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Abstract

We study the welfare properties of a general equilibrium banking model with moral hazard that encompasses incentive mechanisms for bank risk-taking studied in a large partial equilibrium literature. We show that competitive equilibriums maximize welfare and yield an optimal level of banks' risk of failure. This result holds even though the risk of failure of competitive banks is higher than that of banks enjoying monopoly rents, and is robust to the introduction of social costs of bank failures. In this model, there is no trade-off between bank competition and financial stability.

JEL-Code: D500, G210.

Keywords: general equilibrium, bank competition, financial stability.

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I. INTRODUCTION

The issue of whether bank competition is detrimental for financial stability and should be restrained has a long history in the bank regulatory debate, having resurfaced in the aftermath of the recent financial crisis.[1]

The relatively large theoretical banking literature does not offer a clear guidance to this debate, since it has primarily focused on the relationship between competitive conditions and banks' risk of failure using partial equilibrium set-ups, obtaining contrasting results. In models where limited liability banks raise funds from insured depositors, choose the risk of their investment portfolio, and this choice is not observable, more competition results in a higher risk of bank failure, since higher funding costs erode banks' expected profits, prompting banks to choose riskier investments (see e.g. Keeley,1990, Matutes and Vives, 1996, Hellmann, Murdock and Stiglitz, 2000, Allen and Gale, 2000 and 2004a, Cordella and Levi-Yeyati, 2002, and Repullo, 2004, among others). By contrast, when banks compete a là Cournot in both loan and deposit markets and loan returns are perfectly correlated, these results are reversed, as banks' risk of failure declines as competition increases (Boyd and De Nicolò, 2005). However, if loan returns are not perfectly correlated, there might exist a U-shaped relationship between the number of banks and banks' risk of failure (Martinez-Miera and Repullo, 2010).

Yet, these partial equilibrium set-ups are unsuitable to address the key normative issue of whether there is a trade-off between bank competition and financial stability. Is a lower level of risk of bank failure necessarily undesirable in a welfare sense? More generally,

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[1] For an overview of this debate, see Boyd and De Nicolò (2005) and Financial Stability Oversight Council (2011).

what are the welfare rankings of different degrees of bank competition and the relevant levels of banks' risk of failure in a general equilibrium set-up? Addressing these questions is the main objective of this paper.

Policy prescriptions suggesting that bank competition should be restrained seem at variance with the welfare results of some general equilibrium banking models. Allen and Gale (2004b) demonstrate that perfect competition among intermediaries is Pareto optimal under complete markets, and constrained Pareto optimal under incomplete markets. Importantly, in their model an endogenously determined level of financial "instability" is a necessary condition of optimality. Identical results are obtained under low inflation in the general equilibrium monetary economy with aggregate liquidity risk analyzed by Boyd, De Nicolò and Smith (2004). Yet, these general equilibrium models do not feature the type of moral hazard in investment associated with financing choices considered by the partial equilibrium banking literature. This motivates our explicit consideration of these features in a general equilibrium set-up.

In our model, the size of the banking sector and the resource allocated to productive investment are determined endogenously, as risk-neutral agents choose to become either bankers or depositors, with banks established as coalitions of bankers financed by depositors. An important feature of our model is that setting up banks has a resource cost. As a result, a welfare evaluation of equilibriums will balance the costs of bank intermediation with the benefits of increasing available resources for productive investment. This novel modeling feature can be viewed as an extension of general equilibrium constructs where either the distribution of initial resources (see e.g. Holmstrom and Tirole 1997), or the partition of agents in banks or depositors (see e.g. Morrison and White, 2005), are exogenous.

As in the partial equilibrium literature, banks in our model choose the riskiness of their investment incurring higher effort costs to select lower risk investments, and these costs are interpreted as characterizing an intermediation technology that embeds screening and/or monitoring costs. Furthermore, bank risk choices are not observable; hence, there is moral hazard, with depositors taking into account banks' optimal risk choices in their decision to accept deposit terms. Differences in competitive conditions in the economy are simply modeled assuming that banks can choose to operate as monopolists or competitive banks, while depositors incur switching costs to be served by competitive banks. Thus, different degrees of bank competition are indexed by the fractions of bank deposit contracts in the economy priced monopolistically and competitively.

We consider the model under no deposit insurance, as well as the case where a "government" sets-up a deposit insurance scheme that is resource-feasible and partially or totally insures the principal of depositors' investment in a bank. Although there is no explicit rationale for deposit insurance in our model —as there is none in all partial equilibrium models we are aware of [2]—we wish to assess whether, and if so, how, the presence of an arguably realistic deposit insurance scheme affects the welfare ranking of competitive conditions.

The key result of this paper is that the competitive equilibrium in which banks compete a là Bertrand maximizes welfare. This result holds without or with deposit insurance. Notably, the competitive equilibrium maximizes welfare even though competitive

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[2] Most partial equilibrium models assume the existence of deposit insurance either for the sake of realism, or under the implicit assumption that deposit insurance corrects some not explicitly modeled coordination failures, such as the occurrence of runs.

banks exhibit a level of risk of failure higher than banks enjoying monopoly rents: this shows that a particular ranking of banks' risk of failure obtained in partial equilibrium set-ups is neither necessary nor sufficient for welfare maximization. In addition, perfect bank competition maximizes welfare even in the presence of social costs that are consistent with the existence of bank intermediation. Thus, a general equilibrium economy with investment choices subject to moral hazard delivers implications qualitatively similar to those obtained by Allen and Gale (2004b) and Boyd, De Nicolò and Smith (2004) in general equilibrium set-ups that lack these features.

The mechanism that delivers the welfare maximizing property of the competitive equilibrium is simple and intuitive. An increase in bank competition triggers a resource re-allocation mechanism that we term the *general equilibrium effect* of bank competition. As bank competition for funds increase, the return of deposits relative to the return of bank ownership increases, prompting a larger (smaller) fraction of agents to become depositors (bankers). This shift depicts stylistically an economy-wide shift of resources from investment in costly bank intermediation to investment in productive assets intermediated by banks. The resulting increase in economy-wide investment in productive assets generates an increase in expected output net of monitoring and production costs large enough to offset any reduction in the expected return due to the comparatively higher risk of failure of banks operating under more intense competition.

We obtain an additional result that is of independent interest. The introduction of deposit insurance *increases* the risk of failure of competitive banks, since it forces banks to increase deposit rates. The resulting increase in banks' cost of funds decreases their profits, inducing them to choose riskier investments. By contrast, deposit insurance decreases banks' risk of failure in the monopolistic sector, since it increases monopoly rents, which in turn inflate bank profits, inducing banks to choose safer investments. However, different degrees of deposit insurance coverage do not affect the welfare-maximizing property of the competitive equilibrium.

The remainder of the paper is composed of four sections. Section II describes the model. Section III details the bank problems, Section IV the equilibriums, and Section V the welfare rankings of competitive conditions. Section VI concludes. Proofs are in the Appendix.

II. THE MODEL

There are two dates, 0, and 1, and a large number A of risk neutral agents. Each agent is endowed with 1 unit of the date 0 good and with effort, derives disutility from effort, and has preferences over final date consumption. All agents have access to a safe (risk-free) technology which yields $\rho > 1$ per unit invested. At date 0 agents decide either to become bankers or depositors.

Banks

If an agent chooses to become a banker, she forgoes her initial endowment in exchange of the ability to be either a manager or an owner in a coalition of bankers, called bank. A bank composed of N bankers operates as follows. Any banker in the coalition can become a *bank manager* with probability $1-q$, or a *bank owner* with probability q, where draws are independent. Thus, $\mu(N) = 1 - q^N$ is the probability that at least one banker will be a bank manager.

If a banker becomes a bank manager, he has the ability to operate a risky project with borrowed funds by employing effort, and his choices and relevant outcomes are unobservable. If a banker becomes a bank owner, he has no ability of operating a project, but observes the choices of a bank manager and the outcome of his actions. Therefore, in a bank of size N one bank manager operates the project (saving on effort), and $N-1$ agents are or act as (if they are managers) bank owners. Prior to the draw that determines whether a banker is a manager or an owner, bankers agree to share bank profits equally. For simplicity, we assume that the size N of bank coalitions determined in equilibrium is sufficiently large so that the probability $\mu(N)$ that at least one banker becomes a manager is arbitrarily close to one.

A bank (through its bank manager) chooses one among a set of risky projects indexed by the probability of success $P \in [\underline{P},1]$. An investment z in a risky project yields Xz with probability P and 0 otherwise. We assume:

(A1) $\underline{P}X = \rho$.

Assumption (A1) implies that the expected return of any risky project indexed by $P \in (\underline{P},1]$ is higher than that of the safe technology.

A bank (manager) incurs effort costs in choosing P and investment z. The transformation of effort into a probability of project success $P \in [\underline{P},1]$ is interpreted as representing an *intermediation technology* that embeds banks' project screening and/or monitoring. The bank effort cost function is given by $m(P) = \frac{1}{2\alpha} P^2 z$. Therefore, the intermediation technology exhibits constant returns to scale, as the effort cost to implement

P is linearly related to z.[3] The effort cost of operating the project is $c(z) = \frac{1}{2\beta} z^2$. Therefore, the transformation of effort into output is a standard *production technology*.

Competition

To introduce different degrees of competition for funds, we assume that those agents who have chosen to be bankers can move at no cost to one of two unconnected locations, labeled M and C.

In location M, bankers are either unrestricted to communicate and choose to behave cooperatively, or are endowed with the power to set up local monopolies. Thus, each bank in M acts as a monopolist, choosing project risk and deposit rates so as to maximize expected profits subject to depositors' participation constraints. Location M represents the monopolistic banking sector. In location C, bankers do not communicate and compete for depositors' funds à la Bertrand. They set up competitive banks that choose project risk to maximize expected profits and deposit rates that maximize depositors' expected returns. Location C represents the competitive banking sector. As bankers do not incur any cost in moving to either location, there is free entry in the monopolistic and competitive banking sectors.

For simplicity, we assume that project risks are independent *across* locations, but perfectly correlated *within* locations. Denote with P_C and P_M the risk choices in the

[3] The assumption of constant returns to scale in monitoring is fairly standard in the banking literature (see e.g. Besanko and Kanatas, 1993, Boot and Greenbaum, 1993, Boot and Thakor, 2000, Dell'Ariccia and Marquez, 2006, and Allen et al., 2011).

competitive and monopolistic sectors respectively. Then, projects are successful in both sectors with probability $P_C P_M$, successful only in the competitive sector with probability $P_C(1-P_M)$, successful only in the monopolistic sector with probability $(1-P_C)P_M$, and fail in both sectors with probability $(1-P_C)(1-P_M)$.

Depositors

Agents who choose to be depositors will move to location C with probability σ, and to location M with probability $1-\sigma$. Since the remuneration of deposits in the monopolistic sector will be lower than in the competitive banking sector, parameter σ can be viewed as indexing depositors' switching costs to move to the competitive banking sector. These costs are simply modeled as depositors' risk to deposit in the less remunerative monopolistic banking sector. This assumption is germane to the assumption of depositors incurring traveling costs to bank locations in the Salop (1979) tradition (see, e.g. Park and Pennacchi, 2009). Thus, higher values of parameter σ index increasing funding market competition (equivalently, lower switching costs). We assume that relocation risks are independent, so that σ is also the fraction of depositors moving to location C.

Deposit insurance

Deposit insurance is pre-funded by taxation of initial resources A prior to agents' occupational choices to become bankers or depositors. The tax revenues are invested in the safe technology that yields ρ. Let τ denote the tax rate. The total "end-of-period" assets of the deposit insurance fund (DIF) are equal to $\tau A\rho$.

Denote with Z_C and Z_M total investment (deposits) in the competitive and monopolistic banking sectors respectively. A guarantee per unit of deposits $g \in (0,1]$ implies that the DIF will have contingent liabilities as follows: it will pay depositors nothing with probability $P_C P_M$, gZ_M with probability $P_C(1-P_M)$, gZ_C with probability $P_M(1-P_C)$, and $g(Z_C + Z_M)$ with probability $(1-P_M)(1-P_C)$.

If $g = 0$, there is *no* deposit insurance. If $g \in (0,1)$, there is a *partial* guarantee on a fraction g of the principal, while if $g = 1$, the principal is fully guaranteed. Whatever is left in the DIF after payments to depositors is distributed lump-sum to all agents in equal shares.

A credible deposit insurance scheme must be feasible. This requires that the DIF must have total assets whose value covers payments in every contingency. Clearly, the DIF will not raise funds through taxes in order to invest more than what is necessary to honor insurance payments in the worst-case outcome (when all banks fail) since doing that would be inefficient, as the safe technology is dominated in rate of return by the risky technology. Hence, the feasibility of the deposit insurance scheme requires that total DIF assets $\tau A\rho$ equal total payments in the worst-case outcome $g(Z_C + Z_M)$.

Contracts and sequence of decisions

Depositors finance banks with simple debt contracts that pay a fixed amount R per unit invested if the outcome of the investment is successful, and 0 otherwise. *Moral hazard* is introduced by assuming that bank choices of P are not observable by depositors. However, depositors take bank's optimal choice of P into account in their decision to accept the deposit terms offered by the bank.

Denote with x the *fraction* of bankers in location C, with A_B the number of bankers, with n_i the number of banks, with z_i bank size, and with R_i the deposit rates, for $i \in \{C, M\}$. Table 1 summarizes the sequence of decisions and the variables determined in the model.

Table 1. Sequence of decisions and variables

Time	**Agents' sequence of decisions**	**Variables**
t=0	If $g \in (0,1]$, the DIF is established by taxing initial resources Agents choose to become bankers or depositors Bankers choose to locate in M or in C Depositors' locate in M or C according to their location draw The number of banks and the debt equilibrium are determined	$x(1-x)$: fraction of bankers in C (M) A_B : number of bankers $A - A_B$: number of depositors σ : fraction of depositors in C n_C, n_M : number of banks in C and M Z_C, Z_M : total supply of funds (deposits) in the competitive and monopolistic sectors
t=1	Banks choose bank size (fund demand) Debt contract terms between banks and depositors are determined. Banks choose risk. Projects' output is realized and agents' consumption follows. The DIF pays out depositors (if necessary) and distributes remaining funds in equal shares to all agents	z_C, z_M R_C, R_M : deposit rates in the competitive and monopolistic sectors Z_C, Z_M : total investment in the competitive and monopolistic sectors P_C, P_M : risk choices in the competitive and monopolistic sectors

III. BANK PROBLEMS

We solve the model backward, starting with the bank problems.

Competitive banks

The representative competitive bank chooses P_C to maximize

$$\Pi^C \equiv P_C(X - R_C)z_C - \frac{1}{2\alpha} P_C{}^2 z_C - \frac{1}{2\beta} z_C{}^2 \quad (1)$$

The optimal interior solution is given by:

$$P_C^* = \alpha(X - R_C) \quad (2)$$

As we focus on interior solutions, we assume the following sufficient condition for $P_C^* \in (\underline{P},1)$:

(A2) $\underline{P} < \alpha X < 1$.

Bertrand competition implies that R_C maximizes depositors' expected return, with depositors taking into account the optimal bank risk decision given by (2). Depositors' expected return is given by:

$$P_C^* R_C + (1 - P_C^*) g = \alpha(X - R_C)(R_C - g) + g \quad (3).$$

This expected return is a strictly concave function of the deposit rate R_C, with the maximum reached at:

$$R_C^* = \frac{X + g}{2} \quad (4)$$

Substituting (4) in (2), the optimal risk choice of the competitive bank is:

$$P_C^* = \alpha(\frac{X - g}{2}) \qquad (5)$$

Using (4) and (5), the competitive bank expected profits are:

$$\Pi^C(z_C) \equiv \alpha \frac{(X - g)^2}{8} z_C - \frac{1}{2\beta} z_C^{\,2} \, (6)$$

The revenue per unit of investment of the competitive bank is $\pi_C \equiv \alpha \frac{(X - g)^2}{8}$. The optimal bank choice of size (or fund demand) z_C and the expected per-unit bank profits are given respectively by:

$$z_C = \beta \pi_C \quad (7),$$

$$\frac{\Pi^C}{z_C} = \frac{\pi_C}{2} \quad (8).$$

Monopolistic banks

The representative monopolistic bank chooses (P_M, R_M) to maximize

$$\Pi^M \equiv (P_M(X - R_M) - \frac{1}{2\alpha} P_M{}^2) z_M - \frac{1}{2\beta} z_M{}^2 \qquad (9)$$

subject to the depositors' participation constraint

$$P_M{}^* R_M + (1 - P_M{}^*) g \geq \rho \ (10),$$

where $P_M{}^* \in \arg\max \Pi^M$ is given by:

$$P_M^* = \alpha(X - R_M) \ (11),$$

Since the monopolistic bank profit function is strictly decreasing in the deposit rate R_M, constraint (10) is satisfied at equality. Inserting (11) in (10), we can write:

$$(X - R_M) R_M - g(X - R_M) + \alpha^{-1}(g - \rho) = 0 \ (12),$$

Equation (12) is equivalent to the quadratic equation:

$$R_M{}^2 - (X + g) R_M + (gX - \alpha^{-1}(g - \rho)) = 0 \ (13)$$

The solution of the monopolistic bank deposit rate is the smaller root of Equation (13), given by:

$$R_M{}^* = \frac{X + g - \sqrt{X^2 - 4\alpha^{-1}\rho + g(g + 4\alpha^{-1} - 2X)}}{2} \ (14)$$

A necessary condition for existence of equilibriums with monopolistic banks is a strictly positive deposit rate, which holds if $X + g > \sqrt{X^2 - 4\alpha^{-1}\rho + g(g + 4\alpha^{-1} - 2X)}$. This inequality can be easily shown to be equivalent to $\rho > g$, which is satisfied by the assumption $\rho > 1$.

To ensure well-defined deposit rates and existence of equilibriums with monopolistic banks, we introduce the following parametric assumptions. By assumption (A2), $g(g+4\alpha^{-1}-2X) \geq 0$. Therefore, a sufficient condition for a non negative determinant of the solution to the quadratic equation for all $g \in [0,1]$ is

(A3) $X^2 - 4\alpha^{-1}\rho > 0$,

Assumptions (A2) and (A3) imply that the parameter α lies in the interval $[4\rho X^{-2}, X^{-1}]$. This interval is non-empty assuming

(A4) $X > 4\rho$.

The optimal risk choice of the monopolistic bank is thus:

$$P_M^* = \alpha(X - R_M{}^*) = \alpha \frac{X - g + \sqrt{X^2 - 4\alpha^{-1}\rho + g(g - 2X + 4\alpha^{-1})}}{2} \quad (15)$$

Using (14) and (15), the expected profits of the monopolistic bank are:

$$\Pi^M \equiv \alpha \frac{(X - g + \sqrt{X^2 - 4\alpha^{-1}\rho + g(g - 2X + 4\alpha^{-1})})^2}{8} z_M - \frac{1}{2\beta} z_M{}^2 \quad (16)$$

The revenue per unit of investment of the monopolist bank is

$\pi_M \equiv \alpha \dfrac{(X - g + \sqrt{X^2 - 4\alpha^{-1}\rho + g(g - 2X + 4\alpha^{-1})})^2}{8}$. The optimal bank size z_M and the bank expected per-unit profits are respectively given by:

$$z_M = \beta \pi_M \quad (17).$$

$$\frac{\Pi^M}{z_M} = \frac{\pi_M}{2} \quad (18)$$

Comparing bank optimal choices

Recall that the risk of failure of competitive and monopolistic banks are respectively $P_C^* = \alpha(X - R_C^*)$ and $P_M^* = \alpha(X - R_M^*)$. From equations (4) and (14), we see that:

$$R_M^* \equiv R_C^* - \frac{\sqrt{X^2 - 4\alpha^{-1}\rho + g(g + 4\alpha^{-1} - 2X)}}{2} \quad (19),$$

where the term $\frac{\sqrt{X^2 - 4\alpha^{-1}\rho + g(g + 4\alpha^{-1} - 2X)}}{2}$ represents the *monopoly rents*. Since $R_M^* < R_C^*$ for all $g \in [0,1]$, we obtain:

Lemma 1 *For all* $g \in [0,1]$, $P_C^* < P_M^*$

Lemma 1 summarizes the standard result implied by risk-shifting in this type of model: the risk of failure of competitive banks is always strictly higher than that of monopolistic banks. Note that this result holds under no deposit insurance ($g = 0$) and with deposit insurance ($g \in (0,1]$). However, the relationship between deposit insurance coverage and bank risk differs for competitive and monopolistic banks. From Equation (5) we obtain:

Lemma 2 *The risk of failure of competitive banks increases monotonically with deposit insurance coverage*.

By contrast, Equation (15) shows that the risk of failure of the monopolistic bank is made of two terms: the first term is decreasing in g, while the second term—which represents monopoly rents—increases in g, since $g + 4\alpha^{-1} - 2X > 0$ by assumption (A2). It turns out that the net effect of an increase in deposit insurance coverage on bank risk is negative, as shown in:

Lemma 3 The *risk of failure of monopolistic banks declines monotonically with deposit insurance coverage*.

Proof: See Appendix

By Lemmas 2 and 3, the difference between the risk of failures of competitive and monopolistic banks increases with deposit insurance coverage. Yet, as we show below, different levels of deposit insurance coverage do not affect the welfare ranking of equilibriums indexed by the competition parameter σ.

IV. Equilibrium

For any given competition parameter $\sigma \in [0,1]$, and given banks' optimal choices of risk, deposit rates and their demand for funds $(P_i^*, R_i^*, z_i^*)_{i\in(C,M)}$, the characterization of equilibriums is completed by determining the seven-tuple $(\tau, x, A_B, (n_i, Z_i)_{i\in(C,M)})$, using the relevant seven equilibrium conditions.

The first two of equilibrium conditions establish equality between demand for and supply of funds in each sector:

$$n_C z_C = Z_C \qquad (20)$$

$$n_C z_C = Z_C \qquad (21)$$

The third equilibrium condition equates the expected profits per banker in the two sectors, which is implied by free-entry in the competitive and monopolistic sectors. Expected profits per banker in the competitive (monopolistic) sector are given by total profit $n_C \Pi^C$ ($n_M \Pi^M$)

divided by the number of bankers in that sector xA_B ($(1-x)A_B$). Therefore, this equilibrium condition is:

$$\frac{n_C \Pi^C}{xA_B} = \frac{n_M \Pi^M}{(1-x)A_B} \qquad (22)$$

The fourth equilibrium condition determines the number of agents who decide to become bankers, given by equalization of the profit per banker to the expected return of deposits:

$$\frac{n_C \Pi^C}{xA_B} = \sigma(P_C R_C + (1-P_C)g) + (1-\sigma)(P_M R_M + (1-P_M)g) \equiv r(\sigma, g) \qquad (23)$$

Note that in (23), the term $r(\sigma, g)$ denotes the expected return on deposits of an agent who has chosen to be a depositor *prior to moving* to the C or M locations.

The next two equilibrium conditions establish the supply of funds in the two sectors:

$$Z_C = \sigma(A(1-\tau) - A_B) \quad (24)$$

$$Z_M = (1-\sigma)(A(1-\tau) - A_B) \quad (25)$$

Finally, the seventh equilibrium condition determines the tax rate charged to set up the deposit insurance fund (DIF):

$$\rho\tau A = g(Z_C + Z_M) \Leftrightarrow \tau = \frac{gZ}{\rho A} \Rightarrow 1-\tau = \frac{\rho A - gZ}{\rho A} \quad (26)$$

The seven equations (20)-(26) form a linear system that can be easily solved by substitution. Inserting (20) and (21) in (22), and using (8) and (18), the equilibrium fraction of bankers who choose to operate in the competitive sector is given by:

$$x = \frac{\pi_C Z_C}{\pi_C Z_C + \pi_M Z_M} \quad (27)$$

By Equation (27), the fraction of bankers x choosing to operate in the competitive sector is increasing in the ratio of total bank revenues in the competitive sector $\pi_C Z_C$ to total bank revenues $\pi_C Z_C + \pi_M Z_M$.

Inserting (27) in (23) yields the equilibrium number of bankers:

$$A_B = \frac{\pi_C Z_C + \pi_M Z_M}{2r(\sigma, g)} \quad (28).$$

The number of bankers A_B is an increasing function of total bank revenues $\pi_C Z_C + \pi_M Z_M$, and a decreasing function of the expected return of deposits.

Inserting (28) in (24) and (25), we obtain:

$$Z_C = \sigma(A(1-\tau) - \frac{\pi_C Z_C + \pi_M Z_M}{2r(\sigma, g)}) \quad (29)$$

$$Z_M = (1-\sigma)(A(1-\tau) - \frac{\pi_C Z_C + \pi_M Z_M}{2r(\sigma, g)}) \quad (30)$$

The total supply of deposits (investment) in the banking sectors is $Z \equiv Z_C + Z_M$, where $Z_C = \sigma Z$ and $Z_M = (1-\sigma)Z$. Summing (29) and (30), using $Z_C = \sigma Z$ and $Z_M = (1-\sigma)Z$, and solving for Z, we obtain:

$$Z(\sigma, g) = \frac{2r(\sigma, g)\rho}{2r(\sigma, g)(\rho + g) + (\pi_C \sigma + \pi_M (1-\sigma))\rho} A \quad (31)$$

According to Equation (31), total investment in the banking sector can be expressed as a fraction of total available resources A, where this fraction depends on linear combinations of depositors' returns and revenues in the competitive and monopolistic banking sectors.

The following Lemma states a key implication of the model.

Lemma 4 For all $g \in [0,1]$, $\frac{\partial Z}{\partial \sigma} > 0$

Proof: See Appendix

Lemma 4 says that an increase in bank competition increases intermediated investment (total deposits), correspondingly reducing the amount of resources used in setting up banks. This resource shift occurs because an increase in the expected remuneration of deposits resulting from an increase in competition prompts a larger number of agents to become depositors rather than bankers. As detailed momentarily, this mechanism is a key determinant of the general equilibrium effect of bank competition on welfare.

V. WELFARE

As all agents are risk neutral, the welfare metric of an equilibrium indexed by given levels of the competition parameter $\sigma \in [0,1]$ and deposit insurance coverage $g \in [0,1]$ is expected total output net of total effort costs, obtained by summing the expected payoffs of all agents. The welfare function indexed by the pair (σ, g) is defined by:

$$\begin{aligned} Y(\sigma, g) \equiv\ & P_C P_M X(Z_C + Z_M) + P_C(1-P_M)XZ_C + (1-P_C)P_M XZ_M \\ & -(\frac{1}{2\alpha}P_C^2 Z_C + \frac{1}{2\alpha}P_M^2 Z_M) - (n_C c(z_C) + n_M c(z_M)) + \rho\tau A \end{aligned} \quad (32)$$

The first term of Equation (32) is expected output in the competitive and banking sectors, the second and third terms are the sum of monitoring and production costs in the two sectors respectively, and the fourth term is the return of investment of tax receipts of the DIF in the safe asset.

Using the equilibrium conditions (20)-(26), the welfare function of Equation (32) can be written as:

$$Y(\sigma, g) = [(P_C X - \frac{1}{2\alpha}P_C^2 - \frac{1}{2}\pi_C)\sigma + (P_M X - \frac{1}{2\alpha}P_M^2 - \frac{1}{2}\pi_M)(1-\sigma) + g]Z(\sigma, g) \quad (33)$$

The terms in brackets $P_C X - \frac{1}{2\alpha} P_C^2 - \frac{1}{2}\pi_C$ and $P_M X - \frac{1}{2\alpha} P_M^2 - \frac{1}{2}\pi_M$ are the expected outputs *net* of monitoring costs and (one half) bank revenues per unit of investment in the competitive and the monopolistic banking sectors respectively.

Differentiating (33) with respect to the competition parameter σ, we get:

$$\begin{aligned}\frac{\partial Y}{\partial \sigma} &= [(P_C X - \frac{1}{2\alpha} P_C^2 - \frac{1}{2}\pi_C) - (P_M X - \frac{1}{2\alpha} P_M^2 - \frac{1}{2}\pi_M)]Z(\sigma, g) + \\ &[(P_C X - \frac{1}{2\alpha} P_C^2 - \frac{1}{2}\pi_C)\sigma + (P_M X - \frac{1}{2\alpha} P_M^2 - \frac{1}{2}\pi_M)(1-\sigma) + g]\frac{\partial Z}{\partial \sigma}\end{aligned} \tag{34}$$

Note that if $P_C X - \frac{1}{2\alpha} P_C^2 - \frac{1}{2}\pi_C > P_M X - \frac{1}{2\alpha} P_M^2 - \frac{1}{2}\pi_M$, the welfare function (33) would be strictly increasing in the competition parameter as a direct consequence of Lemma 4. However, it is easy to generate numerical examples for which the above inequality is reversed. Nevertheless, we obtain the following

Proposition 1

For all $g \in [0,1]$, $\frac{\partial Y}{\partial \sigma} > 0$: the competitive equilibrium ($\sigma = 1$) maximizes welfare.

Proof: See Appendix.

By Proposition 1, the welfare function under competition reaches a maximum even though the associated banks' risk of failure is higher than the banks' risk of failure under imperfect competition ($\sigma \in [0,1)$). By implication, the level of bank risk of failure attained in the competitive equilibrium maximizes welfare.

The quantitative dominance of the general equilibrium effect of bank competition—captured by the increase in intermediated investment due to more competition illustrated in

Lemma 4—drives this result. Specifically, an increase in the expected returns on deposits due to an increase in competition increases the number of agents that choose to become depositors, and correspondingly decreases the number of agents choosing to become bankers. The increase in the supply of funds and the decrease in resources employed in setting up banks results in higher *total* expected output net of monitoring and production costs. In other words, as competition increases, there is a shift in the allocation of investment from bank intermediation to intermediated investment, which is generated endogenously by agents' optimal occupational choices and free entry into the banking sectors.

Social costs of bank failures

Restrictions on competition, as well as several regulations in banking, are typically justified by the existence of social costs associated with bank failures that are not internalized by banks.[4] Would the welfare ranking of competitive conditions established in Proposition 1 change by introducing social costs?

Assume that there exist social costs not internalized by banks that are an increasing and convex function of intermediated investment as follows: they are 0 with probability $P_C P_M$, $CZ_M{}^{\gamma}$ with probability $P_C(1-P_M)$, $CZ_C{}^{\gamma}$ with probability $P_M(1-P_C)$, and $C(Z_C{}^{\gamma}+Z_M{}^{\gamma})$ with probability $(1-P_M)(1-P_C)$, with $\gamma \geq 1$ and $C>0$. Therefore, expected social costs from bank failures are given by**:**

$$\begin{aligned} SC(\sigma,g) &\equiv P_C(1-P_M)CZ_M{}^{\gamma}+(1-P_C)P_M CZ_C{}^{\gamma}+(1-P_M)(1-P_C)C(Z_C{}^{\gamma}+Z_M{}^{\gamma}) = \\ & C[(1-P_M)(1-\sigma)^{\gamma}+(1-P_C)\sigma^{\gamma}]Z(\sigma,g)^{\gamma} \end{aligned} \tag{35}$$

[4] For a review of externalities in banking and a discussion of the implications for financial policies, see De Nicolò et al. (2012).

A welfare function augmented with the social costs just described is defined by:

$$W(\sigma, g) \equiv Y(\sigma, g) - SC(\sigma, g) \qquad (36)$$

However, social costs cannot be assumed arbitrarily large, since they need to be consistent with the existence of bank intermediation. Thus, we must require that social costs are not too large so as to make bank intermediation inessential. Without this requirement, it might be optimal to invest all resources in the safe asset, rendering a comparison of the welfare properties of different degrees of bank competition infeasible. This requirement implies an upper bound on the social cost function, which must hold for all competitive conditions and all levels of deposit insurance coverage. This upper bound is defined implicitly by the following inequality:

$$W(\sigma, g) \equiv Y(\sigma, g) - SC(\sigma, g) \geq \rho A \text{ for all } \sigma \in [0,1] \text{ and } g \in [0,1] \qquad (37)$$

A social cost function that satisfies (37) is called *admissible*. The following result establishes the welfare maximizing property of perfect bank competition in the presence of increasing and convex social costs of bank failures:

Proposition 2

Given any admissible social cost function that is increasing and convex in investment (deposits),and for all $g \in [0,1], \dfrac{\partial W}{\partial \sigma} > 0$*: a competitive equilibrium (*$\sigma = 1$*) maximizes welfare.*

Proof: See Appendix

The intuition underlying the result of proposition 2 is simple. As noted, the admissibility condition (37) establishes a lower bound to the welfare function. If the welfare function were lower than this bound, then it would not depend on the competition parameter, since no banks would be set up and all resources would be invested in the safe asset. Thus, welfare would still necessarily increase in the competition parameter under any social costs consistent with intermediation, as in the case of Proposition 1.

VI. Conclusion

We studied a general equilibrium model in which banks make their investment and financing decisions under moral hazard. The model exhibits features of a large partial equilibrium banking literature which obtains contrasting results with respect to the ranking of bank's risk of failure according to competitive conditions, and does not address the key normative issue of whether there exists a trade-off between bank competition and financial stability.

We showed that competition in banking maximizes welfare, even though the risk of failure of a competitive bank may be higher than that of a bank operating under imperfect competition, and even when social costs are taken into account. This result suggests that welfare implications derived from implications of partial equilibrium modeling may result in unwarranted normative prescriptions.

A general equilibrium perspective on desirable banking systems' structures and welfare-improving bank regulation has only slowly entered the current policy discourse, with theoretical explorations still limited in numbers. While capturing the essential features of several set-ups studied in a large partial equilibrium banking literature, our model is still highly stylized. Studying richer models of bank competition may be essential to assess the robustness of our conclusions, and this task is already part of our research agenda. Yet, we have shown that the implications of a large partial equilibrium literature on the relationship between banks' risk of failure and competition cannot support the conclusion that bank competition is detrimental for financial stability and should be restrained.

General equilibrium modeling of intermediation appears an essential tool to throw light on the desirable level of financial stability and systemic risk in an economy, and how it could be attained.

APPENDIX

Lemma 3 The *risk of failure of the monopolistic bank declines monotonically with deposit insurance coverage***.**

Proof: Differentiating Equation (14) with respect to g we get:

$$\frac{dR_M^{\ *}}{dg} = \frac{1}{2}(1 - \frac{1}{2}(X^2 - 4\alpha^{-1}\rho + g(g - 2X + 4\alpha^{-1}))^{-1/2}(2g - 2X + 4\alpha^{-1})) \quad \text{(A.1)}$$

Therefore, $sign\{\frac{dR_M^{\ *}}{dg}\} = sign\{1 - \frac{1}{2}(X^2 - 4\alpha^{-1}\rho + g(g - 2X + 4\alpha^{-1}))^{-1/2}(2g - 2X + 4\alpha^{-1})\}$

$\frac{dR_M^{\ *}}{dg} < 0$ is equivalent to the following string of inequalities:

$$\begin{aligned}
&1 < \frac{1}{2}(X^2 - 4\alpha^{-1}\rho + g(g - 2X + 4\alpha^{-1}))^{-1/2}(2g - 2X + 4\alpha^{-1}) \Leftrightarrow \\
&2(X^2 - 4\alpha^{-1}\rho + g(g - 2X + 4\alpha^{-1}))^{1/2} < 2g - 2X + 4\alpha^{-1} \Leftrightarrow \\
&4(X^2 - 4\alpha^{-1}\rho + g(g - 2X + 4\alpha^{-1})) < (2(g - X) + 4\alpha^{-1})^2 \Leftrightarrow \\
&4X^2 - 16\alpha^{-1}\rho + 4g(g - 2X + 4\alpha^{-1}) < 4(g - X)^2 + 16\alpha^{-2} + 16(g - X)\alpha^{-1} \Leftrightarrow \qquad \text{(A.2)} \\
&4X^2 - 16\alpha^{-1}\rho + 4g^2 - 8gX + 16g\alpha^{-1} < 4g^2 + 4X^2 - 8gX + 16\alpha^{-2} + 16\alpha^{-1}g - 16X\alpha^{-1} \Leftrightarrow \\
&-16\alpha^{-1}\rho < 16\alpha^{-2} - 16X\alpha^{-1} \Leftrightarrow \\
&-\rho < \alpha^{-1} - X
\end{aligned}$$

By (A1), $\alpha^{-1} - X \geq 0$. Therefore, $\frac{dR_M^{\ *}}{dg} < 0$, which implies $\frac{dP_M^{\ *}}{dg} > 0$ by Equation (15).

QED

Lemma 4 For all $g \in [0,1]$, $\frac{\partial Z}{\partial \sigma} > 0$

Proof:

Differentiating (31) with respect to σ, we get:

$$\frac{\partial Z}{\partial \sigma} = \frac{2A}{(.)^2}(\frac{\partial r}{\partial \sigma}\rho[2r(\sigma,g)(\rho+g)+(\pi_C\sigma+\pi_M(1-\sigma))\rho]$$
$$-r(\sigma,g)\rho[2\frac{\partial r}{\partial \sigma}(\rho+g)+\rho(\pi_C-\pi_M)\Leftrightarrow \quad \text{(A.3)}$$
$$\frac{2A\rho}{(.)^2}(\frac{\partial r}{\partial \sigma}(\pi_C\sigma+\pi_M(1-\sigma))-r(\sigma,g)\rho(\pi_C-\pi_M))$$

The term $\frac{\partial r}{\partial \sigma}(\pi_C\sigma+\pi_M(1-\sigma))-r(\sigma,g)\rho(\pi_C-\pi_M)$ is strictly positive for all $g \in [0,1]$, since

$\frac{\partial r}{\partial \sigma} = P_C R_C - \rho + g(P_M - P_C) > 0$ and $\pi_C < \pi_M$. Thus, $\frac{\partial Z}{\partial \sigma} > 0$.

QED

Proposition 1

For all $g \in [0,1]$, $\frac{\partial Y}{\partial \sigma} > 0$ *: the competitive equilibrium (* $\sigma = 1$ *) maximizes welfare*

Proof:

Using the bank profit functions in the two sectors, we can write:

$$P_C X - \frac{1}{2\alpha}P_C^2 = \pi^C + P_C R_C \quad \text{(A.4)}$$

$$P_M X - \frac{1}{2\alpha}P_M^2 = \pi^M + \rho \quad \text{(A.5)}$$

Hence, the terms $P_C X - \frac{1}{2\alpha}P_C^2 - \frac{1}{2}\pi_C$ and $P_M X - \frac{1}{2\alpha}P_M^2 - \frac{1}{2}\pi_M$ can be written as:

$$P_C X - \frac{1}{2\alpha}P_C^2 - \frac{1}{2}\pi_C = P_C R_C + \frac{1}{2}\pi_C \quad \text{(A.6)}$$

$$P_M X - \frac{1}{2\alpha}P_M^2 - \frac{1}{2}\pi_M = \pi^M + \rho - \frac{1}{2}\pi_M = \rho + \frac{1}{2}\pi_M \quad \text{(A.7)}$$

Substituting (A.6) and (A.7) in (33), and using (31), we can write:

$$Y(\sigma,g) \equiv [(P_C X - \frac{1}{2\alpha}P_C^2 - \frac{1}{2}\pi_C)\sigma + (P_M X - \frac{1}{2\alpha}P_M^2 - \frac{1}{2}\pi_M)(1-\sigma) + g]Z(\sigma,g) = \frac{[2(P_C R_C \sigma + (1-\sigma)\rho) + \pi_C \sigma + \pi_M (1-\sigma) + g]r(\sigma,g)\rho}{2r(\sigma,g)(\rho+g) + (\pi_C \sigma + \pi_M (1-\sigma))\rho} A \quad \text{(A.8)}$$

It is convenient to consider the cases $g = 0$ and $g > 0$ separately.

Let $g = 0$. Then

$$Y(\sigma,0) = \frac{[2(P_C R_C \sigma + (1-\sigma)\rho) + \pi_C \sigma + \pi_M (1-\sigma)]r(\sigma,0)}{2r(\sigma,0) + (\pi_C \sigma + \pi_M (1-\sigma))} A \quad \text{(A.9)}$$

Since $r(\sigma,0) = \sigma P_C R_C + (1-\sigma)\rho$, $Y(\sigma,0)$ can be written as:

$$Y(\sigma,0) = \frac{[2(\sigma P_C R_C + (1-\sigma)\rho) + \pi_C \sigma + \pi_M)(1-\sigma)]r(\sigma,0)}{2(\sigma P_C R_C + (1-\sigma)\rho) + (\pi_C \sigma + \pi_M (1-\sigma))} A = r(\sigma,0)A \quad \text{(A.10)}$$

Thus, $\frac{\partial Y}{\partial \sigma} = \frac{\partial r}{\partial \sigma}(\sigma,0)A = (P_C R_C - \rho)A > 0$, since $P_C R_C > \rho$.

Let $g \in (0,1]$ and re-write (A.8) as:

$$Y(\sigma,g) = h(\sigma) f(\sigma) A, \quad \text{(A.11)}$$

where

$$f(\sigma) \equiv \frac{r(\sigma,g)\rho}{2r(\sigma,g)(\rho+g) + (\pi_C \sigma + \pi_M (1-\sigma))\rho} \quad \text{(A.12)}$$

$$h(\sigma) \equiv 2(P_C R_C \sigma + (1-\sigma)\rho) + \pi_C \sigma + \pi_M (1-\sigma) + g \quad \text{(A.13)}$$

Next we show that both functions $f(\sigma)$ and $h(\sigma)$ are monotonically increasing in σ.

The derivative of $f(\sigma)$ is given by:

$$f'(\sigma) = \frac{1}{(2r(\sigma,g)(\rho+g)+(\pi_C\sigma+\pi_M(1-\sigma))\rho)^2}x$$
$$\frac{\partial r}{\partial\sigma}\rho 2r(\sigma,g)(\rho+g)+\frac{\partial r}{\partial\sigma}\rho(\pi_C\sigma+\pi_M(1-\sigma))\rho)$$
$$-r(\sigma,g)\rho 2r'(\sigma,g)(\rho+g)-r(\sigma,g)\rho(\pi_C-\pi_M)\rho = \qquad \text{(A.14)}$$
$$\frac{(\frac{\partial r}{\partial\sigma}\rho(\pi_C\sigma+\pi_M(1-\sigma))\rho)-r(\sigma,g)\rho(\pi_C-\pi_M)\rho)}{(2r(\sigma,g)(\rho+g)+(\pi_C\sigma+\pi_M(1-\sigma))\rho)^2}$$

By Lemma 3, $\frac{\partial r}{\partial\sigma}\rho(\pi_C\sigma+\pi_M(1-\sigma))\rho)-r(\sigma,g)\rho(\pi_C-\pi_M)\rho)>0$, hence $f'(\sigma)>0$.

The derivative of $h(\sigma)$ is given by:

$$h'(\sigma)\equiv 2(P_C R_C-\rho)+\pi_C-\pi_M \qquad \text{(A.15)}$$

Plugging in (A.15) equilibrium values, this derivative can be written as:

$$h'(\sigma)=2(P_C R_C-\rho)+\pi_C-\pi_M =$$
$$2\alpha(\frac{X-g}{2})\frac{X+g}{2}+(\alpha\frac{(X-g)^2}{8}-\alpha\frac{(X-g+\sqrt{X^2-4\alpha^{-1}\rho+g(g-2X+4\alpha^{-1})})^2}{8}) \qquad \text{(A.16)}$$

Therefore, $h'(\sigma)>0$ if

$$2\alpha(\frac{X-g}{2})\frac{X+g}{2}>$$
$$\alpha\frac{X^2-4\alpha^{-1}\rho+g(g-2X+4\alpha^{-1})+2(X-g)\sqrt{X^2-4\alpha^{-1}\rho+g(g-2X+4\alpha^{-1})}}{8})+2\rho \qquad \text{(A.17)}$$

By assumption (A1):

$$\sqrt{X^2-4\alpha^{-1}\rho+g(g-2X+4\alpha^{-1})}<X+g \Leftrightarrow$$
$$X^2-4\alpha^{-1}\rho+g(g-2X+4\alpha^{-1})<(X+g)^2 \qquad \text{(A.18)}$$

Therefore, inequality (A.18) implies:

$$\alpha\frac{X^2-4\alpha^{-1}\rho+g(g-2X+4\alpha^{-1})+2(X-g)\sqrt{X^2-4\alpha^{-1}\rho+g(g-2X+4\alpha^{-1})}}{8}<$$
$$\frac{X^2-4\alpha^{-1}\rho+g(g-2X+4\alpha^{-1})+2(X-g)(X+g)}{8} \qquad \text{(A.19)}$$

Hence, $h'(\sigma) > 0$ if:

$$2\alpha(\frac{X-g}{2})\frac{X+g}{2} > \frac{X^2 - 4\alpha^{-1}\rho + g(g-2X+4\alpha^{-1}) + 2(X-g)(X+g)}{8}) + 2\rho \quad (A.20)$$

If (A.20) holds, we can write the following string of inequalities:

$$\begin{aligned}
&2\alpha(\frac{X-g}{2})\frac{X+g}{2} > \frac{X^2 - 4\alpha^{-1}\rho + g(g-2X+4\alpha^{-1}) + 2(X-g)(X+g)}{8}) \Leftrightarrow \\
&4(X^2 - g^2) > X^2 - 4\alpha^{-1}\rho + g(g-2X+4\alpha^{-1}) + 2(X^2 - g^2) + 16\rho \Leftrightarrow \\
&2X^2 - 2g^2 > X^2 - 4\alpha^{-1}\rho + g(g-2X+4\alpha^{-1}) + 16\rho \Leftrightarrow \qquad (A.21)\\
&X^2 > -4\alpha^{-1}\rho + 2g^2 + g(g-2X+4\alpha^{-1}) + 16\rho \Leftrightarrow \\
&X^2 + 4\alpha^{-1}(\rho - g) + g(2X-g) > 2g^2 + 16\rho \Leftrightarrow \\
&X^2 + 4\alpha^{-1}(\rho - g) + g2X > 3g^2 + 16\rho
\end{aligned}$$

By assumptions (A4) and (A1), $g2X > 3g^2$, since $X > 4\rho > \frac{3}{2}g$, and by (A3), $X^2 > 16\rho$.

Hence, $X^2 + 4\alpha^{-1}(\rho - g) + g2X > 3g^2 + 16\rho$ holds, hence inequality (A.21) holds, which implies that $h'(\sigma) > 0$. In conclusion, $Y(\sigma, g) = h(\sigma) f(\sigma) A$ is strictly increasing in σ, since both functions $f(\sigma)$ and $h(\sigma)$ are monotonically increasing in σ. QED

Proposition 2

Given any admissible social cost function that is increasing and convex in investment (deposits),and for all $g \in [0,1]$, $\frac{\partial W}{\partial \sigma} > 0$: the competitive equilibrium ($\sigma = 1$) maximizes welfare.

Proof:

The welfare function (36) can be written as:

$$\begin{aligned}
&W(\sigma, g) = Z(\sigma, g)[(P_C R_C \sigma + \rho(1-\sigma) + \frac{1}{2}\pi_C \sigma + \frac{1}{2}\pi_M (1-\sigma) + g] + \qquad (A.22)\\
&-C[(1-P_M)(1-\sigma)^\gamma + (1-P_C)\sigma^\gamma]Z(\sigma, g)^\gamma
\end{aligned}$$

The upper bound defined by inequality (37) for all $\sigma \in [0,1]$ and $g \in [0,1]$ implies:

$$\begin{aligned} & W(\sigma,g) = Z(\sigma,g)[(P_C R_C \sigma + \rho(1-\sigma) + \frac{1}{2}\pi_C \sigma + \frac{1}{2}\pi_M (1-\sigma) + g] + \\ & -C[(1-P_M)(1-\sigma)^\gamma + (1-P_C)\sigma^\gamma]Z(\sigma,g)^\gamma \geq \rho A \Rightarrow \\ & C \leq \frac{Z(\sigma,g)[(P_C R_C \sigma + \rho(1-\sigma) + \frac{1}{2}\pi_C \sigma + \frac{1}{2}\pi_M (1-\sigma) + g] - \rho A}{[(1-P_M)(1-\sigma)^\gamma + (1-P_C)\sigma^\gamma]Z(\sigma,g)^\gamma} \equiv \bar{C}(\sigma,g) \end{aligned} \quad \text{(A.23)}$$

Function $\bar{C}(\sigma,g)$ is the highest level of social costs consistent with the existence of essential intermediation. Thus, a *lower* bound to any surplus function can be defined as:

$$\begin{aligned} & \underline{W}(\sigma,g) = Z(\sigma,g)[(P_C R_C \sigma + \rho(1-\sigma) + \frac{1}{2}\pi_C \sigma + \frac{1}{2}\pi_M (1-\sigma) + g] + \\ & -\bar{C}(\sigma,g)[(1-P_M)(1-\sigma)^\gamma + (1-P_C)\sigma^\gamma]Z(\sigma,g)^\gamma = \rho A \end{aligned} \quad \text{(A.24)}$$

Therefore,

$$W(\sigma,g) \geq \underline{W}(\sigma,g) \Leftrightarrow Y(\sigma,g) - SC(\sigma,g) \geq \underline{W}(\sigma,g) = \rho A. \quad \text{(A.25)}$$

Differentiation of (A.25) implies that $\frac{\partial Y}{\partial \sigma} - \frac{\partial SC}{\partial \sigma} \geq 0$. If $\frac{\partial Y}{\partial \sigma} - \frac{\partial SC}{\partial \sigma} < 0$ for some $\hat{\sigma} \in (0,1)$, then the maximum welfare function for all $\sigma \in (\hat{\sigma},1]$ would be $\underline{W}(\sigma,g)$, since $\underline{W}(\sigma,g)$ is a lower bound. But if this were the case, investing all resources in the safe asset would be best for all $\sigma \in (\hat{\sigma},1]$, which would imply that bank intermediation inessential. Thus, $\frac{\partial Y}{\partial \sigma} - \frac{\partial SC}{\partial \sigma} = \frac{\partial W}{\partial \sigma} > 0$ for any admissible social cost function that is increasing and convex in investment (deposits) and for all $g \in [0,1]$: the competitive equilibrium $(\sigma = 1)$ maximizes welfare.

QED

References

Allen, Franklin, Elena Carletti and Robert Marquez, 2011, "Credit competition and capital regulation", *Review of Financial Studies*, 24(4) , 983-1018.

Allen, Franklin, and Douglas Gale, 2000, "Comparing Financial Systems" (MIT Press, Cambridge, Massachusetts)

Allen, Franklin, and Douglas Gale, 2004a, "Competition and Financial Stability", *Journal of Money, Credit and Banking*, 36(2), 453-480.

Allen, Franklin, and Douglas Gale, 2004b, "Financial Intermediaries and Markets", *Econometrica*, Vol. 72, 4, 1023-1061.

Besanko, David, and Kanatas, George, 1993, Credit market equilibrium with bank monitoring and moral hazard, *Review of Financial Studies*, Vol. 6, N.1: 213-232.

Boot, Arnoud W., and Greenbaum, Stuart (1993) "Bank regulation, reputation, and rents: Theory and policy implications". In: Mayer, C., and Vives, X. (eds), Capital markets and financial intermediation. Cambridge, UK: Cambridge University Press, 292-318.

Boyd, John H., and Gianni De Nicolò, 2005, "The Theory of Bank Risk Taking and Competition Revisited", *Journal of Finance*, 60, 3, 1329-1343.

Boyd, John H., Gianni De Nicolò and Bruce D. Smith, 2004, "Crises in Competitive versus Monopolistic Banking Systems", *Journal of Money Credit and Banking* 36, 3, 487-506.

Cordella Tito, and Levi-Yeyati, 2002, "Financial Opening, Deposit Insurance, and Risk in A Model of Banking Competition", *European Economic Review*, 46, 693-733.

Dell'Ariccia, Giovanni, and Marquez, Robert, 2006, "Competition among regulators and credit market integration", *Journal of Financial Economics*, 79, :401-430.

De Nicolò, Gianni, Giovanni Favara, and Lev Ratnovski, 2012, "Externalities and Macroprudential Policy", IMF Staff Discussion Note # 12/05.

Financial Stability Oversight Council, 2011, Study of the Effects of Size and Complexity of Financial Institutions on Capital Market Efficiency and Economic Growth, January, Washington D.C.

Hellmann, Thomas, Kevin Murdock and Joseph Stiglitz, 2000, "Liberalization, moral hazard in banking, and prudential regulation: Are capital requirements enough?" *American Economic Review* 90(1), 147–165.

Holmstrom, Bengt, and Jean Tirole, 1997, "Financial Intermediation, Loanable Funds, and the Real Sector" *The Quarterly Journal of Economics*, CXII, 3: 663-691.

Keeley, Michael, 1990, "Deposit Insurance, Risk and Market Power in Banking", *American Economic Review*, 80, 1183–1200.

Klemperer, P., 1995, "Competition when Consumers Have Switching Costs: An Overview with Applications to Industrial Organization, Macroeconomics and International Trade," *Review of Economic Studies*, 62, 515–539.

Matutes, Carmen, and Vives, Xavier, 1996, "Competition for Deposits, Fragility, and Insurance", *Journal of Financial Intermediation* 5, 186-216.

Martinez-Miera, David, and Repullo, Raphael, 2010, "Does Competition Reduce the Risk of Bank Failure?", *Review of Financial Studies* 23 (10), 3638-3664.

Morrison, Alan D., and Lucy White, 2005, "Crises and Capital Requirements in Banking", *American Economic Review*, Vol. 95, No. 5 :1548-1572.

Park, Kwangwoo and George Pennacchi, 2009, "Harming Depositors and Helping Borrowers: The Disparate Impact of Bank Consolidation", *Review of Financial Studies*, 22: 1 1-40

Repullo, Raphael, 2004, "Capital Requirements, Market Power, and Risk-Taking in Banking", *Journal of Financial Intermediation*, Vol. 13: 156-182.

Salop, S., 1979, "Monopolistic Competition with Outside Goods", *Bell Journal of Economics*, 10:141–56.