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Abstract

Wars of conquest and wars of independence are characterized by an asymmetric payoff structure: one party gets aggregate production if it wins, and its own production if it loses, while the other party gets only its own production if it wins, and nothing if it loses. We study a model of war with such an asymmetric payoff structure, and private information about military technologies. We characterize continuous equilibrium strategies and find that the party that gets aggregate production when winning fights aggressively only if its military technology is relatively good, while the other party fights quite aggressively even if its military technology is relatively poor. From an ex ante perspective, this other party is therefore more likely to win the war unless its expected military technology is considerably worse. Our model may thus explain why defending countries and secessionist groups often win against much larger opponents.

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Keywords: conflict, war, asymmetric contests, private information.

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1 Introduction

The most popular models of wars and conflicts focus on the allocation of resources to warfare and production in symmetric conflicts in which the winner gets aggregate production.[1] These standard models are helpful to understand strategic behavior in civil wars for state control, or wars between similarly sized neighboring countries. However they cannot inform us about strategic behavior in wars and conflicts in which the involved parties face fundamentally different incentives due to an asymmetry in the payoff structure.

Wars of conquests and wars of independence are both characterized by an asymmetry in the payoff structure: one party gets aggregate production if it wins, and its own production if it loses, while the other party gets only its own production if it wins, and nothing if it loses. Wars of conquest are fought between an attacking and a defending country. These wars are asymmetric in that the countries only fight for the defending country's production or resources. Wars of conquest were common during the European colonization. The Spanish conquest of the Aztec empire is a prominent example. Both the Spaniards and the Aztecs knew perfectly well that they were only fighting over the Aztecs' production and resources, and not any Spanish production or resources. More recently, it was clear to the Vietnamese communists fighting the U.S. army, and the Afghan mujahideen fighting the Soviets that they could at most win production or resources of their own territory, and no production or resources from the United States or the Soviet Union, respectively. Meanwhile the U.S. army and the Soviets knew that they would not lose any domestic production in case of defeat.

Wars of independence are civil wars fought between a central government and a secessionist group that wants some particular region to become independent. Wars of independence are common and have recently been fought by secessionist groups in, e.g.,

[1] These standard symmetric models are discussed below and in more detail in Garfinkel and Skaperdas (2007, section 3.2). See also Garfinkel and Skaperdas (2007), and Blattman and Miguel (2010) for reviews of the literature on wars and conflicts; and Konrad (2009) for a review of the literature on contests more generally.

Aceh, Chechnya, Eritrea, Kosovo, Northern Sri Lanka, South Sudan, and Timor Leste. In all these conflicts, it was foreseeable from the onset that if the central government wins, it can control total production and resources; but if the secessionist group wins, it only gets production and resources of the newly independent region, while the defeated central government still gets production and resources from the rest of the country.

To the best of our knowledge, there exists no theory of wars that takes the asymmetry inherent in wars of conquest and wars of independence into account. In this paper, we present a model of wars of conquest and independence that does so. In this model, there are two players. Player 1 represents the attacking country in a war of conquest, or the central government in a war of independence. Player 2 represents the defending country in a war of conquest, or the secessionist group in a war of independence.[2] Players are characterized by their resource endowments and their military technologies. Resource endowments are common knowledge, and we focus on the empirically more relevant case in which player 1 has no less resources than player 2. The players' military technologies consist of two components: The publicly observable expected military technology, which may differ across players, and a privately observed component that captures deviations from the expected military technology. Each player can choose how to allocate their resources to production and warfare. The resource allocation and the military technology determine military power and domestic production. The country with the higher military power wins the war for sure. The main innovation of our model is the asymmetric payoff structure, which captures the essence of wars of conquest and independence: If player 1 wins, he can consume aggregate production. If player 2 wins, each player can consume their own production. Hence, player 1 gets *at worst* his own production, while player 2 gets *at best* her own production.

We characterize an equilibrium in which the players' military power is continuous and strictly monotonic with respect to the privately observed component of their mili-

[2]We adopt the convention that player 1 is a "he" and player 2 a "she".

tary technology. Player 1 allocates few resources to warfare if his military technology, in particular its privately observed component, is poor. The reason is that it does not pay for him to forgo own production if his winning chances are small anyway. But if the privately observed component of his military technology is high, he fights aggressively, because his expected returns to warfare are then relatively high compared to his returns to own production. Player 2 fights quite aggressively and desperately even if the privately observed component of her military technology is relatively low, because allocating very few resources to warfare would most likely lead to a defeat and, consequently, a payoff of zero. Moreover, player 2 also fights more aggressively than player 1 if they both have the same expected and the same actual military technology. The reason is that the expected marginal costs of increasing military spending are higher for player 1, who gets his own production for sure, than for player 2, who gets her own production only if she wins. We therefore find that player 2 is from an ex ante perspective more likely to win the war than player 1 unless her expected military technology is considerably worse. These results hold independently of how much more resources player 1 has than player 2.

This equilibrium behavior is consistent with the aggressive and desperate fighting of the Vietnamese communists, the Afghan mujahideen, and many secessionist groups, as well as with their surprisingly frequent victories against much larger opponents. Our model suggests that such aggressive fighting, and victories of small defending countries and small secessionist groups against larger opponents are no coincidence, but the result of the very different incentives faced by the conflicting parties in wars of conquest and independence.

The standard models of symmetric conflicts and wars go back to Haavelmo (1954) and have been popularized by Garfinkel (1990), Grossman (1991), Hirshleifer (1991, 2001), and Skaperdas (1992). They are typically based on the assumptions that a war takes place for exogenous reasons, that each party can choose how to allocate its resources to production and warfare, that the outcome of the war is probabilistic and determined by a contest success function, and that the winner can consume aggregate production. Hodler and

Yektaş (2012) also focus on symmetric conflicts and wars, but drop the assumption that the outcome of the war is probabilistic. Instead they assume that the parties' resource endowments are private information, and that the party with the higher military power wins for sure. We follow their approach of letting the outcome of war be uncertain not because of luck on the battlefield, but because parties lack information about their opponent. We deviate by assuming that parties are imperfectly informed about their opponent's military technology rather than its resource endowment. This assumption may be more realistic as countries routinely keep their opponents' guessing about their military technology.[3] Private information about military technologies could also represent uncertainty about how dedicated and motivated the opponent's people are to fight for their group or their country, respectively.[4] The more important deviation from both the standard models and Hodler and Yektaş (2012) is our focus on asymmetric wars of conquest and independence rather than symmetric wars in which both parties get aggregate production when winning.

Our paper is complementary to contributions that focus on other asymmetries between attacking and defending parties. Building on the standard models discussed above, Grossman and Kim (1995, 1996), and Bester and Konrad (2004) study models of conflict in which both parties can attack or defend, and in which defending parties have a technological advantage modeled by asymmetric contest success functions. Shubik and Weber (1981), Clark and Konrad (2007), and Powell (2007a,b) study models in which a defending party is vulnerable at several points, and needs to defend all these points successfully to win the battle, while the attacker wins if he can surmount the defender at one of these points.[5]

[3]Fearon (1995) argues that countries have a strategic incentive to misrepresent their private information about relative power, and that this misrepresentation of private information can cause warfare. Meirowitz and Sartori (2008) present a model in which countries strategically create uncertainty about their own military capacity even though this uncertainty can lead to warfare.

[4]For example, many observers were surprised that the Iraqis were reluctant to fight when the U.S. army and its allies invaded Iraq to overthrow Saddam Hussein, and also that the U.S. army and its allies encountered fierce resistance in later years (Hodler and Yektaş, 2012).

[5]See Kovenock and Roberson (2012) for a review of models of conflicts with multiple battlefields.

Our model also relates to all-pay auctions with incomplete information, which go back to Amann and Leininger (1996), and Krishna and Morgan (1997). If our player 2 wins, payoffs are very similar as in all-pay auctions, as the winner's and the loser's payoff both decrease in their own bid. But if player 1 wins, payoffs are very different than in all-pay auctions. Then the winner's payoff decreases in his own and the loser's bid, while the loser's payoff is zero independently of her bid.

In addition, our model also relates to Farmer and Pecorino (1999) and, in particular, Baye et al. (2005) who study symmetric litigation contests under different legal systems. We can reinterpret our model as an asymmetric litigation contest under the American rule, which asks the parties to pay their own legal outlays. In this interpretation of our model, the party that presents the stronger case wins the litigation contest, with the strength of the case depending on the expenses for attorneys and the privately known quality of the respective arguments. The asymmetry in our model may represent a litigation environment in which one party (player 2) is currently in possession of the disputed asset, but has no other resources available, while the other, larger party (player 1) has plenty of resources. If the party who only possesses the disputed asset wins, she can keep what is left of the asset after paying her legal outlays. If the larger party wins, he gets what is left of the disputed assets, and the loser gets bankrupt. Our model predicts that the party who only possesses the disputed asset makes high legal outlays even if her argument is of low quality, while the larger party makes high outlays only if his argument is of high quality.

The remainder of the paper is organized as follows: Section 2 introduces the model. Section 3 derives and discusses the equilibrium. Section 4 concludes. The appendix contains all proofs.

2 The Model

There are two players, labeled 1 and 2. These players are at war for exogenous reasons. When thinking about wars of conquest, player 1 represents the attacking country, and player 2 the defending country. When thinking about wars of independence, player 1 represents the central government, and player 2 the secessionist group.

Player $i = 1, 2$ is characterized by the expected military technology $\tau_i \in \mathbb{R}_+$ and the resource endowment r_i, which are both common knowledge. For later use, we define $\tau \equiv \frac{\tau_1}{\tau_2}$, which measures the relative expected military technology of player 1. Further we set $r_2 = 1$, which is without loss of generality, and assume $r_1 \geq 1$. This latter assumption is consistent with typical wars of conquests in which attacking countries are no smaller than defending countries, and typical wars of independence in which secessionist regions are no larger than the rest of the country. It thereby helps us to focus on the potentially paradoxical observation that small defending countries and secessionist regions often win against much larger opponents. From a technical perspective, this assumption will ensure that player 1's resource constraint is not binding.

Each player's actual military technology α_i is private information, and independently and identically drawn from the uniform distribution on $[0, 2\tau_i]$. Equivalently, we can think of the actual military technology as being given by $2\tau_i\lambda_i$, where $\lambda_i \equiv \frac{\alpha_i}{2\tau_i}$, with λ_i drawn from the uniform distribution on $[0, 1]$. While τ_i measures the expected technical sophistication of player i's army, the privately observed component λ_i captures deviations from these expectations and the troops' dedication and motivation to fight hard. Thereby we can interpret $\lambda_i > 1/2$ as positive deviations, and $\lambda_i < 1/2$ as negative deviations.

The players simultaneously decide how to allocate their resources to production and warfare. Given resource endowment r_i, player i chooses to allocate $b_i \in [0, r_i]$ to warfare, and $r_i - b_i$ to production. Player i's military power is then $\alpha_i b_i = 2\tau_i\lambda_i b_i$, and its production $r_i - b_i$. The player with the higher military power wins the war. If player 1 wins, he gets aggregate production. If player 2 wins, each player can keep its own production. Therefore, the players' ex-post payoffs are[6]

$$\tilde{u}_1(b_1, b_2) = r_1 - b_1 + \begin{cases} 1 - b_2 & \text{if} \quad \tau\lambda_1 b_1 > \lambda_2 b_2 \\ 0 & \text{if} \quad \tau\lambda_1 b_1 \leq \lambda_2 b_2 \end{cases} \tag{1}$$

$$\tilde{u}_2(b_1, b_2) = \begin{cases} 1 - b_2 & \text{if} \quad \tau\lambda_1 b_1 \leq \lambda_2 b_2 \\ 0 & \text{if} \quad \tau\lambda_1 b_1 > \lambda_2 b_2. \end{cases} \tag{2}$$

In this game, pure strategies are of the form $b_i = \beta_i(\lambda_i)$: $[0,1] \to [0, r_i]$. We define $f_1(\lambda_1) \equiv \tau\lambda_1\beta_1(\lambda_1)$ and $f_2(\lambda_2) \equiv \lambda_2\beta_2(\lambda_2)$. Subsequently, we call $\beta_1(\lambda_1)$ and $\beta_2(\lambda_2)$ the players' real bidding strategies, and $f_1(\lambda_1)$ and $f_2(\lambda_2)$ their effective bidding strategies. Observe that the player with the higher effective bid always has the higher military power (i.e., $f_1(\lambda_1) > f_2(\lambda_2)$ if and only if $2\tau_1\lambda_1\beta_1(\lambda_1) > 2\tau_2\lambda_2\beta_2(\lambda_2)$). Hence, the player with the higher effective bid always wins the war.

The appropriate solution concept is Bayesian Nash equilibrium, and we look for an equilibrium in which the effective bidding strategies are strictly monotonic, continuous and twice differentiable. We focus on effective bidding strategies as there may exist no equilibrium with monotonic real bidding strategies.

3 Equilibrium analysis

In this section, we first discuss the trade-offs that the players face. We then prove the existence of an equilibrium, and describe the equilibrium real and effective bidding strategies. Finally, we compare the players' equilibrium effective bidding strategies, which yields insights into the likely outcome of the war from an ex-ante perspective.

As the effective bidding strategies are strictly monotonic, the inverse functions exist.

[6] The tie-breaking assumption is without loss of generality as ties are zero-probability events.

We denote them by $\phi_1 = f_1^{-1}$ and $\phi_2 = f_2^{-1}$. Given the opponent's real bidding strategies, we can write the players' ex-post payoffs as

$$
\begin{aligned}
\tilde{u}_1(b_1, \beta_2(\lambda_2)) &= r_1 - b_1 + \begin{cases} 1 - \frac{f_2(\lambda_2)}{\lambda_2} & \text{if} \quad \lambda_2 < \phi_2(\tau\lambda_1 b_1) \\ 0 & \text{if} \quad \lambda_2 \geq \phi_2(\tau\lambda_1 b_1) \end{cases} \\
\tilde{u}_2(\beta_1(\lambda_1), b_2) &= \begin{cases} 1 - b_2 & \text{if} \quad \lambda_1 \leq \phi_1(\lambda_2 b_2) \\ 0 & \text{if} \quad \lambda_1 > \phi_1(\lambda_2 b_2). \end{cases}
\end{aligned}
$$

Then the interim expected payoffs take the form

$$
\begin{aligned}
u_1(\lambda_1, b_1) &= r_1 - b_1 + \int_0^{\phi_2(\tau\lambda_1 b_1)} \left(1 - \frac{f_2(\lambda_2)}{\lambda_2}\right) d\lambda_2 \\
u_2(\lambda_2, b_2) &= (1 - b_2)\phi_1(\lambda_2 b_2).
\end{aligned}
$$

Taking the partial derivatives with respect to the players' bids leads to

$$
\frac{\partial u_1(\lambda_1, b_1)}{\partial b_1} = -1 + \left(1 - \frac{\tau\lambda_1 b_1}{\phi_2(\tau\lambda_1 b_1)}\right) \tau\lambda_1 \phi_2'(\tau\lambda_1 b_1) \tag{3}
$$

$$
\frac{\partial u_2(\lambda_2, b_2)}{\partial b_2} = -\phi_1(\lambda_2 b_2) + (1 - b_2)\lambda_2 \phi_1'(\lambda_2 b_2). \tag{4}
$$

The first terms on the right-hand sides of equations (3) and (4) represent the expected marginal costs of increasing the bids, i.e., military spending, and the second terms the expected marginal benefits from doing so.

Equation (3) and Figure 1 illustrate the trade-off that player 1 faces: Consider a type of player 1, say λ_1, that bids b_1 and thinks about bidding $b_1 + db_1$, such that his effective bid increases from $\tau\lambda_1 b_1$ to $\tau\lambda_1(b_1 + db_1)$. He incurs marginal costs equal to the forgone consumption of one marginal unit of production, as he gets his own production independently of the outcome of war. The benefit from increasing the bid by db_1 occurs if this increase turns him into a winner in which case he gains the amount that player 2 has kept for production, i.e., $1 - \frac{\tau\lambda_1 b_1}{\phi_2(\tau\lambda_1 b_1)}$. This event occurs with probability $\tau\lambda_1\phi_2'(\tau\lambda_1 b_1)$, and generates the expected marginal benefit shown in equation (3).

Equation (4) and Figure 2 similarly illustrate the trade-off that player 2 faces: Consider a type of player 2, say λ_2, that bids b_2 and thinks about bidding $b_2 + db_2$, such that her effective bid would increase from $\lambda_2 b_2$ to $\lambda_2(b_2 + db_2)$. Player 2 incurs the costs of forgone consumption of one marginal unit of production only if she is already a winner, which is the case with probability $\phi_1(\lambda_2 b_2)$. A higher bid implies a benefit equal to her own production $1 - b_2$ if it turns her into a winner. This event occurs with probability $\lambda_2\phi_2'(\lambda_2 b_2)$, and generates the expected marginal benefit shown in equation (4).

We now derive the equilibrium strategies. We thereby rely on two insights. First, the equilibrium strategies $\beta_1(\lambda_1)$ and $\beta_2(\lambda_2)$ must solve the first-order conditions $\frac{\partial u_1(\lambda_1, b_1)}{\partial b_1} = 0$ and $\frac{\partial u_2(\lambda_2, b_2)}{\partial b_2} = 0$, with the partial derivatives given in equations (3) and (4), if the players' resource constraints are not binding for any $\lambda_i \in [0, 1]$.

The second insight is that the equilibrium effective bidding strategies must coincide at the bottom and the top:

Lemma 1 *In any equilibrium with strictly monotonic effective bidding strategies, it holds that $f_1(0) = f_2(0) = 0$, that $f_1(.)$ and $f_2(.)$ are strictly increasing, and that $f_1(1) = f_2(1) = \bar{x}(\tau)$, where $\bar{x}(\tau) < 1$ for any τ.*

Players with a zero military technology obviously have zero military power and, therefore, effective bids of zero. As a consequence, strictly monotonic strategies must be strictly increasing. Moreover, no player ever bids more than necessary to win with probability one because the winner's payoff decreases in the resources he or she allocates to warfare. Effective bids thus coincide if $\lambda_1 = \lambda_2 = 0$, and if $\lambda_1 = \lambda_2 = 1$. The last result in Lemma 1 follows from the observation that player 2 never allocates all resources to warfare. By doing so, she would get a payoff of zero with certainty, while her expected payoff is strictly positive for any slightly lower bid.

Based on these two insights, we can prove the existence of an equilibrium and characterize the equilibrium bidding strategies:

Proposition 1 *There exists a Bayesian Nash equilibrium in strictly monotonic effective bidding strategies. In this equilibrium, player 1's effective bidding strategy is*

$$f_1(\lambda_1) = (\tau\lambda_1)^{-1} e^{\tau^{-1}} \int_{(\tau\lambda_1)^{-1}}^{\infty} \frac{e^{-t}}{t^2} dt, \tag{5}$$

and player 2's effective bidding strategy is implicitly determined by

$$f_2''(\lambda_2) = \frac{f_2(\lambda_2)}{\lambda_2(\lambda_2 - f_2(\lambda_2))} f_2'(\lambda_2), \tag{6}$$

and the boundary conditions $f_2(1) = \bar{x}(\tau)$ and $f_2'(1) = \tau\,(1 - f_2(1))$.

Proposition 1 shows that the equilibrium effective biding strategies $f_1(\lambda_1)$ and $f_2(\lambda_2)$ depend only on the relative expected military technology τ, but neither on the levels of τ_1 and τ_2, nor on player 1's resource endowment r_1. The same must hold true for the equilibrium real biding strategies $\beta_1(\lambda_1)$ and $\beta_2(\lambda_2)$. The players' equilibrium play depends only on their relative, but not their absolute expected military technology because they only care about having the higher military power than their opponent, but not about their military power as such. Their equilibrium play is independent of player 1's resource endowment r_1 because r_1 affects neither the probability that player 1 wins the war (at least in the absence of a binding resource constraint), nor the prize he gets when winning.[7]

Proposition 1 further gives closed-form solutions for player 1's equilibrium bidding strategies $f_1(\lambda_1)$ and $\beta_1(\lambda_1) = \frac{f_1(\lambda_1)}{\tau\lambda_1}$. Closed-form solutions for player 2's equilibrium bidding strategies $f_2(\lambda_2)$ and $\beta_2(\lambda_2) = \frac{f_2(\lambda_2)}{\lambda_2}$ do not exist, but can easily be derived numerically. Figure 3 shows the players' equilibrium bidding strategies for all $\lambda_i \in [0, 1]$

[7] Technically, the equilibrium play is independent of r_1 because player 1's interim expected payoff is linear in r_1.

and various values of the relative expected military technology τ. Note that the figures on the left-hand side show the real bidding strategies, and those on the right-hand side the effective bidding strategies; and that thick lines indicate player 1's strategies, and thin lines player 2's strategies.

We know from Lemma 1 that the players' effective bidding strategies are strictly increasing. We next look at their real bidding strategies:

Proposition 2 *There exists a threshold γ such that player 1's equilibrium real bidding strategy $\beta_1(\lambda_1)$ is strictly increasing in λ_1 if $\tau < \gamma$, and hump-shaped with a single peak at $\frac{\gamma}{\tau}$ if $\tau \geq \gamma$. Player 2's equilibrium real bidding strategy $\beta_2(\lambda_2)$ is strictly increasing in λ_2.*

Proposition 2 implies that player 1's equilibrium real bidding strategy crucially depends on the relative expected military technology τ, and how it compares to the threshold level γ, which is approximately equal to 1.64 (see proof of Proposition 1).

To understand player 1's equilibrium behavior, observe that allocating very few resources to warfare is not too unattractive for him. Sure, he is then likely to lose, but it allows him to produce more, and after all he can keep his own production even when losing. He therefore allocates close to zero resources to warfare if his military technology is poor. This choice explains why we see low real and effective bids in Figure 3 whenever the relative expected military technology τ and the privately observed component λ_1 are low. However, as τ or λ_1 increases, the option of warfare becomes more interesting for player 1, and he therefore allocates more resources to warfare, at least, up to some extent. Once $\tau\lambda_1$ gets sufficiently high, it makes no longer sense for him to increase his military power much, as he is already likely to win. Therefore, he starts reducing his real bid $\beta_1(\lambda_1)$ as soon as $\tau\lambda_1$ exceeds the threshold level γ, i.e., as soon as λ_1 exceeds $\frac{\gamma}{\tau}$. (Note that $\frac{\gamma}{\tau} \approx 0.82$ if $\tau = 2$, as in the bottom figures of Figure 3.)

For player 2, unlike for player 1, an active engagement in warfare is crucial, as she ends up with zero payoff in case of defeat. Therefore, she needs to fight aggressively (and desperately) by allocating a considerable share of her resources to warfare even if her military technology is quite poor, i.e., even if λ_2 is quite low and τ quite high. This necessity explains the relatively high $\beta_2(\lambda_2)$ for any λ_2 and τ, as can be seen in Figure 3. Her equilibrium real bidding strategy $\beta_2(\lambda_2)$ is strictly increasing in λ_2, even though it is already relatively high at low λ_2, because a rise in λ_2 increases the expected marginal benefit of allocating more resources to warfare, without affecting marginal costs.

We next compare the two players' effective bidding strategies, as the player with the higher effective bid wins the war.

Proposition 3 *Given $\tau \leq 1$, it holds that $f_1(\lambda) < f_2(\lambda)$ for all $\lambda \in (0,1)$. Given $\tau > 1$, it holds that $f_1(\lambda) < f_2(\lambda)$ for all $\lambda \in (0, \psi(\tau))$, and $f_1(\lambda) > f_2(\lambda)$ for all $\lambda \in (\psi(\tau), 1)$, where $\psi(\tau)$ satisfies $\psi(\tau) \in (0,1)$ for any $\tau > 1$, and decreases in τ.*

We have argued above that if the privately observed component of the military technology is low, it is attractive for player 1, but not for player 2 to allocate very few resources to warfare. The result that player 2 always chooses the higher effective bid, i.e., the higher military power, at low λ is the direct consequence of this difference in the players' incentives. This difference is also one of the reasons why player 1 chooses the lower real bid than player 2 for all $\lambda \in (0,1)$ if his excepted military technology is relatively poor, i.e., if $\tau < 1$. Another reason is that the expected marginal costs of increasing military spending are higher for player 1, who gets his own production for sure, than for player 2, who gets her own production only if she wins. It is this latter reason that also explains why player 1 chooses the lower effective bid for all $\lambda \in (0,1)$ even if $\tau = 1$, i.e., even if the two players have the same expected military technology. Player 1 only chooses the higher effective bid if both his expected and his actual military technology are good, i.e., if λ is relatively high and $\tau > 1$.

The comparison of the equilibrium effective bidding strategies in Proposition 3 informs us about the likely outcome of the war from an ex-ante perspective. It suggests that player

2 is more likely to win the war if her expected military technology is better or equally good as player 1's expected military technology, i.e., if $\tau \leq 1$. Player 2 is even the likely winner if her expected military technology is slightly worse. Player 1 is more likely to win from an ex-ante perspective only if his expected military technology is considerably better than player 2's.

Conclusions

We have presented a model of wars that takes seriously the asymmetric payoff structure that characterizes wars of conquests and wars of independence. We have shown that defending countries and secessionist groups tend to have stronger incentives to fight aggressively than attacking countries and central governments, and that this difference is most pronounced if actual military technologies are poor relative to what had been expected. From an ex-ante perspective, defending countries and secessionist groups are therefore more likely to win the war unless their expected military technology is considerably worse than their opponents'. It is remarkable that these results hold independently of how much larger the resource endowment of the attacking country and the central government is. They may thus explain why relatively small defending countries and secessionist groups often fight aggressively and desperately, and why they often win against much larger opponents.

Appendix: Proofs

Proofs are currently quite detailed, but could be shortened.

Proof of Lemma 1: It directly follows from the definitions of $f_1(\lambda_1)$ and $f_2(\lambda_2)$ that $f_1(0) = f_2(0) = 0$. Given that $f_i(0) = 0$, that $f_i(\lambda_i) \geq 0$ for any λ_i, and that $f_i(.)$ is strictly monotonic, it follows that $f_i(.)$ must be strictly increasing for $i = 1, 2$. We prove $f_1(1) = f_2(1)$ by contradiction. Suppose $f_1(1) > f_2(1)$. For $\lambda_1 = 1$, player 1 is then better off by deviating and playing $b_1 = \frac{f_2(1)}{\tau}$, because he still wins with probability one, but gets a higher payoff if he wins. Hence, $f_1(1) > f_2(1)$ cannot hold in equilibrium. The proof that $f_2(1) > f_1(1)$ cannot hold in equilibrium is analogous. Finally, we prove that $f_2(1) < 1$ for any τ. Since $f_2(1) = \beta_2(1)$ and $\beta_2(1) \in [0, 1]$, we only need to prove that $\beta_2(1) \neq 1$. We do so by contradiction. Suppose $\beta_2(1) = 1$. For $\lambda_2 = 1$, player 2 gets an expected payoff of zero. She is thus better off by deviating and playing any $\beta_2(1) < 1$ that still leads to a strictly positive winning probability. ■

Proof of Proposition 1: We proceed in two steps: First, we show that the equilibrium effective bidding strategies given in Proposition 1 solve the system of differential equations given by the two first-order conditions, and satisfy the boundary conditions given in Lemma 1. Doing so ensures that these strategies are mutually best responses. Second, we show that the equilibrium effective bidding strategies given in Proposition 1 are indeed strictly increasing (as required by Lemma 1), and satisfy the players' resource constraints.

We start by evaluating equation (3) at $b_1 = \beta_1(\lambda_1)$ and equation (4) at $b_2 = \beta_2(\lambda_2)$ and setting these equations equal to zero to obtain the first-order conditions

$$-1 + \left(1 - \frac{f_1(\lambda_1)}{\phi_2(f_1(\lambda_1))}\right) \tau \lambda_1 \phi_2'(f_1(\lambda_1)) = 0 \tag{7}$$

$$-\phi_1(f_2(\lambda_2)) + \left(1 - \frac{f_2(\lambda_2)}{\lambda_2}\right) \lambda_2 \phi_1'(f_2(\lambda_2)) = 0. \tag{8}$$

We will now transform the system of differential equations given by these first-order conditions in a way that it yields the inverses of the effective bidding strategies, namely $\phi_1 = f_1^{-1}$ and $\phi_2 = f_2^{-1}$, as solution. We therefore rename $f_1(\lambda_1) = y$ and $f_2(\lambda_2) = z$, and rearrange first-order conditions (7) and (8) to obtain

$$\begin{aligned} \tau\phi_1(y)\,(\phi_2(y) - y)\,\frac{\phi_2'(y)}{\phi_2(y)} &= 1 \\ (\phi_2(z) - z)\,\frac{\phi_1'(z)}{\phi_1(z)} &= 1. \end{aligned}$$

Lemma 1 implies that ϕ_1 and ϕ_2 are defined over the same interval $[0, \bar{x}(\tau)]$ for any given τ, which allows to write the system of differential equations as

$$\tau\phi_1(x)\,(\phi_2(x) - x)\,\frac{\phi_2'(x)}{\phi_2(x)} = 1 \tag{9}$$

$$(\phi_2(x) - x)\,\frac{\phi_1'(x)}{\phi_1(x)} = 1, \tag{10}$$

where $x \in [0, \bar{x}(\tau)]$. The boundary conditions given in Lemma 1 can similarly be written as $\phi_1(0) = \phi_2(0) = 0$ and $\phi_1(\bar{x}(\tau)) = \phi_2(\bar{x}(\tau)) = 1$. We evaluate the system of differential equations (9) and (10) at $x = \bar{x}(\tau)$ to obtain

$$\tau\phi_2'(\bar{x}(\tau)) = \phi_1'(\bar{x}(\tau)) = \frac{1}{1 - \bar{x}(\tau)}. \tag{11}$$

Since $\bar{x}(\tau) < 1$, as shown in Lemma 1, equation (11) implies $\tau\phi_2'(\bar{x}(\tau)) = \phi_1'(\bar{x}(\tau)) > 1$, or, equivalently, $f_1'(1) < 1$ and $f_2'(1) < \tau$.

We now derive player 1's strategy. Using equation (10), $\phi_2(x)$ can be written in terms of $\phi_1(x)$ as

$$\phi_2(x) = \frac{\phi_1(x)}{\phi_1'(x)} + x. \tag{12}$$

Differentiating equation (12) yields

$$\phi_2'(x) = 2 - \frac{\phi_1(x)\phi_1''(x)}{\left(\phi_1'(x)\right)^2}. \tag{13}$$

Substituting equations (12) and (13) into equation (9), and rearranging terms leads to

$$\left(2\tau\phi_1^2(x) - \phi_1(x)\right)\left(\phi_1'(x)\right)^2 = x\left(\phi_1'(x)\right)^3 + \tau\phi_1^3(x)\phi_1''(x), \tag{14}$$

which yields ϕ_1 as solution. Without loss of generality, we rename $\phi_1(x) = \lambda_1$ and $x = f_1(\lambda_1)$ to have a differential equation that yields the effective bid function f_1 as solution. Keeping in mind that $\phi_1'(x) = \frac{1}{f_1'(\lambda_1)}$ and $\phi_1''(x) = -\frac{f_1''(\lambda_1)}{\left(f_1'(\lambda_1)\right)^3}$, we can rewrite equation (14) as

$$\tau\lambda_1^3 f_1''(\lambda_1) + \left(2\tau\lambda_1^2 - \lambda_1\right) f_1'(\lambda_1) = f_1(\lambda_1).$$

The solution to this differential equation, together with the boundary conditions $f_1(0) = 0$ and $f_1'(1) = 1 - f_1(1)$, which follow from Lemma 1 and equation (11), is player 1's equilibrium effective bidding strategy $f_1(\lambda_1)$ given in equation (5) in Proposition 1.

We now turn to player 2's equilibrium strategy. Using equation (9), we can write $\phi_1(x)$ in terms of $\phi_2(x)$ as

$$\phi_1(x) = \frac{\phi_2(x)}{\tau\phi_2'(x)\left(\phi_2(x) - x\right)}. \tag{15}$$

Differentiating equation (15) and rearranging terms yields

$$\phi_1'(x) = \frac{-x\left(\phi_2'(x)\right)^2 - \left(\phi_2(x)\right)^2\phi_2''(x) + x\phi_2(x)\phi_2''(x) + \phi_2(x)\phi_2'(x)}{\tau\left(\phi_2'(x)\left(\phi_2(x) - x\right)\right)^2}. \tag{16}$$

Substituting equations (15) and (16) into equation (10) and rearranging terms leads to

$$x\phi_2(x)\phi_2''(x) = x\left(\phi_2'(x)\right)^2 + \left(\phi_2(x)\right)^2\phi_2''(x).$$

Without loss of generality, we rename $\phi_2(x) = \lambda_2$ and $x = f_2(\lambda_2)$ to get a differential equation that yields the effective bid function $f_2(\lambda_2)$ as solution. Again keeping in mind that $\phi_2'(x) = \frac{1}{f_2'(\lambda_2)}$ and $\phi_2''(x) = -\frac{f_2''(\lambda_2)}{\left(f_2'(\lambda_2)\right)^3}$, we get

$$-\lambda_2 f_2(\lambda_2)\frac{f_2''(\lambda_2)}{\left(f_2'(\lambda_2)\right)^3} = f_2(\lambda_2)\left(\frac{1}{f_2'(\lambda_2)}\right)^2 - \lambda_2^2\frac{f_2''(\lambda_2)}{\left(f_2'(\lambda_2)\right)^3},$$

which can be simplified to equation (6) given in Proposition 1. The corresponding boundary conditions given in Proposition 1 follow from Lemma 1 and equation (11).

We next show that the equilibrium effective bidding strategies derived above are strictly increasing, and satisfy the players' resource constraints. We first look at player 1's strategy. Given $f_1(\lambda_1)$ we can compute its first derivative as

$$f_1'(\lambda_1) = e^{\tau^{-1}}\tau(\tau\lambda_1)^{-2}\left(\int_{(\tau\lambda_1)^{-1}}^{\infty}\frac{e^{-t}}{t}dt\right) \tag{17}$$

Since all terms are positive when $\tau > 0$ and $\lambda_1 > 0$, it follows that $f_1(\lambda_1)$ is strictly increasing.

Player 1's resource constraint requires $\beta_1(\lambda_1) \leq r_1$ for each λ_1 and τ. It follows directly from equation (5) and $\beta_1(\lambda_1) = \frac{f_1(\lambda_1)}{\tau\lambda_1}$ that

$$\beta_1(\lambda_1) = (\tau\lambda_1)^{-2}e^{\tau^{-1}}\int_{(\tau\lambda)^{-1}}^{\infty}\frac{e^{-t}}{t^2}dt$$

Differentiate $\beta_1(\lambda_1)$ with respect to λ_1 to obtain

$$\beta_1'(\lambda_1) = \tau e^{\tau^{-1}}(\tau\lambda_1)^{-3}\int_{(\tau\lambda_1)^{-1}}^{\infty}\frac{(t-1)e^{-t}}{t^2}dt \tag{18}$$

Note that $\int_{(\tau\lambda_1)^{-1}}^{\infty}\frac{(t-1)e^{-t}}{t^2}dt$ is positive whenever $\tau\lambda_1 \leq \gamma$, where $\gamma \approx 1.63919$, and negative otherwise. Consequently, $\beta_1(\lambda_1)$ is strictly increasing in λ_1 whenever $\tau < \gamma$. Otherwise, $\beta_1(\lambda_1)$ is increasing for $\lambda_1 \leq \frac{\gamma}{\tau}$ and decreasing for $\lambda_1 \geq \frac{\gamma}{\tau}$, and has a single peak at $\lambda_1 = \frac{\gamma}{\tau}$.

Therefore, for each τ, the highest possible bid that player 1 may make is

$$\bar{b}_1(\tau) \equiv \max_{\lambda_1} \beta_1(\lambda_1) = \begin{cases} \beta_1(1) & \text{if} \quad \tau < \gamma \\ \beta_1\left(\frac{\gamma}{\tau}\right) & \text{if} \quad \tau \geq \gamma \end{cases} = \begin{cases} e^{\tau^{-1}}\left(\tau^{-2}\int_{\tau^{-1}}^{\infty}\frac{e^{-t}}{t^2}dt\right) & \text{if} \quad \tau < \gamma \\ e^{\tau^{-1}}\left(\gamma^{-2}\int_{\gamma^{-1}}^{\infty}\frac{e^{-t}}{t^2}dt\right) & \text{if} \quad \tau \geq \gamma \end{cases}$$

Note that $\bar{b}_1(\tau)$ is decreasing in τ with $\lim_{\tau\to 0}\bar{b}_1(\tau) = 1$. Since $r_1 \geq 1$, it thus follows that player 1's resource constraint is not binding.

We now turn to player 2's strategy. Using equation (5), we can derive

$$\frac{d}{d\lambda_1}\left(\lambda_1 f_1'(\lambda_1)\right) = e^{\tau^{-1}}\tau^{-1}\left(\lambda_1^{-2}\int_{(\tau\lambda_1)^{-1}}^{\infty}\frac{e^{-t}}{t^2}dt + \left(\lambda_1^{-2} - \lambda_1^{-1}\right)e^{-(\tau\lambda_1)^{-1}}\right),$$

where $e^{\tau^{-1}}\tau^{-1}$ and $\lambda_1^{-2}\int_{(\tau\lambda_1)^{-1}}^{\infty}\frac{e^{-t}}{t^2}dt$ are both positive. Moreover, $\left(\lambda_1^{-2} - \lambda_1^{-1}\right)e^{-(\tau\lambda_1)^{-1}}$ must also be positive, as it equals zero for $\lambda_1 = 0$ and $\lambda_1 = 1$, and is hump shaped with a single peak at some $\lambda_1 \in (0,1)$. Hence, $\lambda_1 f_1'(\lambda_1)$ is strictly increasing in λ_1. It then follows from the definition $\phi_1 \equiv f_1^{-1}$ that $\frac{\phi_1(x)}{\phi_1'(x)}$ is strictly increasing in x. Then, it also follows from equation (12) that $\phi_2(x)$ is strictly increasing in x. Hence, $f_2(\lambda_2)$ is strictly increasing in λ_2. Since $\phi_1(x)$, $\phi_2(x)$ and $\phi_2'(x)$ are all positive, equation (15) implies $\phi_2(x) > x$. It follows that $f_2(\lambda_2) < \lambda_2$ and, consequently, $\beta_2(\lambda_2) < 1$. Hence player 2's resource constraint holds for all λ_2. ■

Proof of Proposition 2: The first statement directly follows from the proof of Proposition 1 (see equation (18) and the discussion thereafter). It remains to prove that $\beta_2(\lambda_2)$ is strictly increasing. Remember that $\beta_2(\lambda_2) = \frac{f_2(\lambda_2)}{\lambda_2}$. Differentiating it once yields

$$\beta_2'(\lambda_2) = \frac{f_2'(\lambda_2)\lambda_2 - f_2(\lambda_2)}{(\lambda_2)^2}.$$

Then, since $\lambda_2 > 0$, it holds that $\beta_2'(\lambda_2) > 0$ if and only if $f_2'(\lambda_2)\lambda_2 > f_2(\lambda_2)$, or, equivalently, if and only if $\frac{f_2'(\lambda_2)}{f_2(\lambda_2)} > \frac{1}{\lambda_2}$. Changing the variables using $x = f_2(\lambda_2)$ (and therefore

$\phi_2(x) = \lambda_2$), we obtain $\frac{\phi_2(x)}{x} > \phi_2'(x)$. Substituting equations (12) and (13) into this last inequality and rearranging terms, we get

$$\frac{\phi_1(x)\phi_1'(x) + x\phi_1(x)\phi_1''(x)}{x\left(\phi_1'(x)\right)^2} > 1. \tag{19}$$

Changing the variables using $\lambda_1 = \phi_1(x)$ (and therefore $x = f_1(\lambda_1)$), using $\phi_1'(x) = \frac{1}{f_1'(\lambda_1)}$ and $\phi_1''(x) = -\frac{f_1''(\lambda_1)}{\left(f_1'(\lambda_1)\right)^3}$, and rearranging terms, we can rewrite inequality (19) as

$$\lambda_1\left(f_1'(\lambda_1)\right)^2 > f_1(\lambda_1)f_1'(\lambda_1) + \lambda_1 f_1(\lambda_1)f_1''(\lambda_1). \tag{20}$$

The first derivative of equation (5) is given by equation (17), and the second derivative is $f_1''(\lambda_1) = e^{\tau^{-1}}\tau^2(\tau\lambda_1)^{-4}\int_{(\tau\lambda_1)^{-1}}^{\infty}\left((1-2(\tau\lambda_1))\frac{e^{-t}}{t} - \frac{e^{-t}}{t^2}\right)dt$. Substituting equation (5) and these derivatives into inequality (20), and rearranging terms yields

$$\frac{\int_a^\infty \frac{e^{-t}}{t}dt}{\int_a^\infty \frac{e^{-t}}{t^2}dt} > \frac{\int_a^\infty\left((a-1)\frac{e^{-t}}{t} - a\frac{e^{-t}}{t^2}\right)dt}{\int_a^\infty \frac{e^{-t}}{t}dt}, \tag{21}$$

where $a = (\tau\lambda_1)^{-1} \in [0,\infty)$. Hence, $\beta_2'(\lambda_2) > 0$ if and only if inequality (21) holds.

We now prove that inequality (21) holds for all $a \in [0,\infty)$. Suppose first that $0 \le a \le 1$. Then the numerator on the right-hand side of inequality (21) is negative, while the other three terms are all positive. Hence, inequality (21) must hold. Suppose now that $a > 1$. It holds that

$$\left(\frac{\int_a^\infty \frac{e^{-t}}{t}dt}{\int_a^\infty \frac{e^{-t}}{t^2}dt}\right)' > \left(\frac{\int_a^\infty\left((a-1)\frac{e^{-t}}{t} - a\frac{e^{-t}}{t^2}\right)dt}{\int_a^\infty \frac{e^{-t}}{t}dt}\right)'$$

for all $a > 1$, because the left-hand side is strictly decreasing with $\lim_{a\to\infty}\left(\frac{\int_a^\infty \frac{e^{-t}}{t}dt}{\int_a^\infty \frac{e^{-t}}{t^2}dt}\right)' = 1$, while the right-hand side is strictly increasing with $\lim_{a\to\infty}\left(\frac{\int_a^\infty\left((a-1)\frac{e^{-t}}{t} - a\frac{e^{-t}}{t^2}\right)dt}{\int_a^\infty \frac{e^{-t}}{t}dt}\right)' = 1$. This inequality and the result that inequality (21) holds for $a = 1$ imply that inequality (21) must also hold for any $a > 1$. Therefore, $\beta_2(\lambda_2)$ is strictly increasing. ■

Proof of Proposition 3: We first derive a useful result, and then prove the statements in Proposition 3 separately for the cases $\tau < 1$, $\tau = 1$ and $\tau > 1$. So, note initially that differential equations (9) and (10) imply

$$\tau \frac{\phi_2'(x)}{\phi_2(x)} = \frac{\phi_1'(x)}{(\phi_1(x))^2}. \tag{22}$$

Evaluating this equation at $x = \bar{x}(\tau)$ yields $\tau\phi_2'(\bar{x}(\tau)) = \phi_1'(\bar{x}(\tau))$ or, equivalently,

$$f_2'(1) = \tau f_1'(1). \tag{23}$$

Suppose first that $\tau < 1$. Equation (23) then implies $f_2'(1) < f_1'(1)$. Then, since $f_1(1) = f_2(1)$, it must hold for very small $\varepsilon > 0$ that $f_2(\lambda) > f_1(\lambda)$ for all $\lambda \in (1-\varepsilon, 1)$. It remains to show that $f_2(\lambda) > f_1(\lambda)$ when $\lambda \in (0, 1-\varepsilon]$. Let $\tilde{x}$ be in the interval $(0, \bar{x}(\tau))$. Then, by equation (22), $\phi_2(\tilde{x}) = \phi_1(\tilde{x})$ implies $\tau\phi_1(\tilde{x})\phi_2'(\tilde{x}) = \phi_1'(\tilde{x})$. Since $\phi_1(\tilde{x}) \leq 1$, it follows that $\phi_2'(\tilde{x}) > \phi_1'(\tilde{x})$ whenever $\tau \leq 1$. Equivalently, when $\tilde{\lambda} \in (0,1)$, then $f_2\left(\tilde{\lambda}\right) = f_1\left(\tilde{\lambda}\right)$ implies $f_2'\left(\tilde{\lambda}\right) < f_1'\left(\tilde{\lambda}\right)$. Thus, f_2 and f_1 can intersect at most once. Now suppose, by contradiction, that there is a $\tilde{\lambda} \in (0, 1-\varepsilon]$ such that $f_2\left(\tilde{\lambda}\right) = f_1\left(\tilde{\lambda}\right)$. Then, for small enough $\delta > 0$ it must hold that $f_2\left(\tilde{\lambda}+\delta\right) < f_1\left(\tilde{\lambda}+\delta\right)$. But since both f_2 and f_1 are continuous and since $f_2(\lambda) > f_1(\lambda)$ for $\lambda \in (1-\varepsilon, 1)$, f_1 and f_2 must intersect more than once, which is a contradiction, thereby implying that f_2 and f_1 do not intersect for $\lambda \in (0,1)$. Since f_2 and f_1 do not intersect when $\lambda \in (0,1)$, and since $f_2(\lambda) > f_1(\lambda)$ for $\lambda \in (1-\varepsilon, 1)$, it must hold that $f_2(\lambda) > f_1(\lambda)$ for all $\lambda \in (0,1)$.

Suppose second that $\tau = 1$. We again prove that, for very small $\varepsilon > 0$, $f_2(\lambda) > f_1(\lambda)$ for all $\lambda \in (1-\varepsilon, 1)$. Having done so, the above argument will directly imply that $f_2(\lambda) > f_1(\lambda)$ for all $\lambda \in (0,1)$. By equation (23), for very small $\hat{\varepsilon} > 0$, $\phi_2(\bar{x} - \hat{\varepsilon}) \simeq \phi_1(\bar{x} - \hat{\varepsilon}) < 1$. Hence, $\phi_2(\bar{x} - \hat{\varepsilon}) > [\phi_1(\bar{x} - \hat{\varepsilon})]^2$. This inequality and equation (22) imply $\phi_2'(\bar{x} - \hat{\varepsilon}) > \phi_1'(\bar{x} - \hat{\varepsilon})$. Equivalently, for very small $\varepsilon > 0$, $f_2'(1-\varepsilon) < f_1'(1-\varepsilon)$. This inequality and equation (23) imply $f_2(\lambda) > f_1(\lambda)$ for all $\lambda \in (1-\varepsilon, 1)$, so that the above argument allows us to conclude that $f_2(\lambda) > f_1(\lambda)$ for all $\lambda \in (0,1)$.

Suppose finally that $\tau > 1$. Equation (23) and $\tau > 1$ imply $f_2'(1) > f_1'(1)$. Then, it must hold for very small $\varepsilon > 0$ that $f_2(\lambda) < f_1(\lambda)$ for all $\lambda \in (1-\varepsilon, 1)$. Now suppose, by contradiction, that $f_2\left(\tilde{\lambda}\right) = f_1\left(\tilde{\lambda}\right)$ for some $\tilde{\lambda} > \frac{1}{\tau}$. Then, by equation (22), it holds that $\tau\tilde{\lambda} = \frac{f_2'(\tilde{\lambda})}{f_1'(\tilde{\lambda})} > 1$. But as $f_2(\lambda) < f_1(\lambda)$ for $\lambda \in (1-\varepsilon, 1)$, the two curves have to intersect once more at some $\lambda' > \tilde{\lambda}$. But by equation (22), it also holds that $\tau\lambda' = \frac{f_2'(\lambda')}{f_1'(\lambda')} > 1$. Hence, there is a contradiction as f_1 and f_2 are both continuous. Thus, $f_2(\lambda) < f_1(\lambda)$ for $\lambda \in (\frac{1}{\tau}, 1)$. Moreover, if the two curves intersect at some $\tilde{\lambda}$, then we must have $\tilde{\lambda} \leq \frac{1}{\tau}$ and $\frac{f_2'(\tilde{\lambda})}{f_1'(\tilde{\lambda})} = \tau\tilde{\lambda}$, which implies $\frac{f_2'(\tilde{\lambda})}{f_1'(\tilde{\lambda})} \leq 1$. Since the intersection point $\tilde{\lambda}$ could potentially depend on τ, we denote it by $\psi(\tau)$. Then $f_1(\psi(\tau)) = f_2(\psi(\tau)) = x(\tau)$, or, equivalently, $\phi_1(x(\tau)) = \phi_2(x(\tau)) = \psi(\tau)$. Equation (12) implies that it holds at the intersection point $\psi(\tau)$ that

$$\phi_1'(x(\tau)) = \frac{\phi_1(x(\tau))}{\phi_1(x(\tau)) - x(\tau)}$$

or, equivalently,

$$f_1'(\psi(\tau)) = \frac{\psi(\tau) - f_1(\psi(\tau))}{\psi(\tau)}.$$

Plugging in the functional forms f_1 and f_1' from equations (5) and (17), we obtain

$$e^{\tau^{-1}}\tau(\tau\psi(\tau))^{-2}\int_{(\tau\psi(\tau))^{-1}}^{\infty}\frac{e^{-t}}{t}dt = \frac{\psi(\tau) - (\tau\psi(\tau))^{-1}e^{\tau^{-1}}\int_{(\tau\psi(\tau))^{-1}}^{\infty}\frac{e^{-t}}{t^2}dt}{\psi(\tau)},$$

which can be simplified to

$$\tau^{-1} = \frac{\ln\psi(\tau)}{1-(\psi(\tau))^{-1}}. \tag{24}$$

The left-hand side of equation (24) decreases in τ and satisfies $\tau^{-1} \in (0,1)$. The right-hand side is continuous and strictly increasing in ψ with $\lim_{\psi\to 0}\frac{\ln\psi}{1-\psi^{-1}} = 0$ and $\lim_{\psi\to 1}\frac{\ln\psi}{1-\psi^{-1}} = 1$, and satisfies $\psi < \frac{\ln\psi}{1-\psi^{-1}}$ for all $\psi \in (0,1)$. It follows from these properties that there exists a unique $\psi(\tau)$ that solves equation (24); that $\psi(\tau) \leq \tau^{-1}$; and that $\psi(\tau)$ is decreasing in τ. Since it holds that at $\frac{f_2'(\psi(\tau))}{f_1'(\psi(\tau))} \leq 1$ at the unique intersection point $\psi(\tau)$, that $f_1(0) = f_2(0) = 0$, and that $f_2(\lambda) < f_1(\lambda)$ for $\lambda \in (\tau^{-1}, 1)$, we conclude that $f_1(\lambda) < f_2(\lambda)$ for all $\lambda \in (0, \psi(\tau))$, and $f_1(\lambda) > f_2(\lambda)$ for all $\lambda \in (\psi(\tau), 1)$. ■

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Figures

Figure 1: Player 1's trade-off

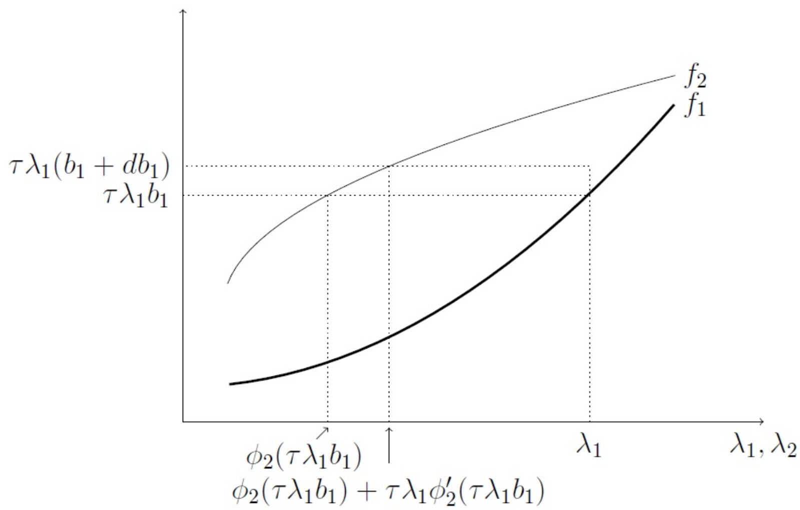
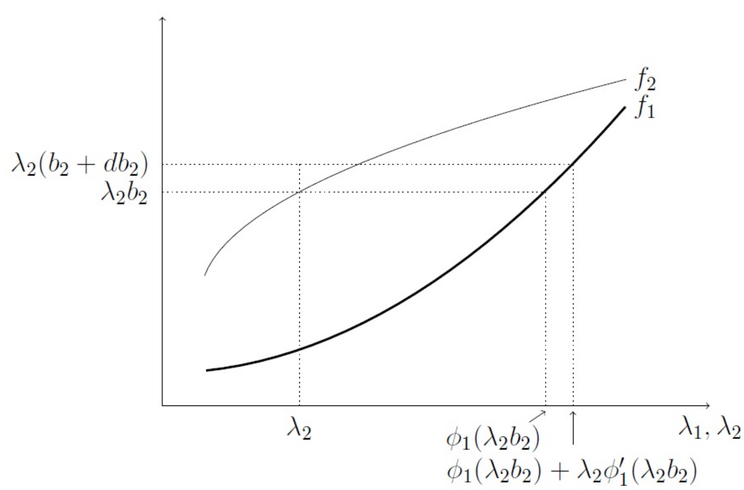
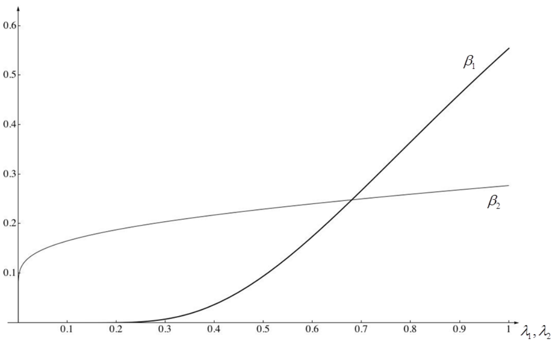
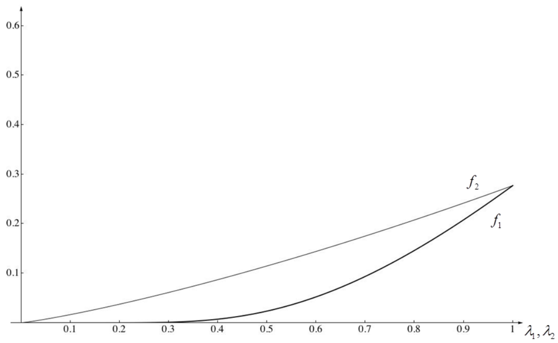


Figure 2: Player 2's trade-off

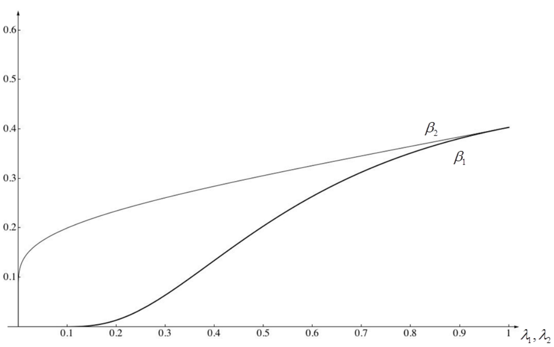




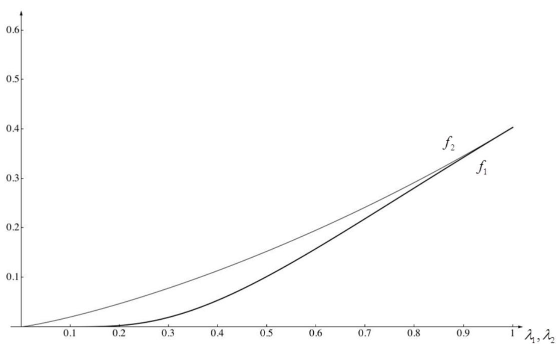
(a) Real bids for $\tau = 0.5$



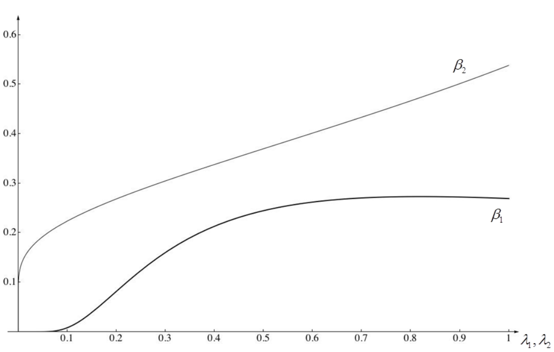
(b) Effective bids for $\tau = 0.5$



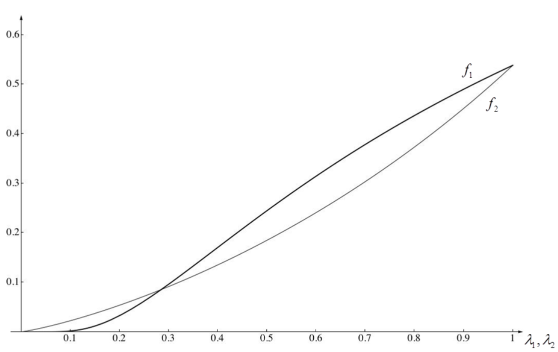
(c) Real bids for $\tau = 1$



(d) Effective bids for $\tau = 1$



(e) Real bids for $\tau = 2$



(f) Effective bids for $\tau = 2$

Figure 3: Real and effective bids for different relative expected military technologies τ