, respectively. □

Proof of Proposition 7. In the sequential-move game with side payments, the equilibrium time allocation choices, ℓ_a^* and ℓ_b^*, simultaneously satisfy the Samuelson condition [eq. (9)] and the home production efficiency condition [eq. (10)] if and only if:

$$\frac{(1+\Delta)MRS^a_{qc}}{1-\tau_a} = MRS^a_{qc} + MRS^b_{qc} = \frac{MRS^b_{qc}}{1-\tau_b}, \tag{A.12}$$

where

$$\Delta = \frac{f_b(\ell)}{f_a(\ell)}\frac{\partial \ell_b}{\partial \ell_a}$$

The proposition follows immediately after solving eq. (A.12) for τ_a and τ_b, respectively. □

Proof of Proposition 8. In order to prove the proposition, it is first necessary to establish how ℓ_i^* and e_i^* vary with τ_i and ϑ_i. Differentiating the first order conditions [eqs. (20) and (21)] with respect to τ_i and ϑ_i, given that time allocation and career choices are set at the equilibrium levels $\ell_i^*(\tau_i, \vartheta_i)$ and $e_i^*(\tau_i, \vartheta_i)$, and then solving for the derivatives of ℓ_i^* and e_i^* with respect to τ_i and ϑ_i, we obtain (for expositional convenience, we suppress the arguments of the functions):

$$\frac{\partial \ell_i^*}{\partial \tau_i} = \frac{(1-\ell_i^*)[U^i_c + (1-\tau_i)w_i e_i^*(1-\ell_i^*)U^i_{cc}]}{(1-\tau_i)\eta_i}, \tag{A.13}$$

$$\frac{\partial \ell_i^*}{\partial \theta_i} = -\frac{(1-\ell_i^*)U^i_c U^i_{cc}}{\eta_i}, \tag{A.14}$$

$$\frac{\partial e_i^*}{\partial \tau_i} = \frac{[(1-\ell_i^*)(U^i_q f_{ii} + U^i_{qq}(f_i)^2) - w_i e_i^*(1-\tau_i)U^i_c][U^i_c + (1-\tau_i)w_i e_i^*(1-\ell_i^*)U^i_{cc}]}{w_i(1-\tau_i)^2\eta_i}, \tag{A.15}$$

$$\frac{\partial e_i^*}{\partial \theta_i} = -\frac{(1-\ell_i^*)(U^i_q f_{ii} + U^i_{qq}(f_i)^2) - w_i e_i^*(1-\tau_i)U^i_c}{w_i(1-\tau_i)\eta_i}, \tag{A.16}$$

where $\eta_i = U^i_{cc}(1-\ell_i^*)[(1-\ell_i^*)(U^i_q f_{ii} + U^i_{qq}(f_i)^2) - 2w_i e_i^* U^i_c(1-\tau_i)] - (U^i_c)^2$. Now consider the first-order conditions for the social planner's problem [eq. (20)] (as before, we suppress the arguments of the functions):

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \tau_a} =& - w_a e_a^*(1-\ell_a^*)U^a_c + \lambda_1 U^b_q f_a \frac{\partial \ell_a^*}{\partial \tau_a} \\ &+ \lambda_2\left[w_a\left((1-\ell_a^*)\left(e_a^* + \tau_a\frac{\partial e_a^*}{\partial \tau_a}\right) - \tau_a e_a^*\frac{\partial \ell_a^*}{\partial \tau_a}\right)\right] = 0\end{aligned} \tag{A.17}$$

$$\frac{\partial \mathcal{L}}{\partial \theta_a} = U^a_c + \lambda_1 U^b_q f_a \frac{\partial \ell_a^*}{\partial \theta_a} + \lambda_2\left[-1 + w_a\left(\tau_a\frac{\partial e_a^*}{\partial \theta_a}(1-\ell_a^*) - \tau_a e_a^*\frac{\partial \ell_a^*}{\partial \theta_a}\right)\right] = 0 \tag{A.18}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \tau_b} =& U^a_q f_b \frac{\partial \ell_b^*}{\partial \tau_b} - \lambda_1 w_b e_b^*(1-\ell_b^*)U^b_c \\ &+ \lambda_2\left[w_b\left((1-\ell_b^*)\left(e_b^* + \tau_b\frac{\partial e_b^*}{\partial \tau_b}\right) - \tau_b e_b^*\frac{\partial \ell_b^*}{\partial \tau_b}\right)\right] = 0\end{aligned} \tag{A.19}$$

$$\frac{\partial \mathcal{L}}{\partial \theta_b} = U^a_q f_b \frac{\partial \ell_b^*}{\partial \theta_b} - \lambda_1 U^b_c + \lambda_2\left[-1 + w_b\left(\tau_b\frac{\partial e_b^*}{\partial \theta_b}(1-\ell_b^*) - \tau_b e_b^*\frac{\partial \ell_b^*}{\partial \theta_b}\right)\right] = 0 \tag{A.20}$$

After making appropriate substitutions using eqs. (A.13) to (A.16), we solve eqs. (A.17) and (A.18) for λ_1 and λ_2 and obtain:

$$\lambda_2 = U^a_c \tag{A.21}$$

$$\lambda_1 = \frac{\tau_a\left[2w_a e_a^*(1-\tau_a)U_c^a - (1-\ell_a^*)\left(U_q^a f_{aa} + U_{qq}^a (f_a)^2\right)\right]}{f_a U_q^b (1-\tau_a)} \tag{A.22}$$

Similarly, solving eqs. (A.19) and (A.20) for λ_1 and λ_2 yields:

$$\lambda_2 = \frac{f_b U_q^a U_c^b (1-\tau_b)}{\tau_b\left[2w_b e_b^*(1-\tau_b)U_c^b - (1-\ell_b^*)\left(U_q^b f_{bb} + U_{qq}^b (f_b)^2\right)\right]} \tag{A.23}$$

$$\lambda_1 = \frac{f_b U_q^a (1-\tau_b)}{\tau_b\left[2w_b e_b^*(1-\tau_b)U_c^b - (1-\ell_b^*)\left(U_q^b f_{bb} + U_{qq}^b (f_b)^2\right)\right]} \tag{A.24}$$

Eqs. (A.21) to (A.24) are simultaneously satisfied if and only if

$$\tau_b\left[MRS_{qc}^b + MRS_{qc}^b\left[1-(1-\ell_b^*)\left(\frac{f_{bb}}{f_b} + f_b\frac{U_{qq}^b}{U_q^b}\right)\right]\right] = MRS_{qc}^a(1-\tau_b) \tag{A.25}$$

and

$$\tau_a\left[MRS_{qc}^a + MRS_{qc}^a\left[1-(1-\ell_a^*)\left(\frac{f_{aa}}{f_a} + f_a\frac{U_{qq}^a}{U_q^a}\right)\right]\right] = MRS_{qc}^b(1-\tau_a) \tag{A.26}$$

where $MRS_{qc}^i = \frac{U_q^i}{U_c^i}$ $(i = a, b)$. Solving eq. (A.25) for τ_b and eq. (A.26) for τ_a yields the result stated in the proposition. □

Proof of Proposition 9. It is first of all necessary to establish how ℓ_i^* and h_i^* vary with τ_i and ϑ_i. We have:

$$\frac{\partial \ell_i^*}{\partial \tau_i} = \frac{w_i U_{hh}^i[w_i(1-\tau_i)(1-\ell_i^*-h_i^*)U_{cc}^i + U_c^i]}{(1-\tau_i)^2\varpi} \tag{A.27}$$

$$\frac{\partial \ell_i^*}{\partial \theta_i} = \frac{w_i(1-\tau_i)U_{hh}^i U_{cc}^i}{(1-\tau_i)^2\varpi} \tag{A.28}$$

$$\frac{\partial h_i^*}{\partial \tau_i} = \frac{w_i[U_q^i f_{ii} + U_{qq}^i (f_i)^2][w_i(1-\tau_i)(1-\ell_i^*-h_i^*)U_{cc}^i + U_c^i]}{(1-\tau_i)^2\varpi} \tag{A.29}$$

$$\frac{\partial h_i^*}{\partial \theta_i} = \frac{w_i(1-\tau_i)U_{cc}^i[U_q^i f_{ii} + U_{qq}^i (f_i)^2]}{(1-\tau_i)^2\varpi} \tag{A.30}$$

where $\varpi = U_{cc}^i[U_{hh}^i + U_q^i f_{ii} + U_{qq}^i (f_i)^2] + U_{hh}^i[U_q^i f_{ii} + U_{qq}^i (f_i)^2]$. Now consider the first-order conditions for the social planner's problem:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \tau_a} =& - w_a(1-\ell_a^*-h_a^*)U_c^a + \lambda_1 U_q^b f_a \frac{\partial \ell_a^*}{\partial \tau_a} \\ &+ \lambda_2\left[w_a(1-\ell_a^*-h_a^*) - w_a\tau_a\left(\frac{\partial \ell_a^*}{\partial \tau_a} + \frac{\partial h_a^*}{\partial \tau_a}\right)\right] = 0\end{aligned} \tag{A.31}$$

$$\frac{\partial \mathcal{L}}{\partial \theta_a} = U_c^a + \lambda_1 U_q^b f_a \frac{\partial \ell_a^*}{\partial \theta_a} - \lambda_2\left[1 + w_a\tau_a\left(\frac{\partial \ell_a^*}{\partial \theta_a} + \frac{\partial h_a^*}{\partial \theta_a}\right)\right] = 0 \tag{A.32}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \tau_b} =& U_q^a f_b \frac{\partial \ell_b^*}{\partial \tau_b} - \lambda_1 w_b(1-\ell_b^*-h_b^*)U_c^b \\ &+ \lambda_2\left[w_b(1-\ell_b^*-h_b^*) - w_b\tau_b\left(\frac{\partial \ell_b^*}{\partial \tau_b} + \frac{\partial h_b^*}{\partial \tau_b}\right)\right] = 0\end{aligned} \tag{A.33}$$

$$\frac{\partial \mathcal{L}}{\partial \theta_b} = U_q^a f_b \frac{\partial \ell_b^*}{\partial \theta_b} - \lambda_1 U_c^b - \lambda_2 \left[1 + w_b \tau_b \left(\frac{\partial \ell_b^*}{\partial \theta_b} + \frac{\partial h_b^*}{\partial \theta_b}\right)\right] = 0 \tag{A.34}$$

After making appropriate substitutions using eqs. (A.27) to (A.30), we solve eqs. (A.31) and (A.32) for λ_1 and λ_2 and obtain:

$$\lambda_2 = U_c^a \tag{A.35}$$

$$\lambda_1 = \frac{w_a \tau_a U_c^a \left[U_q^a f_{aa} + U_{qq}^a (f_a)^2 + U_{hh}^a\right]}{f_a U_b^q U_{hh}^a} \tag{A.36}$$

Similarly, solving eqs. (A.33) and (A.34) for λ and μ yields:

$$\lambda_2 = \frac{f_b U_q^a U_{hh}^b}{\tau_b w_b \left[U_q^b f_{bb} + U_{qq}^b (f_b)^2 + U_{hh}^b\right]} \tag{A.37}$$

$$\lambda_1 = \frac{f_b U_q^a U_{hh}^b}{\tau_b w_b U_c^b \left[U_q^b f_{bb} + U_{qq}^b (f_b)^2 + U_{hh}^b\right]} \tag{A.38}$$

Eqs. (A.35) to (A.38) are simultaneously satisfied if and only if

$$MRS_{qc}^b \left[\frac{\tau_b}{1 - \tau_b}\right] = MRS_{qc}^a \left[\frac{U_{hh}^b}{U_q^b f_{bb} + U_{qq}^b (f_b)^2 + U_{hh}^b}\right] \tag{A.39}$$

and

$$MRS_{qc}^a \left[\frac{\tau_a}{1 - \tau_a}\right] = MRS_{qc}^b \left[\frac{U_{hh}^a}{U_q^a f_{aa} + U_{qq}^a (f_a)^2 + U_{hh}^a}\right] \tag{A.40}$$

where $MRS_{qc}^i = \frac{U_q^i}{U_c^i}$ $(i = a, b)$. Solving eq. (A.39) for τ_b and eq. (A.40) for τ_a yields the result stated in the proposition.

□