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Abstract

This article studies the effects of tax competition on the provision of public goods under business risk and partial irreversibility of investment. As will be shown, the provision of public goods changes over time and also depends on the business cycle. In particular, under source-based taxation, public goods can be optimally provided during a downturn, in the short term. The converse is true during a recovery, when they are underprovided. In the long term however, tax competition does not affect capital accumulation and therefore, the provision of public goods.

JEL-Code: H25, H32.

Keywords: irreversibility, risk, short- and long-term effects, tax competition.

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1 Introduction

Capital mobility is relatively high (see the comprehensive survey by Zodrow, 2010). However, it is not fully cost free. For example greenfield and even brownfield investments are characterized by some irreversibility, which reduces mobility after the investment has been undertaken. Another related cause of partial mobility is the existence of "location-specific capital", which may be relevant when a resident resides in one place for some time (see, e.g., Wildasin and Wilson, 1996).

Despite these well-known characteristics, most of the existing literature on tax competition treats capital as fully mobile. If this assumption fits well with paper profits and intangible assets (see Devereux, 2007), it is less realistic when tangible assets are considered.

There are a few articles that have dealt with the partial mobility of investment. Among these, Lee (1997) uses a two-period framework where firms are free to make an investment abroad and in the second period face exit costs. This induces competing governments to intensify tax competition at time 1 and then raise tax rates at time 2. Lee (1997) also shows that time 2's tax rate increase is positively related to the amount of exit costs. Becker and Fuest (2011) assume two types of firms, mobile and immobile. They then show that the optimal tax policy depends on whether the mobile firms are more or less profitable than the average firm in the economy.

Both articles use a deterministic framework to derive policy implications, although risk is shown to affect the interaction between taxation and investment (see, e.g., Ghinamo et al., 2010). Like partial mobility, volatility is an important characteristic which is seldom considered. To our knowledge, risk has been analyzed in terms of welfare and the main question raised by the relevant literature is to what extent volatility undermines the welfare state. For instance, Wildasin (2000) argues that increased capital mobility reduces the Government's ability to redistribute resources.[1] On the other hand, Lee (2004) states that capital taxation can be used as an insurance against wage fluctuations. To our knowledge, no tax competition article has studied strategic interactions when business conditions change over time because of volatility.

The aim of this article is to investigate fiscal policies under both volatility and partial irreversibility (mobility). To do so, we will use an intertemporal

[1] On this point see also Wilson and Wildasin (2004).

neoclassical model with investment irreversibility and depreciation. By letting capital depreciate we make irreversibility partial, in that obsolescence gives some degree of flexibility to firms that can decide whether and when to re-invest.

Moreover, we will apply this investment framework to the well-known tax competition models, developed by Zodrow and Mieszkowski (1986) and Wilson (1986). We will then show that, when a Government raises revenue by means of a source-based tax on capital, the provision of public goods depends on the state of nature and the time horizon. In particular, we will show that in the short-medium term, during a downturn, public goods can be optimally provided. The reasoning behind this is simple: when business conditions get worse, firms cannot disinvest because of irreversibility (they can only wait for obsolescence). Since capital is given, the source-based tax is equivalent to a lump-sum tax. When however a recovery takes place, taxation discourages capital accumulation and the use of a distortive source-based tax leads to the underprovision of public goods. Results change in the long term. In this case, the distortive effects of taxation vanish, and therefore, public goods can be optimally provided. This finding is in some ways similar to Sinn's (1991) vanishing Harberger triangle.

The is structured as follows. In Section 2, we introduce a standard neoclassical model with investment irreversibility and depreciable capital. Section 3 examines the provision of public goods, in the short term. Section 4 focuses on the long term. Section 5 summarizes our findings and discusses some possible extensions.

2 The model

Let us focus on a representative firm, which is subject to a unit tax. For simplicity, we assume that the price of capital is equal to 1. Denoting capital as K_t, we assume that the production function is $\Theta_t \Psi(K_t)$, where Θ_t is a stochastic productivity variable that follows a geometric Brownian motion

$$\frac{d\Theta_t}{\Theta_t} = \mu_\Theta dt + \sigma dz_t, \tag{1}$$

where μ_Θ is the expected growth, σ is the standard deviation of $\frac{d\Theta_t}{\Theta_t}$, and dz_t is the increment of a Wiener process satisfying the conditions $E(dz_t) = 0$

and $E(dz_t^2) = dt$. Moreover, the function $\Psi(K_t)$ follows the Inada conditions. Finally, the installment of capital is assumed to be irreversible.[2]

In order to make our model more realistic, we also introduce capital risk. By assumption therefore, capital lifetime will follow a Poisson process. This means that over any short period dt, there is a probability λdt that the activity dies. The importance of this assumption is twofold. On the one hand, it makes our analysis more realistic, by adding an important source of uncertainty, i.e., capital risk[3] (e.g., related to obsolescence). On the other hand, depreciation allows us to make the irreversibility assumption weaker. In other words, we state that as long parameter λ is positive, irreversible investments is not eternal and that it may be "made" reversible by technical obsolescence. When the investment project expires in fact, the firm owns a non-depreciable option to restart. As immediate restart may not be profitable, the firm may find it profitable to wait until Π rises. With such an option therefore, at the expiration of the project the firm regains a limited degree of reversibility in its investment strategy.

Given these assumptions our representative firm chooses the stock of capital that maximizes its after-tax profit function:

$$\Pi(K_t, \Theta_t) = \Theta_t \Psi(K_t) - \tau K_t, \tag{2}$$

where τ is a unit tax on capital. Denoting r as the risk-free interest rate, the firm's investment activity is described by the following:

Lemma 1 *The firm invests when the following marginal condition holds:*

$$\Theta_t^* \Psi_K(K_t) \equiv \frac{\beta_1}{\beta_1 - 1} \frac{r + \lambda + \tau}{r + \lambda} (r + \lambda - \mu_\Theta) \tag{3}$$

where Θ_t^* is the maximum value of the stochastic variable reached until time t, i.e., $\Theta_t^* = \{\max_{0 \leq s \leq t} \Theta_s\}$, $\Psi_K(K_t) \equiv \frac{\partial \Psi(K_t)}{\partial K_t}$ and $\beta_1 = \left(\frac{1}{2} - \frac{\mu_\Theta}{\sigma^2}\right) + \sqrt{\left(\frac{1}{2} - \frac{\mu_\Theta}{\sigma^2}\right)^2 + 2\frac{r+\lambda}{\sigma^2}} > 1$.

[2] This means that it owns a compound option to invest, which consists of a continuum of American call options. For any increment dK the firm can exercise a call option to expand capital. After this exercise, the firm obtains another American call option allowing it to undertake a further increment.

[3] Bulow and Summers (1984) argue that capital risk is the most important source of risk involved in holding an asset. Also, notice that the Poisson process may describe political risk, i.e., the risk of expropriation by a foreign government.

Proof. See Appendix A. ■

Lemma 1 derives the optimal investment policy under irreversibility. As can be seen, investment is optimal when the marginal product $\Theta_t^* \Psi_K(K_t)$ (on the LHS) equates to the marginal cost of investment. It is worth noting that under full investment reversibility the term $\frac{\beta_1}{\beta_1 - 1}$ would vanish (as β_1 would go to infinity) and the optimal investment would be reached when the equality $\Theta_t^* \Psi_K(K_t) \equiv \frac{r+\lambda+\tau}{r+\lambda}(r + \lambda - \mu_\Theta)$ holds, irrespective of whether a volatile business cycle exists or not. In this case, any business change would lead to investment (disinvestment) when a recovery (recession) takes place. When however, investment is irreversible the effects of the business cycle are asymmetric.

Since $\frac{\beta_1}{\beta_1 - 1} > 1$ we can say that the marginal cost of investment is higher under irreversibility. Moreover, volatility has an asymmetric effect. During a market expansion, i.e., when at time t, the variable Θ_t is higher than Θ_t^*, investment is made so as to reach equality (3). During a recession, i.e., when $\Theta_t < \Theta_t^*$, the installed capital exceeds the optimal one but cannot be dismantled. In this case, no action takes place and so we can say that capital is immobile.

3 Optimal provision of public goods in the short/medium term

Let us now analyze the provision of public goods. To do so, we will use the well-known models developed by Zodrow and Mieszkowski (1986) and Wilson (1986) where many small countries compete to attract capital but need to use a source-based tax to finance the provision of public goods. By assumption, each competing government chooses its optimal fiscal policy by maximizing the utility function of a representative citizen, i.e., $U(C_t, G_t)$, where C_t and G_t are a private and public good, respectively. The private budget constraint is equal to

$$C_t = \Pi(K_t, \Theta_t) - rK_t + r\Sigma, \tag{4}$$

where Σ is the capital endowment of our representative citizen. Assuming a balanced public budget, the condition

$$\tau K_t = G_t \tag{5}$$

always holds. In order to address the government's policy, let us first analyze the effect of taxation on capital accumulation. If the business cycle is expanding and therefore the optimal condition (3) holds, taxation affects investment. This can be shown by differentiating (3) and rearranging:

$$\frac{\partial K_t}{\partial \tau} = \frac{1}{\Theta_t^* \Psi_{KK}(K_t)} < 0. \tag{6}$$

Given $\Psi_{KK} < 0$, we can therefore say that taxation deters capital accumulation. In this case, the change in public spending, caused by a change in τ, is equal to $dG = K_t d\tau + \tau dK_t$. If however a downturn occurs and so the inequality $\Theta_t < \Theta_t^*$ holds, neither investment nor disinvestment is made (because of irreversibility). Since irreversibility makes capital immobile, we have $\frac{\partial K_t}{\partial \tau} = 0$. Therefore the change in public spending is equal to $dG = K_t d\tau$ and we can say that in this latter case, a source-based tax has the same effect as a lump-sum one. To sum up we can write the following

$$\begin{cases} dG = K_t d\tau + \tau dK_t & if \quad \Theta_t = \Theta_t^*, \\ dG = K_t d\tau & if \quad \Theta_t < \Theta_t^*. \end{cases} \tag{7}$$

More precisely, in the former case (when $\Theta_t \geq \Theta_t^*$) new capital, dK_t, is invested and, due to the absence of installment costs, the equality $\Theta_t = \Theta_t^*$ is immediately reached. In the latter case, the productivity variable Θ_t is less than Θ_t^*. This means that taxation cannot affect investment (and therefore does not affect the tax base) and the revenue change is simply due to the tax rate change $d\tau$. Substituting (6) into (7) gives

$$\begin{cases} \frac{dK}{dG} = \frac{1}{[\Theta_t^* \Psi_{KK}(K_t) K_t + \tau]} & if \quad \Theta_t = \Theta_t^*, \\ \frac{dK}{dG} = 0 & if \quad \Theta_t < \Theta_t^*. \end{cases} \tag{8}$$

Let us next calculate the national budget constraint. Using (4) and (5) we have

$$C_t + G_t = Y\left(K_t\left(G_t\right)\right), \tag{9}$$

where

$$Y\left(K_t\left(G_t\right)\right) \equiv \Theta_t \Psi(K_t\left(G_t\right)) - rK_t\left(G_t\right) + r\Sigma$$

is national income. Therefore, the government's problem will then be:

$$\begin{array}{c} \max_{C_t, G_t} U\left(C_t, G_t\right) \\ s.t.\ (9) \end{array} \tag{10}$$

Using (22) and (23), we thus obtain the following:

Proposition 1 *Under investment irreversibility and uncertain obsolescence the marginal rate of substitution between the public and the private good will be equal to:*

$$MRS \equiv \frac{U_{G_t}(C_t, G_t)}{U_{C_t}(C_t, G_t)} = \begin{cases} \frac{\Theta_t^* \Psi_{KK}(K_t) K_t - \frac{r}{\beta_1 - 1}}{[\Theta_t^* \Psi_{KK}(K_t) K_t + \tau]} > 1 & if \quad \Theta_t = \Theta_t^*, \\ 1 & if \quad \Theta_t < \Theta_t^*. \end{cases} \tag{11}$$

Proof. See Appendix B. ■

The reasoning behind Proposition 1 is straightforward. If $\Theta_t < \Theta_t^*$, no investment is undertaken and given irreversibility no disinvestment occurs. Thus capital is fixed. In this case, tax rate changes have no impact on capital accumulation. Since the source-base tax has the same effect as the one due to lump-sum taxation, public good provision is undistorted. If $\Theta_t = \Theta_t^*$, namely Θ_t reaches or overcomes its previous maximum value, investment is undertaken. In this case taxation discourages capital accumulation and therefore leads to the underprovision of G_t.

Let us next study the effect of risk on public goods provision. We can prove that:

Proposition 2 *If $\tau > r$, the derivative $\frac{\partial MRS}{\partial \sigma}$ is negative. If $\tau < r$, the derivative $\frac{\partial MRS}{\partial \sigma}$ is negative (positive) if the absolute value of elasticity $|\varepsilon| \equiv \left| \frac{\partial K_t}{\partial \tau} \frac{\tau}{K_t} \right|$, is low (high) enough.*

Proof. See Appendix C. ■

The reasoning behind this is that volatility has a twofold effect. On the one hand, it raises the threshold value: this means that, given an initial value Θ_t, the inequality $\Theta_s < \Theta_s^*$ (with $s \geq t$) holds for longer: in other words, the public good is optimally provided for a longer time. On the other hand, for a given threshold value Θ_t^*, the increase in σ makes Θ_s (with $s \geq t$) more volatile. This implies that the equality $\Theta_s = \Theta_s^*$ (with $s \geq t$) is expected to hold for longer. So the public good is underprovided. Proposition 2 therefore shows that if the tax rate is high enough, an increase in volatility reduces MRS. This is due to the fact that the former effect dominates the latter, and hence, the tax distortion is moderate. If however τ is low, results depend on the elasticity of capital with respect to taxation. If capital is moderately sensitive to tax changes, again, the former effect dominates the latter. The converse is true when the absolute value of ε is high enough. In this case, an increase in volatility worsens the underprovision of our public good.

4 The provision in the long term

So far we have focused on the provision of public goods for a given value of Θ_t. This implicitly means that we are focusing on short/medium-term effects. In order to analyze tax effects in the long term, let us rearrange the investment rule (3) as follows:

$$\xi_t = \Theta_t \Psi_K(K_t) \quad \text{for } \xi_t < \hat{\xi}, \tag{12}$$

where the marginal product ξ_t is a regulated process, according to Harrison (1985, ch. 2), and $\hat{\xi} = \frac{\beta_1}{\beta_1 - 1}\frac{r+\lambda+\tau}{r+\lambda}(r + \lambda - \mu_\Theta)$ is its upper reflecting barrier.

When, due to an increase in Θ_t, ξ_t reaches $\hat{\xi}$, the firm finds it profitable to install new capital. New units of capital decrease the marginal product $\Psi_K(K_t)$: for this reason ξ_t cannot overcome $\hat{\xi}$.[4] If, however, the inequality $\xi_t < \hat{\xi}$ holds, the level of Θ_t is too low and no new investment is made. Notice that the existence of a reflecting barrier $\hat{\xi}$ does not mean that there is a finite rate of accumulation over time. Rather, it can simply cause investment inaction for long periods and sudden investment bursts over short periods.

If a steady state distribution for ξ_t exists within the range $(-\infty, \hat{\xi})$, then it is always possible to obtain the corresponding marginal distribution for K_t. As a consequence, we can find the long-term average growth rate of K_t. Following Hartman and Hendrickson (2002) and Di Corato *et al.* (2013) we can prove that:

Proposition 3 *For any initial value of capital K_0, such that $\xi(K_0, \Theta_t) \leq \hat{\xi}$, the expected long-term average rate of capital accumulation can be approximated as follows:*

$$\frac{1}{dt}E\left[d\ln K_t\right] \simeq \begin{cases} -(\mu_\Theta - \frac{1}{2}\sigma^2)\frac{\Psi_K(K_0)}{\Psi_{KK}(K_0)K_0} & \text{for } \mu_\Theta > \frac{1}{2}\sigma^2, \\ 0 & \text{for } \mu_\Theta \leq \frac{1}{2}\sigma^2. \end{cases} \tag{13}$$

Proof. See Appendix D. ■

Proposition 3 shows that the long-term average rate of capital accumulation depends on both the dynamics of Θ_t (i.e. μ_Θ, σ^2), and the characteristics

[4] Since investment is instantaneous, the investment rate is infinite at point $\hat{\xi}$.This is due to the fact that, at point $\hat{\xi}$, neither ξ_t nor K_t are differentiable with respect to time t (see Harrison, 1985, and Dixit, 1993).

of the production function. In particular, if the production function $\Psi(K)$ follows the Inada conditions and the drift parameter is high enough (i.e., $\mu_\Theta > \frac{1}{2}\sigma^2$), the expected long-term growth rate of capital is proportional to $(\mu_\Theta - \frac{1}{2}\sigma^2)$. Otherwise it is nil.

As can be seen, if $\mu_\Theta > \frac{1}{2}\sigma^2$, the expected growth rate of capital accumulation depends on the initial amount K_0, unless the production function is isoelastic. If $\Psi(K_t) = K_t^\gamma$ with $\gamma \in (0,1)$, the long-term growth rate is $\frac{1}{dt}E\left[d\ln K_t\right] = \frac{(\mu_\Theta - \frac{1}{2}\sigma^2)}{1-\gamma}$ and does not depend on K_0.

Using the regulated process (**12**), we can see that when ξ_t hits the barrier, the equality:

$$\ln \Psi_K(K_t) = \ln \hat{\xi} - \ln \Theta_t$$

holds. This means that, since $\ln \hat{\xi}$ is constant, the expected growth of K_t on the boundary is driven by $\ln \Theta_t$. Moreover, since $\ln \xi_t - \ln \Theta_t \leq \ln \hat{\xi} - \ln \Theta_t$ for all t, we can say that in the long term, the average growth rate of K_t cannot be greater than the average growth rate along the boundary.

It is worth noting that the rate in (13) is decreasing in the volatility of future values of Θ_t. A higher volatility has two distinct effects. First, it pushes the barrier $\hat{\xi}$ upward; second, by increasing the negative skewness of the distribution of ξ, it reduces the probability of the barrier being reached.[5] Both effects reduce the rate of capital accumulation in both the short and long term.

As expected, if $\alpha \leq \frac{1}{2}\sigma^2$, the process ξ drives away from $\hat{\xi}$ and the rate falls to zero.

It is worth noting that Proposition 3 has an important implication: i.e., in the long run, taxation does not affect capital accumulation. This means that, given the public budget constraint (5), the long-term level of public goods provision is unaffected by tax competition. Unlike previous work, we have shown that, if $\mu_\Theta > \frac{1}{2}\sigma^2$ public goods are optimally provided. If however $\mu_\Theta < \frac{1}{2}\sigma^2$, the long-term capital (tax base) is nil and this tax tool cannot raise resources to finance the provision of public goods. In neither case, taxation matters and we therefore have a result that echoes Sinn's (1991) vanishing Harberger triangle.

[5] Appendix D shows that the higher the parameter value σ the lower the probability that ξ reaches $\hat{\xi}$ is.

5 Conclusion

In this article we have analyzed the provision of public goods over time, by assuming the partial mobility of capital. More precisely, we have assumed that investment is irreversible but is subject to stochastic obsolescence. In this case, depreciation allows us to consider investments as not eternal. When the investment project expires, the firm indeed owns a non-depreciable option to restart.

As we have shown the provision of public goods changes over time. In the short term, public goods can be optimally provided during a downturn. In this case, the capital stock is fixed and the source-base tax used in our framework has the same effect as a lump-sum one. Only during expansions, the growth of capital is discouraged by taxation and this leads to underprovision.

In the long term, results are different. As we have shown, tax competition affects neither capital accumulation nor public good provision. Moreover, only if the expected growth rate of productivity is high enough, public goods are optimally provided.

A Proof of Lemma 1

The firm's problem is one of choosing the optimal amount of capital:

$$V(K_t, \Theta_t) = \max_{K_t} E_0 \left[\int_0^{\infty} (1 - \lambda dt)\, e^{-rt} [\Pi(K_t, \Theta_t) - dK_t] dt + 0 \cdot \lambda dt \mid K_0 \geq 0,\ \Theta_0 \geq 0 \right], \tag{14}$$

with $dK_t \geq 0$ for all t. Without installation costs, the rate of growth of capital is unbounded and dK is therefore the investment process. The expectation in equation (14) is conditional on the information available at time zero, accounts for the joint distribution of K_t and Θ_t and takes into account the irreversibility constraint.[6]

Assuming that $V(.)$ is twice continuously differentiable, a solution can be obtained starting within a time interval where no new investment occurs. Applying dynamic programming to (14) and rearranging the equation we can write the firm's value as

$$V(K_t, \Theta_t) = \Pi(K_t, \Theta_t)\, dt + e^{-(r+\lambda)dt} E_0 [V(K_t, \Theta_t + d\Theta_t)],$$

[6] As we know, at any interval dt, there is a probability λdt that the business value goes to zero. In this case, the firm can decide whether and when to invest.

Expanding the right-hand side and using Itô's lemma gives

$$(r+\lambda)\,V(K_t,\Theta_t) = \Pi\left(K_t,\Theta_t\right) + \mu_\Theta \Theta_t \frac{\partial V\left(K_t,\Theta_t\right)}{\partial \Theta_t} + \frac{\sigma^2}{2}\Theta_t^2 \frac{\partial^2 V\left(K_t,\Theta_t\right)}{\partial \Theta_t^2}. \tag{15}$$

Differentiating (15) with respect to K_t we obtain

$$(r+\lambda)\,v(K_t,\Theta_t) = \left[\Psi_K(K)\Theta - \tau\right] + \mu_\Theta \Theta_t \frac{\partial \nu\left(K_t,\Theta_t\right)}{\partial \Theta_t} + \frac{\sigma^2}{2}\Theta_t^2 \frac{\partial^2 \nu\left(K_t,\Theta_t\right)}{\partial \Theta_t^2}. \tag{16}$$

where $v(K_t,\Theta_t) \equiv V_K(K_t,\Theta_t)$. The solution of (16) has the following form

$$v(K_t,\Theta_t) = c + \Theta_t f(K_t) + \sum_{i=1}^{2} a_i(K_t)\Theta_t^{\beta_i}, \tag{17}$$

where c is a constant to be found and

$$\begin{aligned}\beta_1 &= \left(\tfrac{1}{2} - \tfrac{\mu_\Theta}{\sigma^2}\right) + \sqrt{\left(\tfrac{1}{2} - \tfrac{\mu_\Theta}{\sigma^2}\right)^2 + 2\tfrac{r+\lambda}{\sigma^2}} > 1,\\ \beta_2 &= \left(\tfrac{1}{2} - \tfrac{\mu_\Theta}{\sigma^2}\right) - \sqrt{\left(\tfrac{1}{2} - \tfrac{\mu_\Theta}{\sigma^2}\right)^2 + 2\tfrac{r+\lambda}{\sigma^2}} < 0\end{aligned}$$

are the roots of the characteristic equation $\frac{\sigma^2}{2}\beta(\beta-1) + \mu_\Theta\beta - (r+\lambda) = 0$. The interpretation of equation (17) is then transparent. The contribution of the Kth unit of capital to the profit flow, when the existing stock of capital is K, is given by

$$\Pi_K\left(K_t,\Theta_t\right) \equiv \Psi_K(K)\Theta - \tau.$$

Calculating the expected present value of this marginal contribution thus gives:

$$v(K_t,\Theta_t) = \frac{\Theta_t \Psi_K(K)}{r+\lambda-\mu_\Theta} - \frac{\tau}{r+\lambda} + \sum_{i=1}^{2} a_i(K_t)\Theta_t^{\beta_i}.$$

Let us next introduce the boundary conditions for (17):

$$v\left(K_t,\Theta_t^*\right) = 1, \tag{18}$$

$$v_\Theta\left(K_t,\Theta_t^*\right) = 0, \tag{19}$$

$$a_2(K_t) = 0. \tag{20}$$

where $\Theta_t^* = \left\{\max_{0\le s\le t}\Theta_s\right\}$. Equations (18) and (19) are the Value Matching Condition and Smooth Pasting Condition for the firm's optimal policy, respectively.[7] Moreover, (20) imposes the irreversibility constraint on capital $dK_t \geq 0$.[8] Substituting (17) into (18) and (19), we have the following two-equation system:

$$\begin{array}{l} \frac{\Theta_t \Psi_K(K)}{r+\lambda-\mu_\Theta} - \frac{\tau}{r+\lambda} + a_1(K_t)(\Theta_t^*)^{\beta_1} = 1, \\ \frac{\Theta_t \Psi_K(K)}{r+\lambda-\mu_\Theta} + \beta_1 a_1(K_t)(\Theta_t^*)^{\beta_1} = 0. \end{array}$$

Rearranging gives the following investment rule:

$$\Theta_t \Psi_K(K) = \frac{\beta_1}{\beta_1 - 1} \frac{r+\lambda+\tau}{r+\lambda} \left(r + \lambda - \mu_\Theta\right).$$

This concludes the proof of Lemma 1.■

B Proof of Proposition 1

To solve problem (10) let us use the following Lagrangian function

$$\mathcal{L} = U\left(C_t, G_t\right) + \lambda \left[Y\left(K_t\left(G_t\right)\right) - \left(C_t + G_t\right)\right]. \tag{21}$$

The f.o.c. of (21) are

$$\frac{\partial \mathcal{L}}{\partial C_t} = U_{C_t}\left(C_t, G_t\right) - \lambda = 0, \tag{22}$$

and

$$\begin{array}{lll} \frac{\partial \mathcal{L}}{\partial G_t} = U_{G_t}\left(C_t, G_t\right) + \lambda \left[\frac{\partial Y(K_t(G_t))}{\partial K_t(G_t)} \frac{\partial K_t(G_t)}{\partial G_t} - 1\right] = 0 & if & \Theta_t = \Theta_t^*, \\ \frac{\partial \mathcal{L}}{\partial G_t} = U_{G_t}\left(C_t, G_t\right) - \lambda = 0 & if & \Theta_t < \Theta_t^*. \end{array} \tag{23}$$

where

$$\frac{\partial Y\left(K_t\left(G_t\right)\right)}{\partial K_t\left(G_t\right)} = \left\{ \begin{array}{lll} \Theta_t^* \Psi_{K_t}(K_t\left(G_t\right)) - r & if & \Theta_t = \Theta_t^*, \\ 0 & if & \Theta_t < \Theta_t^*. \end{array} \right. \tag{24}$$

Substituting (24) into (3) gives

$$\frac{\partial Y\left(K_t\left(G_t\right)\right)}{\partial K_t\left(G_t\right)} = \left\{ \begin{array}{lll} \frac{r}{\beta_1 - 1} + \tau & if & \Theta_t = \Theta_t^*, \\ 0 & if & \Theta_t < \Theta_t^*. \end{array} \right. \tag{25}$$

[7] See Dixit and Pindyck (1994).

[8] In other words, when Θ is very small the expected present value of the last unit of capital installed is close to zero. Therefore, the value of the marginal option to scrap it is almost infinite. For further details see Dixit and Pindyck (1994, Ch. 6).

Using (23) and (25) we thus obtain

$$MRS = \frac{U_{G_t}(C_t,G_t)}{U_{C_t}(C_t,G_t)} = \begin{cases} 1-\frac{\frac{r}{\beta_1-1}+\tau}{[\Theta_t^*\Psi_{KK}(K_t)K_t+\tau]} = \frac{\Theta_t^*\Psi_{KK}(K_t)K_t-\frac{r}{\beta_1-1}}{[\Theta_t^*\Psi_{KK}(K_t)K_t+\tau]} > 1 & if \quad \Theta_t = \Theta_t^*, \\ 1 & if \quad \Theta_t < \Theta_t^*. \end{cases} \tag{26}$$

Proposition 1 is thus proven.■

C Proof of Proposition 2

Let us differentiate (26) with respect to σ. If $\Theta_t = \Theta_t^*$, we obtain

$$\frac{\partial MRS}{\partial\sigma} = \frac{\partial}{\partial\sigma}\left\{\frac{\Theta_t^*\Psi_{KK}(K_t)K_t-\frac{r}{\beta_1-1}}{[\Theta_t^*\Psi_{KK}(K_t)K_t+\tau]}\right\} = \frac{\partial}{\partial\beta_1}\left\{\frac{\Theta_t^*\Psi_{KK}(K_t)K_t-\frac{r}{\beta_1-1}}{[\Theta_t^*\Psi_{KK}(K_t)K_t+\tau]}\right\}\frac{\partial\beta_1}{\partial\sigma} =$$
$$\frac{[\frac{\partial\Theta_t^*}{\partial\beta_1}\Psi_{KK}(K_t)K_t+\frac{r}{(\beta_1-1)^2}][\Theta_t^*\Psi_{KK}(K_t)K_t+\tau]-[\Theta_t^*\Psi_{KK}(K_t)K_t-\frac{r}{\beta_1-1}][\frac{\partial\Theta_t^*}{\partial\beta_1}\Psi_{KK}(K_t)K_t]}{[\Theta_t^*\Psi_{KK}(K_t)K_t+\tau]^2}\cdot\frac{\partial\beta_1}{\partial\sigma} \tag{27}$$

with $\frac{\partial\beta_1}{\partial\sigma} < 0$. Therefore (27) is positive if

$$\frac{\partial}{\partial\beta_1}\left\{1-\frac{\frac{r}{\beta_1-1}+\tau}{[\Theta_t^*\Psi_{KK}(K_t)K_t+\tau]}\right\} = -\frac{-\frac{r}{(\beta_1-1)^2}[\Theta_t^*\Psi_{KK}(K_t)K_t+\tau]-\left(\frac{r}{\beta_1-1}+\tau\right)\frac{\partial\Theta_t^*}{\partial\beta_1}\Psi_{KK}(K_t)K_t}{[\Theta_t^*\Psi_{KK}(K_t)K_t+\tau]^2} < 0, \tag{28}$$

where given (3), we have

$$\Theta_t^*\Psi_{KK}(K_t)K_t = \frac{\beta_1}{\beta_1-1}\frac{r+\lambda+\tau}{r+\lambda}(r+\lambda-\mu_\Theta)\frac{\Psi_{KK}(K_t)K_t}{\Psi_K(K_t)}, \tag{29}$$

and therefore

$$\frac{\partial\Theta_t^*}{\partial\beta_1}\Psi_{KK}(K_t)K_t = \frac{1}{\beta_1(\beta_1-1)}\Theta_t^*\Psi_{KK}(K_t)K_t \tag{30}$$

Using (30) and rearranging (28) gives

$$\left[-\frac{r}{(\beta_1-1)^2}+\left(\frac{r}{\beta_1-1}+\tau\right)\frac{1}{\beta_1(\beta_1-1)}\right]\Theta_t^*\Psi_{KK}(K_t)K_t < \frac{\tau r}{(\beta_1-1)^2}$$

Simplifying this inequality we thus obtain

$$\frac{\tau-r}{\beta_1}\Theta_t^*\Psi_{KK}(K_t)K_t < \frac{\tau r}{\beta_1-1}. \tag{31}$$

As can be seen, if $\tau - r > 0$, then $\frac{\partial MRS}{\partial \sigma} < 0$. If however $\tau - r < 0$, results are ambiguous. Let us the write (31) as follows:

$$\Theta_t^* \Psi_{KK}(K_t) K_t > \frac{\beta_1}{\beta_1 - 1} \frac{\tau r}{\tau - r} \tag{32}$$

Notice that, given (6) ,the elasticity of capital with respect to τ is $\varepsilon \equiv \frac{\partial K_t}{\partial \tau} \frac{\tau}{K_t} = \frac{\tau}{\Theta_t^* \Psi_{KK}(K_t) K_t}$. Therefore, we can rewrite (32) as

$$-\frac{\beta_1 - 1}{\beta_1} \frac{r - \tau}{r} < \varepsilon,$$

or equivalently,

$$\frac{\beta_1 - 1}{\beta_1} \frac{r - \tau}{r} > |\varepsilon| \,.$$

This concludes the proof.■

D Proof of Proposition 3

D.1 Long-term distributions

Let h_t be a linear Brownian motion with parameters μ and σ that evolves according to $dh_t = \mu dt + \sigma dz_t$. Following Harrison (1985, pp. 90-91, and Dixit, 1993, pp. 58-68), the long-term density function for h fluctuating between a lower reflecting barrier, $a \in (-\infty, \infty)$, and an upper reflecting barrier, $b \in (-\infty, \infty)$, is given by the following truncated exponential distribution:

$$f(h_t) = \begin{cases} \frac{2\mu}{\sigma^2} \frac{e^{\frac{2\mu}{\sigma^2} h_t}}{e^{\frac{2\mu}{\sigma^2} b} - e^{\frac{2\mu}{\sigma^2} a}} & \mu \neq 0, \\ \frac{1}{b-a} & \mu = 0. \end{cases} \tag{27}$$

Let us next focus on the limit case where $a \to -\infty$. In this case, from (27), a limiting argument gives:

$$f(h_t) = \begin{cases} \frac{2\mu}{\sigma^2} e^{-\frac{2\mu}{\sigma^2}(b - h_t)} & \mu > 0, \\ 0 & \mu \leq 0. \end{cases} \quad \text{for } -\infty < h_t < b \tag{28}$$

Hence, the long-term average of h_t can be evaluated as $E[h_t] = \int_\Phi h_t f(h_t) \, dh_t$, where Φ depends on the distribution assumed. In a steady-state this gives:

$$E[h_t] = \int_{-\infty}^{b} h_t f(h_t) \, dh_t = \int_{-\infty}^{b} h_t \frac{2\mu}{\sigma^2} e^{-\frac{2\mu}{\sigma^2}(b - h_t)} dh_t = \frac{2\mu}{\sigma^2} e^{-\frac{2\mu}{\sigma^2} b} \int_{-\infty}^{b} h_t e^{\frac{2\mu}{\sigma^2} h_t} dh_t = b - \frac{2\mu}{\sigma^2} \tag{29}$$

D.2 Long-run average rate of accumulation

Let us next take the logarithm of (12):

$$\ln \xi_t = \ln [\Theta_t \Psi_K(K_t)] = \ln \Theta_t + \ln [\Psi_K(K_t)] \tag{30}$$

By Ito's lemma, $\ln \xi_t$ evolves according to $d \ln \xi_t = d \ln \Theta_t = [(\mu_\Theta - \frac{1}{2}\sigma^2)dt + \sigma dz_t]$ with $\ln \hat{\xi}$ is its upper reflecting barrier. Setting $h_t = \ln \xi_t$, the random variable $\ln \xi_t$ follows a linear Brownian motion with parameter $\mu = (\mu_\Theta - \frac{1}{2}\sigma^2)$ and has a long-run distribution with (28) as density function. Solving (30) for $\ln \Psi_K(K_t)$ we obtain:

$$\ln \Psi_K(K_t) = h_t - \ln \Theta_t. \tag{31}$$

Let us next calculate the expected value of (31):

$$E[\ln \Psi_K(K_t)] = E[h_t] - \left[\Theta_0 + (\mu_\Theta - \frac{1}{2}\sigma^2)t\right]$$

Using Taylor's theorem, we can expand $\Psi_K(K_t)$ around the point K_0, thereby obtaining:

$$\begin{aligned} E[\ln(\Psi_K(K_0) - \Psi_{KK}(K_0)K_0 + \Psi_{KK}(K_0)K_t)] &= E[\ln[\Psi_{KK}(K_0)(K_t - \Delta(K_0))]] \\ &\simeq E[h_t] - \left[\Theta_0 + (\mu_\Theta - \tfrac{1}{2}\sigma^2)t\right], \end{aligned} \tag{32}$$

where $\Delta(K_0) = \frac{\Psi_{KK}(K_0)K_0 - \Psi_K(K_0)}{\Psi_{KK}(K_0)}$. Given this result we obtain:

$$E[\ln[(K_t - \Delta(K_0))]] = E[h_t] - \left[\Theta_0 + (\mu_\Theta - \frac{1}{2}\sigma^2)t\right] - \ln \Psi_{KK}(K_0)$$

Rewriting $\ln(K_t - \Delta(K_0))$ as $\ln[x - \hat{x}]$ and expanding it by Taylor's theorem around the point $(\widetilde{\ln \hat{x}}, \widetilde{\ln x})$ gives

$$\ln[x - \hat{x}] \equiv \ln\left[e^{\ln x} - e^{\ln \hat{x}}\right] \simeq v_0 + v_1 \ln x + v_2 \ln \hat{x}$$

where

$$\begin{aligned} v_0 &= \ln\left[e^{\widetilde{\ln x}} - e^{\widetilde{\ln \hat{x}}}\right] - \left[\frac{\widetilde{\ln \hat{x}}}{1 - e^{\widetilde{\ln x} - \widetilde{\ln \hat{x}}}} + \frac{\widetilde{\ln x}}{1 - e^{-(\widetilde{\ln x} - \widetilde{\ln \hat{x}})}}\right], \\ v_1 &= \frac{1}{1 - e^{\widetilde{\ln \hat{x}} - \widetilde{\ln x}}},\ v_2 = \frac{1}{1 - e^{(\widetilde{\ln x} - \widetilde{\ln \hat{x}})}},\ \frac{v_2}{v_1} = \frac{1 - v_1}{v_1} < 0. \end{aligned}$$

Substituting this approximation into (32) we have:

$$E\left[\ln K_t\right] = \frac{E\left[h_t\right] - \left[\Theta_0 + (\mu_\Theta - \frac{1}{2}\sigma^2)t\right]}{v_1} - \frac{v_0 + v_2 \ln \Delta(K_0) + \ln \Psi_{KK}(K_0)}{v_1}. \tag{33}$$

Since by (29) $E(h_t)$ is independent on t, differentiating with respect to t, we obtain:

$$\begin{aligned} \frac{1}{dt} E\left[d \ln K_t\right] &= \frac{-(\mu_\Theta - \frac{1}{2}\sigma^2)}{v_1} \qquad (34) \\ &= -(\mu_\Theta - \frac{1}{2}\sigma^2)(1 - e^{\widetilde{\ln \Delta(K_0)} - \widetilde{\ln K}}). \end{aligned}$$

By the monotonicity property of the logarithm, a level K_0 must exists such that $\ln K_0 = \widetilde{\ln K}$ and $\ln \Delta(K_0) = \widetilde{\ln \Delta(K_0)}$. Therefore, we obtain:

$$\begin{aligned} \frac{1}{dt} E\left[d \ln K_t\right] &= -(\mu_\Theta - \frac{1}{2}\sigma^2)(1 - \frac{K_0}{\Delta(K_0)}) \qquad (35) \\ &= -(\mu_\Theta - \frac{1}{2}\sigma^2)\frac{\Psi_K(K_0)}{\Psi_{KK}(K_0)K_0} \qquad \text{for } \mu_\Theta > \frac{1}{2}\sigma^2. \end{aligned}$$

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