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Abstract

This paper analyses the ways in which product fragmentation (producing part of a product in one country, and a part elsewhere) can be used by multinational firms which have different productivity to serve the market abroad when product chains can be internationally and arbitrarily fragmented. Product fragmentation is thus an additional option to serving markets abroad by either horizontal or vertical FDI. Upon opening a market to trade, firms with the lowest productivity will exit, those with intermediate productivity will export, and those with higher productivity will choose fragmentation. Among the latter, the more productive a firm is, the more product chains are allocated abroad. Firms with the highest productivity will choose horizontal FDI. At a sector level, the more prone to fragmentation a sector is, the lower will be the ratio of exports to FDI sales.

JEL-Code: F120, F230, L250.

Keywords: product fragmentation, heterogeneity, export, FDI.

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1. Introduction

During the past two decades, international fragmentation of production across locations by multinationals has become even more prevalent. Multinationals place different production phases of a product in different countries or regions, and set up a world wide fragmentation network.

Fragmentation is not accounted for in the new trade theory based on Dixit-Stiglitz preferences where all firms export because of consumers' love of variety and the assumption is that there is no fixed cost for exporting. Available evidence indicates, that even within a sector, some firms export while many others do not (e.g. Head and Ries 2003, Eaton, Kortum and Kramarz 2004); a phenomenon not well explained by theory using an assumption of representative firms within industries.

Heterogeneous firms models present explanations for this phenomenon. The Melitz (2003) dynamic industry model with heterogeneous firms and fixed costs of exporting shows that exposure to trade induces only the more productive firms to enter export markets and simultaneously forces the least productive to exit. And among exporters, Melitz's model suggests that firms with higher productivity have greater sales. As a generalization on Melitz, Helpman, Melitz and Yeaple (2004) (hereinafter referred to as HMY) introduce horizontal FDI into Melitz's model, and focus on a firms' choice between exports and horizontal FDI. They concluded that the least productive firms serve only the domestic market, that relatively more productive firms export, and that the most productive firms engage in FDI. In HMY, at a sector

level, the more within-industry dispersion of firm productivity, the lower the ratio of exports to FDI sales. There is, however still no fragmentation in this structure.

This paper extends the HMY model, and analyzes the different ways multinational firms which have different productivity can serve the market abroad when product chains can also be internationally and arbitrarily fragmented. Besides exporting and horizontal FDI, firms face another choice which is to supply abroad through international fragmentation. In HMY, international fragmentation is absent，so firms only choose between exports and horizontal FDI to serve markets abroad. We first consider firms' choice when international fragmentation is possible but has a sole fixed cutoff point. We then analyze firms' choice when international fragmentation is possible but there are two possible fixed cutoff points; and lastly the situation when product chains can be arbitrarily fragmented.

The main conclusions are as the following: On exposure to trade, firms with the lowest productivity will exit, those with intermediate productivity will export, and those with higher productivity will choose fragmentation. Among these, the more productive a firm is the more of the production chains is allocated abroad. Firms with the highest productivity will choose horizontal FDI. At a sector level, the more prone a sector is to be fragmented, the lower the ratio of exports to FDI sales. One of the most significant findings is that the ability of a country to fragment production internationally is a source of comparative advantage.

The remainder of the paper is composed of four sections. The second is a literature review. In the third, we set up a model in which product chains can be

arbitrarily fragmented and study the different choices of heterogeneous firms in supplying abroad. The last part is the conclusion.

2. Related Literature

This paper is related to two strands of literature, one on heterogeneous firms (Melitz (2003), Helpman, Melitz and Yeaple (2004))[①]. The other is fragmentation (Feenstra and Hanson (1996, 1999), Deardorff（2001）, Yamashita (2010)). The former is about the different choices of the firms with different productivity, and the latter focuses on the effect of fragmentation on factors' prices, employment and welfare. This paper is based on the HMY model, and the HMY model is based upon Brainard (1993) and Melitz (2003). Here we briefly review these papers.

Brainard (1993) develops a "proximity-concentration" model to analyze the mode used by representative firms to supply abroad. In her study, there are two countries and two commodities. One is a homogenous product, and the other a differentiated product. The differentiated product sector is characterized by increasing returns at firm level since some inputs that have the characteristic of public goods, and scale economies at plant level so that concentrating production lowers unit costs. There is also a variable transport cost that rises with distance so that investment abroad can save transport costs. The decision to supply abroad via export or horizontal FDI depends on the trade-off between proximity advantage and concentration advantage. When proximity advantage has predominance over concentration advantage, a two-way investment equilibrium will arise. In contrast, when concentration advantage has predominance over proximity advantage, there will be a two-way trade equilibrium.

① Redding(2010) provides a recent review of this literature.

A two-way investment equilibrium is more likely to occur the higher are returns to scale at corporate level relative to plant level; the higher are transport costs across market, the greater is expenditure on differentiated products in the foreign market; and the higher is the elasticity of substitution between varieties in preferences. Under some circumstances, that is, for intermediate ranges of transport costs relative to plant level scale economies and of returns at the corporate level relative to the plant level, there exists a mixed equilibrium in which multinationals and international trade co-exists.

When a mixed equilibrium occurs, in Brainard's model, crowding out effects restrict the number of the firms that can supply abroad via horizontal FDI, so, firms that choose exports and horizontal FDI co-exist (Head and Ries，2004). In Brainard, firms are homogenous. They choose to export or use FDI to supply markets abroad, but we do not know which firms choose to export, and which firms choose FDI.

Melitz (2003) later built a dynamic industry model with heterogeneous firms to analyze the intra-industry effects of international trade. Using heterogeneous firms with different firms have different productivity allows them to choose whether they supply abroad, and, we can distinguish between which firms supply abroad via export or FDI. His model suggests that exposure to trade induces only the most productive firms to enter the export market, the second most productive firms to serve only domestic market, and forces the least productive firms to exit.

To enter the market, firms must make an initial investment, and then draw their productivity from a common distribution. After firms know their productivity, they

choose one of the following three choices: to exit, to supply the domestic market, or supply the domestic market and at the same time export. Firm needs to meet certain productivity requirements to make the two latter choices.

Melitz's model generates the following outcomes: Firms with high enough productivity can supply abroad via export because only these firms can afford the fixed export cost; exposure to trade forces the least productive firms to exit; market shares are reallocated toward more productive firms and contribute to an aggregate productivity increase.

Based on Melitz and following Brainard, Helpman, Melitz and Yeaple (2004) introduce an additional choice for firms into Melitz's model, namely horizontal FDI. They then focus on firm choices between exports and horizontal FDI in supplying abroad. In their structure, each firm decides whether to serve a foreign market, and whether to do so through export or local subsidiary sales. In their model, firms face a proximity-concentration trade-off as in Brainard. The two modes of market access have different relative costs: exporting involves lower fixed costs while FDI involves lower variable costs. They conclude that firms' choices are determined by their productivity, and that the least productive firms serve only the domestic market, that relatively more productive firms export, and that the most productive firms engage in FDI.

Helpman, Melitz and Yeaple (2004) thus highlight the role of within-sector firm productivity differences in explaining the structure of both international trade and foreign investment. In the HMY model, at a sector level, the more within-industry

dispersion of firm productivity there is, the lower the ratio of exports to FDI sales. Helpman, Melitz and Yeaple thus then extend the proximity-concentration model of Brainard by introducing heterogeneous firms, and also extend heterogeneity model of Melitz by introducing horizontal FDI. This paper goes further by extending the HMY model by introducing fragmentation into it.

Many papers have studied the phenomenon of fragmentation which means that various phases of a production process are physically separable and can be allocated in different places or nations. Jones and Kierzkowski (1990) believe that a production process is consisted of a series of production blocks which are connected and coordinated by service links, and that declining of service cost can stimulate the development of fragmentation at a great amount. When fragmentation takes place among different nations, trade in parts and components will happen. So, most of the papers use the imports of intermediate commodities to measure fragmentation. The existed papers in this field mainly concern the effect of fragmentation on factors' prices, employment and welfare. In a series of studies, Feenstra and Hanson develop a model that has the implication of an increase in outsourcing would lead to an increase in the relative wage of skilled labor in both the developed and the developing countries. In this paper, we don't concern the effect of fragmentation but the firms' mode to serve the market abroad.

3. Theoretical Model

3.1 Demand

We assume the world consists of *N* countries, each producing goods in *H+1* sectors. *H* sectors produce differentiated products, and one sector produces a homogenous product which is the numeraire good. Labor is the sole factor of production, with country *i* endowed with L^i units of labor.

A fraction of β_h income is spent on differentiated products of sector *h*, the remaining fraction $1-\sum_h \beta_h$ is spent on the homogenous product. For a differentiated sector *h*, consumers have constant elasticity of substitution preferences. Demand for the differentiated sector *h* is $\left[\int_{v\in n_h} x_h(v)^{\rho_h} dv\right]^{1/\rho_h}$, $0<\rho_h<1$, where $x_h(v)$ is the demand for variety *v* in sector *h*, n_h is the available number of varieties in sector *h*, ρ_h measures the extent of consumers' love of variety, a smaller ρ_h indicates a stronger love of variety, and the elasticity of substitution between varieties is σ_h, $\sigma_h = 1/(1-\rho_h) > 1$.

3.2 Supply

We assume that the sector that produces the homogenous product is competitive. For simplicity, we assume every country has the same productivity in producing the homogenous product. There are no trade costs for the homogenous product in trade between countries. We assume $\sum_h \beta_h$ is small enough that every country produces the homogenous product and given competitive assumption and no trade costs the price is equal in every country. The price of the homogenous product is standardized so that the wage in every country is 1, and the total income of country *i* is L^i.

We next consider differentiated products. For a sector *h* that produces a variety *h*, we assume there are an infinite number of possible production chains which are evenly distributed over [0, 1]. These are arranged in descending order of fixed costs of setting up a plant. A firm can thus choose a cutoff point *s* in [0, 1]. Production chains over [0, s] are allocated to the home country *i* and intermediate products are made at home. Those over [s, 1] are allocated abroad, for example in country *j*[①], and are made in country *j*. Firms export the intermediates produced at home, and production abroad uses these intermediates[②]. Though intermediates are introduced into the production and the whole of the production process is done in two nations, we assume this splitting does not affect a firm's production technology. This treatment appears in Table 2 and is further discussed later.

Table 1 reports the four categories of firms that this treatment implies. Firms in Category 1 only sell commodities at home, and neither export nor invest abroad. Firms in Category 2 supply abroad by exporting the final commodity, but do not invest abroad. Firms in Category 3 supply abroad using fragmentation. This means that production chains over [0, s] are used at home and those over [s, 1] are used abroad. Firms in Category 3 only export intermediates not the finally produced product. We label this fragmentation FDI, because there are fixed costs in setting up production chains in the foreign country. For firms in Category 4, they supply the foreign country with the commodity produced fully abroad. There are no exports, but there is horizontal FDI. Categories 2 and 4 are two special cases of Category 3, with

① This assumption will be clarified in the following analysis.

② When $s = 0$, all production chains are allocated abroad, and no intermediate is exported. When $s = 1$, all production chains are allocated at home, and the final commodity is exported.

the cutoff points are respectively 1 and 0 in these two cases. Here we focus on the Category 3 in most cases.

Table 1 Categories of firms

Category	s	How the commodity is supplied abroad	Export of home production	Category of FDI
1	——	not supply abroad	no export	no FDI
2	$s=1$	commodity produced absolutely at home	the final commodity	no FDI
3	$s \in (0,1)$	production chains over [0, s] produced at home; those over [s, 1] produced abroad	intermediates	FDI for fragmentation
4	$s=0$	commodity produced absolutely abroad	no export	horizontal FDI

Table 2 Fixed and marginal costs of different categories of firms

Category	Fixed cost	Marginal cost of the commodity supplied to home	Marginal cost of the commodity supplied abroad
1	F_D	a	not supplied abroad
2	$F_D + F_I$	a	$a\tau$
3	$F_D + F_I + f(s)F_D$	a	$\tau sa + (1-s)a$
4	$2F_D + F_I$	a	a

To enter a sector *h*, potential entrants should pay a sunk entry cost of F_E units of labor. Once the sunk entry cost is paid, a firm draws its labor-per-unit-output coefficient a from a distribution $G(a)$. Once a firm has observed its productivity, it will choose between exiting and producing. A firm remaining in the industry will always serve its domestic market through domestic production, but it may also serve the foreign market. If so, it can choose to access the foreign market via export, fragmentation or horizontal FDI. This choice is driven by a proximity-concentration trade-off as in earlier literature: the more production chains are allocated abroad, the more transport costs can be saved, but it induces higher fixed costs. After entry, producers engage in monopolistic competition. In equilibrium, one firm chooses only

one way to serve the foreign market.

Firms remaining in the industry should choose one of the four categories listed in Table 1. The fixed cost and the marginal cost of each category are indicated in Table 2. For all these four categories, we assume that there exists a fixed cost of F_D units of labor which is the cost of setting up a factory. For firms in Category 2, 3 and 4 supplying abroad, we assume there exists an additional fixed cost F_I which is needed to selling products abroad. For example, firms may conduct market research in order to supply abroad. Since firms in Category 4 supply abroad with the commodity produced fully abroad, we assume there is an additional fixed cost F_D for the foreign market which is the cost of setting up a factory abroad. Because we assume that production chains are uniformly distributed over [0, 1] in descending order of fixed cost of setting up a plant, we also assume that for firms in Category 3 there exists an additional fixed cost $f(s)F_D$ of setting up abroad, where $0 < f(s) < 1$, $f'(s) \le 0$, when $s \to 0$, $f(s) \to 1$, and when $s \to 1$, $f(s) \to 0$.

The marginal cost of the commodity supplied to home for all four categories is a, since these firms always supply the home market via the domestically produced commodity. We next assume that there exist iceberg transportation costs for both the final commodity and the intermediates, which means τ units need to be exported for 1 unit to arrive[①]. The marginal cost of the commodity supplied abroad by firms in Category 2 is thus τa. For firms in Category 3, the marginal cost of the commodity supplied abroad consists of three parts, the first is sa that is incurred at home, the

① Here we assume a uniform iceberg transportation cost τ for all the intermediates. Therefore, production chains over [0, s] are allocated to the home country and those over [s, 1] are allocated abroad, because production chains over [0, 1] are arranged in descending order of fixed costs for setting up a plant.

second is $(1-s)a$ that is incurred abroad, and the third is transportation costs. The sum of the first two parts is a, since we assume a total of a labor input is needed to produce one unit of a commodity even if there is fragmentation. The sum of the three is $\tau sa+(1-s)a$. For firms in Category 4, the marginal cost of the commodity supplied abroad is a.

Under an assumption of CES preferences, firms will maximize their profits by setting the price as a markup over marginal cost. Here, the markup factor is $\sigma/(\sigma-1)=1/\rho$. For firms in Category 3, the firm with labor-per-unit-output coefficient a will sell the commodity at home at the price of $p_D(a)=a/\rho$, and set the price at $p_I(a)=\left[\tau sa+(1-s)a\right]/\rho$ abroad. Firms in the other three categories also set the price at $p_D(a)=a/\rho$ for the commodity sold at home. Firms in Category 2 and 4 set the price respectively at $a\tau/\rho$ and a/ρ for the commodity sold abroad.

3.3 Choiced mode to serve the abroad market

We consider a firm with the labor-per-unit-output a in sector *h* in country *i*. After it observes its labor-per-unit-output as a, if it chooses to produce commodity, it will obtain profit π_D^i from home production.

$$\pi_D^i=(a)^{1-\sigma}B^i-F_D \tag{1}$$

where $B^i=(1-\rho)A^i/\rho^{1-\sigma}$, $A^i=\beta L^i/\left[\int_0^{n^i}p^i(v)^{1-\sigma}dv\right]$, and A^i is the domestic demand faced by firm *i*.① If the firm is supplying country *j*, a firm in Category 3 will receive an additional profit：

① Helpman, etc (2004) use the expression of A^i, where L^i is the income of country *i*, n^i is the number of varieties of sector *h* available in country *i*, $p^i(v)$ is the price of variety *v*, $\int_0^{n^i}p^i(v)^{1-\sigma}dv$ is the price of composite goods $\left[\int_0^{n^i}x_h(v)^{\rho_h}dv\right]^{1/\rho_h}$.

$$\pi_I^{ij} = \left[\tau^{ij} sa + (1-s)a\right]^{1-\sigma} B^j - F_I - f(s)F_D \tag{2}$$

Because categories 2 and 4 are two special cases of category 3, (2) only reports the profits of firms in category 3.

We firstly consider a firms' choice when international fragmentation is prohibited, which means firms can only choose the category 1, 2 and 3. This is similar to Helpman, Melitz and Yeaple (2004). We next analyze a firm's choice when international fragmentation is possible, but there is only one fixed cutoff point. We then analyze a firms' choice when international fragmentation is possible but there are two fixed cutoff points. Lastly, we consider cases when product chains can be arbitrarily fragmented.

(1) When international fragmentation is prohibited

As in Helpman, Melitz and Yeaple, when there is no international fragmentation, firms only choose among the categories 1, 2 and 4. For a firm in Category 2, the profit from sale to country *j* is

$$\pi_I^{ij}|_{s=1} = (\tau^{ij} a)^{1-\sigma} B^j - F_I \tag{3}$$

For Category 4, the profit from sales in country *j* is

$$\pi_I^{ij}|_{s=0} = (a)^{1-\sigma} B^j - F_I - F_D \tag{4}$$

Figure 1 represents firms' profits from the home market and abroad. In this figure, $(a)^{1-\sigma}$ is represented on the horizontal axis. The variable $(a)^{1-\sigma}$ increases monotonically with labor productivity and can be used as a productivity index. On the vertical axis is profit. π_D^i is the profit from the home market *i*, and $\pi_I^{ij}|_{s=1}$, $\pi_I^{ij}|_{s=0}$ are the respectively profits from country *j* when the firm choose export or horizontal

FDI to supply *j*. All of π_D^i, $\pi_I^{ij}|_{s=1}$ and $\pi_I^{ij}|_{s=0}$ increase monotonically with $(a)^{1-\sigma}$. Under the assumption of equal demands between country *i* and country *j*, from (1), (3) and (4), we can see that $\pi_I^{ij}|_{s=0}$ and π_D^i are parallel, and that the slope of $\pi_I^{ij}|_{s=1}$ is less than $\pi_I^{ij}|_{s=0}$ and π_D^i. The profit lines π_D^i and $\pi_I^{ij}|_{s=1}$ cross the horizontal axis respectively at the point $(a_D^i)^{1-\sigma}$ and $(a_X^i)^{1-\sigma}$. The cross point of profit lines $\pi_I^{ij}|_{s=0}$ and $\pi_I^{ij}|_{s=1}$ have the abscissa of $(a_I^i)^{1-\sigma}$.

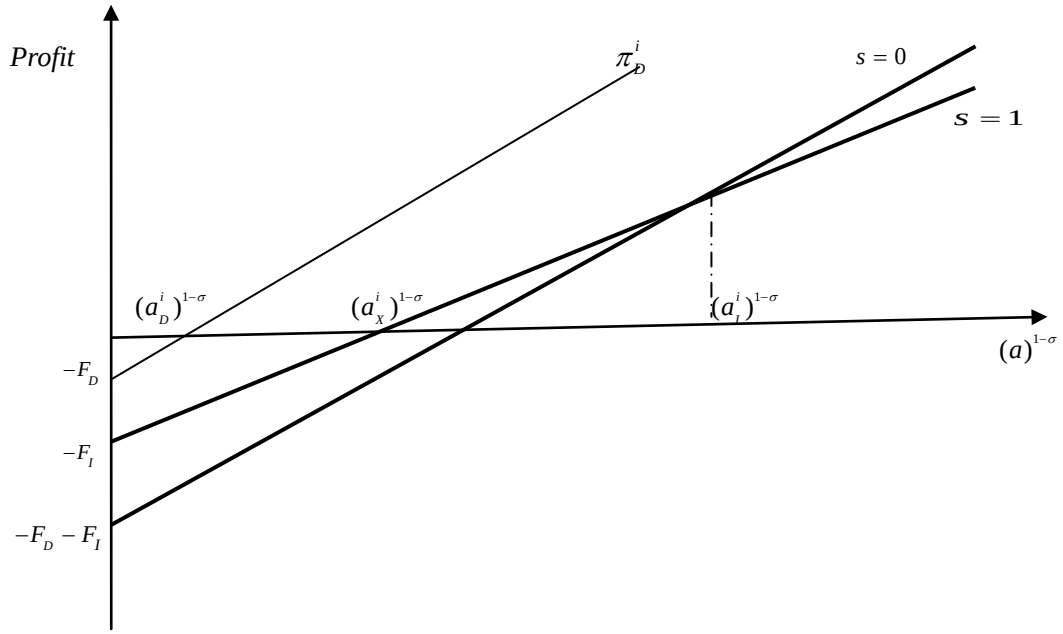

Figure 1 Profits of firms when international fragmentation is prohibited

Notes: Profit lines $s = 0$, $s = 1$ denote respectively profit functions $\pi_I^{ij}|_{s=0}$ and $\pi_I^{ij}|_{s=1}$. This figure is similar to Figure 1 in Helpman, Melitz and Yeaple (2004) with a difference in the treatment of fixed cost and marginal costs relative to HMY.

Helpman, Melitz and Yeaple (2004) use an assumption about fixed costs in order to make the firms' choice tractable, and here we make a similar assumption:

$$F_I(\tau^{\sigma-1}-1) < F_D < F_I\tau^{\sigma-1} \tag{5}$$

In Figure 1, $F_I > F_D$. We also can assume $F_I < F_D$, but if so, (5) should hold.

Under these assumptions, from Figure 1, we can see

$$(a_D^i)^{1-\sigma} < (a_X^i)^{1-\sigma} < (a_I^i)^{1-\sigma} \quad (6)$$

and firms with productivity index below $(a_D^i)^{1-\sigma}$ will exit because of negative net profits. Firms with productivity index between $(a_D^i)^{1-\sigma}$ and $(a_X^i)^{1-\sigma}$ will only supply the domestic market because export and horizontal FDI incur negative net profits. Firms with productivity index between $(a_X^i)^{1-\sigma}$ and $(a_I^i)^{1-\sigma}$ will not only supply domestic markets but also export to supply abroad. They will not adopt horizontal FDI, because exporting brings more profit than horizontal FDI. Firms with productivity index above $(a_I^i)^{1-\sigma}$ will not only supply the domestic market, but also supply abroad via horizontal FDI, because horizontal FDI brings more profit than exporting. These conclusions are as in Helpman, Melitz and Yeaple (2004).

(2) When international fragmentation is possible but only has one fixed cutoff point

Next, we consider cases where international fragmentation is possible but there is only one fixed cutoff point. We assume that a firm can only choose $s = s_1$ as a cutoff point if it chooses international fragmentation, as shown in Figure 2. To make the figure easier to follow, Figure 2 leaves out the line for π_D^i that is included in Figure 1.

From (2), (3) and (4), we can see that the intercept $-F_1$ of the profit line $s = s_1$ is between $-F_I$ and $-F_D - F_I$ which are respectively the intercepts of the profit line $s = 1$ and $s = 0$. Also, the slope of the profit line $s = s_1$ is between the profit line $s = 1$ and profit line $s = 0$. In Figure 2, we can see the inequality

$(a_X^i)^{1-\sigma} < (a_{11}^i)^{1-\sigma} < (a_I^i)^{1-\sigma} < (a_{12}^i)^{1-\sigma}$ is satisfied①. When we consider $s = s_1$ as a possible cutoff point, firms with their productivity between $(a_X^i)^{1-\sigma}$ and $(a_{11}^i)^{1-\sigma}$ still supply abroad via export. Firms with their productivity between $(a_{11}^i)^{1-\sigma}$ and $(a_{12}^i)^{1-\sigma}$ change to supply abroad via fragmentation. Firms with their productivity above $(a_{12}^i)^{1-\sigma}$ still supply abroad via horizontal FDI. So, when international fragmentation is possible, some firms that formerly chose export or horizontal FDI will now choose international fragmentation to supply abroad②.

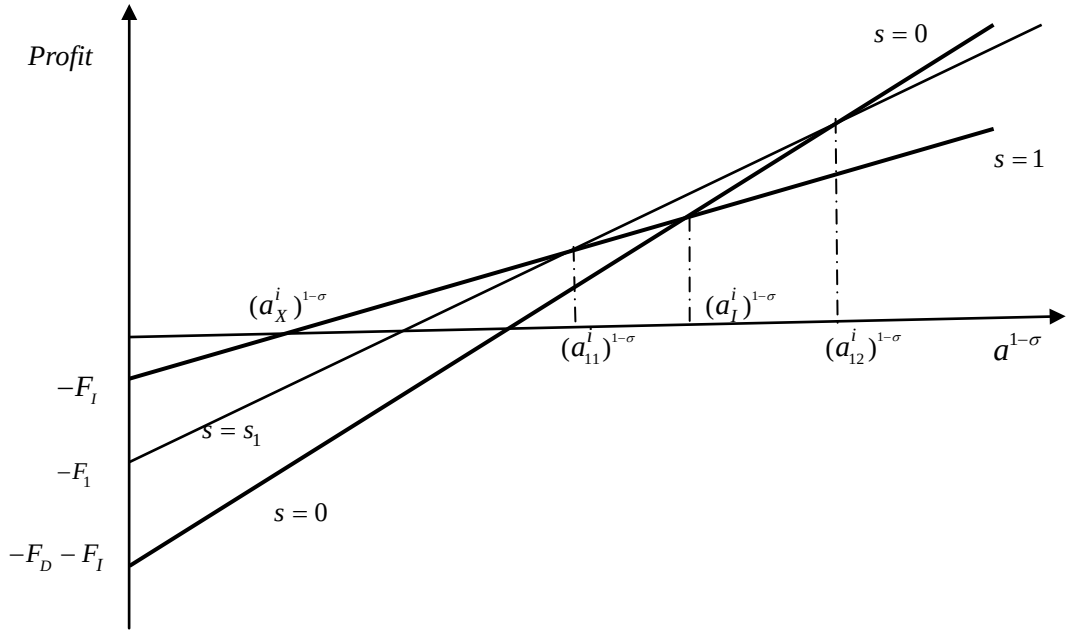

Figure 2 **Profits of firms when international fragmentation is possible but only has one cutoff point**

(3) When international fragmentation is possible but has two fixed cutoff points

Figure 3 depicts firm choices when there are two possible fixed cutoff points.

① There is a possibility that this inequality does not hold. For example, when the cross point of profit line $s = s_1$ and horizontal axis is at the left of $(a_X^i)^{1-\sigma}$, but this does not affect the final conclusion. This paper considers the cases that Figure 2 describes.

② When $(a_{11}^i)^{1-\sigma} < (a_X^i)^{1-\sigma}$, part or all of the firms that previously only sold a commodity at home change to supply abroad via fragmentation; all of the firms that formally exported and part of the firms that formerly chose horizontal FDI change to supply abroad via international fragmentation. When $(a_{11}^i)^{1-\sigma} > (a_I^i)^{1-\sigma}$, the introduction of international fragmentation can not change the choices of firms.

Firms can choose the cutoff point $s = s_1$ or $s = s_2$ ($s_2 < s_1$). From (2), (3) and (4), we can see that the intercept $-F_2$ of profit line $s = s_2$ is between the intercepts of profit line $s = s_1$ and $s = 0$. Furthermore, the slope of the profit line $s = s_2$ is between the slopes of profit line $s = s_1$ and $s = 0$. In Figure 3, the following inequality holds[①]:

$$(a_{11}^i)^{1-\sigma} < (a_{21}^i)^{1-\sigma} < (a_{12}^i)^{1-\sigma} < (a_{22}^i)^{1-\sigma}$$

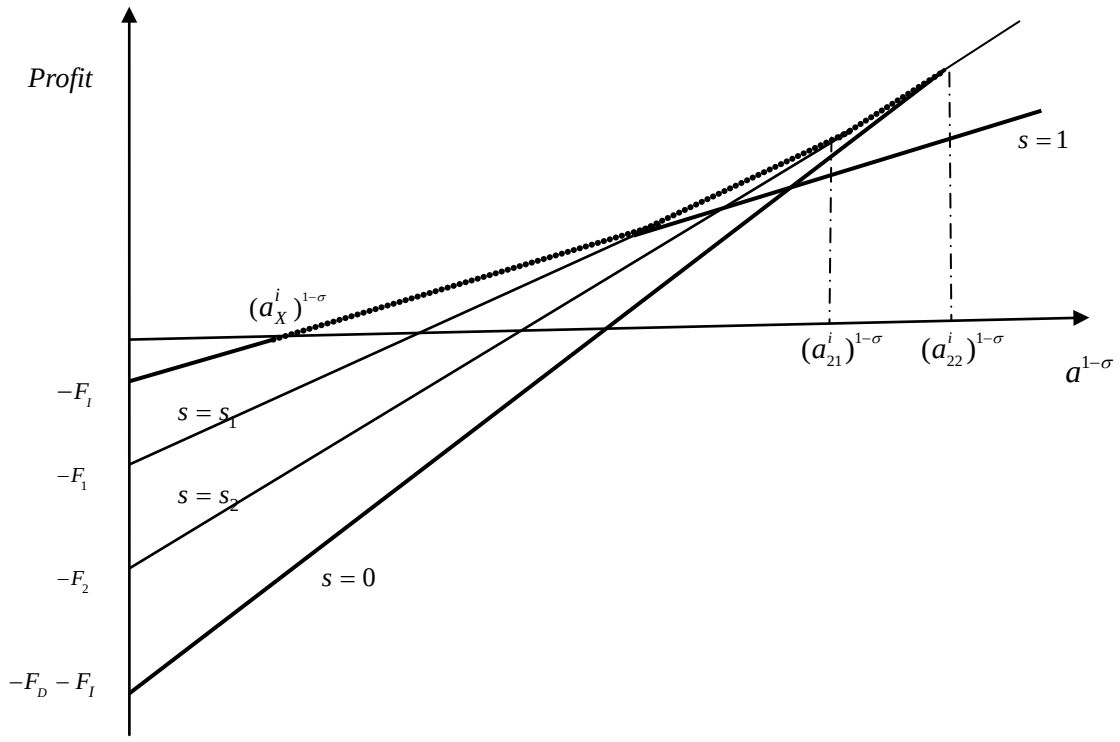

Figure 3 **Profits of firms when international fragmentation is possible and has two possible cutoff points**

When another cutoff point $s = s_2$ is added, firms with their productivity index between $(a_{11}^i)^{1-\sigma}$ and $(a_{21}^i)^{1-\sigma}$ will still supply abroad via international fragmentation and the cutoff point is still s_1. But firms with their productivity index between $(a_{21}^i)^{1-\sigma}$ and $(a_{22}^i)^{1-\sigma}$ that formerly choose the cutoff point s_1 now change to the cutoff point s_2. Firms with productivity index above $(a_{22}^i)^{1-\sigma}$ will

① In order to make Figure 3 concise, $(a_{11}^i)^{1-\sigma}$ and $(a_{12}^i)^{1-\sigma}$ that exist in Figure 2 do not appear in Figure 3.

still supply abroad via horizontal FDI. Thus, an additional cutoff point increases the proportion of the firms that perform fragmentation FDI[①].

(4) When product chains can be arbitrarily fragmented

Under the circumstances of international fragmentation, the more productive a firm is, the more product chains will be allocated abroad. This conclusion follows since: if the cutoff point could be an arbitrary number over (0, 1), in this ideal circumstances, the dashed in Figure 3 would become a smooth curve that is convex to the origin. The more productive a firm is, the more profit it will abtain. Firms with the lowest productivity will exit, and those with lower productivity will choose export. Firms with higher productivity will choose fragmentation, among which, the more productive a firm is, the more product chains are allocated at abroad. Firms with the highest productivity will choose the horizontal FDI. With an increase in the numbers of possible cutoff points, the proportion of firms that choose fragmentation will increase, and the proportion of firms that choose exporting and horizontal FDI will decrease.

3.4 Equilibrium

In equilibrium, for country *i*,

$$(a_D^i)^{1-\sigma} B^i - F_D = 0 \tag{7}$$

where a_D^i is the dividing point of labor-per-unit-output for a firm to survive in the market. A firm with the labor-per-unit-output bigger than a_D^i will exit from the

① When $(a_{21}^i)^{1-\sigma} < (a_{11}^i)^{1-\sigma}$, part or all of the firms that formally exported change to supply abroad via fragmentation and choose the cutoff point s_2. All of the firms that formally choose the cutoff point s_1 and part of the firms that choose horizontal FDI will change to choose the cutoff point s_2. When $(a_{21}^i)^{1-\sigma} > (a_{12}^i)^{1-\sigma}$, the introduction of another cutoff point does not change the firms' choice.

market and firms with labor-per-unit-output a_D^i will earn profits.

For all the $j \neq i$,

$$(\tau^{ij})^{1-\sigma}(a_X^{ij})^{1-\sigma}B^j - F_I = 0 \tag{8}$$

where a_X^{ij} is the dividing point of labor-per-unit-output for a firm in country *i* to supply country *j*. Firms with labor-per-unit-output smaller than a_X^{ij} will choose to supply abroad and firms with labor-per-unit-output a_X^{ij} earn zero profits by supplying country *j*.

The free entry condition is:

$$\int_0^{a_D^i}(a^{1-\sigma}B^i - F_D)dG(a) + \sum_{j\neq i}\int_0^{a_X^{ij}}\left\{\left[\tau^{ij}sa + (1-s)a\right]^{1-\sigma}B^j - F_I - f(s)F_D\right\}dG(a) = F_E \tag{9}$$

The first item on the left side of (9) is the expected profit gained from the domestic market *i* when a potential entrant's labor-per-unit-output is less than a_D^i. The second item on the left side of (9) is the expected profit gained from country *j* when a potential entrant's labor-per-unit-output is less than a_X^{ij}. (9) means that the expected profit is equal to the sunk entry cost F_E, which implies a zero expected profit for a potential entrant.

Firms that supply abroad via fragmentation FDI will choose such an *s* that they can make the highest profit. So, from (2), we can get

$$\partial\pi_I^{ij}/\partial s = a^{1-\sigma}B^j(1-\sigma)(\tau^{ij}-1)\left[1-s+\tau^{ij}s\right]^{-\sigma} - f'(s)F_D = 0 \tag{10}$$

(7) - (10) implicitly implies the dividing point of a_D^i, a_X^{ij}, and the demand levels B^i, B^j.

3.5 The ratio of exports to FDI at the sector level

Next we examine let's look at the ratio of export to FDI for country *i* supplying

country *j* at a sector level. Here FDI include horizontal FDI and fragmentation FDI. When product chains can be arbitrarily fragmented, from Figure 3 nearly all the firms supply abroad via fragmentation FDI and few firms choose export and horizontal FDI. In reality, product chains of a commodity may not be arbitrarily fragmented, which will mean that cutoff points firms can choose are limited.

From Figure 3 it follows that at the sector level, the more prone to be fragmented a sector is, the lower the ratio of exports to FDI sales is. The significance of this is that it suggests that the ability of a country to fragment product internationally is one of the origins of comparative advantage. A country that has a lower ability in fragmenting product internationally will have comparative advantage in this sector.

At a sector level, the ratio of export to FDI for country *i* to supply country *j* is determined by many factors. Among these are two that play an important role. One is the cross point with the minimum abscissa among all the cross points of all possible profit lines and profit line $s=1$. The other is $(a_X^{ij})^{1-\sigma}$ which is the dividing point for firms to supply abroad. The distance along the horizontal axis between these two points determines the ratio of exports to FDI. The larger the distance is, the bigger the ratio of export to FDI①.

From (2) and (3) we get

$$(\tau^{ij}a_{s1}^{ij})^{1-\sigma}B^j - F_I = \left[\tau^{ij}sa_{s1}^{ij} + (1-s)a_{s1}^{ij}\right]^{1-\sigma} B^j - F_I - f(s)F_D$$

which can be rearranged as

① If $(a_{s1}^{ij})^{1-\sigma} < (a_X^i)^{1-\sigma}$, all firms will choose FDI to supply abroad, no firm choose export. To make it general, in this essay, we assume $(a_{s1}^{ij})^{1-\sigma} > (a_X^i)^{1-\sigma}$, so only part of firms choose FDI to supply abroad.

$$(a_{s1}^{ij})^{1-\sigma}=\frac{f(s)F_D}{\left[(1-s+\tau^{ij}s)^{1-\sigma}-(\tau^{ij})^{1-\sigma}\right]B^j} \tag{11}$$

From (11), we see that the bigger τ^{ij} is or the smaller F_D is, the smaller $(a_{s1}^{ij})^{1-\sigma}$ will be.

From (8), we get

$$(a_X^{ij})^{1-\sigma}=\frac{F_I}{(\tau^{ij})^{1-\sigma}B^j} \tag{12}$$

and the bigger τ^{ij} or F_I is, the bigger $(a_X^{ij})^{1-\sigma}$ will be.

The larger τ^{ij} is, the smaller the distance between $(a_{s1}^{ij})^{1-\sigma}$ and $(a_X^{ij})^{1-\sigma}$, and the larger τ^{ij} is, the smaller the ratio of export to FDI.

In (11), the smaller F_D is, the smaller $(a_{s1}^{ij})^{1-\sigma}$ is. When $(a_X^{ij})^{1-\sigma}$ is fixed, the smaller $(a_{s1}^{ij})^{1-\sigma}$ is, the smaller the distance between $(a_{s1}^{ij})^{1-\sigma}$ and $(a_X^{ij})^{1-\sigma}$ is, and hence the smaller is the ratio of export to FDI. In (12), the larger F_I is, the larger $(a_X^{ij})^{1-\sigma}$ is. When $(a_{s1}^{ij})^{1-\sigma}$ is fixed, the larger $(a_X^{ij})^{1-\sigma}$ is, the smaller the distance between $(a_{s1}^{ij})^{1-\sigma}$ and $(a_X^{ij})^{1-\sigma}$ is, and hence the smaller is the ratio of export to FDI.

Also at a sector level, the higher the transportation costs are, or the smaller the fixed cost for setting up a plant is, or the smaller the fixed cost for selling products abroad is, the lower the ratio of exports to FDI sales is. This extends the Proximity-Concentration model of Brainard (1993) in two ways. One is from the firm level to a sector level. The other is that the definition of FDI is extended to include horizontal FDI and fragmentation FDI.

4. Concluding remarks

This paper builds on the HMY model to analyze different ways for multinational firms which have different productivity to serve the market abroad when product chains can be arbitrarily fragmented. Fragmentation is captured through the splitting of production chains across countries to yield a proximity advantage as in Brainard (1993) , but with a fixed cost of fragmentation production in foreign countries.

The analysis generalizes HMY by suggesting that while firms with the lowest productivity will exit, and those with lower productivity will choose to export, and those with higher productivity will choose fragmentation, among which, the more productive a firm is, the more product chains are allocated at abroad. Firms with the highest productivity will then choose horizontal FDI as in HMY. At a sector level, the more prone to be fragmented is an industry, the lower the ratio of exports to FDI sales is. Also at a sector level, the higher are transportation costs, or the smaller the fixed cost for setting up a plant, or the smaller the fixed cost for selling products abroad, the lower the ratio of exports to FDI sales.

By introducing fragmentation into the HMY model, this paper looks at the choices of heterogeneous firms in supplying the market abroad in a product fragmentation world. When product chains can be arbitrarily fragmented, for the firms who choose fragmentation, the more productive a firm is, the more product chains are allocated at abroad. This suggests that the “proximity-concentration” hypothesis developed by Brainard (1993) can be applied even when product fragmentation occurs. For firms who choose fragmentation FDI, because more product chains

allocated abroad incur more fixed costs, only more productive firms can allocate more product chains abroad.

One important implication of this paper is that the ability of a country to fragment product internationally is a source of comparative advantage. At a sector level, the more prone for an industry to be fragmented, the lower the ratio of exports to FDI sales. For any sector, the country that has the most ability relative to others to fragment production internationally is more likely to supply markets abroad via fragmentation. Equally, this country will export fewer commodities for this sector; and the country that has less ability to fragment a product internationally is more likely to export.

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