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Evidence Using Panel Data from Germany

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Abstract

How much retirement income is needed in order to maintain one's living standard at old age? As it is difficult to find a firm basis for an empirical treatment of this question, we employ a novel approach to assessing an adequate replacement rate vis-à-vis income in the pre-retirement period. We subject indications regarding satisfaction with current income as collected in the German Socio-Economic Panel (GSOEP) to longitudinal analyses, using linear fixed-effects models and fixed-effects ordered logit models as our main analytical tools. We obtain a required net replacement rate of about 87% for the year of entry into retirement as a rather robust result, while replacement rates keeping the living standard unchanged may slightly decline over the retirement period.

JEL-Code: D100, D910, H550, J320.

Keywords: retirement, living standard, replacement rate, pensions, saving, satisfaction.

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1 Introduction

Maintaining one's standard of living at old age is usually seen as one of the core goals of old-age provision. Throughout the world, public pensions contribute to reaching this goal in varying degrees. In most countries, however, supplementary private provision – offered by employers or derived from individual saving – is needed to fully meet this task. But what does maintenance of one's living standard actually mean? How much additional cover is needed? These are important questions that individuals are faced with during their entire period of economic activity when making decisions about current consumption and savings. They are also relevant for employers who want to offer suitable benefit packages, or for policy-makers who want to take care of a meaningful framing for individual choices regarding retirement.

Quite interestingly, there is little research on the amount of retirement income, or the replacement rate vis-à-vis end-of-career salaries, which keeps the standard of living of pensioners constant when compared to the active period of their lives. At closer scrutiny, this isn't even surprising, as it turns out to be difficult to find a firm basis for empirical work in this area. In this paper, we will use a novel approach to assessing the percentage of net income accruing shortly before retirement which is needed to maintain one's living standard when retired. The net replacement rate which is required for this purpose is identified based on subjective assessments of satisfaction with income, as reported in a large panel dataset representing the German population living in private households (German Socio-Economic Panel, GSOEP). Using several empirical models and various specifications, the approach yields results that are relatively robust and highly plausible.

The paper is organized as follows. In section 2, we will briefly review the existing literature on the level of retirement income which is required for maintaining one's living standard. We will then explain the equivalence-scale framework that we are going to use in this paper (section 3) and introduce our data and methods (section 4). Section 5 presents the main results deriving from our analyses. Section 6 discusses implications for saving behaviour in the active period of life that arise from our findings. Section 7 concludes, pointing to possible limitations and promising extensions of our work.

2 Existing Literature

Existing work essentially uses three differing approaches to determining the level of income which is needed to maintain one's living standard at old age. In quite a number of contributions, a target replacement rate for retirement income is simply set heuristically, the focus then being on the saving rate that is required during the active period of life in order to top up mandatory old-age provisions accordingly. For instance, Schnabel (2003) refers to the net replacement rate of 70% offered by the German Statutory Pension Scheme between the mid-1970s and the late 1990s, before a continued reduction in benefit levels was legislated. In the US-literature, figures quoted for similar purposes are 80% (e.g., Schulz and Carrin, 1972) or 70% as well (e.g., Boskin and Shoven, 2009; Haveman et al., 2007). While useful for applied purposes, these figures are largely arbitrary.

Further contributions approach the subject mainly from a theoretical angle, attempting to derive an adequate, or even "optimal", replacement rate from the life-cycle model suggested by Modigliani and Brumberg (1954). In this model, consumption during retirement should be basically equal to consumption during the active period of life (i.e., net income minus savings), and savings are used – in addition to public (basic) pensions and/or employer-based provisions – to smooth the consumption profile correspondingly. Replacement rates derived from simple models typically range between 80% and 90% (e.g., Hamermesh, 1984; Bernheim, 1992; Mitchell and Moore, 1998). Up to a point, models of this kind can be calibrated empirically. They can also take into account various types of heterogeneity across individuals or households and further determinants (e.g., subjective discount rates and attitudes towards return risks and longevity risks) producing optimal consumption profiles that are much more complex (see Scholz, Seshadri, and Khitatrakun, 2006, for a very elaborate example leading to an optimal replacement rate of about 66%).[1]

These analyses are theoretically consistent and often formally elegant. However, for some of the parameters needed to determine optimal consumption profiles and savings rates, reliable empirical calibrations are lacking and, in fact, even widely accepted theoretical defaults do not exist. Also, life-cycle theory generally performs weakly in an empirical context, and it is open whether this points to shortcomings in the theory, gaps in existing data, or to "imperfections" in individuals' abilities to act rationally.[2] In the latter case, individuals could be well-advised to behave in line with life-cycle theory in order to provide adequately for their old age, but this proposition cannot easily be validated.

Relatively few contributions address the issue of maintaining the living standard in retirement as an essentially empirical one. This is mainly due to the suspicion, backed by recent research on behavioural finance (for a collection of findings, see Mitchell and Utkus, 2004), that individuals have difficulties in providing adequately for their old age. Therefore, observable saving behaviour or actual consumption profiles before and after retirement may not offer suitable yardsticks for the desired standard of living at old age in terms of revealed preferences – while confirming this suspicion is again difficult. For

[1]The figure quoted here rests on own calculations for the median household. Scholz, Seshadri, and Khitatrakun (2006) present their results in terms of "social security wealth" plus "optimal wealth targets".

[2]One of the controversies that arose is whether there is a "retirement-savings puzzle", as consumption appears to drop unexpectedly when household heads retire (Banks, Blundell, and Tanner, 1998); for a recent attempt to addressing this puzzle using German expenditure data, see Beznoska and Steiner (2012).

an alternative empirical strategy, Binswanger and Schunk (2008) assume that individuals may not be able to actually make adequate provisions, while they do know what adequate targets for doing so would be. Therefore, they collect information on precisely this through a special survey (among individuals living in the Netherlands and the US), putting some effort in making the questionnaire simple and to the point, without provoking biased responses.[3] With desired replacement rates mainly ranging between 75% and 100%, results are not entirely implausible. However, the basic assumption that individuals are able to state meaningful targets ex-ante for their desired living standard at old age remains disputable, and an empirical treatment resting on something that is more directly observable is still on the agenda.

In this paper, we therefore suggest to take (ex-post-)indications regarding individual satisfaction with income shortly before and after retirement as a basis for assessing the level of net income which is needed to maintain the standard of living at old age. Subjective perceptions of this kind have long been neglected, as their reliability seemed doubtful. This has changed through recent trends in economic research (Frey, 2008; Layard, 2011).

3 The Equivalence-Scale Framework

It is usually expected that pensioners need less income to maintain the standard of living they enjoyed before entering retirement. An obvious reason is that they may stop saving for retirement and start drawing on the wealth they have accumulated so far. Other reasons may include changes in their time use, implying a reduction in job-related expenses, e.g., for commuting to their workplace. Equivalence scales provide a measure for assessing how much less (or more) income is needed to reach the same level of well-being, or welfare, based on a comparison of households of different demographic structure. For example, equivalence scales can give an answer to the question how much less income a pensioner household does need to achieve the same standard of living as a non-pensioner household which, apart from labour-force status and age of household members, is similar to the pensioner household. An equivalence scale of 0.7 would mean that the pensioner household needs 70% of the income of the reference household. So if the replacement rate of retirement income were 70%, one could say that it is sufficient for maintaining the living standard.

Methods and applications for assessing equivalence scales which can be found in the literature are mostly concerned with households with children compared to childless households. They can be classified in three broad groups. A normative approach is based on the opinions of experts who assess the relative needs of households of different composition

[3]The authors admit that sampling procedures and the fact that the survey was internet-based may limit representativeness of respondents.

(Hagenaars, de Vos, and Zaidi, 1994). This approach generally lacks a theoretical justification, and different experts come up with different conclusions regarding the relative needs of households and thus with different equivalence scales (Stewart, 2009).

The approach used most often is based on household expenditure data. Well known examples include the approaches of Engel and Rothbarth (e.g., Deaton and Muellbauer, 1986) as well as more refined methods based on expenditure systems (examples using German data are Merz and Faik, 1995; Kohn and Missong, 2003). In general, these approaches require rather strong identifying assumptions, like a specific form of the utility function (Blundell and Lewbel, 1991).

A third group of approaches uses subjective assessments from survey respondents and encompasses different methods. For instance, Koulovatianos, Schröder, and Schmidt (2005) asked survey respondents in Germany and France to estimate the income a household needs to be as well off as a reference household which differs in size, with given income and socio-demographic characteristics. Equivalence scales can then be estimated from the ratios of average responses to reference income. The Leyden-approach uses respondents' assessment of income levels which they consider to be "very good", "good", "sufficient", "insufficient", "bad" and "very bad" for their household (van Praag and Kapteyn, 1973; Kapteyn, 1994). These ratings are used to infer household cost functions from which equivalence scales can be derived. Estimates based on such data are implausibly low in some cases, though (Melenberg and van Soest, 1995).

The method used in this paper takes survey respondents' satisfaction with own household income as a starting point. For example, the German Socio-Economic Panel (GSOEP) includes a question on the respondents' satisfaction with household income on a scale of 0 (lowest) to 10 (highest). Such indications can be used to estimate equivalence scales with cross-sectional or, in the case of repeated measurement, with longitudinal regression methods. Equivalence-scale estimates based on GSOEP data on income satisfaction have been published by Bellemare, Melenberg, and van Soest (2002), Charlier (2002) and Schwarze (2003). Stewart (2002, 2009) uses data of the same nature from the British Household Panel Survey (BHPS) to estimate equivalence scales for households of pensioner couples compared to pensioners living alone. Here, we are mainly interested in income satisfaction of individuals in households of given structure before and after they enter retirement.

The basic idea of equivalence scale estimation based on income satisfaction can be described as follows (see also Charlier, 2002; Stewart, 2009). Let x_{it} denote satisfaction with household income of person i at time t. x_{it} can be modeled as

$$x_{it} = \beta_1 \ln y_{it} + \beta_2 d_{it} + \mathbf{z}'_{it}\boldsymbol{\gamma} + \epsilon_{it}, \tag{1}$$

with $i = 1, \ldots, I$ and $t = 1, \ldots, T$. y_{it} denotes net income of person i at time t, $\mathbf{z}_{it}$

is a column vector of socio-demographic characteristics and ϵ_{it} is a well-behaved error term. d_{it} is a dummy variable capturing a characteristic of central interest (e.g., children, retirement). Given parameter estimates $\hat{\beta}_1$, $\hat{\beta}_2$ and $\hat{\boldsymbol{\gamma}}$, equivalence scales can be calculated by setting values of d and $\mathbf{z}$ for two persons i and j, equating the resulting versions of (1) and solving for y_{it}/y_{jt}. Assuming $d_{it} = 1$, $d_{jt} = 0$ and $\mathbf{z}_{it} = \mathbf{z}_{jt}$ this procedure yields:

$$A = \exp\left(-\frac{\hat{\beta}_2}{\hat{\beta}_1}\right) \tag{2}$$

A can be interpreted as the relative amount of household income person i needs to reach the same satisfaction with household income as person j, where "satisfaction with income" is interpreted as welfare level. If d takes the value 1 for pensioners and 0 for non-pensioners, (2) gives the net replacement rate a pensioner needs to be as well off as an otherwise similar non-pensioner.

4 Data and Methods

4.1 Data

The data used in our analyses are taken from the German Socio-Economic Panel (GSOEP), a representative longitudinal panel survey which was established in 1984. Since its initiation in West Germany, the dataset has been extended several times, and includes East Germany since 1990. At present, the dataset consists of more than 10,000 households, containing over 20,000 individuals. The GSOEP covers a wide range of demograpic, social, and economic variables for individuals living in randomly selected private households in Germany. In addition, it covers a substantial amount of information at the household-level.

Analyses are based on individuals who retired between 1992 and 2011 at ages ranging from 60 to 69. Up to three pre-retirement observations are used per individual to restrict the comparison inherent in a calculation of net replacement rates to pensioners and individuals close to retirement, though only one pre-retirement observation is actually required.[4] Concerning the number of observations in retirement, two variants will be used. The first variant uses all observations in retirement ("Full sample"); the second restricts the number to up to five and therefore imposes further constraints on net replacement rates which are then based only on observations shortly before and after retirement ("Re-

[4]Note that we do not differentiate between members of different schemes of old-age provision, such as the Statutory Pension Scheme covering the vast majority of workers, or special schemes for civil servants or the self-employed. Starting from their retirement at age 60 or higher, we follow any of these individuals for several years into their retirement period obtaining a certain number of "respondents" (indexed i) and a higher number of (annual) "observations" (indexed it).

stricted sample"). Furthermore, individuals who immigrated to Germany after 1991 were excluded from our analyses.

Retirement status is determined based on take-up of pensions. Individuals who report to have received pensions during the last year are identified as pensioners. An alternative criterion would be to use self-classifications of respondents as "pensioners", which leads to very similar results. In some cases, both approaches result in implausible transitions between retirement and non-retirement which were excluded from the analysis.

In all models log net household income and a dummy variable taking the value 1 for pensioners and 0 otherwise are used. Further explanatory variables include age, employment status (employed vs. unemployed/not working), household type (single, couple, couple with child/-ren), employment status of a partner and retirement status of a partner. Health is included via respondents' satisfaction with his/her health status. Wealth is difficult to include because the GSOEP only collects information on types of assets held and on capital income. Both types of information are used, with a focus on types of assets which are included via dummy variables taking the value 1 if a certain type of asset is held by a respondent and 0 otherwise. Relevant types of assets are "savings account", "mortgage savings plan", "life insurance", "securities", and "business assets". All monetary data are adjusted for CPI inflation, based on data provided by the German Federal Statistical Office.

In the longitudinal models we will be using, time-invariant variables are absorbed by individual-level fixed effects. Besides, pooled (i.e., cross-sectional) models will be employed where we control for sex, migration background and education in addition to the variables already described.

Some descriptive results for the full and the restricted sample are given in table 1.

4.2 Methods

Because of the longitudinal nature of the GSOEP, panel-data techniques will be applied. More specifically, fixed-effects models are used which allow for unobserved heterogeneity of individuals and have already been applied to satisfaction with household income in different contexts (e.g., Charlier, 2002; Schwarze, 2003). Irrespective of the choice of using fixed-effects models or not, satisfaction with household income as measured in the GSOEP can be interpreted as a cardinal or ordinal variable (Ferrer-i Carbonell and Frijters, 2004). Assuming cardinality requires responses to be comparable across respondents and that differences between responses can interpreted in such a way that increases in satisfaction form 2 to 4 have the same meaning as increases from 6 to 8. Ordinality only requires respondents to share the same interpretation of satisfaction levels, so that the answer 5 means the same to all respondents. Ferrer-i Carbonell and Frijters (2004) note that the

Table 1: Descriptive Results

	Full Sample	Restricted Sample
Age, mean	66	64
—, min	57	57
—, max	84	74
Age at retirement, mean	63	63
Year of retirement, mean	2002	2002
Net monthly income, mean	2,230	2,320
— before retirement, mean	2,454	2,454
— after retirement, mean	2,142	2,245
Satisfaction with income, mean	6.39	6.41
— before retirement, mean	6.24	6.24
— after retirement, mean	6.46	6.50
Satisfaction with health, mean	6.00	6.11
— before retirement, mean	6.09	6.09
— after retirement, mean	5.96	6.12

Notes: Monthly income is measured in Euro; satisfaction with income and health are measured on scales ranging of 0 (worst) to 10 (best).
Source: GSOEP, waves 1992–2011; authors' calculations.

interpretation of the income-satisfaction scale and the resulting choice between models has only limited effect on the results, while parameter estimates are sensitive to the inclusion of fixed effects. Riedl and Geishecker (2012) reach the same conclusion regarding ratios of parameters which are used for estimating equivalence scales.

Building on these observations, two modeling approaches are used in this paper. The first approach assumes income satisfaction to be a cardinal measure, and a linear fixed-effects (FE) model is used. The second variant assumes ordinality, and a fixed-effects ordered logit (FEOL) model is estimated, using an estimation strategy proposed by Baetschmann, Staub, and Winkelmann (2011).

Both approaches take the following model as a starting point:

$$x_{it}^* = \beta_1 \ln y_{it} + \beta_2 d_{it} + \mathbf{z}_{it}' \boldsymbol{\gamma} + \alpha_i + \epsilon_{it}, \tag{3}$$

where x_{it}^* is cardinal welfare level and α_i captures heterogeneity of individuals due to omitted time-invariant variables, possibly correlated with other explanatory variables. The FE model assumes that observed satisfaction with household income equals x_{it}^* and can easily be estimated by demeaning (within estimator; e.g. Greene, 2012). The FEOL model rests on the assumption that x_{it}^* is a latent variable and only x_{it} can be observed, which is given by

$$x_{it} = k \qquad \text{if} \qquad \tau_k < x^*_{it} \leq \tau_{k+1} \tag{4}$$
$$\tau_k < \tau_{k+1}, \quad \tau_1 = -\infty, \quad \tau_{K+1} = \infty,$$

with $k = 1, \dots, K$ and τ_k being a threshold at which a certain assessment of income satisfaction is given.

Assuming error terms in (3) to follow a standard logistic distribution,

$$f(\epsilon_{it}) = \frac{\exp(-\epsilon_{it})}{(1 + \exp(-\epsilon_{it}))^2}, \tag{5}$$

the probability of observing $x_{it} = k$ conditional on $\mathbf{z}^*_{it} = (\ln y_{it}, d_{it}, \mathbf{z}'_{it})$ and α_i is

$$\Pr(x_{it} = k | \mathbf{z}^*_{it}, \alpha_i) = G(\tau_{k+1} - \mathbf{z}^*_{it}\boldsymbol{\beta} - \alpha_i) - G(\tau_k - \mathbf{z}^*_{it}\boldsymbol{\beta} - \alpha_i), \tag{6}$$

with $G(c) = \exp c/[1+\exp c]$ and $\boldsymbol{\beta} = (\beta_1, \beta_2, \boldsymbol{\gamma}')'$. The problem with (6) is its non-linearity which does not allow for demeaning or differentiating to drop α_i like in the case of the FE model. Furthermore, only $a_{ik} = \tau_k - \alpha_i$ is identified.

A strategy for solving this problem starts from dichotomizing x_{it}. Let $d^k_{it} = I(x_{it} \geq k)$ be an indicator variable, taking values 1 (if $x_{it} \geq k$) and 0 (otherwise). Probabilities of observing $d^k_{it} = 1$ and $d^k_{it} = 0$ are given by:

$$\Pr(d^k_{it} = 1) = G(\mathbf{z}^*_{it}\boldsymbol{\beta} + \alpha_i - \tau_k) \tag{7}$$
$$\Pr(d^k_{it} = 0) = 1 - G(\mathbf{z}^*_{it}\boldsymbol{\beta} + \alpha_i - \tau_k) \tag{8}$$

Estimation of $\boldsymbol{\beta}$ via maximum likelihood is not consistent (e.g., Hsiao, 2003). Nevertheless, a conditional maximum-likelihood approach as suggested by Chamberlain (1979) yields consistent estimates, irrespective of the choice of cut-off point k. As in the case of the FE model time-invariant variables are absorbed in the fixed effects. In addition, individuals with constant d^k_{it}, i.e. $d^k_{i1} = d^k_{i2} = \cdots = d^k_{iT}$, can not be included in the analysis (for details see appendix A; textbook treatments of the conditional fixed-effects binary logit can be found in Hsiao, 2003 and Greene, 2012).

Several suggestions can be found in the literature on how to use this approach and make optimal use of the data at hand. Winkelmann and Winkelmann (1998) and Schwarze (2003) choose a cut-off point k manually. Ferrer-i Carbonell and Frijters (2004) set cut-off points for each individual, k_i. Das and van Soest (1999) estimate parameters for each possible cut-off point $k = 2, \dots, K$ and combine the resulting estimates. The method proposed by Baetschmann, Staub, and Winkelmann (2011) which will be used in this paper also uses all possible cut-off points. For all observations $\kappa = K - 1$ copies are generated, each using a different cut-off point; estimation proceeds by using all resulting copies with varying d_{it}^k at once and clustering standard errors. Baetschmann, Staub, and Winkelmann (2011) call this approach BUC ("Blow up and cluster") estimator. Comparisons of the different approaches show that the finite-sample performance of the BUC estimator is equal or superior to other methods (Baetschmann, Staub, and Winkelmann, 2011; Riedl and Geishecker, 2012). Furthermore, it can easily be implemented with standard statistical software (see the appendix of Baetschmann, Staub, and Winkelmann 2011 for Stata code implementing the BUC estimator).

5 Results

Parameter estimates along with standard errors are given in table 2. For example, the first row of these results corresponds to a linear fixed-effects (FE) model estimated on the full sample as specified in the preceding section. The coefficient of log household income is 1.468 with a standard error of 0.046. Retirement has a positive effect on satisfaction with household income. Combining both estimates yields an equivalence scale, or a required net replacement rate, of 0.870, i.e., pensioners need 87% of the household income of otherwise similar non-pensioners to reach the same welfare level. The approximate standard error amounts to 0.020, giving a 95%-confidence interval of about ± 0.039 (standard errors can be approximated using the delta method; for details see appendix B).[5]

This net replacement rate of 0.87 is based on the comparison of a pensioner to an employed person who is otherwise similar. In particular, this means that the individuals compared are of the same age. Because small differences in age (e.g., by one month) change estimates only slightly, results reported in table 2 can be interpreted as applying to replacement rates immediately after entry into retirement. Estimates including age effects are discussed further below.[6]

Although FE and FEOL models yield quite different parameter estimates for $\hat{\beta}_1$ and $\hat{\beta}_2$, the ratio of these parameters is almost identical in all cases (as noted by Riedl and Geishecker 2012 for simulated data).

The first four models produce approximately the same net replacement rates of about 0.87 and 0.88, irrespective of whether assets are included in addition to socio-demographic

[5]Note that FEOL parameter estimates have no direct interpretation and neither marginal effects nor probabilities can be calculated, because fixed effects α_i and thresholds τ_k are not estimated (see appendix A). Nevertheless the calculation of equivalence scales only requires ratios of parameters.

[6]Also, calculations of net replacement rates via (2) require fixed effects of pensioners and non-pensioners to be similar. Fixed effects implicitly control for sex, education, employment history and so forth, so that this implies no further restrictions on our interpretation of the results. It is impossible, though, to determine replacement rates based on a comparison of persons with different time-invariant characteristics.

Table 2: Model estimates for satisfaction with household income

Model	$\hat{\beta}_1$	s.e.	$\hat{\beta}_2$	s.e.	$\hat{A}$	s.e.	# Resp.	# Obs.	$\log L$
FE, Full Sample	1.468	0.046	0.204	0.031	0.870	0.020	2,891	25,176	-40,432
FEOL, Full Sample	2.181	0.113	0.282	0.056	0.879	0.027	2,804	90,661	-31,672
FE, Full Sample, Assets	1.447	0.046	0.206	0.031	0.867	0.020	2,888	25,148	-40,345
FEOL, Full Sample, Assets	2.147	0.112	0.286	0.056	0.875	0.027	2,801	90,445	-31,576
FE, Full Sample (w/o covariates)	1.436	0.042	0.283	0.020	0.821	0.013	2,894	25,218	-41,081
FEOL, Full Sample (w/o covariates)	2.072	0.099	0.401	0.039	0.824	0.021	2,807	90,824	-32,908
FE, Full Sample, Age dummies	1.461	0.046	0.168	0.034	0.891	0.021	2,891	25,176	-40,408
FEOL, Full Sample, Age dummies	2.171	0.113	0.235	0.060	0.897	0.027	2,804	90,661	-31,627
Binary FEL, Full Sample	2.245	0.126	0.261	0.076	0.890	0.033	1,554	14,528	-5,405
FE, Restricted Sample	1.443	0.051	0.205	0.035	0.868	0.022	2,896	19,722	-31,134
FEOL, Restricted Sample	2.113	0.121	0.289	0.058	0.872	0.029	2,801	62,143	-21,768
Pooled Binary Logit*, Full Sample	2.546	0.089	-0.023	0.058	1.010	0.023	2,868	24,840	-13,656
Pooled Ordered Logit*, Full Sample	2.446	0.070	0.015	0.046	0.993	0.019	2,868	24,840	-47,132

Notes: FE=Linear fixed effects, FEOL=Fixed-effects ordered logit
Unless noted otherwise all models control for age, age squared, employment status, employment status of partner (if any), household type, health status, and year dummies. See appendix D for full listings of parameter estimates.
* Standard errors of pooled estimates are clustered by individuals; pooled models include sex, migrant status and education as additional controls.
Source: GSOEP, waves 1992–2011; authors' estimations.

variables or not.[7] Dropping socio-demographics and assets and including only log household income and retirement reduces net replacement rates by about 0.05, while the order of magnitude is still the same.[8] Using age dummies instead of a quadratic specification yields slightly higher results. The same holds if a fixed-effects binary logit is estimated, using a subjective assessment of 6 as a cut-off point. Other plausible cut-off points yield similar results (not reported here). Using a restricted sample and analyzing only observations close before and after retirement does not change the resulting net replacement rates as well as standard errors.

Finally, pooled estimation of binary and ordered logit models leads to considerably higher net replacement rates. Though the estimates of $\hat{\beta}_1$ reach approximately the same order of magnitude as in the FEOL, the effect of retirement on satisfaction with income is estimated to be close to zero. Dropping age and age squared from the specification turns out to give equivalence estimates comparable to fixed-effect models, though, suggesting that these cross-sectional estimates confound age and retirement effects, while the fixed-effects models are quite robust to exclusion of explanatory variables.[9]

Comparing standard errors to other results in the literature, these seem to be at the lower end of the usual range. For example, Charlier (2002) reports standard errors ranging between 0.013 and 0.084 for fixed-effects models and Stewart (2009) reports standard errors between 0.052 and 0.168 for a wide range of different models and specifications. A still wider range is obtained in the cross-sectional analysis by Bellemare, Melenberg, and van Soest (2002), giving standard errors between 0.017 and 0.381 in the most extreme case. Note, however, that the aforementioned authors study equivalence scales dependent on household size or children, which limits the comparability of results.

Net replacement rates as reported in table 2 compare pensioners with otherwise similar non-pensioners.[10] Figure 1 shows age-specific net replacement rates based on dummy

[7] It appears that better controls for wealth would be desirable. However, replacing assets with capital income or using the latter in addition to assets has no effects on the estimates. Measuring wealth is difficult in any case, also in most other datasets, and we fully exploit what our data are providing. Likewise, using dummies instead of the linear specification of satisfaction with health has no effect on the results.

[8] See also appendix D. Among other things, it is possible that satisfaction with health is an endogenous variable, so that it is disputable whether it should be included in the regressions or not. Another problem with this variable is a potential selection effect (i.e., unhealthy respondents may drop out of the panel with increased probability; longitudinal weights as included in the GSOEP do not control for this problem). Correctly including it may therefore require an additional model for health, or a simultaneous modeling.

[9] Additionally, a binary random effects (RE) model was estimated, yielding an estimate of the net replacement rate of 0.95. A Hausman test was conducted comparing the binary RE and FE logit models (e.g., Greene, 2012), resulting in a p-value of 0.00 and thus providing strong evidence against the RE model.

[10] Parallel estimates for which the sample is divided into three income brackets in a rough fashion (lower quartile, medium quartiles, upper quartile) yield no indications that the net replacement rates required for a constant living standard are declining with income. In fact, results are higher for the upper quartile, while they are basically the same over the three lower quartiles. However, confidence intervals show huge overlaps, and it is possible that lack of better data for assets is particularly relevant for those with higher incomes.

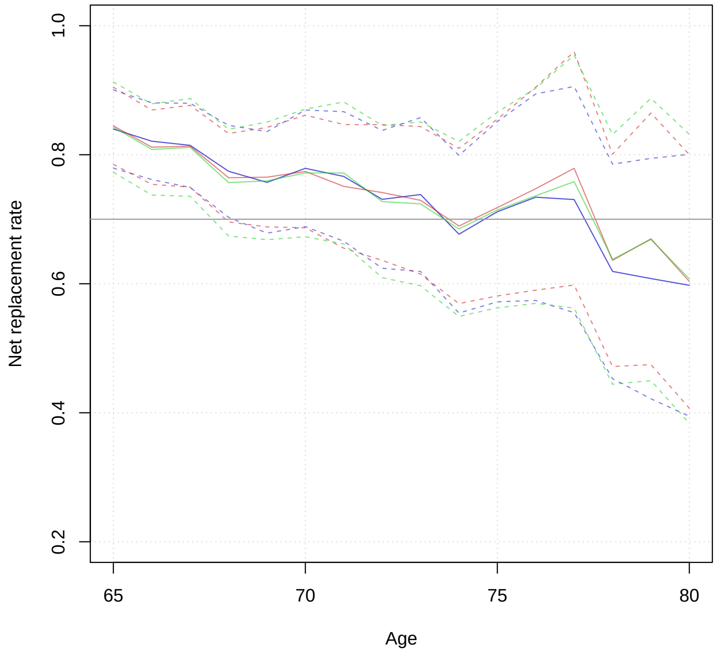

Figure 1: Age-specific net replacement rates

Note: Linear fixed effects = blue line, Linear fixed effects+Assets = red line, Fixed-effects ordered logit = green line; with approximate pointwise 95%-confidence band (dotted lines) and reference net replacement rate of 70% (grey line).

variables for age, comparing non-pensioners at age 64 with pensioners at ages ranging from 65 to 80, so that age effects are included in addition to the retirement effect. Estimates for three models are displayed in the figure: the FE model controlling for socio-demographic characteristics, the FE model controlling for socio-demographics and assets, and the FEOL model including socio-demographics. Additionally, an approximate pointwise 95%-confidence band is provided for each of the models (dotted lines) as well as a reference net replacement rate of 70%.

All three models yield almost identical results, indicating that replacement rates diminish with age. For example, in the FE model controlling for socio-demographics the point estimate for age 65 is about 0.84. At age 80 it has dropped to a value of 0.60. Note, however, that the width of the confidence band increases considerably with age, due to a diminishing number of observations. At age 65 the difference between the upper and lower bound is about 0.12, while at age 80 it has more than tripled to 0.41. At least the estimates for ages higher than 75 should thus be considered quite uncertain.

Weighting point estimates with age-specific survival probabilities leads to mean net replacement rates of 0.74 in the case of the FE models and 0.75 for the FEOL model (for details, see appendix C).

Note that public pensions are typically indexed to prices or wages, so that their replacement rate is constant or even increasing in real terms. Private pensions on the other hand are often nominally fixed amounts of annuitized withdrawals from accumulated wealth, hence their replacement rate is declining through inflation. For instance, if total retirement income were nominally fixed and if the inflation rate were 2% p.a. real replacement rates would decline much in line with the graphs in figure 1. Correcting the weighted average of age-specific point estimates correspondingly leads to mean net replacement rates of about 0.85 and 0.86.

6 Implications for Retirement Saving

Net replacement rates of around 87% (for the first year in retirement) or between 74% and 86% (over a longer time horizon, depending on "automatic" protection of retirement wealth against inflation) that are needed to maintain one's living standard vis-à-vis the years immediately before retirement are a relatively hard result of our analyses. These figures are clearly plausible. At the same time, they are higher than those obtained in most previous studies (see section 2 and, for a discussion, section 7). In other words, individual efforts to make an adequate amount of retirement saving may need to be intensified in the light of our findings.

When income near retirement is known, determining how much an individual should have saved to provide for old age is a simple matter of financial mathematics (e.g., Scholz, Seshadri, and Khitatrakun, 2006, sections II and III). In our context, age-specific replacement rates and actuarial calculations regarding longevity risks can be combined, taking care of a host of details (public pension benefits and other sources of old-age income,[11] e.g., employer-based pensions, taxation, inflation, real interest rates), to calculate additional wealth which is needed in the year of retirement to meet the requirement of a constant living standard as defined in this paper. Ideally, this amount of wealth should then have been accumulated through a continuous flow of annual savings during the active period of life, taking advantage of compound interest over a long accumulation period to keep annual reductions in current consumption low.

In a forward-looking perspective, i.e., with respect to the practical questions regarding an adequate saving rate that individuals have to choose while still active, a number of further uncertainties and unknowns have to be taken into account. Current income (and the income record thus far) may be useful indicators for living standard during the active period of life – better, at least, than any other one. However, possible future spells of non-employment or unemployment, uncertain individual earnings careers, the possibility

[11]Also, expected time trends in these amounts over a given individual's retirement period need to be taken into account. For instance, the level of public benefits may decline over time through reduced up-ratings to keep the pension system financially viable.

of switching to new jobs (with new employers offering other pension plans, or none at all), the individual timing of retirement, all this clearly has an impact on needs and capacities to save for retirement.

Also, at each point in time during one's active period of life, current income tends to be part of a typical life-time profile of earnings and income derived from other sources.[12] If income increases over most of the relevant years, saving enough to maintain the current level as if it were to remain unchanged may not be sufficient. On the other hand, saving enough to maintain a higher future living standard when current income is still low may well overburden younger individuals. Ideally, therefore, one would have to define a non-linear time profile of saving rates which increases in current income, following a typical time profile of income from an ex-ante-perspective, and appears to be adequate in terms of the end-of-career standards developed here.

7 Conclusions

Compared to earlier work on how to maintain one's living standard at old age (see section 2), the approach we employ in this paper is particular with respect to the following features. It addresses the issue in an essentially empirical fashion, but does not rest on individual expectations which may be accurate or not. Instead, it is based on how individuals actually perceive their income at old age, taking satisfaction with income as a reliable indicator for individual welfare.

The results obtained regarding an adequate net replacement rate of retirement income are rather robust over differing estimation strategies and specifications. Taken together, the longitudinal models with various covariates yield a narrow band of point estimates (ranging from 0.86 to 0.90), or for a retirement age of 65 (amounting to 0.84). However, uncertainty about age-specific net replacement rates which are suited to maintain one's living standard increases considerably with age. This is mainly due to a declining number of observations with higher ages.

In any case, these results are higher than those picked heuristically or obtained from various approaches in earlier studies. A potential reason may be that, by tracking individuals who are actually entering retirement, our results are not influenced by subjective discounts on future needs by which the replacement rates required for maintaining one's living standard could be underestimated when it is determined in a purely prospective fashion. This consideration strengthens the case for our empirical approach.

To the extent that the declining pattern of age-specific rates, with an actuarial value

[12] Estimates of typical life-cycle profiles of wage earnings that are based on GSOEP data can be found, e.g., in Fehr (1999, ch. 4) or Fenge, Uebelmesser, and Werding (2006). If capital income is essentially used for accumulating wealth, wages are the main source of current, fresh saving.

of at least 0.74 (but still up to 0.86 correcting for the expected impact of inflation), can be taken to be reliable, it would be interesting to learn more about material questions regarding needs that become less important with age and others that may become more prominent. The data we are using are not suited to answer these questions, as they contain next to no information about the structure of household spending. For this purpose, other data with more detailed indications regarding income and expenditure should be utilized, while living standards may need to be identified in a different fashion then, which may well be a promising issue for future amendments to the present study.

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A The Fixed-Effects (Ordered) Logit Model

Let x_{it}^* be a latent unobserved variable, determined by

$$x_{it}^* = \mathbf{z}_{it}'\boldsymbol{\beta} + \alpha_i + \epsilon_{it}, \tag{A.1}$$

where $\mathbf{z}_{it}$ is a column vector of covariates, α_i is an unobserved, time-invariant individual effect and ϵ_{it} is an error term. Only x_{it} is observed, though, and given by

$$x_{it} = \begin{cases} 1 & \text{if } x_{it}^* > 0 \\ 0 & \text{if } x_{it}^* \leq 0. \end{cases} \tag{A.2}$$

Assuming independence of $\mathbf{z}_{it}$ and ϵ_{it} and a standard logistic distribution for error terms,

$$f(\epsilon_{it}) = \frac{\exp(-\epsilon_{it})}{(1 + \exp(-\epsilon_{it}))^2}, \tag{A.3}$$

it follows that

$$\begin{aligned} \Pr(x_{it} = 1 | \mathbf{z}_{it}, \alpha_i) &= \Pr(x_{it}^* > 0 | \mathbf{z}_{it}, \alpha_i) && \text{(A.4)} \\ &= \Pr(\epsilon_{it} > -\mathbf{z}_{it}'\boldsymbol{\beta} - \alpha_i | \mathbf{z}_{it}, \alpha_i) && \text{(A.5)} \\ &= 1 - G(-\mathbf{z}_{it}'\boldsymbol{\beta} - \alpha_i) && \text{(A.6)} \\ &= G(\mathbf{z}_{it}'\boldsymbol{\beta} + \alpha_i), && \text{(A.7)} \end{aligned}$$

with $G(c) = \exp c/[1 + \exp c]$ being the c.d.f. of the standard logistic distribution, just like in the case of the "usual" logit approach without fixed effects. In contrast to the linear fixed-effects model, there is no simple way of dropping α_i. Estimation via maximum likelihood is possible, but inconsistent (Hsiao, 2003), due to the incidental parameter problem; see Lancaster (2000) for a general discussion. At least for small T, parameter estimates in finite samples are severely biased (Greene, 2004).

A solution to this problem was suggested by Chamberlain (1979). He proposed to use conditional maximum likelihood estimation. The basic idea is to use a likelihood based on conditional instead of unconditional probabilities which do not depend on α_i.

Let $x_i = (x_{i1}, \ldots, x_{iT})$ and $a_i = \sum_{t=1}^{T} x_{it}$. The conditional probability of x_i given a_i is

$$\begin{aligned} \Pr\left(x_i = (j_1, \ldots, j_T) | \sum_t j_t = a_i\right) &= \frac{\Pr(x_i = (j_1, \ldots, j_T), \sum_t j_t = a_i)}{\Pr(\sum_t j_t = a_i)} && \text{(A.8)} \\ &= \frac{\Pr(x_i = (j_1, \ldots, j_T))}{\Pr(\sum_t j_t = a_i)} && \text{(A.9)} \end{aligned}$$

which does not depend on α_i. To see why, first note that the numerator of (A.9) is

$$\Pr(x_i = (j_1, \ldots, j_T)) = \prod_{t=1}^{T} \left[\left(\frac{\exp(\mathbf{z}_{it}'\boldsymbol{\beta} + \alpha_i)}{1 + \exp(\mathbf{z}_{it}'\boldsymbol{\beta} + \alpha_i)} \right)^{j_t} \left(\frac{1}{1 + \exp(\mathbf{z}_{it}'\boldsymbol{\beta} + \alpha_i)} \right)^{1-j_t} \right]. \quad \text{(A.10)}$$

Furthermore, let $\mathcal{J}_a$ be the set of all possible sequences $J = (j_1, \ldots, j_T)$ for which $\sum_t j_t = a$. It follows that the denominator equals

$$\Pr\left(\sum_t j_t = a_i \right) = \sum_{J \in \mathcal{J}_a} \Pr(x_i = J), \quad \text{(A.11)}$$

where $\Pr(x_i = J)$ can be formulated like in (A.10). Now (A.9) can be written as

$$\frac{\prod_{t=1}^{T} \left[\left(\frac{\exp(\mathbf{z}_{it}'\boldsymbol{\beta}+\alpha_i)}{1+\exp(\mathbf{z}_{it}'\boldsymbol{\beta}+\alpha_i)} \right)^{j_t} \left(\frac{1}{1+\exp(\mathbf{z}_{it}'\boldsymbol{\beta}+\alpha_i)} \right)^{1-j_t} \right]}{\sum_{J \in \mathcal{J}_a} \prod_{t=1}^{T} \left[\left(\frac{\exp(\mathbf{z}_{it}'\boldsymbol{\beta}+\alpha_i)}{1+\exp(\mathbf{z}_{it}'\boldsymbol{\beta}+\alpha_i)} \right)^{j_t} \left(\frac{1}{1+\exp(\mathbf{z}_{it}'\boldsymbol{\beta}+\alpha_i)} \right)^{1-j_t} \right]}, \quad \text{(A.12)}$$

where α_i can be factored out. The same holds for constant z_j. Observations with $a_i = 0$ or $a_i = T$, i.e., $x_i = (0, \ldots, 0)$ or $x_i = (1, \ldots, 1)$, have to be dropped from consideration, because (A.9) equals 1 and does not contribute to the likelihood function given below.

Because x_i does not depend on α_i given a_i, a_i is a sufficient statistic for α_i, i.e.,

$$\Pr(x_i | \mathbf{z}_{i1}, \ldots, \mathbf{z}_{iT}, \alpha_i, a_i) = \Pr(x_i | \mathbf{z}_{i1}, \ldots, \mathbf{z}_{iT}, a_i). \quad \text{(A.13)}$$

Chamberlain (1979) showed that using this sufficient statistic and maximizing the conditional likelihood

$$L(\mathbf{b}) = \prod_{i=1}^{I} \Pr(x_i | a_i, \mathbf{b}) \quad \text{(A.14)}$$

with respect to $\mathbf{b}$ yields consistent and asymptotic normal estimates assuming mild regularity conditions. Note, however, that the estimates are not efficient in the sense that the asymptotic variance is equal to the Cramer-Rao lower bound (Chamberlain, 1979; Andersen, 1970).

In case of the dichotomized fixed-effects ordered logit, the probability of observing $d_{it}^k = 1$ is

$$\Pr(d_{it}^k = 1 | \mathbf{z}_{it}, \alpha_i, \tau_k) = G(\mathbf{z}_{it}'\boldsymbol{\beta} + \alpha_i - \tau_k). \quad \text{(A.15)}$$

Conditional on a_i $\Pr(d_i^k = (d_{i1}^k, \ldots, d_{iT}^k))$ is independent of α_i as well as of τ_k

$$\Pr(d_i^k | \mathbf{z}_{i1}, \ldots, \mathbf{z}_{iT}, \alpha_i, \tau_k, a_i) = \Pr(d_i^k | \mathbf{z}_{i1}, \ldots, \mathbf{z}_{iT}, a_i), \tag{A.16}$$

which can be derived in the same way as the result for (A.9). Note also that the threshold τ_k can be made individual-specific, i.e., τ_{ik} can be used instead of τ_k without loss of generality (Ferrer-i Carbonell and Frijters, 2004).

The BUC estimator of Baetschmann, Staub, and Winkelmann (2011) is based on maximization of

$$L_{\text{BUC}}(\mathbf{b}) = \prod_{k=2}^{K} L^k(\mathbf{b}), \tag{A.17}$$

where $L^k(\mathbf{b})$ is the likelihood function for cut-off point k, which can also be shown to yield consistent estimates. Because individuals contribute multiple times to $L_{\text{BUC}}(\mathbf{b})$, Baetschmann, Staub, and Winkelmann (2011) propose to use a robust sandwich variance estimator.

B Approximate Standard Errors and Confidence Intervals for Net Replacement Rates

Estimates of the coefficients of the FE model as well as the FEOL model are asymptotically normal, so that standard errors of nonlinear transformations of these coefficients can be approximated via the delta method, as noted by Charlier (2002). Consider the most basic case in which net replacement rates are given by

$$\hat{A} = \exp\left(-\frac{\hat{\beta}_2}{\hat{\beta}_1}\right), \tag{B.1}$$

where we now write $\hat{A}$ instead of A to make clear that net replacement rates are based on parameter estimates $\hat{\beta}_1$ and $\hat{\beta}_2$.

Given asymptotic normality of $\boldsymbol{\beta} = (\beta_1, \beta_2)'$, application of the Delta Method leads to

$$\hat{A} \to N\left(A, \frac{\delta A}{\delta \boldsymbol{\beta}'} \mathrm{Var}(\hat{\boldsymbol{\beta}}) \frac{\delta A}{\delta \boldsymbol{\beta}}\right) \tag{B.2}$$

from which

$$\mathrm{Var}(\hat{A}) = \frac{\delta \hat{A}}{\delta \hat{\boldsymbol{\beta}}'} \mathrm{Var}(\hat{\boldsymbol{\beta}}) \frac{\delta \hat{A}}{\delta \hat{\boldsymbol{\beta}}} \tag{B.3}$$

follows. For the simple case of (B.1), this yields

$$\mathrm{Var}(\hat{A}) = \frac{\delta \hat{A}}{\delta \hat{\beta}_1}^2 \mathrm{Var}(\hat{\beta}_1) + \frac{\delta \hat{A}}{\delta \hat{\beta}_2}^2 \mathrm{Var}(\hat{\beta}_2) + 2\frac{\delta \hat{A}}{\delta \hat{\beta}_1}\frac{\delta \hat{A}}{\delta \hat{\beta}_2} \mathrm{Cov}(\hat{\beta}_1, \hat{\beta}_2), \tag{B.4}$$

with

$$\frac{\delta \hat{A}}{\delta \hat{\beta}_1} = \exp\left(-\frac{\hat{\beta}_2}{\hat{\beta}_1}\right) \frac{\hat{\beta}_2}{\hat{\beta}_1^{\,2}} \quad \text{and} \quad \frac{\delta \hat{A}}{\delta \hat{\beta}_2} = \exp\left(-\frac{\hat{\beta}_2}{\hat{\beta}_1}\right) \frac{-1}{\hat{\beta}_1}, \tag{B.5}$$

which can be evaluated at parameter estimates.

For the more general case of

$$\hat{A} = \exp\left(\sum_{j=2}^{J} \frac{\hat{\beta}_j}{\hat{\beta}_1}(z_{jr} - z_j)\right) \tag{B.6}$$

this procedure leads to

$$\hat{A} = \sum_{j=1}^{J} \frac{\delta \hat{A}}{\delta \hat{\beta}_j}^2 \mathrm{Var}(\hat{\beta}_j) + \sum_{j=1}^{J} \sum_{k=j+1}^{J} 2 \frac{\delta \hat{A}}{\delta \hat{\beta}_j} \frac{\delta \hat{A}}{\delta \hat{\beta}_k} \mathrm{Cov}(\hat{\beta}_j, \hat{\beta}_k). \tag{B.7}$$

Setting $d_j = z_{jr} - z_j$, the derivates are

$$\frac{\delta \hat{A}}{\delta \hat{\beta}_1} = \exp\left(\sum_{j=2}^{J} \frac{\hat{\beta}_j}{\hat{\beta}_1} d_j\right)\left(\sum_{j=2}^{J} \frac{-\hat{\beta}_j d_j}{\hat{\beta}_1^2}\right) \quad \text{and} \quad \frac{\delta \hat{A}}{\delta \hat{\beta}_j} = \exp\left(\sum_{k=2}^{J} \frac{\hat{\beta}_k}{\hat{\beta}_1} d_k\right) \frac{d_j}{\beta_1}. \tag{B.8}$$

Given estimates of the variance and assuming normality, $\alpha\%$ confidence intervals can be calculated in the usual way and can be used to construct pointwise confidence bands.

C Mortality-weighted Net Replacement Rate and Correcting for Inflation

Let $\hat{A}_j$ be the estimate of the age-specific net replacement rate at age j. Further, let S_j denote the probability of surviving from age 65 to age j, with $S_{65} = 1$. Then the mortality-weighted net replacement rate can be calculated as

$$\bar{\hat{A}} = \frac{\sum\limits_{j=65}^{\rho} \hat{A}_j S_j}{\sum\limits_{j=65}^{\rho} S_j}, \tag{C.1}$$

where ρ is the maximum age under consideration.

The calculation in figure 1 uses $\rho = 80$ and values for S_j based on the life table for German males for the years 2009 to 2011, as published by the Federal Statistical Office.

Weighting for inflation proceeds in a similar fashion. Let φ be the annual inflation rate. Then the following formula is used:

$$\bar{\hat{A}}_{\varphi} = \frac{\sum\limits_{j=65}^{\rho} \hat{A}_j S_j \frac{1}{(1-\varphi)^{j-65}}}{\sum\limits_{j=65}^{\rho} S_j}. \tag{C.2}$$

D Tables

Table 3: Model estimates for the linear fixed-effects models

Coefficient	Full Sample (1)	Full Sample (2)	Full Sample (3)	Restricted Sample
log Income	1.436	1.468	1.447	1.443
	(0.042)	(0.046)	(0.046)	(0.051)
Retired (Dummy)	0.283	0.204	0.206	0.205
	(0.020)	(0.031)	(0.031)	(0.035)
Age	—	0.219	0.219	0.329
		(0.056)	(0.057)	(0.119)
Age^2 ($\times$ 100)	—	-0.135	-0.135	-0.229
		(0.040)	(0.041)	(0.091)
Satisfaction w/ health	—	0.161	0.161	0.167
		(0.006)	(0.006)	(0.007)
Unemployed/Not working	—	-0.223	-0.228	-0.241
		(0.033)	(0.033)	(0.035)
Household: Couple	—	-0.470	-0.477	-0.498
		(0.065)	(0.065)	(0.088)
Household: Couple w/ child	—	-0.841	-0.853	-0.891
		(0.087)	(0.088)	(0.112)
Partner: Working	—	0.078	0.084	0.105
		(0.040)	(0.040)	(0.044)
Partner: Retired	—	0.026	0.032	0.056
		(0.036)	(0.036)	(0.041)
Year (1992-1994)	—	0.090	0.089	-0.053
		(0.187)	(0.187)	(0.206)
Year (1995-1997)	—	0.155	0.148	0.038
		(0.155)	(0.155)	(0.172)
Year (1998-2000)	—	0.189	0.177	0.109
		(0.124)	(0.124)	(0.139)
Year (2001-2003)	—	0.169	0.155	0.134
		(0.093)	(0.093)	(0.106)
Year (2004-2006)	—	-0.132	-0.142	-0.124
		(0.064)	(0.064)	(0.075)
Year (2007-2009)	—	-0.156	-0.160	-0.135
		(0.038)	(0.038)	(0.047)
Home ownership	—	—	0.038	—
			(0.061)	
Savings account	—	—	0.118	—
			(0.030)	
Mortgage savings plan	—	—	0.055	—
			(0.029)	
Life insurance	—	—	0.016	—
			(0.026)	
Securities	—	—	0.091	—
			(0.029)	
Business assets	—	—	-0.103	—
			(0.069)	
$\log L$	-41,081	-40,432	-40,345	-31,134
# Respondents	2,894	2,891	2,888	2,896
# Observations	25,218	25,176	25,148	19,722

Note: Standard errors in parentheses. Coefficients and standard errors for age squared are reported $\times$100.
Source: GSOEP, waves 1992–2011; authors' estimations.

Table 4: Model estimates for the fixed-effects ordered logit models

Coefficient	Full Sample (1)	Full Sample (2)	Full Sample (3)	Restricted Sample
log Income	2.072	2.181	2.147	2.113
	(0.099)	(0.113)	(0.112)	(0.121)
Retired (Dummy)	0.401	0.282	0.286	0.289
	(0.039)	(0.056)	(0.056)	(0.058)
Age	—	0.297	0.296	0.382
		(0.124)	(0.124)	(0.226)
Age^2 ($\times$ 100)	—	-0.178	-0.177	-0.259
		(0.091)	(0.091)	(0.174)
Satisfaction w/ health	—	0.227	0.227	0.233
		(0.012)	(0.012)	(0.013)
Unemployed/Not working	—	-0.329	-0.337	-0.355
		(0.060)	(0.059)	(0.062)
Household: Couple	—	-0.693	-0.698	-0.711
		(0.151)	(0.150)	(0.169)
Household: Couple w/ child	—	-1.254	-1.268	-1.313
		(0.196)	(0.195)	(0.215)
Partner: Working	—	0.121	0.129	0.154
		(0.074)	(0.074)	(0.077)
Partner: Retired	—	0.049	0.054	0.094
		(0.072)	(0.072)	(0.073)
Year (1992-1994)	—	0.187	0.179	-0.053
		(0.280)	(0.280)	(0.320)
Year (1995-1997)	—	0.246	0.234	0.047
		(0.228)	(0.228)	(0.265)
Year (1998-2000)	—	0.283	0.266	0.153
		(0.185)	(0.185)	(0.215)
Year (2001-2003)	—	0.257	0.234	0.207
		(0.141)	(0.141)	(0.166)
Year (2004-2006)	—	-0.208	-0.225	-0.192
		(0.100)	(0.100)	(0.121)
Year (2007-2009)	—	-0.261	-0.268	-0.217
		(0.061)	(0.061)	(0.077)
Home ownership	—	—	0.050	—
			(0.130)	
Savings account	—	—	0.167	—
			(0.051)	
Mortgage savings plan	—	—	0.081	—
			(0.056)	
Life insurance	—	—	0.014	—
			(0.047)	
Securities	—	—	0.150	—
			(0.051)	
Business assets	—	—	-0.112	—
			(0.121)	
$\log L$	-32,908	-31,672	-31,576	-21,768
# Respondents	2,807	2,804	2,801	2,801
# Observations	90,824	90,661	90,445	62,143

Note: Standard errors in parentheses. Coefficients and standard errors for age squared are reported $\times$100.
Source: GSOEP, waves 1992–2011; authors' estimations.

Table 5: Model estimates for the binary fixed-effects ordered logit and the pooled models

Coefficient	Binary FEOL	Binary Logit	Ordered Logit
log Income	2.245	2.546	2.446
	(0.126)	(0.089)	(0.070)
Retired (Dummy)	0.261	-0.023	0.015
	(0.076)	(0.058)	(0.046)
Age	0.285	0.264	0.208
	(0.137)	(0.111)	(0.093)
Age^2 ($\times$ 100)	-0.183	-0.159	-0.113
	(0.098)	(0.081)	(0.068)
Satisfaction w/ health	0.226	0.310	0.324
	(0.014)	(0.013)	(0.012)
Unemployed/Not working	-0.260	0.081	0.047
	(0.080)	(0.067)	(0.056)
Household: Couple	-0.778	-0.723	-0.679
	(0.150)	(0.093)	(0.083)
Household: Couple w/ child	-1.350	-1.614	-1.591
	(0.206)	(0.137)	(0.117)
Partner: Working	0.117	-0.437	-0.389
	(0.100)	(0.075)	(0.061)
Partner: Retired	0.053	-0.181	-0.178
	(0.088)	(0.064)	(0.053)
Year (1992-1994)	-0.152	0.185	0.329
	(0.448)	(0.127)	(0.114)
Year (1995-1997)	-0.193	0.025	0.249
	(0.371)	(0.105)	(0.088)
Year (1998-2000)	0.022	0.066	0.193
	(0.299)	(0.089)	(0.073)
Year (2001-2003)	0.037	0.003	0.100
	(0.225)	(0.074)	(0.059)
Year (2004-2006)	-0.371	-0.268	-0.145
	(0.155)	(0.061)	(0.047)
Year (2007-2009)	-0.298	-0.198	-0.148
	(0.095)	(0.047)	(0.034)
Sex (Female)	—	0.191	0.237
		(0.064)	(0.054)
Migrant	—	0.084	0.052
		(0.102)	(0.091)
Education (Yrs)	—	-0.057	-0.049
		(0.014)	(0.012)
$\log L$	-5,405	-13,656	-47,132
# Respondents	1,554	2,868	2,868
# Observations	14,528	24,840	24,840

Note: FEOL=Fixed-effects ordered logit
Standard errors in parentheses. Coefficients and standard errors for age squared are reported $\times$100.
Source: GSOEP, waves 1992–2011; authors' estimations.

Table 6: Model estimates for age dummies

Coefficient	Linear FE	Linear FE (/w assets)	FEOL
log Income	1.461	1.440	2.171
	(0.046)	(0.046)	(0.113)
Retired (Dummy)	0.168	0.171	0.235
	(0.034)	(0.034)	(0.060)
Age 65	0.078	0.080	0.136
	(0.040)	(0.040)	(0.059)
Age 66	0.136	0.137	0.227
	(0.046)	0.046	(0.070)
Age 67	0.134	0.133	0.219
	(0.053)	(0.053)	(0.080)
Age 68	0.224	0.221	0.370
	(0.061)	(0.061)	(0.094)
Age 69	0.222	0.222	0.362
	(0.070)	(0.070)	(0.108)
Age 70	0.206	0.208	0.327
	(0.080)	(0.080)	(0.121)
Age 71	0.250	0.253	0.399
	(0.091)	(0.091)	(0.141)
Age 72	0.269	0.274	0.456
	(0.102)	(0.102)	(0.157)
Age 73	0.293	0.298	0.467
	(0.113)	(0.114)	(0.174)
Age 74	0.375	0.382	0.586
	(0.127)	(0.127)	(0.194)
Age 75	0.315	0.313	0.495
	(0.140)	(0.140)	(0.217)
Age 76	0.257	0.258	0.429
	(0.155)	(0.155)	(0.235)
Age 77	0.196	0.199	0.366
	(0.172)	(0.172)	(0.276)
Age 78	0.492	0.509	0.741
	(0.190)	(0.190)	(0.315)
Age 79	0.417	0.434	0.639
	(0.215)	(0.215)	(0.351)
Age 80	0.569	0.592	0.846
	(0.240)	(0.241)	(0.392)
$\log L$	-40,408	-40,321	-31,627
# Respondents	2,891	2,888	2,804
# Observations	25,176	25,148	90,661

Note: FE=Fixed effects; FEOL=Fixed-effects ordered logit
Standard errors in parentheses. All models control for socio-demographic characteristics (parameter estimates available from the authors on request).
Source: GSOEP, waves 1992–2011; authors' estimations.