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Private Provision of Public Goods and Information Diffusion in Social Groups

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Abstract

We describe a dynamic model of costly information sharing, where private information affecting collective-value actions is transmitted by social proximity. Individuals make voluntary contributions towards the provision of a pure public good, and information transmission about quality of provision is a necessary condition for collective provision to take place in a stationary equilibrium. We show that, unlike in the case of private goods, better informed individuals face positive incentives to incur a cost to share information with their neighbours; and that these incentives are stronger, and provision of the pure public good greater, the smaller are individuals' social neighbourhoods.

JEL-Code: H100, L300, D600, D700.

Keywords: private provision of public goods, information transmission, social learning.

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1 Introduction

There is evidence that social interactions matter for voluntary contributions towards collective goods (Brown and Ferris, 2007). Casual empiricism suggests the same – being targeted by co-workers and acquaintances for fundraising towards various charitable causes is a commonplace experience. Sociological research has long stressed the role of social links in motivating individual behaviour (e.g. Wasserman and Faust, 1994), but has paid little attention to the role of social connections in voluntary giving – a notable exception being Galaskiewicz (1985). Although much recent work in economics has focused on the role of interpersonal links in motivating economic behaviour in general, and voluntary giving in particular, to the best of our knowledge there has been no investigation of how social interactions affect giving towards third parties, and specifically of the question of whether more social interaction leads to more voluntary giving.

This paper tries to fill this gap, formalizing the relationship between voluntary social interactions, information sharing and contribution equilibria. It describes a social proximity-based mechanism of information transmission in groups of individuals who consume a pure public good. In the mechanism we study, information about quality for alternative modes of provision of a public good can spread from one individual to the next just as it does for private goods. However, unlike in the case of private goods, better informed individuals face positive incentives to incur private costs in order to transmit information to their less informed neighbours, because this can bring about an increase in collective provision, the benefits of which they partake in. In this setting, the sharing of information has the characteristics of a local public good that is confined within individual social neighbourhoods, even when voluntary contributions fund the provision of a pure public good that spans all neighbourhoods. Thus, incentives to engage in costly information sharing are stronger when social neighbourhoods are smaller; consequently, large societies composed of comparatively small social neighbourhoods can sustain comparatively higher levels of private provision of collective goods.

These conclusions are in line with the view that individuals are more "engaged" in collective choices in smaller communities than they are in larger communities;[1] but our analysis delivers a new theoretical argument for why the same conclusions may extend to collective choices that are not local in nature.

Understanding how and why social connections can shape voluntary giving also has

[1]For example, a 2010 report on volunteering and charitable giving by the UK Department for Communities and Local Government concludes that rural dwellers are significantly more likely to engage in volunteering than are urban dwellers.

implications for understanding how government policies affect private giving. As many developed countries are increasing their reliance on the private sector to meet collective needs, we see a shift in the use of public resources from the funding of public provision to the subsidization of private provision. Our findings suggest that, in designing such subsidies, policymakers may be able to leverage on the relationship between private giving and social structure to maximize their impact.

Our paper is related to three main strands of literature: the literature on private contributions towards collective consumption (Malinvaud, 1972; Bergstrom, Blume and Varian, 1986; Andreoni, 1990; and subsequent contributions); the literature on social learning (Banerjee, 1992; Ellison and Fudenberg, 1995; Banerjee and Fudenberg, 2001; and subsequent contributions); and the literature on strategic information transmission (originating with Crawford and Sobel, 1982). Two recent papers that are somewhat related to ours are Dutta and Chatterjee (2011), and Bramoullé and Kranton (2007). The first paper looks at costless information transmission across consumers for the case of private goods; as we have already noted, the public good case is fundamentally different from the private good case – where no costly transmission of information can occur. The second paper focuses on the provision of public goods in networks, with the structure of network links determining the scope of local public goods, and it fully abstracts from voluntary transmission of information. Our line of questioning is quite different: we specifically study voluntary information sharing in a dynamic environment with repeated signaling, and focus on a scenario where the collective good provided is a pure public good – i.e. a public good whose scope is independent of social links – and where social links are only relevant for information transmission and social learning.

The paper is organized as follows. Section 2 develops a dynamic model of private giving and costly information sharing in social groups. Section 3 derives results concerning the relationship between neighbourhood size, information, and information sharing. Section 4 discusses an alternative but equivalent specification, where private giving is uniquely motivated by private "warm-glow" effects. It also presents a number of other extensions: preference heterogeneity with respect to collective consumption, strictly convex preferences. Section 5 derives implications for policy design. Section 6 summarizes and concludes. An extension to the case of forward-looking agents is presented in the Appendix.

2 A model of private giving and information sharing in social neighbourhoods

Theoretical literature on voluntary provision of public goods has highlighted two main types of motives for private giving: consumption motives and outwardly orientated motives (e.g. "warm glow" or status signaling), with the latter typically being invoked whenever the former is unable to account for giving in large groups. In this paper we describe an information-transmission mechanism of contagion in private giving that can be related to both private consumption motives and warm-glow motives.

We examine information-related incentives on both sides of a given social link between two socially connected individuals, and then derive implications for the diffusion of information and giving behaviour in a dynamic (indefinitely-repeated) setting.

Our analysis deliberately abstracts (by way of suitable assumptions) from the topological structure of social links,[2] and builds on assumptions that lead to stationary outcomes, making it possible to characterize the "long-run" relationship between the density of social interactions, information transmission and collective provision outcomes. The model is highly stylized and abstracts from many other dimensions of heterogeneity that are relevant to real-world voluntary giving in order to concentrate on how the density of social interactions affects the sharing of provision-relevant information that is not publicly observable.

Preferences, technologies and information structure

There is an economy with N individuals, $i \in I \equiv \{1, \ldots, N\}$, and a countable number of periods indexed by $t = 1, 2, \ldots$. Each individual consumes both a private good and a pure public good in each period, in amounts that are respectively denoted by $x_{i,t}$ and G_t. Provision of the public good in period t is funded by private contributions, $v_{i,t}$, made in each period by each individual $i \in I$, out of her exogenously given period-t income, $y_{i,t}$.

The public good is produced at a marginal cost of unity by M private non-profit providers (charities), $j \in J \equiv \{1, \ldots, M\}$, and sold at marginal cost. Individuals can make contributions to a single provider in each period. Providers differ from each other only with respect to the quality of their provision, $q_{j,t} \in \{\overline{q}, \underline{q}\}$, with $\overline{q} > \underline{q}$, which measures units of quality-adjusted provision for each dollar's worth of provision through provider

[2] These considerations are likely to be important in reality; but the mechanism we highlight would also underlie information diffusion in a social network with a stable, non-regular topological structure (see the discussion on this point in Section 4.5).

j.[3] Thus, given each individual i's contribution, $v_{i,t}$, to her selected provider at t, $j(i,t)$, and given period-t quality realizations, $q_{j,t}$, the effective level of collective consumption experienced by all individuals at t equals $G_t = \sum_i q_{j(i,t),t} v_{i,t}$. Without loss of generality, we assume $\overline{q} = 1$ and $\underline{q} = 0$.

The quality of provision of provider j at time t is *ex-ante* unobservable but is observable *ex post* to individuals who have made positive contributions to provider j at t.[4] Quality of provision for provider j evolves stochastically over time, according to the following conditional distribution: $\Pr\left(q_{j,t} = \underline{q} \mid q_{j,t-1} = \underline{q}\right) = \Pr\left(q_{j,t} = \overline{q} \mid q_{j,t-1} = \overline{q}\right) = \rho > 1/2$; i.e. quality remains the same from one period to the next with probability $\rho > 1/2$ and changes with probability $1 - \rho$. Any given individual can only identify provider j if she has contributed to it at $t - 1$. This means that the only portion of i's history that determines her information set at t is the pair $\left(j(i,t-1), q_{j(i,t-1),t-1}\right)$, where $j(i,t)$ denotes the provider selected by i in period t. As we are concerned here with transmission of private information on provision quality, we can abstract from other determinants of quality that are non-stochastic or public information.[5]

We assume that quality evolves independently across providers. Thus, with M large, and given our assumption of symmetric transition probabilities, the proportion of quality types in the population will converge through time to $1/2$ for each type.

All individuals have identical, risk-neutral preferences for private and collective consumption within each period. These are represented by

$$U(x_{i,t}, G_t, e_{i,t}) = x_{i,t} + \theta G_t - e_{i,t}, \tag{1}$$

where $e_{i,t}$ is private effort directed towards information sharing (see below). Individuals have disposable income $y_{i,t} = y = 1 + \mu$ ($\mu > 0$) in all periods. Consumption is bounded below to unity and therefore contributions must lie between zero and μ. We assume $2 > \theta > 1/\rho$ (the role of this assumption is explained below). In this specification, individuals only care about their own contributions towards G_t and those of others because of a consumption motive; in Section 4, we discuss how our analysis and results

[3] Heterogeneity with respect to quality could equivalently be modelled in terms of unobservable marginal costs.

[4] Individuals that direct contributions towards a certain provider may not able to perfectly observe quality, but they would still receive more information about quality than individuals who do not make contributions towards that provider. Thus, our assumption represents an extreme case that is presentationally convenient.

[5] There may be persistent heterogeneity across providers, which would ultimately become public information; but there are also new varieties being introduced and new providers of unknown quality, which implies that information about providers depreciates and new learning occurs.

carry over to a setting where individuals are motivated by warm-glow effects.

Within each period, t, nature assigns each individual $i \in I$ a social neighbourhood, $S_{i,t}$, consisting of b individuals (excluding the individual herself), with $1 \leq b < N$. Neighbourhoods can be overlapping, but the structure of social neighbourhoods is such that each individual has exactly b neighbours. The b individuals that form individual i's neighbourhood at t are newly sampled at random from the population of N individuals in each period. The configuration of social links in each period can therefore be represented as a regular graph of degree b, randomly sampled in each period from the set of all possible regular graphs of degree b – a specification that can be thought of as a generalization of repeated N-person random matching or repeated random sampling (in turn, random sampling is commonly featured in analyses of social learning; e.g. Ellison and Fudenberg, 1992).

In each period t, individuals can each choose to inform their neighbours, $i' \in S_{i,t}$, of the quality they have experienced from provider $j(i, t-1)$, sending each a signal $\sigma_{i,i',t} = (j, q)$, with $j = j(i, t-1)$. If they do so, they incur a cost c – translating into a higher $e_{i,j}$ in (1) – for each neighbour they inform. A signal is truthful if the quality level reported corresponds to the quality actually experienced by i, i.e. the signal reports $q = q_{j(i,t-1),t-1}$.

Individuals are indefinitely-lived and myopic. This means that choices with respect to sending costly signals in any given period only account for how those signals may affect provision in that period (an extension to forward-looking signaling choices is presented in the Appendix).

Sequence of actions and events

The sequence of actions and information sets in each time period t are as follows: (i) at the beginning of each time period t, nature generates social neighbourhoods, $S_{i,t}$, and updates providers' quality types, $q_{j,t}$; (ii) each individual $i \in I$ for whom $v_{i,t-1} > 0$ is fully informed about $q_{j(i,t-1),t-1}$, and can choose to send signals $\sigma_{i,i',t}$, $i' \in S_{i,t}$, simultaneously with the signaling choices of other individuals; (iii) each individual, i, receives signals $\sigma_{i',i,t}$ from her neighbours, $i' \in S_{i,t}$, and updates her information set; (iv) each individual $i \in I$ selects a provider, $j(i, t)$; (v) once they have selected a provider, individuals then simultaneously make contributions, $v_{i,t}$, to providers $j(i, t)$; (vi) individuals who have made positive contributions ($v_{i,t} > 0$) observe provision qualities, $q_{j(i,t),t}$, and everyone experiences collective consumption G_t.[6]

[6]This means that average quality is observable by everyone, but only individuals that have made contributions can link quality of provision of their chosen supplier to its identity.

We will discuss within-period actions starting from contribution choices (v), and then move backwards to provider selection choices (iv), and signaling choices (ii).

Contribution choices

Suppose individual i has selected provider $j(i,t) = j'$, and let $E_i[q_{j',t}] \equiv \tilde{q}$ be i's expected level of quality from provider j' in period t given i's information. Expected quality can take one of three values, depending on the information i has. If j' is a provider from whom i has experienced high quality at $t-1$, or if it is a provider that one of i's neighbours has signaled as being of high quality at $t-1$, then $\tilde{q} = \rho > 1/2$; if it is a randomly selected provider, then, for t large enough (implying that the proportions of suppliers of each quality type are the same types for both types), expected quality is $\tilde{q} = 1/2$; if it is a provider from whom i has experienced low quality at $t-1$, expected quality is $\tilde{q} = 1 - \rho < 1/2$.

Constrained utility maximization in period t then gives $v_{i,t} = 0$ or $v_{i,t} = \mu$ depending on whether $\tilde{q}$ is less than or greater than $1/\theta$. Given our earlier assumption that $2 > \theta > 1/\rho$ (or $\rho > 1/\theta > 1/2$), contributions are $v_{i,t} = \mu$ for $\tilde{q} = \rho$ and $v_{i,t} = 0$ otherwise, i.e. individuals only make positive contributions if the expected quality of provision from their selected provider exceeds average quality, $1/2$.

Selection of providers

Consider first individuals who have experienced high quality from their chosen provider at $t-1$, $j(i,t-1)$. If they receive no additional information, then, given that $\rho > 1/2$, they would elect to go back to the same provider – since doing so yields a higher expected quality than selecting a new supplier at random. If they are tipped off by a neighbour concerning a supplier that has delivered high quality at $t-1$, and they believe the information to be truthful, they are indifferent between switching to this alternative supplier and sticking to $j(i,t-1)$.

Consider next individuals who have experienced low quality from their chosen provider at $t-1$. If they receive no additional information, given that $1-\rho < 1/2$, they would choose a new supplier – since doing so yields a higher expected quality than going back to the same provider. If they are tipped off by a neighbour about a supplier that has delivered high quality at $t-1$, i.e. if they receive a signal, $\sigma_{i',i,t} = (j', q')$ from one of their neighbours, they must choose between acting on the advice received or selecting a new supplier at random. As long as they take any signal received to be truthful (an issue that we examine below), i.e. if $j' = j(i,t-1)$ and $q' = q_{j(i,t-1),t-1}$, selecting j' will deliver a higher expected quality than selecting a supplier at random; so they will select j'.

If an individual simultaneously receives multiple signals from different neighbours, then, as long as those signals can be taken to be truthful, it is irrelevant which particular signal the receiver acts upon. Thus we will assume that one signal is selected at random. This is without loss of generality: below we show that information-sharing equilibria feature only truthful signals, implying that the choice of tie-breaking rule is immaterial.

Information-sharing choices

An individual who has experienced high quality from her chosen supplier at $t-1$ may be willing to incur a cost to share information about her provision experience with her neighbours because she will benefit from the effect that better information has on her neighbours' contribution choices: if any given one of i's neighbours, $i' \in S_{i,t}$, has no information, her contribution is $v_{i',t} = 0$, whereas, if i' has information about a supplier that has delivered high quality at $t-1$, then she makes a direct a contribution $v_{i',t} = \mu$ towards that supplier, resulting in a higher payoff for i. Denoting by $E[G_t^{-i'}]$ the period-t expected, quality-adjusted level of provision from the contributions of all individuals other than i', the public good-related component of the payoff experienced by i when i' is uninformed is $\theta E[G_t^{-i'}] \equiv \underline{\Phi}$; when i' is informed, the corresponding level is $\theta\big(E[G_t^{-i'}] + \rho\mu\big) \equiv \overline{\Phi} = \underline{\Phi} + \theta\rho\mu > \underline{\Phi}$. So, if the effort cost, c, of sending an informative signal to her neighbours is not too high relative to the expected gain $\theta\rho\mu \equiv \Psi$, i may choose to voluntarily incur that cost – a choice that may be interpreted as "fundraising". In an analogous setting where private information about supplier quality concerns private consumption, individuals would never incur a private cost to inform their neighbours about their own consumption experience. It is only in the case of contributions to collective consumption that the actions of uninformed individuals are of concern to better informed individuals, inducing them to actively share their information with others.[7]

In the calculation of whether or not to send a signal to her neighbours, each informed individual also has to consider the likelihood that her signal will make a difference for the neighbour who receives it. As discussed above, any neighbour receiving the signal will only find it valuable if she has experienced low quality in period $t-1$ and is therefore uninformed. Also, since additional informative signals convey no additional information, the signal will only be valuable if the uninformed neighbour does not also receive an-

[7]In principle, an individual, i, could also choose to send signals about a provider $j(i, t-1)$ from whom she has experienced low quality in order to prevent neighbours from selecting this provider. However, as long as M is sufficiently large, the probability of a neighbour selecting $j(i, t-1)$ at random is negligible, and so i will never have an incentive to incur a cost c to send a signal in this case.

other signal from another neighbour. Thus, information-sharing decisions involve both an assessment of the likelihood that neighbouring individuals are uninformed and of the likelihood that they might also be targeted by others.

Let k_t be the proportion of individuals in the population who, having experienced high quality at $t-1$, are able to identify a provider who was a high-quality supplier in period $t-1$, i.e. the proportion of individuals that have no need of further information. Assume for now that k_t is publicly known. Then, given that an individual's neighbours are a random sample of the whole population, the probability that each neighbour will be able to make use of an informative signal is $1-k_t$, and the probability that each of her b neighbours will be informed – and therefore will be in a position to send the signal herself – is k_t. Suppose then that all informed individuals other than i send a signal at the beginning of period t with probability $s_{-i,t}$. From the point of view of an informed individual, i, the probability that any given neighbour will benefit from her signal is equal to the probability of the neighbour finding *any* signal valuable, which equals $1-k_t$, times the probability that this neighbour will not receive a signal from an informed neighbour other than i, which equals $(1-k_t s_{-i,t})^{b-1}$. So, the expected net value to i of sending a signal to one of her neighbours is

$$(1-k_t)(1-k_t s_{-i,t})^{b-1}\Psi - c \equiv \Lambda(s_{-i,t}, k_t). \tag{2}$$

An individual i will then always send a signal ($s_{i,t}=1$) if $\Lambda(s_{-i,t}, k_t) > 0$, will never do so ($s_{i,t}=0$) if $\Lambda(s_{-i,t}, k_t) < 0$, and will be indifferent between sending and not sending a signal ($0 < s_{i,t} < 1$) if $\Lambda(s_{-i,t}, k_t) = 0$.[8]

All of this presumes that individuals only send truthful signals. But an individual might also have an incentive to send an untruthful signal, i.e. to signal high quality for a provider that has delivered low quality. This is because, given that all individuals benefit from the provision of collective consumption, and given that the contribution level selected by an uninformed individual is zero, deceitfully inducing an uninformed individual to make a positive contribution to a randomly selected provider of expected quality $1/2$ yields an expected public good-related benefit $\theta\mu/2 > 0$. However, we show below that untruthful signals can be ruled out in equilibrium.

[8]Here i is assumed to randomize her signaling choices independently for each of her neighbours; however, this is fully equivalent to a setup where i's signaling choices are perfectly correlated across receivers, i.e. where i sends signals to all of her neighbours with probability $s_{i,t}$ or to none of her neighbours with probability $1-s_{i,t}$.

3 Information sharing, information diffusion, and neighbourhood size

In this section we address the question of how information sharing and private provision are affected by neighbourhood size, b. For this purpose, we first characterize equilibria in signaling and contribution choices in any given period (for a given k_t), and then derive stationary state conditions for an equilibrium of the dynamic game (with k_t endogenous).

3.1 Within-period equilibria

Equilibrium information-sharing choices

We want to study how neighbourhood size affects information-sharing effort, $s_{i,t}$, on the intensive margin, i.e. for $s_{i,t}$ strictly between zero and unity. For a given, publicly known k_t, a symmetric mixed-strategy equilibrium in signaling choices with $s_{i,t} = s_{-i,t} = s_t$, $0 < s_t < 1$ (giving rise to contribution choices as described in the previous section) requires $\Lambda(s_t, k_t) = 0$.[9] A corner (a pure-strategy equilibrium) where $s_t = 0$ can be ruled out as long as signaling costs, c, are not too large; and a corner where $s_t = 1$ can be ruled out as long as signaling costs, c, are not too small. Setting (2) equal to zero and solving

[9] Symmetric equilibria are most natural in the context we are examining; but this framework also admits asymmetric equilibria – in pure strategies only; and if we were to account for those, our results would be stated in terms of weak rather than strong monotonicity. However, in our model the possibility of asymmetric equilibria only stems from assuming that actions are discrete, a modelling choice we have made purely for presentational simplicity. Suppose, for example, that instead we assumed effort, e, to be continuous and to result in a signal being received with probability $s(e) = f(e)/(1 + f(e))$, with $f(e)$ being any increasing function such that $f(0) = 0$, implying $s(0) = 0$, $\lim_{e\to\infty} s(e) = 1$, and $\lim_{e\to 0} s'(e) = \infty$. Then, in the case $b = 2$ (and omitting the time subscript), the payoff for an informed player, i, would be $(1-k)s(e_i)(1 - k\,s(e_{-i}))\Psi - e_i \equiv \Pi_i, \;\; i = 1, 2$. Here, the unique equilibrium would be a symmetric equilibrium with positive e_i's (identified by $d\Pi_i/de_i = 0$, $i = 1, 2$) which is analogous to a symmetric equilibrium in mixed strategies for the discrete choice case. Moreover, it can be argued that, in a setting with anonymous players, asymmetric equilibria are ”non-focal”. In order to plausibly incorporate asymmetric play in an anonymous setting, one would need to invoke the notion of a *correlated equilibrium* (Aumann, 1974), whereby players would ”take turns” about who sends the signal on the basis of public random signal (effectively restoring symmetry and typically attaining a higher symmetric payoffs than otherwise possible). However, the institutional conditions for players to be able to correlate their strategies in this way are quite stringent, which is why correlated equilibria tend to be viewed as a theoretical curiosum in the applied literature.

for s_t, we obtain

$$s_t = \frac{1}{k_t}\left(1 - \left(\frac{c}{(1-k_t)\Psi}\right)^{1/(b-1)}\right). \tag{3}$$

One can verify that this is strictly between zero and unity if and only if $1 - k_t > c/\Psi > (1-k_t)^b$.

Untruthful signals can be ruled out in equilibrium. Suppose that someone were to send an untruthful signal when everyone else is sending truthful signals. Then, starting from a belief that the signals that are sent are truthful, an uninformed individual receiving a single signal would act upon it; and an uninformed individual receiving multiple signals would have to decide which particular signal to act upon. Earlier, we stated the assumption that, when multiple signals are received, one particular signal is selected at random; but we can generalize this tie-breaking rule and simply posit that, if a signal is sent, and the receiver believes signals to be truthful, then that signal will be selected and acted upon with a probability that is either positive or zero. Then, assuming that all informed individuals other than i are sending truthful signals, the expected net value to i of sending an untruthful signal to one of her neighbours is

$$(1-k_t)(1-k_t s_t)^{b-1}\theta\mu/2 + \Upsilon - c, \tag{4}$$

where the first term represents the expected gain associated with the possibility that an individual not receiving a signal from anyone else would act on the untruthful signal, and $\Upsilon \leq 0$ is the expected loss associated with the possibility that an individual receiving multiple signals selects i's untruthful signal over the truthful signal of a different neighbour, resulting in a lower expected level of quality-adjusted provision.[10] In an equilibrium in strictly mixed strategies where $\Lambda(s_t, k_t) = 0$, the signaling cost, c, must equal $(1-k_t)(1-k_t s_t)^{b-1}\theta\mu\rho$; and, since $\theta\mu/2 < \theta\mu\rho = \Psi$, and $\Upsilon \leq 0$, we can conclude that expression (4) must be negative. This implies that an equilibrium outcome must be one where only truthful signals are sent, and where receivers have no reason to doubt the signals' truthfulness.[11]

[10]If sending an untruthful signal when all other signals are truthful generates a non-zero probability of a receiver selecting a provider j for whom $q_{j,t-1} = \underline{q}$, then Υ is strictly negative, otherwise it is zero.

[11]Since signals are costly to send, an outcome where all signals are untruthful and are disregarded cannot be an equilibrium outcome – if signals are disregarded, then senders would opt to send no signals and save the cost. An outcome where some signals are truthful and others are not can also be ruled out by analogous arguments – i.e. given any mix of truthful and untruthful signals, and receivers' beliefs consistent with that mix, then withholding an untruthful signal is strictly payoff-improving for the sender. In our analysis, we

Information sharing and neighbourhood size

For $b = 1$, the only value of k_t compatible with an equilibrium in strictly mixed strategies is $k_t = c/\Psi$; in this case the equilibrium level of s_t is indeterminate in the short run (but, as we will show in the next section, not in the long run, when s_t and k_t are determined jointly in equilibrium). For $b > 1$, we can rearrange condition (3) for a symmetric equilibrium in strictly mixed strategies and express it as

$$k_t s_t = 1 - \left(\frac{c}{(1-k_t)\Psi} \right)^{1/(b-1)}. \tag{5}$$

The left-hand side of (5) is a measure of information-sharing intensity in the economy – information sharing per head per neighbour. This is negatively related to signaling costs, c. Let $r_t \equiv k_t s_t$. Trivially, for a given s_t, a higher k_t results in a higher r_t; but r_t, taken as a whole, varies with k_t as described by the right-hand side of (5):

Proposition 1 *For $b > 1$, information-sharing intensity, $r_t = k_t s_t$, is negatively related to the stock of information, k_t, and to neighbourhood size, b.*

PROOF: Denote the right-hand side of (5) with $\Omega(k_t, b)$, and differentiate this with respect to k_t and b. Noting that $\Lambda(s_t, k_t) = 0$ implies $c/\big((1-k_t)\Psi\big) = (1-r_t)^{b-1} < 1$, we obtain, after substitution,

$$\Omega_{k_t} = -\frac{1-r_t}{(b-1)(1-k_t)} < 0; \tag{6}$$

and

$$\Omega_b = \frac{1-r_t}{(b-1)^2} \ln\left((1-r_t)^{b-1}\right) < 0. \tag{7}$$

The result that r_t decreases in b for a given k_t, also implies that s_t decreases with b. □

The intuition for this result is straightforward. Although private contributions are made towards provision of a pure public good, information sharing with one's neighbours has the characteristics of a local public good. Specifically, it is a locally provided

abstract from asymmetric equilibria (for the reasons discussed in Footnote 9) as well as from pure-strategy equilibria, but the same logic would apply to those: in an equilibrium where at least some of the players who send signals send truthful signals, no individual would have an incentive to send an untruthful signal; in an equilibrium where the only signals that are sent are untruthful, they would not be acted upon by receivers, and hence senders would have no incentive to send them in the first place.

good that is only indirectly purely public: an individual can only receive signals from her neighbours, and so provision of information to neighbours is in itself not a pure public good despite the fact that its benefits ultimately flow through the provision of a pure public good. Since only neighbours can provide information to an uninformed individual, free-riding incentives with respect to information sharing remain contained within a given neighbourhood. The larger an individual's social neighbourhood, then, the greater the free-riding incentives in information sharing.

3.2 Information diffusion

Information dynamics

Proposition 1 deals with the relationship between neighbourhood size and signaling choices for a given stock of information, k_t. However, in a multi-period economy where individuals make repeated signaling and contributions choices – as detailed in Section 2 – the stock of information is endogenous. In what follows, we look at the relationship between neighbourhood size and information sharing when the endogeneity of k_t is accounted for.

The stock of information, k_t, evolves through time as follows:

$$k_{t+1} = k_t\rho + (1-k_t)\big(1-(1-k_t s_t)^b\big)\rho \equiv \Gamma(s_t, k_t). \tag{8}$$

The term $k_t\rho$ in (8) represents the fraction of individuals who, having experienced high quality at $t-1$, do a repeat purchase at t and again experience high quality at t; the next term represents the fraction of individuals who, having experienced low quality at $t-1$, receive an informative signal at t (which occurs with probability $1-(1-k_t s_t)^b$), and, having acted on it, experience high quality; since only informed individuals participate in provision (whether they have personally gathered information or they have received signals from neighbours), no new information can be gained by uninformed individuals who receive no signals from neighbours.

Up to this point, we have assumed that the proportion of informed individuals, k_t, is publicly known. But given that quality realizations are only observable by contributors, k_t is not directly observable. However, since in a large economy the realized level of public good provision, G_t equals ρk_t with probability one, and since individuals experience (and thus observe) the level of public good provision, they can perfectly infer k_t from G_t at the end of period t (i.e. after signaling and contribution choices for period t have been made). On the basis of their knowledge of k_t, individuals can then determine the level of k_{t+1} from (8) before making their choices at $t+1$. This amounts to k_t being publicly observable at the beginning of each period.

In this setting, knowledge of good suppliers spreads to others – at a speed that depends on the size of social neighbourhoods and the cost of information transmission. But information always remains incomplete due to the fact that suppliers undergo quality shocks; i.e. there is social learning, but it never leads to information being complete as information "depreciates".

Stationary states

A stationary state consists of an indefinite sequence of periods where information-sharing choices are in equilibrium within each period given the amount of information about high-quality providers in the economy, and where the amount of information stays the same through time, i.e. where $k_{t+1} = k_t = \hat{k}$.

A stationary state can thus be characterized with reference to a fixed point, $(\hat{s}, \hat{k})$, such that

$$\Lambda(\hat{s}, \hat{k}) = 0, \tag{9}$$

and

$$\hat{k} = \Gamma(\hat{s}, \hat{k}). \tag{10}$$

This definition implicitly incorporates a condition on the consistent updating of beliefs about k_t, as previously discussed. We abstract from questions of global stability;[12] however, numerical simulations for a parameterized example (discussed below) are indicative of fast convergence.

Focusing on (9) and (10), we can totally differentiate the equilibrium conditions with respect to $\hat{r} \equiv \hat{k}\hat{s}$ and b in order to derive an expression for the total derivative $\mathrm{d}\hat{r}/\mathrm{d}b$. This is unambiguously negative:

Proposition 2 *In a stationary state, information-sharing intensity, $\hat{r} = \hat{k}\hat{s}$, is negatively related to neighbourhood size, b.*

PROOF: Solving for $\hat{k}$ from (10) as a function of $\hat{r}$, we obtain

$$\hat{k} = \rho \frac{1 - (1 - \hat{r})^b}{1 - \rho(1 - \hat{r})^b} \equiv \Theta(\hat{r}, b). \tag{11}$$

[12] Gale and Kariv (2003) derive global convergence results for a setup that is somewhat related to ours but does not share the same formal structure.

Expressing (9) as a function of $\hat{k}$ and $\hat{r}$ (rather than of $\hat{k}$ and $\hat{s}$), i.e. as $\tilde{\Lambda}(\hat{r},\hat{k}) \equiv (1-\hat{k})(1-\hat{r})^{b-1}\Psi - c = 0$, substituting (11) into it, and totally differentiating, we obtain

$$\frac{\mathrm{d}\hat{r}}{\mathrm{d}b} = -\frac{\tilde{\Lambda}_{\hat{k}}\Theta_b + \tilde{\Lambda}_b}{\tilde{\Lambda}_{\hat{k}}\Theta_{\hat{r}} + \tilde{\Lambda}_{\hat{r}}} = \frac{(1-\hat{r})\ln(1-\hat{r})}{b-1+\rho(1-\hat{r})^b} < 0. \tag{12}$$

□

The conclusion that a larger neighbourhood size results in lower information-sharing intensity thus also applies to comparisons across steady states that take into into account information dynamics.

The above result does not immediately imply that collective consumption is negatively affected by an increase in neighbourhood size. The total expected surplus associated with collective consumption – gross of signaling costs – increases with the information stock $\hat{k}$. As shown earlier, for a given b, the stock of information is directly related to information-sharing intensity, $\hat{r}$; and, as just shown, information-sharing intensity decreases with b. However a larger b also has a direct positive effect of information diffusion – the expression Θ_b is positive (we show this in the proof below); i.e., for a constant level of information-sharing intensity, an increase in the number of social connections raises learning and thus the stock of information.[13] In principle, if this latter effect were to dominate the former, information on supplier quality could, in a stationary equilibrium, be positively related to neighbourhood size even if it is associated with a lower level of information-sharing intensity. Our next result shows that this can never be the case:

Proposition 3 *In a stationary state, the stock of information, $\hat{k}$, about supplier quality, as well as expected, quality-adjusted provision, $E[\hat{G}]$, are decreasing in the size of social neighbourhoods, b.*

PROOF: The total effect of an increase in b on $\hat{k}$ is expressed by the total derivative

$$\frac{\mathrm{d}\hat{k}}{\mathrm{d}b} = \Theta_b + \Theta_{\hat{r}}\frac{\mathrm{d}\hat{r}}{\mathrm{d}b}. \tag{13}$$

We have

$$\Theta_{\hat{r}} = \frac{\rho(1-\rho)(1-\hat{r})^{b-1}b}{\left(1-\rho(1-\hat{r})^b\right)^2} > 0; \tag{14}$$

[13]For a constant $\hat{s}$ (or, equivalently, at a corner where $\hat{s} = 1$) the model becomes a pure social learning model with no endogenous information-sharing effort.

$$\Theta_b = -\frac{\rho(1-\rho)(1-\hat{r})^b \ln(1-\hat{r})}{\big(1-\rho(1-\hat{r})^b\big)^2} > 0. \tag{15}$$

We can substitute these and (12) into (13); after simplification, we obtain

$$\frac{\mathrm{d}\hat{k}}{\mathrm{d}b} = \frac{\rho(1-\rho)(1-\hat{r})^b \ln(1-\hat{r})}{\big(1-\rho(1-\hat{r})^b\big)\big(b-1+\rho(1-\hat{r})^b\big)} < 0. \tag{16}$$

Since expected collective provision at t equals $Nk_{t+1}\mu$, which in turn equals $Nk_t\mu$ in a stationary equilibrium, an increase in neighbourhood size also decreases expected provision of the collective good. Information-sharing effort, $\hat{s}$, is also decreasing in b: using (12) and (13), we obtain

$$\frac{\mathrm{d}\hat{s}}{\mathrm{d}b} = \left(\frac{\mathrm{d}\hat{r}}{\mathrm{d}b} - \hat{s}\,\frac{\mathrm{d}\hat{k}}{\mathrm{d}b}\right)\Big/\,\hat{k} = \frac{1-(1-\hat{r})^{b-1}\rho\big(1-\hat{r}+(1-\rho)\hat{s}\big)}{\rho(1-\rho)(1-\hat{r})^{b-1}\,\hat{k}}\;\frac{\mathrm{d}\hat{k}}{\mathrm{d}b}. \tag{17}$$

The expression in the denominator on the right-hand side of (17) is positive, and the expression in the numerator is also positive for $\hat{s} \in (0,1)$, $\hat{k} \in (0,1)$ and $\rho \in (1/2,1)$, implying $\mathrm{d}\hat{s}/\mathrm{d}b < 0$. As anticipated earlier, (9) and (10) together can deliver an interior solution for $\hat{s}$ even when $b = 1$, and so (17) may also apply to the case $b = 1$. □

Thus, larger social neighbourhoods unambiguously result in a lower level of expected collective consumption – although this remains strictly positive as long as some signaling takes place (i.e. as long as $\hat{s} > 0$).[14] It should be noted, however, that in this model there is no collective provision without learning, and there is no learning with empty neighbourhoods: for $b = 0$, any individual who is initially contributing through a high-quality supplier would at some point experience low quality from that supplier and, in the absence of any informative signals from others, would permanently stop contributing. So, learning and collective provision are promoted by small neighbourhoods but not by empty ones.

This result can be illustrated by the following parameterized example. Consider a scenario with $\rho = 2/3$, $\theta = 7/4 > 1/\rho$, $\mu = 1/2$, $c = \Psi/2 = 7/24$. For $b = 2$, we have $\hat{s} \simeq 0.45$, $\hat{k} \simeq 0.39$, $\hat{r} = \hat{s}\hat{k} \simeq 0.18$, $E[\hat{G}]/N \simeq 0.195$. Convergence to the stationary state starting from $k_0 \neq \hat{k}$ is relatively fast: starting from $k_0 = 0.01$, convergence to $\hat{k}$, up to the third decimal digit, is attained at $t = 40$; starting from $k_0 = 0.99$ convergence is even faster, with k_t reaching $\hat{k}$, up to the third decimal digit, at $t = 21$. Doubling neighbourhood size to $b = 4$ results in $\hat{s} \simeq 0.22$, $\hat{k} \simeq 0.36$, $\hat{r} \simeq 0.08$, $E[\hat{G}]/N \simeq 0.18$. Finally, for $b = 8$, we

[14] If signaling effort goes to zero, any individual who experiences low quality, even once, will permanently cease to contribute – implying that both the stock of information and the level of provision will decline to zero.

have $\hat{s} \simeq 0.11$, $\hat{k} \simeq 0.345$, $\hat{r} \simeq 0.04$, $E[\hat{G}]/N \simeq 0.17$.

Collective consumption is lower when social neighbourhoods are larger even if, as in this setup, spillovers from collective provision are independent of neighbourhood size. Social neighbourhoods are only relevant here for voluntary information sharing. An increase in neighbourhood size increases free-riding incentives in information sharing, reducing information diffusion about provider quality and thus expected provision, despite the potential for a higher number of signals reaching any given individual in larger neighbourhoods.

4 Extensions

So far the analysis has been carried out in a model where private contributions come about purely from consumption motives that are perfectly aligned across individuals. In this section we show how our analysis and results extend to alternative characterizations of the motives underlying giving and information-sharing behaviour.

4.1 Warm-glow motives

In the linear specification of preferences that we have assumed, although the public good is a pure public good, the choice of whether or not to contribute is structurally independent of the level of contributions by others, i.e. donations by one individual do not crowd out donations by others. This also implies that individual donations are independent of group size. Our arguments, however, do not crucially hinge on ruling out free riding; we are simply modeling situations where individuals have positive incentives to give (as we observe them to do), given the contribution choices of others, and where acquiring information can affect such incentives. In fact, our specification is fully equivalent to one where preferences are strictly convex – implying that, in principle, free riding can occur – consumption is bounded below to unity, and the marginal rate of substitution of private for public consumption (U_G/U_x) is greater than ρ for ($x_{i,t} = 1, G_t = N\mu$), and is less than $1/2$ for ($x_{i,t} = 1 + \mu, G_t = 0$) – which means that the individual selects a donation $v_{i,t} = \mu$ if $\tilde{q} \geq \rho$ and a donation $v_{i,t} = 0$ if $\tilde{q} \leq 1/2$. Thus, our arguments only require that individuals make positive donations (conditional on the information they hold) in the ”region of interest”.

Nevertheless, with strictly convex preferences and in the absence of any further restrictions, private giving will tend to vanish as the number of donors gets larger. In order to rationalize non-vanishing contributions in large economies, economists have hypothesized that individuals may also experience a private benefit ("warm glow") from their donations (Andreoni, 1990). In the following, we show that the above analysis and results are robust with respect to the inclusion of warm-glow motives.

If warm glow only relates to the donation an individual makes, then it would simply involve an additional, private benefit term, $\gamma g_{i,t}$, with $g_{i,t} \equiv q_{j(i,t),t} v_{i,t}$, in (1), which would lower the minimum level of the marginal rate of substitution between public and private consumption (represented by θ in our linear specification) required to rationalize positive donations. The rest of the model and analysis would be qualitatively unaffected. On the other hand, if warm glow only arises from the donation individuals make *and* is their only reason for giving, i.e. if $\gamma > 0$ and $\theta = 0$, then no individual would have an incentive to engage in costly signaling.

However, it seems plausible that individuals who derive warm glow from their own direct contributions to collective consumption would also derive warm glow from any positive effect on collective consumption that they brought about with their own information-sharing efforts, i.e. from other individuals' contributions for which they can "take credit". Then, if an individual sends an informative signal of quality to a neighbour, she would also experience warm glow from that neighbour's contributions in those realizations where the signal has value to the neighbour and affects her giving choice.

This idea can be modelled as follows. Let σ^0 denote the empty signal, $\mathcal{E}(s_{i,t}, i \in I)$ denote the event space given the mixed signaling strategies chosen by individuals, and let $\mathcal{E}_{i,h,t}$ be the subset of realizations where $\sigma_{i,h,t} \neq \sigma^0$ and $\sigma_{i',h,t} = \sigma^0$, $i' \in S_{h,t}$, $i' \neq i$, i.e. those realizations where individual h receives a signal from i and not from anyone else in her social neighbourhood, $S_{h,t}$. Also denote with $\varepsilon_{i,h,t} \in \mathcal{E}_{i,h,t}$ a specific realization in this set. We then write the payoff of an informed individual i at t as

$$x_{i,t} + \gamma\left(g_{i,t} + \sum_{h \in \left\{h \in S_{i,t} | \varepsilon_{i,h,t} \in \mathcal{E}_{i,h,t}\right\}} g_{h,t}\right); \tag{18}$$

i.e., the individual assigns a marginal valuation of γ on her own quality-adjusted donations as well as on the donations of those neighbours for whom her signaling efforts have "made a difference".

In this case an informed individual's expected payoff from sending a signal to a neighbour is equal to the probability that the neighbour is uninformed times the probability that the neighbour has not received a signal from someone else, times $\gamma\rho\mu$, minus the cost of signaling, c; i.e. an expression that is identical to $\Lambda(s_t, k_t)$, as defined in (2), but for the fact that the scalar θ is now replaced by γ. The analysis of information sharing equilibria can then proceed along the exact same lines as in the case where giving stems purely from consumption motives, and the results and conclusions are also identical.[15]

The only difference between this alternative characterization and the one developed in the previous sections lies in the interpretation of the neighbourhood size effect. As noted earlier, when the benefits from information sharing derive from a collective consumption motive, information sharing can be thought of as a local public good; accordingly, an increase in the size of social neighbourhoods induces free riding in the provision of this public good – the larger the size of social neighbourhoods, the more individuals can rely on others to inform their neighbours. In contrast, when the benefits from information sharing stem from a warm-glow motive, information sharing can be thought of as a private good subject to local congestion; in this case an increase in the size of social neighbourhoods directly reduces the warm-glow value of information sharing – the larger the size of social neighbourhoods, the more difficult it is for an individual who sends an informative signal to take credit for her neighbours' contributions.

4.2 Heterogeneous preferences for collective consumption

An important aspect of information-sharing activities is the potential for informative signals to steer other people's contributions towards goals that the sender favours. This is a key dimension of fundraising behaviour: individuals who engage in fundraising towards charitable causes do not just do it to share information with others, but also in the hope of affecting the kind of charitable activities that other individuals will support.

We can incorporate this motive into our analysis by allowing for preference heterogeneity with respect to collective consumption. Suppose that there are two forms of collective provision, 1 and 2, and that individuals have heterogeneous preferences with respect to the two forms of provision; namely, half of the population (type 1) have a period-t payoff equal to

$$x_{i,t} + 2\theta\big(\alpha G_t^1 + (1-\alpha) G_t^2\big) - e_{i,t}, \tag{19}$$

[15] This also applies to the conclusion that only truthful signals are sent in a mixed-strategy equilibrium: sending an untruthful signal can generate a warm-glow benefit, but sending a truthful signal generates a larger benefit; thus, in an equilibrium where the net expected benefit of sending a truthful signal is zero, the net expected benefit of sending an untruthful signal is negative, and so no untruthful signal will be sent.

whereas the other half (type 2) have a payoff equal to

$$x_{i,t} + 2\theta\big((1-\alpha)G_t^1 + \alpha G_t^2\big) - e_{i,t}, \tag{20}$$

with $\alpha > 1/2$ – meaning that, quality being constant, the first half would prefer to contribute towards collective good 1, and the other half would prefer to contribute towards collective good 2. In addition, we assume that

$$\rho(1-\alpha)\theta > 1 > (1/2)\alpha\theta, \tag{21}$$

implying that an individual of a given type would make positive contributions towards the collective good she does not favour through a known high-quality supplier but would not contribute anything towards the good she favours through a supplier of unknown quality: these are conditions under which informative signals have the potential to "sway" contributors away from one collective good towards the other.[16]

Then, an individual, i, of a given type who has information about quality for a supplier providing the collective good she favours would face even stronger incentives to engage in costly signaling. This is for two reasons: (i) information sharing also increases the chance that an uninformed neighbour, h, of a different type would opt to contribute to the good she (individual i) favours, if the only signal h receives is i's signal; and (ii) it also increases the chance that an individual of the same type as i, who is only informed about provision of the good she does not favour, would switch to her favoured good, if she receives a signal from i. In other words, a fundraiser can be pivotal in her neighbours' choice about which form of collective provision to support.

In addition, if individuals who are informed about the good that they favour engage in information sharing with a positive probability that is less than unity (i.e. in an interior outcome in strictly mixed strategies), individuals who have information about the good they do not favour will not do so – since the expected gain from sending a signal in this case is strictly less than for individuals who have information about their favoured good. Thus, a smaller proportion of informed individuals will engage in information sharing, which will in turn increase signaling incentives for those who do so.

As a result, information-sharing intensity and the overall level of collective provision will rise. This result is formalized in the proof of the following proposition.

[16]When this condition is not met, the only relevant interactions are those between individuals who are of the same preference type. Such a scenario has the same structure and properties as the single-good scenario that we have already analyzed.

Proposition 4 *In a stationary equilibrium, if (21) is satisfied at $\alpha = 1/2$, and for α lying in a right-hand neighbourhood of 1/2: (i) information-sharing intensity, $\hat{r}$, is increasing in α; and (ii) expected, quality-adjusted provision, $E[\hat{G}^1 + \hat{G}^2]$, is increasing in α and decreasing in b.*

PROOF: Consider the signaling incentives for an individual of type 1. Let $\Lambda_t^{\tau\tau}$ be the expected net payoff in period t of an individual of type τ ($\tau \in \{1,2\}$) for sending a signal about quality of the good she favours, and $\Lambda_t^{\tau\tau'}$ ($\tau' \neq \tau$) be the corresponding payoff for a signal about the good she does not favour. Because the good she favours is valued more, it must be the case that if $\Lambda_t^{\tau\tau} = 0$, then $\Lambda_t^{\tau\tau'} < 0$, and so in a symmetric mixed-strategy equilibrium individuals will only send truthful signals of quality about the good they favour (we state this conclusion informally for the time being, but we verify it formally later on in our proof). The expected payoff for an informed individual of type 1 from sending a truthful signal about provision of good 1, if the only signals are truthful signals sent by informed individuals of each type about the good that type favours, is then

$$\left(1 - \frac{k_t^{11} + k_t^{12} + k_t^{22} + k_t^{21}}{2}\right) 2^{1-b} \sum_{h=0}^{b-1} \binom{b-1}{h} \left(1 - s_t^1 k_t^{11}\right)^h \left(1 - s_t^2 k_t^{22}\right)^{b-1-h} 2\alpha\Psi$$

$$+ \frac{1 - k_t^{11} - k_t^{12}}{2} 2^{1-b} \sum_{h=0}^{b-1} \binom{b-1}{h} \left(1 - s_t^1 k_t^{11}\right)^h \left(1 - (1 - s_t^2 k_t^{22})^{b-1-h}\right) 2(2\alpha - 1)\Psi$$

$$+ \frac{k_t^{12}}{2} 2^{1-b} \sum_{h=0}^{b-1} \binom{b-1}{h} \left(1 - s_t^1 k_t^{11}\right)^h 2(2\alpha - 1)\Psi - c \equiv \Lambda_t^{11}, \tag{22}$$

where k_t^{11} and k_t^{22} are the proportion of individuals of each type who, in the previous period, have experienced a good outcome about the good they favour; k_t^{12} and k_t^{21} are the proportion of individuals of each type who, in the previous period, have experienced a good outcome about the good they do not favour; s^1 and s^2 are each type's mixed signaling strategy; and $\binom{b-1}{h}$ is the binomial coefficient ${}_{b-1}C_h = (b-1)!/\big((b-1-h)!h!\big)$. The first term in Λ_t^{11} represents the expected gain related to the possibility that, by sending the signal, the type-1 individual induces a fully uninformed individual (of either type) to contribute towards good 1; the second term represents the expected gain related to the possibility that a signal about good 1 induces a same-type individual (of type 1) who has only received signals about good 2 at t to switch to good 1 from good 2; the third term represents the expected gain related to the possibility that a signal about good 1 induces a same-type individual (of type 1) who has experienced high quality in the provision of good 2 at $t-1$ and has not received any informative signal at t to switch from good 2 to good 1 (which type 1 individuals value $2\big(\alpha - (1-\alpha)\big) = 2(2\alpha - 1)$ more than good 2). The corresponding expected payoff for sending a truthful signal about quality of provision of good 2 would have a similar structure, but the first term would feature a factor $1 - \alpha$ rather than $\alpha > 1 - \alpha$, and the last two terms would involve type-2 rather than type-1 receivers, and would feature a factor $1 - 2\alpha < 0$ rather than $2\alpha - 1$ – implying an expected loss. Thus, as initially posited, $\Lambda_t^{\tau\tau'} < \Lambda_t^{\tau\tau}$.

Stocks of information for type-1 individuals evolve as follows:

$$k_{t+1}^{11} = \rho k_t^{11} + \rho(1 - k_t^{11})2^{-b} \sum_{h=0}^{b} \binom{b}{h} \big(1 - (1 - s_t^1 k_t^{11})^h\big); \tag{23}$$

$$k_{t+1}^{12} = \rho k_t^{12} 2^{-b} \sum_{h=0}^{b} \binom{b}{h} \big(1 - s_t^1 k_t^{11}\big)^h + \rho(1 - k_t^{11} - k_t^{12})2^{-b} \sum_{h=0}^{b} \binom{b}{h} \big(1 - s_t^1 k_t^{11}\big)^h \big(1 - (1 - s_t^2 k_t^{22})^{b-h}\big). \tag{24}$$

The corresponding expected payoff, Λ_t^{22}, for a type-2 sender, and the equalities defining the evolution of information stocks for type-2 individuals are defined analogously.

In a symmetric stationary equilibrium, it will be the case that $\hat{k}^{11} = \hat{k}^{22} = \hat{k}^{\tau\tau}$, $\hat{k}^{12} = \hat{k}^{21} = \hat{k}^{\tau\tau'}$, and $\hat{s}^1 = \hat{s}^2 = \hat{s}$.[17] Carrying out these substitutions, using the identities $\sum_{h=0}^{n} \binom{n}{h} = 2^n$ and $2^{-n} \sum_{h=0}^{n} \binom{n}{h} \big(1 - x\big)^h = 2^{-n}(2 - x)^n = (1 - x/2)^n$ – the latter derived from the identity $\sum_{h=0}^{n} \binom{n}{h} x^n = (1 + x)^n$ – and further simplifying, the conditions for a stationary equilibrium in strictly mixed strategies can be re-written as

$$\Big((1 - \hat{k}^{\tau\tau} - \hat{k}^{\tau\tau'})\big(1 - \hat{s}\hat{k}^{\tau\tau}\big)^{b-1} + (1 - \hat{k}^{\tau\tau})(1 - \hat{s}\hat{k}^{\tau\tau}/2)^{b-1}(2\alpha - 1)\Big)\Psi - c = 0; \tag{25}$$

$$\hat{k}^{\tau\tau} - \rho\Big(\hat{k}^{\tau\tau} + (1 - \hat{k}^{\tau\tau})\big(1 - (1 - \hat{s}\hat{k}^{\tau\tau}/2)^b\big)\Big) = 0; \tag{26}$$

$$\hat{k}^{\tau\tau'} - \rho\Big(\hat{k}^{\tau\tau'}(1 - \hat{s}\hat{k}^{\tau\tau})^b + (1 - \hat{k}^{\tau\tau})\big((1 - \hat{s}\hat{k}^{\tau\tau}/2)^b - (1 - \hat{s}\hat{k}^{\tau\tau})^b\big)\Big) = 0. \tag{27}$$

Letting $\hat{s}\hat{k}^{\tau\tau} = \hat{r}$ and solving for $\hat{k}^{\tau\tau}$ and $\hat{k}^{\tau\tau'}$ as a function of $\hat{r}$ from (26) and (27), we obtain

$$\hat{k}^{\tau\tau} = 1 - \frac{1 - \rho}{1 - \rho(1 - \hat{r}/2)^b}, \tag{28}$$

$$\hat{k}^{\tau\tau'} = 1 - \hat{k}^{\tau\tau} - \frac{1 - \rho}{1 - \rho(1 - \hat{r})^b}, \tag{29}$$

$$\hat{k}^{\tau\tau} + \hat{k}^{\tau\tau'} = 1 - \frac{1 - \rho}{1 - \rho(1 - \hat{r})^b}. \tag{30}$$

The total in (30) equals the expression obtained from solving for $\hat{k}$ as a function of $\hat{r}$ in a scenario with homogeneous provision.

For $\alpha = 1/2$, both $\hat{k}^{\tau\tau}$ and $\hat{k}^{\tau\tau'}$ come to refer to a common homogeneous stock of information;

[17] Asymmetric stationary equilibria in which there are positive stocks of information for both forms of provision but where only one agent type engages in costly information sharing cannot exist: if one agent type does not send costly signals, the information stock pertaining to the good which that agent type favours must be zero in a stationary equilibrium (as in the linear case, there is always a stationary equilibrium with zero information stocks). Starting from a positive information stock for only one form of provision (say good 1), an outcome where only one agent type signals and where the stock of information for the other form of provision remains zero can also be an equilibrium. Signalling incentives in that case would still be captured by (22) – with $k^{12} = k^{22} = 0$. The same intuition with respect to effects of neighbourhood size would apply.

so we can write $\hat{k}^{\tau\tau} + \hat{k}^{\tau\tau'} = \hat{k}$. Moreover, for $\alpha = 1/2$, (25) becomes

$$(1-\hat{k})(1-\hat{r})^{b-1}\Psi - c = 0, \tag{31}$$

which coincides with the corresponding condition for a scenario with homogeneous provision. So, we conclude that, for $\alpha = 1/2$, the stationary equilibrium values $\hat{k}$ and $\hat{r}$ coincide with those in a scenario with homogeneous provision, and so do those for α approaching $1/2$ from the right – the only difference being that, for α close to but not equal to $1/2$, only the stock $\hat{k}^{\tau\tau}$ gives rise to information sharing and so $\hat{s} = \hat{r}/\hat{k}^{\tau\tau}$, whereas for $\alpha = 1/2$ all informed individuals engage in signaling and so $\hat{s} = \hat{r}/\hat{k}$.

Substituting (28) and (30) into (25) and totally differentiating with respect to $\hat{r}$ and α, we obtain

$$\left[\frac{\mathrm{d}\hat{r}}{\mathrm{d}\alpha}\right]_{\alpha=1/2} = \frac{2(1-r)^{2-b}(1-\hat{r}/2)^{b-1}\big(1-\rho(1-r/2)^b\big)^2}{\big(b-1+\rho(1-\hat{r})^b\big)\big(1-\rho(1-r/2)^b\big)} > 0. \tag{32}$$

Multiplying the above by $\partial(\hat{k}^{\tau\tau} + \hat{k}^{\tau\tau'})/\partial\hat{r}$ (as derived from (30)), and evaluating the resulting expression at $\alpha = 1/2$, we obtain

$$\left[\frac{\mathrm{d}(\hat{k}^{\tau\tau} + \hat{k}^{\tau\tau'})}{\mathrm{d}\alpha}\right]_{\alpha=1/2} = \frac{2b\rho(1-\rho)(1-r)(1-\hat{r}/2)^{b-1}}{\big(b-1+\rho(1-\hat{r})^b\big)\big(1-\rho(1-r/2)^b\big)} > 0. \tag{33}$$

Finally, the expression for $\mathrm{d}\hat{k}/\mathrm{d}b$ (obtained as in the proof of Proposition 3) is continuously differentiable in α; this implies that, for α approaching $1/2$ from the right, $\mathrm{d}\hat{k}/\mathrm{d}b$ approaches (16). By continuity, we can then conclude that there exists a right-hand neighbourhood of $1/2$, $\mathcal{N}_\epsilon(1/2)$, for some $\epsilon > 0$, in which $\mathrm{d}\hat{k}/\mathrm{d}b < 0$. □

Thus, with reference to the implications of neighbourhood size for information-sharing intensity and provision levels, this scenario yields qualitatively analogous predictions as the basic specification we examined earlier – albeit with stronger signaling incentives. The main virtue of this variant, however, is that it incorporates motives that are typically thought of as being central to fundraising, namely the drive by fundraisers to steer others' contributions towards forms of provision favoured by the fundraisers themselves.

Referring back to the example presented at the end of Section 3, for $b = 2$ and α approaching $1/2$ we obtain the same level of expected provision as in the homogeneous good case, with stocks respectively equal to $\hat{k}^{\tau\tau} \simeq 0.25$, $\hat{k}^{\tau\tau} \simeq 0.14$ – adding up to the same information stock level, $\hat{k} \simeq 0.39$, as in the homogeneous good case – and $\hat{s} = 0.7$ – giving the same level of information-sharing intensity, $\hat{r} = \hat{s}\hat{k}^{\tau\tau} \simeq 0.18$, as in the homogeneous good case – and a level of expected provision $E[\hat{G}^1 + \hat{G}^2]/N \simeq 0.195$. For $b = 2$ and $\alpha = 5/8 > 1/2$, we have $\hat{k}^{\tau\tau} \simeq 0.35$, $\hat{k}^{\tau\tau} \simeq 0.15$, $\hat{r} \simeq 0.29$, $E[\hat{G}^1 + \hat{G}^2]/N \simeq 0.25 > 0.195$: other things equal, heterogeneity in preferences results in a higher level of expected provision.

4.3 Strictly convex preferences

In the preceding discussion, payoffs have been assumed to be linear in the level of collective provision. Linearity is not at odds with the presence of a diffuse positive externality (a public good) that entails suboptimal levels of provision. But linearity *does* imply that individual provision incentives are unaffected by the the provision choices of others; specifically, it implies that any increase in the supply of the collective good, either coming from government or from other private agents, does not "crowd out" provision by the remaining agents – as is the case with convex preferences (as reflected by a concave utility function).

Abstracting for the time being from information sharing, and omitting time subscripts, suppose that preferences are represented by the following strictly concave utility function:

$$\ln(y - v_i) + \theta \ln(G); \tag{34}$$

and consider a scenario with a finite N and where there is no uncertainty in provision outcomes. The first-order condition identifying the optimal level of contributions for individual i can be then written as

$$-\frac{1}{y - v_i} + \theta \frac{1}{G_{-i} + v_i} = 0, \tag{35}$$

where $G_{-i} = \sum_{h \neq i} v_h$. This gives $v^i = \big(\theta y - G_{-i}\big)/(1+\theta)$ and so $\mathrm{d}v_i/\mathrm{d}G_{-i} < 0$. Thus, with concave preferences, i's individually optimal level of provision is decreasing with the level of provision of others, i.e. private contributions are strategic substitutes.

It can be shown that the arguments and results we have developed for the case with linear preferences generalize to a scenario with concave utility. Focusing still on the logarithmic utility case,[18] in a scenario where only informed individuals contribute (as in the linear case), we can derive the following result:

Proposition 5 *With loglinear utility, an increase in neighbourhood size lowers information-sharing intensity as well as the stationary-state level of expected collective provision.*

PROOF: We assume the following concave payoff specification: $U(x_i, G, e_i) = \ln x_i + \theta \ln G - e_i$

[18]This is analytically convenient, as it yields closed-form solutions for contribution levels. Nevertheless, the result we prove below for the logarithmic utility case can be generalized to arbitrary quasilinear preferences – albeit at the cost of employing extra analytical machinery.

(omitting time subscripts for the time being), under the parameterization $\theta = N\zeta$, with $\zeta > 0$.[19] We begin by supposing first that N is finite, and by abstracting from information-sharing choices. The expected payoff of individual i, gross of information-sharing costs, is

$$\ln(y - v_i) + \theta E\big[\ln(G_{-i} + q_i v_i)\big], \tag{36}$$

where $E[.]$ is the expectation operator and where G_{-i} is the realization of total provision resulting from the contributions of all individuals other than i. The first-order condition for an interior optimal choice of v_i is

$$-\frac{1}{y - v_i} + \theta E\left[\frac{q_i}{G_{-i} + q_i v_i}\right] = 0, \tag{37}$$

We focus here on a scenario where ρ is large enough (and so $1 - \rho$ small enough) that uninformed individuals choose a zero contribution $\underline{v} = 0$, which is analogous to the scenario we examined for the linear case.[20] In a symmetric equilibrium where all informed individuals each select a contribution level $\overline{v} > 0$ and all uninformed individuals each select a contribution level $\underline{v} = 0$, and for N approaching infinity we can write $\lim_{N\to\infty} N\zeta E\big[q_i/(G_{-i} + q_i v_i)\big] = \lim_{N\to\infty} N\zeta\rho/\big(E[G_{-i}] + \overline{v}\big) = \lim_{N\to\infty} N\zeta\rho/\big((Nk-1)\rho\overline{v} + \overline{v}\big) = \zeta/(k\overline{v})$; and so (37) approaches $-1/(y - \overline{v}) + \zeta/(k\overline{v}) = 0$.[21] This gives

$$\overline{v} = \frac{\zeta}{\zeta + k}\, y \equiv \overline{v}(k). \tag{38}$$

Individual contributions are thus decreasing in k, i.e. they are smaller the greater the proportion of individuals making positive contributions. Nevertheless, as in the linear case a higher k translates into higher collective consumption: the effect of an increase in k on expected provision per head, $\rho k\overline{v}$, is $\rho y\zeta^2/(\zeta + k)^2 > 0$.

To characterize information-sharing choices, we must consider the value to an informed donor of having an additional informed donor, which is found by comparing the expected provision outcome with Nk informed individuals with the corresponding outcome with $Nk - 1$ informed

[19] This parameterization implies that, consistently with the linear case, payoff effects of signaling choices remain finite for $N \to \infty$.

[20] The relevant condition is $\rho \geq (k + \zeta)/(2k)$ ($\geq 1/2$). This is found by solving (37) for both agent types, together with the condition $G = \rho Nk\overline{v} + (1/2)N(1-k)\underline{v}$, for $N \to \infty$ – which gives $\overline{v}(k) = y\left(1 - \left(1/2 + k(\rho - 1/2)\right)/\left(\rho(1+\zeta)\right)\right)$, $\underline{v}(k) = y\left(\zeta - 2k(\rho - 1/2)\right)/(1+\zeta)$ – and then solving $\underline{v}(k) = 0$ for ρ.

[21] Contribution choices remain strategic substitutes even for $N \to \infty$: letting $\rho = 1$, re-writing (37) as $-1/(y - v_i) + \theta/\big((N-1)\bar{g}_{-i} + v_i\big) = 0$ (taking provision per head by other players, $\bar{g}_{-i}$, as exogenous), replacing θ with $N\zeta$, taking the limit for $N \to \infty$, and solving for v_i, we obtain $v_i = y - \bar{g}_{-i}/\zeta$, which is decreasing in $\bar{g}_{-i}$.

individuals. The value to another donor of the marginal donor becoming informed is

$$\lim_{N\to\infty} N\zeta \Big(\rho \ln\big(\rho(Nk-1)\overline{v}(k)+\overline{v}(k)\big) + (1-\rho)\ln\big(\rho(Nk-1)\overline{v}(k)\big) - \ln\big(\rho(Nk-1)\overline{v}(k)\big)\Big) = \frac{\zeta}{k} \equiv \Psi(k). \tag{39}$$

The net expected payoff to an informed individual of sending a signal at t is then

$$(1-k_t)(1-k_t s_{-i,t})^{b-1}\Psi(k_t) - c \equiv \Lambda(s_{-i,t}, k_t), \tag{40}$$

i.e. an expression analogous to (2) but with $\Psi(k_t) = \zeta/k_t$ now depending inversely on k_t rather than being constant.

Proceeding just as in the linear case, we can then derive results analogous to Propositions 1-3. In particular, with reference to the effect on $\hat{k}$ of a change in neighbourhood size, we obtain

$$\frac{\mathrm{d}\hat{k}}{\mathrm{d}b} = \frac{1-(1-\hat{r})^b}{1-\rho(1-\hat{r})^b}\,\frac{\rho(1-\rho)(1-\hat{r})^b \ln(1-\hat{r})}{\big(1-\rho(1-\hat{r})^b\big)\big(b-1+\rho(1-\hat{r})^b\big)} < 0. \tag{41}$$

Since the first ratio after the equality in (41) is less than unity, the above effect is smaller in absolute value than the corresponding effect (16) for the linear case. □

In the concave utility case, changes in the stock information affect information-sharing intensity through a second channel: a higher k lowers individual contributions by all types and hence lowers the value of having an additional individual becoming informed, $\Psi(k)$. This is in addition to the effect directly associated with signaling incentives, just as analyzed for the linear case – i.e. a higher k lowers the probability that individual signals will be valuable to receivers. Thus, with concave utility, the negative first-order effect on $\hat{k}$ of an increase in b is dampened by a positive second-order effect stemming from the fact that a lower $\hat{k}$ raises the value of informative signals to senders.

5 Government subsidies

Many developed countries offer government support to private giving – typically delivered in the form of tax relief for donations, but also involving direct support for charities and for fundraising activities.

In the equilibrium described in the previous sections private provision is the combined result of private contribution choices and private information-sharing choices. A natural question to ask is then whether government policies that aim to encourage private provision should be targeted towards voluntary contributions or towards fundraising (or both).

We can look at this question in the context of a variant of our model in which private signaling costs are monetary costs that reduce private disposable income rather than directly appearing as a non-monetary component of individuals' payoffs. This means that, if we assume that donations are bounded above to a maximum of μ, private consumption by individual i in period t equals $1 + \mu - e_{i,t}$, where $e_{i,t}$ includes all signaling costs incurred. Other than for this difference in the interpretation of signaling costs, the resulting expression for individual payoffs is exactly as before (Section 2) – and so is the rest of the analysis. We will use this framework to ask what is the maximum, aggregate level of private provision that government can achieve by subsidizing contributions and then compare it with what it can achieve by subsidizing fundraising instead.

Suppose first that the government subsidizes giving. Given that the maximum contribution per individual is μ, that informed individuals make contributions μ, and that uninformed individuals make zero contributions, the best the government can do is to offer a subsidy that induces uninformed individuals to give, i.e. such that the net of subsidy price is $\theta/2$. By doing so, it will induce all individuals (informed and uninformed) to contribute, which raises the level of expected collective provision. This, however, lowers the benefit from information sharing: the expected payoff from signaling now becomes $(1 - k_t)(1 - k_t s_t)^{b-1}\theta(\rho - 1/2)\mu - c < \Lambda(s_t, k_t)$; so, as long as c is not negligible, individuals do not share information: if $(1 - k_t)\theta\rho\mu > c > (1 - k_t)\theta(\rho - 1/2)\mu$, then the conditions for an interior mixed-strategy equilibrium in signaling choices are met when contributions are not subsidized, whereas with subsidization of contributions the optimal signaling strategy is always $s_t = 0$ for any $c > 0$. Accordingly, when contributions are subsidized information will evolve as $k_{t+1} = k_t\rho + (1 - k_t)/2$. Imposing $k_{t+1} = k_t$, we can then identify a stationary equilibrium information level equal to $\hat{k}' = 1/(3 - 2\rho)$. Thus, government subsidies to private contributions can "crowd out" private fundraising efforts; on the other hand, by inducing uninformed individuals to give, government subsidies to contributions induce them to experiment, which results in more information being gathered through direct experimentation.

Suppose that, instead of subsidizing contributions, the government subsidizes signaling (fundraising) costs. Specifically, suppose that the government offers a subsidy z_t that is just high enough to induce individuals to choose $s_t = 1$, i.e. a subsidy z_t that reduces the net-of-subsidy cost of signaling, $(1 - z_t)c$, to a level such that $\Lambda(1, k_t) = 0$ (once c has been replaced by $(1 - z_t)c$). By ensuring that $\Lambda(1, k_t) = 0$, such a subsidy ensures that all signals are truthful. One can verify that this requires a subsidy such that $c(1 - z_t) = (1 - k_t)^b$. In this case, the stationary equilibrium condition becomes $\hat{k} = \Gamma(1, \hat{k}) = \rho\big(1 - (1 - \hat{k})^{b+1}\big)$; this identifies a stationary equilibrium level of information $\hat{k}''$. Note that, as b gets large, $\Gamma(1,\hat{k})$ converges to $\hat{k}$; this gives $\hat{k}'' = \rho$, which, for $\rho > 1/2$, is greater than $\hat{k}' = 1/(3-2\rho)$. So, for b large enough, subsidizing fundraising results in a higher level of collective consumption than does subsidizing contributions. This result extends to finite values of b, as the following example shows.

Going back to the parameterization detailed at the end of Section 3, suppose now that government subsidizes contributions. The best it can do is offer a subsidy of 1/8, which results in all individuals giving. In this case, signaling effort is zero, independently of the value of b, and we have $\hat{k} = 0.6$, and $E[\hat{G}] \simeq 0.3$. For this value of $\hat{k}$, the expected payoff from signaling is negative.

Now suppose instead that the government subsidizes signaling costs, with a subsidy just large enough to induce $\hat{s} = 1$ and $\Lambda(1,\hat{k}) = 0$ for any given value of $\hat{k}$. Then, for $b = 2$, we have $\hat{k} \simeq 0.635$, $E[\hat{G}]/N \simeq 0.32 > 0.3$. For $b = 4$, we have $\hat{k} \simeq 0.664$, $E[\hat{G}]/N \simeq 0.331$; for $b = 8$, we have $\hat{k} \simeq 0.666$, $E[\hat{G}]/N \simeq 0.333 > 0.3$.

The above conclusion does not imply that subsidies to fundraising are necessarily a more *effective* way of promoting private provision using a given amount of public funds. While a systematic characterization of optimal government policies under budgetary constraints is beyond the scope of this paper – and the present setup is in any case too abstract for studying this question – it is easy to point to scenarios where subsidizing information sharing is comparatively more effective than subsidizing giving. Suppose, for example that the per-capita public funds, F, required to grant government subsidies attract a premium, $N\varphi(F)$, reflecting the (unmodeled) efficiency losses associated with collecting public funds through taxation (the derivative $\varphi'(F)$ is often referred to as the *marginal cost of public funds* in the public finance literature); and suppose that $\varphi'(0) = 0$ and $\varphi''(F) > 0$. Also let F^{CS} be the per-capita amount required to fund a general (anonymous) contribution subsidy that induces giving by uninformed individuals, i.e. $F^{CS} = \mu(1-\theta/2)$, and let $\Delta\bar{U}^{CS}$ denote the change in the mean payoff associated with this change in contributions. If the function $\varphi(F)$ is such that $\varphi(F^{CS}) > \Delta\bar{U}^{CS}/N$, a contribution subsidy will not be worthwhile, whereas there will always be some positive level of subsidization of information-sharing costs that is worthwhile: for $N \to \infty$, the marginal effect $d\bar{U}/N$ of a marginal increase in an information-sharing subsidy ξ that reduces the signaling cost from c to $c-\xi$, starting from $\xi = 0$, equals $-(\mathrm{d}\hat{k}/\mathrm{d}c)\Psi - (\mathrm{d}\hat{r}/\mathrm{d}c)c$; this is positive (the stationary-state, non-cooperative level of signalling is suboptimal[22]) and hence always greater than $\varphi'(0) = 0$ – implying there will always be an interior optimum with $\xi > 0$.

[22]Totally differentiating the equilibrium conditions and simplifying the resulting expressions, we obtain $-(\mathrm{d}\hat{k}/\mathrm{d}c)\Psi - (\mathrm{d}\hat{r}/\mathrm{d}c)c = \left(1 + b\rho(1-\rho)/\left((1-\hat{k})(1-\rho(1-\hat{r})^b)^2\right)\right) c > 0$.

6 Summary and discussion

We have described a model of private provision choices in the presence of interpersonal information transmission, where private information on modes of collective provision can be transmitted by social proximity, but where sending signals is an endogenous, costly decision on the art of the sender. Unlike in the case of private goods, informed individuals face positive incentives to engage in costly information transmission towards less informed social neighbours. In this setting information sharing has the characteristics of a local public good, even if contributions are directed towards the provision of a pure collective good.

We have shown that in this model information-sharing incentives are stronger the smaller are individuals' social neighbourhoods, and this effect always dominates any advantage that larger neighbourhoods may have with respect to the diffusion of information, resulting in a lower level of provision of collective consumption at the economy level. It is worthwhile noting that, in interpreting this result in the context of real-world social connections, the size of communities need not be understood in a geographical sense. What is relevant to our arguments is the size of social communities – which include communities of co-workers, on-line communities, and the like. Thus, for example, while an increase in population density might be thought of as implying an increase in the size of geographical social neighbourhoods, it might actually lead to more rather than less information sharing if it is accompanied by a rise in the relevance of smaller, non-geographical social communities in individuals' social lives.

We have shown that these results generalize to a variety of settings: when there are heterogeneous preferences, when preferences are strictly convex, and when agents are forward-looking (discussed in the Appendix). To the extent that individuals are forward-looking (patient) rather than myopic, they will also account for the non-local effects of their information sharing efforts on future information stocks and future provision outcomes – both directly and indirectly through the effect of information stocks on further learning. In the Appendix we show that, other things equal, attaching a positive weight to future outcomes raises information-sharing incentives and the stationary-state stock of information, $\hat{k}$; but the effect of changes in neighbourhood size is qualitatively the same as in the myopic case, i.e. the results stated in Propositions 2 and 3 generalize to the forward-looking case.

We have abstracted from the complications that would arise in a social networks with stable links: the presence of stable links in a large network of individuals with repeated signaling results in a high-dimensional mapping from individual histories (of observed signals) to beliefs about the distribution of information in the network. This makes it difficult to provide a sharp characterization of equilibrium information-sharing choices with information dynamics for the specific problem we are analyzing (beyond just describing the structure of such an equilibrium in general terms).[23] Moreover, with stable links, it is not the number of links but their relative locations (the topological structure of the network) that matters for how much information is eventually accumulated and transmitted. Thus, with stable links, one cannot just ask what the effect of neighbourhood size is on long-run information transmission; rather, the implications of alternative network topologies must be considered for any given number of links. Nevertheless, it can be shown that a mechanism analogous to the free-riding mechanism that we have highlighted in Proposition 1 will be at work in every round of the relevant sequential game even when links are stable.

Our results have been derived in a framework where private contributions to collective consumption arise solely from consumption motives, but we have shown that results carry over to the case where giving stems from warm-glow motives. There are other possible interpretations of the motives for private giving that we have not examined here – such as status signaling or reciprocity effects. It should be noted, however, that these alternative interpretations of giving motives do not automatically extend to information-sharing motives. Developing status signaling-based and/or reciprocity-based theories of information sharing is left for future research.

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[23]Static strategic information transmission problems in networks (without repeated choice) are more tractable; see Jackson (2008) for a survey of the relevant literature.

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Appendix: Forward-looking agents

Proposition 6 *When agents are forward-looking: (i) an increase in neighbourhood size lowers information-sharing intensity and the expected level of collective provision; (ii) for a given neighbourhood size, an increase in the discount factor increases information-sharing incentives and the expected level of collective provision.*

PROOF: Let preferences be represented by an intertemporally separable utility function expressed as the present value of instantaneous utility discounted by a discount factor $\delta \in (0, 1]$. We develop our argument starting from a scenario where N is finite and then derive limit results for $N \to \infty$ (consistently with our analysis of the myopic case). Also, for the sake of expositional simplicity, we assume from the outset that i's signaling choice is the same vis-à-vis all her neighbours, i.e. i either sends a signal to all of her neighbours or to none of them (for $N \to \infty$, this is equivalent to assuming that signaling choices are independent across receivers; see Footnote 8).

Let $a_{i,t} \in \{0,1\}$ denote a realization of the signaling strategy of agent i at t , with $a_{i,t} = 0$ representing no signal and $a_{i,t} = 1$ representing an informative signal. We can then write the discounted expected indirect utility for an informed individual i at time t – leaving out i's income and direct contributions made by i – as a function of the current total stock of information, $K_t = Nk_t$, and of the realization, $a_{i,t} \in \{0,1\}$, of her current information-sharing, mixed-strategy choice, $s_{i,t}$, for given current (symmetric) mixed-strategy information choices, $s_{-i,t}$, made by other informed individuals:

$$\tilde{V}_{i,t}(K_t; a_{i,t}, s_{-i,t}) = \theta\mu E\left[K_{t+1}\right] - a_{i,t}bc + \delta E\left[\bar{V}_{i,t+1}(K_{t+1})\right], \tag{42}$$

where $E[.]$ is the expectation operator. The first term on the right-hand side of (42) is the expected value of current collective consumption; this involves the stock K_{t+1}, which evolves from K_t according to $K_{t+1} = \tilde{K}_{t+1}(K_t; a_{i,t}, s_{-i,t})$ (the structure of this mapping is discussed below for the case $N \to \infty$, the focus of this proof). The second term on the right-hand side of (42) is the cost of signaling. The third term on the right-hand side of (42) is expected, discounted continuation utility from $t+1$ onwards, given K_{t+1} and the mixed-strategy choice $\tilde{s}_{t+1}(K_{t+1})$, i.e.

$$\bar{V}_{i,t+1}\left(K_{t+1}\right) = \tilde{s}_{t+1}(K_{t+1})V_{i,t+1}\left(K_{t+1}; 1, \tilde{s}_{t+1}(K_{t+1})\right) + \left(1 - \tilde{s}(K_{t+1})\right)V_{i,t+1}\left(K_{t+1}; 0, \tilde{s}_{t+1}(K_{t+1})\right), \tag{43}$$

where $\tilde{s}_{t+l}(K_{t+l}), l \geq 1$, denotes symmetric mixed-strategy information choices by all players in the relevant sub-games;[24] and where we are making use of the fact that $E\left[G_t\right] = \mu E\left[K_{t+1}\right]$.

Using (42), a symmetric, subgame perfect, mixed-strategy equilibrium in signalling choices with mixed strategies, $s_{i,t} = s_{-i,t} = s_t \equiv \tilde{s}_t(K_t)$, $\forall t$, is recursively identified by the condition

$$\tilde{V}_{i,t}(K_t; 1, s_t) - \tilde{V}_{i,t}(K_t; 0, s_t) - \Delta EPVC = 0, \tag{44}$$

where $\Delta EPVC$ is a term that accounts for the effect of the realization $a_{i,t}$ on the expected present value of contributions made by i (the role of this term for $N \to \infty$ is discussed below). Substituting the expressions on the right-hand side of (42), we can write

$$\begin{aligned}\tilde{V}_{i,t}(K_t; 1, s_t) - \tilde{V}_{i,t}(K_t; 0, s_t) = {} & \Psi E\left[\tilde{K}_{t+1}(K_t; 1, s_t) - \tilde{K}_{t+1}(K_t; 0, s_t)\right] \\ & + \delta E\left[\bar{V}_{i,t+1}\left(\tilde{K}_{t+1}(K_t; 1, s_t)\right) - \bar{V}_{i,t+1}\left(\tilde{K}_{t+1}(K_t; 0, s_t)\right)\right] - bc.\end{aligned} \tag{45}$$

In turn, for N approaching infinity, the term $\tilde{K}_{t+1}(K_t; a_{i,t}, s_t)$ can be approximated by the following expres-

[24]In a subgame perfect equilibrium, equilibrium strategies must account for the relationship between current strategic choices and the equilibrium strategies that result from such choices in the continuation game.

sion:[25]

$$
\begin{aligned}
&K_t\rho + (N - K_t - b(1-K_t/N))\big(1-(1-(K_t/N)s_t)^b\big)\rho \\
&+b(1-K_t/N)\Big(a_{i,t} + (1-a_{i,t})\big(1-(1-(K_t/N)s_t)^{b-1}\big)\Big)\rho \;\equiv \Xi(K_t, a_{i,t}, s_t).
\end{aligned} \tag{46}
$$

Thus, for N approaching infinity, the first expectation on the right-hand side of (45) approaches

$$
\lim_{N\to\infty} E\big[\tilde{K}_{t+1}(K_t;1,s_t) - \tilde{K}_{t+1}(K_t;0,s_t)\big] = \lim_{N\to\infty} E\big[\Xi(K_t;1,s_t) - \Xi(K_t;0,s_t)\big] = \rho\, b(1-k_t)(1-k_t s_t)^{b-1}. \tag{47}
$$

The first term on the right-hand-side of (45) is thus equal to b times the first term of (2) as defined earlier for the myopic case.

If we next expand the second term in the right-hand side of (45) we obtain an infinite series, the first term of which can be written as

$$
\begin{aligned}
&\delta\theta\mu E\Big[\bar{K}_{t+2}\big(\tilde{K}_{t+1}(K_t;1,s_t);\tilde{s}_{t+1}(\tilde{K}_{t+1}(K_t;1,s_t))\big) - \bar{K}_{t+2}\big(\tilde{K}_{t+1}(K_t;0,s_t);\tilde{s}_{t+1}(\tilde{K}_{t+1}(K_t;0,s_t))\big)\Big] \\
&+\delta\Big(\tilde{s}_{t+1}(\tilde{K}_{t+1}(K_t;1,s_t)) - \tilde{s}_{t+1}(\tilde{K}_{t+1}(K_t;0,s_t))\Big)c,
\end{aligned} \tag{48}
$$

where

$$
\bar{K}_{t+1}(K_t;s_t) = s_t\tilde{K}_{t+1}(K_t;1,s_t) + (1-s_t)\tilde{K}_{t+1}(K_t;0,s_t). \tag{49}
$$

For N approaching infinity, $\tilde{s}_{t+1}\big(\tilde{K}_{t+1}(K_t;1,s_t)\big)$ and $\tilde{s}_{t+1}\big(\tilde{K}_{t+1}(K_t;0,s_t)\big)$ converge to a common limit, $\bar{s}_{t+1}(k_{t+1})$, and so the second term on the right-hand side of (48) vanishes. Expanding the $t+2$ period difference within the expectation operator of the first term of (48) by using (46) in conjunction with (49), noticing that for N approaching infinity the ratio K_{t+l}/N becomes independent of $a_{i,t+l}$, factoring common terms and simplifying the resulting expression, we can express (48) as

$$
\delta\Psi \lim_{N\to\infty} \big(\tilde{K}_{t+1}(K_t,1,s_t) - \tilde{K}_{t+1}(K_t,0,s_t)\big)\big(1-k_{t+1}\bar{s}_{t+1}(k_{t+1})\big)^b. \tag{50}
$$

Now use (47) to substitute for $\lim_{N\to\infty}\big(\tilde{K}_{t+1}(K_t,1,s_t) - \tilde{K}_{t+1}(K_t,0,s_t)\big)$; (50) becomes

$$
\delta\Psi\rho b(1-k_t)(1-k_t s_t)^{b-1}\big(1-k_{t+1}\bar{s}_{t+1}(k_{t+1})\big)^b. \tag{51}
$$

Proceeding in the same way, the next term in the (series) expansion of the second term on the right-hand side of (45) can be shown to equal

$$
\delta^2\Psi\rho^2 b(1-k_t)(1-k_t s_t)^{b-1}\big(1-k_{t+1}\bar{s}_{t+1}(k_{t+1})\big)^b\big(1-k_{t+2}\bar{s}_{t+2}(k_{t+2})\big)^b; \tag{52}
$$

and so on for all subsequent terms in the series – i.e. the $1+l$th term in the series ($l = 0,\ldots,\infty$) is

$$
\delta^l\Psi\rho^l b(1-k_t)(1-k_t s_t)^{b-1}\prod_{j=0}^{l}\big(1-k_{t+l}\bar{s}_{t+l}(k_{t+l})\big)^b. \tag{53}
$$

[25]The structure of (46) is analogous to that of (8). The second term on the right-hand side refers to those individuals who are not in i's neighbourhood at t – and are thus unaffected by i's actions at t; the last term refers to i's neighbours at t.

In a stationary equilibrium, with $k_t = k_{t+l} = \hat{k}$ and $s_t = \bar{s}_{t+l}(k_{t+l}) = \hat{s},\ l \geq 1$, we can thus write (45) as

$$b(1-\hat{k})(1-\hat{k}\hat{s})^{b-1}\sum_{l=0}^{\infty}\delta^l\rho^l(1-\hat{k}\hat{s})^{lb}\,\Psi - b\,c = \frac{b(1-\hat{k})(1-\hat{k}\hat{s})^{b-1}}{1-\delta\rho(1-\hat{k}\hat{s})^b}\,\Psi - b\,c, \tag{54}$$

(The last equality obtains because $\delta \in [0,1)$, $\rho \in (0,1)$, $(1-\hat{k}\hat{s})^b \in (0,1)$, and so $\delta\rho(1-\hat{k}\hat{s})^b < 1$.) Also, for $N \to \infty$, i's signalling choice at t has a vanishing effect on the likelihood of i herself being informed (and thus making positive contributions) in each of the following periods, and so the term $\Delta EPVC$ in (44) also vanishes. Dividing through by b, the indifference condition for an equilibrium in strictly mixed strategies can be then written as

$$\frac{(1-\hat{k})(1-\hat{k}\hat{s})^{b-1}}{1-\delta\rho(1-\hat{k}\hat{s})^b}\,\Psi - c \equiv \Lambda(\hat{s},\hat{k}) = 0, \tag{55}$$

where the left-hand side coincides with the expression that was previously given in (2) when $\delta = 0$ (the myopic case). Note that the assumed random assignment structure implies that information sharers attach a vanishingly small probability of interacting again with their social neighbours in future periods. Therefore, forward-looking incentives have the same structure as myopic ones – with the exception that forward-looking agents face a higher incentive to signal, as the stock of information is persistent over time.

Comparative statics on the steady state with respect to changes in b can then be obtained as before (proofs of Propositions 2 and 3), using the condition $\tilde{\Lambda}(\hat{r},\hat{k}) = 0$ (obtained from (55)) in conjunction with (11). Proceeding as in the proof of Proposition 2, and manipulating terms, we obtain

$$\frac{\mathrm{d}\hat{r}}{\mathrm{d}b} = \frac{(1-\hat{r})(1-\delta Z^2)\ln(1-\hat{r})}{W}, \tag{56}$$

where $Z \equiv \rho(1-\hat{r})^b$, and $W \equiv (b-1)(1-\delta Z^2) + Z(1-\delta Z) + \delta(Z - Z^2)$. Since $Z \in (0,1)$ and $\delta \in (0,1)$, the expression W is positive. The numerator of (56) is negative, and so $\mathrm{d}\hat{r}/\mathrm{d}b < 0$, as in the myopic case. Moving on to the effect of a change in b on $\hat{k}$, proceeding as in the proof of Proposition 3, we obtain

$$\frac{\mathrm{d}\hat{k}}{\mathrm{d}b} = \frac{(1-\rho)Z\big(1-\delta Z\big)\ln(1-\hat{r})}{\big(1-Z\big)W} < 0. \tag{57}$$

Next we look at how forward-looking preferences affect information sharing. We can first focus on the total derivative

$$\frac{\mathrm{d}\hat{k}}{\mathrm{d}\delta} = \Theta_{\hat{r}}\frac{\mathrm{d}\hat{r}}{\mathrm{d}\delta}, \tag{58}$$

where

$$\frac{\mathrm{d}\hat{r}}{\mathrm{d}\delta} = -\frac{\tilde{\Lambda}_{\hat{k}}\Theta_\delta + \tilde{\Lambda}_\delta}{\tilde{\Lambda}_{\hat{k}}\Theta_{\hat{r}} + \tilde{\Lambda}_{\hat{r}}} = \frac{(1-\hat{r})(1-Z)}{W} > 0; \tag{59}$$

since $\Theta_{\hat{r}} > 0$ (as shown in the proof of Proposition 3), we conclude that an increase in δ raises the stationary-state stock of information and collective provision. □