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Brown Growth, Green Growth, and the Efficiency of Urbanization

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Abstract

We analyze the efficiency of urbanization patterns in a stylized dynamic model of urban growth with three sectors of production. Pollution, as a force that discourages agglomeration, is caused by domestic production. We show that cities are too large and too few in number in uncoordinated equilibrium if economic growth implies increasing pollution ('brown growth'). If, however, production becomes cleaner over time ('green growth') the equilibrium urbanization path reaches the efficient urbanization path after finite time without need of a coordinating mechanism. The results may be generalized to take other forms of congestion into account.

JEL-Code: O180, Q560, R120.

Keywords: urbanization, green growth, migration, pollution, congestion.

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1 Introduction

There is a widely held concern in the urban economics literature that cities tend to be too large in equilibrium. In particular environmental pollution is assumed to be exceedingly high in large cities, expecially in developing countries (Henderson 1977; Henderson 2002a; Henderson 2002b; Trolley 1979; Shah and Nagpal 1997; UNFPA 2001; UN-Habitat 2003).[1] This becomes even more of an issue in a dynamic context where economic activity and urban population grow over time. Yet, urban pollution may either increase or decrease over time, even in a growing economy. As the extensive literature on the so-called Environmental Kuznets Curve indicates, the environmental load with some pollutants, e.g. SO_2, increases with output at early stages of economic development, but may ultimately decrease again (e.g., Andreoni and Levinson 2001, Egli and Steger 2007, Grossman and Krueger 1995, Lieb 2002; 2003, Plassmann and Khanna 2006, Stern 2004). Recent evidence shows increasing urban air quality in California despite increasing traffic due to an increasingly 'green' technology (Kahn and Schwartz 2008). In light of these observations the question is, if cities are currently too large compared to their optimal size, will this problem become worse or better in the course economic development?

Seminal contributions by Henderson (1974,1988) provide a theoretical foundation of the concern that cities are too large in equilibrium. He showed that in an uncoordinated migration equilibrium, cities are too few in number and too large each, because individual households and firms have little incentives to found new cities and rather stay in inefficiently large existing cities. As a solution to this coordination failure Henderson proposed that

[1] With strong migration restrictions, as the prevail in many regions in China, cities may on the other hand be inefficiently small (Au and Henderson 2006).

powerful land developers should be given the right to found new cities and receive all land rents households and firms pay in the new city. The competition of such land developers for tenants would then lead to efficient city sizes. Current dynamic theories of urban growth assume that such powerful land developers exist and ensure efficient city sizes (Black and Henderson 1999, Rossi-Hansberg and Wright 2007, Henderson and Venables 2009). In contrast to this view, Helsley and Strange (1997) argue that in practice developers only have limited power and thus may fail to implement efficient city sizes. But also the very diagnosis has been challenged that without coordination equilibrium city sizes are inefficient. In a model of two regions with an exogenously growing total population, Anas and Xiong (2005) show that an efficient population distribution over two cities may emerge without developers if there are positive externalities between cities.

In this paper, we re-examine the question of whether and under which conditions city sizes are inefficient without coordination in a dynamic context. For this sake, we develop a simple and analytically solvable model of urban growth. Growth is driven by human capital accumulation, and an endogenous number of cities forms as the result of (aggregate) increasing returns to scale in urban production. The force that limits city sizes in this model is (local) urban pollution, resulting as an unwanted joint product of urban production. The pollutants we have in mind include air pollution with particulate matter, which is of high economic importance (e.g. Muller and Mendelsohn 2009, Kahn 2006). Other congestion externalities associated with commuting or crime could be incorprated easily in our framework. Unlike other theories our model can explain a *decreasing* number of cities even in a growing economy. This is the case when pollution decreases, such that equilibrium city sizes increase.

Our main result is that cities may be of efficient sizes or inefficiently large in an equilibrium development path, depending on the nature of economic growth. If economic growth is accompanied by increasing pollution, cities are inefficiently large in the uncoordinated equilibrium. Under conditions of 'green growth', with decreasing pollution, city sizes in the uncoordinated market equilibrium become efficient. This, however, does not happen immediately, but there is a hysteresis effect. It takes a sufficiently large reduction of pollution, and, correspondingly, a sufficiently long (but finite) interval in time over which pollution is reduced, until the efficient urbanization pattern is reached.

The next section develops the model of urban growth. In Section 3 we consider a city in isolation and derive results on optimal urban environmental policy and how the citizens' well-being depends on the city's population size. Section 4 contains the dynamic analysis of the whole urbanization pattern, and derives the equilibrium and Pareto-efficient paths of urban growth. In the final Section 5 we summarize our results and discuss how they can be extended to take other forms of congestion explicitly into account.

2 A simple model of urban growth

We consider a small open economy that trades a primary resource and a final consumption good on world markets at given prices. The price of the resource is normalized to unity, the price of the consumption good is P. At these prices, the primary resource and the consumption good are also traded between cities within the economy. By contrast, intermediate goods that are used to produce the final consumption good are non-tradable and can be used only within the city where they are produced. One may imagine the intermediate goods as specific sub-contracted production services that are provided locally. Thus, we neglect transport cost for the consumption good and assume that transport cost are infinite for the intermediate goods. Transport costs are important to consider if one is interested in global pollutants for example by greenhouse gases (Gaigné et al. 2012), but they are not in the core interest when studying the effects of local pollution.

The economy is divided into a large number M of regions. In each region $i = 1, \ldots, M$, a city could possibly exist. A city, in our model, is a region that is inhabited by a minimum number of (urban) residents. While the number of regions is exogenous, the number of cities is an endogenous variable of particular interest. In a city, two sectors of production exist: an industrial sector which produces the final consumption good and a small and medium-sized enterprises (SME) sector with a large variety of firms. Each firm in the SME-sector produces a particular variety of the intermediate good, using the resource and sector-specific human capital.

To operate a business in the SME-sector, one unit of sector-specific human capital has to be employed at a rental rate of r_i. In addition, for each unit of output, $\alpha_i > 0$ units of the resource are consumed. Hence, a firm's total costs of producing x units of output are:

$$C_i(x) = r_i + \alpha_i\, x \tag{1}$$

The industrial sector in city i uses a composite X_i of intermediate goods and specific human capital H_i to produce a quantity Y_i of the final consumption good with a technology described by the production function[2]

$$Y_i = H_i^{\mu_i}\, X_i^{1-\mu_i}, \tag{2}$$

[2]We allow all technological parameter values to be region-specific.

where H_i is the firm's employment of human capital and

$$X_i = \left(\int_0^{J_i} x_i(j)^{\frac{\sigma_i - 1}{\sigma_i}} \, dj \right)^{\frac{\sigma_i}{\sigma_i - 1}} \tag{3}$$

is the aggregate of a mass J_i of intermediate input varieties with quantities $x_i(j)$. The different varieties are assumed to be (imperfect) substitutes in production, i.e. the elasticity of substitution exceeds one, $\sigma_i > 1$.

By symmetry, all firms in the SME sector will produce the same quantity $x_i(j) \equiv x_i$ of output. As every firm needs one unit of human capital input, the mass of varieties equals city i's endowment with human capital specific to the SME sector, $J_i = K_i$. Aggregate output thus is

$$Y_i = H_i^{\mu_i} \, K_i^{(1-\mu_i)\frac{\sigma_i}{\sigma_i - 1}} \, x_i^{1-\mu_i} \tag{4}$$

Urban pollution is an unwanted by-product of the production of intermediate goods. Pollution load is assumed to be proportional to the aggregate input of the primary resource, $R_i = \alpha_i \, K_i \, x_i$ (cf. equation 1), with a proportionality factor $e_i(t)$,

$$E_i = e_i(t) \, R_i = e_i(t) \, \alpha_i \, K_i \, x_i. \tag{5}$$

Given the technology of production, in particular (2), it is possible to abate emissions by substituting human capital for intermediate goods. It is however not possible to 'export' pollution, as intermediate goods cannot be traded between cities.

We assume an exogenous 'pollution-saving' technical progress. That means, the amount of pollution per unit of resource input decreases over time, $de_i(t)/dt \leq 0$. Thus, even with increasing output, local pollution does not necessarily increase at the same rate. Actually, it may even decrease if the rate at which $e(t)$ decreases is large enough. We call such a situation 'green growth'.

All individuals have the same preferences over consumption c_i and environmental quality. The instantaneous utility function is

$$u(c_i, E_i) = c_i \cdot \left(1 - \frac{E_i}{\bar{E}_i}\right)^{\phi} \tag{6}$$

where c_i is the amount of goods consumption, E_i is pollution in the individual's city of residence. Beyond a maximum tolerable level of pollution $\bar{E}_i$ life at the place of residence is impossible. The parameter $\phi > 0$ is the weighting factor of environmental pollution in utility.

In each region the consumption good can also be produced by labor input only, causing no environmental damage. The technology in this 'traditional' sector is characterized by constant returns to scale, such that one unit of labor input generates β_i units of output. Without loss of generality, the regions are ordered according to their productivity, i.e. $\beta_1 > \beta_2 > \ldots > \beta_M$.

Each individual inelastically supplies $k(t)$ units of human capital specific to the SME-sector and $h(t)$ units of human capital specific to the industrial sector. We assume that both types of human capital grow at the same rate $\theta(t)$ and both are perfectly mobile across regions.[3]

Total urban population $N(t)$ is time-dependent, capturing natural growth (or decline) of urban population. Each individual decides in which city she lives. We do not consider any migration costs. We use $n_i \in [0, N(t)]$ to denote the population of region i, with $\sum_i n_i = N(t)$. If an individual in region i works in the traditional sector, her consumption is β_i, as all current income

[3]It is possible to endogenize the rates of human capital growth and the distribution of the two types of human capital across the two sectors. We skip this complication to focus the analysis on the question of efficiency.

is used for the consumption of goods. If she works in city i her consumption is $c_i = (k(t)\, r_i + h(t)\, w_i)/P$.

3 Urban environmental policy and optimal city size

In this section we derive the results about locally optimal environmental policy and how utility of an individual depends on the population size of the city she lives in.

Profits in both sectors are zero – in the industrial sector because of constant returns to scale (cf. Equation 2), and in the SME-sector because of free entry. Hence, total revenues equal total cost, $P\, M_i = r_i\, K_i + w_i\, H_i + \alpha_i\, K_i\, x_i$. Aggregate capital stocks are $K_i = n_i\, k(t)$ and $H_i = n_i\, h(t)$. Hence, per capita consumption is equal to the value of net output per urban worker, which is equal to the value of gross output minus the value of imported resource inputs:

$$c_i = \frac{P\, Y_i - \alpha_i\, K_i\, x_i}{n_i}. \tag{7}$$

Using (4), we can express per-capita consumption as a function of the city's population and aggregate resource input R_i:

$$c_i(t) = \frac{1}{n_i}\, \left(\left(d(t)\, n_i^{1+\psi_i} \right)^{\mu_i}\, R_i^{1-\mu_i} - R_i \right) \tag{8}$$

where $\psi_i = \frac{1-\mu_i}{\mu_i}\, \frac{1}{\sigma_i - 1}$ is a measure for the degree of increasing returns to scale in urban production and $d_i(t) = P^{\frac{1}{\mu_i}}\, \alpha_i^{-\frac{1-\mu_i}{\mu_i}}\, h(t)\, k(t)^{\psi_i}$ is a measure for the level of economic development in city i.

In order to focus on the potential inefficiencies that arise from the migration process we assume that an urban government internalizes the external effects from pollution. For this sake, the locally optimal level of pollution is determined given the urban population size n_i. This level of pollution is found by solving the problem

$$\max_{R_i} v_i = \max_{R_i} \left\{ \left(\left(d_i(t)\, n_i^{1+\psi_i} \right)^{\mu_i} R_i^{1-\mu_i} - R_i \right) \left(1 - \frac{e_i(t)\, R_i}{\bar{E}_i} \right)^{\phi} \right\}$$

From the first order condition for this optimization problem, we obtain the condition[4]

$$\left(d_i(t)\, n_i^{1+\psi_i} \right)^{\mu_i} = \left(\hat{R}_i(n_i) \right)^{\mu_i} \frac{1}{1-\mu_i} \frac{\bar{E}_i - e_i(t)\,(1+\phi)\, \hat{R}_i(n_i)}{\bar{E}_i - e_i(t) \left(1 + \frac{\phi}{1-\mu_i} \right) \hat{R}_i(n_i)}. \quad (9)$$

Given the population of city i the efficient level $\hat{R}_i(n_i)$ of input of the polluting resource is implicitly given by equation (9). The locally optimal level of resource input is monotonically increasing with urban population from $\hat{R}_i(0) = 0$ to $\lim_{n_i \to \infty} \hat{R}_i(n_i) = \bar{E}_i / (e_i(t)\,(1 + \frac{\phi}{1-\mu_i}))$ (see Appendix A.2).

Using (9) in the utility function, we obtain utility under environmental policy as a function of the city's population alone,

$$v_i(n_i) = \frac{1}{n_i} \left(\left(d_i(t)\, n_i^{1+\psi_i} \right)^{\mu_i} \left(\hat{R}_i(n_i) \right)^{1-\mu_i} - \hat{R}_i(n_i) \right) \left(1 - \frac{e_i(t)\, \hat{R}_i(n_i)}{\bar{E}_i} \right)^{\phi}. \quad (10)$$

The following result states how utility depends on the city's population size.

Result 1. *If $\sigma_i > 2$, and when pollution is at a locally optimal level, utility has a unique interior maximum at a population size of*

$$n_i^{opt}(t) = \left(\frac{\bar{E}_i}{e_i(t)\, d_i(t)} \frac{\psi_i}{(\phi + \psi_i + \phi\, \psi_i)\, (1 - \mu_i\,(1+\psi_i))^{\frac{1}{\mu_i}}} \right)^{\frac{1}{1+\psi_i}} \quad (11)$$

Furthermore, utility $v_i(n_i)$ tends to zero for $n_i \to 0$ and $n_i \to \infty$.

The optimal population size decreases with the level of development, $d(t)$, and with the amount of pollution per resource input, $e_i(t)$.

[4] See Appendix A.1. There we also show that the solution of (9) exists and is unique.

Proof. see appendix A.3. □

Result 1 states that utility in city i is a hump-shaped function of the city's population n_i. In accordance with intuition, utility is increasing with population in small cities, because pollution is still small and a larger population generates a higher income per capita due to increasing returns. For large cities, the increasing environmental damage dominates such that utility decreases with the city's population. Utility is maximal at an intermediate population size (precisely at $n_i^{\text{opt}}(t)$) where increasing returns to scale in production and damage costs are in an appropriate balance. This, however, holds only true if the condition $\sigma_i > 2$ is met, which means that increasing returns to scale are not too high. Otherwise, utility would be monotonically increasing with population. We concentrate on the relevant case $\sigma_i > 2$ for all i in the following.

We assume that technologies are such that in every city utility at the optimal population size is larger than utility from the traditional sector in this region for all times, $v_i(n_i^{\text{opt}}) > \beta_i$ for all $i = 1, \ldots, M$ and all $t \geq 0$.

The optimal population size $n_i^{\text{opt}}(t)$ depends in particular on (i) the state of economic development ($d_i(t)$) and (ii) the amount of polluting emissions per unit of resource input ($e_i(t)$). As stated in Result 1, the optimal population size decreases with $d_i(t)$. The intuitive reason for this result is that with a higher level of economic development, increasing returns to scale pay off already at a smaller population size. Hence smaller and cleaner cities become optimal. The optimal population size also decreases with $e_i(t)$, as the negative effect of pollution plays a bigger role. Over time, optimal population may decrease or increase, depending on whether the joint effect $e_i(t)\, d_i(t)$ decreases or increases. These dynamics drive the results on the efficiency of urbanization dynamics derived in the following section. Figure 1 illustrates

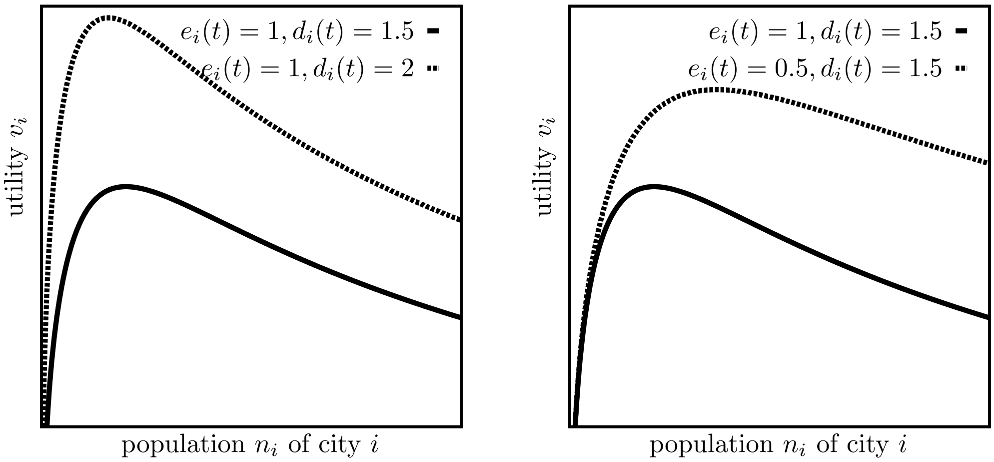

Figure 1: Instantaneous utility $v_i(n_i)$ of an individual living in city i as a function of this city's population n_i. Parameter specification: $\mu_i = 0.5$, $\sigma_i = 5$, $\phi = 0.5$, $\bar{E}_i = 10$.

the properties of the utility function $v_i(n_i)$ and how they depend on $d_i(t)$ and $e_i(t)$.

4 Efficiency of urbanization dynamics

In this section we study the efficiency of urbanization dynamics. We thereby do not consider a single city in isolation, but how the economy's urban population is distributed across cities. We focus on stable equilibrium urbanization patterns, which we define as follows:

An *equilibrium urbanization pattern* $\{n_1, \ldots, n_m\}$ with $n_i > 0$ for $i \in \{1, \ldots, m\}$ is a distribution of urban population $N(t)$ over $m < M$ cities such that (i) every urban resident lives in one of the cities, $\sum_{i=1}^{m} n_i = N(t)$, (ii) utility is the same in all cities

$$v_i(n_i) = v_j(n_j) \quad \text{for all cities } i, j \in \{1, \ldots, m\} \tag{12}$$

and (iii) no urban resident has an incentive to move to a village,

$$\min_{i\in\{1,\ldots,m\}} \{v_i(n_i)\} \geq \beta_{m+1} \tag{13}$$

An equilibrium urbanization pattern $\{n_1, \ldots, n_m\}$ is *stable* if migration of a small number dn of people creates an incentive to move back to the equilibrium urbanization pattern

$$v_i(n_i - dn) > v_j(n_j + dn) \quad \text{for all cities } i, j \in \{1, \ldots, m\} \tag{14}$$

From these definitions and Result 1, we immediately have the following result:

Result 2. *An equilibrium urbanization pattern* $\{n_1, \ldots, n_m\}$ *with* $n_i > n_i^{opt}(t)$ *for all* $i \in \{1, \ldots, m\}$ *is stable.*

Hence, an urbanization pattern is stable if all cities are larger than optimal. The intuitive reason for this is that if there was a movement from one of the large cities to another one, the latter would increase further while the former would decrease. Because the larger city would become more polluted, and the smaller city would be come cleaner, this would create an incentive to move back to the original equilibrium. This result holds irrespective of how much too large the cities are compared to the optimum.

In general many stable urbanization patterns may exist. The set of all stable equilibrium urbanization patterns we denote by Ω. In formal terms Ω is the set of all $\{n_1, \ldots, n_m\} \in \mathbb{R}_+^m$ with $m \in \{1, \ldots, M\}$ for which (i) $\sum_i n_i = N(t)$, (ii) $v_i(n_i) = v_j(n_j)$ for all $i, j = 1, \ldots, m$, (iii) $\min\limits_{i\in\{1,\ldots,m\}} \{v_i(n_i)\} \geq \beta_{M-m}$ and (iv) $n_i > n_i^{\text{opt}}(t)$ for all $i \in \{1, \ldots, m\}$. Out of these stable equilibrium urbanization patters, one is Pareto efficient.

Result 3. *Pareto efficient is the urbanization pattern* $\{n_1, \ldots, n_{m^\star}\}$ *with* $m^\star = \max\{m \in \mathbb{N} \,|\, \{n_1, \ldots, n_m\} \in \Omega\}$.

Proof. Consider an urbanization pattern $\{n_1, \ldots, n_{\tilde{m}}\} \in \Omega$ with $\tilde{m} < m^\star$. Since $v_i'(n_i) < 0$ for all $i = 1, \ldots, \tilde{m}$, the pattern with $m^\star$ cities which are smaller each involves a higher level of utility. Hence, a pattern with $\tilde{m} < m^\star$ may not be a Pareto optimum. □

It is a matter of history which of the many stable urbanization patterns actually prevails. In order to study this history and the efficiency of urbanization involved, we consider three stylized phases of growth.

1. a 'Malthusian' phase with $\dot{N}(t) > 0$, while $\dot{d}_i(t) = 0$ and $\dot{e}_i(t) = 0$ for all i.

During the 'Malthusian' phase, total urban population grows, while human capital per capita and the amount of emissions per unit of resource input remain constant.

2. a 'brown growth' phase with $\dot{d}_i(t) > 0$, while $\dot{N}(t) = 0$ and $\dot{e}_i(t) = 0$ for all i.

During the 'brown growth' phase, total urban population is constant, while human capital per capita in creases. It is a phase of 'brown growth', as the emissions per unit of resource input stay constant, which leads to increasing aggregate pollution (see Appendix A.2).

3. a 'green growth' phase with $\frac{d}{dt}(e_i(t)\, d_i(t)) < 0$ for all i, while $\dot{N}(t) = 0$.

During the 'green growth' phase, human capital increases, but emissions per unit of resource input decrease in such a way that the joint effect implies a 'greening' of the production process on aggregate. Total urban population stays constant.

In the following we study how the number and sizes of cities, utility, and the efficiency of the emerging urbanization pattern evolve over time.

In order to derive clear-cut results, we assume that all cities have the same technology, i.e. $\mu_i = \mu$, $\sigma_i = \sigma$, $e_i(t) = e(t)$, $\alpha_i = \alpha$ for all $i = 1, \ldots, M$. Thus, $m^\star$ identical cities with population $N(t)/m^\star$ each are an equilibrium urbanization pattern that is stable if $N(t)/m^\star > n^{\rm opt}(t)$ (cf. Result 2) and that is Pareto-efficient if $m^\star = \max\{m \in \mathbb{N}\,|\,N(t)/m \geq n^{\rm opt}(t)\}$ (cf. Result 3).

We first consider the 'Malthusian' phase. During this phase, the shape of the utility function $v_i(n_i)$ is unchanged. In particular $n^{\rm opt}$ is constant during this phase. We assume

$$v(2\,n^{\rm opt}) > \beta_2, \tag{15}$$

with $n^{\rm opt}$ given by (11). The implications of this assumption will become apparent below.

The dynamics during the Malthusian phase is driven by the fact that the growing urban population has to be accommodated. Consider an initial situation without any city. Because the traditional sector in region 1 is most productive, the whole urban population will be concentrated there.[5] The increasing urban population will continue to work in the traditional sector of region 1 until it becomes profitable to start an industrial sector, which happens at time t_0 when $v(N(t_0)) > \beta_1$. The additional urban population will move in the city in region 1 until this city's population has grown to a level $\overline{n}_1$ at which utility in this city is equal to utility in the region with the second most productive traditional sector, i.e. $v(\overline{n}_1) = \beta_2$. Note that by Result 1 the equation $v(n) = \beta_2$ has two solutions, $\underline{n}_1$ and $\overline{n}_1$, with $\underline{n}_1 < n^{\rm opt} < \overline{n}_1$.

At the point in time where $N(t)$ is marginally higher than $\overline{n}_1$, utility is marginally higher for individuals working in the traditional sector in the unpolluted region 2. Thus, the city stops growing and further urban population will move to the traditional sector of region 2 rather than to the polluted city in region 1, until region 2's population has reached the size $\underline{n}_1$ where an

[5]The assumed productivity differences in the traditional sectors coordinate migration, as workers would be indifferent between regions with identical productivities.

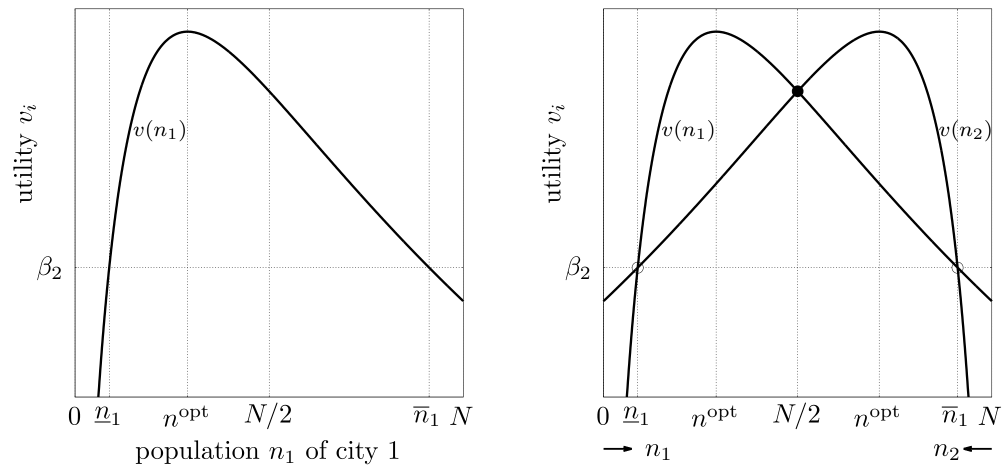

Figure 2: Illustration of the dynamics in the Malthusian phase with one city (left panel) or two cities (right panel). In the right panel, the population of city 1 (city 2) is measured from left to right (from right to left). The filled circle marks a stable equilibrium, while the open circles mark unstable equilibria.

industrial sector becomes profitable. This situtation is illustrated in the left panel of Figure 2. If β_2 is small enough (i.e. $\overline{n}_1$ is large enough) and $\psi < 1$, this new situation is unstable (see Appendix A.3): inhabitants of the large polluted city will move to the emerging clean city, as illustrated in the right panel of Figure 2. This process will increase utility in both cities, but the utility increase is stronger in the smaller city, thus the incentive for migrating there persists. Finally, population will be equally distributed over two cities. Condition (15) guarantees that both of these cities have population sizes larger than n^{opt}, as it implies $\overline{n}_1 + \underline{n}_1 > 2\,n^{\text{opt}}$. The new equilibrium is stable (cf. Result 2).[6]

[6]We can not exclude the possibility that there is another stable equilibrium with one smaller and one larger city. Considering the right panel of Figure 2, this would correspond to a situation where the two curves intersect more than three times. However, with

With further growing population, the two cities will grow until they reach population sizes $\overline{n}_2 < \overline{n}_1$ at which utilities are equal to utility in the region with the third most productive traditional sector, i.e. $v(\overline{n}_2) = \beta_3$. With growing urban population an industrial sector in region 3 will eventually become profitable. Again, this situation is not a stable equilibrium, and individuals will move from the two larger cities to the smaller one until all three cities have reached the same population size, which all are larger than n^{opt} by assumption (15).[7]

The growth pattern continues with m cities as illustrated in Figure 3, panel (a). Between t_1 and t_2, the number m of cities is constant, and utility decreases. At time t_3, the previous urbanization pattern becomes unstable. Let $\underline{n}_m$ and $\overline{n}_m$ be the two solutions of the equation $v(n) = \beta_{m+1}$. Total urban population then is $m\,\overline{n}_m$, and further urban population moves to the traditional sector in region $m+1$. A new city emerges when total population is $N(t) = m\,\overline{n}_m + \underline{n}_m$, as from then onwards the industrial sector in region m becomes more profitable than the traditional sector. Finally a stable urbanization pattern with $m+1$ cities with population sizes larger than n^{opt} will emerge. It will remain stable, until total urban population has reached the size $N(t) = (m+1)\,\underline{n}_{m+1}$, and the increasing urban population would prefer to move to the traditional sector in region $m+2$.

The following result summarizes these results how the number of cities and utility evolve over time during the 'Malthusian' phase.

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increasing $N(t)$, the two curves move apart and eventually the asymmetric equilibrium will disappear, while the symmetric equilibrium still exists and is stable. Thus, an asymmetric equilibrium can only exist for small $N(t)$: as $N(t)$ grows big enough, eventually only the symmetric equilibrium is stable.

[7]Note that this condition implies $2\,\overline{n}_2 > 3\,n^{\text{opt}}$, or equivalently, $v(\frac{3}{2}\,n^{\text{opt}}) > \beta_3$, because $v(\frac{3}{2}\,n^{\text{opt}}) > v(2\,n^{\text{opt}}) > \beta_2 > \beta_3$.

(a) (b)

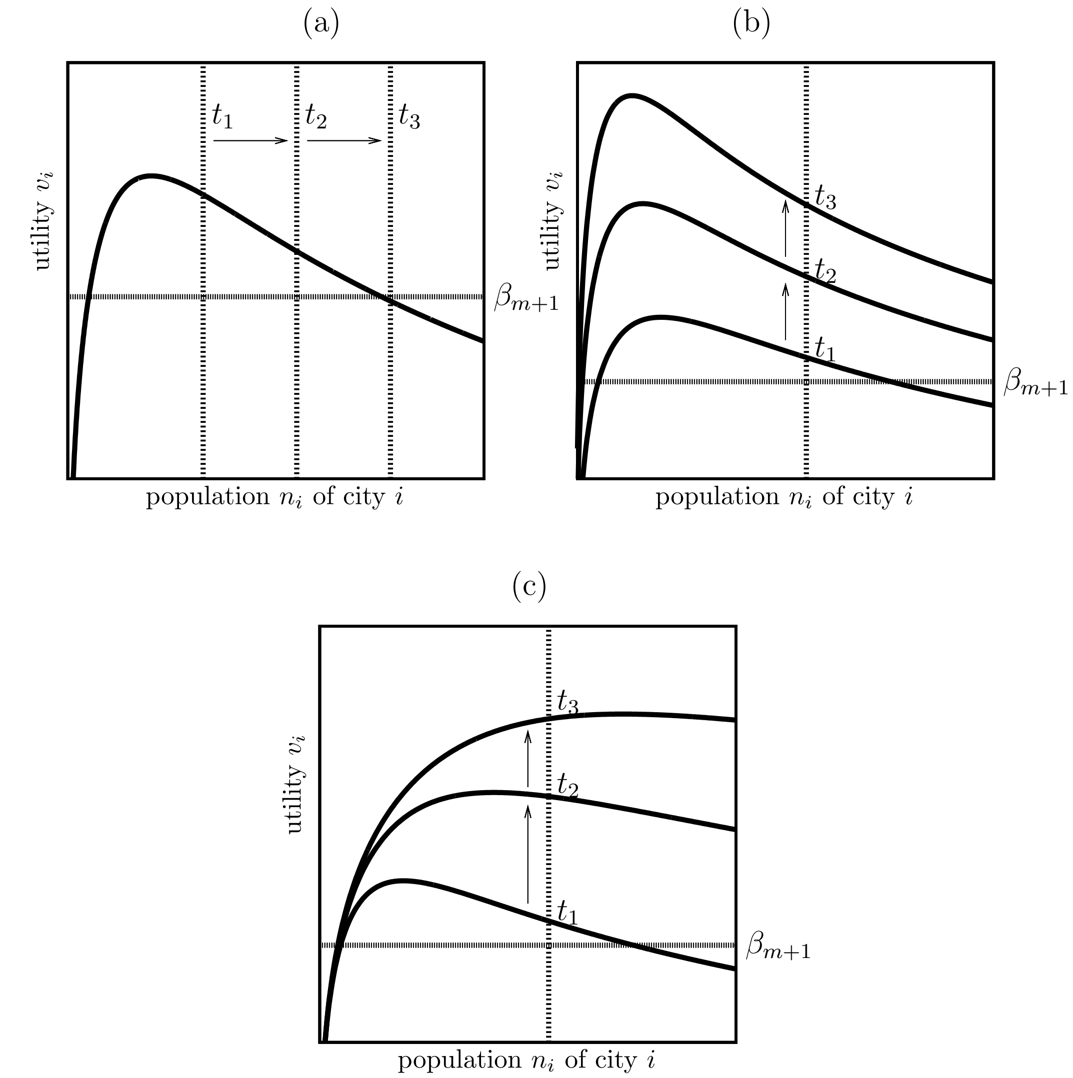

Figure 3: Illustration of the stylized phases of urban growth: (a) Malthusian growth, (b) brown growth, and (c) green growth. The parameter values are as in Figure 1, with $d(t_1) = 1.5$ and $e(t_1) = 1.0$ in all sub-figures. In sub-figure (b), $d(t_2) = 2.0$ and $d(t_3) = 2.5$. In sub-figure (c), $e(t_2)\, d(t_2) = 0.6$, and $e(t_3)\, d(t_3) = 0.3$.

Result 4. *Assume that* (15) *holds. During the 'Malthusian' phase, (i) the number of cities (weakly) increases. (ii) Utility weakly decreases in every existing city,* $\dot{v}_i(n_i) \leq 0$ *for all* i *during phases where the number of cities is constant, while utility increases at points in time when a new city emerges.*

So, if people move one by one (Anas and Xiong 2005 call this "laissez-faire"), with growing urban population the equilibrium number of cities will increase, but stay as small as possible. The lack of coordination in the laissez-faire outcome involves too few cities that are too large each, compared to the Pareto-optimum. This is the result typically found in the literature (e.g. Henderson 1974). We next analyze the effects of economic growth, assuming that urban population is constant.

During the 'brown growth' phase with constant urban population the current urbanization pattern remains stable: as $n^{\text{opt}}(t)$ decreases, $v_i'(n_i) < 0$ remains true. Hence, the stability properties of the urbanization pattern are unchanged. As utility increases there is also no incentive to move to a village, where utility is constant. In conclusion, the number of cities and population of every city are constant. Utility increases with $d(t)$, as $\partial v_i(n_i)/\partial d(t) > 0$ (cf. Equation 10). As the optimal population size decreases (Result 1), the difference between the (constant) actual and the (decreasing) optimal population increases. This argument is illustrated in Figure 3 (b). We summarize these findings as the following result:

Result 5. *During the 'brown growth' phase, (i) the number of cities remains constant, (ii) utility increases in every city,* $\dot{v}_i(n_i) > 0$ *for all* i*, and (iii) the difference between the equilibrium and optimal population increases in every city,* $d/dt\ (n_i - n^{opt}(t)) > 0$.

Economic growth is beneficial even if it causes increasing pollution, as utility increases. Due to increasing pollution and the lack of a coordinating mechanism, however, economic growth is not as beneficial as it potentially could: the difference between the actual and the optimal population size increases over time. Hence, also the 'opportunity cost' of remaining in the stable urbanization pattern increase over time. The picture is quite different in a phase of 'green growth' where pollution per unit of resource input decreases at a sufficiently high rate.

During the 'green growth' phase utility increases due to both increasing efficiency of labor, as human capital per capita increases, and decreasing pollution. The second characteristic of the 'green growth' phase is that optimal population size increases (cf. Result 1). Hence, the current urbanization pattern with m cities remains stable until $N(t)/m < n^{\text{opt}}(t)$. Then, the urbanization pattern is unstable and the number of cities decreases by one, such that the number of cities is $m - 1 = \max\{m \in \mathbb{N} \,|\, N(t)/m \geq n^{\text{opt}}(t)\}$, i.e. the new pattern is a Pareto optimum. The argument is illustrated in Figure 3 (c). From t_1 to t_2, the number of cities is constant, and utility increases. At time t_3, the previous urbanization pattern becomes unstable, and a city vanishes. Thus, we have the following result.

Result 6. *During the 'green growth' phase, (i) the number of cities (weakly) decreases, (ii) utility increases in every city, $\dot{v}_i(n_i) > 0$ for all i, and (iii) the difference between the equilibrium and optimum population decreases in every city, $d/dt\ (n_i - n^{opt}(t)) \leq 0$. If the 'green growth' phase continues long enough, the Pareto-efficient urbanization pattern is reached and sustained.*

The result shows that 'green growth' is beneficial in two respects: first, utility increases. Secondly, the coordination failure present in the phases of 'Malthusian' growth and 'brown growth' disappears and the efficient urbanization pattern is reached and sustained after finite time even in the uncoordinated equilibrium development.

5 Conclusion and discussion

In this paper we have studied the dynamic development of urbanization patterns in a dynamic model of urban growth. Equilibrium and optimal city sizes are determined by the balance between increasing returns to scale as a positive feedback mechanism that favors agglomeration and environmental pollution as a negative feedback mechanism that discourages agglomeration.

Our main result is that cities may be of efficient sizes or inefficiently large in an equilibrium development path, depending on the nature of economic growth. If economic growth is accompanied by increasing urban pollution, cities are inefficiently large and too few in number in the uncoordinated equilibrium. Thus, the equilibrium growth path is not efficient. This result confirms the concern that urbanization patterns may involve inefficiently large cities. The outcome is remarkably different from previous findings in the case of decreasing pollution, i.e. if there is 'green growth': First, our model predicts a *decreasing* number of cities in a growing economy if urban pollution decreases over time. Then, the equilibrium city sizes increase and the economy's population will distribute over a smaller number of cities. In such a situation of 'green growth' the equilibrium city sizes and number approach the efficient urbanization pattern, i.e. the uncoordinated market equilibrium becomes efficient.

The policy conclusion is that a continuous reduction of polluting by-products of production will ultimately lead to an efficient urbanization pattern. The point in time when the efficient urbanization pattern is reached can be observed, as it is the moment when the number of cities starts to decline. Between the moment when urban pollution starts to decline and the moment when the efficient urbanization pattern is reached there is a time lag, however. Accordingly it may require some degree of patience until under a 'green growth' policy the efficient urbanization pattern is reached.

We have not explicitly considered any other types of congestion, but it is straightforward to generalize the model in this respect. For example, if housing needs space, and production takes place in a central business district to which workers have to commute, aggregate commuting costs would increase with the city's size. In such a setting, the phase of 'green growth' would correspond to an improving transport infrastructure associated with significantly decreasing commuting costs. Ultimately, an efficient urban growth path would be obtained without need for a coordinating mechanism.

A Appendix

A.1 Derivation of Equation (9)

The first-order condition for the problem to maximize v_i over R_i is

$$\frac{dv_i}{dR_i} = (1-\mu_i)\left(\left(d_i(t)\, n_i^{1+\psi_i}\right)^{\mu_i} R_i^{-\mu_i} - 1\right)\left(1 - \frac{e_i(t)\, R_i}{\bar{E}_i}\right)^{\phi} + \left(\left(d_i(t)\, n_i^{1+\psi_i}\right)^{\mu_i} R_i^{1-\mu_i} - R_i\right)\frac{\phi\, e_i(t)}{\bar{E}_i}\left(1 - \frac{e_i(t)\, R_i}{\bar{E}_i}\right)^{\phi-1} = 0.$$

Rearranging leads to (9). Note that (9) has a unique solution, as the right-hand side is monotonically increasing in R_i from zero at $R_i = 0$ to infinity as $R_i \to \bar{E}_i/(e_i(t)\,(1 + \frac{\phi}{1-\mu_i}))$.

A.2 Proof that efficient resource use increases with population

Taking the total differential of equation (9) yields

$$d_i(t)^{\mu_i}\,(1+\psi_i)\,\mu_i\,n_i^{(1+\psi_i)\,\mu_i-1}\,dn_i$$
$$= \left(\frac{\mu_i\,\hat{R}_i^{\mu_i-1}}{1-\mu_i}\,\frac{\bar{E}_i - e_i(t)\,(1+\phi)\,\hat{R}_i}{\bar{E}_i - e_i(t)\,(1+\frac{\phi}{1-\mu_i})\,\hat{R}_i} + \frac{\hat{R}_i^{\mu_i}}{1-\mu_i}\,\frac{e_i(t)\,\phi\,\frac{\mu_i}{1-\mu_i}\,\bar{E}_i}{\left(\bar{E}_i - e_i(t)\,(1+\frac{\phi}{1-\mu_i})\,\hat{R}_i\right)^2} \right) dR_i \tag{A.1}$$

Both the term in front of dn_i on the left hand side and the term in front of dR_i on the right hand side are strictly positive. Hence, $dR_i/dn_i > 0$. For $R_i = 0$, we have $n_i = 0$; for $R_i \to \bar{E}_i/\left(e_i(t)(1+\frac{\phi}{1-\mu_i})\right)$, we have $n_i \to \infty$ from condition (9). A similar argument leads to the conclusion that $\hat{R}_i$ increases if $d_i(t)$ increases, while n_i and $e_i(t)$ remain constant.

A.3 Proof of Result 1

By the envelope theorem, the total derivative of utility with respect to population equals the partial derivative of utility with respect to population. Hence,

$$\frac{dv_i(n_i)}{dn_i} = \frac{\partial v_i(n_i)}{\partial n_i} = -\frac{1}{n_i^2}\,\left(\left(d_i(t)\,n_i^{1+\psi_i}\right)^{\mu_i}\,R_i^{1-\mu_i} - R_i\right)\,\left(1-\frac{e_i(t)\,R_i}{\bar{E}_i}\right)^{\phi}$$
$$+\frac{R_i^{1-\mu_i}}{n_i^2}\,\mu_i\,(1+\psi_i)\,\left(d_i(t)\,n_i^{1+\psi_i}\right)^{\mu_i}\,\left(1-\frac{e_i(t)\,R_i}{\bar{E}_i}\right)^{\phi}$$
$$= -\frac{1}{n_i^2}\,\left((1-\mu_i\,(1+\psi_i))\,\left(d_i(t)\,n_i^{1+\psi_i}\right)^{\mu_i}\,R_i^{1-\mu_i} - R_i\right)\,\left(1-\frac{e_i(t)\,R_i}{\bar{E}_i}\right)^{\phi}$$
$$= \frac{1}{n_i}\left[-1+\frac{\mu\,(1+\psi)\,\left(d(t)\,n_i^{1+\psi}\right)^{\mu}\,R_i^{1-\mu}}{\left(d(t)\,n_i^{1+\psi}\right)^{\mu}\,R_i^{1-\mu} - R_i}\right]\,v(n_i) \tag{A.2}$$

Thus, utility has an interior maximum, if $1 - \mu_i\,(1+\psi_i) = 1 - \mu_i + \frac{1-\mu_i}{\sigma_i - 1} = \frac{1-\mu_i}{\sigma_i-1}\,(\sigma_i - 2) > 0$. Otherwise, utility would be monotonically increasing with n_i due to very strongly increasing returns to scale. Assuming $\sigma_i > 2$, the condition for an interior extremum is

$$(1 - \mu_i\,(1+\psi_i))\,\left(d_i(t)\,n_i^{1+\psi_i}\right)^{\mu_i} = R_i^{\mu_i} \tag{A.3}$$

Using (9) this condition becomes

$$\frac{1-\mu_i\,(1+\psi_i)}{1-\mu_i}\,\frac{\bar{E}_i - e_i(t)\,(1+\phi)\,R_i}{\bar{E}_i - e_i(t)\,(1+\frac{\phi}{1-\mu_i})\,R_i} = 1 \tag{A.4}$$

We rearrange this equation to obtain R_i as follows

$$\begin{aligned}(1-\mu_i\,(1+\psi_i))\,&\left(\bar{E}_i - e_i(t)\,(1+\phi)\,R_i\right) \\ &= (1-\mu_i)\left(\bar{E}_i - e_i(t)\left(1+\frac{\phi}{1-\mu_i}\right)R_i\right)\end{aligned} \tag{A.5}$$

$$\Leftrightarrow \qquad \mu_i\,(\phi + \psi_i + \phi\,\psi_i)\,e_i(t)\,R_i = \mu_i\,\psi_i\,\bar{E}_i \tag{A.6}$$

$$\Leftrightarrow \qquad R_i = \frac{\bar{E}_i}{e_i(t)}\,\frac{\psi_i}{\phi+\psi_i+\phi\,\psi_i} \tag{A.7}$$

Plugging into (A.3), we obtain the result (11).

To show that (11) is a maximum of utility, we consider the second derivative of $v(n_i)$. In the following we use the abbreviation

$$\Gamma \equiv \frac{\mu\,(1+\psi)\,\left(d(t)\,n_i^{1+\psi}\right)^{\mu}\,R_i^{1-\mu}}{\left(d(t)\,n_i^{1+\psi}\right)^{\mu}\,R_i^{1-\mu} - R_i} \tag{A.8}$$

Note that $\Gamma = 1$ for $n_i = n^{\text{opt}}$. Thus, $v'(n_i) = v(n_i)\,(\Gamma - 1)/n_i$ and

$$v''(n_i) = -\frac{v(n_i)\,(\Gamma-1)}{n_i^2} + \frac{v(n_i)\,(\Gamma-1)^2}{n_i^2} + \frac{v(n_i)}{n_i}\,\frac{d\Gamma}{dn_i} \tag{A.9}$$

Using (9), we obtain the following

$$\left(d(t)\, n_i^{1+\psi}\right)^{\mu} R_i^{1-\mu} = \frac{\Gamma\, R_i}{\Gamma - \mu\,(1+\psi)} \tag{A.10}$$

$$\Gamma = (1+\psi)\,\frac{\bar{E} - e(t)\,(1+\phi)\,R_i}{\bar{E} - e(t)\,R_i} \tag{A.11}$$

$$R_i = \frac{\bar{E}}{e(t)}\,\frac{1+\psi-\Gamma}{(1+\psi)\,(1+\phi)-\Gamma} \tag{A.12}$$

For $n_i \to 0$, we have $R_i = 0$, thus from (A.11) that $\Gamma = 1+\psi > 1$. For $n_i \to \infty$, we have from (A.8) that $\Gamma = \mu\,(1+\psi) < 1$.

The derivative of Γ with respect to n_i is

$$\frac{d\Gamma}{dn_i} = -\frac{(1+\psi)\,e(t)\,\phi\,\bar{E}}{(\bar{E} - e(r)\,R_i)^2} R_i'(n_i) \tag{A.13}$$

Using this in (A.9) shows that $v''(n^{\mathrm{opt}}) < 0$. Further, from equation (9),

$$n_i = \left(\frac{R_i}{d(t)}\right)^{\frac{1}{1+\psi}} \left(\frac{1}{1-\mu}\,\frac{\bar{E} - e(t)\,(1+\phi)\,R_i}{\bar{E} - e(t)\,\left(1+\frac{\phi}{1-\mu}\right)\,R_i}\right)^{\frac{1}{\mu\,(1+\psi)}}$$

Thus,

$$\begin{aligned} v(n_i) &= \frac{R_i}{n_i}\,\frac{\mu\,\bar{E}^{-\phi}}{1-\mu}\,\frac{\left(\bar{E} - e(t)\,R_i\right)^{1+\phi}}{\bar{E} - e(t)\,\left(1+\frac{\phi}{1-\mu}\right)\,R_i} \\ &= \frac{\mu\,(1-\mu)^{\frac{1-\mu\,(1+\psi)}{\mu\,(1+\psi)}}\,d(t)^{\frac{1}{1+\psi}}}{\bar{E}^{\phi}}\,R_i^{\frac{\psi}{1+\psi}}\,\frac{\left(\bar{E} - e(t)\,\left(1+\frac{\phi}{1-\mu}\right)\,R_i\right)^{\frac{1-\mu\,(1+\psi)}{\mu\,(1+\psi)}}\,\left(\bar{E} - e(t)\,R_i\right)^{1+\phi}}{\left(\bar{E} - e(t)\,(1+\phi)\,R_i\right)^{\frac{1}{\mu\,(1+\psi)}}}. \end{aligned}$$

This shows that $v(0) = 0$ and $\lim_{n_i\to\infty} v(n_i) = 0$. Similarly,

$$v'(n_i) = \frac{v(n_i)\,(1-\Gamma)}{n_i}$$

$$= (1-\Gamma)\,\frac{\mu\,(1-\mu)^{\frac{2-\mu\,(1+\psi)}{\mu\,(1+\psi)}}\,d(t)^{\frac{2}{1+\psi}}}{\bar{E}^{\phi}}\,R_i^{\frac{\psi-1}{\psi+1}}\,\frac{\left(\bar{E} - e(t)\,\left(1+\frac{\phi}{1-\mu}\right)\,R_i\right)^{\frac{2-\mu\,(1+\psi)}{\mu\,(1+\psi)}}\,\left(\bar{E} - e(t)\,R_i\right)^{1+\phi}}{\left(\bar{E} - e(t)\,(1+\phi)\,R_i\right)^{\frac{2}{\mu\,(1+\psi)}}}$$

Thus, $\lim_{n_i\to 0} v'(n_i) = \infty$ for $\psi < 1$ and $\lim_{n_i\to\infty} v'(n_i) = 0$.

References

Anas, A., Xiong, K., 2005. The formation and growth of specialized cities: efficiency without developers or malthusian traps. Regional Science and Urban Economics 35, 445–470.

Andreoni, J., Levinson, A., 2001. The simple analytics of the environmental kuznets curve. Journal of Public Economics 80, 269–286.

Au, C.-C., Henderson, J. V., 2006. Are chinese cities too small? Review of Economic Studies 73 (3), 549 – 576.

Black, D., Henderson, J. V., 1999. A theory of urban growth. Journal of Political Economy 107 (2), 252–284.

Egli, H., Steger, T. M., 2007. A dynamic model of the environmental kuznets curve: Turning point and public policy. Environmental and Resource Economics 36 (1), 15–34.

Gaigné, C., Riou, S., Thisse, J.-F., 2012. Are compact cities environmentally friendly? Journal of Urban Economics 72 (2-3), 123–136.

Grossman, G., Krueger, A., 1995. Economic growth and the environment. Quarterly Journal of Economics 110 (2), 353–377.

Helsley, R. W., Strange, W. C., 1997. Limited developers. Canadian Journal of Economics 30 (3), 329–348.

Henderson, J. V., 1974. The sizes and types of cities. American Economic Review 64 (4), 640–656.

Henderson, J. V., 1977. Externalities in a spatial context: The case of air pollution. Journal of Public Economics 7, 89–110.

Henderson, J. V., 1988. Urban Development: Theory, Fact, and Illusion. Oxford University Press, Oxford.

Henderson, J. V., 2002a. Urban primacy, external costs, and quality of life. Resource and Energy Economics 24, 95–106.

Henderson, J. V., 2002b. Urbanization in developing countries. The World Bank Research Observer 17 (1), 89–112.

Henderson, J. V., Venables, A. J., 2009. The dynamics of city formation. Review of Economic Dynamics 12 (2), 233 – 254.

Kahn, M. E., 2006. Green Cities: Urban growth and the environment. Brookings Institution Press, Washington, D.C.

Kahn, M. E., Schwartz, J., 2008. Urban air pollution progress despite sprawl: The 'greening' of the vehicle fleet. Journal of Urban Economics 63 (3), 775 – 787.

Lieb, C. M., 2002. The environmental kuznets curve and satiation: A simple static model. Environment and Development Economics 7 (3), 429–448.

Lieb, C. M., 2003. The environmental kuznets curve: a survey of the empirical literature and of possible causes. University of Heidelberg, Department of Economics, discussion paper 391.

Muller, N., Mendelsohn, R., 2009. Efficient pollution regulation: Getting the prices right. American Economic Review 99 (5), 1714–1739.

Plassmann, F., Khanna, N., 2006. A note on 'the simple analytics of the environmental kuznets curve'. Environment and Development Economics 11 (6), p697–707.

Rossi-Hansberg, E. A., Wright, M. L., 2007. Urban structure and growth. Review of Economic Studies 74, 597–624.

Shah, J. J., Nagpal, T., Dec. 1997. Urban air quality management strategy in asia - greater mumbai report. World Bank Technical Paper 381, World Bank, Washington.

Stern, D., 2004. The rise and fall of the environmental kuznets curve. World Development 32, 1419–1439.

Trolley, G., 1979. Urban Growth in a Market Economy. Academic Press, New York.

UN-Habitat, 2003. The challenge of slums: Global Report on Human Settlements. United Nations Human Settlements Programme, Nairobi.

UNFPA, 2001. Footprints and Milestones - Population and Environmental Change. United Nations Population Fund.