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Abstract

This paper merges two branches of the literature. On one hand we study a heterogeneous agents framework to model exchange rates and stock prices. On the other hand we model the relationship between these two series through a DSGE model. Investors choose one of two rules to form their expectations. One rule is based on an open economy model, which reacts to the information from the financial markets. The second rule follows a backward looking approach. We find that when DSGE agents misinterpret the information coming from the financial markets as exogenous productivity shocks they unknowingly amplify the volatility of these markets. The simulated series replicate the stylized facts of real data. We also estimate the DSGE and chartists expectations, and we find that our DSGE agents make output forecasts that are not qualitatively different than the DSGE forecasts from the recent Bayesian literature.

JEL-Code: A110, E200, F300, F310, F370.

Keywords: heterogeneous agents, DSGE, exchange rates, stock prices.

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1 Introduction

The subject of asset prices and exchange rates recently received a lot of attention by many economic experts. We can think of several reasons for this fact. On the one hand the asset prices and exchange rates show several analogies. This makes it possible to use similar statistical tools to study both markets. On the other hand a full understanding of the economy is essential in order to model the interaction between the two markets. This second factor is what makes the subject attractive for theoretical and macroeconomic contributions as well as for empirical and financial analysis. In both cases, a paper studying the relationship between exchange rates and stock prices has to make important economic assumptions. The contribution of this paper is to study the consequences of making some of these assumptions.

This work introduces a new approach that combines the literature of behavioral finance with the literature on DSGE modeling. We use a DSGE model to capture the fundamental expectations of a heterogeneous agents model.

In order to place this paper within the modern literature, we make a short survey of papers in two separate fields. The first one is the behavioral finance literature. The second one studies specifically the relationship between the stock prices and the exchange rates.

Within the field of behavioral finance, a few papers lay the groundwork for the model presented in the next paragraph. The econometric process to the switch between rules has been introduced by Brock and Hommes (1997 and 1998). These papers develop the tools to study systems with endogenously alternating rules. Frankel and Froot (1998) analyze the exchange rate market trading and relate it to a fundamentalist chartist model.

Finally, De Grauwe and Grimaldi (2006) summarizes most of the findings to date and shows new evolutions for the model.

In the second field, the area of stock prices and exchange rates, there are a number of theoretical approaches. The three most relevant models with respect to this paper are first Kollmann in 2001 who develops and solves a DSGE model that includes the capital market. For the first time, the equity flows are studied in conjunction with the exchange rate created by an open economy two country market of the style of Obstfeld and Rogoff (1998). In a later contribution, Coeurdacier, Kollmann and Martin (2009) specify the financial markets in a formal way, which includes more complex, agent based portfolio choice rules. This paper combines previous DSGE literature with a new way of studying portfolio choice issues. The approach taken in this paper is building a dynamics of a similar kind. Within the class of New Keynesian models, bounded rational expectations have been introduced to create a NK Phillips curve with lags. More specifically, Gali and Gertler (1999) and Steinsson (2003) include a (static) percentage of agents being rational and a percentage which uses a simple backward looking rule (of the type used in this paper).

Di Giorgio and Nistico' (2007) develop an open economy model with stock prices. They model is a perpetual youth economy. They conclude that central banks can increase welfare by looking at the stock prices, especially when they are affected by relatie productivity shocks.

Finally, Rey and Hau (2006) study the exchange rate as the result of order flows for foreign currency from equity trade in an international two country setting. Their partial equilibrium model is complemented by a supportive empirical application to over 20 countries.

The remainder of this paper is structured as follows. The next section introduces the theoretical framework. Here we describe the heterogeneous agents model in detail. Section 3 analyzes the macroeconomic expectations. We use the assumptions and basic equations of the DSGE model that is solved to a log-linearized solution. Section 4 focusses on the connection between the financial market and the macroeconomic model. A clear mechanism is outlined that binds the two in a non-linear system.

Section 5 describes the properties of this system via simulations. We focus on the new results of the paper. We also add an empirical analysis, that shows the consistency between our simulated series and the real financial markets. A short conclusion emphasizes some further points for research.

2 Theory

The model focusses on a small open economy. This country has private equity stocks and a floating exchange rate. There are no transaction costs. Within this setting we study the two financial time series. These are the home stock prices and the effective exchange rate between the home currency and the rest of the world currencies.

The heterogeneous agents model is based on two alternative rules agents may use to create expectations. We will refer to the first rule as chartist. This is a backward looking formula that only considers past returns. We take a traditional autoregressive process with decreasing memory. The second rule is a mean-reverting, forward looking one. This is a fundamentalist rule, as agents formulate it by measuring the distance between the realization of the price and the value that price should have, if based on its fundamentals.

In our model the fundamentalist rule is replaced by a DSGE model, entailing that the fundamentalist agents believe that a DSGE model best describes the connection between the two markets. The consequence is that every time these agents see an unexpected change in the market, they believe that there was an unexpected shock in their model, and use the impulse response functions as a forecast of the two series in the following periods.

Each rule is continuously checked according to its profitability. If it is profitable agents keep it for the next period, otherwise they switch to the other one. While the intuition is relatively simple, we will see that the dynamics created is complex and it is source of non-linearities as well as heteroscedasticity and time varying behavior.

Most importantly, this paper adds a macroeconomic dimension to the traditional fundamentalist expectation. We show that this results into DSGE formulated expectations that may be as volatile as the chartists, and create a bubble-like behavior completely supported by their interpretation of the DSGE model.

2.1 Stock Prices

The stock price series is determined by the average of the different individuals' expectations plus a white noise. There are two types of rules to form expectations. One is generally called a fundamentalist rule. This rule is characterized by a mean reverting dynamics that brings the system closer every period to what the agents believe to be the fundamental price. In this paper we assume that there is no disagreement on this fundamental price[1]. The second expectation rule is chartist. Chartists look at the past information on the time series, without taking any reference value in consideration. Since these two rules use different sets of information, the agents choose from time to time what is relevant. They do this by continuously checking the profitability of their rule, and switching to the other one if they find it more profitable.

Stock prices can be derived from the Gordon Model. We postulate them to be defined by the following specification:

$$\Delta S_{t+1} = m^s_{f,t}[Rule1Forecast] + m^s_{c,t}[Rule2Forecast] + \varepsilon_{t+1} \tag{1}$$

where ΔS_{t+1}, the change in the stock price of period $t+1$, is given by the first rule's forecast weighted by the agents that believe in this rule plus the forecast from the second rule, weighted by the agents believing in this second rule. ε represents a white noise shock, or news, that is not incorporated in either rule.

Equation 1 makes it clear that expectations are crucial in the evolution of the stock prices. This shows the self-fulfilling nature of expectations.

The backward looking or chartist rule is given by:

$$E_{c,t}(\Delta S_{t+1}) = \beta_s \sum_{k=1}^{T} \upsilon(1-\upsilon)^k \Delta(S_{t-k}) \qquad with \qquad 0 < \beta_s < 1 \tag{2}$$

The parameter υ shows the memory of the system, and its dependence on past shocks. The higher this parameter, the longer is the memory of the system.

The fundamentalist rule, or forward looking, calculates the distance to some value of the stocks believed to be the "fundamental" value, and adjusts this dis-

[1] This assumption could be relaxed, and then we would have a dynamics similar to the one described in De Grauwe and Rovira Kaltwasser, 2007.

tance period after period. Formally, this rule follows a mean reverting dynamics given by the impulse response functions from the DSGE model.

This DSGE forecast rule will follow a dynamics of the type:

$$E_{f,t}(\Delta S_{t+1}) = \phi(S_t - \bar{S}) \qquad with \qquad 0 < \phi < 1 \tag{3}$$

We will analyze the fundamental expectation in depth in the next paragraph.

2.2 Switching between the rules

A key feature of this model is that the agents are not static, but keep evaluating their returns and check if their rule is still the most profitable one. If the rule performed well they keep it, otherwise they will switch to the other rule, in the belief that it will continue to yield higher returns. This happens in both markets and independently from one another. We apply the fitness criterion based on discrete choice theory, as developed by Brock and Hommes (1997a) and Brock and Hommes (1998a). This criterion provides the fractions of population at each point in time which use each of the rules. These percentages add up to 1 and they are functions of the (risk adjusted) profitability of each rule. Formally:

$$m_{f,t} = \frac{exp\,\lambda\Pi'_{f,t}}{exp\,\lambda\Pi'_{f,t} \,+\, exp\,\lambda\Pi'_{c,t}} \qquad ; \qquad m_{c,t} = \frac{exp\,\lambda\Pi'_{c,t}}{exp\,\lambda\Pi'_{f,t} \,+\, exp\,\lambda\Pi'_{c,t}} \tag{4}$$

$m_{f,t}$ and $m_{c,t}$ are the fractions of population that at time t are following the fundamentalist (f) and chartist (c) rules. These fractions are calculated for each market independently. The variables $\Pi'_{f,t}$ and $\Pi'_{c,t}$ are the risk adjusted profits realized through the use of each of the rules. We could think of them as $\Pi'_{f,t} = \Pi_{f,t} - \rho\sigma^2_{f,t}$ and $\Pi'_{c,t} = \Pi_{c,t} - \rho\sigma^2_{c,t}$ where the symbol Π represents the profits made by each of the rules, and the σ are the standard deviations of the prices. For this purpose forecast errors represent an unconditional measure of risk. Finally ρ is a coefficient of risk aversion.

Equations 5 show how the agents revise their decision making rules as one of the two rules becomes more attractive (i.e. more profitable or less risky). An important factor in this picture is how quick the agents are to revise and eventually correct their rules, concept that is present in the model through the parameter λ. λ is known in the literature as the " intensity of choice". When the agents are insensitive to the profitability of their rules a good starting point could be a 0.5 fraction of the population using each rule.

All the parameters outlined above may take different values in the exchange rate market and in the stock market. It is possible that the same agent at time t has a fundamental expectation in one market and a chartist one in the other.

Now we formalize how to calculate the profits $\Pi'_{i,t}$ for every rule i. We start by defining the profits non adjusted for the risk, as the single period returns from 1 unit of local currency invested into a foreign asset:

$$\Pi_{i,t} = \Delta S_t \; sgn[E^i_{t-1}(S_t) - S_{t-1}] \tag{5}$$

$$\text{with } sgn(x) = \begin{cases} 1, & \text{for } x > 0 \\ 0, & \text{for } x = 0 \\ -1 & \text{for } x < 0 \end{cases} \quad \text{and } i = c, f$$

Therefore the profit for every rule goes up whenever the forecast is right, and goes down when the sign of the forecast is wrong. In order to calculate the risk associated with every rule we are going to look at the most general case in which it is the forecast error of the previous periods weighted by a discount factor. Therefore, formally:

$$\sigma^2_{i,t} = \sum_{k=1}^{T} \rho(1 - rho)^k (E^i_{t-1-k}(S_{t-k}) - S_{t-k})^2 \tag{6}$$

We can now re-write Equation 1 more precisely, by adding the chartist expectation rule:

$$E_t \Delta S_{t+1} = m_{f,t}[Rule1] + m_{c,t}[\beta_s \Delta S_t] + \varepsilon_{t+1} \tag{7}$$

The actual change of the stock price, is formally written as:

$$\Delta S_{t+1} = m_{f,t}[Rule1] + m_{c,t}[\beta_s \Delta S_t] + \varepsilon_{t+1} \tag{8}$$

Therefore the market is governed by this dynamics and by a white noise shock occurring in time $t + 1$.

2.3 Exchange Rate

The exchange rate in this model is the equilibrium price of a market with two types of demand. On the one hand there is the demand from good traders, which is related to the difference between the exchange rate and the ratios of the prices at home and abroad. On the the other hand there is the speculative demand, given by the difference of the current exchange rate with the expected future exchange rate. The two demands are defined by:

$$d_{gt} : e_t - (p_t - p_t^*)$$
$$d_{sp} : \frac{E_t^i(\Delta e_{t+1})}{\rho\sigma_i^2}; \quad for\, i = fundamentalist, chartist$$

where P_t is the level of Home prices and P_t^* is the average of the price levels of all the foreign countries (in foreign currency). The exchange rate expectation is given by the weighted average of the two rules of expectations.

We impose the condition of no excess demand:

$$\gamma(d_{gt}) + (1-\gamma)[m_f d_{sp,f} + (1-m_f)d_{sp,c}] = 0 \quad (9)$$

Substituting the two demand functions in the previous equation, we can solve for the exchange rate as a function of its expectations and of the variance of the two rules. Thus we obtain the following expression:

$$e_t = (1-\gamma) * \frac{m_{f,t}E_t^f(e_{t+1})\sigma_c^2 + m_{c,t}E_t^c(e_{t+1})\sigma_f^2}{\Theta_1} + \gamma(p_t - p_t^*)\Theta_2 + \varepsilon_{t+1} \quad (10)$$

with Θ_1 and Θ_2 being two functions of the variances of the fundamentalist and chartist rules.

This specification shows that in the steady state, where the exchange rate reaches an equilibrium, the speculative demand disappears and the value will be defined by purchasing power parity. This is the same steady state reached by the DSGE model described in the following paragraphs. Therefore it is possible for the agents to mistake the underlying model for the DSGE model, since the two are equivalent in the steady state.

3 The macroeconomics in finance: fundamental expectations

The structure of the fundamentalist expectations comes from the macroeconomic literature. Specifically, we will include within our heterogeneous agents framework, expectations drawn from a traditional model featuring a small open economy, of the type of Gali, Monacelli (2005), augmented with stock prices. The setting and dynamics of stock prices follows a standard asset pricing approach, as developed in Di Giorgio and Nistico' (2007).

3.1 General equilibrium model

We proceed now to the description of the small open economy model used for fundamental expectations.

3.1.1 Households

Households maximize a utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{11}$$

where N_t are the hours of work in period t and C_t is a consumption index defining the composition of the consumption basket for every level of income:

$$C_t \equiv [(1-\alpha)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + (\alpha)^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}} \tag{12}$$

with $C_{H,t}$ being an index of consumption of domestic goods that has a constant elasticity of substitution and $C_{F,t}$ is the sum of goods produced in all the foreign countries and consumed by the home country. α is a parameter that shows the openness of the home country.

Moreover, the family is subject to a budget constraint, that may be written as:

$$\int_0^1 P_{H,t}(j) C_{H,t}(j) dj + \int_0^1 \int_0^1 P_{i,t}(j) C_{i,t}(j) dj di + E_t\{\Omega_{t,t+1} D_{t+1}\} \leq D_t + W_t N_t + T_t \tag{13}$$

with $P_{H,t}(j)$ being the price of good j produced at home, $P_{i,t}(j)$ is good j that is produced in country i and consumed at home. D_{t+1} is the payoff in period $t+1$ of the portfolio held in t (which can include stocks of the firms), $\Omega_{t,t+1}$ is the stochastic discount factor and W_t is the salary per hour. The markets are complete because we assume that we can trade Arrow securities for all risks. The system may be solved to get demand functions of the different types of goods. This is when all expenditure levels are optimized by spending in the different allocations. These demand functions are:

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} C_{i,t} \qquad ; \qquad C_{i,t}(j) = \left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\varepsilon} C_{i,t} \tag{14}$$

and the optimal allocations between home produced and imported goods are:

$$C_{H,t}(j) = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \qquad ; \qquad C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{15}$$

where α is the share of imported goods. This allows optimization of consumption expenditures for $P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_tC_t$. Overall, now we can simplify the budget constraint to be equal to:

$$P_tC_t + E_t\{\Omega_{t,t+1}D_{t+1}\} \leq D_t + W_tN_t + T_t \tag{16}$$

We may specify the Utility function to be equal to the following one:

$$U(C,N) \equiv \frac{C^{1-\sigma_{u}u}}{1-\sigma_u} - \frac{N^{1+\varphi}}{1+\varphi} \tag{17}$$

where we see the utility coming from consumption and the disutility arising from hours of labor. Other first order inter-temporal conditions are given by:

$$\Omega_{t,t+1} = \beta\frac{P_tU_c(C_{t+1})}{P_{t+1}U_c(C_t)} = \beta\frac{P_tC_{t+1}}{P_{t+1}C_t} \tag{18}$$

$$Q_t = E_t\{\Omega_{t,t+1}[Q_{t+1} + Div_{t+1}]\} \tag{19}$$

The stochastic discount factor behaves according to the non-arbitrage condition:

$$(1+i_t)E_t\{\Omega_{t,t+1}\} = 1 \tag{20}$$

Since the firms' dividends amount to all the excess profits, it holds that:

$$Div_t = \frac{P_t^H}{P_t} Y_t - \frac{W_t}{P_t} N_t \tag{21}$$

In log linear terms this dynamics can be summarized in two equations:

$$q_t = \beta E_t q_{t+1} + (1-\beta) E_t div_{t+1} - (i_t - r_t^n) \tag{22}$$

$$div_t = y_t - \frac{\beta Y/Q}{\mu(1-\beta)} mc_t \tag{23}$$

3.1.2 Terms of trade, Exchange rate, Inflation

Having defined most of the features of our representative household choice domain, we look at the international side of the economy. Following is a form for the effective terms of trade:

$$\varepsilon_t \equiv \frac{P_{F,t}}{P_{H,t}} = \left(\int_0^1 \varepsilon_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \tag{24}$$

where $P_{F,t}$ is the foreign level of prices. This may be linearized around a symmetric steady state where $\varepsilon_t = 1$ for all foreign countries. In logarithms:

$$\varsigma_t = \int_0^1 \varsigma_{i,t} di \tag{25}$$

Inflation is composed of domestic inflation and changes to terms of trade (from here forward we proceed in logarithms):

$$\pi_t = \pi_{H,t} + \alpha \Delta \varsigma_t \tag{26}$$

Domestic inflation, indicated by $\pi_{H,t}$, is the inflation on home produced goods[2]

Furthermore the law of one price holds for all goods. The following is an expression connecting foreign prices to the domestic price index for the different foreign countries (in terms of the foreign currency) and to the nominal exchange rate.

$$p_{F,t} = e_t + p_t^* \tag{28}$$

We also have a form of uncovered interest rate parity, as shown below:

$$i_t = i_t^* + E_t\{\Delta e_{t+1}\} \tag{29}$$

[2]Another way of writing is

$$\pi_t = (1-\alpha)\pi_{H,t} + \alpha \pi_{F,t} \tag{27}$$

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3.1.3 Firms

The representative firm produces a differentiated good and has a production function of the following type:

$$Y_t(j) = A_t N_t(j) \tag{30}$$

where A_t is a linear technology autoregressive process, with coefficient ρ and innovation ϵ_t. This leads to a marginal cost that is common to all firms and of the form:

$$mc_t = -v + w_t - p_{H,t} - a_t \tag{31}$$

Overall this model assumes Calvo pricing, where each period a fraction θ of the firms lets its price fixed, and a fraction equal to $1-\theta$ fixes a new price. This leads to a pricing dynamics that is described by:

$$\overline{p}_{H,t} = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\ \{mc_{t+k} + p_{H,t+k}\} \tag{32}$$

where $\overline{p}_{H,t}$ is the log of the prices that are set new in period t, and μ is the log of the gross mark up in the steady state.

3.1.4 Monetary policy and equilibrium

The model laid out in this paragraph may be solved around a steady state and log linearized in order to get a closed form solution. When this is done we can solve for a Dynamic IS curve, a New Keynesian Phillips curve, and a monetary policy rule.

We get the following New Keynesian Phillips curve:

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + k_\alpha \tilde{y}_t \tag{33}$$

where $tildey_t$ is the output gap and for $\alpha = 0$ we get a closed economy version of the same relationship. We also get a dynamic IS curve, defined as follows:

$$\tilde{y}_t = E_t\tilde{y}_{t+1} - \frac{1}{\sigma_{u\alpha}}(i_t - E_t\{\pi_{H,t+1}\} - r_t^n) \tag{34}$$

with the natural interest rate:

$$r_t^n \equiv \rho - \sigma_{u\alpha}\Gamma_a(1-\rho_a)a_t + \frac{\alpha\Theta\sigma_{u\alpha}\varphi}{\sigma_{u\alpha}+\varphi}E_t\{\Delta\tilde{y}^*_{t+1}\} \tag{35}$$

Finally we choose a monetary policy rule. We have a Taylor rule of the type:

$$i_t = \rho + \phi_\pi \pi_t \tag{36}$$

The set of equations above is used by the fundamentalist agents to create expectations. Therefore when they invest in the financial markets, they will use the forecasts produced by this model.

4 Merging two literatures

The core characteristic of the DSGE literature is that a large number of economic variables is made endogenous. These models become so complex that have no closed form solution. In order to study the dynamics of the economy, it is common to log linearize all the relationships around steady state values of the macroeconomics variables. This is true also of the section above. This log linearization allows to study reactions of the whole system to shocks in productivity (or technology) shocks. After a shock the dynamics of the system of equations brings the economy back to its initial equilibrium.

There are four scenarios that we will study, following this literature. These are a positive and negative shock on Home productivity, and a positive and negative shock to the Rest of the World productivity.

The identification of the shocks is the way that agents interpret unexpected errors in their predictions. When agents realize that the fundamentalist expectations are wrong, they interpret their mistake as due to a combination of the home and foreign productivity shocks. An identification mechanism allows them to univocally go from the data to the model and vice-versa. When the agents have computed their errors in both markets, they run a VAR regression on the data using the DSGE model, and compute the combination of shocks that is consistent with the data they observe. Based on this combination of shocks, they compute impulse response functions for the next period, in order to calculate their expectations for $t + 1$.

H shock	RoW shock	Δq	Δe
H=0	F=-1	+	-
H=0	F=1	-	+
H=-1	F=0	-	+
H=1	F=0	+	-
H=-1	F=-1	-	+
H=1	F=1	-	+
H=-1	F=1	-	+
H=1	F=-1	+	-

Table 1: Responses from shocks of size 0.01 on Home and RoW productivity

4.1 Macroeconomists at work

Because of the identification procedure, the VAR provides a combination of home and foreign productivity shocks consistent with the data observed at each point in time. Therefore it is useful to observe the four scenarios in a simplified context, before simulating the complete model. The mapping in Table 1 shows that a positive foreign tech shock will cause a rise in exchange rates. Viceversa a negative foreign tech shock will provoke a negative change in exchange rates.

The fundamentalist expectations are the sum of all the past impulse response functions from all the past shocks. The value that we get is the proportional distance from the steady state value of the relevant financial price.

The next period, we consider only the distance between the new realized value and the sum of impulse responses from old shocks. That distance is used to compute the new shock vector to the DSGE model.

Section 2 shows that the model is non-linear[3]. This does not allow a study of a closed form solution without making stronger assumptions. While this complexity is a strength of these models, it also means that we will mainly proceed by numerical simulations.

[3]It can be shown that the model is stable for a range of parameters.

5 Model specification and simulation dynamics

In what the parameters are: β_s and β_e, describing the autoregressive coefficient of the chartist rule, are set both equal to 0.9. υ and υ_e, setting the memory of the chartist rules, are equal to 0.5. These parameters allow the chartists to take into account in their forecast past returns on stocks, with decreasing weight. The values of λ, λ_e describe the intensity of choice. For low values we see systems in which the fundamental expectation is usually present in the market, without bubbles. The intuition behind this is that though some noise may bring the system temporarily further from the fundamental value, people do not adjust to that difference, and so they keep using their initial rule. This behavior is affected by the intensity of choice parameter[4]. These parameters are also weighted as a function of the initial values. So the values of $\lambda = 1$ and $\lambda_e = 50$ let the two financial markets exposed to bubbles, while allow for the system to eventually go back to the fundamental value.

Another parameter in the model is the traditional risk aversion. This is seen when we calculate the way of discounting absolute profits of a rule by the variance of those profits. The higher the variance, the lower the final, risk weighted profits will be considered. The weight that sets how important the variance is, the parameter ρ, is we set at 0.5.

Within this paper we study the effects of the parameters connecting macroeconomic expectations and heterogeneous agents. This is also the novice part of the paper, so we can start by looking at some simulations under different parameters, and we study the first and second moments of the data.

Finally, within the DSGE model we assume a steady state of no growth and a monetary policy regime of CPI inflation targeting.

5.1 Some Simulations

Summarizing, the link between the financial and the macroeconomic model is through the fundamental expectation channel. Agents working in the financial

[4]For examples of the impact of the intensity of choice parameter on the Brock and Hommes non-linear switching mechanism, please refer to Brock and Hommes (1998)

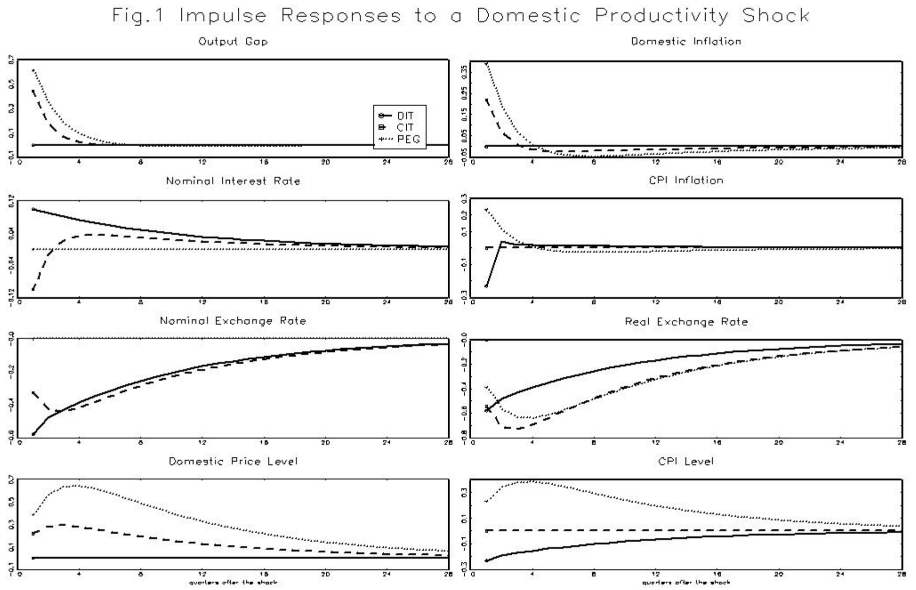

Figure 1: Impulse responses from a positive shock on H productivity of size 1

markets look at the unexpected changes in the series, and believe that they were unable to expect these changes because they were caused by exogenous shocks.

Figure 1 shows an impulse response function of the DSGE model to a shock in the Home productivity process. This is in accordance with Table 1, which shows how a positive shock of this type would provoke a temporary rise of the stock prices, and a temporary decrease (or appreciation) of the exchange rate.

Figure 2 shows the impulse response function for a shock to foreign technology. Once again the findings are in line with Table 1.

5.2 New heterogeneous agents dynamics: "fundamentalist bubbles"

In this section we study the intuition of what we call "fundamentalist bubbles". In the past models the two rules for expectations are characterized by different volatilities. The fundamentalist rule is usually less variable. Therefore the realization of the final price has two distinct regimes: one which is dominated

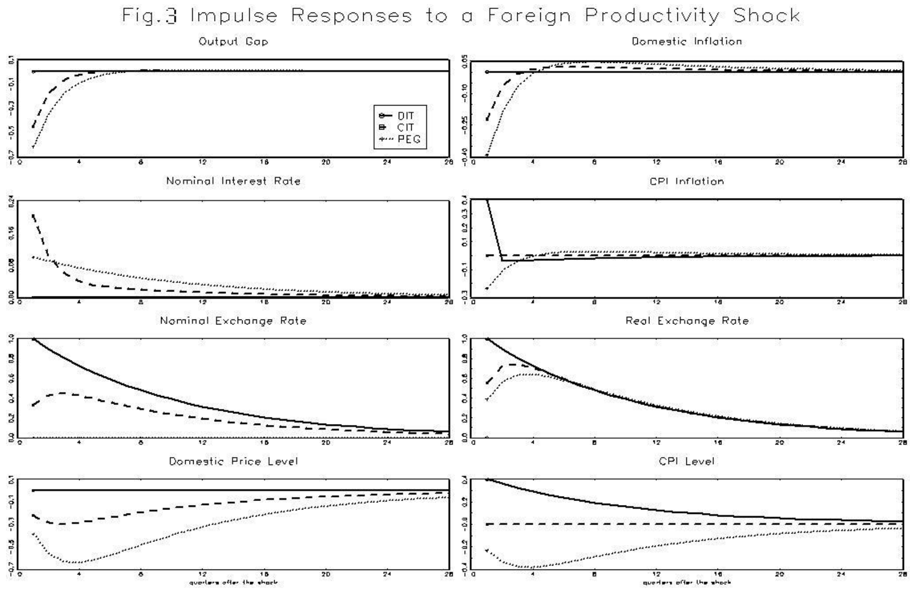

Figure 2: Impulse responses from a positive shock on F productivity of size 1

by chartists, which exhibits a higher volatility and momentum, while the second regime is closer in value to the fundamentalist price and is characterized by lower volatility, and a strong presence of fundamental agents.

However in our model markets following a fundamental expectation may still exhibit a bubble-like behavior: we refer to bubbles as market realizations that are increasingly volatile and away from what would be suggested by the fundamentals. Even with a strong presence of DSGE believers, the market can display high variance, not backed by fundamentals. This is shown in figure 3. The exchange rate market is always reflecting its fundamental expectation price. However this fundamental expectation includes a backward looking component, through the shocks that the agents deduce and plug into their model. For this reason, it will always reflect changes in any financial market that may have impact on the macroeconomic variables of the system. This new mechanism can in some cases create highly volatile fundamental expectations, and it is shown in Figure 5.2.

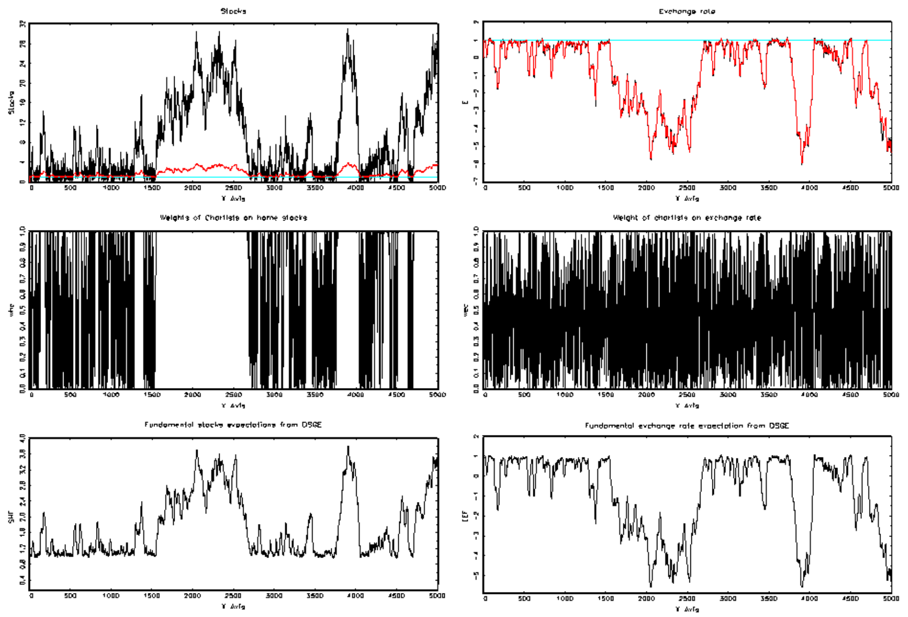

Figure 3: Stochastic simulation from low λ_e

Figure 5.2 shows a market in which the value of λ_e (intensity of choice for the exchange rate market) is very low to allow the exchange rate to follow mainly the fundamental realization. The first panel on the left side shows the realization of the stock price series of 5000 iterations. The second panel on the left shows the weight of chartists during each of the 5000 periods. The weight of chartists and fundamentalists adds up to 1. What we usually see when the series follows a fundamentalist value is that the weight is alternating quickly between the two rules. This is because they are both at similar levels of profitability.

The difference between the two rules arises whenever the realization of the price differs from the fundamentalist value. What happens in this case is that the chartist rule becomes progressively more profitable, and therefore a larger number of people switches to the chartist rule. When we have bubbles, these are caused by a great majority of chartists (or "momentum traders"). In the second panel on the left side, we can see this particular behavior. In the bubbles, the weight of chartists increases to close to 1.

The first panel on the right shows the realizations of the exchange rate for the same 5000 iterations. The fundamental value here, like in the stock prices, is set by the impulse response of the DSGE model, being shocked for stock price changes. We also see cases in which the chartists take over the market, creating variance to the exchange rate that is not motivated by a change in the fundamental value. This is shown in the second panel on the right, where the fraction of chartists is plotted.

The third row of panels shows the fundamental expectation levels. This allows to see that the fundamental expectations of the stock prices and exchange rates are negatively correlated to each other in returns. The reason for this is that the DSGE moves the exchange rate in exactly the opposite way than the stock prices, when it comes down to shocks coming from home productivity. The intuition behind the model is that a temporary shock on productivity causes a higher GDP. This leads to a growing home economy which is reflected in the price of the currency.

This example allows to describe the "fundamental bubbles". Raising λ_e produces a system in which "fundamental bubbles" will be present in the system as well as normal momentum traders bubbles, characterized by a prevalence of chartists. This is the case in the exchange rate series of figure 4.

5.3 Empirical study

We look at the empirical data for stock markets and exchange rates and verify whether they exhibit a similar behavior to the model that we described. We do this with two exercises. In the first exercise we check the validity of the model's implications. For this purpose we study some of the main world markets. Specifically we use the exchange rate against the US dollar and the index of the stock prices for 4 countries (The United Kingdom, Australia, Brazil and Mexico). We have daily data from June 1986 to June 2011. We test all the series for unit roots. The results are uniform across the sample. They are all integrated of order 1. We perform the same test for the simulated data from our model and have the same outcome. We move to check whether the direction of correlation predicted by the theory is supported by the data. The

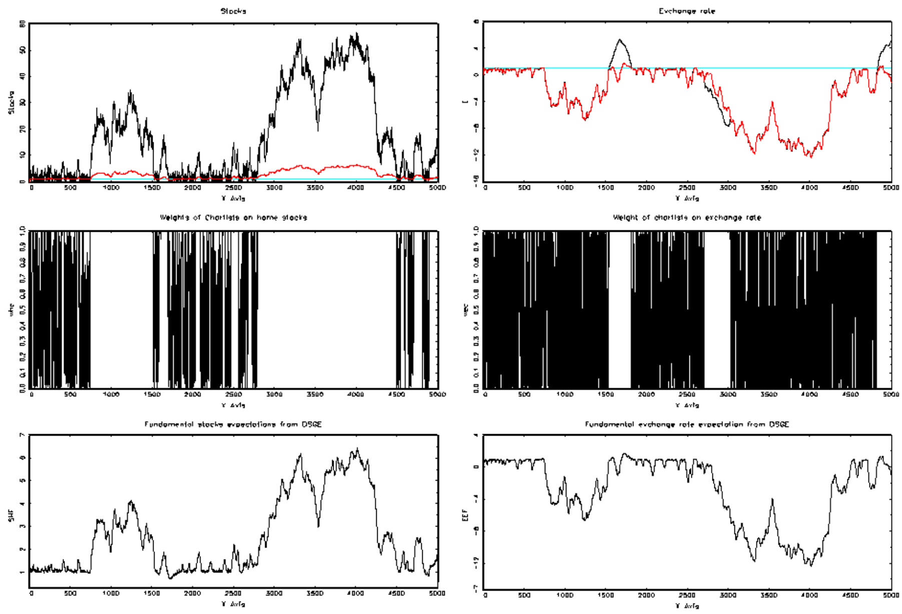

Figure 4: Stochastic simulation with baseline parameters

theory predicts a negative correlation between the exchange rate and the stocks when the "fundamentalist" bubbles are prevailing. This is due to the channel of transmission of fundamental expectations. Therefore if the theory has any relevance in the actual data, we expect to find the same negative relationship in the financial series. We regress stock returns on exchange rates differences, to verify whether the relationship of the simulated series is comparable to the relationship found in the data.

Table 2 gives the results. In all the markets studied we find a negative and significant (at the 1% level) coefficient between the stock market and the corresponding currency exchange rate with the US dollar. This is the same relationship that we predicted above, therefore the data supports the dynamics of the model.

Further, we can also compare the markets by looking at the statistical properties of the single series. Traditionally the financial markets exhibit heteroscedasticity and fat tails. This exercise has been carried out in the past (De Grauwe and Grimaldi and De Grauwe and Rovira-Kaltwasser) for models of

Market	Variables	Coef.	t-stat.	Prob.
UK	C	0.00	-0.02	0.98
	Stock	-0.03	-4.06	0.00
Australia	C	0.00	-0.25	0.81
	Stock	-0.40	-20.95	0.00
Brazil	C	0.00	1.53	0.13
	Stock	-0.12	-15.35	0.00
Mexico	C	0.00	2.98	0.00
	Stock	-0.21	-20.24	0.00

Table 2: Coefficients of the regressions of several stock markets on the local exchange rate with the US Dollar

exchange rates with a similar expectation structure. It can be shown that this model also creates the properties shown in these papers (especially high kurtosis and unconditional heteroscedasticity) in both the simulated exchange rates and stock prices.

The second exercise that we carry out is to estimate the chartist and fundamental expectations in order to deduce the percentage of chartists and fundamental believers in the market at each point in time. For this purpose we use data from the US economy[5]. The results are illustrated in figure 5.3. Just like in the simulations, the fundamental expectations react to the news in the market in a smaller measure. The second row of plots shows the percentage of chartists for respectively the stock market and the exchange rate. The chartists do not seem to gain dominance over the market, except briefly at the end of the sample on the exchange rate market. These results remain substantially similar for a range of values of the parameters of the intensity of choice λ and λ_e.

The fact that fundamental agents are a sizable portion of the market for the majority of the sample is due to the fact that this model allows for fundamental

[5]We also carry out this exercise for the UK economy, to keep a continuity with the exercise in Table 2. The results for the British case are shown in the Appendix.

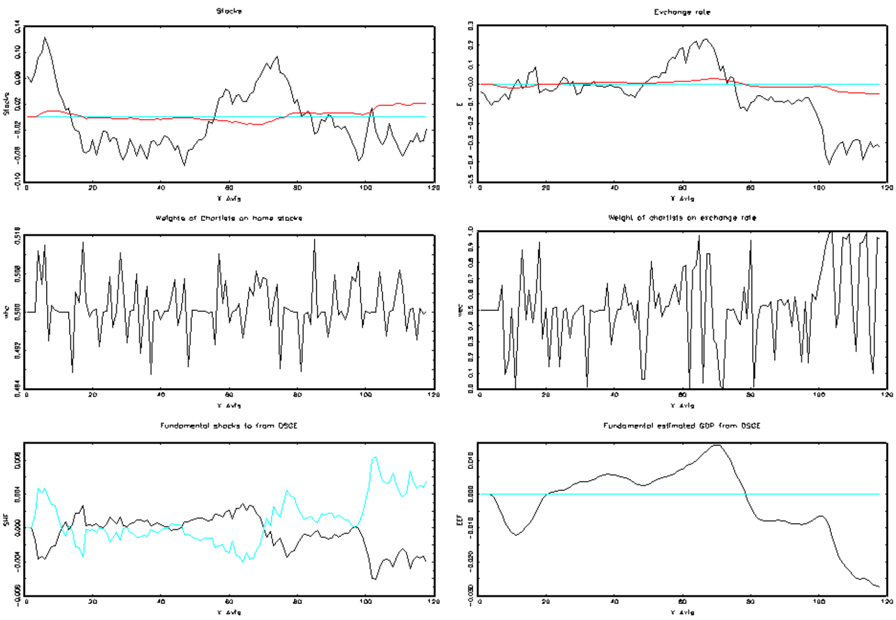

Figure 5: Estimation using US data

expectations to adjust to the news from the market. This feature of our model allows the fundamental agents to not be easily outdone in the market by the alternative chartist rule, since in effect they incorporate also a backward looking component into their rule. However this strong presence does not cause the market to be less volatile, since in some cases the DGSE believers can originate the "fundamental bubbles" as seen above.

Finally, we conclude this empirical exercise by observing the implied GDP forecasts from the DSGE believers. What this means is that when the DSGE believers make a forecast of the stock prices or the exchange rate, they also implicitly make one for the GDP. This variable is shown in the last panel on the right side. The GDP forecast adjusts over time due to the shocks that the agents plug into their model. The shocks to the home productivity (in black) and to the foreign productivity (in blue) are illustrated in the last panel on the left side. The GDP forecasts become particularly interesting since they behave in a very similar fashion to the behavior observed in Wieland and Wolters (2010) and Wieland and Wolters (2012). The authors observe the nowcasts of 5 DSGE

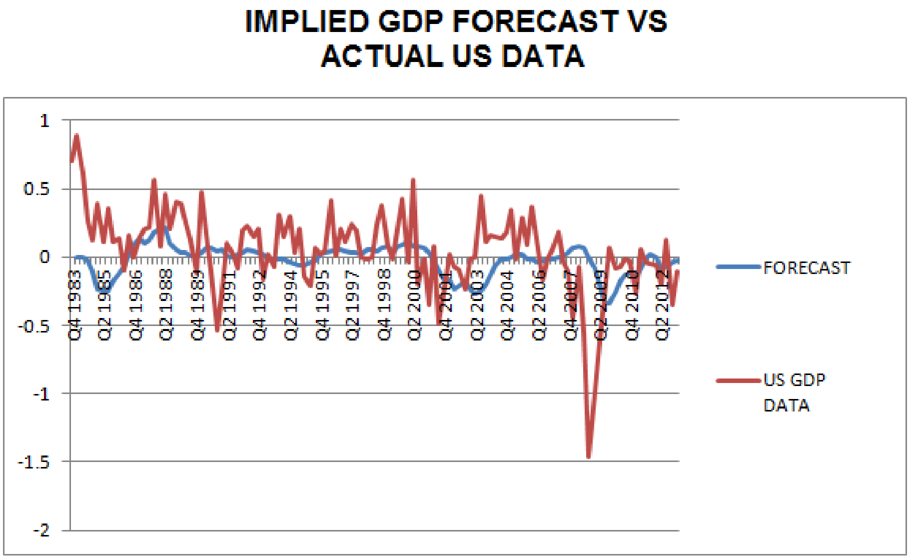

Figure 6: Forecast of US GDP data

models, to conclude that unfortunately the reaction is common to all models as being too small (not deep enough recession forecasts for the crisis) and too late (the performance is more successful in the recovery stage than in the first part of the recession).

Figure 5.3 shows the GDP growth as forecasted by the model using the financial markets data. This forecast is compared to the actual data for GDP (adjusted by steady state growth). While this exercise is not meant to check the performance of the model in forecasting GDP using financial data, the conclusions reached in Wieland and Wolters (2010) and Wieland and Wolters (2012) are confirmed despite the smaller amount of data used and the simpler estimation techniques. Once again the recession is not as deep as the DSGE model predicts, and the prediction adjusts to the data with a lag.

6 Conclusion

This paper described a model of exchange rates and stock prices. While using a financial model to set the dynamics of the two markets, this work draws a strong connection with the current macroeconomic framework. The aim is to analyze the macroeconomic connection between the two financial markets.

We constructed a model that merges two branches of the economic literature. The first one is the behavioral finance branch, featuring expectations that are not following the rational expectations paradigm, and a setting of incomplete information. The second branch is New Keynesian models. This creates the fundamental expectations from our financial markets.

The result is a model that accounts for high and time dependent variance and the statistical traits that are typical of the financial markets, while taking into account the macroeconomic pressures coming from a complete economy model. We study the dynamics that is entailed. We realize that our financial model has some new dynamics with respect to other papers. In particular, the fundamental expectations described in this paper are derived from a macroeconomic model and received feedback from past variations on both markets.

We show that the data of four financial markets support the dynamics predicted by the model. The consistency of the generated series with the real markets is not only in the individual series, but is reflected also in the relationship between the two financial markets. Secondly, the DSGE expectations behave in a similar fashion to what is observed in fully estimated DSGE models.

During the work on this paper, we came across some new ideas for future research. It is possible to invert the intuition of the model presented here and solve a New Keynesian DSGE model using heterogeneous agents from the microfoundations. Recent new papers by Kurz (2011) and Kurz, Piccillo, Wu (2012) do just this.

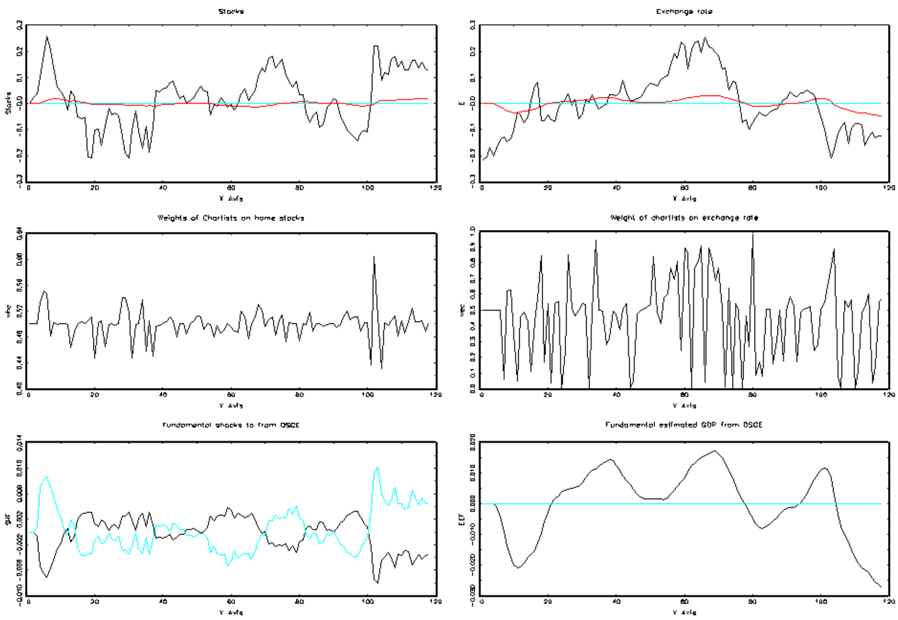

Figure 7: Estimation using UK data

Appendix A - UK empirical data

We carry out the same exercise that we showed in the main body of the paper, for the UK economy. Figures 6 and 6 show the results. These are not qualitatively different than the US scenario, though the UK is probably closer to a small open economy than the US economy.

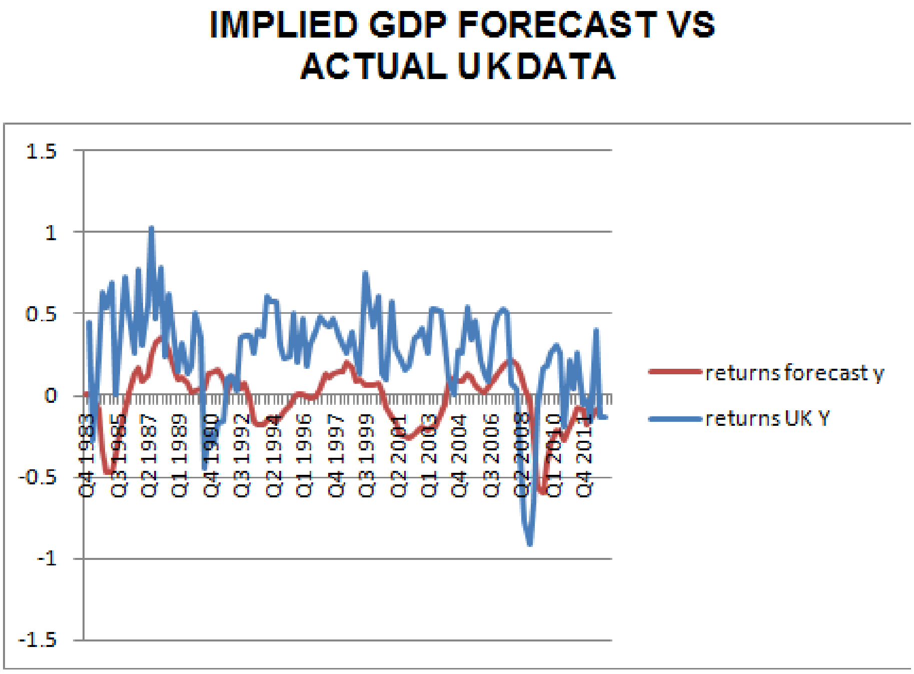

Figure 8: Forecast of UK GDP data

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