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Emissions Taxes and Abatement Regulation under Uncertainty

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Abstract

We consider environmental regulation in a context where firms invest in abatement technology under conditions of uncertainty about subsequent abatement cost, but can subsequently adjust output in the light of true marginal abatement cost. Where an emission tax is the only available instrument, policy faces a trade-off between the incentive to invest in abatement technology and efficiency in subsequent output decisions. More efficient outcomes can be achieved through combined use of tax and mandated use of a given abatement technology or through combining the tax with an abatement technology investment subsidy. We compare the properties of the two potential supplements to the emissions tax.

JEL-Code: H230.

Keywords: externalities, Pigouvian taxes, subsidies, regulation.

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1. Introduction

It is well known that uncertainty about the costs of pollution abatement has implications for the choice between alternative forms of pollution regulation. Weitzman's celebrated analysis (Weitzman, 1974) sets out conditions under which regulation using a price-based instrument such as an environmental tax on emissions would be more or less efficient than regulation by quantity, such as a system of tradeable emission quotas. Developing this line of analysis, Roberts and Spence (1976) show that in fact a mixed instrument, combining elements of price and quantity regulation, would out-perform either price or quantity regulation alone. In particular, they demonstrate that the expected welfare costs of quantity regulation based on tradeable emission quotas will be reduced if supplemented by an element of price regulation, in the form of upper and lower bounds to the permitted range of permit prices. Judged against the ex post optimum this mixed regulatory regime – with appropriately-chosen parameters - performs better than either price or quantity regulation alone.

In this paper, we explore the implications of uncertainty for optimal environmental regulation in a context which extends, and in some respects contrasts with the approach taken by these earlier papers.

Like Roberts and Spence we are particularly interested in characterising cases where combined use of two regulatory instruments may be able to achieve outcomes which mitigate the potentially-extreme *ex post* inefficiencies that arise from dependence on a single regulatory approach. Unlike Roberts and Spence, however, we focus on cases where efficient quantity regulation, in the form of tradeable emissions quotas, is not an available option. This is a case of considerable practical relevance, especially in contexts where the number of firms to be regulated is too small to sustain a competitive market for tradable permits, or where the transactions costs of emissions trading are too high. We therefore focus on an initial regulatory approach based on an emissions tax, supplemented by other instruments with certain second-best limitations. Nevertheless it would be reasonable to anticipate that there would be some symmetry between our analysis and that of Roberts and Spence, and we explore the extent to which this is the case.

A second respect in which our analysis develops this earlier literature is that we provide a clearer characterisation of the uncertainties and informational asymmetries, and the way in which these constrain the operation of environmental regulation. Like this earlier literature we consider cases where uncertainty differentially affects governments and firms, and unlike policy-makers, the firms in our model are able to make certain decisions in the light of the true realised state. However, our model involves a richer set of decisions, which provide a clearer context for this asymmetry. In particular, both governments and firms are forced to make decisions about one form of abatement, based on long-term investment, before the uncertainty is resolved. Firms then make a second set of decisions that also affect emissions, and these decisions are made after the uncertainty has been resolved. We argue later that this context provides a clearer motivation for the presence of uncertainty, and its differential impact on policy and business decision-makers than the rather ad hoc uncertainty in the earlier literature.

The model we consider is one in which a regulatory agency seeks to control pollution from firms which produce a commodity that they sell at an exogenous price. The assumption that the price is exogenous allows us to neglect general equilibrium effects, and to focus on the issues of direct interest. We might for example think of this as a commodity which is traded in a global market, but other interpretations are possible.

We represent uncertainty in our model as uncertainty about the output price[1]. This is unknown to the authorities at the time the regulatory parameters are set, and is unknown to the firm at the time they make long-term irreversible investments in abatement technology, but firms make their output decisions based on the true price realisation. When the world price for the product is high, firms will wish to produce more, and will generate additional pollution. To the extent that pollution abatement can be partly achieved through output reductions, a higher output price translates into a higher marginal cost of pollution abatement.

Uncertainty about the costs of pollution abatement can take different forms. We can imagine a once-and-for-all scenario where at some future date a state of the world will unfold and stay unchanged for a sufficiently long period to allow us to disregard what happens beyond this horizon. A priori, it is uncertain which state of the world will emerge. Alternatively, we can

[1] A similar representation of abatement cost uncertainty appears in Quirion (2005), who compares separately the merits of three possible policy instruments, a tax, an absolute emissions cap, and a relative emissions cap.

conceive of a vibrating system with variation between states over time[2]. We may know the set of states that will occur but may be unable to monitor continuously the randomly generated states, and even if that were possible the authorities might be unable to adjust policy to the current transitions between states as policy-making takes time. With policy responses to variation in states precluded, the policy problem will be to set available instruments so as to maximise average social efficiency over time or across contingencies. In the former once-and-for-all case, it is harder to justify that one will have to stick to the same policy after the realisation of the uncertainty, but a case for a once-and-for-all solution may still be made as argued by Roberts and Spence (1976, p. 193)[3]. In what follows we shall not distinguish between the two cases but treat both the single uncertain event and the sequence of randomly generated states as permissible interpretations of our analysis. A third type of uncertainty may be that the government lacks information about circumstances that are crucial for the effects of policy decisions ("inadequate information" or "an information gap" in the words of Weitzman (1974, p. 480.)

Related literature includes a number of studies of instrument choice, and studies of irreversible abatement investment under uncertainty.

In the first category, Christiansen and Smith (2011) consider how second-best limitations on available instruments give rise to a case for combined use of multiple instruments, even in the absence of uncertainty. The instrument choice literature also includes discussion of the properties of *relative* emissions quotas - in other words, quotas defined relative to output - which will tend to encourage abatement, while having a lower impact on output decisions than absolute quotas. Ebert (1998) for example, considers a model of relative standards in a context with no uncertainty and identical firms. He characterises the optimal relative standard where it is the only instrument employed and shows that it is never equivalent to an optimally chosen environmental tax.

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[2] This is a possibility alluded to by Weitzman (1974): He refers to "elements of genuine randomness" (op. cit. p. 480) and adds: "like day-to-day fluctuations" (op. cit. footnote 1).

[3] "…we are rejecting the idea that the government can 'feel out' the 'optimum' by successively announcing and revising its policies…" "Given these circumstances, we have opted for the once-and –for –all-problem…" Op. cit. p. 193.

The literature on irreversibility in abatement investment includes a number of papers looking at the timing of abatement investments, assuming a stochastic time path for demand or output price (see eg Xepapadeas, 2001, and Chao and Wilson, 1993). In this context investors have the option to delay irreversible investment, and will choose to do so in states where prices are unfavourable. As a result, the market price for emissions allowances will exceed the marginal cost of abatement investment by an amount which can be intrepreted as an option value. In our model, these issues do not arise, as we assume that a single investment (commitment) decision is made, and that the firm has no choice about timing.

Baldursson and Fehr (2004) set up a model where profits are uncertain due to uncertain future number of firms and uncertain polluting emissions prior to abatement. They consider a long-term irreversible investment under uncertainty and a short-term abatement decision after the uncertainty is resolved, with both decisions affecting polluting emissions. This is a distinction similar to ours. Most of the analysis is devoted to quota regimes, but they also briefly address a tax regime. Since, by the assumptions of their model, the marginal profitability of investment is deterministic cost efficiency in emission reductions is achieved as one replicates the conventional result that all abatement activities are carried out up to the point where the respective marginal abatement costs are equated to the tax.

In the next section we set up the basic economic model. We characterise the first best solution in Section 3, and describe the behaviour of the producer in Section 4. In Section 5 we consider regulation when the government employs an emission tax as the sole instrument. Sections 6 and 7 are devoted to the cases where the emissions tax can be suppemented by direct regulation of abatement technology or subsidies to investment in abatement technology. In section 8 we turn our attention to the case where the investment cost is perceived as uncertain by the government. Section 9 concludes.

2. The economic setting

Our model is one in which we have a number of firms which produce a good, using a production process which generates emissions. At an initial stage, when the firms must make a decision about the abatement technology they will use, they face uncertainty about output prices. Since reducing output reduces emissions, this means they also face *ex ante* uncertainty about abatement costs. However, at a subsequent stage, when the firms come to make decisions about output levels, these are made in full knowledge of the output price.

We formalise this in terms of a representative firm that produces x units of a good. The good is sold at an exogenous price p, which is assumed to be stochastic with density $f(p)$. Our assumption that the price is exogenous is made to avoid unnecessary complexity; it might, for example, be understood to reflect the case of firms which are price-takers in international trade. To get a tractable model, we also assume simple functional forms. The (private) cost of production is $cx^2/2$ where c is a positive parameter. Production generates emissions of $(1-\gamma)$ per unit produced. γ parameterises the choice of abatement technology, which is made in the first stage and is assumed to be irreversible; it can be interpreted as the scale of an abatement activity. In the absence of any abatement activity γ =0. In general, $0 \le \gamma \le 1$. The cost of abatement is $k(\gamma)x$ where $k(0) = 0$, $k'(\gamma)$ >0 and $k''(\gamma)$ >0.

In general, firms have different cost structures and may choose different output and abatement levels. We use subscripts when referring to a specific firm, but omit them when referring to a single unspecified firm. Firm i has a (private) cost of production $c_i x_i^2/2$, and its abatement technology is γ_i. Aggregate emissions are then $\sum_i (1-\gamma_i)x_i$. The emissions cause external effects. The external cost is an increasing and strictly convex function of aggregate emissions expressed as $\frac{1}{2}e\left(\sum_i (1-\gamma_i)x_i\right)^2$ where e is a positive parameter. We see that the external cost is increasing in output (x_i) and decreasing in abatement effort (γ_i).

Two features of our model may be noted. Firstly, abatement is possible after the uncertainty about output prices has been resolved, but the cost of this abatement is uncertain at the time of the technology investment decision. Secondly, adopting a more expensive abatement technology increases the cost of producing each unit of output; we may think of this as a cost of operating the abatement technology, such as, for example, the purchase of more or less sophisticated filters which wear out in proportion to the amount produced.

One possible practical interpretation of our model is that it represents the type of decisions that have to be made about pollution abatement in the power sector. Typically, acid rain policies include some major investments in pollution abatement facilities at power stations, such as flue gas desulphurisation “scrubbers. These investments are large and irreversible, and have to be made in the face of uncertainty about many relevant variables, including the future level of the electricity price. They also significantly affect the costs of future changes in output and emissions. Where the electricity price in future is low, a “dirty” power station that has not made much investment in abatement facilities might be able to reduce emissions relatively cheaply by reducing its level of output. In other words, the initial abatement technology decision, made before future prices are known, will have implications for the costs of reducing emissions during future operation.

Given our assumptions, the social surplus arising from the activities of the firms is the gross output value net of social (private and external) costs:

$$\Psi = \sum_i p x_i - \sum_i \frac{1}{2} c_i x_i^2 - \frac{1}{2} e \left(\sum_i (1-\gamma_i) x_i \right)^2 - \sum_i k(\gamma_i) x_i \qquad (1)$$

Emissions can be diminished either by increasing γ or lowering x. We note that the marginal cost of abatement by lowering x is $p - cx - k(\gamma)$ which is stochastic due to the randomness of p.

3. The first best optimum

As a benchmark we begin by characterising the first best optimum. This is defined as the values of x_i that maximise the social surplus conditional on p and γ_i and the γ_i values that maximise the expected social surplus since there will be uncertainty at the time of decision. Maximising

$$\Psi = \sum_i px_i - \sum_i \frac{1}{2} c_i x_i^2 - \frac{1}{2} e \left(\sum_i (1-\gamma_i) x_i \right)^2 - \sum_i k(\gamma_i) x_i$$

with respect to x_j, yields the first order condition

$$\frac{\partial \Psi}{\partial x_j} = p - c_j x_j - k(\gamma_j) - e \left(\sum_i (1-\gamma_i) x_i \right) (1-\gamma_j) = 0 \qquad (2)$$

The marginal cost is composed of the production cost, the clean-up cost and the external cost, and this is then equated to the marginal value of a unit of output given by its price.

Denote by $f(p)$ the density of the probability distribution of the stochastic price. The expected social surplus is

$$E(\Psi) = \int \left(\sum_i px_i - \sum_i \frac{1}{2} c_i x_i^2 - \frac{1}{2} e \left(\sum_i (1-\gamma_i) x_i \right)^2 - \sum_i k(\gamma_i) x_i \right) f(p) dp \qquad (3)$$

The optimal choice of γ_j is characterised by the first order condition

$$\frac{\partial E(\Psi)}{\partial \gamma_j} = \int \left(-e \left(\sum_i (1-\gamma_i) x_i \right) (-x_j) - k'(\gamma_j) x_j \right) f(p) dp = 0$$

A couple of manipulations yield

$$\frac{\partial E(\Psi)}{\partial \gamma_j} = \int \left(e \left(\sum_i (1-\gamma_i) x_i \right) - k'(\gamma_j) \right) x_j f(p) dp = 0 \qquad (4)$$

This equation has a straightforward interpretation. It states that the marginal cost of choosing a better abatement technology in firm *j* should be equated to the marginal benefit it generates. In general, different technologies will be chosen for different firms.

Making use of standard statistical concepts, we can alternatively write the condition

$$\frac{\partial E(\Psi)}{\partial \gamma_j} = \sum_i e(1-\gamma_i)\int x_i x_j f(p)dp - k'(\gamma_j)E(x_j)$$
$$= \sum_i e(1-\gamma_i)E(x_i x_j) - k'(\gamma_j)E(x_j)$$
$$= \sum_i e(1-\gamma_i)\operatorname{cov}(x_i, x_j) + eE(x_j)\sum_i (1-\gamma_i)E(x_i) - k'(\gamma_j)E(x_j) = 0$$

and furthermore

$$\sum_i e(1-\gamma_i)\operatorname{cov}(x_i, x_j)/E(x_j) + \left(e\sum_i (1-\gamma_i)E(x_i) - k'(\gamma_j)\right) = 0$$

The optimal values of γ_j are then characterised by

$$k'(\gamma_j) = e\sum_i (1-\gamma_i)E(x_i) + \sum_i e(1-\gamma_i)\operatorname{cov}(x_i, x_j)/E(x_j) \qquad (5)$$

Since a higher price will induce all firms to produce higher output the covariances are positive. The marginal cost of choosing a better abatement technology should be equated to its marginal benefit which exceeds the marginal benefit at the expected output levels (the first term on the right hand side). The reason is that a superior abatement technology will have a bigger impact when output is high, which is when emissions and the marginal external cost are greater.

4. Producer behaviour

Recall that producers initially face uncertainty about the output price, and during this period must nonetheless commit to a decision about the abatement technology they will employ, γ_i.

The profits of each producer *i* are

$$\Pi_i = px_i - \frac{1}{2}cx_i^2 - \tau(1-\gamma_i)x_i - k(\gamma_i)x_i \qquad (6)$$

After the price uncertainty has been resolved, the firm chooses an output level to maximise profits. This yields the first order condition

$$\frac{\partial \Pi}{\partial x_i} = p - cx_i - \tau(1-\gamma_i) - k(\gamma_i) = 0 \ ,$$

and it follows that

$$x_i = \frac{p - \tau(1-\gamma_i) - k(\gamma_i)}{c_i}. \quad (7)$$

Prior to the price uncertainty being resolved, expected profits are

$$E(\Pi_i) = \int \Pi_i f(p)dp = \int \left[px_i - \frac{1}{2}cx_i^2 - \tau(1-\gamma_i)x_i - k(\gamma_i)x_i \right] f(p)dp \quad (8)$$

The firm chooses γ_i, the abatement technology, to maximise expected profits. This yields the first order condition

$$\begin{aligned} \frac{\partial}{\partial \gamma_i} E(\Pi_i) &= \frac{\partial}{\partial \gamma_i} \int \Pi_i f(p)dp = \frac{\partial}{\partial \gamma_i} \int \left[px_i - \frac{1}{2}cx_i^2 - \tau(1-\gamma_i)x_i - k(\gamma_i)x_i \right] f(p)dp \\ &= \int \left[\tau x_i - k'(\gamma_i)x_i \right] f(p)dp = (\tau - k'(\gamma_i))E(x_i) = 0 \end{aligned} \quad (9)$$

It follows that $k'(\gamma_i) = \tau$, which implies that all producers will set the same value of γ_i:

$$\gamma_i = \gamma = k'^{-1}(\tau). \quad (10)$$

Then

$$x_i = \frac{p - \tau(1-\gamma) - k(\gamma)}{c_i} \quad (11)$$

$$\frac{\partial x_i}{\partial \tau} = -\frac{1-\gamma}{c_i} \quad (12)$$

$$\frac{\partial x_i}{\partial \gamma} = \frac{\tau - k'(\gamma)}{c_i} = -\frac{\tau - k'(\gamma)}{1-\gamma} \frac{\partial x_i}{\partial \tau} \quad (13)$$

We note that for the optimum choice of γ_i, $\partial x_i / \partial \gamma = 0$

$$\partial\gamma / \partial\tau = 1 / k''(\gamma) \tag{14}$$

A larger tax will diminish output and encourage investment in abatement technology. Requiring firms to use a cleaner abatement technology will diminish output by imposing requirements that makes production more expensive.

5. The government sets τ only

In this and subsequent sections we consider various government policies. We start with the case where the government sets an emissions tax but does not directly interfere with the producer's choice of abatement technology. Our assumption is that this tax must be set prior to output price uncertainty being resolved, and that it then must remain fixed. In setting the emissions tax τ the government takes into account that this will affect the producer's choice of output once the price becomes known, and also the producer's choice of abatement technology prior to knowing the price.

The government maximises the expected social surplus

$$E(\Psi) = \int \left(\sum_i \left(px_i - \frac{1}{2}cx_i^2 - k(\gamma)x_i \right) - \frac{1}{2}e(1-\gamma)^2 \left(\sum_j x_j \right)^2 \right) f(p)dp$$

The first order condition is

$$\begin{aligned} \frac{dE(\Psi)}{d\tau} &= \int \left(\sum_i \left(p - cx_i - k(\gamma) \right) \frac{\partial x_i}{\partial \tau} - e(1-\gamma)^2 \left(\sum_j x_j \right) \sum_i \frac{\partial x_i}{\partial \tau} \right) f(p)dp \\ &+ \int \left(e(1-\gamma) \left(\sum_j x_j \right)^2 - k'(\gamma) \sum_i x_i \right) \frac{\partial \gamma}{\partial \tau} f(p)dp = 0 \end{aligned} \tag{15}$$

where $dE(\Psi)/d\tau$ is the total derivative allowing for the effects via both x and γ. Performing a number of manipulations shown in the appendix, part A, and defing the aggregate $X = \sum_i x_i$, we get

$$(e(1-\gamma)E(X) - \tau)(1-\gamma)\sum_i \frac{1}{c_i} + \left(e(1-\gamma)\operatorname{var}(X) + (e(1-\gamma)E(X) - \tau)E(X) \right) \frac{\partial \gamma}{\partial \tau} / (1-\gamma) = 0 \tag{16}$$

Since $\text{var}(X) > 0$ and $\partial \gamma / \partial \tau > 0$ it follows that

$$e(1-\gamma)E(X) - \tau < 0 . \tag{17}$$

This means that the former main term on the left hand side is negative, and as a consequence the latter main term must be positive:

$$e(1-\gamma)\,\text{var}(X) + \left(e(1-\gamma)E(X) - \tau\right)E(X) > 0 \tag{18}$$

To get some intuition for the result, first note that the effect of a tax increase will be to discourage production and also to encourage investment in abatement technology. For the sake of the argument, neglect for a moment the latter effect. As is well known from the literature on Pigouvian taxes one would like to internalise the external cost by facing the polluter with a tax that reflects the marginal damage caused by its activity. The marginal damage will vary across possible states but since one has to impose a uniform tax it will only reflect the expected marginal damage, or, in other words, the marginal damage that occurs at the expected output level $\left(e(1-\gamma)E(X)\right)$. The private economic incentive will induce the producer to invest in abatement technology to the extent that the investment cost is offset by the expected tax savings from reduced emissions. Taking as given that the tax is set without allowing for the *marginal* effect on investment in abatement technology, the expected tax savings will reflect the marginal damage reduction evaluated at the expected production level. This is equivalent to saying that if the investment level were fixed the tax should reflect the expected marginal damage.

However, the tax will also encourage abatement investment. We then note that investing in a better abatement technology will have a larger impact on emissions where output is large and the (marginal) damage is large. In other words, since the damage function is strictly convex (quadratic) in total output level the expected damage reduction from investment in abatement technology will exceed the expected tax savings of the producer, and there will be underinvestment in abatement from a social perspective. To correct this, the tax must be set above the expected marginal damage in order to induce further investment. The drawback is that there will then be a downward distortion of output for a given abatement technology as the external cost is now being over-internalised[4]. For a *given* investment level the output is too small. The beneficial effect on investment must then be traded off against the

[4] The tax-inclusive cost facing the producers will overstate the social cost of production (including the external cost).

harmful effect on output. Where a tax is the only instrument it fails to achieve both the desired output level and the desired investment, and the trade-off implies too small output and too little investment as reflected by the opposite signs of (17) and (18). We summarise our findings in this case as follows:

Proposition 1

i. Where the emission tax, τ , is the only instrument employed, setting τ at the level of expected marginal damage will lead to underinvestment in abatement technology.

ii. Where the emission tax, τ , is the only instrument employed, τ should optimally be set above the expected marginal damage in order to induce investment in abatement technology.

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6. The government sets τ and γ .

Let us now consider whether the government could get closer to the first-best optimum by directly regulating abatement investment in addition to setting the tax. We start with the case above where the government has chosen the value of τ , and consider whether the outcome could be improved by regulating the value of γ . Keeping τ fixed, the effect of increasing γ from the privately optimal value is given by

$$\begin{aligned} \frac{\partial E(\Psi)}{\partial \gamma} &= \int \left(e(1-\gamma)\left(\sum_j x_j \right)^2 - k'(\gamma) \sum_i x_i \right) f(p)dp \\ &= e(1-\gamma)E\left(X^2 \right) - \tau E(X) \\ &= e(1-\gamma)\operatorname{var}(X) + \left(e(1-\gamma)E(X) - \tau \right) E(X) \end{aligned} \qquad (19)$$

We see from (18) above that $\partial E(\Psi)/\partial\gamma > 0$. It would be socially desirable to enhance γ . Assuming that the government can set both τ and γ , we obtain the first order conditions

$$\frac{\partial}{\partial \tau} E(\Psi) = \int \left(\sum_i \left(p \frac{\partial x_i}{\partial \tau} - c x_i \frac{\partial x_i}{\partial \tau} - k(\gamma) \frac{\partial x_i}{\partial \tau} \right) - e(1-\gamma)^2 \sum_i \sum_j x_j \frac{\partial x_i}{\partial \tau} \right) f(p)dp = 0 \qquad (20)$$

$$\frac{\partial}{\partial\gamma}E(\Psi)=\int\sum_i\left(p-c_ix_i-k(\gamma)-e(1-\gamma)^2\sum_j x_j\right)\frac{\partial x_i}{\partial\gamma}f(p)dp$$
$$+\int\left(\sum_i\left(-k'(\gamma)x_i\right)+e(1-\gamma)\left(\sum_j x_j\right)^2\right)f(p)dp=0 \tag{21}$$

Further manipulations conducted in the appendix, part B yields.

$$\tau=E\left(e\sum_j(1-\gamma)x_j\right) \tag{22}$$

The tax rate is set equal to the expected marginal damage, unlike the case described by (17). The important insight is that having access to investment regulation does not only add more investment, but changes the way the tax is set so that the output distortion is removed.

Moreover, as shown in the appendix, part B:

$$k'(\gamma)=e(1-\gamma)\left(E(X)+\operatorname{var}X\,/\,E\left(X\right)\right)$$

or equivalently

$$\frac{k'(\gamma)}{1-\gamma}=e\left(E(x)+\operatorname{var}X\,/\,E\left(X\right)\right) \tag{23}$$

The left hand side is increasing in γ. It follows that γ is larger the larger is the damage parameter *e*, and γ is larger the larger is the variance of X for a given expected value. The latter finding is easy to understand since the beneficial property of the investment that its impact is stronger where output and (marginal) damage is large is accentuated when the output level does indeed vary a lot across states.

Proposition 2

Where the government sets the emission tax, τ *, and regulates the abatement technology* γ *, the optimal use of the instruments is characterised by* $\tau = E\left(e\sum_j (1-\gamma)x_j \right)$ *and* $k'(\gamma) = e(1-\gamma)E\left((X)^2\right)/E(X)$*. The tax is set equal to the expected marginal damage, and the investment in abatement technology is set to reflect the output (and damage) variance across states.*

We may note that if the regulation of abatement technology were the only instrument we would get an outcome similar to the one described in Proposition 1. Investment in abatement technology would directly mitigate the external cost but would also make production more costly and hence diminish emissions through lower output. In order to achieve the latter effect there would be overinvestment in abatement technology. Real resources would be used up in order to discourage production.

7. Subsidising abatement

An alternative way to induce additional investment in abatement technology would be to subsidise it. Consider a subsidy *s* to acquisition of abatement technology so that the private cost becomes $k(\gamma) - s\gamma$. Such schemes exist or are often proposed either as explicit subsidies or in the form of a tax rebate to reward abatement[5].

Given our assumed subsidy scheme the profits of each producer *i* are

$$\Pi_i = px_i - \frac{1}{2}cx_i^2 - \tau(1-\gamma_i)x_i - k(\gamma_i)x_i + s\gamma_i x_i$$

As before, the producer is assumed to maximise profits after the price uncertainty has been resolved. Maximising profits wrt output yields the first order condition

[5] Rajah and Smith (1993)

$$\frac{\partial \Pi}{\partial x_i} = p - cx_i - \tau(1-\gamma_i) - k(\gamma_i) + s\gamma_i = 0,$$

and output is given by

$$x_i = \frac{p - \tau(1-\gamma_i) - k(\gamma_i) + s\gamma_i}{c_i}. \tag{24}$$

γ_i is chosen to maximise expected profits, which requires that

$$k'(\gamma_i) = \tau + s, \tag{25}$$

implying that $\gamma_i = \gamma = k'^{-1}(\tau + s)$.

Moreover it follows that

$$\frac{\partial x_i}{\partial \tau} = -\frac{1-\gamma}{c_i} \tag{26}$$

$$\frac{\partial x_i}{\partial s} = \frac{\gamma}{c_i} = -\frac{\gamma}{1-\gamma}\frac{\partial x_i}{\partial \tau} \tag{27}$$

$$\frac{\partial \gamma}{\partial \tau} = \frac{\partial \gamma}{\partial s} = \frac{1}{k''(\gamma)} \tag{28}$$

We note that τ and *s* have the same effect on the choice of abatement scale γ. The fact that subsidising abatement technology provides a stimulus to choosing larger abatement scale that is no different from that of an emission tax may throw doubt on the gains from adding this instrument. Moreover a tax discourages polluting production while a subsidy makes production cheaper and increases output, which is the well-known disadvantage of subsidising abatement rather than taxing emissions. However, the *combined* use of the two instruments can yield additional benefits.

We start by showing that using the proper combination of the two instruments one can in fact reproduce the allocation achieved by means of direct regulation and a tax. Denote this allocation by γ^* and x_i^* (for all *i*) and denote by τ^* the optimal tax used in conjunction with direct regulation.

We use the notation $\tilde{\tau}$ and $\tilde{s}$ for the tax and the subsidy when that combinations is being used. From (25) we realise that to obtain γ^* one has to set $\tilde{\tau} + \tilde{s} = k'(\gamma^*)$. Combining the supply functions for the two cases from (11) and (24) we have that in order to obtain x_i^* we must have

$$x_i^* = \frac{p - \tau^*(1-\gamma^*) - k(\gamma^*)}{c_i} = \frac{p - \tilde{\tau}(1-\gamma^*) - k(\gamma^*) + \tilde{s}\gamma^*}{c_i},$$

which requires that $\tilde{\tau}(1-\gamma^*) - \tilde{s}\gamma^* = \tau^*(1-\gamma^*)$. Adding this to $\tilde{\tau} + \tilde{s} = k'(\gamma^*)$ then give us two equations to determine the instruments needed to implement the same allocation as the one achieved with a tax and direct regulation.

An important implication is that one would always supplement a tax with a subsidy if those were the only available instruments, as we know from Proposition 1 that τ alone is an inadequate instrument. We realised above that the problem with sole use of a tax is that in order to induce investment in abatement technology subsidy a too low output is chosen and the externality is being overinternalised for a given abatement technology. As the tax and the subsidy have the same effect on investment in abatement technology but differ in their effect on output, the gain from introducing a subsidy is that it mitigates the excessive output effect without weakening the incentive to invest in abatement technology. When the choice is between using either a tax or a subsidy to encourage investment in abatement technology, the well-known disadvantage of the subsidy is that it encourages polluting emissions by lowering the cost of production and hence raises output. However, when the two instruments are deployed together, the output stimulus is in fact beneficial because in the absence of a subsidy there is excessive discouragement of the output and the associated externality for a given abatement technology.

Proposition 3

Where an optimal emission tax is used in the absence of regulation of abatement technology the social surplus can be enhanced by introducing a subsidy to investment in abatement technology, which mitigates the overinternalisation of output externalites for a given abatement technology.

Having shown that one can do as well using a tax and a subsidy as when deploying a tax and direct regulation, the remaining question is whether one could even perform strictly better. We now turn to this question.

Starting from an arbitrary allocation, the effects of changing the instruments are

$$\frac{\partial E(\Psi)}{\partial \tau} = \int \left(\sum_i \left(p - cx_i - k(\gamma) \right) \frac{\partial x_i}{\partial \tau} - e(1-\gamma)^2 \left(\sum_j x_j \right) \sum_i \frac{\partial x_i}{\partial \tau} \right) f(p)dp$$
$$+ \int \left(e(1-\gamma) \left(\sum_j x_j \right)^2 - k'(\gamma) \sum_i x_i \right) \frac{\partial \gamma}{\partial \tau} f(p)dp$$

$$\frac{\partial E(\Psi)}{\partial s} = \int \left(\sum_i \left(p - cx_i - k(\gamma) \right) \frac{\partial x_i}{\partial \tau} - e(1-\gamma)^2 \left(\sum_j x_j \right) \sum_i \frac{\partial x_i}{\partial s} \right) f(p)dp$$
$$+ \int \left(e(1-\gamma) \left(\sum_j x_j \right)^2 - k'(\gamma) \sum_i x_i \right) \frac{\partial \gamma}{\partial s} f(p)dp$$

Invoking (24), we get

$$\frac{\partial E(\Psi)}{\partial \tau} = \int \left(\sum_i \left[\tau(1-\gamma) - s\gamma - e(1-\gamma)^2 X \right] \frac{\partial x_i}{\partial \tau} \right) f(p)dp$$
$$+ \int \left(e(1-\gamma) X^2 - k'(\gamma) X \right) \frac{\partial \gamma}{\partial \tau} f(p)dp$$

$$\frac{\partial E(\Psi)}{\partial s} = \int \left(\sum_i \left[\tau(1-\gamma) - s\gamma - e(1-\gamma)^2 X \right] \frac{\partial x_i}{\partial s} \right) f(p)dp$$
$$+ \int \left(e(1-\gamma) X^2 - k'(\gamma) X \right) \frac{\partial \gamma}{\partial s} f(p)dp$$

and moreover

$$\frac{\partial E(\Psi)}{\partial \tau} = \int \left(\sum_i \left[\tau(1-\gamma) - s\gamma - e(1-\gamma)^2 X \right] \frac{\partial x_i}{\partial \tau} \right) f(p)dp$$
$$+ \left(e(1-\gamma) E\left(X^2 \right) - k'(\gamma) E(X) \right) \frac{\partial \gamma}{\partial \tau}$$

$$\frac{\partial E(\Psi)}{\partial \tau} = \int\left(\sum_i \left[\tau(1-\gamma) - s\gamma - e(1-\gamma)^2 X \right] \frac{\partial x_i}{\partial s} \right) f(p)dp$$

$$+\left(e(1-\gamma)E\left(X^2\right) - k'(\gamma)E(X)\right)\frac{\partial \gamma}{\partial s}$$

Inserting γ^* and X^*, the second main term on the right hand side vanishes in both expressions due to (23) . Also setting $s = \tilde{s}$ and $\tau = \tilde{\tau}$ and making use of $\frac{\partial x_i}{\partial s} = \frac{\gamma}{c_i} = -\frac{\gamma}{1-\gamma}\frac{\partial x_i}{\partial \tau}$, we obtain

$$\frac{\partial E(\Psi)}{\partial \tau} = \int\left(\sum_i \left[\tilde{\tau}(1-\gamma^*) - \tilde{s}\gamma^* - e(1-\gamma^*)^2 X^* \right] \frac{\partial x_i}{\partial \tau} \right) f(p)dp$$

$$\frac{\partial E(\Psi)}{\partial s} = \int\left(\sum_i \left[\tilde{\tau}(1-\gamma^*) - \tilde{s}\gamma^* - e(1-\gamma^*)^2 X^* \right] \frac{\gamma^*}{1-\gamma^*}\frac{\partial x_i}{\partial \tau} \right) f(p)dp$$

Then invoking $\tilde{\tau}(1-\gamma^*) - \tilde{s}\gamma^* = \tau^*(1-\gamma^*)$ and (22), which is fulfilled at τ^*, γ^*, X^*, it follows that

$$\frac{\partial E(\Psi)}{\partial \tau} = \int\left(\sum_i \left[\tilde{\tau}(1-\gamma^*) - \tilde{s}\gamma^* - e(1-\gamma^*)^2 X^* \right] \frac{1-\gamma^*}{c_i} \right) f(p)dp = 0 \qquad (29)$$

$$\frac{\partial E(\Psi)}{\partial s} = \int\left(\sum_i \left[\tilde{\tau}(1-\gamma^*) - \tilde{s}\gamma^* - e(1-\gamma^*)^2 X^* \right] \frac{1-\gamma^*}{c_i} \right) f(p)dp \frac{\gamma^*}{1-\gamma^*} = 0 \qquad (30)$$

which demonstrates that the optimal allocation in a tax/subsidy regime is achieved by setting the tax and the subsidy that implement the optimal allocation from the tax/direct regulation regime. The two sets of instruments are therefore equivalent in the sense that they yield the same allocation when set optimally.

We may note that $\tilde{\tau} = \tau^* + \tilde{s}\gamma^*/(1-\gamma^*)$. The interesting implication is that when deploying a subsidy rather than direct regulation one will set a larger tax.

Proposition 4

The same second best optimum can be achieved by using either a combination of an emission tax and direct regulation of abatement technology, or alternatively a larger emission tax combined with a subsidy to investment in abatement technology.

8. Subsidising abatement technology with uncertain cost.

Now we will add the further complication that the governent may have to determine its policy without knowing the cost of investing in abatement, which we now express as $k(\gamma)+\theta\gamma$ where previously $\theta=0$. In this case, the government must make its decisions on the basis of a probability distribution for the value of θ with density $g(\theta)$, while the firms are still assumed to be fully informed about the cost, which now means that they know the value of θ with certainty.

Given our assumed subsidy scheme, the profits of each producer *i* are

$$\Pi_i = px_i - \frac{1}{2}c_i x_i^2 - \tau(1-\gamma_i)x_i - k(\gamma_i)x_i - \theta\gamma_i x_i + s\gamma_i x_i$$

The producer is assumed to maximise profits after the price uncertainty has been resolved. Maximising profits wrt output yields the first order condition

$$\frac{\partial \Pi}{\partial x_i} = p - c_i x_i - \tau(1-\gamma_i) - k(\gamma_i) - \theta\gamma_i + s\gamma_i = 0\,, \tag{31}$$

It follows that

$$x_i = \frac{p - \tau(1-\gamma_i) - k(\gamma_i) - \theta\gamma_i + s\gamma_i}{c_i}.$$

γ_i is chosen to maximise expected profits. The following condition must hold

$$k'(\gamma_i) = \tau + s - \theta \tag{32}$$

which implies that $\gamma_i = \gamma = k'^{-1}(\tau + s - \theta)$

$$\frac{\partial x_i}{\partial \gamma_i} = \frac{\tau - k'(\gamma_i) - \theta + s}{c_i} \tag{33}$$

We note that for the optimal choice of γ_i given by (32) $\frac{\partial x_i}{\partial \gamma_i} = 0$.

Moreover it follows that

$$\frac{\partial x_i}{\partial \tau} = -\frac{1-\gamma}{c_i} \tag{34}$$

$$\frac{\partial x_i}{\partial s} = \frac{\gamma}{c_i} = -\frac{\gamma}{1-\gamma}\frac{\partial x_i}{\partial \tau} \tag{35}$$

$$\frac{\partial x_i}{\partial \theta} = \frac{-\gamma}{c_i} = \frac{\gamma}{1-\gamma}\frac{\partial x_i}{\partial \tau} \tag{36}$$

$$\frac{\partial \gamma}{\partial \tau} = \frac{\partial \gamma}{\partial s} = \frac{1}{k''(\gamma)} \tag{37}$$

$$\frac{\partial \gamma}{\partial \theta} = -\frac{1}{k''(\gamma)} \tag{38}$$

We note that τ and *s* have the same effect on the choice of abatement level but differ in their effect on output. A tax discourages production while a subsidy makes production cheaper and increases output, which is the disadvantage of subsidising abatement rather than taxing emissions.

$$\Psi = \sum_i \left(px_i - \frac{1}{2}c_i x_i^2 - k(\gamma)x_i - \theta\gamma x_i \right) - \frac{1}{2}e(1-\gamma)^2 \left(\sum_j x_j \right)^2$$

and the expected social surplus is

$$E(\Psi) = \int\int \left(\sum_i \left(px_i - \frac{1}{2}c_i x_i^2 - k(\gamma)x_i - \theta\gamma x_i \right) - \frac{1}{2}e(1-\gamma)^2 \left(\sum_j x_j \right)^2 \right) f(p)g(\theta)dpd\theta$$

Now consider the optimal choice of τ where $s=0$ so that the tax is the only instrument in use. The first order condition is

$$\begin{aligned} \frac{dE(\Psi)}{d\tau} &= \int\int \left(\sum_i \left(p - cx_i - k(\gamma) - \theta\gamma \right) \frac{\partial x_i}{\partial \tau} - e(1-\gamma)^2 \left(\sum_j x_j \right) \sum_i \frac{\partial x_i}{\partial \tau} \right) f(p)g(\theta)d\theta dp \\ &+ \int\int \left(e(1-\gamma) \left(\sum_j x_j \right)^2 - \left(k'(\gamma) + \theta \right) \sum_i x_i \right) \frac{\partial \gamma}{\partial \tau} f(p)g(\theta)d\theta dp = 0 \end{aligned} \quad (39)$$

where $dE(\Psi)/d\tau$ is the total derivative allowing for the effects via both output and γ.

From (c6) in the appendix we have the following characterisation of the optimum tax:

$$\begin{aligned} &E_g\left((1-\gamma)^2\right)\left[eE_g\left((1-\gamma)E(X|\theta)\right) - \tau\right]\sum_i \frac{1}{c_i} + \left[eE_g\left((1-\gamma)E(X|\theta)\right) - \tau\right]E_g\left(\left(E(X|\theta)\right)\frac{\partial \gamma}{\partial \tau}\right) \\ &+ eE_g\left((1-\gamma)\left(\text{var}(X|\theta)\right)\frac{\partial \gamma}{\partial \tau}\right) \\ &+ e\,\text{cov}\left((1-\gamma)^2\sum_i \frac{1}{c_i}, (1-\gamma)E(X|\theta)\right) + e\,\text{cov}\left((1-\gamma)E(X|\theta), \left(E(X|\theta)\right)\frac{\partial \gamma}{\partial \tau}\right) = 0 \end{aligned} \quad (40)$$

E is used as the expectation operator when conditioning on θ, while E_g denotes the expectation due to the randomness of θ and accordingly also the randomness of γ. To get some intuition for the condition, let us start by neglecting the two covariances in the bottom line. We are then very close to the condition we obtained in the case of a certain cost of abatement technology; cf. eq. (16) and Proposition 2. The difference is that rather than considering γ as fixed it is now perceived as stochastic and we have to consider expectations due to the randomness of θ (and accordingly of γ).

But the basic interpretaion is the same. Where the variance in line two, and accordingly its expectation, is positive it follows that $eE_g\left((1-\gamma)E(X|\theta)\right)-\tau<0$. That is, the tax is set above the expected marginal damage allowing both for the uncertain *p* and the uncertain θ (and accordingly γ) from the government's point of view. It follows that

$$\left[eE_g\left((1-\gamma)E(X|\theta)\right)-\tau\right]E_g\left(\left(E(X|\theta)\right)\frac{\partial\gamma}{\partial\tau}\right)+eE_g\left((1-\gamma)\left(\mathrm{var}(X|\theta)\right)\frac{\partial\gamma}{\partial\tau}\right)>0$$

which means that the stimulating effect of τ on γ has a positive effect on the social surplus. We should note that the impact on γ and its effect on the social surplus is now stochastic due to the randomness of θ. Again the question is whether an instrument targeted at inducing investment in abatement technology can be used. In the case of a certain θ, the most direct instrument was to regulate γ directly. Where the cost of abatement technology is uncertain for the government, fixing the value of γ will have the disadvantage that one does not make use of the information held by the firms allowing them to set the value of γ based on their superior information. A more promising potential instrument is therefore to subsidise γ.

But before we proceed we need to consider also the covariances in the bottom line of (40).

Consider first the partial effect of $e\,\mathrm{cov}\left((1-\gamma)^2\sum_i\frac{1}{c_i},(1-\gamma)E(X|\theta)\right)$. $E(X|\theta)(1-\gamma)$ is the expected emission level conditional on θ. $(1-\gamma)^2\sum_i\frac{1}{c_i}$ is a measure of the impact on emissions of the decline in *X* caused by a higher tax. We note that $(1-\gamma)$ has a double (ie squared) effect. A larger $(1-\gamma)$ increases the output effect of a larger tax (cf. eq. 34) and it strengthens the impact on the emissions from a given output. Moreover, the change in emissions has a larger impact on the external cost the larger is the initial emission level $(1-\gamma)X$. It follows that if the covariance is positive the impact on emissions is larger in states where the externality problem is more serious, which enlarges the gain from taxing emisssions. The opposite holds if the covariance is negative. We note that larger θ will lower the abatement scale γ and increase $(1-\gamma)^2$. It is also plausible that emissions will increase when the abatement technology beomes more expensive and accordingly $\mathrm{cov}\left((1-\gamma)^2\sum_i\frac{1}{c_i},(1-\gamma)E(X|\theta)\right)$ would be positive. This term will then reinforce the effect in the second line of (40) . The two effects are of the same nature; one caused by the stochastic variation in p and the other by the stochastic variation in θ. We can immediately conclude that where the first term in the bottom line of (40) is sufficiently small the sign of $eE_g\left((1-\gamma)E(X\,|\theta)\right)-\tau$ in our previous model carries over to the case where θ is stochastic.

Then consider in further detail the partial effect of $\operatorname{cov}\left((1-\gamma)E(X\,|\theta),\left(E(X\,|\theta)\right)\frac{\partial\gamma}{\partial\tau}\right)$. This is the covariance between the change in the emission level generated by the induced change in γ and the expected emission level (conditional on θ). If this covariance is positive it has a beneficial effect on expected social surplus because a fall in the emission level is more valuable where the existing emission level is large. This will reinforce the case for a tax as well as the beneficial effect of γ. A negative covariance would have the opposite effect.

Consider the special case where $\frac{\partial\gamma}{\partial\tau}$ is constant. Then the sign of $\operatorname{cov}\left((1-\gamma)E(X\,|\theta),\left(E(X\,|\theta)\right)\frac{\partial\gamma}{\partial\tau}\right)$ is determined by the sign of the covariance between $E(X\,|\theta)$ and $(1-\gamma)E(X\,|\theta)$, ie. the covariance between expected output and expected emission level. Letting θ increase, $E(X\,|\theta)$ will diminish as production becomes more expensive. There will be opposing effects on $(1-\gamma)E(X\,|\theta)$. The output effect will diminish emissions. The opposite effect is that when abatement becomes more expensive a smaller abatement scale, γ, is chosen and emissions will increase. It seems the more plausible outcome that the net effect of more expensive abatement technology is to increase emissions. Then the covariance is negative. As a consequence the two covariances will have opposite signs and in general the sign of the bottom line of (40) is ambiguous.

We can summarise the discussion of these effects in a proposition.

Proposition 5.

Where the output price is uncertain, and the government faces additional uncertainty about the cost of abatement investment, the level at which the emissions tax would be set, if employed as the sole instrument, will depend on how these two sources of uncertainty affect emissions reductions and abatement investment. We can define three partial effects as follows:

a) *Uncertainty about θ has the effect that the impact on emissions of the decline in output due to a higher tax is positively correlated with the emissions level;*

b) *Uncertainty about p has the effect that the impact on emissions of abatement investment is positively correlated with the emissions level;*

c) *Uncertainty about θ has the effect that the impact on emissions of abatement investment is negatively correlated with the emissions level;*

Effects a) and b) will tend to imply that the tax should optimally be set <u>*above*</u> *the level of expected marginal damage, while effect c) will tend to imply that the the tax should be set* <u>*below*</u> *expected marginal damage.*

Now consider if a social gain is achievable by letting *s* deviate slightly from zero. In the appendix, we derive in (c10) the result that

$$\left(\frac{d}{ds}E(\Psi)\right)/\sum_i \frac{1}{c_i} = E_g(1-\gamma)\Big(\tau - eE_g\big((1-\gamma)E(X|\theta)\big)\Big) - \operatorname{cov}\big((1-\gamma)E(X|\theta),(1-\gamma)\big) \qquad (41)$$

We note that the former term on the right hand side is positive if the tax exceeds the expected marginal damage, and then constitutes a partial case for a subsidy. The basic argument is the same as in condition (30) of the certainty case. A positive sign means that given the expected values the external effect of X is overinternalised, and the role of a subsidy is to lower the cost of production and counteract the excessive discouragement of output level. However, the uncertainty about θ and γ can make a difference in two ways. Firstly, we realised above that the sign of the former term on the right hand side is not unambiguously determined. Secondly, we have to allow for the covariance between the emission coefficient $1-\gamma$ and the emission level. Making the plausible assumption that the emission level rises when abatement becomes more expensive, the covariance is positive and taking the minus sign into account, the latter term on the right hand side of (41) is negative. This means that there is a partial argument against a subsidy and hence in favour of a tax on investment in abatement. To interpret this result, we note that the effect of a tax depends on the emission level and the emission coefficient $1-\gamma$. If a tax is introduced on investment and τ is lowered the net effect is to lower emissions by $(1-\gamma)\sum_i \frac{1}{c_i} \left(= ((1-\gamma)^2 + \gamma(1-\gamma))\sum_i \frac{1}{c_i} \right)$. This effect is larger when θ is larger (and γ is smaller) but then, plausibly, existing emissions are larger and the marginal damage reduction is larger. In brief, the effect of shifting some of the tax from output to abatement investment is to shift more of the emission reduction to states of the world where emissions are high, which is clearly a beneficial effect where there is increasing marginal damage. If the former term on the right hand side is negative or zero there is an unambiguous case for a tax on abatement investment. But perhaps it is more likely that the former term is positive, in which case there are opposing effects and the overall case for a tax or a subsidy is indeterminate.

We can summarise as follows.

Proposition 6.

Where the output price is uncertain, and the government faces additional uncertainty about the cost of abatement investment, and where an abatement investment subsidy is available to supplement the emissions tax set as in Proposition 5:

i. *If the emissions tax is optimally set above the expected marginal damage, and if the emission coefficient $1-\gamma$ is negatively correlated with the emission level, then the emissions tax should be augmented by a subsidy to abatement investment.*

ii. *If the emissions tax is optimally set below the expected marginal damage, and if the emission coefficient $1-\gamma$ is positively correlated with the emissions level, then the emissions tax should be augmented by a tax on abatement investment (ie the optimal subsidy is negative).*

iii. *In the other cases no clear-cut result for the subsidy can be derived.*

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9. Concluding remarks

It is well known that uncertainty about the costs of emissions abatement can influence the choice between quantity-based environmental regulation such as a tradeable emissions quota and price-based regulation such as an emissions tax, and that combined use of both may do better than either alone (Roberts and Spence, 1976). However, tradeable quotas will not always be practicable, for example when small numbers of firms are involved or where transactions costs are high. This motivates our discussion of a regime where an emissions tax is the primary instrument, and where the other instruments available cannot be fully differentiated to reflect the heterogeneity of emitters. In particular we consider abatement standards which mandate adoption of certain abatement technologies, and subsidies to abatement investments.

The conventional wisdom, based on deterministic cases, is that, where a tax reflects the external cost, agents will make the appropriate trade-off between the benefits and costs associated with investment in abatement or clean-up technology, and that there is no need for the tax to be supplemented by instruments bearing directly on abatement investment. We show that the optimal policy may be different where abatement costs are uncertain or fluctuating over time and the tax instrument is not sufficiently flexible to be adjusted to all conceivable circumstances.

In our model the externality tax must be set at a rate determined prior to the realisation of the uncertain cost of abatement. If the tax is employed as the sole instrument it must be set taking into account how it affects emissions via output as well as its inducement to investment in abatement technology. As the tax is fixed and therefore uniform across contingencies there will (normally) be ex post inefficiencies compared to the hypothetical use of state-adjusted taxes. The question is whether supplementing the tax with other instruments will mitigate the expected inefficiencies. We first showed that the addition of direct regulation of abatement technology can achieve efficient abatement investment with less damage to output levels than the tax alone. Second, we showed that a small subsidy to investment in abatement technology will have a beneficial effect, but, perhaps surprisingly, the reason is its stimulating effect on output rather than on investment. However, the combined use of tax and subsidy will enable a higher investment level at the optimum. Finally, we considered circumstances where the cost of abatement investment is also unknown to the government. We show that this complicates the case for subsidising investment in abatement technology, and that in some circumstances there might conceivably be a case for taxing investment in abatement technology.

Appendix

Part A

Starting from (15) and making use of (7) and (10), yields

$$\frac{dE(\Psi)}{d\tau} = \int \left(\sum_i \tau\left(1-\gamma\right)\frac{\partial x_i}{\partial \tau} - e(1-\gamma)^2 \left(\sum_j x_j \right) \sum_i \frac{\partial x_i}{\partial \tau} \right) f(p)dp$$
$$+\int \left(e(1-\gamma)\left(\sum_j x_j \right)^2 - \tau \sum_i x_i \right) \frac{\partial \gamma}{\partial \tau} f(p)dp = 0 \quad \text{(a1)}$$

Then make use of (12) and, for convenience, define the aggregate $X = \sum_i x_i$.

$$\frac{dE(\Psi)}{d\tau} = \int \left(\tau\left(1-\gamma\right)(-\left(1-\gamma\right)) - e(1-\gamma)^2 X(-(1-\gamma)) \right) \sum_i \frac{1}{c_i} f(p)dp$$
$$+\int \left(e(1-\gamma)X^2 - \tau X \right) \frac{\partial \gamma}{\partial \tau} f(p)dp = 0 \quad \text{(a2)}$$

Further simple manipulations yield

$$\frac{d}{d\tau} E(\Psi) = \int (e(1-\gamma)X - \tau) f(p)dp(1-\gamma)\sum_i \frac{1}{c_i}$$
$$+\int \left(e(1-\gamma)X^2 - \tau X \right) f(p)dp \frac{\partial \gamma}{\partial \tau} / \left(1-\gamma\right) = 0 \quad \text{(a3)}$$

Introducing statistical concepts and doing some further manipulations, we get

$$(e(1-\gamma)E(X) - \tau)(1-\gamma)\sum_i \frac{1}{c_i} + \left(e(1-\gamma)E\left(X^2\right) - \tau E(X) \right) \frac{\partial \gamma}{\partial \tau} / \left(1-\gamma\right) = 0$$

$$\left(e(1-\gamma)E(X) - \tau \right)(1-\gamma)\sum_i \frac{1}{c_i} + \left(e(1-\gamma)\,\mathrm{var}(X) + e(1-\gamma)(E(X))^2 - \tau E(X) \right) \frac{\partial \gamma}{\partial \tau} / \left(1-\gamma\right) = 0$$

$$(e(1-\gamma)E(X) - \tau)(1-\gamma)\sum_i \frac{1}{c_i} + \left(e(1-\gamma)\,\mathrm{var}(X) + (e(1-\gamma)E(X) - \tau)E(X) \right) \frac{\partial \gamma}{\partial \tau} / \left(1-\gamma\right) = 0 \quad \text{(a4)}$$

Part B

Consider equations (20) and (21) from the main text.

$$\frac{\partial}{\partial\tau}E(\Psi)=\int\left(\sum_i\left(p\frac{\partial x_i}{\partial\tau}-cx_i\frac{\partial x_i}{\partial\tau}-k(\gamma)\frac{\partial x_i}{\partial\tau}\right)-e(1-\gamma)^2\sum_i\sum_j x_j\frac{\partial x_i}{\partial\tau}\right)f(p)dp=0 \tag{20}$$

$$\begin{aligned}\frac{\partial}{\partial\gamma}E(\Psi)&=\int\sum_i\left(p-c_ix_i-k(\gamma)-e(1-\gamma)^2\sum_j x_j\right)\frac{\partial x_i}{\partial\gamma}f(p)dp\\&+\int\left(\sum_i\left(-k'(\gamma)x_i\right)+e(1-\gamma)\left(\sum_j x_j\right)^2\right)f(p)dp=0\end{aligned} \tag{21}$$

Consider $\partial E(\Psi)/\partial\tau$ in detail. From (7) above $p-cx_i-k(\gamma)=\tau(1-\gamma)$. Making use of this equation and substituting by means of (12), we get

$$\begin{aligned}\frac{\partial}{\partial\tau}E(\Psi)&=\int\left(\sum_i\left(\tau(1-\gamma)\frac{\partial x_i}{\partial\tau}\right)-e(1-\gamma)^2\sum_i\sum_j x_j\frac{\partial x_i}{\partial\tau}\right)f(p)dp\\&=\int\left(-\sum_i\left(\tau(1-\gamma)\frac{1-\gamma}{c_i}\right)+e(1-\gamma)^2\sum_i\sum_j x_j\frac{1-\gamma}{c_i}\right)f(p)dp=0\end{aligned} \tag{b1}$$

It follows that

$$\int\left(-\tau(1-\gamma)+e(1-\gamma)^2\sum_j x_j\right)f(p)dp=0$$

$$\tau=E\left(e\sum_j(1-\gamma)x_j\right) \tag{b2}$$

Now consider $\partial E(\Psi)/\partial\gamma$ in detail. From (13) above $\frac{\partial x_i}{\partial\gamma}=\frac{\tau-k'(\gamma)}{c_i}=-\frac{\tau-k'(\gamma)}{1-\gamma}\frac{\partial x_i}{\partial\tau}$. The first term on the right hand side of (20) vanishes owing to (19), and we get.

$$k'(\gamma)E\left(\sum_i x_i\right)=e(1-\gamma)E\left(\left(\sum_i x_i\right)^2\right) \tag{b3}$$

which we can rewrite as

$$k'(\gamma)=e(1-\gamma)E\left((X)^2\right)/E(X) \tag{b4}$$

and furthermore

$$k'(\gamma)=e(1-\gamma)\left(\operatorname{var}x+(E(X))^2\right)/E(X) \tag{b5}$$

Part C

Take as point of departure equation (39) from the main text.

$$\frac{dE(\Psi)}{d\tau}=\int\int\left(\sum_i\left(p-cx_i-k(\gamma)-\theta\gamma\right)\frac{\partial x_i}{\partial\tau}-e(1-\gamma)^2\left(\sum_j x_j\right)\sum_i\frac{\partial x_i}{\partial\tau}\right)f(p)g(\theta)d\theta dp$$
$$+\int\int\left(e(1-\gamma)\left(\sum_j x_j\right)^2-\left(k'(\gamma)+\theta\right)\sum_i x_i\right)\frac{\partial\gamma}{\partial\tau}f(p)g(\theta)d\theta dp=0$$

where $dE(\Psi)/d\tau$ is the total derivate allowing for the effects via both output and γ. Invoking (32) and (34), we get

$$\frac{dE(\Psi)}{d\tau}=\int\int\left(\sum_i\left(\tau(1-\gamma)\right)\frac{\partial x_i}{\partial\tau}-e(1-\gamma)^2\left(\sum_j x_j\right)\sum_i\frac{\partial x_i}{\partial\tau}\right)f(p)g(\theta)d\theta dp$$
$$+\int\int\left(e(1-\gamma)\left(\sum_j x_j\right)^2-\tau\sum_i x_i\right)\frac{\partial\gamma}{\partial\tau}f(p)dp=0 \qquad \text{(c1)}$$

Due to (34)

$$\frac{d}{d\tau}E(\Psi)=$$

$$-\int\tau(1-\gamma)^2g(\theta)d\theta\sum_i\frac{1}{c_i}+\int e(1-\gamma)^2E(X|\theta)(1-\gamma)\sum_i\frac{1}{c_i}g(\theta)d\theta$$
$$-\int\tau E(X|\theta)\frac{\partial\gamma}{\partial\tau}g(\theta)d\theta+\int e(1-\gamma)E(X^2|\theta)\frac{\partial\gamma}{\partial\tau}g(\theta)d\theta=0 \qquad \text{(c2)}$$

where we have made use of $k'(\gamma_i)+\theta=\tau$. Here expectations with operator *E* are expectations conditional on θ. Now denote by E_g the expectation operator where the expectation is due to the randomness of θ and accordingly the randomness of γ. Further manipulations yield

$$dE(\Psi)/d\tau=-\tau E_g\left((1-\gamma)\sum_i\frac{1-\gamma}{c_i}\right)+eE_g\left((1-\gamma)^2E(X|\theta)\sum_i\frac{(1-\gamma)}{c_i}\right)$$
$$-\tau E_g\left(E(X|\theta)\frac{\partial\gamma}{\partial\tau}\right)+eE_g\left((1-\gamma)\left(E(X^2|\theta)\right)\frac{\partial\gamma}{\partial\tau}\right)=0 \qquad \text{(c3)}$$

$$dE(\Psi)/d\tau = -\tau E_g\left((1-\gamma)^2\right)\sum_i \frac{1}{c_i} + eE_g\left((1-\gamma)^2 \sum_i \frac{1}{c_i}(1-\gamma)E(X|\theta \right)$$

$$-\tau E_g\left(E(X\left|\theta\right)\frac{\partial\gamma}{\partial\tau}\right) + eE_g\left((1-\gamma)\operatorname{var}(X|\theta)\frac{\partial\gamma}{\partial\tau}\right) + eE_g\left((1-\gamma)\left(E(X|\theta)\right)^2\frac{\partial\gamma}{\partial\tau}\right) = 0 \tag{c4}$$

$$-\tau E_g\left((1-\gamma)^2\right)\sum_i \frac{1}{c_i} + eE_g\left((1-\gamma)^2 \sum_i \frac{1}{c_i}\right)E_g\left((1-\gamma)E(X|\theta)\right) + e\operatorname{cov}\left((1-\gamma)^2\sum_i \frac{1}{c_i}, (1-\gamma)E(X|\theta)\right)$$

$$-\tau E_g\left(E(X\left|\theta\right)\frac{\partial\gamma}{\partial\tau}\right) + eE_g\left((1-\gamma)\operatorname{var}(X|\theta)\frac{\partial\gamma}{\partial\tau}\right)$$

$$+eE_g\left((1-\gamma)E(X|\theta)\right)E_g\left(E(X|\theta)\frac{\partial\gamma}{\partial\tau}\right) + e\operatorname{cov}\left((1-\gamma)E(X\left|\theta\right), E(X\left|\theta\right)\frac{\partial\gamma}{\partial\tau}\right) = 0 \tag{c5}$$

$$E_g\left((1-\gamma)^2\right)\left[eE_g\left((1-\gamma)E(X|\theta)\right)-\tau\right]\sum_i \frac{1}{c_i} + \left[eE_g\left((1-\gamma)E(X|\theta)\right)-\tau\right]E_g\left(\left(E(X|\theta)\right)\frac{\partial\gamma}{\partial\tau}\right)$$

$$+eE_g\left((1-\gamma)\left(\operatorname{var}(X|\theta)\right)\frac{\partial\gamma}{\partial\tau}\right) \tag{c6}$$

$$+e\operatorname{cov}\left((1-\gamma)^2\sum_i \frac{1}{c_i}, (1-\gamma)E(X|\theta)\right) + e\operatorname{cov}\left((1-\gamma)E(X|\theta), \left(E(X|\theta)\right)\frac{\partial\gamma}{\partial\tau}\right) = 0$$

Now consider the effect of letting the subsidy deviate from zero. Making use of (31), (32) and (35), we obtain

$$\frac{d}{ds}E(\Psi)=$$

$$\iint\sum_i\left(\left(\tau(1-\gamma)\right)-e(1-\gamma)^2\left(\sum_j x_j\right)\right)\frac{\partial x_i}{\partial s}\,f(p)g(\theta)dpd\theta$$

$$+\iint\left(-\sum_i\tau x_i+e(1-\gamma)\left(\sum_j x_j\right)^2\right)\frac{\partial\gamma}{\partial s}\,f(p)g(\theta)dpd\theta$$

$$=\iint\tau(1-\gamma)\sum_i\frac{\gamma}{c_i}f(p)g(\theta)dpd\theta-\iint e(1-\gamma)^2X\sum_i\frac{\gamma}{c_i}f(p)g(\theta)dpd\theta \tag{c7}$$

$$-\iint\tau X\frac{\partial\gamma}{\partial s}\,f(p)g(\theta)dpd\theta+\iint e(1-\gamma)X^2\frac{\partial\gamma}{\partial s}\,f(p)g(\theta)dpd\theta$$

$$=\tau E_g\left((1-\gamma)\gamma\sum_i\frac{1}{c_i}\right)-eE_g\left((1-\gamma)^2\gamma E(X\,|\theta)\right)\sum_i\frac{1}{c_i}$$

$$-\tau E_g\left(E(X\,|\theta)\frac{\partial\gamma}{\partial s}\right)+eE_g\left((1-\gamma)E(X^2\,|\theta)\frac{\partial\gamma}{\partial s}\right)$$

We can reformulate (d3) as

$$\frac{d}{d\tau}E(\Psi)=$$

$$-\tau E_g\left((1-\gamma)\sum_i\frac{1}{c_i}\right)+eE_g\left((1-\gamma)^2E(X\,|\theta)\sum_i\frac{1}{c_i}\right)$$

$$+\tau E_g\left((1-\gamma)\sum_i\frac{\gamma}{c_i}\right)-eE_g\left((1-\gamma)^2E(X\,|\theta)\sum_i\frac{\gamma}{c_i}\right) \tag{c8}$$

$$-\tau E_g\left(E(X\left|\theta\right)\frac{\partial\gamma}{\partial\tau}\right)+eE_g\left((1-\gamma)\left(E(X^2\,|\theta)\right)\frac{\partial\gamma}{\partial\tau}\right)=0$$

We know from (37) above that $\frac{\partial\gamma}{\partial s}=\frac{\partial\gamma}{\partial\tau}$. Taking as our point of departure the optimal tax, we find

$$\frac{d}{ds}E(\Psi)=\frac{d}{ds}E(\Psi)-\frac{d}{d\tau}E(\Psi)=$$

$$\tau E_g(1-\gamma)\sum_i\frac{1}{c_i}-eE_g\left((1-\gamma)^2E(X\,|\theta)\right)\sum_i\frac{1}{c_i}$$

$$=\left(\tau E_g(1-\gamma)-eE_g(1-\gamma)E_g\left((1-\gamma)E(X\,|\theta)\right)-\operatorname{cov}\left((1-\gamma)E(X\,|\theta),(1-\gamma)\right)\right)\sum_i\frac{1}{c_i} \tag{c9}$$

and by performing simple reformulations

$$\left(\frac{d}{ds}E(\Psi)\right)/\sum_i \frac{1}{c_i}$$

$$= \tau E_g(1-\gamma) - eE_g(1-\gamma)E_g\left((1-\gamma)E(X|\theta)\right) - \operatorname{cov}\left((1-\gamma)E(X|\theta),(1-\gamma)\right)$$

$$\left(\frac{d}{ds}E(\Psi)\right)/\sum_i \frac{1}{c_i} = E_g(1-\gamma)\left(\tau - eE_g\left((1-\gamma)E(X|\theta)\right)\right) - \operatorname{cov}\left((1-\gamma)E(X|\theta),(1-\gamma)\right) \qquad \text{(c10)}$$

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