CESifo Working Papers
www.cesifo.org/wp

Strategic Complexities in the Combinatorial Clock Auction

Vitali Gretschko
Stephan Knapek
Achim Wambach

CESifo Working Paper No. 3983
Category 11: Industrial Organisation
Original Version: October 2012
This Version: July 2016

CESifo *Working Paper No. 3983*

Strategic Complexities in the Combinatorial Clock Auction

Abstract

In recent years, regulatory bodies in Europe and around the world implemented Combinatorial Clock Auctions (CCA) to allocate scarce and valuable spectrum frequencies usage rights. Although the auction design is complex, the promise is that bidding becomes simple. More precisely, bidders may bid on the profit-maximizing package (truthful bidding) during the clock phase and submit bids that are equal to their valuations on only a handful of relevant packages (truncated bidding) in the supplementary phase. While this might be correct with the ideal implementation of the CCA for perfectly rational, profit-maximizing bidders with private values, our experience with consulting bidders shows that practical implementations of the CCA and the concerns of the bidders lead to severe complexities in determining the right bidding behavior. We provide simple examples that illustrate how "truthful and truncated" bidding may be harmful to bidders and thereby illustrate the complexities bidders face in preparing for a CCA.

JEL-Code: D440, L960.

Keywords: combinatorial clock auction.

*Vitali Gretschko**
University of Cologne
Albertus Magnus Platz
Germany – 50923 Cologne
gretschko@wiso.uni-koeln.de

Stephan Knapek
TWS Partners
Munich / Germany
knapek@tws-partners.com

Achim Wambach
University of Cologne
Albertus Magnus Platz
Germany – 50923 Cologne
wambach@wiso.uni-koeln.de

*corresponding author

We thank Nicolas Fugger for helpful comments and suggestions.

1 Introduction

The Combinatorial Clock Auction (CCA) is an innovative auction design that has been used in many recent auctions of spectrum for telecommunication use.[1] The CCA is based on ideas from modern microeconomic theory and combines package bidding with dynamic price discovery in a two stage design.

In the *clock phase,* bidders express their demands at increasing prices in each of the auctioned categories of spectrum lots until the indicated demand matches the available supply. In the *supplementary phase,* bidders can improve their bids from the clock phase and submit additional bids for other desired combinations of lots. To induce truthful bidding in the clock phase, bids in the supplementary phase are constrained by a cap that is based on the clock bids. To determine winnings and prices all bids of a particular bidder are treated as mutually exclusive package bids and the combination of packages that maximizes the value as expressed by the bids is the winning allocation. The prices are determined through (a variant of) the *Vickrey* (second-price) rule by calculating the opportunity cost imposed by each bidder on her competitors. In general, there exist many more additional details like reserve prices, caps, or activity rules that have to be considered when designing a CCA.[2]

While the design is quite complex, the promise of the CCA is that bidding is simple. Regulators argue that in a CCA *truthful bidding* is close to optimal independent of the bidding strategy of the competitors. Regulators claim that the CCA "allows bidders to use a simple strategy" and "allows the participants to evaluate the spectrum without [...] shadow bids."[3] If truthful bidding is indeed close to optimal independent of the competitors behavior, this would be useful for the participants. In this case there is no need for strategizing and bidders could simply quote on the packages that lead to largest profit in the clock phase. In particular, bidders could focus their resources on determining the correct valuations and would not need to worry about the preferences of the competitors or their potentially erratic or spiteful behavior.

The goal of this article is to demonstrate with clear and simple examples that truthful bidding is neither a dominant strategy nor close to optimal. Thus, bidding truthfully can significantly hurt bidders. We will provide two examples in which a bidder can either gain by overstating or understating her demand during the clock phase. In both cases, whether understating or overstating demand is preferable to truthful bidding or not crucially depends on the behavior

[1] The CCA was used among other auctions in the United Kingdom 2008 and 2013, Austria 2010 and 2013, Denmark 2010, Netherlands 2010, Ireland 2012, Switzerland 2012, Czech Republic 2012, Australia 2013, and Canada 2014.

[2] See Ausubel and Baranov (2014) for an excellent overview. A detailed description of the rules used in this paper can be found in Section 2.1.

[3] See for example, Cramton (2013) and ComReg (2010). The quotes are from CTU (2011).

of the competitors and thereby their preferences. More precisely, whenever her competitor incurs a (small) cost of submitting bids in the supplementary round, the bidder can gain by expanding her demand in the clock phase and deter supplementary bids of her competitor. Whenever her competitor has a preference for raising the payments of his rivals, she can gain by understating her demand in the clock phase and limit the opportunity of riskless bids aimed at increasing the price. The two examples demonstrate that the success of a bidding strategy in the CCA may crucially depend on the bidding behavior of the competitors. Moreover, there is no straightforward way to adjust the bidding strategy. Thus, the CCA cannot deliver on the promise that bidding is simple. Bidders face complex strategic decisions when preparing for the CCA.[4]

The idea that truthful bidding may be a dominant strategy is based on the opportunity cost pricing rule. This pricing rule is adapted from static Vickrey auctions in which truthful bidding is a dominant strategy. However, as our examples will demonstrate, the dynamic nature of the CCA can make bidding truthfully suboptimal. Both examples exploit the fact that the bidders can observe the bidding and outcome of the clock-phase and condition their strategies on this outcome. Such a strategy is clearly not possible in a static Vickrey auction. In the first example, the competitor is deterred from submitting bids in the supplementary round after observing in the clock-phase that he is not able to win a profitable package. In the second example, the raising of the cost of a particular bidder is only riskless if the spiteful bidders can observe the allocation and prices in the clock phase.

Another advertised feature of the CCA is that once the clock phase is over and the Final Cap rule is in place, it is optimal for bidders to submit bids only on a handful of packages (*truncated bidding*).[5] More precisely, if demand equals supply at the end of the clock phase the final allocation cannot change and thus no further bids are required.[6] On one hand, this is a very desirable feature for mostly three reasons. First, there is no need to calculate a precise value for all possible packages. Second, if bidders need to secure a minimal package to ensure business continuity, they value greatly to be in full control of the final allocation. Third, there is no need for bidders to reveal full valuations to the regulator, as this information might be used against them in the future. On the other hand, bidders might be concerned that if the final allocation cannot change after the clock phase, they will loose flexibility. Furthermore, their rival's strategies aimed at raising the costs become riskless if such bids have no effect on

[4]To keep the examples as simple as possible, we restrict ourselves to an environment in which bidders are fully able to observe each other bids in the clock phase. A usual implementation of the CCA does not provide such information which makes manipulation more difficult but not impossible.

[5]For example, see ComReg (2010).

[6]If demand is less then supply at the end of the clock phase, bidders can secure the final allocation by raising their bids on the final package by the value of the unsold blocks at the final round prices.

the final allocation. Thus, in some implementations of the CCA the Final Cap is replaced by a Relative Cap.[7] We provide a simple example in which if only a Relative Cap is imposed, the final allocation can change dramatically if the bidders submit truncated bids as described above. We then argue that with a Relative Cap the bidders lack the relevant information to calculate the sufficient bids to secure the final clock-round allocation. Moreover, as was demonstrated by Ausubel and Baranov (2014), the bidders would need to place bids that exceed the final clock-phase bids by up to several orders of magnitude.

Overall, preparing a bidding strategy for a CCA is a difficult task. It is not sufficient to focus on the derivation of the own valuations and rely on truthful bidding. Depending on the preferences and valuations of the competing bidders different strategies become optimal. Thus, when preparing for a CCA a strong focus on the values and preferences of the competition is needed. However, such a focus implies a level of complexity that is hard to handle. A good way to address this complexity is to use simulation software that allows to quickly generate a vast variety of bidding scenarios. At this simulations should not be viewed as a way to forecast outcomes but rather as a tool to facilitate the understanding of how bidding behavior could be influenced by preferences. After the derivation of bidding strategies, the overall complexity makes it difficult to educate the top-level management and the supervisory board on the insights of the project team. Mock auctions, time-lapse versions of the actual auction, can in this case be used to familiarize the decision makers with different scenarios and the potential impact of their decisions.

The paper proceeds as follows. In Section 2 we outline a simplified version of the rules and summarize the recommendations of the regulators with regard to the bidding behavior. In Section 3 we show that truthful bidding is not a dominant strategy and argue that depending on the preference of the competitors a bidder may gain from either overstating or understating her demand. In Section 4 we demonstrate how the final allocation may change if merely a Relative Cap is imposed. In Section 5 we discuss further issues in the CCA that complicate the development of a bidding strategy. In Section 6 we conclude by briefly discussing the implications of the findings for the preparation of a bidding strategy in the CCA. Throughout the article we will use simple examples and verbal explanations to illustrate our points and refer the readers to other articles for a more formal analysis of the discussed issues.

[7] The precise description of the cap rules can be found in Section 2.1.

2 Auction rules and recommended behavior

In what follows, we describe a simplified version of the rules of a Combinatorial Clock Auction and summarize the recommendations for bidding behavior given by regulators. Our goal in this article is to demonstrate with simple examples that deriving a bidding strategy in a CCA is not a simple task. Thus, we focus on rules that are essential for our examples and comment on further rules and resulting complexities in Section 5. Moreover, we restrict our attention to a simplified setting with at most three bidders and two categories of spectrum licenses. This approach is without loss with respect to the goals of this paper as the omitted rules do not simplify the bidders decision but rather add another layer of complexity.

2.1 Rules of the Combinatorial Clock Auction

We consider the sale of spectrum licenses to up to three bidders: bidder A, bidder B and bidder C. The licenses are organized in up to 2 categories: category α and category β. Category α contains n_α identical lots and category β contains n_β identical lots.[8] The auction consist of two phases, the clock phase and the supplementary phase.

Clock phase. The clock phase consists of multiple rounds. In each round t the auctioneer announces prices for each of the categories. That is, she announces $p^t = (p_\alpha, p_\beta)$ with p_α (p_β) denoting the price of one lot in category α (category β). After the announcement of the auctioneer, the bidders decide how many lots they desire at the announced prices in each of the categories. That is, each bidder decides on $q^t = (n^t_\alpha, n^t_\beta)$ with n^t_α (n^t_β) denoting the number of lots she desires in category α (category β).[9] Bidders are constrained by the *eligibility rule*: the overall demand in round t needs to be below or equal to the overall demand in round $t-1$ ($t-2, \ldots, 1$). That is, $n^t_\alpha + n^t_\beta \leq n^{t-1}_\alpha + n^{t-1}_\beta$. Let $N^t := n^{t-1}_\alpha + n^{t-1}_\beta$ denote the *eligibility* of the bidder in round t. The initial eligibility is the sum of all available lots, i.e., $N^1 = n_\alpha + n_\beta$. If in round t the overall demand (i.e., the sum of the demands of the three bidders) in some category exceeds the available supply in this category, the auction moves on to round $t+1$ and the auctioneer increases the price in this category. The clock phase closes as soon as the overall demand in each category is below (or equal to) the available supply in the category. Denote the final round as round f and the desired demand of a bidder in round f as q^f.

Supplementary phase. The supplementary phase commences after the closing of the clock phase. In the supplementary phase, each bidder can make an arbitrary number of additional

[8] For example, in most European auctions the spectrum was organized in nationwide licenses of 2×5 MHz blocks in predefined bands. In this case, the categories correspond to spectrum bands (e.g., 800 MHz band) and the lots correspond to blocks of spectrum in this band (e.g., six 2×5 MHz blocks in the 800 MHz band).

[9] To economize on notation, we forgo to index q^t with respect to the bidder whenever it does not cause confusion.

bids on any possible combination of lots. That is, a bid in the supplementary phase is a package $q^s = (n^s_\alpha, n^s_\beta)$ and a (maximum) price $b(q^s)$ that a bidder would pay for this package. Moreover, all packages that the bidder bid on during the clock phase are considered as supplementary bids if the bidder does not increase her bid for such a package. That is, if in round t the bidder indicated demand $q^t = (n^t_\alpha, n^t_\beta)$, her bid on this package is $b(q^t) = q^t \times p^t = n^t_\alpha p^t_\alpha + n^t_\beta p^t_\beta$. The bid on the final round package $b(q^f)$ is not constrained. The bid on any other package is constrained by a *cap*. In this article we will consider two different caps:[10]

Relative cap. If the Relative Cap is imposed, only packages with a smaller eligibility than the final clock package are constrained by inequality (2). If a bidder wants to bid on a package q^x which requires a higher eligibility than the final package, say $N_x > N_f$, than her bid is constrained by the bid she placed in the last clock round where her eligibility was sufficient to bid on q^x. To illustrate, suppose that during the clock phase in round t the bidder reduced her demand from N_x to $N_t < N_x \leq N_{t-1}$. Her bid on package q^x is then constrained by

$$b(q^x) \leq b(q^t) + (q^x - q^t) \times p^t. \tag{1}$$

Inequation (1) implies that the bid for package q^x is constrained by the bid for the package q^t plus the difference in package prices at the prices in round t. The idea of the Relative Cap is that as the bidder was not willing to bid on the larger bundle in round t, when she was still allowed to do so, she 'revealed' that she was at most willing to pay this differences in prices to obtain a bundle of size N_x compared to the smaller bundle of size N_t. Note that if $N_t > N_f$, the price the bidder can bid on bundles of size N_t is constrained in a similar manner. Recursively, the maximum bid on q^x is determined by the maximum bid on the final clock-round bundle.

Relative cap plus Final cap. If additionally to a Relative Cap a Final Cap is imposed, the maximum bid for a package q is constrained by the bid on the final clock package:

$$b(q) \leq b(q^f) + (q - q^f) \times p^f. \tag{2}$$

Inequation (2) implies that the bid for a package q is constrained by the bid for the final clock round package plus the difference in package prices at the final round of the clock phase. The idea of the Final Cap is that it is unreasonable for a bidder to claim a higher value for a smaller package than the package she bid on in the final round, considered she could have bid on this smaller package in the final round. Moreover, for a larger package, it is unreasonable to claim a value higher than the price difference to the final package at final prices, considered she had

[10]See Ausubel and Baranov (2014) for more possible cap rules.

decreased her demand at lower prices.

Winner determination and payments. Once all bids are collected, the auctioneer determines the combination of bids that maximizes the overall values as reflected by the bids. The sum of demands in each category reflected in the winning bids has to be below or equal to the available supply. We call this allocation the winning allocation.

In the logic of a second-price auction (Vickrey auction), the price a winning bidder has to pay is equal to the lowest bid that she would have needed to bid in order to make the winning allocation the winning allocation. That is, the auctioneer determines the value of a hypothetical second best allocation, where all bids of this particular bidder are excluded. This bidder has to pay the difference of the value of this hypothetical second-best allocation minus the value of the bids of all the other winning bidders in the final allocation, as long as this expression is larger than zero. Otherwise, she has to pay zero. Thus, each winner pays her opportunity cost. To illustrate, denote by q_A^w, q_B^w, and q_C^w the packages bidders A, B, and C receive in the winning allocation with their respective bids $b(q_A^w)$, $b(q_B^w)$, and $b(q_C^w)$. Moreover, denote by q_B^{sb} and q_C^{sb} the packages bidders B and C receive in the second best allocation if A was excluded with their respective bids $b(q_B^{sb})$, and $b(q_C^{sb})$. The price bidder A has to pay is then

$$P_A = b(q_B^{sb}) - b(q_B^w) + b(q_C^{sb}) - b(q_C^w)$$

if P_A is larger than zero. Otherwise, she does not have to pay anything.

2.2 Recommended behavior

Truthful bidding. Even though the rules of the CCA are complex, even in the simplified version presented above, the promise is that bidding in the CCA becomes simple. As the rules entail many features of the Vickrey auction, a straightforward recommendation to a particular bidder would actually be to bid truthfully independent of the bidding behavior of the competitors. Bidding truthfully implies that during the clock phase the bidder should bid on an eligible combination of frequencies which maximizes her (hypothetical) profit at the given clock prices. In the supplementary phase, truthful bidding is also a sensible recommendation as the caps on the supplementary bids are such that truthful bids are possible given truthful bidding in the clock phase.

The optimality of truthful bidding would be a very desirable feature from the bidders perspective as in preparing the auctions the bidders could fully concentrate on determining the right valuations and would not need to worry about the competitors preferences or their potentially erratic behavior. Thus, we would like to stress, that truthful bidding being a dominant strategy is of great importance as compared to truthful bidding merely being an equilibrium.

Truncated bidding. Even if bidders do not have to worry about competitors behavior, bidding on all possible combinations of lots in the supplementary phase might be still very demanding. Thus, if a Final Cap is imposed, the following recommendation would simplify bidding significantly. If at the end of the clock phase, all lots have been allocated, i.e., in all categories demand exactly equals supply, no additional bid in the supplementary bid is required as this allocation cannot be changed by any combination of bids in the supplementary phase. If, however, at the end of the clock phase in some category the demand is strictly below the supply, the bidders who are bidding on lots in this category have to increase their bids by the values of the unsold lots at the final clock prices.[11] That the allocation cannot change in the supplementary phase not only reduces the complexity, it is also important to bidders who need to secure a minimal package to ensure business continuity after the auction.[12]

3 Truthful bidding in the clock round is not a dominant strategy

As stated above, the optimality of truthful bidding irrespective of the competitors strategy would be a desirable feature for bidders in a CCA. In this case, bidders would not have to worry about the behavior and preferences of their competitors. Unfortunately, this property of the Vickrey auction does not extend to the CCA. Janssen and Karamychev (2013) and Levin and Skrzypacz (2014) showed that truthful bidding is not dominant but merely an equilibrium (one of many possible equilibria). That is, the optimal bidding behavior depends on the bids of the other bidders.

One of the crucial differences between the static Vickrey auction and the CCA is that in the CCA bidders can condition their strategies on the outcomes of the clock phase. Whenever the allocation cannot change after the clock phase, the second-price rule implies that bidders can only change the prices paid by their competitors but do not influence their own payoffs. Thus, if bidders only care about their profits as measured by willingness to pay minus the price paid, they are indifferent among all bids in the supplementary phase. The optimal bidding in the clock phase of a profit maximizing bidders then crucially depends on how her competitors resolve this indifference. This is an important observation as it demonstrates that even if a bidder has

[11] The optimality of truncated bidding with a final cap was first observed by Bichler et al. (2013).

[12] DotEcon writes on this issue on behalf of the Irish regulator ComReg (ComReg (2010)): The problem of business continuity risks and the difficulty for an incumbent bidder in valuing the retention of spectrum to serve existing GSM customers would be better addressed by a CCA than a SBCA [Sealed Bid Combinatorial Auction] (provided the appropriate activity rule is used for the CCA). This issue was not considered in our previous report as we judged that there the probability of an incumbent not bidding to reflect a high value on business continuity was insignificant. However, incumbents have taken a contrary view about this probability in their consultation responses.

straightforward profit maximizing preferences she still has to worry about the preferences of her competitors.

In what follows we discuss two examples in which either overstating or understating the demand is optimal depending on the preferences of the competitors. In each subsection we will start with the description of the setting and the outcome of truthful bidding. We then introduce a particular preference on how to resolve the indifference about bids in the supplementary phase for one of the bidders and show how the other bidders should optimally change their bids as a reaction to this preference. To keep the examples as simple as possible, we restrict ourselves to an environment in which bidders are fully able to observe each other bids in the clock phase.

3.1 Overstating demand in the clock phase can be optimal if bidding in the supplementary phase is costly

In this section we provide an example in which one bidder may profit from overstating her values in the clock phase depending on the bidding behavior of the other bidder. Suppose two bidders A and B bid on two lots in one category α. The willingness to pay of bidder A for two lots is 14 and zero for a single lot. Bidder B is willing to pay up to 15 for two lots and up to 11 for one lot.

Truthful bidding. If both bidders bid truthfully in the clock phase, bidder B reduces her demand from 2 to 1 when the price of a lot of α reaches 5. Bidder A reduces her demand from 2 to zero as soon as the clock price of α reaches 8. In the supplementary phase, bidder B increases her bid for a single lot to 11. Moreover, she raises her bid for two lots to 15. As a result, bidder B wins both lots and pays a price of 14. This is summarized in Table 1.

Round	Price of α	Demand of A	Demand of B
1	0	2	2
...	...	2	2
5	4	2	2
6	5	2	1
7	6	2	1
8	7	2	1
9	8	0	1

Clock phase

Package of	Bid of A	Bid of B
1 lot	–	11
2 lots	–	15

Supplementary phase

	Bidder A	Bidder B
Allocation	0	2
Price	0	14

Outcome

Table 1: Bids and outcomes if both bidders bid truthfully

Bidding cost. To illustrate that bidding truthfully is not always optimal for bidder A, we induce bidder B with a slightly different bidding behavior by assuming that for bidder B it is costly to submit bids in the supplementary phase. More precisely, the cost of submitting a bid in the supplementary phase for bidder B is arbitrary small but strictly positive. That is, whenever bidder B could strictly gain from submitting a bid in the supplementary phase, she will do so. That bidding is (slightly) costly in the supplementary phase is not an unreasonable assumption in real-life auctions. The preparation of the bid documents for the supplementary phase requires at the very least some time by an employee who could have been productive performing a different task. Moreover, all bids have to be approved by the board which typically requires the board to meet and to go through the determination of the bid. Another way to think about the cost of submitting a bid is to consider the drawbacks from exposing the true valuation on a combination of spectrum lots. This information might at a later stage be used by the regulator or the competitors to extract rents from the bidder in question.

Now suppose that bidder B bids truthfully in the clock phase and reduces her demand from 2 to 1 when the price of a lot of α reaches 5. Bidder A, however, deviates from truthful bidding and reduces her demand from 2 to zero when the clock price of α reaches 9 instead of 8. In this case, there exists no profitable bid in the supplementary phase for bidder B that would make him win one of the lots. As A has placed a bid of 16 on two lots in the clock phase, neither bidding 15 for two lots or bidding 11 for one lot will change the allocation in favor of B. Thus, as bidding in the supplementary phase is costly, bidder B would refrain from placing her truthful bids. It follows that bidder A wins both objects at a price of 8. This is summarized in Table 2.

Round	Price of α	Demand of A	Demand of B
1	0	2	2
...	...	2	2
5	4	2	2
6	5	2	1
7	6	2	1
8	7	2	1
9	8	2	1
10	9	0	1

Clock phase

Package	Bid of A	Bid of B
1	–	–
2	–	–

Supplementary phase

	Bidder A	Bidder B
Allocation	2	0
Price	8	0

Outcome

Table 2: Bids and outcomes if bidder B has costs of submitting supplementary bids

Summing up, compared to truthful bidding, bidder A has profited from overstating her demand in the clock phase. Thus, deterring bidder B from placing additional bids in the supplementary phase. As stated above, this is due to the two-stage nature of the CCA; bidder B could only be deterred from submitting truthful bids by giving him the opportunity to observe the outcome of the clock phase.

3.2 Understating demand in the clock phase can be optimal if competitors try to raise costs

In this section we provide an example in which a bidder may significantly profit from understating her values in the clock phase depending on the bidding behavior of the other bidders. Suppose three bidders A, B, and C, bid on two lots in each of two categories α and β. The willingness to pay of bidder A is 15 for two lots of α, 8 for one lot of α, and zero for any number of lots of β. Bidder B values one lot of α at 8 and has a willingness to pay of zero for any other package. The willingness to pay of bidder C is 15 for two lots of β and zero for any other package.

Truthful bidding. If all bidders bid truthfully, bidder A reduces her demand in the clock phase from 2 lots of α to 1 lot at the price of 8. Bidders B and C do not change their demand throughout the clock-phase. The clock phase ends as soon as bidder A reduces her demand. In the supplementary phase bidder A submits a bid of 15 for two lots of α and bidder C submits a bid of 15 on two blocks of β. Bidder B does not need to submit any additional bids as her values are reflected in her clock-phase bid. As a result, bidders A and B receive one lot of α each and bidder C receives both lots of β. Bidders A and C pay 0, bidder B pays 6.[13] The results are summarized in Table 3.

Round	Price of α	Price of β	Demand of A	Demand of B	Demand of C
1	0	0	(2,0)	(1,0)	(0,2)
...	...	0	(2,0)	(1,0)	(0,2)
7	7	0	(2,0)	(1,0)	(0,2)
8	8	0	(1,0)	(1,0)	(0,2)

Clock phase

Package	Bid of A	Bid of B	Bid of C
(1,0)	–	–	–
(2,0)	15	–	–
(0,2)	–	–	15

Supplementary phase

	Bidder A	Bidder B	Bidder C
Allocation	(1,0)	(1,0)	(0,2)
Price	0	6	0

Outcome

Table 3: Bids and outcomes with truthful bidding.

Raising rivals cost. To illustrate that reducing demand in the clock phase may be beneficial for bidder A, we induce bidder C with a different bidding behavior by assuming that bidder C

[13] The second best allocation for the other bidders without either bidder A or bidder C is the same as the resulting allocation. Thus, by the Vickrey pricing rule the prices for both bidders must be 0. The second best allocation without bidder B would be that bidder A receives 2 lots of α at a price of 14. Thus the opportunity cost imposed by bidder B is $14 - 8 = 6$.

weakly prefers if bidder A has to pay more in the auction. More precisely, if bidder C has the choice between two bids that give him the same profit, she will choose the bid that increases the payment of A. That is, bidder C is not willing to sacrifice some of her profits to make bidder A pay more. However, whenever increasing the payment of A has no impact on bidder C's profits, she will increase the payment of bidder A.[14]

Suppose first, bidder A does not change her bidding behavior in the clock phase and demands two lots of α up to a price of 7 and reduces her demand to one lot of α at a price of 8. In this case, bidder C can increase the price bidder A has to pay without any risk for himself by submitting a supplementary bid of 16 for $(2, 0)$. Thus, bidder C changes the next best alternative for the calculation of the payment of bidder A. The allocation remains unchanged as compared to truthful bidding. However, bidder A has to pay 8 instead of 0 due to the malicious bid of bidder C. By the same token, the amount that bidder B has to pay increases from 6 to 8. That is, bidders A and B bid the full amount of their bids. The situation is depicted in Table 4.

Round	Price of α	Price of β	Demand of A	Demand of B	Demand of$C$
1	0	0	(2,0)	(1,0)	(0,2)
...	...	0	(2,0)	(1,0)	(0,2)
7	7	0	(2,0)	(1,0)	(0,2)
8	8	0	(1,0)	(1,0)	(0,2)

Clock phase

Package	Bid of A	Bid of B	Bid of C
(1,0)	–	–	–
(2,0)	15	–	16
(0,2)	–	–	15

Supplementary phase

	Bidder A	Bidder B	Bidder C
Allocation	(1,0)	(1,0)	(0,2)
Price	8	8	0

Outcome

Table 4: Bids and outcomes if Bidder C prefers to raise rivals cost.

Given the preference of bidder C to raise her competitors cost, bidder A benefits from reducing her demand in the first round of the clock phase followed by placing a truthful bid on one lot of α. In this case, bidder C has no maneuver to increase the bid of A without bearing the risk of winning a worthless package. Thus, bidder C refrains from a malicious bid and bids truthfully in the supplementary phase. This is depicted in Table 5.

[14] Janssen and Karamychev (2013) provide an equilibrium analysis of bidding in the CCA with such preferences. The point we make here is much simpler: even if a bidder is only concerned with profit maximization she needs to take into account the preferences of their rivals. Marsden and Sorensen (2016) provide further examples on how a preference for good relative outcomes changes bidding in a CCA.

Round	Price of α	Price of β	Demand of A	Demand of B	Demand of C
1	0	0	(1,0)	(1,0)	(0,2)

Clock phase

Package	Bid of A	Bid of B	Bid of C
(1,0)	8	8	–
(2,0)	8	–	–
(0,2)	–	–	15

Supplementary phase

	Bidder A	Bidder B	Bidder C
Allocation	(1,0)	(1,0)	(0,2)
Price	0	0	0

Outcome

Table 5: Bids and outcomes if Bidder A anticipates the preference of Bidder C for raising rivals cost.

Note that reducing the demand in the clock phase comes at a cost for bidder A: she is not able to submit a truthful bid on two lots of α. Thus, bidder A is giving up on the opportunity to outbid bidder B on the second lot of α. Overall, bidder A faces a complex trade-off that in general depends on her belief about her competitors preferences.

That bidders may benefit from raising the payments of their rivals is a reasonable assumption for real-life spectrum auctions given that all bidders will compete in the aftermarket. Moreover, the management of companies as well as operative project teams involved in the preparation of the bidding are very much interested in not paying more for comparable frequencies than its competitors. One of the reasons is that management tries to limit the available budget of its competitors for necessary infrastructure investments following the auction. Another reason is that involved management and project members carry a significant personal risk for their careers, if the own company seemingly overpaid in the auction. In fact, Ofcom (2012) decided against a Final Cap in the UK 4G auction because "[A Final Cap] allows bidders to place bids in the Supplementary Bids Round that they know cannot win but that might raise the prices other bidders pay for spectrum." Unfortunately, as we have demonstrated above, the ability to risklessly raise the cost of the competitors is not only driven by the cap regulation but rather by the dynamic nature of the CCA. Bidders could observe the competitors demand in the clock phase and taylor their bids to the sole purpose to raise rivals cost without running the risk of winning undesirable packages.

4 If a Relative Cap is imposed, truncated bids are not tractable

In this section we focus on the feasibility of truncated bidding. That is, the property that to secure the final allocation in the supplementary phase it is sufficient to bid only on a handful of packages. As stated above, truncated bidding is desirable for bidders for various reasons. For example, to reduce complexity, to limit the exposure of the true valuation or to secure business continuity.[15] However, truncated bidding is only optimal if a Final Cap is imposed on the bids in the supplementary phase. Nevertheless, imposing a Final Cap has also drawbacks. For example, bidders worry that if the allocation cannot change in the supplementary phase, their rivals would be able to risklessly raise their payments.[16] However, if instead a Relative Cap is imposed, truncated bidding may not be sufficient to secure the final package. In this section we provide a simple example on how the final allocation can change in the supplementary phase if a Relative Cap is imposed. Moreover, we comment on the difficulties that bidders face if they want to secure the final allocation in the final round. A more general discussion on different cap rules beyond the final and the Relative Cap can be found in Ausubel and Baranov (2014).

Suppose three bidders A, B, and C, bid on two lots in each of two categories α and β. Suppose furthermore that the clock phase consists of two rounds of bidding. Bidder A demands a package of $(2, 1)$ in either round. Bidder B demands a package of $(0, 1)$ in each round. Bidder C demands a package of (0,2) in round 1 and after the price of β rises to 10 in round 2 drops her demand to (0,0).[17]

In the supplementary phase, bidders A and B engage in truncated bidding and make no additional bid, i.e., the final bid of A is 10 for the package (2,1). B bids 10 for the package (0,1). Bidder C, however, bids 20 on the package (0,2). This is feasible as the last round when she was able to bid on this package is the final round and 20 is the price for this package at the final round prices. Moreover, bidder C hands in a bid of 20 on the package (2,1). This is also feasible as the first round when she was bidding on a bundle with eligibility smaller than three was round one. Thus, the bid she can place on (2,1) is 20 which is the bid for the package (0,2) of round one plus the difference in the value of the packages (2,1) and (0,2) at the prices of round one. This difference, however, is equal to zero. With this bids, bidder C outbids bidder A and changes the allocation of the final round. In the end, bidder C receives the package (2,1) at a price of 10, bidder B receives the package (0,1) at the price of 0 and bidder A does not receive anything. This situation is depicted in Table 6.

[15]There are several further reasons why bidders would like to be able to reduce uncertainty about the final allocation. For example, bidders valuations are usually based on the revenue that they can achieve in the telecommunications market after the auction. Thus, these values crucially depend on the packages that are won by the competitors. If the individual demands of the bidders are transparent in the clock phase, bidders can condition their bids on the demand of their competitors. This is clearly not possible in the supplementary phase.

[16]See Ofcom (2012).

[17]As in this section we are only concerned with the feasibility of truncated bidding, we will abstract from true valuations to simplify exposition.

Round	Price of α	Price of β	Demand of A	Demand of B	Demand of C
1	0	0	(2,1)	(0,1)	(0,2)
2	0	10	(2,1)	(0,1)	(0,0)

Clock phase

Package	Bid of A	Bid of B	Bid of C
(0,2)	–	–	20
(2,1)	–	–	20

Supplementary phase

	Bidder A	Bidder B	Bidder C
Allocation	(0,0)	(0,1)	(2,1)
Price	0	0	10

Outcome

Table 6: The allocation can change in the supplementary phase.

The example demonstrates that truncated bidding as described in Section 2.2 is not sufficient to secure the final allocation of the clock phase. This is always the case if one of the bidders in some round reduces his overall eligibility but increases his demand in at least one category and the overall price of the package. However, this does not mean that securing the final allocation from the clock phase is generally impossible. In principle, bidders could use the Relative Cap constraints as described in Section 1 to calculate the bids that are sufficient to secure the final clock phase package. However, there are two complications with this. First, this would require the exact knowledge on the eligibility of all the other bidders at the start of the auction and full transparency of all demands during the clock phase which is not given in most implementations of the CCA. Second, the bids needed may be well above the final clock phase prices. Ausubel and Baranov (2014) calculated the sufficient bids on the final clock round package in the UK 4G auction as a percentage of the final clock phase prices. Their results are reported in Table 7.

Bidder	Sufficient bid [in % of final prices]
Vodafone	245 %
Telefonica	282 %
EE	697 %
Niche	1092 %

Table 7: Required bids in the supplementary phase to secure the final clock phase package as a percentage of the final prices in the UK 4G auction 2013.

5 Discussion of further complexities

In the previous sections we have demonstrated that deciding on a bidding strategy is not as straightforward as the usual recommendations suggest. Truthful and truncated bidding may be harmful for bidders. Moreover, the optimal bidding strategy may crucially depend on the bidding strategies and thereby on the preferences of the competitors. In this section we will comment upon further complications that may arise during the preparation of a bidding strategy. These complications arise due to particular design features or due to particularities of decision making inside firms.

Threats and coercion. In the context of spectrum auctions, it is well conceivable that there are lots which a specific telecommunication company needs to acquire. For example, if the regulator is not able to freely rearrange spectrum after the auction, some of the spectrum will be organized in specific lots rather than generic lots.[18] In this case, companies owning lots that are adjacent to the lots at auction may have a significantly higher valuation for such lots than other lots in the same band and this potentially be known to other bidders. This knowledge can be leveraged by their competitors in the clock phase. Punitive bidding on such lots could be used to coerce a bidder to reduce her demand on lots in other categories that are desired by her competitors. While this is true for any dynamic format, the Vickrey pricing of the CCA may make such a strategy almost costless as it is possible to increase the prices of a competitor without influencing own payments.

Close-to-Vickrey core pricing. In most implementations of the CCA payments are not determined purely according to the Vickrey pricing rule. It is further required that the sum of the final payments should not be smaller than the sum of bids of an alternative set of bidders. That is, it is required that the outcome is in the core. This potentially leads to an increase in the payments of one or more winning bidders. Thus, as the own bid is an upper limit for the own payment, a bidder can now influence her own payments by her own bids. This in turn destroys the logic behind truthful bidding.[19] Even worse, Goeree and Lien (2015) demonstrate that if the outcome of the Vickrey auction is not in the core, there exists no auction with a dominant strategy equilibrium that is in the core.

CFOs want to have control over their expenses. Typically there is a lot at stake for bidders participating in CCAs and the corresponding valuations amount to significant investments for companies if they would have to be spent. Therefore, during the auction, bidders usually have to report to their supervisory board on the development of the auction and especially on the momentary expenses at that stage of the auction. Additionally, CFOs usually want to keep close control over their expenses and the value at stake. However, in the CCA, the Vickrey pricing rule (or even more complex the 'Close-to-Vickrey core prices') makes this requirement hard to fulfill. Any computation of the actual payments would require knowledge of individual bids of competitors which is not available. Thus, in most cases, only estimates of varying reliability (and

[18] In a CCA this would imply having more categories of lots within the same band.
[19] See Marsden and Sorensen (2016) for comprehensive examples of this effect.

hence often very limited usefulness) can be provided. Despite the limited relevance of auction expenses for optimal bidding behavior (apart from budget limitations), bidders nevertheless tend to put huge effort in determining actual expenses or even act according to actual expenses and not clock prices.

Budget constraints. In many cases, bidding according to valuations is limited by available resources. From a bid strategy point of view, budget limitations force bidders to change to ”cheaper” packages if the absolute budget limit is reached during the auction, although another package would still be more profitable. Additional problems might occur due to a change of a budget limit during the auction, which might lead to ex-ante suboptimal bids. Note that these problems are not specific to the CCA.However, there are two problems that are CCA specific. First, bidders do not like to gamble in spectrum auctions. Thus, the budget constraint is usually interpreted such that no bid can be placed above the budget. However, as Vickrey pricing implies that bidders hardly ever need to pay their bids, bidders do not utilize significant parts of their budget. Thus, the budget constraint is more restricting than in other formats with a more direct payment rule. Second, even with a Final Cap, there might be not enough budget to secure the end-of-clock-phase allocation. Thus, to gain certainty, it may be beneficial for bidders to reduce their demand in the clock-phase even if her current preferred package is well within the budget constraints.[20]

6 Conclusion

We have demonstrated with simple examples that optimal bidding in a CCA crucially depends on the behavior and thereby preferences of the competitors. Thus, when preparing for a CCA it is not sufficient to focus on the determination of the own valuations and rely on truthful bidding. There is no straightforward way to adjust the bidding strategy. Depending on the preferences and valuations of the competitors different strategies become optimal. For example, we demonstrated in Section 3 that both increasing the demand and decreasing the demand in the clock phase might be optimal.

Our results imply that when preparing for a CCA the project team needs to focus on the competing bidders. It should have a model in place to estimate the valuations of the competitors and use it to understand potential final outcomes of the auction. Moreover, different scenarios should be considered with respect to the competitors preferences that are not captured in the valuations. At this, the preference for raising rivals cost should take a prominent role. Once

[20]See Janssen et al. (2016) and McKenzie and Fookes (2016) for a thorough analysis of the impact of budget constraints on the CAA. Moreover, Marsden and Sorensen (2016) provide some comprehensive examples on how budget constraints may affect bidding.

there is a basic understanding of the competition, the next step would be to understand how the derived preferences might translate into bidding behavior in the auction at hand. For this it is important to understand which degrees of freedom the specific set-up gives to the competition to act according to their potential preferences.

Conditioning the strategy on the behavior of the competitors is in general a very delicate task, since a CCA with many packages quickly reaches a complexity level that is difficult to handle. One way to address this challenge is to use simulations with automated bidders. Such simulations allow to generate many different bidding scenarios and to test the sensitivities of the made assumptions on the valuations and preferences.

Once the project team has understood the different scenarios they usually need to explain the insights to the top-level management and the supervisory board. The complexity of the CCA makes it difficult to achieve a sufficient understanding in the limited amount of time that is available when dealing with the top-level decision makers of a company. Mock auctions can help to facilitate this process. Mock auctions are a time-lapse version of the actual auction and can be used to generate different decision scenarios. This gives the management the opportunity to understand the workings of the CCA and the potential impact of their decisions concerning budgets and bidding strategy.

Summing up, in the CCA the optimal bidding strategy crucially depends on the behavior of the competitors and the derivation of the optimal bid strategy is not straightforward. Thus, the derivation of an optimal bidding strategy is a complex task that requires careful preparations and analysis.

References

Ausubel, L. M. and O. V. Baranov (2014): "A Practical guide to the Combinatorial Clock Auction," *mimeo.*

Bichler, M., P. Shabalin, and J. Wolf (2013): "Do core-selecting Combinatorial Clock Auctions always lead to high efficiency? An experimental analysis of spectrum auction designs," *Experimental Economics*, 16, 511–545.

ComReg (2010): "Award of liberalised spectrum in the 900MHz and other bands," *ComReg Document 10/71a.*

Cramton, P. (2013): "Spectrum auction design," *Review of Industrial Organization*, 42, 161–190.

CTU (2011): "Basic principles of tender/auction for the assignment of rights to use radio frequencies in the 800 MHz, 1800 MHz and 2600 MHz bands," *Ref: CTU-80 070/2011-20.*

Goeree, J. K. and Y. Lien (2015): "On the impossibility of core-selecting auctions," *Theoretical Economics, forthcomming.*

Janssen, M. and V. Karamychev (2013): "Gaming in Combinatorial Clock Auctions," *mimeo.*

Janssen, M., V. Karamychev, and B. Kasberger (2016): "Budget constraints in VCG and the CCA," in *Handbook of Spectrum Auction Design*, Cambridge University Press.

Levin, J. and A. Skrzypacz (2014): "Are dynamic Vickrey Auctions practical?: Properties of the Combinatorial Clock Auction," *mimeo.*

Marsden, R. and S. Sorensen (2016): "The Combinatorial Clock Auction from a bidder perspective," in *Handbook of Spectrum Auction Design*, Cambridge University Press.

McKenzie, S. and N. Fookes (2016): "Impact of budget constraints on the efficiency of combinatorial auctions," in *Handbook of Spectrum Auction Design*, Cambridge University Press.

Ofcom (2012): "Assesment of future mobile competition and award of 800 MHz and 2.6 GHz," *http://stakeholders.ofcom.org.uk/consultations/award-800mhz-2.6ghz/statement/.*