CESifo Working Papers

Cyclical Asset Returns in the Consumption and Investment Goods Sector

Burkhard Heer
Alfred Maußner
Bernd Süssmuth

CESifo
Center for Economic Studies & Ifo Institute

Cyclical Asset Returns in the Consumption and Investment Goods Sector

Abstract

We document the empirical fact that asset prices in the consumption-goods and investment-goods sector behave almost identically in the US economy. In order to derive the cyclical behavior of the equity returns in these two sectors, we consider a standard two-sector real-business cycle model with habit formation and sector-specific adjustment costs of capital. The model is able to replicate the equity premium and the Sharpe values observed empirically. In addition, we are able to match the empirical fact that equity returns in the two sectors are not correlated with output.

JEL-Code: G120, C630, E220, E320.

Keywords: asset prices, business cycles, equity premium, investment sector, consumption sector.

Burkhard Heer
University of Augsburg
Department of Economics
Universitätsstraße 16
Germany – 86159 Augsburg
burkhard.heer@wiwi.uni-augsburg.de

Alfred Maußner
University of Augsburg
Department of Economics
Universitätsstraße 16
Germany – 86159 Augsburg
alfred.maussner@wiwi.uni-augsburg.de

Bernd Süssmuth
University of Leipzig
Institute for Empirical Research
in Economics
Grimmaische Straße 12
Germany – 04109 Leipzig
suessmuth@wifa.uni-leipzig.de

1 Introduction

Recent extensions of the standard representative-agent models of a production econ-
omy have been successful in matching the equity premium implied by the model with
the empirical one[1].[2] Among others, Boldrin, Christiano and Fisher (2001) (BCF for
short) propose a two-sector model where labor is immobile between the investment
and consumption goods sector for one period after the observation of the shock. In
their model, the equity premium results from the variation in the relative price of the
two goods. In Uhlig (2007), a sizeable equity premium is generated if real wages are
sticky to a considerable degree. Wage stickiness is introduced as in Blanchard and
Galí (2005).[3] Most recently, Albuquerque, Eichenbaum, and Rebelo (2012) introduce
preference shocks in the production economy in order to successfully model the weak
correlation of stock returns with consumption and output growth.

All these models above only consider the effects of a supply-side or demand-side shock
on aggregate stock returns. In the present paper, we explicitly study the dynamics
of the sectoral asset prices in the two production sectors of the economy, a capital
goods and consumption goods sector. We first document in Section 2 the empirical
regularities that both asset returns and the Sharpe value are moderately higher in
the capital goods sector than in the consumption goods sector using data from the US
economy in the period 1980-2009. The same applies to the more narrow period ranging
from 1980 to 1999 excluding equity price crashes due to asset price bubbles. In both
cases, the differences are not significantly different from zero at the conventional levels.
In addition, the correlation of stock returns with output are found to be insignificant
in both sectors.

In Section 3, we propose a two-sector business cycle model that is able to replicate these
qualitative findings. The model is an extension of the BCF (2001) model. In particular,
we use CES-technologies with sector specific technology shocks and introduce sector

---

[1]In their seminal paper, Mehra and Prescott (1985) estimate an equity premium of 6.18% p.a. for
the United States over the period 1889-1979.

[2]A pioneering work in this area is the production-based asset pricing model of Jerman (1998).
Assuming exogenous labor supply, his model has been demonstrated to replicate the empirically
observed equity premium successfully. Heer and Maußner (2013) compare the relative performance
of one- and two-sector models of production economies with endogenous labor supply with respect to
their asset price and labor market implications.

[3]In addition, Uhlig introduces habits in consumption and leisure.

specific adjustment costs of capital, which allows us to identify the asset price (Tobin's q) in each sector. The essential feature of the model are frictions in the allocation of labor and capital. Without these frictions and with identical technology shocks in both sectors, the model reduces to the standard one-sector model. We thus demonstrate that relatively small departures from the standard model are sufficient to explain our stylized facts on asset returns.

Our findings can be related to recent results in the literature on asset pricing: 1) Gomme, Ravikamur, and Rupert (2011) show that the standard real business (RBC) cycle model produces a volatility of the return to capital relative to output that is too low and only 50% of values observed empirically. One of the most promising ingredients for the RBC model in order to align its second moments of the return to capital data with the asset returns computed from the S&P 500 index is the consideration of stochastic taxes on capital and labor income. They show that the model with a joint stochastic process for total factor productivity, the capital income tax, and the labor income tax can explain almost 80% of the volatility of the return to capital.

2) Covas and Den Haan (2007) consider an economy with small and large firms that have different access (costs) to bank debt markets. As a consequence, they are able to explain different behavior of these firms with regard to equity issuance, asset prices, and the prices of risk. However, in their model, the required rate of return for investors is specified as an exogenous process.

3) Ireland and Schuh (2008) study a real business cycle model with two production sectors similar to those in the model of the present paper. They identify the sources of the changes in total factor productivity in the postwar US economy and show that the main and persistent contributor is the slowdown of the consumption goods sector. In addition, they also introduce a preference shock and find, in accordance with the study of Albuquerque, Eichenbaum, and Rebelo (2012), that this shock helps to reconcile the business-cycle properties of the model with the data. However, Ireland and Schuh do not study the asset price implications of their model.

The paper is organized as follows. In Section 2, we present the empirical asset price statistics for the investment and consumption goods sector in the US economy during the period 1980-2009. The two-sector model is described in Section 3. We present the results from simulations of this model in Section 4 and conclude in Section 5. The interest reader finds the detailed description of the model together with its business-

cycle properties in the Appendix.

2 Business-cycle behavior of sectoral asset returns in the US

The time series used in this subsection are of quarterly frequency and refer to the U.S. economy. The data consists of time series covering the period from the first quarter of 1980 to the fourth quarter of 2011. In order to avoid a downward bias due to equity price crashes in the 2000s, we also consider a narrowed period ranging from 1980:Q1 to 1999:Q4. Asset price series are drawn from the Datastream Global Equity Indices (GEI) database. The source for the time series to construct an adequate deflator is the U.S. Bureau of Economic Analysis (BEA). The series are individually described as follows.

Asset prices for the consumer goods sector are obtained from an equally weighted average of the breakdown of the Datastream GEI at its Industry Classification Benchmark (ICB) Level 2 "Consumer Goods (CNSMGUS)" and "Consumer Services (CNSMSUS)," respectively. Notice, all indices contained in the GEI database are constructed based on a representative sample of stocks covering a minimum 75-80% of total market capitalization.

Stock prices for the capital goods sector are drawn from the ICB-2 series "Industrials (INDUSUS)." It comprises the ICB-4 levels: Construction and materials, aerospace and defense, general industrials, electronic and electric equipment, industrial engineering, industrial transportation, and support services.[4]

As our measure of nominal yield on relatively riskless securities over our period of observation we rely on the most frequently used Treasury Yield (USGBOND) series adjusted to constant maturity.

In order to calculate real returns, we use a consumption deflator series that we obtain by dividing nominal consumption, i.e. nominal personal consumption expenditures, at current prices through real consumption in 2005 dollars (source: BEA).[5] Real series are

---

[4]As an alternative the Standard and Poor's 500 Capital Goods index series (SP5GCAP) could have been used. However, in order to avoid introducing some scale bias as, in contrast to our consumption sector ICB series, the S&P 500 series is based on the market capitalizations of the 500 leading companies only, we decided to use this ICB series for the capital good sector index.

[5]The reason for using a consumption deflator for both sectors instead of, for example, considering

Table 2.1

Equity premium by sector

Index	Mean	Median	Standard deviation	Sharpe ratio
1980:Q1 to 2009:Q4				
Capital goods	0.0096	0.0182	0.0991	0.0976
Consumption goods	0.0074	0.0184	0.0981	0.0753
	(0.319)			(0.863)
1980:Q1 to 1999:Q4				
Capital goods	0.0176	0.0210	0.0880	0.2009
Consumption goods	0.0169	0.0371	0.0993	0.1715
	(0.453)			(0.853)

Notes: Authors' own calculations based on quarterly real returns from the data described in the body of the paper. Values in parentheses are p-values: In the 'Mean' column they refer to a one-sided test for a positive mean of equity premium difference between capital goods sector index and consumption goods sector index. In the 'Sharpe ratio' column they refer to a two-sided Wald test of a zero difference in the respective Sharpe ratio values.

constructed dividing nominal series by this deflator. Returns are calculated in the usual way. Risk or equity premia are calculated as the difference between the real return on the respective sectoral index and the real return on a riskless security as defined above. Sharpe Ratios are the ratios between the equity premium and the standard deviation of the respective asset's returns.

In Table 2.1, we present the statistics for the equity premium and the Sharpe ratio. The average quarterly equity premium in the capital goods sector amounts to 0.0096 (0.0176 leaving out the 2000s) or around 3.9% (6.8%) annually. The equity premium of the consumption goods sector behaves similarly and amounts to 3.0% (6.4%) annually. Notice that the equity premia are marked smaller than those values found, for example, by Mehra and Prescott (1985). As pointed out by Jagannathan, McGrattan and Scherbina (2000), the US equity premium has been declining markedly since 1970 due

the PPI in the case of the capital goods sector series is twofold. First, the consumption deflator is the one used in the seminal studies on the equity premium, i.e. in Mehra and Prescott (1985) and in Kocherlakota (1996), that also contain capital sector stocks in their considered indices. Secondly, it seems straightforward to deflate shares that are ususally held in the same portfolios, though stemming from different sectors, with the same deflator.

Table 2.2

Asset return volatility and correlations: 1980:Q1-2009:Q4

Index	Standard deviation	Correlations	
		Output	Consumption
Capital goods	0.0986	0.0111	0.00584
		(0.94)	(0.98)
Consumption goods	0.0980	-0.00264	0.00136
		(0.99)	(0.99)

Notes: Authors own calculations based on quarterly returns and growth rates, respectively. HAC consistent p-values in parentheses.

to declining transaction costs and higher participation rates in the financial markets.

In Table 2.2, the asset return volatility and correlations are presented. The standard deviations of the equity returns in the two sector coincide and amount to approximately 10%. In addition, the correlations of the asset returns with output and consumption are both small in size and statistically insignificant.

3   The model

We consider an extended version of the two-sector model of Boldrin, Christiano, and Fisher (2001). In particular, we depart from the standard Cobb-Douglas production function and use CES technologies, and we introduce adjustment costs of capital in both sectors. Furthermore, we assume sector-specific, stationary technology shocks, modeled as AR(1) processes in the (natural) logarithm of total factor productivity, whereas BCF consider labor-augmenting technical progress driven by a random walk with drift.

3.1   Production

A consumption good C and an investment good I are produced in two different sectors. The consumption goods sector employs the technology

$$C_t = Z_{Ct} \left[(1-\alpha) N_{Ct}^\rho + \alpha K_{Ct}^\rho \right]^{\frac{1}{\rho}}, \quad \alpha \in (0,1), \rho = \frac{\sigma - 1}{\sigma}, \sigma \in (0, \infty), \tag{3.1a}$$

where N_{Ct} and K_{Ct} denote labor and capital employed in this sector. Z_{Ct} denotes the total factor productivity in the consumption good sector.

The investment goods sector (subscript I) uses the same technology so that

$$I_t = Z_{It}\left[(1-\alpha)N_{It}^\rho + \alpha K_{It}^\rho\right]^{\frac{1}{\rho}}, \tag{3.1b}$$

is the amount of investment goods which sell at the relative price p_t. Accordingly,

$$Y_t = C_t + p_t I_t \tag{3.2}$$

is the economy's output at current prices. Total labor and capital in the economy equal

$$N_t = N_{Ct} + N_{It}, \tag{3.3a}$$

$$K_t = K_{Ct} + K_{It}. \tag{3.3b}$$

The level of total factor productivity Z_{Xt} in both sectors, $X \in \{C, I\}$, is governed by an AR(1)-process

$$\ln Z_{Xt} = \rho_X \ln Z_{Xt-1} + \epsilon_{Xt}, \quad \epsilon_{Xt} \text{ iid } \sim \mathcal{N}\left(0, (\sigma_X)^2\right). \tag{3.4}$$

3.2 Households

A representative household supplies labor N_{Ct} and N_{It} at the real wage w_{Ct} and w_{It} in the consumption and investment goods sector, respectively. His total labor supply amounts to $N_t = N_{Ct} + N_{It}$ and he chooses his labor supply and its allocation to the two sectors prior to the realization of the technology shocks Z_{Xt} in period t, $X \in \{C, I\}$. Besides labor income the household receives dividends d_{Xt} per unit of share S_{Xt} he holds of the representative firm in the sector $X \in \{C, I\}$. The current price of shares in units of the consumption good is v_{Xt}. His current period utility function u depends on current and past consumption, C_t and C_{t-1}, and labor N_t. Given his initial stock of shares $\{S_{Ct}, S_{Xt}\}$ the households maximizes his intertemporal utility

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left\{ \frac{(C_{t+s} - \chi C_{t+s-1})^{1-\eta} - 1}{1-\eta} - \nu_0 \frac{N_{t+s}^{1+\nu_1}}{1+\nu_1} \right\}, \quad \beta \in (0,1),\ b,\ \eta \geq 0. \tag{3.5}$$

subject to his budget constraint:

$$v_{Ct}(S_{Ct+1} - S_{Ct}) + v_{It}(S_{It+1} - S_{It}) \leq w_{Ct}N_{Ct} + w_{It}N_{It} + d_{Ct}S_{Ct} + d_{It}S_{It}. \tag{3.6}$$

The first-order conditions are given by

$$\Lambda_t = (C_t - \chi C_{t-1})^{-\eta} - \beta\chi\mathbb{E}_t(C_{t+1} - bC_t)^{-\eta}, \tag{3.7a}$$

$$\mathbb{E}_t\nu_0 N_{t+1}^{\nu_1} = \mathbb{E}_t\Lambda_{t+1}w_{Ct+1}, \tag{3.7b}$$

$$\mathbb{E}_t\nu_0 N_{t+1}^{\nu_1} = \mathbb{E}_t\Lambda_{t+1}w_{It+1}, \tag{3.7c}$$

$$1 = \beta\mathbb{E}_t\frac{\Lambda_{t+1}}{\Lambda_t}\frac{d_{Ct+1} + v_{Ct+1}}{v_{Ct}}, \tag{3.7d}$$

$$1 = \beta\mathbb{E}_t\frac{\Lambda_{t+1}}{\Lambda_t}\frac{d_{It+1} + v_{It+1}}{v_{It}}, \tag{3.7e}$$

where Λ_t is the multiplier of the household's budget constraint. Equations (3.7d) and (3.7e) determine his portfolio allocation.

3.3   Firms

The representative firm in the consumption goods sector maximizes

$$V_{Ct} = \mathbb{E}_t\sum_{s=0}^{\infty}\beta^s\frac{\Lambda_{t+s}}{\Lambda_t}\left[C_{t+s} - w_{Ct+s}N_{Ct+s} - p_{t+s}I_{t+s}\right] \tag{3.8}$$

subject to (3.1a) and

$$K_{Ct+1} = \Phi_I(I_{Ct}/K_{Ct})K_{Ct} + (1-\delta)K_{Ct}, \quad \delta \in (0,1], \tag{3.9}$$

where δ denotes the rate of capital depreciation. We parameterize the capital adjustment cost function Φ_i for $i \in \{C, I\}$ as

$$\Phi_i(I_t/K_t) := \frac{a_{1i}}{1-\zeta_i}\left(\frac{I_t}{K_t}\right)^{1-\zeta_i} + a_{2i}, \quad \zeta_i > 0. \tag{3.10}$$

The first-order conditions for the optimal choice of N_{Ct}, I_{Ct} and K_{Ct+1} are:

$$w_{Ct} = (1-\alpha)Z_{Ct}^{-\rho}\left(\frac{C_t}{N_{Ct}}\right)^{\sigma}, \tag{3.11a}$$

$$q_{Ct} = \frac{p_t}{\Phi_C'(I_{Ct}/K_{Ct})}, \tag{3.11b}$$

$$q_{Ct} = \beta\mathbb{E}_t\frac{\Lambda_{t+1}}{\Lambda_t}\Bigg\{\alpha Z_{Ct+1}^{-\rho}\left(\frac{C_{t+1}}{K_{Ct+1}}\right)^{\sigma} - \frac{p_{t+1}I_{Ct+1}}{K_{Ct+1}} \tag{3.11c}$$

$$+ q_{Ct+1}\left[\Phi_C\left(I_{Ct+1}/K_{Ct+1}\right) + 1 - \delta\right]\Bigg\},$$

where q_{Ct} (Tobin's q) is the Lagrange multiplier on the equation governing capital accumulation. In addition, the transversality condition

$$\lim_{s \to \infty} \mathbb{E}_t \beta^s \Lambda_{t+s} q_{Ct+s} K_{Ct+s+1} = 0$$

must hold. In this case, one can show (see Heer and Maußner (2009), p. 317) that $V_{Ct+1} = q_{Ct} K_{Ct+1}$.

Analogously, the representative firm in the investment goods sector maximizes

$$V_{It} = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \left[p_{t+s} I_{t+s} - w_{It+s} N_{It+s} - p_{t+s} I_{It+s} \right] \tag{3.12}$$

subject to (3.1b) and

$$K_{It+1} = \Phi_I(I_{It}/K_{It}) K_{It} + (1 - \delta_I) K_{It}, \quad \delta \in (0, 1]. \tag{3.13}$$

The respective first-order conditions are:

$$w_{It} = (1 - \alpha) p_t Z_{It}^{-\rho} \left(\frac{I_t}{N_{It}} \right)^{\sigma}, \tag{3.14a}$$

$$q_{It} = \frac{p_t}{\Phi_I'(I_{It}/K_{It})}, \tag{3.14b}$$

$$q_{It} = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left\{ p_{t+1} \alpha Z_{It+1}^{-\rho} \left(\frac{I_{t+1}}{K_{It+1}} \right)^{\sigma} - \frac{p_{t+1} I_{It+1}}{K_{It+1}} \right. \tag{3.14c}$$

$$\left. + q_{It+1} \left[\Phi_I \left(I_{It+1}/K_{It+1} \right) + 1 - \delta \right] \right\},$$

and the transversality condition is

$$\lim_{s \to \infty} \mathbb{E}_t \beta^s \Lambda_{t+s} q_{It+s} K_{It+s+1} = 0.$$

Firms from both sectors $X \in \{C, I\}$ transfer their profits less retained earnings as dividends to the household sector and finance the remaining part of their investment expenditures from issuing new equity:

$$d_{Xt} S_{Xt} = Y_{Xt} - w_{Xt} N_{Xt} - RE_{Xt}, \quad Y_{Ct} = C_t, \ Y_{It} = p_t I_t$$

$$v_{Xt}(S_{Xt+1} - S_{Xt}) = p_t I_{Xt} - RE_{Xt}.$$

Thus, in equilibrium, the budget constraint of the household implies the definition of GDP given in equation (3.2).

3.4 Stationary Equilibrium and Calibration

The equations characterizing the stationary equilibrium are summarized in the Appendix. In order to calibrate the model, we distinguish three sets of parameters. For the first set, $\{\alpha, \delta, \nu_1, \sigma\}$, we use direct observations from the US economy. For the second set, $\{\beta, \nu_0\}$, we use the equilibrium conditions characterizing the steady state. The final set of parameters, $\{\eta, \chi, \rho_Z, \rho_I, \sigma_Z, \sigma_I, \zeta_C, \zeta_I\}$, is chosen to optimize the match of the model statistics with the following empirical observations presented in Section 2 of the paper.

1. A quarterly equity premium of the returns equal to 0.74% and 0.96% in the sectors $X \in \{C, I\}$, respectively.

2. A Sharpe ratio equal to 0.075 and 0.0972 in the sectors $X \in \{C, I\}$, respectively.

3. A correlation of the returns in the two sectors with quarterly output growth equal to 0 in both sectors.

We use the parameter values specified in Table 3.1 to simulate the model. We follow Heer and Schubert (2012) and set $\sigma = 0.75$. As BCF, we employ $\delta_C = \delta_I = 0.021$. We determine α so that the steady state capital share in output equals 0.36, the value employed by BCF. Our value of $\nu_1 = 3.33$ implies a Frisch elasticity of labor supply equal to 0.30. In addition, we specify ν_0 so that $N = 0.33$. Our choice of $\beta = 0.994$ implies an annual risk-free rate of 2.5% while the average real annual return of US-treasury bills found in our data set is 3.4%.

Table 3.1

Benchmark calibration

Preferences	β=0.994	χ=0.75	η=1.0	N=0.33
	ν_1=3.33			
Production	$\alpha(Y/K)^{-\rho}$=0.36	δ=0.021	ζ_C=7.8	ζ_I=9.6
	ρ_C=0.92	ρ_I=0.76	σ_C=0.0072	σ_I=0.0288

The values of the remaining parameters in Table 3.1 minimize the sum of squared relative deviations between our six targets and their model implied equivalents.[6] Although

---

[6]In the case of the correlation coefficients we compute the absolute deviations.

we have more free parameters than targets the non-linearity of the model does not allow for a perfect match. We searched over a coarse grid with the following boundaries:

$$\eta \in [1, 5],$$

$$\chi \in [0.1, 0.9]$$

$$\zeta_X \in [0.5, 9.8]$$

$$\rho^X \in [0.5, 0.95]$$

$$\sigma^I \in [0.5\sigma^Z, 4\sigma^Z].$$

The minimizer yields a score of 0.1463.

Two further comments are in order. First, the search routine implies log-utility with respect to consumption $\eta = 1$, an assumption frequently used in the related literature. Second, the model requires that total factor productivity is four times more volatile in the investment than in the consumption goods sector. While the equity premia and the Sharpe ratios are not too sensitive with respect to this choice, the model is unable to replicate the zero correlations between the equity returns and output growth for smaller values of σ^X. The intuition behind this result is that a shock in the consumption goods sector depresses the equity return in both sectors while a shock in the investment good sector boost the equity return in both sectors. Since the investment sector is much smaller than the consumption goods sector, larger shocks to the latter sector are needed to offset an otherwise negative correlation between output growth and equity returns.[7]

3.5 Computation of the Equity Premium

Heer and Maußner (2009) demonstrate that the real one-period gross rate of return $R_{Xt+1} = (v_{Xt+1} + d_{Xt+1})/v_{Xt}$ with $X \in \{C, I\}$ is independent of the firm's dividend policy and equals

$$R_{Ct+1} = \frac{C_{t+1} - w_{Ct+1}N_{Ct+1} - p_{t+1}I_{Ct+1} + q_{Ct+1}K_{Ct+2}}{q_{Ct}K_{Ct+1}}, \tag{3.15a}$$

$$R_{It+1} = \frac{p_{t+1}I_{t+1} - w_{It+1}N_{It+1} - p_{t+1}I_{It+1} + q_{It+1}K_{It+2}}{q_{It}K_{It+1}} \tag{3.15b}$$

---

[7]We give a more detailed explanation on the impulse responses in the Appendix.

in the consumption goods and the investment goods sector, respectively.

The risk free rate of return is given by

$$r_t = \frac{\Lambda_t}{\beta \mathbb{E}_t \Lambda_{t+1}}.$$ (3.16)

We solve and simulate the model in two steps, following Kliem and Uhlig (2011). In the first step, we obtain second-order approximate solutions of the agent's decision rules at the deterministic stationary solution. We then simulate the model for many time periods. We employ 1,000,000 periods to ensure that the model settles on the stationary distribution. In the second step, we use the average values from this simulation for a new second-order perturbation solution and simulate the model again for 1,000,000 periods. We split the entire period into subperiods of length 128 (the number of observations in our data) and compute our targets for each subperiod. Table 4.1 reports the averages over the subperiods and the associated standard deviations.[8]

4   Results

Table 4.1 presents the empirical values of the statistics and those implied by the model.[9] Obviously, the model is able to fit the empirical facts closely. The empirical equity premium in the consumption sector is 8 basis points smaller than those implied by the model. In the investment sector the model predicts an equity premium which is 7 basis points above the premium observed in the data. The Sharpe ratios generated by the model are somewhat smaller than their empirical counterparts. In addition, the correlation of the equity returns in both sectors with quarterly output growth is not significantly different from zero.

5   Conclusion

Our research is motivated by previous findings of Greenwood, Hercowitz, and Krusell (1997, 2000), Fisher (2003), and Marquis and Trehan (2005) that the productivities

---

[8]To compute the risk-free rate from (3.16), we have to compute the expectation value in (3.16) over a two-dimensional grid using Gauss-Hermite quadrature.

[9]In the Appendix, we present impulse response functions for the two technology shocks and second moments from the simulation of the model for various variables of the model such as the output Y_t, consumption C_t, investment I_t, the relative price p_t, and average wages W_t and hours N_t.

Table 4.1

Quarterly Asset Return Statistics

Statistic	US economy	Model	
		Mean	Std.dev.
Consumption Sector			
Equity Premium	0.740%	0.828%	0.811%
Sharpe Ratio	0.075	0.062	0.061
Correlation with output growth	−0.003	0.181	0.097
Investment Sector			
Equity Premium	0.960%	1.032%	0.874%
Sharpe Ratio	0.097	0.076	0.065
Correlation with output growth	0.011	0.138	0.098

Notes: Empirical values are taken from Tables 2.1 and 2.2. Standard deviations are from the simulation of the model.

in the consumption goods and investment goods producing sectors behave differently both in the short and in the long run. In order to derive their results these authors use data on the relative price of investment goods to distinguish between technology shocks to the consumption and investment goods sector. Their results give support to the view that to distinguish between a consumption goods and a capital goods sector is justified and warranted by the different behavior of these sectors. As a consequence, asset prices in these two sectors are likely to behave differently.

In the present paper, we have presented empirical evidence for the return to equity in the consumption goods and capital goods sector. While close to each other, both the equity premium and the Sharpe value are moderately higher in the investment goods sector. The returns in both sectors are uncorrelated with output growth. We have presented a two-sector model that replicates the zero correlations, slightly overestimates the equity premia, and comes close to the Sharpe ratios. Our model is based upon Boldrin, Christiano and Fisher (2001) and extended for sector-specific adjustment costs in capital and sector-specific TFP shocks. On its balanced growth path the model reduces to a standard one-sector growth model. Fluctuations around this path are driven by sector-specific shocks and propagated by frictions in the allocation of labor and capital. We are thus able to demonstrate that these slight departures from

the benchmark business cycle model are consistent with the small and insignificant differences in sectoral asset price statistics.

References

Albuquerque, R., M.S. Eichenbaum, and S. Rebelo, 2012, Valuation Risk and Asset Pricing, *National Bureau of Economics Research (NBER) Working Paper*, No. 18617.

Blanchard, O.J., and J. Galí, 2005, Real Wage Rigidities and the New Keynesian Model, *Centre for Economic Policy Research (CEPR) Discussion Paper*, No. 5375.

Boldrin, M., L.J. Christiano, and J.D.M. Fisher, 2001, Habit Persistence, Asset Return, and the Business Cycle, *American Economic Review*, vol. 91, 149-166.

Covas, F. and W. Den Haan, 2007, The Role of Debt and Equity Finance over the Business Cycle, *CEPR Discussion Papers* No. 6145.

Fisher, J.D.M., 2003, Technology Shocks Matter, Manuscript. Federal Reserve Bank of Chicago, December 2003.

Gomme, P., B. Ravikumar, and P. Rupert, 2011, The Return to Capital and the Business Cycle, *Review of Economic Dynamics*, vol. 14(2), 262-278.

Greenwood, J., Z. Hercowitz, and P. Krusell, 1997, Long-Run Implications of Investment-Specific Technological Change, *American Economic Review*, vol. 87, 342-362.

Greenwood, J., Z. Hercowitz, and P. Krusell, 2000, The Role of Investment-Specific Technological Change in the Business Cycle, *European Economic Review*, vol. 44, 91-115.

Heer, B., and A. Maußner, 2009, *Dynamic General Equilibrium Models: Computation and Applications*, 2nd edition, Heidelberg: Springer.

Heer, B., and A. Maußner, 2013, Asset Returns, the Business Cycle, and the Labor Market, German Economic Review, vol. 14(3), 2013, 372-397.

Heer, B. and S.F. Schubert, 2012, Unemployment and Debt Dynamics in a Highly Indebted Small Open Economy, *Journal of International Money and Finance*, vol. 31(6), 1392-1413

Ireland, P., and S. Schuh, 2008, Productivity and U.S. Macroeconomic Performance: Interpreting the Past and Predicting the Future with a Two-Sector Real Business Cycle Model, *Review of Economic Dynamics*, vol. 11(3), 473-492.

Jagannathan, R., E.R. McGrattan, and A. Scherbina, 2000, The declining U.S. equity premium, *Quarterly Review, Federal Reserve Bank of Minneapolis*, Fall, 3-19.

Jermann, U.J., 1998, Asset Pricing in Production Economies, *Journal of Monetary Economics*, vol. 41, 257-275.

Kliem, M. and H. Uhlig, 2011, Bayesian Estimation of a DSGE Model with Asset Prices, mimeo

Kocherlakota, N.R., 1996, The Equity Premium: It's Still a Puzzle, *Journal of Economic Literature*, vol. XXXIV, 42-71.

Marquis, M., and B. Trehan, 2005, On Using Relative Prices toMeasure Capital-Specific Technological Progress, *Federal Reserve Bank of San Francisco Working Paper*, No. 2005-02, March 2005.

Mehra, R., and E.C. Prescott, 1985, The Equity Premium: A Puzzle, *Journal of Monetary Economics*, vol. 15. 145-161.

Uhlig, H., 2007, Explaining Asset Prices with External Habits and Wage Rigidities in a DSGE Model, *American Economic Review, Papers & Proceedings*, vol. 97, 239-243.

Appendix

A.1 Stationary equilibrium in the model of Section 3

In the following, we summarize the equilibrium conditions for the model of Section 3 consisting of 18 equations.[10]

$$w_{Ct} = (1 - \alpha)Z_{Ct}^{-\rho}\left(\frac{C_t}{N_{Ct}}\right)^{\sigma}, \tag{A.1.1a}$$

$$w_{It} = p_t(1 - \alpha)Z_{It}\left(\frac{I_t}{N_{It}}\right)^{\sigma}, , \tag{A.1.1b}$$

$$q_{Ct} = \frac{p_t}{\Phi'(I_{Ct}/K_{Ct})}, \tag{A.1.1c}$$

$$q_{It} = \frac{p_t}{\Phi'(I_{It}/K_{It})}, \tag{A.1.1d}$$

$$N_t = N_{Ct} + N_{It}, \tag{A.1.1e}$$

$$K_t = K_{Ct} + K_{It}, \tag{A.1.1f}$$

$$w_t = \frac{N_{Ct}}{N_t}w_{Ct} + \frac{N_{It}}{N_t}w_{It}, \tag{A.1.1g}$$

$$C_t = Z_{Ct}\left[(1 - \alpha)N_{Ct}^{\rho} + \alpha K_{Ct}^{\rho}\right]^{\frac{1}{\rho}}, \tag{A.1.1h}$$

$$I_t = Z_{It}\left[(1 - \alpha)N_{It}^{\rho} + \alpha K_{It}^{\rho}\right]^{\frac{1}{\rho}}, \tag{A.1.1i}$$

$$Y_t = C_t + I_t, \tag{A.1.1j}$$

$$I_t = I_{Ct} + I_{It}, \tag{A.1.1k}$$

$$\Lambda_t = (C_t - \chi C_{t-1})^{-\eta} - \beta\chi\mathbb{E}_t(C_{t+1} - bC_t)^{-\eta}, \tag{A.1.1l}$$

$$\mathbb{E}_t\nu_0 N_{t+1}^{\nu_1} = \mathbb{E}_t\Lambda_{t+1}w_{Ct+1}, \tag{A.1.1m}$$

$$\mathbb{E}_t\nu_0 N_{t+1}^{\nu_1} = \mathbb{E}_t\Lambda_{t+1}w_{It+1}, \tag{A.1.1n}$$

$$q_{Ct} = \beta\mathbb{E}_t\frac{\Lambda_{t+1}}{\Lambda_t}\left\{\alpha Z_{Ct+1}^{-\rho}\left(\frac{C_{t+1}}{K_{Ct+1}}\right)^{\sigma} - \frac{p_{t+1}I_{Ct+1}}{K_{Ct+1}}\right.$$
$$\left. + q_{Ct+1}\left[(\Phi\left(I_{Ct+1}/K_{Ct+1}\right) + 1 - \delta\right]\right\}, \tag{A.1.1o}$$

$$q_{It} = \beta\mathbb{E}_t\frac{\Lambda_{t+1}}{\Lambda_t}\left\{p_{t+1}\alpha Z_{It+1}^{-\rho}\left(\frac{I_{t+1}}{K_{It+1}}\right)^{\sigma} - \frac{p_{t+1}I_{It+1}}{K_{It+1}}\right.$$
$$\left. + q_{It+1}\left[(\Phi\left(I_{It+1}/K_{It+1}\right) + 1 - \delta\right]\right\}, \tag{A.1.1p}$$

$$K_{Ct+1} = \Phi(I_{Ct}/K_{Ct})K_{Ct} + (1 - \delta)K_{Ct}, \tag{A.1.1q}$$

$$K_{It+1} = \Phi(I_{It}/K_{It})K_{It} + (1 - \delta)K_{It}. \tag{A.1.1r}$$

---

[10]Note that our measure of aggregate output is $Y_t = C_t + I_t$ instead of $Y_t = C_t + p_t I_t$. We thus follow the practice in the National Income and Product Accounts where real output is defined as output at constant relative prices. We employ $p = 1$ as the base price.

In order to calibrate the model, we need to compute the steady state. The steady state is free of adjustment costs, $\Phi'(I_X/K_X) = \Phi'(I_C/K_C) = \Phi'(\delta) = 1$ and $\Phi(\delta) = \delta$, and features $Z_C = Z_X = 1$. Therefore, $q = p$ (from (A.1.1c) and (A.1.1d)) and the Euler equations (A.1.1o) and (A.1.1p) reduce to

$$p = \beta(\alpha(C/K_C)^\sigma + p(1 - \delta)),$$
$$p = \beta(\alpha(I/K_I)^\sigma + p(1 - \delta))$$

implying $C/K_C = I/K_I$ and, thus, $K_C/N_C = K_I/N_I$ via (A.1.1h) and (A.1.1i). Equations (A.1.1m) and (A.1.1n) imply equal wages in the steady state so that from equations (A.1.1a) and (A.1.1b)

$$(1 - \alpha)(C/N_C)^\sigma = (1 - \alpha)p(I/N_I)^\sigma.$$

Since both, C/N_C and I/N_I are functions of $k = K_C/N_C = K_I/N_I$, the previous equations will only hold if $p = 1$.

The capital-labor ratio $k = K/N$ can now be inferred from

$$\frac{C}{K_C} = \frac{I}{K_I} = \left(\frac{1 - \beta(1 - \delta)}{\alpha\beta}\right)^\sigma$$

yielding

$$k = \left[\frac{(C/K_C)^\rho - \alpha}{1 - \alpha}\right]^{\frac{-1}{\rho}}. \tag{A.1.2}$$

For the US economy, we set $N = 0.33$ so that, given the values of the parameters $\{\alpha, \beta, \delta\}$, we can infer the stationary value of the capital stock from (A.1.2). From $I = \delta K$ and $I = N_I(K/N)^\alpha$ we get the stationary value of N_I. Analogously, $I = K_I(K/N)^{\alpha-1}$ implies K_I. Given N_I and K_I we are able to compute $N_C = N - N_I$ and $K_C = K - K_I$, which allows us to find the stationary vale of C from the production function for consumption goods (3.1a). In the last step, we compute Λ from the stationary version of (3.7a).


A.2   Business-cycle properties of the model in section 3

In Figures A.1 and A.2, we present the impulse responses of the model variables to a technology shock of one standard deviation in the consumption and investment good sector, respectively.

A TFP shock in the consumption goods sector shifts the production function outward so that both the marginal product of labor and the marginal product of capital increase. On impact, the supply of factors is predetermined so that aggregate investment cannot adjust. This explains the sharp spikes in the relative price of investment goods, the prices of installed capital, and the real wage paid to workers in the consumption sector. Since the shock is highly correlated, workers move to the consumption goods sector and

investment goods will remain expensive. As a consequence, installed capital remains to be a scarce factor. The more expensive investment goods dominate the outward shift of the production function on the cash flow in the consumption goods sector (see the right panel in the third row of Figure A.1). In the investment goods sector the higher selling price increases the cash flow. Yet this effect is more than outweighed by the higher price of installed capital (Tobin's q). As a result, the expected return on equity deviates from its stationary value at about the same size in both sectors (see the lower right panel in Figure A.1).

Figure A.1: Impulse Responses to a Technology Shock in the Consumption Goods Sector
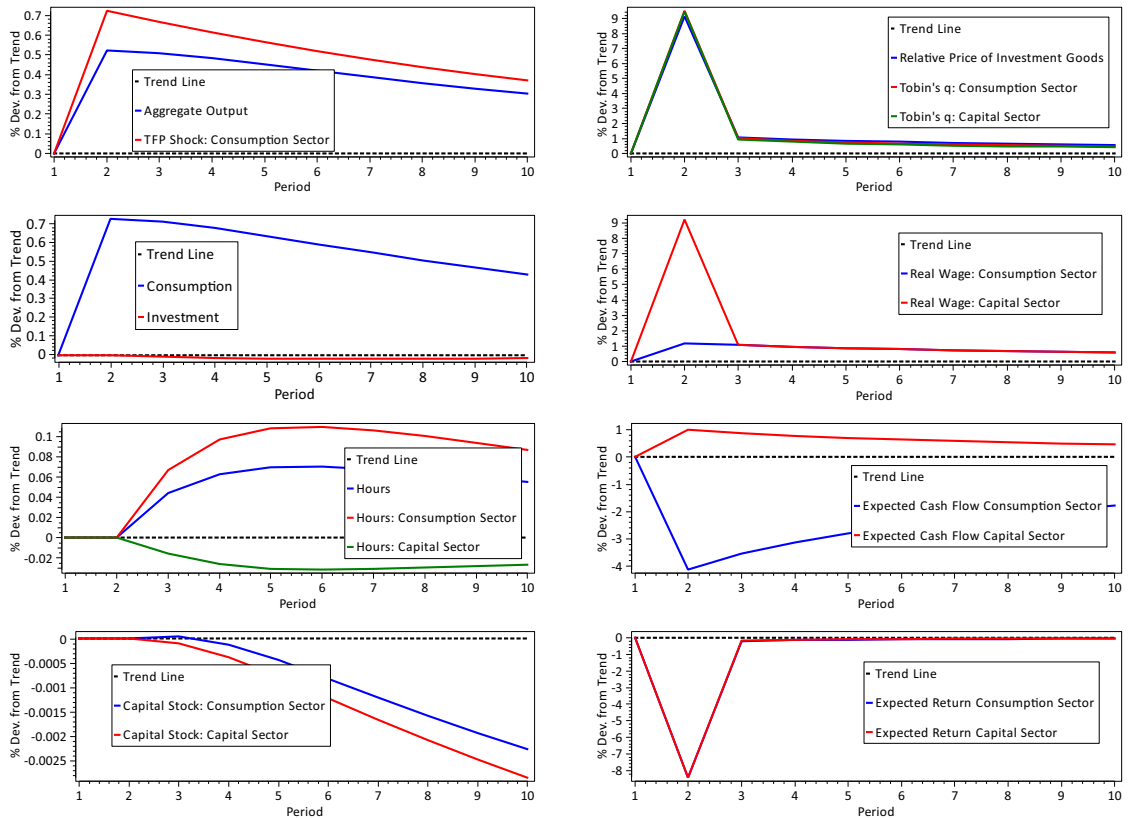
A TFP Shock in the investment goods has opposite effects. The magnitude of these effects is larger since the shock is more pronounced than the corresponding shock in the consumption goods sector.[11] The increased production of investment goods causes a sharp decrease in the relative price of these goods and an associated drop in the price of installed capital. However, the size of the latter effect is much smaller than the size of the former so that investment and the stock of capital increase in the periods thereafter. While the expansion in the wake of a shock in the consumption goods sector is driven by an increase in hours, the expansion following the shock in the investment goods sector is driven by an increase of capital. Aggregate hours even decline since the

---

[11]In our baseline calibration presented in Table 3.1, $\sigma_I = 4\sigma_C$.

lower price of investment goods more than offsets the outward shift of the marginal product of labor so that aggregate real wages fall. The lower price of investment goods increases the expected cash flow in the consumption goods sector and decreases the cash flow in the investment goods sector. The drop in Tobin's q further increases the expected equity return in the consumption goods sector and more than outweighs the negative effect on the cash flow in the investment goods sector so that in this sector, too, the return on equity increases. Again, the percentage deviation from the stationary return has about the same size in both sectors. The opposite effects of TFP shocks in the two sectors on the return on equity explains the zero correlation between output growth and the equity returns. Since the investment sector is much smaller than the consumption sector, it requires relatively larger shocks to the former to deliver this result.

Figure A.2: Impulse Responses to a Technology Shock in the Investment Goods Sector
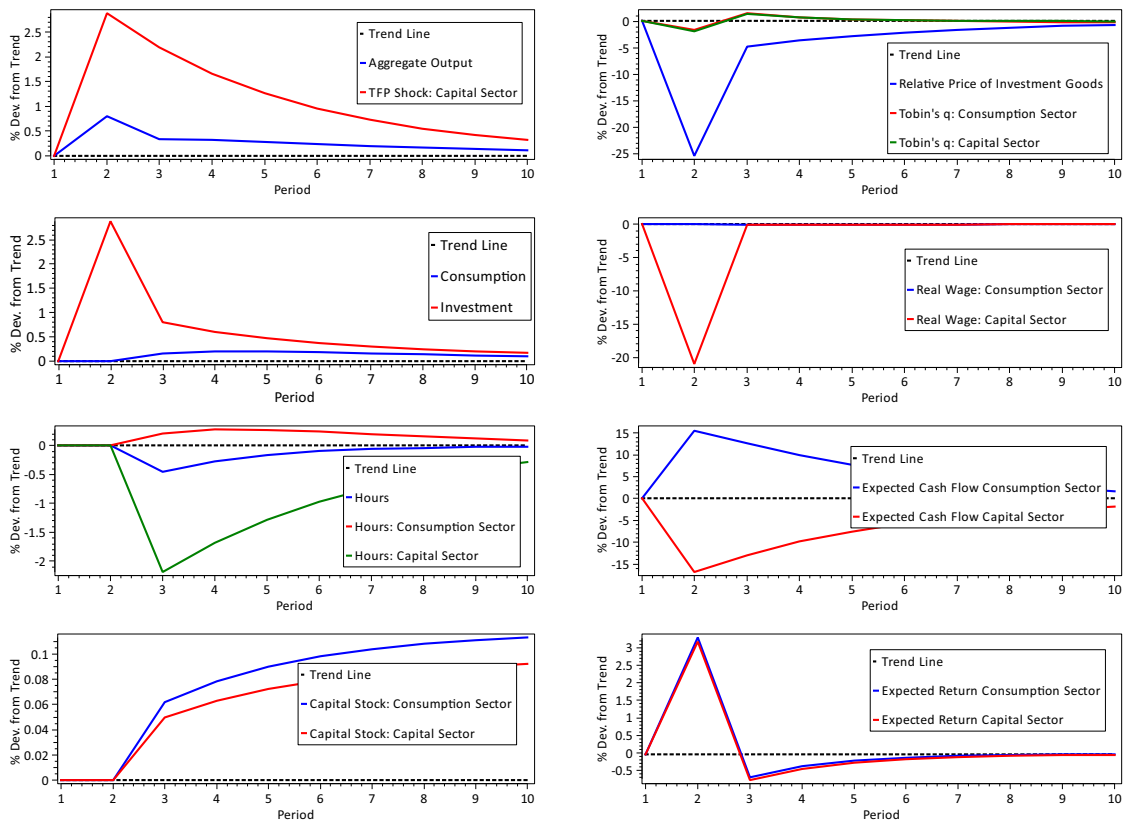

Table A.1 presents second moments from simulations of the model. They are averages from 500 simulations with 128 periods each. Note the following observations for the model with respect to its business-cycle moments:

1) The model predicts that consumption and output have about the same standard deviation, which is well in line with empirical observation.

Table A.1
Second Moments

Variable	s_x	s_x/s_y	r_{xy}	r_{xn}	r_x
Output	1.04	1.00	1.00	−0.16	0.46
Consumption	0.99	0.95	0.68	−0.09	0.71
Investment	2.71	2.61	0.73	−0.15	0.14
Total Hours	0.49	0.47	−0.16	1.00	0.44
Total Capital	0.10	0.10	−0.02	−0.36	0.84
Real Wage	6.32	6.09	−0.31	−0.12	−0.03
Relative Price	4.19	23.30	−0.55	0.04	0.04

Notes: s_x:=Standard deviation of HP-filtered simulated time series x, s_x/s_y:=standard deviation of variable x relative to standard deviation of output y, r_{xy}:=Cross-correlation of variable x with output y, r_{xn}:=Cross-correlation of variable x with hours N , r_x:=First order autocorrelation of variable x.

2) The model is also in line with the fact that aggregate investment is about 2-3 times more volatile than output.

3) The model predicts a weak negative but insignificant correlation between output and hours.[12]

4) The negative correlation between the average real wage and average working hours, predicted by the BCF model, however, almost vanishes and becomes insignificant.[13]

---

[12]95% of the simulated correlations are within the interval [-0.39; 0.10]. On the correlation between hours and output see also Uhlig (2007) who tries to match the empirical equity premium and the Sharpe ratio together with statistics from the labor market in a standard one-sector RBC model with wage rigidities. A further study on the joint behavior of asset price and labor market statistics in production economies is Heer and Maußner (2013).

[13]95% of the simulated correlations are within the interval [-0.28; 0.06].