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Abstract

We extend the model of Fullerton, Karney, and Baylis (2012 working paper) to explore cost-effectiveness of unilateral climate policy in the presence of leakage. We ignore the welfare gain from reducing greenhouse gas emissions and focus on the welfare cost of the emissions tax or permit scheme. Whereas that prior paper solves for changes in emissions quantities and finds that leakage maybe negative, we show here that all cases with negative leakage in that model are cases where a unilateral carbon tax results in a welfare loss. With positive leakage, however, a unilateral policy can improve welfare.

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Keywords: trade and environment, pollution havens, unilateral climate policy.

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Leakage, Welfare, and Cost-Effectiveness of Carbon Policy

By Kathy Baylis, Don Fullerton, and Daniel H. Karney*

Policymakers fear that a unilateral carbon policy will reduce competitiveness, increase imports, and lead to higher carbon emissions elsewhere ("leakage"). In Don Fullerton, Daniel H. Karney and Kathy Baylis (2012), we show that it may actually reduce emissions in other sectors ("negative leakage"). But reducing emissions in both sectors may merely reflect welfare costs of carbon policy that reduce real income and thus reduce consumption of both outputs. These possibilities capture the concern that unilateral carbon policy might have a high cost per global unit of carbon abated (that is, low "cost-effectiveness").

Based on Harberger (1962), the two-input, two-output analytical general equilibrium model of Fullerton *et al* (2012) could represent two countries or two sectors of a closed economy. Each sector has some initial carbon tax or price, and the paper solves for the effect of a small increase in one sector's carbon tax on the quantity of emissions in each sector. But it does not solve for welfare effects. Here, we use the same model but derive expressions for the cost-effectiveness of a unilateral carbon tax – the welfare cost per ton of emission reduction. We show that higher leakage does not always mean lower welfare. If one sector is already taxed at a higher rate, then an increase in the other sector's tax might reduce deadweight loss from pre-existing misallocations. Thus, abatement can have negative cost. The welfare cost most directly depends on the relative levels of tax in the two sectors. We show that negative leakage always corresponds to a negative income effect, but negative income effects can also arise with positive leakage. Conversely, positive leakage does not always mean positive welfare cost.

Actual carbon policy is not likely to be applied uniformly across all countries and sectors. The EU Emission Trading Scheme (EU-ETS) only covers about 40 percent of emissions (http://ec.europa.eu/clima/policies/ets/index_en.htm). In the U.S., the Waxman-Markey bill proposed carbon policy primarily in the electricity sector. Metcalf and Weisbach (2009) estimate that even a very broad carbon policy can only include 80 to 90 percent of emissions, so applied carbon policy will likely leave some sectors uncovered. Raising one sector's carbon tax may have welfare costs if the other sector has no carbon tax, but on the other hand, that other sector

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may face an indirect price of carbon through taxes on fossil fuels such as gasoline. Those fuels may serve as substitutes for electricity, so a new carbon tax in the electricity sector may shift consumption back somewhat from the low-taxed electricity sector into other fuels. In that case, a new carbon tax just in the electricity sector may increase welfare despite positive leakage.

This paper makes several contributions. First, we demonstrate the generality of the Fullerton *et al* (2012) model by showing cases where leakage can exceed 100%. We solve for conditions under which total emissions increase or decrease. We also solve for welfare effects, and for "cost effectiveness" (the additional welfare cost per ton of net abatement). And we explore the relationship between the sign of leakage and the sign of the effect on welfare.

In addition, we decompose the change in deadweight loss into two components. First, the unilateral increase in carbon tax worsens a production distortion, as that sector substitutes from carbon to other inputs (such as labor or capital for abatement). Second, it affects a consumption distortion, the existing misallocation between the two outputs. Depending on the other sector's pre-existing carbon tax rate and carbon intensity, this consumption distortion may rise or fall.

Our prior paper shows that negative leakage occurs when the elasticity of substitution in utility is small and the elasticity of substitution in production is large. Here, we show that these are the same conditions that lead to higher deadweight loss from an increased carbon tax in one sector: a low elasticity in utility means that any reduction in the consumption distortion is relatively small, while any increase in the production distortion is relatively large. However, positive leakage may be associated either with welfare gains or losses. The intuition is that welfare cost most directly relates to the relative levels of tax in the two sectors, rather than to the relative changes in emissions. That is, a high cost per ton of carbon abatement can be associated with either negative or positive leakage. All proofs and derivations are in the Appendix.

I. The Change in Carbon Emissions

Using the model of Fullerton *et al* (2012), we demonstrate here the conditions under which an increase in one sector's carbon price may increase total emissions, and the conditions under which it is certain to decrease total emissions. The two competitive sectors have constant returns to scale production, $X = X(K_X, C_X)$ and $Y = Y(K_Y, C_Y)$, where a clean input K_i and carbon emissions C_i have decreasing marginal products ($i = X, Y$). The clean input can be labor, capital, or a composite of the two, with fixed total supply $\left(\overline{K} = K_X + K_Y\right)$. That input is mobile and earns the same factor price p_K in both sectors. Sector i can use any positive amount of C_i, given price τ_i (which can be a tax rate or permit price). Either sector might initially have the higher carbon price. Reducing total carbon emissions $C \equiv C_X + C_Y$ can have separable benefits in homothetic utility, $U(X, Y; C)$, but we focus only on the cost of the policy. Permit or tax revenue is $R \equiv \tau_X C_X + \tau_Y C_Y$, rebated in a lump sum. Many identical consumers use income $p_K \overline{K} + R$ to maximize utility by their choice of X and Y (facing prices p_X, p_Y, and p_K).

The simple version of this model assumes the supply of fossil fuel is perfectly elastic. It does not model traded oil in limited supply, so it misses the positive leakage caused when a carbon tax reduces one sector's demand, thereby reducing the price of oil and increasing use elsewhere. Instead, think of τ_Y applying to coal-fired power plants where coal is not scarce. The model does have positive leakage from the terms of trade effect (TTE) and negative leakage from the abatement resource effect (ARE). The goal in Fullerton *et al* (2012) is not to measure leakage but to demonstrate the ARE in a simple model that abstracts from other issues. That paper lists citations to discussion of these other issues.

The model is used to derive effects of a small increase in τ_Y, with no change in τ_X, where firms in sector Y can substitute away from carbon by additional use of abatement capital (K_Y) such as natural gas plants, wind turbines, or solar power. The model ignores any transition but instead compares initial allocations to those in a new long run equilibrium.

Given this set-up, Fullerton *et al* (2012) differentiate all equations above to derive a set of n linear equations with n unknowns, using a hat for each proportional change (e.g. $\hat{X} \equiv dX/X$). They differentiate production to get $\hat{Y} = \theta_{YK}\hat{K}_Y + \theta_{YC}\hat{C}_Y$, where θ_{ij} is a factor share [e.g. θ_{YK} = $(p_K K_Y)/(p_Y Y)$]. Define σ_Y as the elasticity of substitution in Y, to get $\hat{C}_Y - \hat{K}_Y = \sigma_Y(\hat{p}_K - \hat{\tau}_Y)$. The definition of σ_U implies $\hat{X} - \hat{Y} = \sigma_U(\hat{p}_Y - \hat{p}_X)$. Then, given a small exogenous increase in one carbon tax ($\hat{\tau}_Y > 0$), the system of linear equations is solved for the general equilibrium impact on each price and quantity as a function of parameters.

For sector Y, the increase in tax always raises the equilibrium price ($\hat{p}_Y = \theta_{YC}\hat{\tau}_Y > 0$) and reduces the equilibrium quantity ($\hat{Y} = -[\alpha_X\sigma_U + \alpha_Y\sigma_Y]\theta_{YC}\hat{\tau}_Y < 0$, where $\alpha_i = K_i/K$). The tax unequivocally reduces that sector's carbon emissions ($\hat{C}_Y < 0$). To calculate the total effect on carbon, we need to know the amount of leakage. As derived in our prior paper:

$$(1) \qquad \hat{C}_X = \alpha_Y(\sigma_U - \sigma_Y)\theta_{YC}\hat{\tau}_Y = \left[\underbrace{\sigma_U\alpha_Y\theta_{YC}}_{\text{TTE}} - \underbrace{\sigma_Y\alpha_Y\theta_{YC}}_{\text{ARE}}\right]\hat{\tau}_Y \quad \gtrless 0$$

The first term in equation (1) is the terms-of-trade effect (TTE), where the higher price of Y induces households to substitute into X (by an amount that depends on σ_U). This effect by itself increases production of X and emissions C_X. This positive leakage term is offset by a negative second term, the abatement resource effect (ARE), where the higher price of carbon induces firms to substitute into K_Y (by an amount that depends on σ_Y). If sector Y increases its use of capital, then sector X must reduce its use of capital, its output, and its emissions. (The price of carbon in sector X does not change relative to the cost of other inputs, so those firms do not change their ratio of inputs; less capital in X therefore means less emissions and less output.)

Theorem 1 (Fullerton *et al* 2012)*: Net leakage is negative when* $\sigma_Y > \sigma_U$. Equation (1) provides this result. When consumer substitution is low, they want to buy almost as much of the taxed output Y after the tax increase (such as with inelastic electricity demand). Producer substitution is high, so firms reduce carbon and use more capital, drawing capital from X.

From here, we develop several new theorems to characterize the conditions for total carbon emissions to fall. All proofs and derivations are in the Appendix.

Theorem 2: *Net negative leakage in this model implies that total carbon falls.* An increased carbon tax in sector Y clearly cuts the emissions of that sector. If the increase in τ_Y also reduces emissions of sector X, then total carbon emissions definitely fall.

Theorem 3: *If sector Y is carbon intensive* ($C_Y/K_Y > C_X/K_X$), *then total carbon falls*. Intuitively, increasing the carbon tax in the sector that uses carbon intensively creates a large decrease in emissions that overcomes any possible positive leakage. Importantly, these two theorems only provide sufficient conditions for a decrease in total carbon, as other parameter combinations may also lead to reductions of total carbon emissions.

Next, we identify necessary and sufficient conditions for an *increase* in total carbon emissions. For total emissions to rise, carbon leakage must be positive and large enough to exceed the reduction in C_Y. Thus, substitution in utility must be larger than substitution in sector Y production ($\sigma_U > \sigma_Y$), and sector X must be more carbon-intensive than sector Y (that is, $\alpha_Y > \beta_Y$, where $\alpha_Y \equiv K_Y / K$ and $\beta_Y \equiv C_Y / C$).

Theorem 4: *A necessary and sufficient condition for total carbon to increase* ($\hat{C} > 0$) *is* $(\sigma_U / \sigma_Y) > (\alpha_Y \theta_{YC} + \beta_Y \theta_{YK})/[(\alpha_Y - \beta_Y)\theta_{YC}] > 1$. An increase in total carbon requires not only that leakage is positive ($\sigma_U > \sigma_Y$). It also requires the denominator in the middle term to be positive, which means that Y must be relatively capital-intensive, while X is carbon intensive. Intuitively, increasing the carbon tax in a *capital*-intensive sector has little direct effect on carbon, while it does raise the relative price of Y. If σ_U is sufficiently high, consumers switch consumption from Y to X. Since the direct effect on C_Y is small, and substitution in consumption is large, carbon leakage can more than offset the direct reduction in emissions of the taxed sector.

II. The Change in Deadweight Loss

In Fullerton *et al* (2012), both sectors have non-zero pre-existing carbon tax rates. Here, we show that these taxes cause deadweight loss (DWL) via two channels. The first is a production distortion, since firms use too little carbon. Second, differential carbon tax rates change relative output prices and create a consumption distortion. We assume that environmental damages from carbon are separable in utility, so that we can focus on the loss in utility from consumption (the cost of abatement). We consider utility of our one worldwide consumer, not separate nations.

To quantify the change in deadweight loss (ΔDWL), we totally differentiate utility and follow the steps in our Appendix. Intuitively, the policy's utility cost is the difference in the bundle of X and Y that can be consumed before and after the tax change, where those changes in outputs can be written as changes in inputs. Then we can re-write ΔDWL as:

$$(2) \qquad -dU/\lambda = \Delta DWL = -(\tau_X C_X \hat{C}_X + \tau_Y C_Y \hat{C}_Y) \gtreqless 0$$

where λ is the marginal utility of income, so dU/λ is the monetary value of the change in utility. Thus, the sign of the change in deadweight loss is a function not only of the pre-existing tax rates but the sectors' relative carbon use. Furthermore, we can decompose the welfare loss into the consumption distortion and the production distortion:

$$(3) \qquad \Delta DWL = R\{\sigma_U[\alpha_X - \delta_X]\theta_{YC} + \sigma_Y[\alpha_Y\theta_{YC} + \delta_Y\theta_{YK}]\}\hat{\tau}_Y$$

where R is total tax revenue, $\delta_X \equiv \tau_X C_X/R$, and $\delta_Y \equiv \tau_Y C_Y/R$. Inside the curly brackets, the consumption distortion is the term that depends on σ_U, while the production distortion is the term that depends on σ_Y. An increase in τ_Y always worsens the production distortion in that sector (as firms switch from C_Y to K_Y). Also, the magnitude of this welfare effect increases with the initial tax rate τ_Y [through R in equation (3)]. Finally, ΔDWL is zero when $\sigma_U = \sigma_Y = 0$, because then τ_Y is essentially a lump-sum tax (with a lump-sum revenue rebate).

Theorem 5: *If sector Y has a higher carbon-weighted tax rate than sector X, then an increase in* τ_Y *raises deadweight loss. That is,* $\tau_Y(C_Y/K_Y) > \tau_X(C_X/K_X)$ *implies* $\Delta DWL > 0$. When the carbon-weighted tax rate in sector Y exceeds that in sector X, the further increase in τ_Y has welfare cost. The ΔDWL is positive because an increase τ_Y moves the weighted tax rates farther apart and thus increases distortions. Equation (2) also implies that if both sectors reduce use of carbon, the deadweight loss of the tax increase must be positive. In other words:

Theorem 6: *Negative leakage means a positive change in deadweight loss. That is,* $\hat{C}_X < 0$ *implies ΔDWL*>0. The increase in τ_Y always shrinks *Y*. If it also shrinks *X*, then utility of consumption must fall. The converse does not hold, however: *ΔDWL*>0 does not imply negative leakage. The reason is that *ΔDWL* also depends on initial tax rates. We next explore whether and when an increase in one sector's carbon tax leads to a decrease in deadweight loss.

For an increase in τ_Y to provide a welfare gain (*ΔDWL* < 0), Theorem 6 says that leakage must be positive (so we need $\sigma_U > \sigma_Y$). Further, in equation (3), the loss from the production distortion (the term in σ_Y) must be offset by a gain from reduced consumption distortion (the term in σ_U). That requires $\alpha_X < \delta_X$ (the share of carbon in sector *X* is smaller than the share of carbon revenue from *X*). From these two conditions and equation (3) above, we have:

Theorem 7: *The ΔDWL<0 if and only if* $\left[\alpha_Y\theta_{YC} + \delta_Y\theta_{YK}\right]/[(\delta_X - \alpha_X)\theta_{YC}] > \sigma_U/\sigma_Y > 1$.

Note that this condition requires $\sigma_U > \sigma_Y$ and $\alpha_X < \delta_X$. It looks similar to the condition for an increase in carbon (Theorem 4), except that the big ratio here must exceed the ratio of elasticities, and δ_i (shares of revenue) replace β_i (shares of carbon). The intuition is similar to that of Theorem 5: for DWL to fall, the carbon-weighted carbon tax in sector *X* must be larger than the carbon-weighted tax *Y*, so that an increase in τ_Y reduces the consumption distortion.

In summary, an increase in one sector's carbon tax can have negative marginal abatement cost, if it reduces deadweight loss by raising the *low* carbon tax rate. Next, we use *ΔDWL* and the quantity of carbon reduction to calculate of the cost-effectiveness of the policy.

III. Cost-Effectiveness

We measure the cost-effectiveness of a policy change as the "marginal cost of abatement" (MCA), the dollar value of the change utility divided by the change in carbon emissions:

$$\text{(4)} \qquad \text{MCA} = \frac{dU/\lambda}{dC} = \left[\frac{\alpha_Y(\sigma_U - \sigma_Y)\theta_{YC} - \delta_Y(\sigma_U\theta_{YC} + \sigma_Y\theta_{YK})}{\alpha_Y(\sigma_U - \sigma_Y)\theta_{YC} - \beta_Y(\sigma_U\theta_{YC} + \sigma_Y\theta_{YK})}\right]\left(\frac{R}{C}\right).$$

The fraction R/C is the *average* tax paid by firms per unit of carbon emissions at the initial tax rates; this ratio is always positive. The scalar in square brackets contains just elasticity and share parameters; it reflects the distortions in production and consumption. As demonstrated above, the sign of the numerator is ambiguous (*ΔDWL* ⋛ 0), as is the sign of the denominator (*dC* ⋛ 0). In fact, raising one tax rate may have welfare gain or loss even as *dC* approaches zero in the denominator, so the MCA approaches positive or negative infinity. In the "normal" case, the increase in carbon tax reduces carbon emissions, so the denominator is negative and we have:

Theorem 8: *If* $dC<0$, *then* $\tau_Y < \tau_X$ *implies the scalar in* (4) *is less than one (the MCA is less than the average cost,* R/C*).* In the normal case, a relatively low τ_Y can be increased with little welfare cost. Conversely, increasing a relatively high τ_Y means MCA larger than the average cost. To further explore this intuition, we consider two special cases.

A. Special case where $\tau_X = \tau_Y$

Assume both sectors have the same initial tax rate, $\tau_X = \tau_Y = \tau_C > 0$. Then the share of revenue from sector Y matches its share of carbon emissions ($\delta_Y = \beta_Y$), and from equation (4) we have MCA = $R/C = \tau_C$. That is, all firms in both sectors abate until the MCA equals the tax rate, common to all those firms, so the equi-marginal principle guarantees efficient allocation of abatement. Moreover, a higher initial tax rate means higher marginal cost of abatement.

B. Special case with no leakage

Assume $\sigma_U = \sigma_Y$, so $\hat{C}_X = 0$ from equation (1). The MCA can be written as the change in utility [$-\Delta DWL$ from equation (2)] over $dC = C_X\hat{C}_X + C_Y\hat{C}_Y$. Rearrangement yields MCA = τ_Y. Since leakage is zero, and input prices in sector X remains constant, all consumption changes are reductions in Y. Thus, the dollar-equivalent utility cost is the carbon tax rate in Y.

IV. The Relationship between Leakage and Welfare

We now explore the relationship between leakage and welfare effects of unilateral climate policy, using a numerical example and figure to help with intuition. When does the sign of the welfare effect match the sign of leakage? Two key parameters for both outcomes are σ_Y and σ_U, so Figure 1 shows the elasticity of substitution in production (σ_Y) on the horizontal axis and the elasticity of substitution in utility (σ_U) on the vertical axis. We know that leakage is zero when these two parameters equal each other, so the 45° line shows the boundary between cases where leakage is positive ($\sigma_U > \sigma_Y$) or negative ($\sigma_U < \sigma_Y$).

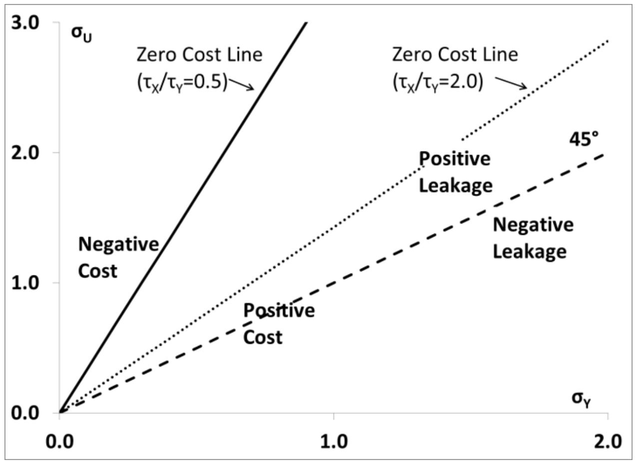
To get the boundary for the sign of the welfare effect, we set ΔDWL to zero in equation (3) above, and solve for σ_U in terms of σ_Y (see the Appendix):

$$(5) \qquad \sigma_U = \sigma_Y\left[1 + \frac{\delta_Y}{(\alpha_Y - \delta_Y)\theta_{YC}}\right].$$

Thus, the ΔDWL=0 line always goes through the origin. Also, Theorem 6 says that negative leakage implies positive ΔDWL. Therefore the ΔDWL=0 line must have a slope greater than one.

We then plot ΔDWL=0 lines for two different values of τ_X/τ_Y. When the initial τ_X is high relative to τ_Y, the policy to raise τ_Y is more likely to improve efficiency.

Figure: 1: The Sign of Leakage and the Sign of ΔDWL (C_X/K_X = 1.00 and C_Y/K_Y = 0.25)



Since $\delta_Y \equiv \tau_Y C_Y / R$, the slope of the ΔDWL=0 line is also determined partly by relative carbon intensity. If sector Y were carbon intensive, then τ_X must always exceed τ_Y for the increase in τ_Y to reduce deadweight loss. But if X is carbon intensive as in Figure 1, then raising τ_Y can improve welfare even when the initial $\tau_X < \tau_Y$. The solid line indicates ΔDWL=0 when the initial τ_X/τ_Y is only 0.5, so the area above that line shows combinations of σ_U and σ_Y where raising τ_Y has negative cost. When τ_X/τ_Y is 2.0, the dotted line shows an even wider area where raising τ_Y has negative cost. A larger initial τ_X/τ_Y means larger initial consumption distortion, which can be improved by raising τ_Y. The implication, as shown in the figure, is that the change in deadweight loss can be either sign when leakage is positive.

V. Conclusions

For unilateral climate policy, this paper uses a simple two-sector, two-input general equilibrium model to explore how leakage is related to welfare changes from consumption and the cost per ton of abatement (cost effectiveness). Even with this simple model, Fullerton *et al* (2012) find that leakage can be negative. Here, we find that positive leakage can more than offset the direct abatement achieved by the tax. We also explore the effect of the tax change on deadweight loss (the cost of abatement). As it turns out, the conditions that give rise to negative leakage always result in welfare costs. Yet positive leakage can be associated either with gains or losses.

In addition, we show that a policy without leakage is not necessarily more cost efficient than a policy with leakage. One sector's tax increase can reduce a consumption distortion by more than it increases the production distortion, if the initial carbon tax in the *other* sector is relatively high. A higher elasticity of substitution in consumption increases this welfare gain, but it also increases leakage. In other words, when the tax increase cuts the gap between the two tax rates, the conditions that give rise to a welfare gain also give rise to leakage.

These results have important policy implications for two reasons. First, carbon policy proposals can cover only a fraction of emissions. Even if the same tax could apply to electricity and other sectors, it could not apply to all emissions (e.g. homeowners can cut their own firewood for heat, which is difficult to monitor). Second, most sectors already face an implicit price on carbon. For example, the EU-ETS covers only "major industries" such as electricity, cement, and some manufacturing (only 40 percent of emissions). Yet other sectors also face a price of carbon (such as gasoline taxes in the transportation sector or BTU taxes on home heating fuel). Even if an explicit carbon tax is imposed only in one sector, with positive leakage, it may still raise welfare by reducing the consumption distortion from high fuel taxes in other sectors.

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Appendix

Proof of Theorem 1: See Fullerton *et al* (2012).

Proof of Theorem 2: Totally differentiate $C = C_X + C_Y$ to get $dC = dC_X + dC_Y$, and then multiply and divide through by appropriate terms, using the hat notation:

$$\hat{C} = \beta_X \hat{C}_X + \beta_Y \hat{C}_Y$$

where $\beta_Y = C_Y / C$ and $1 = \beta_X + \beta_Y$. Recall that $\hat{\tau}_Y > 0$ implies $\hat{C}_Y < 0$ always, and thus negative leakage $\left(\hat{C}_X < 0\right)$ clearly leads to a fall in total carbon emissions. ■

Proof of Theorem 3: Using $\hat{C} = \beta_X \hat{C}_X + \beta_Y \hat{C}_Y$, insert solutions of Fullerton *et al* (2012):

$$\hat{C} = \beta_X [\alpha_Y (\sigma_U - \sigma_Y)\theta_{YC}]\hat{\tau}_Y + \beta_Y [-(\alpha_X \sigma_U + \alpha_Y \sigma_Y)\theta_{YC} - \theta_{YK}\sigma_Y]\hat{\tau}_Y$$

and simplify to yield:

$$\hat{C} = \{\sigma_U (\alpha_Y - \beta_Y)\theta_{YC} - \sigma_Y (\alpha_Y \theta_{YC} + \beta_Y \theta_{YK})\}\hat{\tau}_Y .$$

By inspection, $\alpha_Y < \beta_Y$ is a sufficient condition for $\hat{C} < 0$. However, the condition $\alpha_Y < \beta_Y$ implies both $(C_Y/K_Y) > (C/K)$ and $(C/K) > (C_X/K_X)$. ■

Proof of Theorem 4: We know $\hat{C} = \{\sigma_U (\alpha_Y - \beta_Y)\theta_{YC} - \sigma_Y (\alpha_Y \theta_{YC} + \beta_Y \theta_{YK})\}\hat{\tau}_Y$. For total carbon emissions to increase, it follows that:

$$\hat{C} = \{\sigma_U (\alpha_Y - \beta_Y)\theta_{YC} - \sigma_Y (\alpha_Y \theta_{YC} + \beta_Y \theta_{YK})\}\hat{\tau}_Y > 0$$

$$\Leftrightarrow \frac{\sigma_U}{\sigma_Y} > \frac{(\alpha_Y \theta_{YC} + \beta_Y \theta_{YK})}{(\alpha_Y - \beta_Y)\theta_{YC}}.$$

From the proof of Theorem 3, we know that $\hat{C} > 0$ requires $\alpha_Y > \beta_Y$, and so the denominator of that last expression is positive. But the positive numerator necessarily exceeds the positive denominator, so we have $\frac{\sigma_U}{\sigma_Y} > \frac{(\alpha_Y \theta_{YC} + \beta_Y \theta_{YK})}{(\alpha_Y - \beta_Y)\theta_{YC}} > 1$. ■

Derivation of Equation 2: Totally differentiate the utility function to yield:

$$dU = U_X dX + U_Y dY + U_C dC$$

Next, substitute in the first-order conditions from the utility maximization problem:

$$dU = \lambda p_X dX + \lambda p_Y dY + U_C dC$$

where λ is the multiplier on the budget constraint. Divide through by λ, so the left-hand side is the dollar value of the change in utility:

$$\frac{dU}{\lambda} = p_X dX + p_Y dY + \frac{U_C}{\lambda} dC$$

Continue by substituting in the totally differentiated production functions for *X* and *Y*, and the totally differentiated definition of total carbon:

$$\frac{dU}{\lambda} = p_X (X_K dK_X + X_C dC_X) + p_Y (Y_K dK_Y + Y_C dC_Y) + \frac{U_C}{\lambda}(dC_X + dC_Y)$$

where X_K is the marginal product of X (K_X, C_X) with respect to K_X. Other terms are similarly defined. Next, use first-order conditions from profit maximization (e.g. $p_K = p_X X_K$):

$$\frac{dU}{\lambda} = p_K dK_X + \tau_X dC_X + p_K dK_Y + \tau_Y dC_Y + \frac{U_C}{\lambda}(dC_X + dC_Y)$$

Recall that $dK_X = -dK_Y$, so:

$$\frac{dU}{\lambda} = \tau_X dC_X + \tau_Y dC_Y + \frac{U_C}{\lambda}(dC_X + dC_Y)$$

Define $\mu_C \equiv -U_C/\lambda$ as the marginal environmental damage from carbon, and rearrange terms:

$$\frac{dU}{\lambda} = (\tau_X - \mu_C)dC_X + (\tau_Y - \mu_C)dC_Y$$

Multiply and divide though by C_X and C_Y, respectively:

$$\frac{dU}{\lambda} = (\tau_X - \mu_C)C_X\hat{C}_X + (\tau_Y - \mu_C)C_Y\hat{C}_Y$$

In the first term, any increase in C_X reduces welfare if the existing τ_X is not high enough to correct for carbon damages μ_C (and similarly for C_Y). However, our measure of the policy's cost ignores benefits from reduced carbon damages, and so we delete the μ_C parameter:

$$\frac{dU}{\lambda} = \tau_X C_X\hat{C}_X + \tau_Y C_Y\hat{C}_Y .$$

Finally, multiply by –1, which converts a negative gain to a positive change in deadweight loss.

Derivation of Equation 3: From $\Delta DWL = -\left(\tau_X C_X\hat{C}_X + \tau_Y C_Y\hat{C}_Y\right)$, insert solutions for $\hat{C}_X$ and $\hat{C}_Y$:

$$\Delta DWL = -\left(\tau_X C_X\left[\alpha_Y(\sigma_U - \sigma_Y)\theta_{YC}\right]\hat{\tau}_Y + \tau_Y C_Y\left[-(\alpha_X\sigma_U + \alpha_Y\sigma_Y)\theta_{YC} - \theta_{YK}\sigma_Y\right]\hat{\tau}_Y\right)$$

Note that $\delta_i \equiv \tau_i C_i/R$. We can replace $\tau_i C_i = \delta_i R$, where R is total tax revenue:

$$\Delta DWL = -R\left\{\delta_X\left[\alpha_Y(\sigma_U - \sigma_Y)\theta_{YC}\right] - \delta_Y\left[(\alpha_X\sigma_U + \alpha_Y\sigma_Y)\theta_{YC} + \theta_{YK}\sigma_Y\right]\right\}\hat{\tau}_Y$$

Rearranging to collect the terms σ_U and σ_Y we obtain

$$\Delta DWL = R\left\{\sigma_U\left[\alpha_X - \delta_X\right]\theta_{YC} + \sigma_Y\left[\alpha_Y\theta_{YC} + \delta_Y\theta_{YK}\right]\right\}\hat{\tau}_Y .$$

Proof of Theorem 5: From $\Delta DWL = R\left\{\sigma_U\left[\alpha_X - \delta_X\right]\theta_{YC} + \sigma_Y\left[\alpha_Y\theta_{YC} + \delta_Y\theta_{YK}\right]\right\}\hat{\tau}_Y$, by inspection, $\alpha_X > \delta_X$ is a sufficient condition for $\Delta DWL>0$. However, the condition $\alpha_X > \delta_X$ necessarily means both $(\tau_Y C_Y/K_Y) > (R/K)$ and $(R/K) > (\tau_X C_X/K_X)$). ■

Proof of Theorem 6: Use $\Delta DWL = -\left(\tau_X C_X\hat{C}_X + \tau_Y C_Y\hat{C}_Y\right)$ and $\hat{C}_Y < 0$. Thus, negative leakage $\left(\hat{C}_X < 0\right)$ clearly leads to an increase in deadweight loss. ■

Proof of Theorem 7: From equation (3), a reduction in deadweight loss requires:

$$\Delta DWL = R\{\sigma_U[\alpha_X - \delta_X]\theta_{YC} + \sigma_Y[\alpha_Y\theta_{YC} + \delta_Y\theta_{YK}]\}\hat{\tau}_Y < 0$$

$$\Leftrightarrow \frac{\sigma_U}{\sigma_Y} < -\frac{[\alpha_Y\theta_{YC} + \delta_Y\theta_{YK}]}{[\alpha_X - \delta_X]\theta_{YC}}, \text{ where the latter term equals } \frac{[\alpha_Y\theta_{YC} + \delta_Y\theta_{YK}]}{[\delta_X - \alpha_X]\theta_{YC}}.$$

Also, Theorem 6 says that negative leakage always means a positive change in deadweight loss. Thus, ΔDWL<0 requires that leakage be positive ($\sigma_U > \sigma_Y$). Accordingly, σ_U/σ_Y exceeds one, and:

$$\Leftrightarrow 1 < \frac{\sigma_U}{\sigma_Y} < \frac{[\alpha_Y\theta_{YC} + \delta_Y\theta_{YK}]}{[\delta_X - \alpha_X]\theta_{YC}}. \blacksquare$$

Derivation of Equation 4: We construct the MCA by dividing -ΔDWL by the totally differentiated change in total carbon emissions to yield:

$$\text{MCA} = \frac{(dU/\lambda)}{dC} = \frac{\tau_X C_X \hat{C}_X + \tau_Y C_Y \hat{C}_Y}{C_X \hat{C}_X + C_Y \hat{C}_Y}.$$

Next, substitute to closed-form expressions for the baseline $\hat{C}_X$ and $\hat{C}_Y$:

$$\frac{(dU/\lambda)}{dC} = \frac{\tau_X C_X[\alpha_Y(\sigma_U - \sigma_Y)\theta_{YC}\hat{\tau}_Y] - \tau_Y C_Y[(\alpha_X(\sigma_U - \sigma_Y)\theta_{YC} + \sigma_Y)\hat{\tau}_Y]}{C_X[\alpha_Y(\sigma_U - \sigma_Y)\theta_{YC}\hat{\tau}_Y] - C_Y[(\alpha_X(\sigma_U - \sigma_Y)\theta_{YC} + \sigma_Y)\hat{\tau}_Y]}$$

and simplify:

$$\frac{(dU/\lambda)}{dC} = \frac{\theta_{YC}(\sigma_U - \sigma_Y)[\alpha_Y\tau_X C_X - \alpha_X\tau_Y C_Y] - \sigma_Y\tau_Y C_Y}{\theta_{YC}(\sigma_U - \sigma_Y)[\alpha_Y C_X - \alpha_X C_Y] - \sigma_Y C_Y}.$$

noting that $\hat{\tau}_Y$ cancels from numerator and denominator. Continuing, define $\delta_X \equiv \tau_X C_X / R$ and $\delta_Y \equiv \tau_Y C_Y / R$, where $\delta_X + \delta_Y = 1$ and R is the total tax revenue; that is, δ_X is the share of tax revenue collected from sector X. Similarly, define the carbon share in sector X as $\beta_X \equiv C_X / C$ and define β_Y analogously. Rewrite the MCA in terms delta- and beta-shares:

$$\frac{(dU/\lambda)}{dC} = \frac{\theta_{YC}(\sigma_U - \sigma_Y)[\alpha_Y\delta_X - \alpha_X\delta_Y]R - \sigma_Y\delta_Y R}{\theta_{YC}(\sigma_U - \sigma_Y)[\alpha_Y\beta_X - \alpha_X\beta_Y]C - \sigma_Y\beta_Y C}.$$

Finally, factor (R/C) and simplify the term in square brackets to yield equation (4) in the text.

Proof of Theorem 8: To start, observe $\tau_X > \tau_Y$ if and only if $\beta_Y > \delta_Y$ and $\delta_X > \beta_X$. In that case, the theorem to be proven says that:

$$\text{MCA} = \frac{\tau_X C_X \hat{C}_X + \tau_Y C_Y \hat{C}_Y}{C_X \hat{C}_X + C_Y \hat{C}_Y} < \frac{R}{C}.$$

The denominator is negative by assumption, so we need to show:

$$\tau_X C_X \hat{C}_X + \tau_Y C_Y \hat{C}_Y > (C_X \hat{C}_X + C_Y \hat{C}_Y)\left(\frac{R}{C}\right)$$

$$\Leftrightarrow \left(\tau_X C_X \hat{C}_X + \tau_Y C_Y \hat{C}_Y\right)\left(\frac{1}{R}\right) > \left(C_X \hat{C}_X + C_Y \hat{C}_Y\right)\left(\frac{1}{C}\right)$$

$$\Leftrightarrow \delta_X \hat{C}_X + \delta_Y \hat{C}_Y > \beta_X \hat{C}_X + \beta_Y \hat{C}_Y$$

$$\Leftrightarrow \hat{C}_X > \frac{(\beta_Y - \delta_Y)}{(\delta_X - \beta_X)}\hat{C}_Y \qquad \Leftrightarrow \hat{C}_X > \frac{((1-\beta_X)-(1-\delta_X))}{(\delta_X - \beta_X)}\hat{C}_Y$$

$$\Leftrightarrow \hat{C}_X > \frac{(-\beta_X + \delta_X)}{(\delta_X - \beta_X)}\hat{C}_Y \qquad \Leftrightarrow \hat{C}_X > \hat{C}_Y .$$

Insert solutions for $\hat{C}_X$ and $\hat{C}_Y$, and we still need to show that $\hat{C}_X > \hat{C}_Y$:

$$[\sigma_U \alpha_Y \theta_{YC} - \sigma_Y \alpha_Y \theta_{YC}]\hat{\tau}_Y > [-(\alpha_X \sigma_U + \alpha_Y \sigma_Y)\theta_{YC} - \theta_{YK}\sigma_Y]\hat{\tau}_Y$$

$$\Leftrightarrow \alpha_Y \sigma_U \theta_{YC} + \alpha_X \sigma_U \theta_{YC} > -\theta_{YK}\sigma_Y$$

$$\Leftrightarrow \theta_{YC}\sigma_U > -\theta_{YK}\sigma_Y .$$

Thus, it is always true that $\hat{C}_X > \hat{C}_Y$. ■

Derivation of Equation 5: We use $\Delta DWL = R\{\sigma_U[\alpha_X - \delta_X]\theta_{YC} + \sigma_Y[\alpha_Y\theta_{YC} + \delta_Y\theta_{YK}]\}\hat{\tau}_Y$ and set it equal to zero, which means:

$$\sigma_U = \sigma_Y \frac{[\alpha_Y\theta_{YC} + \delta_Y\theta_{YK}]}{[\alpha_Y - \delta_Y]\theta_{YC}} \quad = \quad \sigma_Y \frac{[\alpha_Y\theta_{YC} + \delta_Y(1-\theta_{YC})]}{[\alpha_Y - \delta_Y]\theta_{YC}} \quad = \quad \sigma_Y\left[1 + \frac{\delta_Y}{(\alpha_Y - \delta_Y)\theta_{YC}}\right].$$