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Ben J. Heijdra
Laurie S. M. Reijnders

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Abstract

How do population ageing shocks affect the long-run macroeconomic performance of an economy? To answer this question we build a general equilibrium overlapping generations model of a closed economy featuring endogenous factor prices. Finitely-lived individuals are endowed with perfect foresight and make optimal choices over the life cycle. In addition to selecting age profiles for consumption and the hours of time supplied to the labour market, they also choose their schooling level and retirement age. Human capital is accumulated as a result of work experience, the extent of which is determined by the intensity of labour supply. As the agent gets older, biological deterioration sets in and human capital depreciates at an increasing rate. This ultimately prompts the agent to withdraw from the labour market. The microeconomic and macroeconomic effects of three ageing shocks are studied, namely a *biological longevity boost*, a *comprehensive longevity boost*, and a *baby bust*. Robustness checks are performed by allowing for capital market imperfections and indivisibility of labour supply.

JEL-Code: E100, D910, O400, F410, J110.

Keywords: human capital, education, experience effects, borrowing constraints, indivisible labour, retirement, overlapping generations, demography.

Ben J. Heijdra
Faculty of Economics and Business
University of Groningen
9700 AV Groningen / The Netherlands
b.j.heijdra@rug.nl

Laurie S. M. Reijnders
Faculty of Economics and Business
University of Groningen
9700 AV Groningen / The Netherlands
l.s.m.reijnders@rug.nl

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1 Introduction

Over the last half century most advanced economies have experienced substantial demographic changes leading to an overall ageing of the population. Table 1 provides some data for the nine economically most important OECD economies. For each country the first row gives the crude birth rate expressed as a percentage of the total population. In the immediate post-war period these economies typically had relatively high fertility rates with Canada even realizing a birth rate of almost 3 percent in the period 1950-55. After the 1960s, however, a downward trend in the crude birth rate is clearly visible in many of these countries. The most recent figures show that the fertility rate ranges between 0.8 and 1.4.

In addition to this baby bust, these same countries have also experienced an increase in life expectancy. For each country the second row gives the data for life expectancy at birth. The increase in biological longevity is nothing short of spectacular. Whereas in the 1950s life expectancy in all but one of the OECD countries was below 70 years, at the end of the sample period it ranges between 78 and 82.7 years. So in the interval of six decades advances in medical technology and increased hygiene have managed to increase individuals' length of life by roughly ten years.

The combination of a falling birth rate and increased longevity has led to an ageing of the population. The third row for each country provides a simple measure for the age composition of the population. This is the old-age dependency ratio which represents the fraction of old people (65 years and older) to people of working age (ages between 20 and 64). Whereas this ratio ranged between 10 to 14 percent for most countries in the early 1950s, at the start of the twenty-first century it lies between 20.5 and 32.5 percent.

The objective of this paper is to study the macroeconomic general equilibrium effects of the demographic shocks mentioned above. Since these shocks affect the richest countries in the world in a more or less synchronized fashion we adopt a closed economy perspective, i.e. factor price changes form an important mechanism by which ageing affects the macroeconomic outcome.

To capture the notion of biological longevity we formulate an overlapping generations model populated by finitely-lived, utility-maximizing individuals who are blessed with perfect foresight. We abstract from mortality risk and use a generalized version of the classic "certain death" model of Cass and Yaari (1967). We extend their model *inter alia* by assuming that individual agents accumulate both physical and human capital. In the tradition of the recent human capital literature we assume that individuals engage in full-time educational activities at the start of economic life and choose the optimal age of labour market entry. Incorporating the insights of the empirical labour economics literature on Mincerian wage equations, we assume that participating in the labour market boosts a worker's human capital stock and wage income as valuable experience is gained. As a worker gets older, economic ageing sets in because human capital starts to depreciate at an increasing rate. This

Table 1: Demographic indicators for large OECD countries‡

	1950	1960	1970	1980	1990	2000	2005
United States	2.4	2.2	1.6	1.5	1.5	1.4	1.1
	68.6	70.2	71.5	74.5	75.6	77.2	78.0
	14.3	17.5	18.7	19.8	21.3	21.0	20.5
Japan	2.4	1.7	1.9	1.3	1.0	0.9	0.9
	62.2	69.0	73.1	77.0	79.5	81.8	82.7
	10.0	10.6	11.7	15.0	19.4	27.6	32.5
Germany	1.6	1.8	1.1	1.1	1.0	0.9	0.8
	69.5	70.3	71.0	73.8	76.0	78.7	79.9
	16.2	19.2	24.3	27.2	23.5	26.1	31.4
France	1.9	1.8	1.6	1.4	1.3	1.3	1.3
	67.3	70.8	72.4	74.8	77.4	79.6	81.0
	19.5	20.8	23.8	25.0	24.0	27.5	28.0
United Kingdom	1.5	1.8	1.4	1.3	1.3	1.1	1.2
	69.3	71.0	72.2	74.1	76.2	78.4	79.6
	17.9	20.2	23.3	26.8	26.9	26.8	26.9
Italy	1.8	1.8	1.6	1.1	1.0	0.9	0.9
	66.3	69.6	72.1	74.8	77.4	80.2	81.4
	14.3	16.4	19.4	23.7	24.5	29.4	32.0
Canada	2.8	2.5	1.6	1.5	1.4	1.1	1.1
	69.0	71.3	73.0	75.8	77.8	79.7	80.5
	14.0	14.6	15.1	16.2	18.5	20.4	20.9
Australia	2.3	2.2	2.0	5.5	1.5	1.3	1.4
	69.3	70.9	71.7	75.1	77.5	80.3	81.4
	14.0	15.9	15.5	17.1	19.1	20.8	21.4
Netherlands	2.2	2.1	1.5	1.2	1.3	1.2	1.1
	71.9	73.5	74.1	76.1	77.3	78.7	80.2
	14.0	16.8	18.8	20.0	20.8	21.9	22.8

‡Source: United Nations (2011). *World Population Prospects: The 2010 Revision*, Files 3, 5-1, and 5-3B. First row: crude birth rate (births per 100 population). Second row: life expectancy at birth. Third row: old-age dependency ratio (population of 65^+ as a fraction of population $20-64$. Rows 1–2 are averages over the 5-year interval starting in year as indicated. Row 3 is observation in the indicated year.

prompt the rational individual to ultimately retire from the labour force.

The microeconomic and macroeconomic effects of three stylized demographic shocks are studied. The first shock is a *biological longevity boost*, consisting of a substantial increase in the length of life accompanied by a constant population growth rate and an unchanged human capital depreciation schedule. The second shock includes both economic and biological ageing and is called a *comprehensive longevity boost*. This case combines the biological longevity boost with an increase in economic longevity which we model as an outward shift of the depreciation schedule for human capital. Finally, the third shock is a pure *baby bust* scenario, in which the crude birth falls but the length of biological and economic life both remain unchanged.

Our main findings are as follows. As a result of a biological longevity boost, agents respond to the longer lifetime by choosing slightly more schooling, retiring somewhat later, and marginally increasing the length of the work career. Almost all of the additional years of life are consumed in the form of leisure as the retirement period increases substantially. Matters are quite different under a comprehensive longevity boost. In this case the stocks of physical and human capital both rise sharply, with the latter increase dominating the former. Schooling is increased only slightly, but as retirement takes place much later on in life, the length of the work career rises dramatically. Surprisingly, the optimal retirement period is decreased under this shock, i.e. the additional years of biological life are not spent on leisure but on working. Finally, a baby bust shock results in a slight increase in the physical capital stock accompanied by a decrease in the stock of human capital and a small reduction in macroeconomic output.

Our paper is intended as a contribution to the extensive literature on the macroeconomics of ageing. One strand of this literature studies ageing shocks using models in which physical capital is the only accumulable production input. For recent examples, see Heijdra and Ligthart (2006), Bloom et al. (2007), Prettner and Canning (2012), and the references accompanying these papers. There is a large diversity of models, depending on *inter alia* the endogeneity or exogeneity of the agent's labour market retirement decision and the inclusion or exclusion of a public pension system. But the typical conclusion that emerges from these physical-capital-only models is that ageing results in an increase in the capital intensity of production accompanied by an increase in real wages and a drop in the real rate of return on physical capital.

A second, more recent strand of the literature adds human capital as an additional accumulable production factor. See, for example Kalamli-Ozcan *et al.* (2000), Heijdra and Romp (2009), and Ludwig *et al.* (2012) and the references therein. One of the key features common to all these models is the endogeneity of the agent's schooling decision. Again many different model varieties exist, depending on the assumptions made regarding retirement and public pension systems. The typical conclusion resulting from these human-plus-physical-capital models is that a longevity shock raises both stocks, but still increases the ratio of physical to human capital employed in production, though by a smaller amount than is the case in models without human capital – see, for example, Kalemli-Ozcan *et al.* (2000, p. 15) and Ludwig *et al.* (2012, p. 106). In contrast, in our model we find that the increase in human capital dominates the boost to physical capital under the comprehensive longevity boost so that ageing leads to a lower wage and a higher real interest rate. By separately distinguishing biological and economic longevity we are thus able to demonstrate that factor prices may move in a direction opposite to the one accepted as conventional wisdom in the existing literature.

Our paper is complementary to the recent study by Ludwig *et al.* (2012). Like them, we model a closed economy featuring exogenous technological progress, and assume that agents have finite lives. Their model includes uninsured individual longevity risk and a public pension system, but excludes an endogenous retirement decision and does not make a distinction between economic and biological longevity. The difference between the two papers is mainly one of focus. Ludwig *et al.* (2012) is a carefully constructed simulation model containing many real-life features that are deemed to aid in the calibration, such as a realistic demographic process, public pensions, and individual longevity risk. In contrast, our paper is much more stylized and studies ageing shocks in the most parsimonious setting possible.

The remainder of this paper is organized as follows. In Section 2 we formulate our base model. Section 3 uses a calibrated version of the model to compute and visualize the microeconomic and macroeconomic effects of stylized shocks in biological and economic longevity and the crude birth rate. Section 4 investigates the robustness of our conclusions by including capital market imperfections in the form of borrowing constraints and labour market imperfections resulting from the indivisibility of labour. Section 5 summarizes and concludes.

2 Model

In this section we develop a dynamic microfounded general equilibrium model of a closed economy featuring exogenous growth. All agents in the economy are endowed with perfect foresight. The population is made up of overlapping generations of finitely-lived agents who make optimal decisions over the life cycle. There is no longevity risk.[1] Firms use stocks

[1]By postulating a rectangular mortality profile we are able to sidestep troublesome issues relating to annuitization that are tangential to the topic of this paper. Indeed, in the presence of longevity risk, economic theory strongly predicts that full annuitization of assets is optimal to an individual. In reality, however, people hardly buy annuities at all – the so-called "annuity puzzle". In the absence of annuities, uninsured agents leave accidental bequests which – in the context of a general equilibrium model – must be redistributed somehow. Ludwig *et al.*, for example, assume that these bequests are transferred in equal amounts to all surviving agents (2012, p. 97). As is shown in Heijdra et al. (2010) and Heijdra and Mierau (2012), however, the specific age-profile of

of physical and human capital in order to produce the homogeneous good. Individuals accumulate the two types of capital by saving, by enjoying formal education at the start of economic life, and by gaining experience on the work floor. In the first two subsections we discuss the optimal decisions made by firms and individuals, and in the final subsection we characterize the steady-state macroeconomic equilibrium.

2.1 Firms

Technology features constant returns to scale, the goods market is perfectly competitive, and we postulate the existence of a single representative firm facing a linear homogeneous production function of the Cobb-Douglas form:

$$Y(t) = \Omega_0 K(t)^{\varepsilon_k} \left[Z(t) HC(t)\right]^{1-\varepsilon_k}, \qquad 0 < \varepsilon_k < 1, \quad \Omega_0 > 0, \tag{1a}$$

where $Y(t)$ is aggregate output, $Z(t) = Ze^{gt}$ represents Harrod-neutral technological progress, g is the exogenous growth rate ($g \geq 0$), and $K(t)$ and $HC(t)$ stand for the inputs of, respectively, physical and human capital. Aggregate profit is given by:

$$\Pi(t) \equiv Y(t) - w(t) HC(t) - (r(t) + \delta_k) K(t),$$

where $r(t)$ is the real interest rate, δ_k is the constant depreciation rate of physical capital, and $w(t)$ is the rental rate on standardized units of human capital. The firm chooses its inputs $K(t)$ and $HC(t)$ in order to maximize profits. This gives the usual marginal productivity conditions:

$$r(t) + \delta_k = \varepsilon_k \Omega_0 \left(\frac{k(t)}{hc(t)}\right)^{-(1-\varepsilon_k)}, \tag{1b}$$

$$\tilde{w}(t) \equiv \frac{w(t)}{Z(t)} = (1-\varepsilon_k)\,\Omega_0 \left(\frac{k(t)}{hc(t)}\right)^{\varepsilon_k}, \tag{1c}$$

where $k(t) \equiv K(t) / [P(t) Z(t)]$, $hc(t) \equiv HC(t) / P(t)$, and $P(t)$ is the total population (see below). Profits are equal to zero as a result of the linear homogeneity of the production technology.

In the remainder of the paper we restrict attention to the balanced growth path for which $hc(t) = hc$, $k(t) = k$, $r(t) = r$, $\tilde{w}(t) \equiv \tilde{w}$, and $y(t) \equiv Y(t) / [P(t) Z(t)] = y$ are all time-invariant, and the rental rate on human capital grows exponentially at the constant rate g, i.e. $w(t) = w(v) e^{g(t-v)}$.

transfers exerts a major influence on allocation and welfare effects.

2.2 Individuals

We follow Cass and Yaari (1967) by assuming that individuals have finite lives but experience no longevity risk. The lifetime utility function of a vintage-v newborn is given by:

$$\Lambda(v) \equiv \int_{v}^{v+D} \Phi(c(v,t), z(v,t)) e^{-\rho(t-v)} dt, \tag{2a}$$

where $\Phi(\cdot)$ is instantaneous utility (or "felicity"), $c(v,t)$ is consumption of a person of vintage v at time t, $z(v,t)$ is leisure, ρ is the pure rate of time preference, and D is the age at which sudden death occurs. The felicity function takes the following convenient form:

$$\Phi(c(v,t), z(v,t)) \equiv \frac{\left(c(v,t)^{1-\varepsilon_z} z(v,t)^{\varepsilon_z}\right)^{1-1/\sigma} - 1}{1 - 1/\sigma}, \tag{2b}$$

with $0 < \varepsilon_z < 1$ and $0 < \sigma < 1$. The parameter σ represents the *intertemporal* substitution elasticity for subfelicity which itself is a Cobb-Douglas function of consumption and leisure, i.e. the *intratemporal* substitution elasticity between consumption and leisure is equal to unity. Although our theoretical expressions are valid for values of σ exceeding unity, we restrict attention in this paper to the empirically plausible case featuring $0 < \sigma < 1$. It is convenient to state the following key properties of the felicity function upfront.

Lemma 1. *(Properties of the felicity function). Let $\Phi(c(v,t), z(v,t))$ be defined as in (2b). The following properties can be established.*

(i) Felicity is non-separable in consumption and leisure in that the marginal felicity of consumption depends on leisure and vice versa:

$$\begin{aligned} \Phi_c(c(v,t), z(v,t)) &\equiv \frac{1-\varepsilon_z}{c(v,t)} \left(c(v,t)^{1-\varepsilon_z} z(v,t)^{\varepsilon_z}\right)^{1-1/\sigma} > 0, \\ \Phi_z(c(v,t), z(v,t)) &\equiv \frac{\varepsilon_z}{z(v,t)} \left(c(v,t)^{1-\varepsilon_z} z(v,t)^{\varepsilon_z}\right)^{1-1/\sigma} > 0; \end{aligned}$$

(ii) Felicity features a negative cross derivative, and consumption and leisure are direct substitutes (Heckman, 1974):

$$\Phi_{cz}(c(v,t), z(v,t)) \equiv -\frac{1-\sigma}{\sigma} \frac{\varepsilon_z}{z(v,t)} \Phi_c(c(v,t), z(v,t)) < 0;$$

(iii) The marginal rate of substitution between consumption and leisure is not affected by the intertemporal substitution elasticity:

$$\frac{\Phi_z(c(v,t), z(v,t))}{\Phi_c(c(v,t), z(v,t))} = \frac{\varepsilon_z}{1-\varepsilon_z} \frac{c(v,t)}{z(v,t)};$$

(iv) The intertemporal substitution elasticity for consumption is given by:

$$\sigma^* \equiv -\frac{\Phi_c(c(v,t),z(v,t))}{c(v,t)\Phi_{cc}(c(v,t),z(v,t))} = \frac{\sigma}{1-\varepsilon_z(1-\sigma)},$$

where $\sigma < \sigma^* < 1$.

Proof: Straightforward by differentiation. □

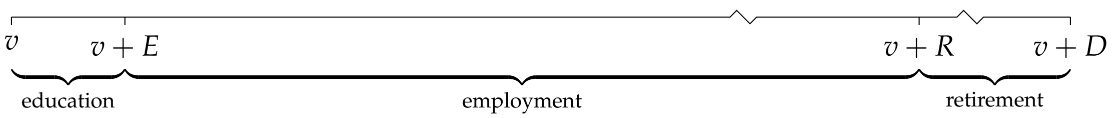
The accumulation of financial assets, $a\,(v,t)$, proceeds according to:

$$\dot{a}(v,t) = ra(v,t) + w\,(t)\,l(v,t)h(v,t) - c(v,t),$$

where $\dot{a}(v,t) \equiv da\,(v,t)\,/dt$ is net asset accumulation, $l\,(v,t)$ is labour supply, and $h\,(v,t)$ is the stock of human capital. In the absence of a bequest motive, individuals are born without any financial assets ($a(v,v) = 0$) and will choose to expire bare of such assets as well ($a(v,v+D) = 0$). It thus follows that in the presence of perfect capital markets the lifetime budget constraint is given by:

$$\int_v^{v+D} w\,(t)\,l(v,t)h(v,t)e^{-r(t-v)}dt = \int_v^{v+D} c(v,t)e^{-r(t-v)}dt. \tag{2c}$$

The accumulation of human capital, $h\,(v,t)$, depends critically on the use of time over the life cycle.[2] The agent's choices of E and R constitute important life-cycle decisions. The time line of a person's life takes the following form.



Throughout life the individual has a time endowment of one unit. At the start of economic life, for ages $0 \leq t-v < E$, the individual does not supply any labour to the market ($l\,(v,t) = 0$) but engages in full-time training which we take to involve e_0 units of time. Startup education and working in the market are thus modeled as mutually exclusive activities.[3] Assuming that the individual retires from the labour force at age R, it follows that

[2]Following the seminal paper by Ben-Porath (1967), a large literature on endogenous human capital formation has emerged. Key contributions are by Rosen (1972), Blinder and Weiss (1976), Heckman (1976), and Driffill (1980).

[3]The assumption of a start-up education phase of endogenous length is a standard one in the macroeconomic literature. See, for example, de la Croix and Licandro (1999), Kalemli-Ozcan *et al.* (2000), Boucekkine *et al.* (2002), and Heijdra and Romp (2009).

leisure is given by:

$$z(v,t) \equiv \begin{cases} 1-e_0 & \text{for } 0 \le t-v < E \\ 1-l(v,t) & \text{for } E \le t-v < R \\ 1 & \text{for } R \le t-v \le D \end{cases} \tag{2d}$$

Depending on the length of the schooling period, the agent enters the labour market with a stock of marketable human capital equal to:

$$h(v, v+E) = e^{G(E)},$$

where $G(E)$ is a function such that there exist positive but diminishing returns to education ($\partial h(v, v+E)/\partial E > 0$ and $\partial^2 h(v, v+E)/\partial E^2 < 0$). See Figure 1(a) for an illustration of the training function that we use in our quantitative analysis below.

Following labour market entry at age $t-v=E$, the individual starts accumulating human according to the following schedule:

$$\dot{h}(v,t) = \left[\gamma l(v,t) - \delta_h(t-v)\right] h(v,t), \qquad \text{for } E \le t-v \le D, \quad \gamma > 0,$$

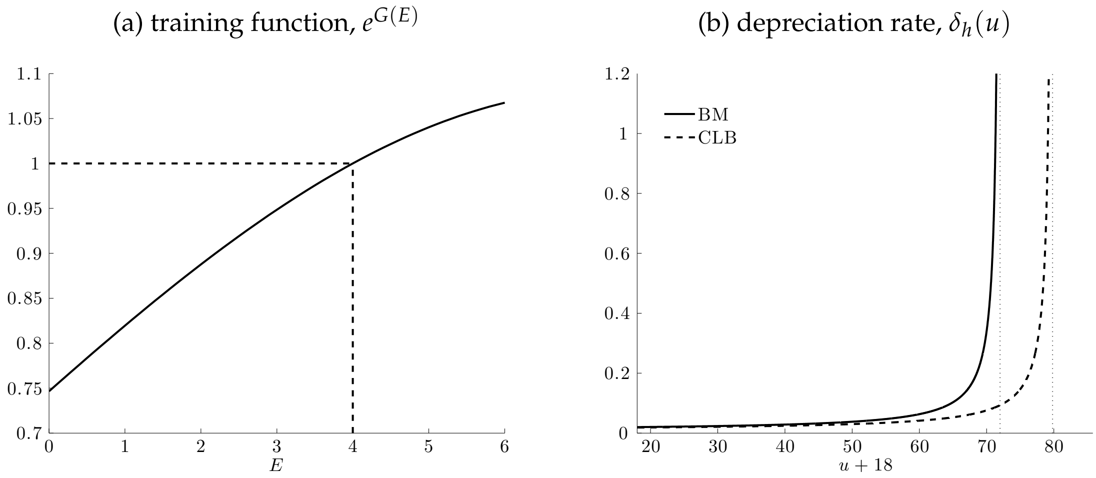
where $\delta_h(t-v)$ is the age-dependent depreciation rate of human capital. The accumulation function combines two distinct mechanisms. First, by working in the market the agent gains experience which we assume to be proportional to the number of hours worked times the available stock of human capital, i.e. a smart worker gains more experience per unit of work time than a less accomplished colleague and the higher is labour supply the larger is the gain per unit of available human capital. The parameter γ regulates the strength of this *experience effect*. The second mechanism captures the *economic ageing effect*. Old age is postulated to be associated with a loss of skills and physical abilities which we model by taking the depreciation rate to be increasing in age ($\delta_h'(t-v) > 0$). We assume that $\lim_{u \to \bar{R}} \delta_h'(s) = +\infty$ for $E < \bar{R} < D$, i.e. the depreciation function features a vertical asymptote at $u = \bar{R}$ which we call the age of economic death.[4] Figure 1(b) depicts the depreciation function we use in our quantitative analysis below.

By solving the differential equation for human capital subject to the initial condition we find:

$$h(v,t) = e^{G(E) + \int_{v+E}^{t} [\gamma l(v,\tau) - \delta_h(\tau - v)] d\tau}, \tag{2e}$$

[4] If economic and biological death were to coincide, then the agent's incentives to retire are typically not strong enough to lead to withdrawal from the labour market.

Figure 1: Features of the human capital accumulation process‡



‡ For the training function we postulate $G(E) = \beta_0 + \beta_1 E - \beta_2 E^2$ whilst for the human capital depreciation function we use $\delta_h(u) = \delta_0 + \frac{1}{\delta_1(\bar{R}-u)}$. The choice of $\bar{R}$ and the β and δ parameters is explained in the parameterization section below.

and thus:

$$\frac{\partial h(v,t)}{\partial l(v,\tau)} = \begin{cases} \gamma h(v,t) & \text{for } \tau < t \\ 0 & \text{for } \tau \geq t \end{cases}$$

$$\frac{\partial h(v,t)}{\partial E} = \left[G'(E) - (\gamma l(v,v+E) - \delta_h(E))\right] h(v,v+E)$$

By its very nature, human capital is a predetermined variable in that it is only affected by choices made in the past. This explains why only past labour supply decisions influence the stock at any moment in time. Note furthermore that the proportional effect on the human capital stock of a marginally higher level of education is captured by the difference between the return to education ($G'(E)$) and the net return to job market experience ($\gamma l(v,v+E) - \delta_h(E)$).

The agent chooses the education level E, the retirement age R, and time profiles for consumption $\{c(v,\tau)\}_{\tau=v}^{v+D}$, and labour supply $\{l(v,\tau)\}_{\tau=v}^{v+D}$ in order to maximize lifetime utility (2a) subject to the lifetime budget constraint (2c) and the human capital equation (2e). The first-order condition for consumption during life is given by:

$$\Phi_c(c(v,t),z(v,t)) = \lambda(v)\, e^{-(r-\rho)(t-v)}, \qquad \text{for } 0 \leq t-v \leq D, \tag{3a}$$

where $\lambda(v)$ is the (optimized value of the) Lagrange multiplier for the lifetime budget constraint (2c) representing the marginal utility of wealth. At each moment in time, the agent chooses consumption such that the marginal utility cost (right-hand side of (3a)) is equated to the marginal benefit (left-hand side of (3a)) of additional consumption.

One feature of the optimal consumption path can be deduced already. Since the right-hand side of (3a) is continuous in the consumer's age, it follows from the indivisibility of the schooling decision and the perfect divisibility of labour supply that there generally exists a discrete jump in consumption at age E satisfying:[5][6]

$$\frac{c(v,v+E)}{c(v,v+E^{-})} = \left(\frac{1-e_0}{1-l(v,v+E)}\right)^{\frac{(1-\sigma)\varepsilon_Z}{1-\varepsilon_Z(1-\sigma)}}. \tag{3b}$$

If, for example, the optimally chosen supply of hours at labour market entry exceeds the time requirement of schooling ($l(v,v+E) > e_0$), then it follows from (3b) that consumption features an upward jump at age E ($c(v,v+E) > c(v,v+E^{-})$) which compensates for the downward jump in leisure occurring at that point in the life cycle.

The first-order condition for labour supply during the work career is given by:

$$\Phi_z(c(v,t),z(v,t)) = [w(t)\,h(v,t) + \gamma V_H(v,t)]\,\lambda(v)\,e^{-(r-\rho)(t-v)}, \qquad \text{for } E \le t-v < R, \tag{3c}$$

where $V_H(v,t)$ is the present value of future wage income:

$$V_H(v,t) \equiv \int_t^{v+R} w(\tau)\,l(v,\tau)\,h(v,\tau)\,e^{-r(\tau-t)}d\tau. \tag{3d}$$

Again the optimal decision rule equates marginal costs and benefits of increasing leisure consumption. By working a little less the agent obtains more leisure, the marginal benefits of which are given by the left-hand side of (3c). The marginal cost of leisure is given by the right-hand side of (3c) and consists of forgone direct wage income $w(t)h(v,t)$ (backward-looking) plus the imputed value of forgone human capital $\gamma V_H(v,t)$ (forward-looking), all expressed in utility terms.

The first-order condition for the years of schooling (and age of labour market entry) is

[5]Note that we write $f(x^{-})$ for the left limit of f at x. Hence, $f(x^{-}) \equiv \lim_{z \nearrow x} f(z)$.

[6]Heckman (1974) was the first to stress the importance of non-separable preferences in a life-cycle model with endogenous consumption and labour supply choices.

given by:

$$\begin{aligned}
&\Phi(c(v,v+E^-),1-e_0)-\Phi(c(v,v+E),1-l(v,v+E))\\
&\quad=\lambda\left(v\right)e^{-(r-\rho)E}\Bigg[w\left(v+E\right)l(v,v+E)h(v,v+E)\\
&\qquad-\int_{v+E}^{v+R}w\left(t\right)l(v,t)\frac{\partial h(v,t)}{\partial E}e^{-r(t-v-E)}dt+\Big(c(v,v+E^-)-c(v,v+E)\Big)\Bigg]. \qquad (3e)
\end{aligned}$$

This optimality condition equalizes the marginal gains and losses from additional schooling. The left-hand side represents the change in felicity that occurs the moment the agent leaves school and enters the labour market. The right-hand side states the three components that make up the marginal utility cost of increasing E. The first term in square brackets shows that the agent foregoes wage earnings by staying in school a bit longer. The second term is a negative cost, namely the net return to education relative to gaining experience on the job. The third term gives the effect of the consumption jump at age E.

Finally, the first-order condition for the optimal retirement age is given by:

$$\begin{aligned}
&\Phi(c(v,v+R^-),1-l(v,v+R^-))-\Phi(c(v,v+R),1-l(v,v+R))\\
&\quad=\lambda\left(v\right)e^{-(r-\rho)R}\Bigg[-w\left(v+R^-\right)l(v,v+R^-)l(v,v+R^-)\\
&\qquad+\Big(c(v,v+R^-)-c(v,v+R)\Big)\Bigg]. \qquad (3f)
\end{aligned}$$

The left-hand side gives the change in felicity experienced at labour market exit. The right-hand represents the utility cost of retiring slightly later on in life. The first term in square brackets is a negative cost consisting of the wage income that is earned by staying in the work force somewhat longer. The second term accounts for a possible consumption jump at age R. It follows from (3a) that:

$$\frac{c\left(v,v+R\right)}{c\left(v,v+R^-\right)}=\left(\frac{1-l\left(v,v+R^-\right)}{1-l\left(v,v+R\right)}\right)^{\frac{(1-\sigma)\varepsilon_Z}{1-\varepsilon_Z(1-\sigma)}}. \qquad (3g)$$

Because labour is perfectly divisible and labour supply is zero at retirement ($l\left(v,v+R\right)=0$), it follows that the consumption path is continuous at age R and that $l(v,v+R^-)=0$, i.e. there is no jump in labour supply at retirement either.

We summarize the key microeconomic relationships in panel (a) of Table 2. The micro model is written entirely in the agent's age $u\equiv t-v$. Scaled variables are defined as:

$$\tilde{c}\left(u\right)\equiv\frac{c\left(v,v+u\right)}{Z\left(v\right)},\qquad\tilde{V}_H\left(u\right)\equiv\frac{V_H\left(v,v+u\right)}{Z\left(v\right)},\qquad\mu\equiv\frac{1}{Z(v)}\left(\frac{\varepsilon_c}{\lambda(v)}\right)^{\sigma^*},$$

Table 2: Model equations[‡]

(a) Microeconomic relationships:

$$\tilde{c}\,(u) = \mu z\,(u)^{\sigma^* \varepsilon_z (1-1/\sigma)}\, e^{\sigma^*(r-\rho)u}, \quad 0 \leq u \leq D \tag{T1.1}$$

$$l(u) = 1 - \frac{\varepsilon_z}{1-\varepsilon_z} \frac{\tilde{c}(u)}{\tilde{w} e^{gu} h(u) + \gamma \tilde{V}_H(u)}, \quad E \leq u < R \tag{T1.2}$$

$$z\,(u) \equiv \begin{cases} 1 - e_0 & \text{for } 0 \leq u < E \\ 1 - l(u) & \text{for } E \leq u < R \\ 1 & \text{for } R \leq u \leq D \end{cases} \tag{T1.3}$$

$$h(u) = e^{G(E)} e^{\int_E^u [\gamma l(s) - \delta_h(s)] ds}, \quad E \leq u < \bar{R} \tag{T1.4}$$

$$\tilde{V}_H(u) = \tilde{w}\, e^{gu} \int_u^R l(s) h(s) e^{-(r-g)(s-u)} ds, \qquad E \leq u < R \tag{T1.5}$$

$$e^{-rE} \tilde{V}_H\,(E) = \int_0^D \tilde{c}\,(u)\, e^{-ru} du \tag{T1.6}$$

$$\tilde{w} e^{gE} l(E) h(E) = \frac{\tilde{c}(E)}{\sigma^*(1-\varepsilon_z)} \frac{\left(\frac{1-e_0}{1-l(E)}\right)^{\sigma^* \varepsilon_z (1-1/\sigma)} - 1}{1 - 1/\sigma} + \left[G'(E) - \gamma l(E) + \delta_h(E)\right] \tilde{V}_H(E) \tag{T1.7}$$

(b) Macroeconomic relationships:

$$c = \frac{1}{\Delta\,(n, D)} \int_0^D \tilde{c}\,(u)\, e^{-(n+g)u} du \tag{T1.8}$$

$$hc \equiv \frac{1}{\Delta\,(n, D)} \int_E^R l\,(u)\, h\,(u)\, e^{-nu} du \tag{T1.9}$$

$$c = (r - g - n)\, k + \tilde{w}\, hc \tag{T1.10}$$

$$\tilde{w} = (1 - \varepsilon_k)\, \Omega_0 \left(\frac{k}{hc}\right)^{\varepsilon_k} \tag{T1.11}$$

$$r + \delta_k = \varepsilon_k \Omega_0 \left(\frac{k}{hc}\right)^{-(1-\varepsilon_k)} \tag{T1.12}$$

[‡]The properties of $\Delta(n, D)$ are covered in Lemma 2. Redefined variables: $\tilde{c}(u) \equiv c(v,t)/Z(v)$, $\tilde{V}_H(u) \equiv V_H(v,t)/Z(v)$, $z(u) \equiv z(v, v{+}u)$, $l(u) \equiv l(v, v{+}u)$, $h(u) \equiv h(v, v{+}u)$, and $\mu \equiv Z(v)^{-1}((1-\varepsilon_z)/\lambda(v))^{\sigma^*}$. Auxiliary parameter: $\sigma^* \equiv \frac{\sigma}{1-\varepsilon_z(1-\sigma)}$.

and with a slight abuse of notation we write:

$$l(u) \equiv l(v, v+u), \qquad h(u) \equiv h(v, v+u), \qquad z(u) \equiv c(v, v+u).$$

By virtue of the structure of the felicity function, the microeconomic relationships provide time-invariant solutions for the life-cycle dates E and R, and age-dependent solutions for scaled consumption, $\tilde{c}(u)$, the scaled value of the stock of human capital, $\tilde{V}_H(u)$, labour supply, $l(u)$, and the stock of human capital, $h(u)$.

Equation (T1.1) expresses the optimal path for consumption over the life cycle conditional on a time-invariant term involving the transformed marginal utility of wealth (μ), the amount of leisure consumption ($z(u)$), and an exponential term ($e^{\sigma^*(r-\rho)u}$). It is obtained by using (2b) and (3a). Equation (T1.2) gives the expression for optimal labour supply over the life cycle. It is obtained by combining (3a) and (3c), using (2b), and noting that $w(t) = w(v)\,e^{g(t-v)}$ and $w(t) = \tilde{w}Z(t)$. Equations (T1.3)–(T1.6) are rewritten versions of, respectively, (2d), (2e), (3d), and (2c). Finally, equation (T1.7) gives the implicit function determining the optimal amount of schooling. It is obtained by substituting (3a)–(3b) and the expression for $\partial h(v,t)/\partial E$ in (3e) and rewriting.

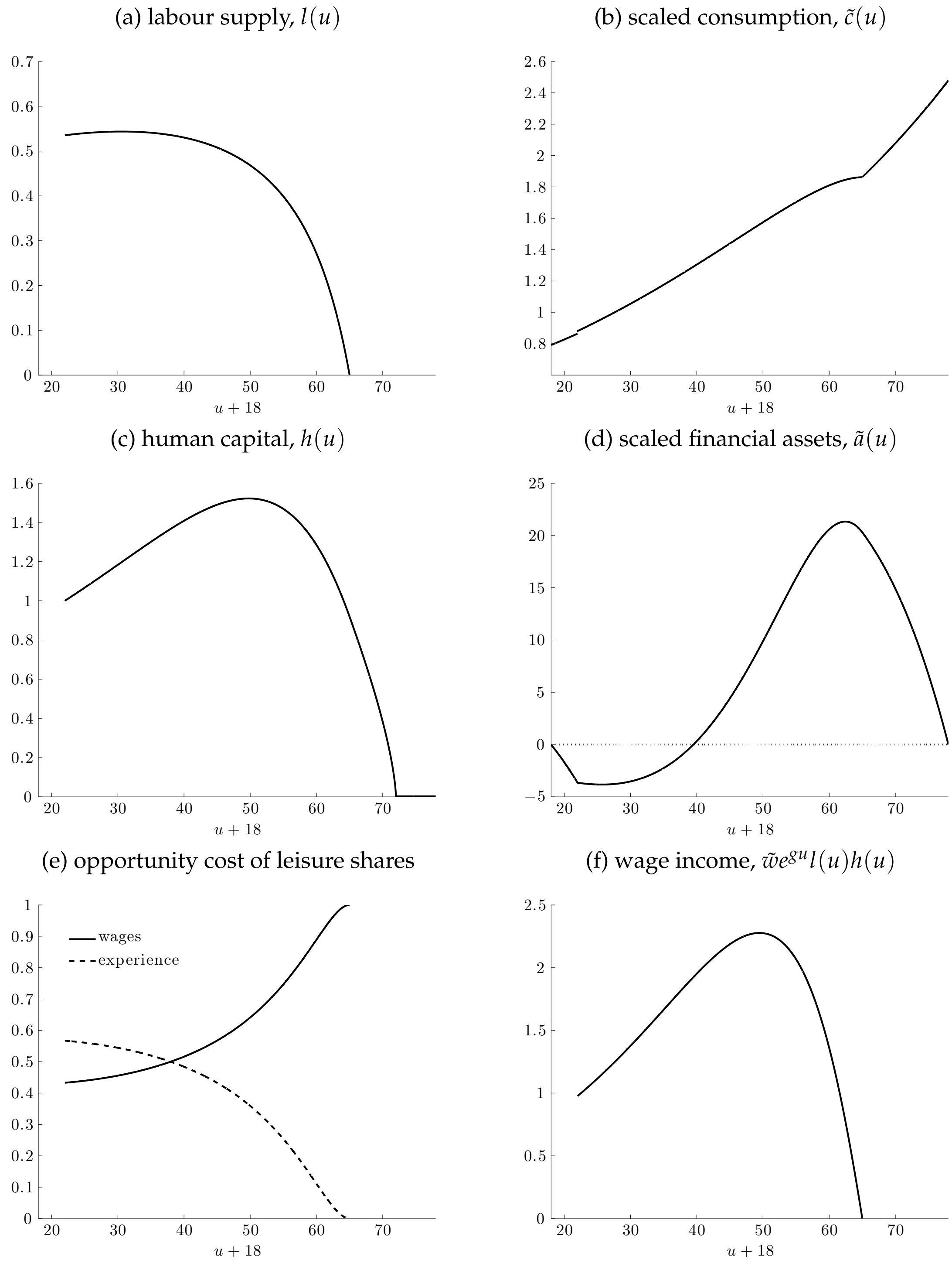
Anticipating our quantitative results, we visualize the individual life-cycle profiles for the most important variables in Figure 2. To facilitate interpretation, biological age $u+18$ rather than economic age $u \equiv t - v$ is plotted along the horizontal axes. Panel (a) depicts the calibrated life-cycle profile of labour supply. Labour market entry occurs at age 22 whilst retirement takes place at age 65. Labour supply is fairly constant for much of the work period but after age 60 the agent quickly reduces working hours and retires. Panel (e) depicts the changing role of explicit and implicit remuneration of labour over the life cycle. A young individual is willing to work for relatively low wages because a large part of the work remuneration accrues in the form of experience effects. The opposite holds for an old individual, who has little use for building up experience due to the brief remaining lifetime and instead is affected principally by wages in his/her labour supply decision.[7]

Panel (b) depicts the life-cycle path for scaled consumption. During the schooling and retirement periods leisure is constant and it follows from (T1.1) that consumption grows at an exponential rate $\sigma^*(r-\rho)$ which is positive in the parameterization (see Table 2). During the employment period the time change in leisure augments the growth rate of consumption somewhat. Consistent with (3b) there is a (small) upward jump in consumption at labour market entry.

Panel (c) illustrates the optimal life-cycle path of human capital. The parameterization procedure ensures that the stock of human capital at labour market entry equals unity. During much of working life experience effects are strong and depreciation is low, resulting in a sharply increasing profile for human capital. The peak is reached around the age of 53 after

[7] Imai and Keane (2004, p. 602) find similar effects in their stochastic partial equilibrium simulation model.

Figure 2: Life-cycle profiles



which economic ageing in the form of increasing depreciation starts to dominate resulting in the depletion of the stock at the calibrated age of 72. Of course, as panel (a) shows, the agent has long since left the labour force at that age.

Panel (d) depicts the other stock that an individual accumulates over the life cycle, namely financial assets. Note that the asset path supporting (and originating from) an individual's optimal choices can be written as follows:

$$\tilde{a}(u) \equiv \frac{a(v, v+u)}{Z(v)} = \int_0^u \left[\tilde{w} e^{gs} l(s) h(s) - \tilde{c}(s)\right] e^{r(u-s)} ds.$$

As the diagram shows, assets are zero at the beginning and end of life and the agent is a net debtor until about age 40. Assets peak just before withdrawing from the labour market and are gradually decumulated during the retirement phase.

2.3 Demography and macroeconomic equilibrium

We allow for a positive growth rate of the population, which we denote by n, and assume that the economy is in a demographic steady state. Agents are fertile for ages $0 \leq u < B < D$ and the *net* birth rate b is age-independent. The population level at time t is $P(t) = P(v) e^{n(t-v)}$ and the size of the new cohort born at time v is given by $P(v, v) = b \int_{v-B}^{v} P(s, v) \, ds$. Since $P(s, v) = P(v, v)$ for $0 \leq v - s \leq D$ and $P(t) \equiv \int_{t-D}^{t} P(v, v) \, dv$ it follows that:

$$1 = b\Delta(n, B), \tag{4a}$$

where $\Delta(\cdot)$ is a two-parameter "demographic" function implied by the postulated birth and mortality processes. Since this function is quite useful we summarize its properties in the following Lemma.

Lemma 2. *(Properties of the $\Delta(\theta, T)$ function). Define the function (for $0 < T \ll \infty$):*

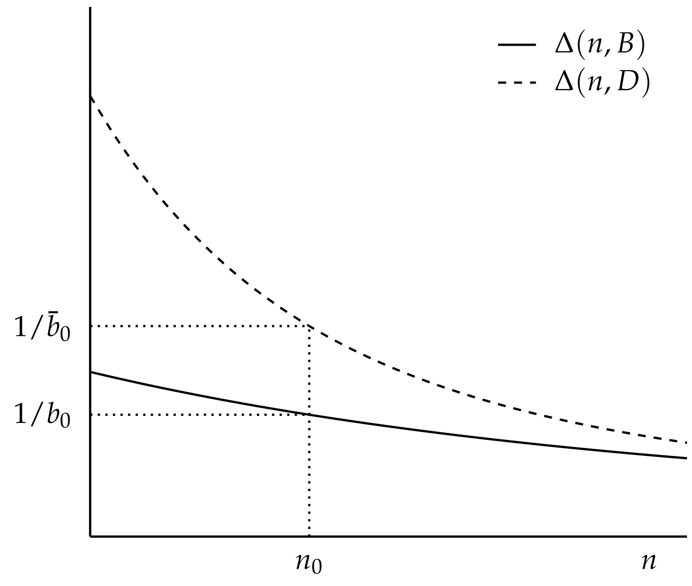
$$\Delta(\theta, T) \equiv \begin{cases} \dfrac{1 - e^{-\theta T}}{\theta} & \text{for } \theta \neq 0 \\ T & \text{for } \theta = 0 \end{cases}$$

Then the following properties can be established: (i) Positive, $\Delta(\theta, T) > 0$; (ii) Decreasing in θ, $\Delta_\theta(\theta, T) \equiv \partial \Delta(\theta, T) / \partial \theta < 0$; (iii) Increasing in T, $\Delta_T(\theta, T) \equiv \partial \Delta(\theta, T) / \partial T > 0$.

Proof: Straightforward by differentiation and application of L'Hôpital's Rule. □

Equation (4a) is the demographic equilibrium condition providing an implicit relationship between the net birth rate b, the growth rate of the population n, and the maximum

Figure 3: Demographic equilibrium



fertile age B. Once two of these parameters have been set, the third is implied by (4a). In Figure 3, the solid line depicts $\Delta(n, B)$. By selecting a particular value for the net birth rate, say $b = b_0$, the equilibrium population growth rate n_0 is obtained.

The age composition of the population is derived as follows. Absolute cohort sizes satisfy $P(v,t) = P(v,v)$ for $v \le t \le v + D$ and $P(v,t) = 0$ for $t > v + D$ and $P(t,t) = P(v,v)e^{n(t-v)}$ we easily find that relative cohort sizes in the age domain are given by:

$$p(v, v+u) \equiv \begin{cases} \dfrac{e^{-nu}}{\Delta(n,D)} & \text{for } 0 \le u \le D \\ 0 & \text{for } u > D \end{cases} \tag{4b}$$

where $\bar{b} \equiv P(t,t)/P(t) = 1/\Delta(n,D)$ represents the *crude* birth rate. In Figure 3 the equilibrium crude birth rate associated with the net rate b_0 is given by $\bar{b}_0$. The relative size of a cohort falls with its age not because of ongoing mortality but rather because the population as a whole is growing over time.

Armed with the relative cohort weights (4b) it is possible to derive the key macroeconomic relationships – see equations (T1.8)–(T1.12) in Table 2. Equation (T1.8) is obtained as follows. First, note that total consumption is defined as $C(t) \equiv P(t) \int_{t-D}^{t} c(v,t)\, p(v,t)\, dv$, where we have used the fact that $P(v,t) \equiv P(t)\, p(v,t)$. By exploiting the stationarity property of scaled consumption, $\tilde{c}(t-v)$, and substituting the balanced growth path for productivity, $Z(t) = Z(v)\, e^{g(t-v)}$, as well as (4a)–(4b) we obtain:

$$c(t) \equiv \frac{C(t)}{P(t)\, Z(t)} = \frac{1}{\Delta(n, D)} \int_0^D \tilde{c}(s) e^{-(n+g)s} ds, \tag{4c}$$

where $c(t)$ denotes consumption in terms of raw efficiency units. Since the right-hand side of (4c) is independent of the generations index v, it follows that $c(t)$ is time-invariant, as is indicated in (T1.8).

Equation (T1.9) is obtained by noting that $HC(t) \equiv P(t) \int_{t-R}^{t-E} l(v,t)\, h(v,t)\, p(v,t)\, dv$ and recalling that $l(v,t)$, $h(v,t)$, and $p(v,t)$ depend only on age. Finally, equation (T1.10) follows in a straightforward fashion from the aggregate asset accumulation equation,

$$\dot{A}(t) = rA(t) + w(t)\, HC(t) - C(t), \tag{4d}$$

in combination with the capital market equilibrium condition $A(t) = K(t)$, where $A(t) \equiv P(t) \int_{t-D}^{t} a(v,t)\, p(v,t)\, dv$, and the steady-state condition $\dot{k}(t) / k(t) = \dot{K}(t) / K(t) - (g+n) = 0$.

The model features a two-way interaction between the microeconomic and macroeconomic parts. For given values of the macroeconomic equilibrium factor prices ($\tilde{w}$ and r), the microeconomic part determines optimal values for $\tilde{c}(u)$, $\tilde{V}_H(u)$, $l(u)$, $h(u)$, E, and R. Using these micro variables in the macroeconomic part gives general equilibrium solutions for c, hc, k, $\tilde{w}$, and r. The functional form of the felicity function (2b) satisfies the King-Plosser-Rebelo conditions ensuring that income and substitution effects on labour supply of changes in $\tilde{w}$ exactly cancel out (King et al., 2002). In the context of the model studied here, the following proposition can be establised.

Proposition 1. *Optimal choices for E, R, $l(u)$, and $h(u)$ are independent of the rental rate on human capital, $\tilde{w}$. Optimal choices for $\tilde{c}(u)$ and $\tilde{V}_H(u)$ are homogeneous of degree one in $\tilde{w}$.*

Proof: Straightforward by multiplying $\tilde{w}$ with a given constant and substituting in (T1.1)–(T1.7). □

2.4 Parameterization and model solution

The structural parameters of the model are given in Table 3. The model is parameterized in the following way. In the base model we assume that individuals live for 78 years ($D = 60$) and that the population grows at 1 percent per annum ($n = 1.0000 \cdot 10^{-2}$). For these values, the crude birth rate is just over 2 percent per annum ($\bar{b} = 2.2164 \cdot 10^{-2}$), whilst the average age of the population is $\bar{u} = 27.02$. We parameterize the model in such a way that the individual chooses 4 years of schooling – thus entering the labour market at age 22 ($E = 4$)

Table 3: Parameter values

Targeted steady-state life-cycle dates		
E	4.0000	Years of education
R	47.0000	Retirement age
Targeted steady-state macro outcomes		
y	1.0000	Steady-state output
r	0.0500	Interest rate
Chosen parameters		
$\bar{R}$	54.0000	Maximum feasible retirement age
D	60.0000	Age at certain death
σ	0.7000	Intertemporal substitution elasticity
ρ	0.0250	Pure rate of time preference
γ	0.0800	Return to experience
δ_0	$0.7654 \cdot 10^{-2}$	Parameter of the human capital depreciation profile
δ_1	1.5000	Parameter of the human capital depreciation profile
β_1	0.1000	Parameter of the return-to-education function
e_0	0.5000	Time spent studying during education
n	0.0100	Population growth rate
g	0.0200	Economic growth rate
δ_k	0.0700	Depreciation rate of physical capital
Calibrated parameters		
ε_c	0.3198	Preference parameter for consumption
β_0	-0.3046	Parameter of the return-to-education function
β_2	$0.5960 \cdot 10^{-2}$	Parameter of the return-to-education function
$\bar{b}$	$2.2164 \cdot 10^{-2}$	Crude birth rate
ε_k	0.2351	Capital efficiency parameter
Ω_0	1.5610	Scale factor production function

– and retires at biological age 65 ($R = 47$). We furthermore ensure that the equilibrium interest rate is 5 percent per annum ($r = 0.05$) and that per capita output is normalized to unity ($y = 1$). The individual uses half of the time endowment for studying during the schooling period ($e_0 = 0.5$). Interpreting the net-of-sleeping day to last for 16 hours, this means that each student spends 8 hours per day on formal education.

The return to education is governed by a simple quadratic function:

$$G(E) = \beta_0 + \beta_1 E - \beta_2 E^2, \quad \beta_1 > 0, \quad \beta_2 > 0.$$

We choose the β-parameters to ensure that $e^{G(E)}$ is increasing and concave in E for a range of plausible values. We fix $\beta_1 = 0.1$, set β_2 in such a way that the first-order condition for schooling (given in (T1.7)) is satisfied for $E = 4$, and choose the value for β_0 such that $G(4) = 0$. This approach yields the concave training function that is illustrated in Figure 1(a).

Regarding the depreciation rate of human capital we propose:

$$\delta_h(u) = \delta_0 + \frac{1}{\delta_1(\bar{R} - u)} \quad \text{for } 0 \leq u < \bar{R}, \quad \delta_0 > 0, \quad \delta_1 > 0.$$

This function has a vertical asymptote at (what we call) the age of *economic death*, $\bar{R}$. This means that the human capital stock is driven to zero at this age.[8] The functional form allows us to easily control both the level and the shape of the depreciation profile. We choose δ_0 and δ_1 so that $\delta_h(u)$ remains nearly flat initially but increases rapidly when approaching $\bar{R}$. This curvature guarantees an interior solution for the retirement age R. In particular, it will always lie to the left of $\bar{R}$. In the base model we set $\bar{R} = 54$ which, in combination with a suitable choice for ε_z, gives rise to a calibrated equilibrium featuring retirement at age 65 ($R = 47$). The resulting depreciation function is depicted in Figure 1(b).

As we cannot obtain an analytical solution to the system of equations given in Table 2 we have to solve it numerically. The main difficulty lies in the fact that labour supply at any point in time depends both on past (via human capital) and future (through experience effects) labour supply choices. This means that we have to loop over the age profile for labour supply in order to obtain a consistent solution. At the same time, the non-separability of leisure and consumption in the felicity function means that we cannot solve for labour supply choices separate from consumption allocations. To speed up the computations we program the model in Fortran 90 and use spline interpolation and root finding techniques. Further details of the parameterization and solution method can be found in Heijdra and Reijnders (2012).

Key features of the general equilibrium attained in the base model (BM hereafter) are

[8] Pfeiffer and Reuss (2008, p. 633) use a special case of this function by setting $\delta_0 = 0$ and $\bar{R} = D$. Hence, they do not make a distinction between economic and biological death. See also footnote 4 above.

presented in column (a) of Table 4. The corresponding life-cycle profiles are depicted in Figure 2 and have already been discussed. The consumption-output ratio is 79.1%, and the capital-output ratio is 1.960. Both are quite plausible values.

3 Demographic shocks

In this section we study the quantitative effects of stylized demographic ageing shocks on microeconomic behaviour and macroeconomic outcomes. We restrict attention to three types of demographic shocks. The first shock is a *biological longevity boost* (BLB hereafter), consisting of a substantial increase in the length of life accompanied by a constant population growth rate and an unchanged human capital depreciation schedule. The second shock combines economic and biological ageing and is called a *comprehensive longevity boost* (CLB hereafter). This case combines the BLB with an increase in economic longevity which we model as an outward shift of the depreciation schedule for human capital. Finally, the third shock is a pure *baby bust* (BB hereafter) scenario, in which the crude birth falls but the length of biological and economic life both remain unchanged.[9]

To streamline the discussion we first state the following proposition.

Proposition 2. *(Features of the demographic structure). Consider the demography as stated in equations (4a)–(4b) and define the crude birth rate and average age of the population as:*

$$\bar{b} \equiv \frac{1}{\Delta(n,D)}, \qquad \bar{u} \equiv \int_0^D u p\,(v, v+u)\, du = -\frac{\partial \ln \Delta\,(n,D)}{\partial n}.$$

Then the following properties can be established:

(i) For a given population growth rate n, an increase in the length of life D leads to an decrease in the crude birth rate and an increase in the average age of the population:

$$\left.\frac{d\bar{b}}{dD}\right|_n < 0, \qquad \left.\frac{d\bar{u}}{dD}\right|_n > 0;$$

(ii) For a given length of life D, an increase in the crude birth rate $\bar{b}$ leads to an increase in the population growth rate and a decrease in the average age of the population:

$$\left.\frac{dn}{d\bar{b}}\right|_D > 0, \qquad \left.\frac{d\bar{u}}{d\bar{b}}\right|_D < 0;$$

[9] For convenience and without loss of generality we model the baby bust as a drop in the crude birth rate. Of course, by using equation (4a) we find that a drop in b or B results in a fall in n. Holding constant D it then follows from the definition of $\bar{b}$ that the crude birth rate also falls. Hence we can always engineer a given change in $\bar{b}$ by changing b and/or B.

Table 4: Quantitative results‡

	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)
	BM	*BLB*		*CLB*		*BB*	*BC*		*IL*	
Microeconomic choices										
E	4.000	4.121	4.395	4.426	4.146	4.114	3.988	4.032	3.848	3.787
F							7.815	8.396		
R	47.000	47.602	48.878	55.409	53.550	47.575	46.873	47.083	42.265	41.776
$\tilde{c}(0)$	0.791	0.797	0.897	0.945	0.832	0.829	0.785	0.798	0.748	0.730
$\tilde{\imath}$							0.192	0.196		
$l\,(E)$	0.535	0.535	0.536	0.535	0.534	0.532	0.535	0.549	0.500	0.500
$h\,(E)$	1.000	1.006	1.020	1.021	1.008	1.006	0.999	1.002	0.992	0.989
$h\,(R)$	0.919	0.926	0.877	1.115	1.165	0.901	0.920	0.913	1.305	1.317
$e^{-rE}\tilde{V}_H\,(E)$	22.572	23.737	27.148	28.512	24.733	23.773	22.433	22.874	21.208	20.638
$\Lambda(v_0)$	−11.591	−11.562	−11.514	−10.683	−10.807	−11.559	−11.593	−11.583	−11.749	−11.767
Macroeconomic outcomes										
y	1.000		1.044		1.159	0.990		1.001		0.936
k	1.960		2.113		2.207	1.965		1.972		1.821
hc	0.454		0.470		0.531	0.448		0.454		0.426
c	0.804		0.833		0.938	0.809		0.804		0.754
$\tilde{w}$	1.684		1.701		1.669	1.690		1.686		1.680
$r\ (\times 100\%)$	5.000		4.615		5.350	4.850		4.938		5.087

‡Column (a) is the calibrated base model with $D = 60$, $\bar{R} = 54$, $\bar{b} = 2.2164 \cdot 10^{-2}$, and $n = 1.0000 \cdot 10^{-2}$. Columns (b)–(c): biological longevity boost (partial and general equilibrium). $D = 67.8$, constant n, $\bar{b} = 2.0310 \cdot 10^{-2}$; Column (d)–(e): comprehensive longevity boost (partial and general equilibrium). As in column (b)–(c) but $\bar{R} = 61.8$. Column (f): baby bust. $\bar{b} = 1.7731 \cdot 10^{-2}$, $n = 0.2085 \cdot 10^{-2}$, $D = 60$, and $\bar{R} = 54$. Columns (g)–(h): model solution under borrowing constraints (partial and general equilibrium). Columns (i)–(j): model solution with indivisible labour (partial and general equilibrium).

Proof: See Heijdra and Reijnders (2012) for details. □

3.1 Biological longevity boost

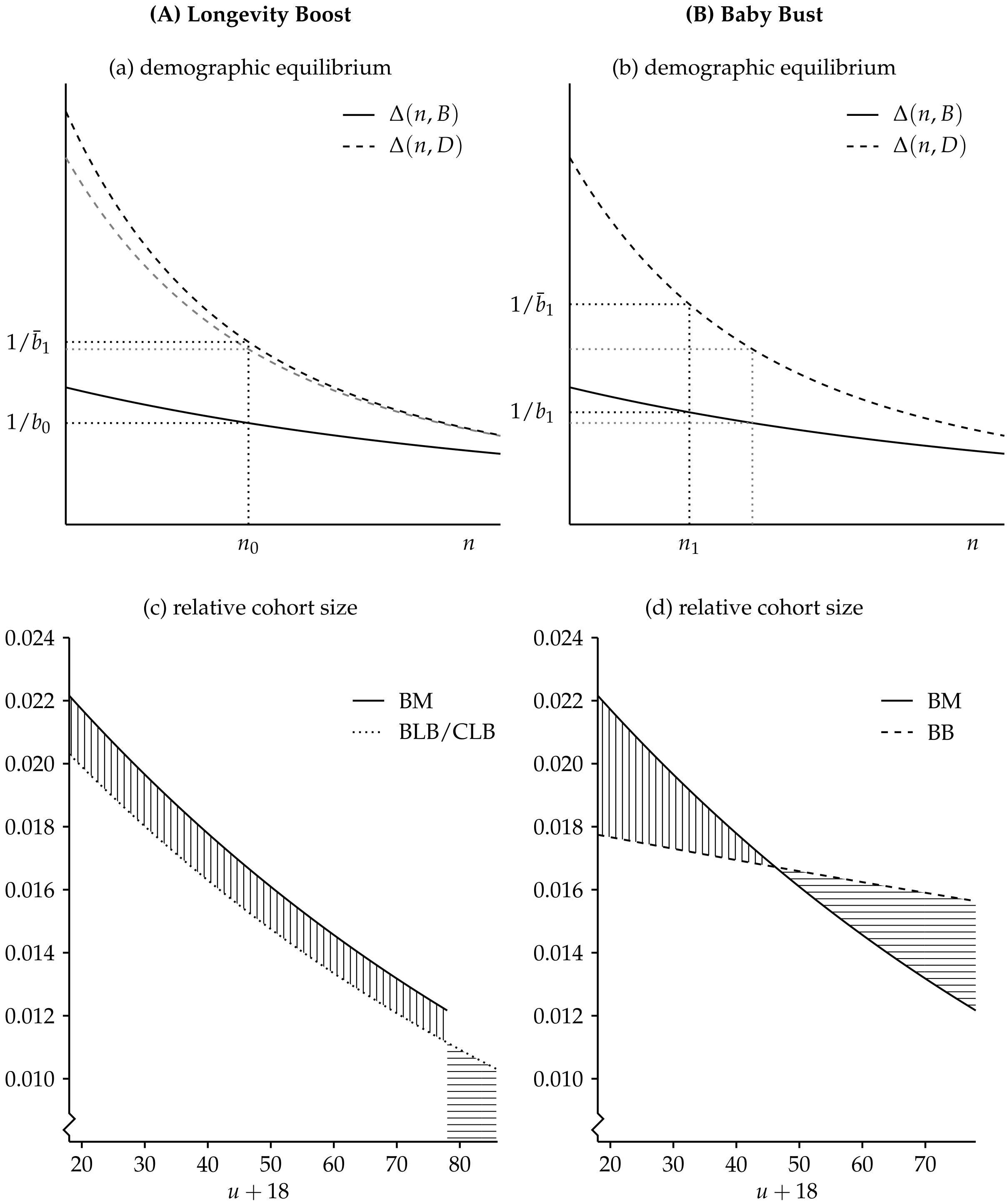
A biological longevity boost (BLB) consists of an exogenous increase in D. Holding constant the fertility parameters (b and B) and thus the population growth rate n, it follows from Proposition 2(i) that the crude birth rate $\bar{b}$ falls and the average age of the population increases. In Figure 4(a) the longevity boost rotates the $\Delta(n, D)$ line in a clockwise fashion and the implied crude birth rate falls. Figure 4(c) illustrates how the longevity shock affects the composition of the population. In that figure, the thick solid line represents the base case scenario (BM). The dashed line shows how the population half-pyramid changes as a result of a 7.8 year increase in the length of life from $D = 60$ to $D = 67.8$. The population ages because mass of the distribution is shifted from the young to the old. In the new demographic equilibrium $\bar{b} = 2.0310 \cdot 10^{-2}$ and $\bar{u} = 30.10$.

Blessed with a longer lifetime, individuals will make different choices than in the base case scenario, even if we hold factor prices constant at their pre-shock levels. In column (b) of Table 4 we present the *partial equilibrium* effects of the longevity boost. Schooling and the retirement age increase by, respectively, 0.12 and 0.60 years, whilst the working career $(R - E)$ is extended by almost half of one year. In view of the fact that life is extended by 7.8 years, these effects are modest indeed. Much of the extra years of life will be enjoyed in the form of leisure, i.e. the retirement period $(D - R)$ increases by more than 7 years.

But this is not the end of the story because the adjustments of individual plans result in significant changes in factor supplies and thus in equilibrium factor prices. The *general equilibrium* effects of the longevity boost are reported in column (c) of Table 4. The stocks of physical and human capital increase by, respectively, 7.85% and 3.35% resulting in an increase in output of 4.39%, a 39 basis point drop in the interest rate, and a 1.01% increase in the rental rate on human capital. The general equilibrium effects on E, R, and $R - E$ are in the same direction but significantly larger than their partial equilibrium counterparts. For future reference we summarize the general equilibrium effects on the key variables in column (a) of Table 5.

The intuition behind the effects on factor prices is as follows. Even though the length of an individual's biological life increases dramatically, the duration of economic life is unaffected because the human capital depreciation schedule is held constant by assumption under the BLB scenario. Agents have very little incentive to enjoy more schooling and to retire later (compared to the BM) because economic ageing still sets in at exactly the same phase of life. As a result, the stock of human capital increases only slightly. The increased length of biological life, however, necessitates higher savings resulting in a substantial increase in the stock of physical capital. Human capital thus becomes relatively scarce under

Figure 4: Demographic shocks



the BLB leading to an increase in its rental rate and an decrease in the rate of interest.

3.2 Comprehensive longevity boost

A comprehensive longevity boost (CLB) features exogenous increases in both D and $\bar{R}$, i.e. the biological and economic lifespan are both increased under this shock. The biological shock is parameterized in the same way as before; D increases from $D = 60$ to $D = 67.8$, and the population ages as is illustrated by the dashed line in Figure 4(c). The increase in economic longevity consists of an increase in the age of economic death from $\bar{R} = 54$ to $\bar{R} = 61.8$. In terms of Figure 1(b) the human capital depreciation function shifts to the right.

In column (d) of Table 4 we present the partial equilibrium effects of the CLB. Schooling and the retirement age increase by, respectively, 0.43 and 8.41 years, the working career ($R - E$) is extended by almost eight years, and the retirement period ($D - R$) is *reduced* by more than half of one year.

The *general equilibrium* effects of the CLB are reported in column (e) of Table 4 and summarized in column (d) of Table 5. The stocks of physical and human capital increase by, respectively, 12.63% and 16.95% resulting in an increase in output of 15.92%, a 35 basis point increase in the interest rate, and a 0.88% decrease in the rental rate on human capital. Interestingly, the general equilibrium effects on E, R, and $R - E$ are in the same direction but significantly smaller than their partial equilibrium counterparts. In contrast, the change in the retirement period is in the opposite direction.

The intuition behind the effects on factor prices is as follows. Both biological and economic life spans are increased. The former gives rise to increased saving and a boost to the stock of physical capital. The latter stimulates the accumulation of human capital and slows down its rate of depreciation during the active period. Since the working career increases, the economy-wide stock of human capital increases substantially. Human capital thus becomes relatively abundant under the CLB leading to a decrease in its rental rate and an increase in the rate of interest.

3.2.1 Offsetting forces of biological and economic longevity

As the comparison between columns (c) and (e) of Table 4 reveals, factor price changes under the CLB are in the opposite direction to those resulting from the BLB. Recall that this result is obtained for the case in which economic and biological longevity are increased by the same amount of years, i.e. $\Delta\bar{R} = \Delta D = 7.8$. Whilst changes in biological longevity of that magnitude are well documented (see Table 1), the same cannot be said for the boost to economic longevity which is – in any case – inherently hard to measure. In our view the scenario with $\Delta\bar{R} = \Delta D$ must be regarded as an extremely optimistic one. The fact that nowadays the medical profession can keep people alive much longer does not – in and of itself – imply that their embodied stock of human capital is similarly granted an "extension of life."

This prompts the following question. For a given change in D, by how many years must the date of economic death be postponed for factor prices to be unaffected by the resulting comprehensive increase in longevity? Our computations reveal that this result is obtained for a CLB featuring $\Delta D = 7.8$ and $\Delta \bar{R} = 3.73$, i.e. for $\phi \equiv \frac{\Delta \bar{R}}{\Delta D} = 0.4782$.[10] It follows that for any $\Delta \bar{R} > \phi \Delta D$ human capital becomes relatively abundant and the interest rate rises, whereas the opposite holds for $\Delta \bar{R} < \phi \Delta D$.

3.3 Baby bust

A baby bust (BB) consists of a decrease in the crude birth rate $\bar{b}$ resulting from, for example, an exogenous decrease in the net birth rate. For a given age of death D, it follows from Proposition 2(ii) that the population growth rate declines and the average age of the population increases. Figure 4(b) illustrates how the demographic equilibrium changes as a result of a drop in the net birth rate. In Figure 4(d) the dashed line illustrates how the composition of the population changes compared to the base case scenario if the crude birth rate decreases by twenty percent, from $\bar{b} = 2.2164 \cdot 10^{-2}$ to $\bar{b} = 1.7731 \cdot 10^{-2}$. Again, as in the previous ageing shock, mass of the distribution is shifted from young to old agents. In the new demographic equilibrium $n = 0.2085 \cdot 10^{-2}$ and $\bar{u} = 29.37$.

There is no direct effect on individual behavior: D is unchanged and $\bar{b}$ and n do not affect individual plans as is clear from equations (T1.1)–(T1.7) in Table 2. There are nevertheless general equilibrium effects because the baby bust results in a decrease in the population growth rate which features prominently in the macroeconomic part of the model consisting of equations (T1.8)–(T1.12) in Table 2. Key indicators of the equilibrium attained after the baby bust are reported in column (f) of Table 4 and in column (g) of Table 5. In contrast to the longevity boost, the baby bust produces only relatively small effects on most variables. Schooling and the retirement age increase by, respectively, 0.11 and 0.58 years, and the working career ($R - E$) is extended by about half of one year. The stock of physical capital increases by 0.29% whilst the human capital stock and output fall by, respectively, 1.35% and 0.97%.

4 Robustness checks

In the previous section we have shown that of the prototypical ageing shocks that have hit modern economies in recent times, the longevity boosts exert quantitatively major effects

[10] Solving the model for this shock yields: $E = 4.271$, $R = 51.252$, $\Lambda(v_0) = -11.135$, $y = 1.102$, $k = 2.159$, $hc = 0.500$, $\tilde{w} = 1.684$, and $100 \cdot r = 5.000$. Compared to the base model, output and the stocks of physical and human capital all incease by 10.17%.

on factors supplies and on output, whereas the effects of a baby bust are very modest. In this section we examine the robustness of these conclusions by relaxing a number of the key assumptions underlying the base model. First, in subsection 4.1 we drop the assumption of perfect capital markets and postulate that individuals face binding borrowing constraints after leaving school. Second, in subsection 4.2 we introduce indivisibility of labour into the base model and show how life-cycle plans and macroeconomic outcomes are affected both qualitatively and quantitatively.

4.1 Borrowing constraints

As is clear from the scaled asset profile in Figure 2(d), an optimizing individual is indebted for more than twenty years during the first part of his/her life. In the base model capital markets are perfect and individuals are freely able to borrow against future wage income. In this subsection we investigate the qualitative and quantitative importance of this capital market assumption.[11] In the trivial case, without any borrowing allowed whatsoever, the constraint is bound to have major effects because a positive amount of schooling would be prohibitively unattractive as the agent has no income during the schooling period and consumption is essential.[12]

We study a less drastic capital market imperfection by assuming that there exists a perfect[13] system of study loans during the schooling period and a prohibition on borrowing after leaving school. Two regimes can be distinguished, depending on whether or not the borrowing constraint is binding. If the constraint does not bind, then the optimal paths for consumption and leisure are limited only by a *present-value constraint* just as in the base model. We refer to this as the (Irving) Fisher Regime, after its principal architect. In contrast, if the borrowing constraint is binding, then the paths for consumption and leisure are limited by the availability of non-asset income, i.e. a *flow constraint* is relevant in that case. We call this the (John Maynard) Keynes Regime.[14]

In our extended model the life-cycles dates are as follows. The educational phase is Fisherian and lasts for ages $0 \leq u < E$. It is followed by the Keynesian employment phase for $E \leq u < F$, where F stands for the age at which the constraint on borrowing first ceases

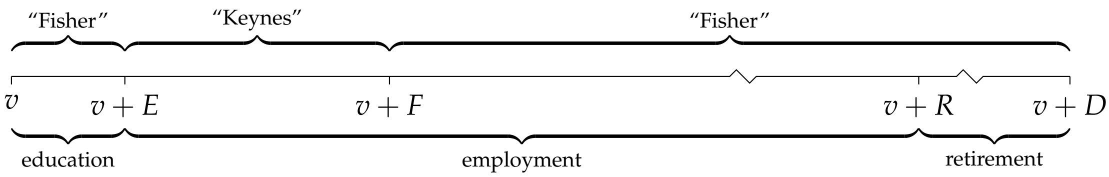
[11] An earlier macroeconomic literature investigates the effects of borrowing constraints on the economic growth rate. See, for example, Jappelli and Pagano (1994, 1999) and De Gregorio (1996).

[12] Solving the base model under the life-time borrowing constraint yields: $E = 0$, $R = 48.875$, $\Lambda(v_0) = -12.715$, $y = 0.839$, $k = 1.854$, $hc = 0.367$, $\tilde{w} = 1.747$, and $100 \cdot r = 3.637$. Compared to the base model, physical and human capital stocks fall by, respectively, 5.37% and 19.17%, whilst output drops by 16.12%. Wages rise because human capital get scarcer. The borrowing constraint ceases to be binding at biological age 32.382, i.e. agents are constrained throughout their youth.

[13] We call this a perfect system because students can borrow as much as they like at the going market rate of interest, restrained only by the requirement that they redeem their study debt upon leaving school. In the absence of borrowing constraints after leaving school the study loan system is completely neutral – see below.

[14] Weiss (1972) uses the terminology *Jevons phase* and *Ramsey phase* for, respectively, our Keynesian and Fisherian Regimes. In our view the notion of consumption being constrained by the flow of income is most closely associated with Keynes. Furthermore, the dynamic approach to individual saving was pioneered by Fisher.

to bind. The remainder of life is Fisherian and consists of an employment phase ($F \leq u < R$) and a retirement phase ($R \leq u \leq D$).



The study loans system works as follows. During the schooling period ($0 \leq t - v < E$) the agent can borrow freely at the given interest rate r and thus accumulates study debt according to:

$$\dot{s}(v,t) = rs(v,t) + c(v,t),$$

where $s(v,v) = 0$ as the students enters the system with a clean slate. Upon completion of the educational period the agent is faced with a study debt of $s(v,v+E)$ which must be paid off over the remaining lifetime. Hence, for $E \leq t - v < D$ the accumulation identity for study loans is given by:

$$\dot{s}(v,t) = rs(v,t) - i(v), \quad \text{such that} \quad s(v,v+D) = 0,$$

where $i(v)$ is the periodic redemption payment which we assume for simplicity sake to be age-independent. In present value terms the system of study loans imposes the following constraint of the agent:

$$i(v) = \frac{r}{1 - e^{-r(D-E)}} \int_{v}^{v+E} c(v,t) e^{-r(t-v-E)} dt. \tag{5a}$$

In the Fisherian regime ($F \leq t - v \leq D$) regular financial assets evolve according to:

$$\dot{a}(v,t) = ra(v,t) + w(t)\, l(v,t) h(v,t) - c(v,t) - i(v),$$

with $a(v,v+F) = a(v,v+D) = 0$ so that the present-value constraint is given by:

$$\int_{v+F}^{v+R} w(t)\, l(v,t) h(v,t) e^{-r(t-v)} dt = \int_{v+F}^{v+D} [c(v,t) + i(v)] e^{-r(t-v)} dt. \tag{5b}$$

Equation (5b) replaces (2c) and differs from it because (i) $F > E > 0$ and (ii) redemption payments feature on the right-hand side.[15]

[15] If there were no constraint on the sign of regular assets then the study loan system would be completely neutral. Indeed, in such a case the integrals in (5b) would run from E instead of F. In combination with (5a) the resulting expression would yield (2c).

The agent chooses E, F, R, $i\left(v\right)$, $\left\{c\left(v,\tau\right)\right\}_{\tau=v}^{v+D}$, and $\left\{l\left(v,\tau\right)\right\}_{\tau=v}^{v+D}$ in order to maximize lifetime utility (2a) subject to the present-value budget constraints (5a) and (5b), and the human capital equation (2e). The optimal choices for consumption are given by:

$$\Phi_c(c(v,t),z(v,t)) = \xi\left(v\right)\frac{re^{rE}}{1-e^{-r(D-E)}}e^{-(r-\rho)(t-v)} \qquad 0 \le t-v < E, \tag{5c}$$

$$c\left(v,t\right) = w\left(t\right)l(v,t)h(v,t) - i(v) \qquad E \le t-v < F, \tag{5d}$$

$$\Phi_c(c(v,t),z(v,t)) = \lambda\left(v\right)e^{-(r-\rho)(t-v)} \qquad F \le t-v \le D, \tag{5e}$$

where $\xi\left(v\right)$ and $\lambda\left(v\right)$ are the Lagrange multipliers for, respectively, (5a) and (5b). Consumption grows at the exponential rate $\sigma^*(r-\rho)$ in the two Fisherian regimes, just as in the base model, but during the Keynesian employment regime wage income net of redemption payments dictates the path of consumption.

The first-order condition for the redemption payment furnishes the following relationship between $\xi\left(v\right)$ and $\lambda\left(v\right)$:

$$\xi\left(v\right) = \lambda(v)\left[\int_{v+E}^{v+F}\Psi\left(v,\tau\right)e^{-r(\tau-v)}d\tau + e^{-rE}\frac{1-e^{-r(D-E)}}{r}\right], \tag{5f}$$

where $\Psi\left(v,\tau\right)$ is a "distortion" term originating from the borrowing constraint:

$$\Psi\left(v,\tau\right) \equiv \frac{\Phi_c(c(v,\tau),z(v,\tau)) - \lambda\left(v\right)e^{-(r-\rho)(\tau-v)}}{\lambda(v)e^{-(r-\rho)(\tau-v)}}, \qquad E \le t-v \le D. \tag{5g}$$

Clearly, in view of (5d)–(5e), $\Psi\left(v,\tau\right) > 0$ for $E \le t-v < F$ and $\Psi\left(v,\tau\right) = 0$ for $F \le t-v \le D$. Intuitively, the agent would like to consume more in the Keynesian employment regime but the borrowing constraint prohibits this from happening. In contrast, in the Fisherian employment and retirement regimes the agent faces no borrowing constraint and there is thus no distortion in the consumption choice.

The first-order condition for labour supply can be written as follows:

$$\begin{aligned}\Phi_z(c(v,t),z(v,t)) = \lambda\left(v\right)e^{-(r-\rho)(t-v)}\Big[&w\left(t\right)h(v,t) + \gamma V_H\left(v,t\right) \\ &+ \gamma\int_t^{v+D}\Psi\left(v,\tau\right)w\left(\tau\right)l(v,\tau)h(v,\tau)e^{-r(\tau-t)}d\tau \\ &+ \Psi\left(v,t\right)w\left(t\right)h(v,t)\Big], \qquad E \le t-v \le R.\end{aligned} \tag{5h}$$

The first line coincides with the expression obtained for the base model (see equation (3b) above), whilst the second and third line show the effects of the borrowing constraint on labour supply in the Keynesian regime. In this regime the agent partially relaxes the consumption-reducing effect of the borrowing constraint by supplying more hours of labour than he/she would in the absence of this restriction.

The first-order condition for the education decision is given by:

$$\begin{aligned}
&\Phi(c(v,v+E^-),1-e_0)-\Phi(c(v,v+E),1-l(v,v+E))\\
&\quad=\lambda\left(v\right)e^{-(r-\rho)E}\Bigg[w\left(v+E\right)l\left(v,v+E\right)h\left(v,v+E\right)+\Big(c\left(v,v+E^-\right)-c\left(v,v+E\right)\Big)\\
&\qquad-\int_{v+E}^{v+R}w\left(\tau\right)l(v,\tau)\frac{\partial h(v,\tau)}{\partial E}e^{-r(\tau-v-E)}d\tau\\
&\qquad+\Psi\left(v,v+E^-\right)\Big[w\left(v+E\right)l\left(v,v+E\right)h\left(v,v+E\right)+\Big(c\left(v,v+E^-\right)-c\left(v,v+E\right)\Big)\Big]\\
&\qquad-\int_{v+E}^{v+F}\Psi\left(v,\tau\right)w\left(\tau\right)l(v,\tau)\frac{\partial h(v,\tau)}{\partial E}e^{-r(\tau-v-E)}d\tau\Bigg].
\end{aligned}\tag{5i}$$

Compared to the corresponding condition in the base model ((3e)), the expression in (5i) is much more complicated because the distortion term $\Psi(v,\tau)$ affects all three components constituting the marginal utility cost of schooling.

Finally, the first-order conditions for F and R just impose there to be no discontinuities in consumption and labour supply at these points in the life cycle.

The quantitative effects of borrowing constraints can be gleaned from columns (g)–(h) in Table 4.[16] For ease of comparison, column (a) restates the calibrated base model. Comparing columns (a) and (g) we find the partial equilibrium effects of the capital market imperfection. Optimal schooling is virtually unaffected and retirement occurs at a slightly earlier age. Even though the agent faces a binding borrowing constraint and is forced to underconsume for almost 4.5 years, life-time utility is reduced only by a tiny amount.

Not suprisingly, in iew of the minute partial equilibrium effects, factor price changes are rather modest and the partial and general equilibrium effects of the capital market imperfection are very similar. Comparing columns (a) and (h) we find that output and the stock of physical capital increase by, respectively, 0.13% and 0.65%, whereas the stock of human capital falls by 0.03%. Life-time utility rises slightly because the increase in the ratio between physical and human capital leads to a reduction in the interest rate thus bringing the economy closer to the Golden Rule point (featuring $r=n$).

From a quantitative perspective the conclusion seems warranted that the system of study loans postulated here virtually eliminates the unpleasant aspects of the constraints on borrowing. It removes the teeth from the dragon of capital market imperfections. By insulating young agents from this form of market imperfection, the process of human capital formation is saved from otherwise inevitable destruction. The borrowing constraints that hinder agents during the employment period are quantitatively unimportant.

Figure 5 visualizes some of the life-cycle profiles in the presence of borrowing constraints.

[16] In that table, scaled redemption payments are given by $\tilde{\imath}\equiv i(v)/Z(v)$.

Panel (a) shows that the agent supplies more hours of labour between ages E and F. Intuitively, in doing so he/she builds up human capital more rapidly (see panel (c)) and partially alleviates the constraint on consumption. Panel (b) shows that there is a downward jump in consumption jump at age E, for reasons explained above, and that there exists a smooth connection between the Keynesian and Fisherian consumption paths at age F. Finally, panel (d) depicts the profiles for scaled regular assets and study debt. An agent's net financial assets become positive a little before reaching biological age 40.

The quantitative effects of demographic shocks under borrowing constrains are summarized Table 5. Columns (b), (e), and (h) summarize the general equilibrium solutions for, respectively, BLB, CLB, and BB. Interestingly, as the comparison with columns (a), (d), and (g) (the base model results) reveals, the effects of demographic shocks are quantitatively very similar with and without borrowing constraints. Indeed, even in the absence of study loans, with agents facing life-time borrowing constraints, exactly the same conclusion is obtained – see Heijdra and Reijnders (2012). Economies with and without borrowing constraints react in a very similar way to the ageing shocks that we have considered in this paper.

4.2 Indivisible labour

In the base model labour supply is fairly constant over much of the agent's life but declines sharply after age 50. Labour is perfectly divisible so retirement from the labour force takes the form of a gradual reduction in working hours. In reality, however, part-time jobs of the right magnitude are very hard to find or even completely absent from the economic menu. In this subsection we therefore investigate the qualitative and quantitative importance of the assumption regarding divisibility of labour.[17] In particular we assume that an individual can either work full time (and choose $l(v,t) = l_F$) or not at all (by setting $l(v,t) = 0$). Whilst it is – in principle – straightforward to recognize a finer grid of employment choices (such as half-time jobs), we employ the 2-state model because, first, it is well established in the macroeconomic literature and, second, it gives the cleanest possible representation of the notion of indivisibility of labour supply. We reinstate the assumption of perfect capital markets.

We can already anticipate that in the current (deterministic) set-up the agent will start working full-time directly upon finishing school and will work until a certain age R at which he or she retires completely from the labour force. In the Indivisible Labour (IL hereafter) model, leisure is no longer a choice variable in itself, as it is determined by choices of E and

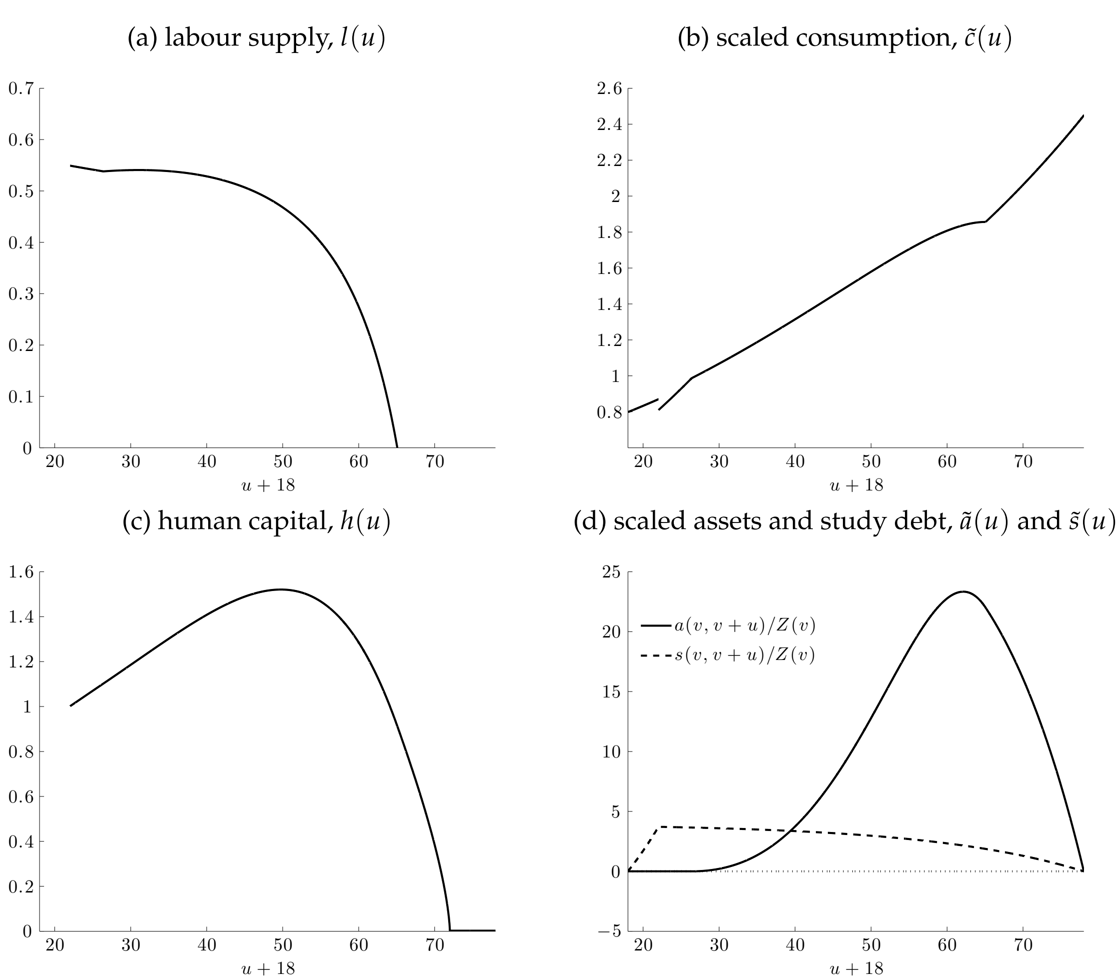
[17] See also Mulligan (2001), Prescott et al. (2009), and Sargent and Ljungqvist (2011) on this issue.

Table 5: Quantitative general equilibrium effects of demographic shocks

	BLB			CLB			BB		
	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
	BM	BC	IL	BM	BC	IL	BM	BC	IL
$E^{\ddagger}$	0.40	0.46	0.34	0.15	0.21	0.12	0.11	0.14	0.20
$F^{\ddagger}$		3.58			5.09			1.82	
$R^{\ddagger}$	1.88	1.98	3.23	6.55	6.84	6.22	0.58	0.63	0.88
$(R-E)^{\ddagger}$	1.48	1.51	2.89	6.40	6.63	6.10	0.46	0.49	0.68
$(D-R)^{\ddagger}$	5.92	5.82	4.58	1.25	0.96	1.58	−0.58	−0.63	−0.88
$y^{\sharp}$	4.39	4.69	−0.54	15.92	16.25	13.17	−0.97	−0.75	−1.21
$k^{\sharp}$	7.85	9.23	3.48	12.63	14.24	10.38	0.29	0.02	0.15
$hc^{\sharp}$	3.35	3.16	−1.74	16.95	16.68	14.04	−1.35	−0.01	−1.58
$r^{\S}$	−38.85	−51.16	−46.89	34.99	19.47	30.57	−14.99	−20.41	−16.00
$\tilde{w}^{\sharp}$	1.01	1.36	1.22	−0.88	−0.50	−0.77	0.39	0.01	0.41

Notes: ‡change in years; $^{\sharp}$percentage change; §change in base points.

Figure 5: Life-cycle profiles in the presence of borrowing constraints



R:

$$z(v,t) = \begin{cases} 1-e_0 & \text{for } 0 \le t-v < E \\ 1-l_F & \text{for } E \le t-v < R \\ 1 & \text{for } R \le t-v \le D \end{cases}, \tag{6a}$$

whilst the human capital stock evolves according to:

$$h(v,t) = \begin{cases} e^{G(E)} e^{\gamma l_F (t-v-E) - \int_{v+E}^{t} \delta_h(\tau - v) d\tau} & \text{for } E \le t-v < R \\ h(v,v+R) e^{-\int_{v+R}^{t} \delta_h(\tau-v) d\tau} & \text{for } R \le t-v \le \bar{R} \end{cases} \tag{6b}$$

The lifetime budget constraint is given by:

$$\int_{v+E}^{v+R} w\left(t\right) l_F h(v,t) e^{-r(t-v)} dt = \int_{v}^{v+D} c(v,t) e^{-r(t-v)} dt. \tag{6c}$$

In the presence of perfect capital markets, the individual's time line is the same as in the base model. The agent chooses the education level E, the retirement age R, and a time profile for consumption $\{c\left(v,\tau\right)\}_{\tau=v}^{v+D}$ in order to maximize lifetime utility (2a) subject to the lifetime budget constraint (6c) and the human capital equation (6b). Not surprisingly, since capital markets are perfect, the first-order condition for consumption is still as given in (3a). The optimality conditions for schooling and the retirement age are obtained by setting $l\left(v,t\right) = l_F$ for $E \le t-v < R$ and $l\left(v,t\right) = 0$ for $R \le t-v \le D$ in, respectively, (3e) and (3f):

$$\begin{aligned} &\Phi(c(v,v+E^-),1-e_0) - \Phi(c(v,v+E),1-l_F) \\ &\quad = \lambda\left(v\right) e^{-(r-\rho)E} \Bigg[w\left(v+E\right) l_F h(v,v+E) \\ &\qquad - \int_{v+E}^{v+R} w\left(t\right) l_F \frac{\partial h(v,t)}{\partial E} e^{-r(t-v-E)} dt + \Big(c(v,v+E^-) - c(v,v+E) \Big) \Bigg], \end{aligned} \tag{6d}$$

$$\begin{aligned} &\Phi(c(v,v+R^-),1-l_F) - \Phi(c(v,v+R),1) \\ &\quad = \lambda\left(v\right) e^{-(r-\rho)R} \Big[-w\left(v+R^-\right) l_F h(v,v+R^-) \\ &\qquad + \Big(c(v,v+R^-) - c(v,v+R) \Big) \Big]. \end{aligned} \tag{6e}$$

As a consequence of the indivisibility of labour, the consumption jumps (3b) and (3g) are replaced by:

$$\frac{c\left(v,v+E\right)}{c\left(v,v+E^-\right)} = \left(\frac{1-e_0}{1-l_F}\right)^{\frac{(1-\sigma)\varepsilon_Z}{1-\varepsilon_Z(1-\sigma)}}, \quad \frac{c\left(v,v+R\right)}{c\left(v,v+R^-\right)} = \left(\frac{1-l_F}{1}\right)^{\frac{(1-\sigma)\varepsilon_Z}{1-\varepsilon_Z(1-\sigma)}}. \tag{6f}$$

The quantitative effects of the indivisibility constraints can be gleaned from columns (i)–(j) in Table 4. We assume that $l_F = 0.5$ represents a fultime job. For ease of comparison, column (a) restates the calibrated base model. Comparing columns (a) and (i) we find the partial equilibrium effects of the labour market market imperfection. Optimal schooling is reduced by a mere 0.15 years but retirement takes place as much as 4.74 years earlier. Since gradual retirement is ruled out, consumption features a sizeable downward jump at age R and life-time utility is reduced substantially.

The general equilibrium effects of the labour market imperfection on E and R are very similar to their partial equilibrium counterparts. Comparing columns (a) and (j) we find that output and the stocks of physical and human capital decrease by, respectively, 6.40%, 7.04% and 6.16%. Since the ratio between physical and human capital falls, there is a slight increase in the interest rate and decrease in the scaled wage rate.

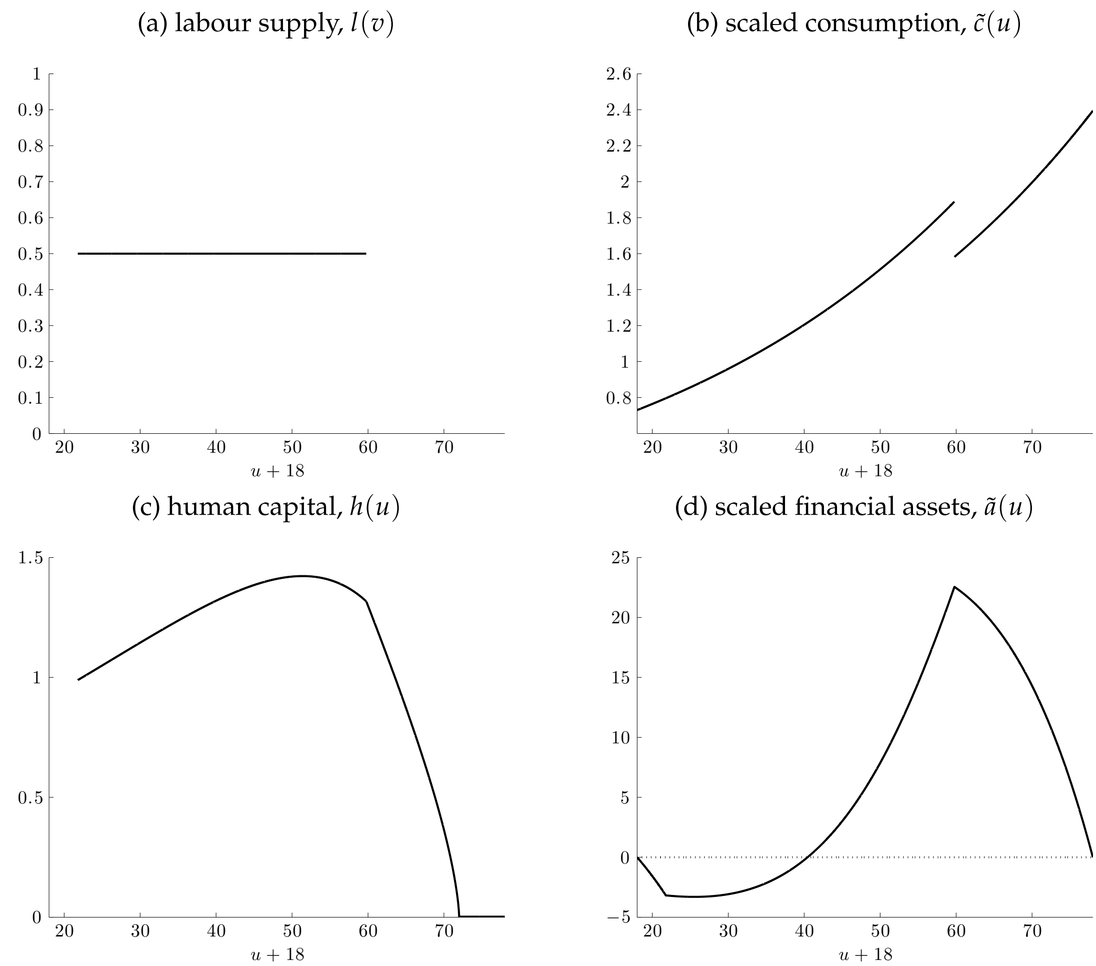
Figure 6 visualizes some of the life-cycle profiles in the presence of IL constraints. Panel (a) shows that the agent supplies l_F hours of labour between ages E and R. Panel (b) shows that there is a substantial downward jump in consumption jump at age R, for reasons explained above – see (6f). The consumption path is continuous at age E because – by assumption – students spend a full work day on education, so that $e_0 = l_F$. Panels (c) and (d) show that the indivisibility of labour introduces sharp kinks in the profiles for human capital and financial wealth.

The quantitative effects of demographic shocks under indivisible labour constraints are given in Table 5. Columns (c), (f), and (i) summarize the general equilibrium solutions for, respectively, BLB, CLB, and BB. The comparison with columns (a), (d), and (g) reveal two main conclusions. First, the effects of the CLB and BB are quantitatively rather similar with and without the labour market imperfection. Second, the effects of the BLB are somewhat different under the IL constraint. Compared to the base model retirement is later, the increase in the capital stock is much smaller, and both human capital and output fall. The movement of factor prices is still in line with the results obtained in the base model. We conslude that even though economies with and without perfect divisibility of labour are quite different, they react in a very similar way to the ageing shocks that we have studied in this paper.

5 Conclusions

We study the steady-state general equilibrium effects of a number of stylized ageing shocks that have hit most western economies in one form or another over the last half century. In order to do so we formulate an overlapping generations model of a closed economy featuring endogenous factor prices. We generalize the classic ”certain death” framework of Cass and Yaari (1967) by assuming that individual agents accumulate both physical and human capital. In the tradition of the recent human capital literature we assume that individuals

Figure 6: Life-cycle profiles with indivisible labour



engage in full-time educational activities at the start of economic life and choose the optimal age of labour market entry. Incorporating the insights of the empirical literature on Mincerian wage equations, we assume that participating in the labour market boosts a worker's human capital stock as valuable experience is gained. As a worker gets older, economic ageing sets in because human capital starts to depreciate at an increasing rate. This prompt the rational individual to ultimately retire from the labour force.

The first ageing shock that we consider is a purely biological one taking the form of an increase in the age of death. Holding constant the growth rate of the population this results in a decrease in the crude birth rate and increase in the old-age dependency ratio. In response to the longer lifetime, agents choose more schooling, retire later, and increase the length of the work career. Almost all of the additional years of life are consumed in the form of leisure as the retirement period increases substantially. At the macroeconomic level the stocks of physical and human capital both rise, with the former increase dominating the latter. Human capital becomes relatively scarce under the biological longevity boost (BLB) prompting an increase in its rental rate and a decrease in the interest rate. The intuition behind this result is that the human capital depreciation schedule is kept unchanged under the BLB. So even though individuals enjoy a longer biologial life, their economic life is unaffected.

The second ageing shock is a comprehensive longevity boost (CLB) in which both biological and economic lifetimes are lengthened. In this case the stocks of physical and human capital both rise sharply, with the latter increase dominating the former. Schooling is increased only slightly, but as retirement takes place much later on in life, the length of the work career rises dramatically. Surprisingly, the optimal retirement period is decreased under the CLB, i.e. the additional years of biological life are not spent on leisure but on working.

The third ageing shock is a baby bust (BB) scenario in which there is an exogenous decrease in the crude birth rate. For a given length of biological life, this implies that the growth rate of the population decreases and the old-age dependency ratio increases. Interestingly, the quantitative effects of this shock are rather small. The slight increase in the physical capital stock is accompanied by a small decrease in the stock of human capital leaving macroeconomic output virtually unchanged.

Our results are based on a plausibly calibrated numerical version of our theoretical model. To demonstrate the robustness of our conclusions to alternative modelling assumptions we also study some extensions. The first extension introduces a capital market imperfection. Whereas the resulting constraints on borrowing are potentially disastrous to the educational process, we show that much of the damage can be removed by incorporating a system of study loans for young individuals. Provided such a system exists, the effects of the ageing shocks are very similar to those for the base model.

A second extension introduces a labour market imperfection taking the form of an indivisibility of labour supply. Under the assumption that agents can either work full time or not at all, ageing shocks can only affect the worker's participation, i.e. changes take place via the extensive margin. The indivisibility contraint itself causes non-trivial welfare losses to the agent because the desired gradual retirement process is not feasible. Interestingly, the conclusions with respect to the ageing shocks are qualitatively the same and quantitatively rather similar to the ones obtained in the base model.

Our paper studies the impact of ageing shocks on physical and human capital accumulation in a highly stylized model of the economy. Many real world features have been deliberately left out in order to focus on some of the key mechanisms. In future work we aim to address at least two aspects of economic life that are not covered in the current model. First, we wish to adopt a more realistic labour market model – one which allows for the possibility of unemployment spells. In such a setting the accumulation of human capital is risky, a feature that is likely to have a nontrivial efffect on the schooling decisions of risk-averse agents. The second major real life feature that we wish to add to our model concerns the large pensions systems that are in place in many western economies. In this context we expect that both the accumulation decisions and the optimal retirement age are significantly affected by details of the pension system.

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