The international platform of Ludwig-Maximilians University's Center for Economic Studies and the Ifo Institute

CESifo Working Papers

www.cesifo.org/wp

An Environmental-Economic Measure of Sustainable Development

Robert D. Cairns
Vincent Martinet

CESIFO WORKING PAPER NO. 4327
CATEGORY 9: RESOURCE AND ENVIRONMENT ECONOMICS
JULY 2013

CESifo
Center for Economic Studies & Ifo Institute

CESifo *Working Paper No. 4327*

An Environmental-Economic Measure of Sustainable Development

Abstract

A central issue in the study of sustainable development is the interplay of growth and sacrifice in a dynamic economy. This paper investigates the relationship among current consumption, sacrifice, and sustainability improvement in a general context and in two canonical, stylized economies. We argue that the maximin value of utility measures what is sustainable and provides the limit to growth. Maximin value is interpreted as a dynamic environmental-economic carrying capacity and current utility as an environmental-economic footprint. The time derivative of maximin value is interpreted as net investment in sustainability improvement. It is called durable savings to distinguish it from genuine savings, usually computed with discounted-utilitarian prices.

JEL-Code: O440, Q560.

Keywords: sustainable development, growth, maximin, sustainability indicator.

Robert D. Cairns
Department of Economics
McGill University
Montréal H3A 0E6
Québec / Canada
robert.cairns@mcgill.ca

*Vincent Martinet**
INRA, UMR210 Économie Publique
78850 Thiverval-Grignon
France
vincent.martinet@grignon.inra.fr

June 27, 2013
*corresponding author

1 Introduction

Sustainable development describes growth out of poverty toward a developed state that can be sustained for what Solow (1993) calls the very long run. What is *sustained* (supported from below) along a feasible path of the economy is the minimum level of utility of any generation, looking forward from the present. The maximum attainable such minimum level, the so-called maximin value, is what we call the *sustainable* level.

Let social utility at time t be represented by U_t. The sustainable or maximin level of utility at time t is given by

$$\max \bar{U} \;\; \text{s.t. } U_s \geq \bar{U} \;, \forall s \geq t \; . \tag{1}$$

On a regular maximin path, utility remains constant and equal to the maximin value over time (Burmeister and Hammond, 1977; Cairns and Long, 2006).

An important criticism of applying maximin as a social objective in a poor economy is that future generations may be mired in a "poverty trap." Poverty may be sustained. This criticism implies that the sustainable (maximin) level of utility is considered to be so low that economic development is called for. Development, or growth, entails the diversion of resources from consumption by the current generation to investment that will increase productivity in the future. For sustainable growth to occur the standard of living of the present must be reduced to an *even lower* level than that of the poverty trap. Moreover, the development path followed by the economy must be within environmental and technological constraints. The issue is how to grow out of poverty while improving what can be sustained.

The present paper formalizes the relationship among current consumption, sacrifice, and sustainability improvement. Current decisions reduce what is sustainable if the maximin value decreases. Sustainable development is defined as non-decreasing of the current maximin value.

We examine the conditions for the sacrifice of present generations to improve sustainability. Except for a non-regular case, the current level of utility is unsustainable if it is greater than the maximin value. Conversely, if the level of utility is lower than the maximin value on an interval, *sustainable growth* is possible, with both current utility and the sustainable level of utility of the economy increasing through time. Once utility catches up with the dynamic maximin indicator's level, utility can be sustained only at the maximin level.

Our results are illustrated in two canonical models that have been prominent in the study of sustainability: the simple fishery and the *Dasgupta-Heal-Solow* (DHS) model (Dasgupta and Heal, 1974; Solow, 1974). Each addresses a fundamental issue in environmental economics. Each implies that growth is subject to environmental constraints. Open access in the fishery leads to a tragedy of the commons. The DHS model illustrates the fact that sustaining an economy may not involve a steady state. Each of open access and growth can lead to unsustainability and to a poverty trap.

Future decisions are unpredictable and it is difficult to project the path of an economy. Our results are related to the current generation's decisions only; we make no assumptions about decisions in the future. Our contribution to the economic analysis of sustainable development is to use the current maximin value as the sustainability indicator along any trajectory, optimal or not, efficient or not. This value, which depends only on the stocks of resources available and not on the current economic decisions, characterizes the dynamic limit to growth. Sustainability improvement is measured by non-negative net invesment at maximin accounting prices.

2 Maximin value and sustainability

For a vector of available resource stocks $X \in \mathbb{R}^n_+$ and a vector of decisions within the set of feasible controls, $c \in C(X) \subseteq \mathbb{R}^p$, let utility at time t be represented by $U(X(t), c(t))$ and maximin value by $m(X(t))$.[1] Sustainability has sometimes been defined (e.g., Pezzey, 1997) as the requirement that utility be no greater than the maximal sustainable util-

[1]We restrict our analysis to economic models for which a maximin value function is well-defined. This means that, if a maximin optimization problem is solved for such an economic model, an optimal maximin path actually achieves the maximin value. Assuming that the maximin value is actually achieved allows us to consider a max min problem instead of a sup inf problem. Mitra et al. (2013) provide conditions on the technology for the existence of a maximin solution in the Dasgupta-Heal-Solow model.

ity: $U(X(t), c(t)) \leq m(X(t))$. We argue instead that sustainability is defined by non-decreasing of the maximin value.

A formal juxtaposition of the two criteria is instructive in studying the properties of sustainable as opposed to unsustainable development. Let $M(X, c) \equiv \frac{dm(X)}{dt}|_c$ denote the change in the maximin value for given current economic decisions $c = (c_1, \ldots, c_p)$. The following table summarizes the nine possibilities of combinations of the conditions $U(X(t), c(t)) \lesseqgtr m(X(t))$ and $M(X(t), c(t)) \lesseqgtr 0$.[2]

Table 1: Utility, maximin value, and net maximin investment

	$U(X,c) > m(X)$	$U(X,c) = m(X)$	$U(X,c) < m(X)$
$M(X,c) > 0$	1. Impossibility	2. Non-regularity "Bounded utility"	6. Sustainability improvement
$M(X,c) = 0$	3. Non-regularity "Bounded investment"	4. Regularity (or inefficiency)	7. Inefficiency
$M(X,c) < 0$	5. Unsustainability (even with efficiency)	8. Unsustainability due to inefficiency	9. Inefficiency and Unsustainability

We start by proving the impossibility of case 1 and then characterize the two non-regular cases 2 and 3. Then, we characterize the sustainability and unsustainability of the regular case. These are done given the transition equation for the stocks,

$$\dot{X}_i = F_i\left(X, c\right) \ , i = 1, \ldots, n \ . \tag{2}$$

and the following three assumptions.

Assumption 1 *The functions $F_i(X, c)$ are continuous and differentiable.*

Assumption 2 *Utility $U(X, c)$ is continuous and differentiable.*

Assumption 3 *A maximin-value function $m(X)$ exists and is differentiable.*

[2]For the sake of simplicity, the time argument is omitted in what follows.

We do not assume that the economy follows the maximin path, or any other optimal or efficient path. We study the evolution of the maximin value over time for any feasible vector of decisions.

Let $\mu_i(X) \equiv \frac{\partial m(X)}{\partial X_i}$. Cairns and Long (2006, Proposition 1) show that these partial derivatives are the co-state variables, or shadow-values, of a maximin problem. They are defined with respect to a potential maximin path and depend only on the current state X and not the decisions c. They are independent of the trajectory determined by the functions $F_i(X, c)$. The maximin shadow values are the accounting prices of the present paper.

Definition 1 (Net maximin investment) *The time derivative of the maximin value measures net maximin investment:*

$$M(X, c) = \sum_{i=1}^{n} \frac{\partial m(X)}{\partial X_i} \dot{X}_i = \sum_{i=1}^{n} \mu_i(X) F_i(X, c) \ . \tag{3}$$

This definition of net investment applies to any feasible vector of decisions $c = (c_1, \ldots, c_p)$.

The following lemma characterizes maximin decisions.

Lemma 1 (Maximin decisions) *Any vector of decisions c such that $U(X, c) \geq m(X)$ and $M(X, c) \geq 0$ is consistent with a maximin path and thus corresponds to maximin decisions.*

Proof of Lemma 1 *Consider a state $X(t)$ and the associated maximin value $m(X(t))$, as well as a vector of decisions $c(t)$ such that $U(X(t), c(t)) \geq m(X(t))$ and $M(X(t), c(t)) \geq 0$. For an arbitrarily small time interval dt, the condition $M(X(t), c(t)) \geq 0$ implies $m(X(t + dt)) \geq m(X(t))$. From time $t + dt$, there exists a maximin path determined by some decisions $c^*(\cdot) : [t + dt; \infty[\to \mathbb{R}^p$ such that $U(X(s), c^*(s)) \geq m(X(t + dt)) \geq m(X(t))$ for all $s \geq t + dt$. Now, consider the path starting from $X(t)$ and generated by the decision vectors $c(s)$, $s \in [t, t + dt]$ and $c^*(s)$, $s \in]t + dt; \infty[$, with $dt \to 0$. This paths satisfies $U(X(s), c(s)) \geq m(X(t))$, for all $s \geq t$. Decisions $c(t)$ are thus maximin decisions.*

Cases 1 to 4 in Table 1 correspond to maximin decisions. We show in Theorem 1 that it is not possible to have both a utility level greater than the maximin value and a positive net maximin investment. This impossibility theorem is key to the discussion of the other cases in Table 1.

Theorem 1 (Maximin impossibility theorem) *: Under Assumptions 1-3, for any state X, there is no vector of decisions c such that $U(X,c) > m(X)$ and $M(X,c) > 0$.*

Proof of Theorem 1 *Consider any state vector X_0 at time t_0 and the associated maximin value $m(X_0) \equiv \max_{[c(t)]_{t_0}^{\infty}} \min_{t \geq t_0} U(X(t), c(t))$, given the dynamics $\dot{X}(t) = F(X(t), c(t))$ starting from state X_0. Suppose that there exists a vector of decisions c such that $U(X_0, c) > m(X_0)$ and $M(X_0, c) > 0$. Let $U(X_0, c) - m(X_0) = \epsilon_0 > 0$. There is a time $\tilde{t} > t_0$ along the continuous path generated by decisions c such that $m(X(\tilde{t})) > m(X_0)$. From time $\tilde{t}$, there is thus $\epsilon_1 > 0$ such that $U(X(t), c(t)) \geq m(X_0) + \epsilon_1$, $\forall t \geq \tilde{t}$. It is thus possible to define a path starting from X_0 such that $U(X(t), c(t)) \geq m(X_0) + \min(\epsilon_0, \epsilon_1)$ at all $t \geq t_0$. This contradicts the definition of $m(X_0)$.*

We now turn to case 2, which is non-regular. Proposition 2 states that the maximin value can increase when utility is equal to the maximin value only if it is not possible at the margin to increase the utility above the maximin value.[3]

Proposition 2 (Non-regularity due to locally bounded utility) *: It is possible to have both $U(X,c) = m(X)$ and $M(X,c) > 0$ only if $\frac{\partial U(X,c)}{\partial c_j} = 0$ for all $c_j \in c$, wherever these partial derivatives are defined.*

Proof of Proposition 2 *Consider a state vector X and a vector of decisions c such that $U(X,c) = m(X)$* and *$M(X,c) > 0$. Suppose that there is a decision $c_j \in c$ such that $\frac{\partial U(X,c)}{\partial c_j} \neq 0$ and c_j is not on the boundary of $C(X)$, so that it is possible to increase utility*

[3] Utility is not necessarily globally bounded from above. There may be decisions such that $U(X,c) > m(X)$, but these decisions cannot be marginally close to maximin decisions, and they necessarily imply $M(X,c) \leq 0$, in accordance with Theorem 1.

above $m(X)$ by marginally changing decision c_j.[4] *Even if $\frac{\partial M(X,c)}{\partial c_j} = \sum_{i=1}^{n} \mu_i(X) \frac{\partial F_i(X,c)}{\partial c_j} < 0$, by continuity of the $F_i(X,c)$ and of $U(X,c)$, there is a vector of decisions $\tilde{c}$ such that $U(X,\tilde{c}) > m(X)$ while $M(X,\tilde{c}) > 0$. This contradicts Theorem 1.*

In this case, for the given state, utility is locally bounded from above in the neighborhood of the maximin decisions considered. This corresponds to a particular case of non-regularity in maximin problems. An example has been described by Cairns and Tian (2010).[5]

Corollary 2 *In case 2, it is not possible to increase $U(X,c)$ by decreasing $M(X,c)$ at the margin.*

Proof of Corollary 2 *Obvious since $\frac{\partial U(X,c)}{\partial c_j} = 0$ for all c_j.*

A main result below is that, apart from the non-regular case 2, net maximin investment cannot be positive unless current utility is lower than the maximin value. There must be a sacrifice of utility by present generations to increase the sustainable level of utility.

We now characterize another type of non-regularity. Proposition 3 states that utility can exceed the maximin value without implying a decrease in that value only if no decision c_j marginally affects net maximin investment.

Proposition 3 (Non-regularity due to locally bounded investment) *: It is possible to have both $U(X,c) > m(X)$ and $M(X,c) = 0$ only if $\frac{\partial M(X,c)}{\partial c_j} = 0$ for all $c_j \in c$.*

Proof of Proposition 3 *Consider a state vector X and a vector of decisions c such that $U(X,c) > m(X)$ and $M(X,c) = 0$. Suppose that there is a decision $c_j \in c$ such that*

[4] If some controls are on the boundary of the admissibility set $C(X)$, the derivatives of these controls are defined on only one side. The condition is then that the derivative is non-positive (resp. non-negative) on the right-hand (resp. left-hand) side when the control is bounded from below (resp. above).

[5] In Cairns and Tian (2010), non-regularity arises in states for which the utility is locally bounded from above. The maximin value is equal to the maximal utility given the state vector, and the maximin path corresponds to a myopic behavior of instantaneous utility maximization. Along this path, the maximin value increases as the state evolves.

$\frac{\partial M(X,c)}{\partial c_j} = \sum_{i=1}^{n} \mu_i(X) \frac{\partial F_i(X,c)}{\partial c_j} \neq 0$ *and* c_j *is not on the boundary of* $C(X)$*, so that it is possible to increase net maximin investment above zero by marginally changing decision* c_j*. Even if marginally changing* c_j *reduces utility, by continuity of the* $F_i(X,c)$ *and of* $U(X,c)$*, there is a vector of decisions* $\tilde{c}$ *such that* $M(X,\tilde{c}) > 0$ *while* $U(X,\tilde{c}) > m(X)$*. This contradicts Theorem 1.*

This type of non regularity includes as a main particular case the situation in which all the elements of the sum $\sum_{i=1}^{n} \mu_i(X) \frac{\partial F_i(X,c)}{\partial c_j}$ are equal to zero, i.e., $\mu_i(X) = 0$ for any X_i for which $\frac{\partial F_i(X,c)}{\partial c_j} \neq 0$ for some control c_j. All the capital stocks that are locally influenced by (at least) a decision have no marginal contribution to the maximin value. These stocks are redundant from a maximin point of view.[6] This particular case was studied by Asako (1980).

Corollary 3 *In case 3, it is not possible to increase* $M(X,c)$ *above zero by reducing utility at the margin.*

Proof of Corollary 3 *Obvious since* $\frac{\partial M(X,c)}{\partial c_j} = 0$ *for all* c_j*.*

If the two types of non-regularity are ruled out, maximin decisions belong to case 4 and are regular, as stated in the following proposition.[7]

Proposition 4 (Regularity) *For a state vector* X *and a vector of maximin decisions* c*, if there is a decision* c_j *such that* $\frac{\partial U(X,c)}{\partial c_j} \neq 0$ *and a decision* c_k *such that* $\frac{\partial M(X,c)}{\partial c_k} \neq 0$*, then the vector of decisions* c *necessarily satisfies* $U(X,c) = m(X)$ *and* $M(X,c) = 0$*.*

Proof of Proposition 4 *Consider a state vector* X *and any associated vector of maximin decisions* c*. One has* $U(X,c) \geq m(X)$ *and* $M(X,c) \geq 0$ *(Lemma 1). It is not possible to have* $U(X,c) > m(X)$ *and* $M(X,c) > 0$ *(Theorem 1). If there is a decision* $c_j \in c$ *such*

[6] An even more restrictive case is when all the maximin shadow values are equal to zero at the considered state. This is the case in the simple fishery or in the Ramsey (1928) model when the single capital stock is above the golden rule level.

[7] Case 4 could also occur if there is inefficiency in non-regular cases, in the sense that potential maximin investment is wasted (case 2) or potential utility is wasted (case 3).

that $\frac{\partial U(X,c)}{\partial c_j} \neq 0$, one cannot have $U(X,c) = m(X)$ and $M(X,c) > 0$ (Proposition 2). If there is a decision $c_k \in c$ such that $\frac{\partial M(X,c)}{\partial c_k} \neq 0$, one cannot have $U(X,c) > m(X)$ and $M(X,c) = 0$ (Proposition 3). One necessarily has $U(X,c) = m(X)$ and $M(X,c) = 0$.

Regularity has been understood as the ability "to spread" utility equally over time (Solow, 1974; Burmeister and Hammond, 1977; Cairns and Long, 2006). The two types of non-regularity arise if spreading is restricted locally. The restriction in case 2 is that current utility cannot be increased by reducing the positive net maximin investment (Corollary 2). The restriction in case 3 is that net maximin investment cannot be increased by marginally reducing utility (Corollary 3). These two conditions allow the deducing of local conditions for regularity: maximin decisions must be able to influence both current utility and net maximin investment. Corrollary 4 formalizes this property.[8]

Corollary 4 *For any state vector X, if for any vector of maximin decisions c, there is a decision c_j such that $\frac{\partial U(X,c)}{\partial c_j} \neq 0$ and a decision c_k such that $\frac{\partial M(X,c)}{\partial c_k} \neq 0$, then $\frac{\partial U(X,c)}{\partial c_k} \neq 0$ and $\frac{\partial M(X,c)}{\partial c_j} \neq 0$, and it is possible to smooth the current utility to the maximin value.*

Proof of Corollary 4 *Assume that $\frac{\partial U(X,c)}{\partial c_j} \neq 0$ and $\frac{\partial M(X,c)}{\partial c_j} = \sum_{i=1}^{n} \mu_i(X) \frac{\partial F_i(X,c)}{\partial c_j} = 0$. By continuity of the $F_i(X,c)$ and $U(X,c)$, it would be possible to increase current utility (by changing decisions c_j and c_k) and the maximin investment (by changing decision c_k) to define a vector of decisions $\tilde{c}$ such that $U(X,\tilde{c}) > m(X)$ and $M(X,\tilde{c}) > 0$. This contradicts Theorem 1. (A similar argument holds if $\frac{\partial M(X,c)}{\partial c_k} \neq 0$ and $\frac{\partial U(X,c)}{\partial c_k} = 0$.) Decision c_j thus satisfies $\frac{\partial U(X,c)}{\partial c_j}\frac{\partial M(X,c)}{\partial c_j} \neq 0$. The product cannot be strictly positive (again, by Theorem 1). Therefore, if $\frac{\partial U(X,c)}{\partial c_j} > 0$, one has $\frac{\partial M(X,c)}{\partial c_j} < 0$, and vice versa. It is possible to increase (decrease) current utility and decrease (increase) maximin investment at the margin.*

We now characterize unsustainability (case 5). Except in the non-regular case 3, realizing a utility greater than the maximin value necessarily reduces maximin value, i.e., comes at the cost of reducing the sustainable level.

[8] The condition we derive in Corollary 4 is related to the concept of "eventual productivity" (Asheim et al., 2001).

Proposition 5 (Unsustainability) *: If there is a control c_j such that $\frac{\partial M(X,c)}{\partial c_j} \neq 0$, then $U(X,c) > m(X) \Rightarrow M(X,c) < 0$.*

Proof of Proposition 5 *A direct consequence of Theorem 1 and Proposition 3.*

We now characterize sustainability improvement (case 6). Except in the non-regular case 2, to increase the maximin value ($M(X,c) > 0$), there must be a sacrifice of utility by the current generation ($U(X,c) < m(X)$). This is stated in part i) of Theorem 6. This condition is not sufficient, however. The sacrifice results in a sustainability improvement only if the applied decisions result in a positive net maximin investment, as stated in part ii) of Theorem 6, which rules out case 3.[9]

Theorem 6 (Sustainability improvement) *:*

i) If, for a state vector X and vector of decisions c, there is a decision $c_j \in c$ such that $\frac{\partial U(X,c)}{\partial c_j} \neq 0$, then $M(X,c) > 0 \Rightarrow U(X,c) < m(X)$.

ii) Let a vector of maximin decisions for state X be denoted by $c^m(X) = (c_1^m, \dots, c_p^m)$. If there is a decision c_j such that, on an interval I containing c_j^m, one has $\frac{\partial U(X,(c_1^m,\dots,c_j,\dots,c_p^m))}{\partial c_j} \neq 0$ and $\frac{\partial M(X,(c_1^m,\dots,c_j,\dots,c_p^m))}{\partial c_j} \neq 0$, then there are decisions $\tilde{c}$ by which $U(X,\tilde{c}) < m(X)$ and $M(X,\tilde{c}) > 0$ on that interval. The result holds also if the two signs are reversed.

Proof of Theorem 6 *i) A direct consequence of Theorem 1 and Proposition 2.*
ii) We demonstrate that is it possible to deviate from a maximin path by reducing current utility and increasing maximin investment.

Consider a vector of maximin decisions $c^m(X) = \left(c_1^m, \dots, c_p^m\right)$ for which there is a decision c_j such that, on an interval I containing c_j^m, one has $\frac{\partial U(X,(c_1^m,\dots,c_j,\dots,c_p^m))}{\partial c_j} \neq 0$ and $\frac{\partial M(X,(c_1^m,\dots,c_j,\dots,c_p^m))}{\partial c_j} \neq 0$. In particular, $\frac{\partial U(X,c^m)}{\partial c_j} \neq 0$ and $\frac{\partial M(X,c^m)}{\partial c_j} \neq 0$. According to Proposition 4, c^m is a vector of regular maximin decisions and satisfies $U(X,c^m) = m(X)$ and $M(X,c^m) = 0$. Moreover, $\frac{\partial U(X,c^m)}{\partial c_j}\frac{\partial M(X,c^m)}{\partial c_j} < 0$.[10]

[9] A sacrifice cannot increase the maximin value in case 3 (e.g., in a fishery), as stated in Corollary 3. Note, however, that part ii) of Theorem 6 holds for non-regular case 2.

[10] See the proof of Corollary 4.

Because $\frac{\partial U(X,(c_1^m,\dots,c_j,\dots,c_p^m))}{\partial c_j} \neq 0$ *on the interval, as* $U(X,c)$ *is continuous in* c_j*,* $U(X,c)$ *is also monotone in* c_j *on the interval. The same holds for* $M(X,c)$*. Since* c_j *has an opposite effect on* $U(X,c^m)$ *and* $M(X,c^m)$ *and the functions are monotone, this opposite effect holds on the whole interval. By choosing* $\tilde{c}_j - c_j^m > 0$ *if* $\frac{\partial U(X,c^m)}{\partial c_j} < 0$ *and* $\tilde{c}_j - c_j^m < 0$ *if* $\frac{\partial U(X,c^m)}{\partial c_j} > 0$*, one can define a vector of decisions* $\tilde{c} = (c_1^m, \dots, \tilde{c}_j, \dots, c_p^m)$ *such that* $U(X,\tilde{c}) < m(X)$ *and* $M(X,\tilde{c}) > 0$*. A reversal of the sign of* $\tilde{c}_j - c_j^m$ *entails that* $U(X,\tilde{c}) > m(X)$ *and* $M(X,\tilde{c}) < 0$*.*

According to Theorem 6, in the regular case, it is possible to improve sustainability (to increase $m(X)$ over time) by reducing utility. This is not a sufficient condition, however, as the resources freed up by utility reduction have to be reinvested so as to increase the maximin value, i.e., net maximin investment must be positive. Depending on the sacrifice of utility, there may be many different vectors of decisions for which $M(X,c) > 0$. The notion of sustainability improvement is not limited to efficient paths.[11]

The remaining three cases in Table 1 are inefficient. The following Proposition is obvious from Theorems 1 and 6 and Propositions 2 to 5.

Proposition 7 (Inefficiency) *:*

i) Sustainable inefficiency (Case 7): A sacrifice of current utility with respect to the maximin sustainable level ($U(X,c) < m(X)$*) may not result in sustainability improvement if investment decisions are such that* $M(X,c) = 0$ *(including the non-regular case 3).*

ii) Unsustainability due to inefficiency (Case 8): Current decisions may result in a reduction of the maximin value ($M(X,c) < 0$*) even if utility is equal to the maximin value (*$U(X,c) = m(X)$*). Inefficiency may induce non-sustainability.*

[11] The assumption that $\frac{\partial U(X,c^m)}{\partial c_j} \neq 0$ rules out non-regular case 2 of part ii) in Theorem 6. If $\frac{\partial U(X,c^m)}{\partial c_j} = 0$, net maximin investment can be positive ($M(X,c^m) > 0$) without decreasing utility in Case 2. The result on sustainability improvement may, however, hold even for this non-regular case if $\frac{\partial U(X,\tilde{c})}{\partial c_j} \neq 0$ for $\tilde{c}_j \in I - \{c_j^m\}$. A sacrifice of utility makes it possible to increase net investment more than the non-regular maximin decision, i.e., $M(X,\tilde{c}) > M(X,c^m) > 0$.

iii) Unsustainability and inefficiency (Case 9): A sacrifice of current utility with respect to the maximin sustainable level ($U(X,c) < m(X)$) may result in unsustainability if investment decisions are inefficient ($M(X,c) < 0$).

Notwithstanding the non-regular case 3 in which consumption is wasted, the outcome of both cases 7 and 9 is driven by (poor) investment choices that are harmful for society and lead to $M(X,c) \leq 0$ while $M(X,c) > 0$ is possible. It has no link with the condition $U \lesseqgtr m(X)$. Cases 8 and 9 emphasize that the condition $U(X,c) \leq m(X)$ fails to characterize sustainability when net investment at maximin shadow values is not efficient.[12] The condition $U \lesseqgtr m(X)$ is a property of sustainability or unsustainability only for optimal paths in regular cases. For non-regular cases, and more importantly for inefficient economies, this is a misleading indicator. In a non-efficient or non-optimal setting, the sign of $M(X,c)$ is the authentic sustainability indicator.

3 Sustainability and unsustainability in two canonical economies

3.1 The Fishery

The simple fishery model involves one renewable resource stock S and one economic decision, the fishing effort $E \geq 0$. This model illustrates cases 3, 4, 5 and 6, including regular and non-regular cases.[13]

The natural rate of growth of the stock is given by $S(1-S)$ and the consumption (catch level) by $C = SE$. The evolution of the stock is then given by $\dot{S} = S(1-S) - SE$. The highest sustainable level of consumption is called the "maximum sustainable yield" (MSY); its value is $C_{MSY} = \max_S [S(1-S)] = \frac{1}{4}$. The associated stock is $S_{MSY} = \frac{1}{2}$ and the level of effort is $E_{MSY} = \frac{1}{2}$.

[12] The condition $U(t) > m(t)$ also fails to characterize unsustainability for non-regular case 3.

[13] As a single decision determines consumption (the catch) and investment (the growth rate of the stock) simultaneously, there is no possible inefficiency. We cannot use this model to illustrate cases 7–9.

If the initial stock S_0 is less than S_{MSY}, the maximin criterion (1) prescribes a constant harvest, $C(t) = S_0 (1 - S_0)$. If the initial stock is greater than S_{MSY}, the maximin value is C_{MSY}. The maximin value is thus given by

$$m(S) = \begin{cases} S_{MSY} (1 - S_{MSY}) & \text{if } S > S_{MSY}, \\ S(1 - S) & \text{if } S \leq S_{MSY}. \end{cases}$$

Consider a harvesting schedule with four time intervals which correspond with the conditions of cases 3, 5, 4 and 6, respectively. Let $S(0) = 1$. For simplicity, let the fishing effort be constant within each interval.[14] The four intervals are defined as follows and depicted in Fig. 1.

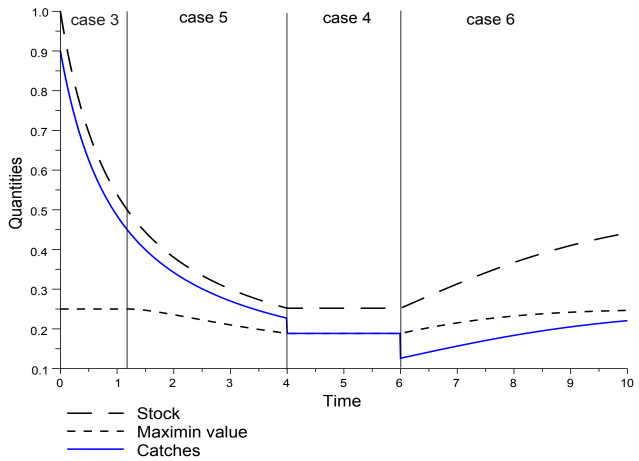

Figure 1: Sustainability and unsustainability in the fishery

- (Case 3): The first interval is characterized by a constant fishing effort $E_0 > E_{MSY}$ ($E_0 = 0.9$ in Fig. 1) and by a fish stock $S(t) > S_{MSY}$. Consumption is $C(t) =$

[14]Along a constant-effort path with effort $E_0 \in]0, 1[$, consumption at time t is given by $C(t) = E_0 S(t)$ and the dynamics of the resource by $\dot{S}(t) = S(t) (1 - E_0 - S(t))$. The stock evolves as $S(t) = \left[\frac{1}{1-E_0} + \left(\frac{1}{S_0} - \frac{1}{1-E_0}\right) e^{-(1-E_0)t}\right]^{-1}$. The stock tends toward a limit $S_\infty = 1 - E_0$ if the effort is maintained.

$E_0 S(t) > C_{MSY}$. The stock declines over time to S_{MSY} at the end of the interval. As long as $S(t) > S_{MSY}$, one has $\frac{dm(S)}{dS} = 0$: the maximin value remains constant at the MSY level. In this non-regular case, consuming more than the maximin value does not reduce this value (Proposition 3). At the end of the interval, t_1, $S(t_1) = S_{MSY} = \frac{1}{2}$ and $m(S(t_1)) = \frac{1}{4}$.

- (Case 5): Once the MSY stock is overshot, the maximin value decreases. The second interval begins at t_1, where the stock declines below S_{MSY}. The effort level is kept constant at E_0 and the stock keeps decreasing. Also, $\frac{dm(S)}{dS} > 0$. The maximin value decreases as the stock decreases. This interval illustrates the unsustainability described in Proposition 5. It corresponds to the "tragedy of the commons" for a fishery in open access.

- (Case 4): At the beginning of the third time interval, t_2, a limitation of the fishing effort is implemented to maintain the stock at $S(t_2)$. On the interval, the trajectory follows the maximin path. Net investment is $\dot{S}(t) = 0$ (the catch equals natural growth). The catch stays constant at $m(S(t_2))$. This part of the path illustrates sustainability as described in Proposition 4. If the catch is low, this part of the program corresponds to a poverty trap.

- (Case 6): On the last time interval, beginning at t_3, a recovery strategy is adopted. Effort is set at $E(t) = E_{MSY}$, which is lower than the maximin level of effort $E^{mm} \equiv 1 - S(t) > 1/2$ for $S < 1/2$. The stock size increases toward S_{MSY} and the maximin value increases toward $m(S_{MSY})$. Consumption is $C(t) = \frac{1}{2}S(t) < m(S(t))$. It increases toward C_{MSY} as the stock increases. This part of the path illustrates sustainability improvement as described in Theorem 6.

In intervals 2 and 4, where the program deviates from the maximin path, an optimum is not defined. All that is required in intervals 1 and 2 is that the level of effort remain greater than 1/2. In this case, interval 1 is part of a maximin path whatever the level of effort; the maximin path is not unique. Also, except on interval 3, where the maximin path is regular and is followed, the levels of effort need not be constant. The times t_2 and

t_3 are arbitrary but t_2 determines the stock size in interval 3. In interval 4, effort could have been chosen in the interval $[1/2, E^{mm}(S(t))]$ and, say, tend to a limit. The paths of $C(t)$ and $S(t)$ would be determined by these choices.

3.2 The Dasgupta-Heal-Solow model

The DHS model can be used to illustrate cases 4, 5, and 6, as well as case 9 on inefficiency and unsustainability. Consider a society that has stocks of a nonrenewable resource, S, and of a manufactured capital good, K, at its disposal. It produces output (consumption c and investment $\dot{K}$) by using the capital stock and depleting the resource stock at rate $\dot{S}(t) = -r(t)$, according to a Cobb-Douglas production function,

$$c + \dot{K} = F\left(K, r\right) = K^{\alpha} r^{\beta}, \text{ with } 0 < \beta < \alpha, \text{ and } \alpha + \beta \leq 1 \; .$$

If the discounted-utility criterion with a constant, positive discount rate is applied to this economy, consumption decreases asymptotically toward zero (Dasgupta and Heal 1974, 1979). Analysis of how consumption can be sustained requires a different approach.

For given levels of the capital and resource stocks, Solow (1974) and Dasgupta and Heal (1979) show that the maximin consumption is given by

$$m(S,K) = (1-\beta)\left(\alpha-\beta\right)^{\frac{\beta}{1-\beta}} S^{\frac{\beta}{1-\beta}} K^{\frac{\alpha-\beta}{1-\beta}} \; . \tag{4}$$

This increasing function of the two stocks measures the capacity of the economy to sustain the standard of living $m(S,K)$ for the long term. Sustaining consumption at this level requires that investment in manufactured capital offset the depletion of the resource (Hartwick, 1977).

To illustrate the interplay of consumption, investment and maximin value, we choose a feasible trajectory and study the evolution of the maximin value. The path depicted in Fig. 2 is composed of four time intervals, corresponding to the conditions of cases 5, 9, 6 and 4, respectively. Each interval is characterized by an illustrative consumption pattern and extraction rule. For simplicity, we consider a constant rate of change of consumption in each interval.

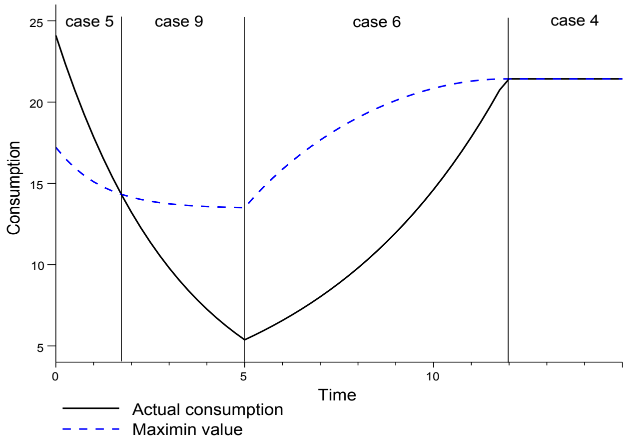

Figure 2: Sustainability and unsustainability in the DHS model

- (Case 5): At first let consumption be greater than the maximin value ($c_0 > m(S_0, K_0)$) and decrease at a constant rate $\gamma > 0$, so that $c(t) = c_0 e^{-\gamma t}$. Extraction is determined such that production is equal to consumption.[15] Investment in manufactured capital, $\dot{K}$, is zero but the resource stock is depleted; therefore, net investment $\frac{dm(S,K)}{dt} = M((S,K),(r,c))$ is negative. The maximin value decreases in accordance with Proposition 5. This program is inefficient: the same level of consumption could be maintained with a higher level of net investment, so that the maximin value would not fall so fast. But overconsumption alone would have also led to a decrease of the maximin value and unsustainability.

- (Case 9): The second time interval starts once the consumption decreases below the maximin value, at t_1. The consumption and extraction decisions are unchanged. Consumption is lower than the maximin value, but still net investment is negative and the maximin value continues to decrease. This interval illustrates Proposition 7, where unsustainability is not related to overconsumption but to inadequate in-

[15]We thus have the feedback rule $r(c,K) = c^{1/\beta} K^{-\alpha/\beta}$. As there is no investment in manufactured capital, we can express the extraction as an open-loop decision, $r(t) = c_0 e^{-\gamma t} K_0^{-\alpha/\beta}$.

vestment.

- (Case 6): At the beginning of the third interval, t_2, consumption has reached a low level, $c(t_2) = c_0 e^{-\gamma t_2}$. A decision is made to improve the level of sustainability. Having a positive level of net investment can improve sustainability in accordance with Theorem 6. The consumption pattern changes; it is now defined by a positive growth rate g, so that $c(t) = c(t_2)e^{g(t-t_2)}$. Since $c(t) < m(S(t), K(t))$, net investment can be positive. The extraction rule is modified so that production is sufficient to have a positive net investment.[16] Consumption growth can be maintained as long as the consumption remains below the maximin value.

- (Case 4): The fourth time interval starts once consumption has caught up with the maximin value, at t_3. To avoid the unsustainability of interval 1, the consumption pattern must change from the constant-growth path to the maximin path with consumption constant at $c^m(t) = m(S(t_3), K(t_3))$. Net investment is nil. Extraction and investment in capital are determined by the maximin solution. This is a sustainable path as described in Proposition 4.

At any time, the society can choose to follow a regular maximin path with a maximin value determined by the stocks at that time, or to deviate from it. We have examined some particular cases that are illustrative. On intervals 1, 2 and 3, society deviates from the maximin paths at each point of the intervals and hence from optimality as defined by maximin. On intervals 1 and 2, "degrowth" (negative growth) is an unsustainable policy. On interval 2, the maximin value decreases even though consumption is lower than the sustainable level. On interval 3, the maximin value increases even though consumption is growing at a constant, positive rate. Growth may be sustainable.

The analysis of these two models illustrates that the maximin value can be used as an indicator of sustainability, even when the policy objective is not to sustain utility by

[16] In Fig. 2, the extraction rule is arbitrarily defined so as to maximize $M((S, K), (r, c))$ given the current stock levels and current consumption. It is in fact the feedback extraction rule of a maximin program.

following the maximin path. What is sustainable in the long-run is enventually defined by the maximin value, which is dynamic and depends on investment. Unsustainability occurs when the maximin value decreases.

4 A General Measure of Sustainability

4.1 Sustainable growth

The path of the economy can be said to be a *sustainable growth* at time t if $M(X(t), c(t)) \geq 0$ and $\frac{dU(X(t),c(t))}{dt} \geq 0$.

Sustainability improvement and unsustainability are departures from the maximin path that have implications for the maximin value. The following propositions show that it is possible in regular cases for utility to improve and to catch up with the maximin value under sustainable growth and to reduce utility faster than the maximin value, until the former catches the latter, in an unsustainable economy.

Proposition 10 (Sustainable growth) *Assume that there is a control c_j such that $\frac{\partial U(X,c)}{\partial c_j} \neq 0$. If current utility is less than the maximin value ($U(X,c) < m(X)$), it is possible to choose controls such that utility rises faster than the maximin value ($dU(X,c)/dt \geq M(X,c) \geq 0$).*

Proof of Proposition 10 *The change in $U(X,c)$ is given by*

$$\frac{dU(X,c)}{dt} = \sum_{i=1}^{n} \frac{\partial U(X,c)}{\partial X_i} F_i(X,c) \ + \sum_{j=1}^{p} \frac{\partial U(X,c)}{\partial c_j} \dot{c}_j \ .$$

The first summation on the RHS is dependent only on the values of c and X at time t, and not on the $\dot{c}_j$. Let $\operatorname{sgn} \dot{c}_j = \operatorname{sgn} \frac{\partial U(X,c)}{\partial c_j}$ for any c_j for which $\frac{\partial U(X,c)}{\partial c_j} \neq 0$. Then $\sum_{j=1}^{p} \frac{\partial U(X,c)}{\partial c_j} \dot{c}_j > 0$. If c_j is not on the boundary of $C(X)$ then there is no bound on $\dot{c}_j$. (There are necessarily such c_j as $U(X,c) = m(X)$ is feasible and $U(X,c) < m(X)$.) Choose the $\dot{c}_j$ such that $\frac{dU(X,c)}{dt} \geq M(X,c) \geq 0$.

Since it assumes that $U(X,c) < m(X)$, Proposition 10 is not restricted by the non-regularity considerations of Propositions 2 and 3 that apply to maximin decisions defined in Lemma 1. Once utility catches up with the dynamic maximin indicator's level, utility can be sustained only at the maximin level.

Our results can also be used to give a rigorous meaning to the notion of "sustainable degrowth" from an initial utility level that is larger than the maximin value. So long as the utility level remains above the maximin level, the latter decreases. The utility level can be decreased fast enough that it reaches the maximin level, and thereafter can be held constant at the maximin level. Sustainable degrowth consists of decreasing utility until it reaches the maximin value.

Corollary 11 (Sustainable degrowth) *If current utility is greater than the maximin value, it is possible to choose controls such that utility falls faster than the maximin value.*

Proof of Corollary 11 *Let* $\operatorname{sgn} \dot{c}_j = -\operatorname{sgn} \frac{\partial U(X,c)}{\partial c_j}$ *for any* j *for which* $\frac{\partial U(X,c)}{\partial c_j} \neq 0$. *The same proof as that of Proposition 10 goes through for appropriate choices of* $\dot{c}_j$.

Obviously, on inefficient paths there is scope to reduce the inefficiency.[17] The model can also be extended to include uncertainty, including technological progress or regress, following Cairns and Long (2006).

We have not stressed technological progress, which is often viewed as a major source of continuing improvement in the human condition. In the general model of this paper, endogenous or exogenous technological progress can be introduced by defining stocks of knowledge or R&D among the n states. Investments in the associated stocks then have maximin prices. Technological progress is thus factored into the maximin value.[18]

Propositions 6 shows that apart from case 3, so long as utility is less than the maximin value, the maximin value can be increased. It is possible to choose the vector of decisions

[17]Llavador et al. (2011) find that sustainable consumption for the USA was higher than actual consumption in 2000. A possible reason is inefficiency. For them, the long-term solution is to address the inefficiency, not necessarily to invest more in the present. Our results stress that the two issues are linked.

[18]As regards unanticipated exogenous technological change, it is not possible to include it directly in a deterministic approach. The possibility of such technological progress, however, does not invalidate our results. Such technological progress acts like manna from heaven. When occurring, there is a "jump" in the maximin value, offering room for growth by increasing the limit to growth.

c such that both utility and the maximin value increase (Proposition 10). The path so followed can be considered to be sustainable growth.[19]

4.2 Practical implications for sustainability accounting based on investment at maximin shadow values

The maximin indicator is a very-long-run indicator of what is sustainable, of the sort that Solow (1993) seeks. At least two other indicators have been proposed to evaluate sustainability, namely the ecological footprint and genuine savings.

The ecological footprint has been proposed as an indicator of the environmental limit to sustainable output. It seeks to compare the level of current utilization of environmental resources (the ecological footprint) with the available flow of environmental services (the ecological carrying capacity), evaluated in terms of land of a given quality. If the level of utilization is greater than the flow of available services, the society depletes the stock and is considered to be unsustainable at its current level of utilization. The ecological footprint has no explicit objective, although an implicit objective is some form of ecological sustainability. This lack of an explicit objective is what leads to the derivation of accounting prices from the (natural) constraints facing the society.[20]

Maximin analysis puts the insights of the ecological footprint on a sounder, more comprehensive footing, based not on land capacity but on "generalized capacity to produce economic well-being" (Solow, 1993). In the present paper, the idea of the footprint is made more comprehensive through the analysis of evolving environmental *and* technological constraints. The current level of utility corresponds with the environmental-economic footprint. The maximin value may be considered to be a dynamic, *environmental-economic*

[19]In the DHS model of section 3.2, a deviation downward from maximin path can allow for growth at a parametric rate through investment (d'Autume and Schubert, 2008). Asheim *et al.* (2007) show that it is possible, with what they call quasi-arithmetic growth, for the maximin value to increase indefinitely and for the consumption level to approach it asymptotically. It would also be possible for the utility level to catch up with the maximin value in finite time. Once $U(X, c) = m(X)$, the only sustainable program is for current utility to remain equal to the maximin value, forever.

[20]Through its set of explicit trade-offs that make land the numeraire, ecological footprint analysis has implied a form of substitutability among natural and other stocks.

limit to growth. Current decisions modify this limit. In regular cases, as predicted by analyses of the ecological footprint, society faces diminishing long-run prospects, or diminishing sustainability, if utility exceeds the limit.

The indicator in Definition 1 closely resembles the genuine-savings indicator as determined from the extension of the national accounts (e.g., World Bank, 2006; Dasgupta, 2009). Genuine savings (sometimes called genuine investment) generalizes the concept of savings in the national accounts to include changes in the quantities of capital goods, especially environmental goods, that do not have market prices. It is equal to the current change in social welfare, defined to be the integral of discounted utility. An increase in this integral implies that genuine savings computed at competitive prices is positive at a given instant. Non-negative genuine-savings is sometimes considered to be an indicator of sustainability because current welfare does not decrease. For example, the World Bank (2006: 41) argues that "Economic theory tells us that there is a strong link between changes in wealth and the sustainability of development – if a country (or a household, for that matter) is running down its assets, it is not on a sustainable path. For the link to hold, however, the notion of wealth must be truly comprehensive."

The issue regarding sustainability turns not solely on the assets to be included but also on the shadow or accounting prices at which investment is evaluated.[21] If there is a suspicion that the market is not producing a sustainable result, the prices derived from national accounts should not be used for sustainability accounting. An increase of welfare signaled by positive genuine savings may not be lasting or durable. Rather, the genuine savings indicator can be positive along a competitive path even though consumption is not sustainable (Asheim, 1994). The welfare integral can increase at the current moment but eventually decrease, even if the environment is incorporated into optimal decisions (Dasgupta and Heal, 1979, Pezzey, 2004). Genuine savings with a discounted utility objective functional is not the long-run measure sought in considering sustainability.

According to the generalized concept of genuine savings indicator formalized by

[21] The comprehensive vector of capital stocks accounted for in the genuine savings approach is the same as the vector of capital stocks used to define the maximin value. The value of each stock is, however, different.

Asheim (2007), non-negative net investment, accounted at the shadow values of a given welfare function, is associated with non-decreasing welfare at the current time.[22] Maximizing discounted utility though time does not require non-negative investment. There is thus no normative reason to pursue a non-negative net investment when welfare is defined as discounted utility. Non-negative investment at maximin prices is a characteristic of the maximin approach.[23] Pursuing non-negative investment at maximin prices, even in a sub-optimal economy, is consistent with sustainability and with the optimality concept of maximin.

We distinguish genuine investment, be it applied to maximized social welfare or the level of welfare generated by a resource–allocation mechanism describing the economy (Dasgupta and Mäler, 2000), from investment calculated from the maximin value by calling the latter maximin or *durable investment* (from the French term, *développement durable*). Durable investment is the indicator of the current change in sustainability. It is comprehensive investment evaluated at maximin shadow prices, along any particular path of the economy. It is the statistic that is appropriate in expressing sustainability improvement. For sustainable *development* at time t the economy must have $M(X(t), c(t)) \geq 0$. This condition means that the maximal sustainable utility does not decrease at the current time. Sustainable *growth* requires $\frac{dU(X(t),c(t)}{dt} \geq 0$ and $M(X(t), c(t)) \geq 0$. Current growth does not jeopardize the capacity of future generations to sustain utility.

[22]If the welfare function is denoted by $W(X)$, the associated shadow values are $\frac{\partial W(X)}{\partial X_i}$, and generalized genuine savings is defined as $\sum_{i=1}^{n} \frac{\partial W(X)}{\partial X_i} \dot{X}_i$. When welfare is defined as discounted utility, i.e., $W(X) \equiv V(X) = \max_{c(\cdot)} \int_0^\infty U(X(t), c(t))e^{-\delta t} dt$, where δ is the positive, constant utility discount rate, the shadow values are $\frac{\partial V(X)}{\partial X_i}$, and genuine savings correspond to the usual genuine savings indicator. When welfare is defined as the maximin value, i.e., $W(X) \equiv m(X)$, one obtains net maximin investment as characterized in Definition 1.

[23]The objective of a maximin problem can mathematically be expressed as the maximization of the Hamiltonian $H(X, c) \equiv \sum_{i=1}^{n} \mu_i \dot{X}_i$, subject to the constraint $U(X, c) \geq m(X)$ (Cairns and Long, 2006). The maximin problem is thus tantamount to maximizing the net investment at maximin shadow values, i.e., $M(X, c)$, subject to the constraint that consumption is no less than the maximin value.

5 Conclusion

Our discussion stresses a property of a growth path that is not stressed by proponents of sustainable development out of poverty. If the maximin path is not pursued, but instead some growth path is followed, then earlier generations must be deprived in order to divert toward investment the resources needed to development. Growth is possible only at a cost. Open access, which in abstract terms is the main environmental problem facing humanity, is an inefficiency that cannot be overcome without current sacrifice. Growth is possible only within limits given by the technology and the environment. Otherwise, it can cause overshooting.

Our contribution to the literature on sustainability is to use the maximin value as an indicator of sustainability along any development path (efficient or not, optimal or not). The maximin value is a dynamic environmental and economic indicator of the prospect for sustainable growth, which, when increasing, indicates sustainability improvement.

The definition of durable savings holds for any resource-allocation mechanism. Durable savings must be evaluated at "the right prices," the maximin shadow values. How to get the maximin prices is a difficult question, even in simple models. The difficulty is no reason to use genuine savings with discounted utilitarian prices to measure long-term sustainability. This practice can be misleading and send an incorrect message, as genuine savings can be positive even if current utility exceeds the maximal sustainable utility and the maximin value indicator is decreasing.

The indicator of sustainability on any program, optimal or not, is the maximin value. Durable investment, the change in the maximin value, is the indicator of whether or not the level of well-being that can be sustained is increasing or decreasing.

References

[1] Asheim, G. (1994), "Net National Product as an Indicator of Sustainability," *Scandinavian Journal of Economics* 96:257-265.

[2] Asheim, G. (2007), “Can NNP be used for welfare comparisons?,” *Environment and Development Economics*, 12(1):11-31.

[3] Asheim, G., W. Buchholz, and D. Tungodden (2001) “Justifying sustainability,” *Journal of Environmental Economics and Management* 41: 252–268.

[4] Asheim, G., W. Buchholz, J. Hartwick, T. Mitra and C. Withagen (2007), “Constant Savings Rates and Quasi-Arithmetic Population Growth under Exhaustible Resource Constraints,” *Journal of Environmental Economics and Management* 53(2):213-229.

[5] D’Autume, A. and K. Schubert (2008), “Zero Discounting and Optimal Paths of Depletion of an Exhaustible Resource with an Amenity Value,” *Revue d’Economie Politique* 119(6):827-845.

[6] Burmeister, E. and P. Hammond (1977), “Maximin Paths of Heterogeneous Capital Accumulation and the Instability of Paradoxical Steady States,” *Econometrica* 45:853-870.

[7] Cairns, R. and N. V. Long (2006), “Maximin: A Direct Approach to Sustainability”, *Environment and Development Economics* 11:275-300.

[8] Cairns, R. and H. Tian (2010), “Sustained Development of a Society with a Renewable Resource”, *Journal of Economic Dynamics and Control* 34(6):1048-1061.

[9] Dasgupta, P. (2009), “The Welfare Economic Theory of Green National Accounts,” *Environmental and Resource Economics* 42:3-48.

[10] Dasgupta, P. and G. Heal (1974), “The Optimal Depletion of Exhaustible Resources,” *Review of Economic Studies*, Symposium Issue, 41:3-28.

[11] Dasgupta, P. and G. Heal (1979), *The Economics of Exhaustible Resources*, Nisbet, Cambridge.

[12] Dasgupta, P. and K.-G. Mäler (2000), “Net National Product, Wealth and Social Well Being, *Environment and Development Economics* 5:69-94.

[13] Doyen, L. and V. Martinet (2012), Maximin, Viability and Sustainability, *Journal of Economic Dynamics and Control*, 36(9):1414-1430.

[14] Hartwick, J. (1977), "Intergenerational Equity and the Investing of Rents from Exhaustible Resources", *American Economic Review* 67:972-974.

[15] Llavador, H., J. Roemer and J. Silvestre (2011), "A Dynamic Analysis of Human Welfare in a Warming Planet," *Journal of Public Economics*, 95(11-12):1607-1620.

[16] Martinet, V. and L. Doyen (2007), "Sustainability of an Economy with an Exhaustible Resource: A Viable Control Approach", *Resource and Energy Economics* 29:17-39.

[17] Mitra, T., G. Asheim, W. Buchholz and C. Withagen (2013), "Characterizing the sustainability problem in an exhaustible resource model", *Journal of Economic Theory*, forthcoming.

[18] Pezzey, J. (1997), "Sustainability Constraints versus 'Optimality' versus Intertemporal Concern and Axioms vs. Data," *Land Economics* 73(4):448-466.

[19] Pezzey, J. (2004), "One-sided sustainability tests with amenities, and changes in technology, trade and population," *Journal of Environmental Economics and Management* 48(1):613-631.

[20] Ramsey, F. (1928), "A Mathematical Theory of Saving," *Economic Journal* 38:543-559.

[21] Solow, R. (1974), "Intergenerational Equity and Exhaustible Resources," *Review of Economic Studies*, Symposium Issue 41:29-45.

[22] Solow, R. (1993), "An Almost Practical Step Toward Sustainability," *Resources Policy* 19:162-172.

[23] World Bank (2006), *Where is the Wealth of Nations? Measuring Capital for the Twenty-First Century*, Washington DC: The World Bank.