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Unemployment Risk and Wage Differentials

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Abstract

Workers in less secure jobs are often paid less than identical-looking workers in more secure jobs. We show that this lack of compensating differentials for unemployment risk can arise in equilibrium when all workers are identical and firms differ only in job security (i.e. the probability that the worker is not sent into unemployment). In a setting where workers search for new positions both on and off the job, the worker's marginal willingness to pay for job security is endogenous: it depends on the behavior of all firms in the labor market and increases with the rent the employing firm leaves to the worker. We solve for the labor market equilibrium, finding that wages increase with job security for at least all firms in the risky tail of the distribution of firm-level unemployment risk. Meanwhile, unemployment becomes persistent for low-wage and unemployed workers, a seeming pattern of 'unemployment scarring' created entirely by firm heterogeneity. Higher in the wage distribution, workers can take wage cuts to move to more stable employment.

JEL-Code: J310, J630.

Keywords: layoff rates, unemployment risk, wage differentials, unemployment scarring.

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1 Introduction

When transitions to unemployment inflict a loss of income or utility, workers are willing to give up a part of their wages in exchange for more job security. At first glance, this risk of becoming unemployed might seem as just another job attribute that workers dislike, e.g. the need to spend a lot of additional time commuting or traveling or a requirement to put in higher levels of effort. However, job security, i.e. the absence of unemployment risk, is quite different from typical amenities, as we describe below.

For one, job security is naturally complementary to the desirability of the job along all dimensions, including, prominently, the wage. For workers, it is one thing to lose a job that is marginally better than unemployment; it is quite another thing to lose the best possible job out there. In a frictional labor market, a job loss incorporates not only the immediate drop in income when becoming unemployed, but also its impact on the worker's subsequent labor market outcomes. It is much more difficult and time-consuming to recover to the best job than it is to find a marginally attractive job. We find that a given incremental increase in job security is valued more in high-wage employment than low-wage employment, ceteris paribus. Moreover, the valuation of job security grows with the wage earned at an increasing rate. Put differently, the marginal willingness to pay for job security is an increasing and convex function of the firm's wages. By contrast, in most hedonic wage models the marginal willingness to pay is taken to be exogenously given and *constant* across wage levels. (See e.g. Bonhomme and Jolivet (2009), Hwang, Mortensen, and Reed (1998), and Sullivan and To (2013)[1]).

A higher unemployment risk can affect firms in several ways. First, as sketched above, it lowers the value of employment for the firm's employees, increasing the risk of losing workers to other firms, for a given wage. Therefore, retention becomes a problem, and firms offering less secure jobs would need to pay higher wages if they would want to keep worker outflows at the same level. Second, a higher unemployment risk also affects the profit maximization of the firm directly, because safer firms have longer employment relationships, ceteris paribus. The impact of unemployment risk on the expected length of the employment relationship changes the firms' recruitment and retention incentives when they consider which wage to set. We show that firms offering less secure jobs have less incentives to compete with other firms, both when fending off poaching by competing firms as well as when trying to recruit employed workers. This additional effect on firms' wage setting behavior cannot be captured in some general utility specification on the workers' side. In order to fully capture the impact of job security and job loss, one needs to bring both sides of the market together and solve for the equilibrium.

Overall, the resulting cost of a job loss depends on market characteristics, prominently the extent of frictions, and how these affect firms' optimal wage setting responses (which, in turn, take into account the endogenous valuation of job security on the workers' side). Given that it takes time for a laid-off worker to climb the job ladder to his previous position, he clearly loses earned wages during the process. But, in a market where different firms offer jobs with distinct levels of job security, a job loss may also imply an increase in future layoff risk

[1] In order to a hedonic wage model deliver this complementarity, we would need to assume a complementarity between wages and job amenities *within* the utility function. However, this leads one to easily ignore that anything that influences firms' behavior, labor market flows, or labor market conditions in general, will affect this utility relation.

since the displaced worker may take a job at a firm with lower job security. This potential increase in probability of future unemployment spells is another component of the cost of job loss.[2].

In this paper, we propose an equilibrium theory of frictional labor markets that incorporates the different, endogenous, nature of the valuation of job security. In our model, we follow Burdett and Mortensen (1998), henceforth referred to as BM, and introduce search frictions and on-the-job search into an otherwise competitive setting. Our sole deviation from the BM setup is that firms differ in the job security they provide.[3] In our model, job security enters directly both in the firm's optimization and also in the worker's valuation in an endogenous, nonlinear way. Even though this complicates the setup significantly, we are able to characterize the equilibrium almost as easily as the standard BM model.

In our model's equilibrium, riskier firms need to increase their wages more than safer firms when offering higher values, while they do not gain as much in firm size as safer firms would. As a result, riskier firms will find it optimal to offer lower values to workers, and workers move, job by job, towards increasingly safe firms. Wages will increase with job security at the bottom of the wage/job security distribution, causing the firms with the lowest job security to be the ones with the lowest wages, contrary to competitive compensating wage theory. While on the workers' side the marginal willingness to pay for job security is increasing and convex in wages, competition on the firms' side pushes wages upwards with job security throughout the firm distribution. The firm-competition component dominates the workers increasing marginal willingness to pay for job security as long as the density of the firm-level job security distribution is increasing, constant, or not decreasing too sharply. Only when the density of the firm-level job security distribution decreases sharply enough *could* wages drop with job security, in which case some workers would be observed to take wage cuts. However, whether wage cuts in fact occur depend also on the rate at which workers receive alternative offers on the job; the higher the incidence of on-the-job search, the less likely we are to observe a negative relationship between job security and wages. In fact, we show that wages can be strictly increasing with job security for any continuous distribution of firm-level unemployment risk if the arrival rate of job offers to employed workers is high enough.[4]

Overall, our equilibrium is consistent with the lack of compensating wage differentials and a positive correlation between wages and job security. This pattern has been documented e.g. in Bonhomme and Jolivet (2009) and Mayo and Murray (1991). The model is also consistent with 'unemployment scarring' through both repeated unemployment spells and lower wages, as found e.g. by Stevens (1997) and Kletzer (1998). Surprisingly, such a pattern is observed even though the heterogeneity lies entirely on the firm's side of the market.[5]

[2]The empirical literature finds that a layoff raises the prospect of shortened employment spells in the future, increasing the probability of future job losses (e.g. Stevens 1997, Kletzer 1998). This reduction in subsequent employment durations is responsible for a significant part of the cost of a transition into unemployment (Eliason and Storrie 2006, Boheim and Taylor 2002, Arulampalam et al. 2000). Moreover, for displaced workers of a given quality, commonly, new jobs come with lower wages, and simultaneously with a higher risk of renewed unemployment (Cappellari and Jenkins 2008, Uhlendorff 2006, Stewart 2007).

[3]In order to isolate the impact of the job security channel, firms will not differ in e.g. the instantaneous productivity of jobs.

[4]Given that the density has full support.

[5]Additionally, the model produces a correlation between firm size and job security through the same channel as Burdett and Mortensen (1998), once the ranking of values provided to workers is taken into account.

The presence of heterogeneous degrees of unemployment risk across firms is the unique feature in our model. In search models[6], exogenous shocks that separate workers from their jobs at a rate that is independent of firm identity, can naturally be interpreted as supply side shocks. In this category, job matches are randomly broken as workers relocate for personal or family issues, among other reasons. Differently, in our setup, the unemployment risk may arise from the relationship between the firm and its employees through the productive process. For example, differences in the quality of managerial practices (see Syverson, 2004) can generate different unemployment risk across firms. To spell this out in more detail, we can interpret the firm as a problem-solving production function, in which workers use their knowledge to solve regularly faced tasks, as in the knowledge hierarchies literature (See Garicano, 2000 and Garicano and Rossi-Hansberg, 2006). As the nature of the tasks changes, the worker may eventually face a recurring problem whose solution that he cannot solve. In this case, in the absence of any additional help, the worker becomes unproductive and the firm's optimal choice is to lay him off. However, better managerial practices and codes inside the firm may expand the total knowledge available to the worker, reducing the probability that displacement may occur (In the online Appendix D, we present an extended version of this argument). Therefore, a firm with better managerial practices – what Cremer(1993) calls *Corporate Culture* or even what Prescott and Visscher (1980)[7] labels as a type of *Organization Capital* – would have lower layoff rates. The empirical literature has corroborated the importance of managerial practices and organization capital. Bloom and Reenen (2007) and Huselid (1995) show that different managerial practices are at the core of differences in firms' economic performance.[8]

Finally, if firm heterogeneity is a leading factor behind the equilibrium patterns described, a typical worker can face more risk in the labor market than he does according to theories with a constant firm-level unemployment risk. In our theory, after a worker falls off the job ladder into unemployment, the first rungs of the wage ladder will be more slippery than the higher rungs. Therefore bad labor market outcomes can persist, and time and luck is needed before a recently unemployed worker can find traction on the job ladder. Until then, unemployment begets low wages and unemployment. Overall, bad outcomes will be correlated over time, as will be good outcomes, increasing the distance between best-case and worst-case outcomes in the labor market in terms of life-time discounted utility. This increase in risk might have implications for policies to insure the worker against labor market risks.[9] For example, consider the case of policies which aim to increase the earnings capacity of

[6]Separations into unemployment in standard models of on-the-job search can capture both supply and demand side shocks that lead to break up of the match. Here we focus on heterogeneity in this break-up rate that originates on the firm side. In any case, our analysis is consistent with a homogenous amount of unemployment risk originating on the workers' side which is added to the firm-specific unemployment risk, to yield overall risk δ in a firm.

[7]A formal argument for different job destruction rates can be made using Prescott and Visscher' first definition of organization capital, i.e., the firm's knowledge of the macth quality between worker abilities and tasks. For a discussion about Organization Capital and labor turnover, see Carlin, Chowdhry, and Garmaise, 2012.

[8]Similarly, Atkeson and Kehoe (2005) show that *organization rents* represent one third of the size of rents received by physical capital.

[9]It also has further implications for consumption and saving choices of workers who have just become unemployed, and, ex ante, for employed workers who face differing risks of becoming unemployed, with more dire consequences if they do fall off the job ladder into unemployment. See Lise (2013) for a model linking the climbs of the wage ladder through on-the-job search, and the drops from the wage ladder, to consumption and savings decisions. In his model, following BM, the risk of dropping from the ladder is constant across

unemployed workers and are often thought to decrease the probability that these workers will subsequently slide back into unemployment. If, however, firm heterogeneity is a key determinant of the probability of flowing back into unemployment, and the workers in question keep taking jobs in the same risky firms, the foreseen reduction in repeat unemployment might not fully come to pass.

2 Model

A measure 1 of risk-neutral firms and a measure m of risk-neutral workers live forever in continuous time, discounting the future at rate r. When not matched with a firm, a worker receives unemployment benefits b. When matched with a firm, the worker produces output p, which is the same in any firm. Firms, however, differ in the probability δ with which they send workers back into unemployment. We index firms by this probability, and will refer to a high-δ firm as a "risky firm" and a low-δ firm as a "safe firm." One can think of the heterogeneity in δ as standing in for the quality of the firm's management, manifested, for example, when the tasks that workers' do occasionally become obsolete and they need to be given new tasks. In a safe firm, managers are more likely to provide good guidance for the new task, such that the worker is likely to maintain his productivity level p. In a risky firm, managers are less likely to be able to support the worker's transition to the new task and hence, they are more likely to dismiss the worker.[10]

The distribution function of firm types is $H(\delta)$; the distribution may (but does not have to) contain mass points. Apart from the differences in firms' layoff risk δ, the remaining setup follows Burdett and Mortensen (1998). This means that search frictions in the labor market prevent workers from instantaneously matching with the best job offer in the market. Instead, from time to time workers meet randomly with one of the firms in the market. In fact, unemployed workers receive a single job offer at a time at Poisson arrival rate λ_0, while employed workers do so at rate λ_1. An offer is a wage-layoff risk combination (w, δ) which specifies the wage w that the firm, which has layoff rate δ, commits to pay as long as the match lasts (it is the same as the wage of workers already employed at the firm). The job offer must be accepted or rejected on the spot, and when rejected it cannot later be recalled. Firms are able to hire everyone who accepts their wage, and they choose this wage to maximize their steady state profits. In setting profit maximizing wages, firms take into account both the distribution of wages posted by the other firms in the market and also how workers compare wage offers from firms with different layoff risks. We first turn to *workers*' decisions in the face of differentially risky firms posting different wages.

workers and jobs.

[10]See also appendix D in which we have spelled out these microfoundations more elaborately. In this interpretation, unproductive tasks by definition have a productivity low enough that no firm wants to keep the worker in it, just in case any future change in the task portfolio render the worker productive again. Additionally, all productive tasks have productivity p for simplicity. We take elements here from the 'knowledge hierarchy' literature, following Garicano (2000) and Garicano and Rossi-Hansberg (2006). The environment as we sketch it can also be, loosely, considered to be a normalization with respect to trend, so becoming 'unproductive' means becoming unproductive relative to trend.

2.1 Workers' Job Offer Acceptance Decisions

Due to the frictions in the labor market, a worker receives a new offer of employment only occasionally (at Poisson arrival rate λ_0 when unemployed, or λ_1 when employed). The worker is offered a wage w by a firm with layoff risk δ, and he will accept any offer that improves his life-time expected discounted income. Because a new offer could come with higher job security but also with a lower wage (or vice versa), the worker needs to fully rank these two-dimensional job offers by their life-time expected discounted income.

Call V_0 the life-time expected discounted income of a worker who is currently unemployed, and $V(w, \delta)$ the value for a worker currently employed at a firm that pays wage w and has a layoff risk δ. To establish notation, consider that firms with layoff risk δ symmetrically post according to a possibly pure strategy with cdf $\hat{F}(w|\delta)$. We can express the value functions of workers as follows: for an unemployed worker, the flow Bellman equation is

$$rV_0 = b + \lambda_0 \int\int \max\{V(w, \delta) - V_0, 0\}\mathrm{d}\hat{F}(w|\delta)\mathrm{d}H(\delta). \tag{1}$$

The flow value of searching for a job when unemployed, rV_0, equals the benefit flow b and the expected capital gain of the job search. The latter, in the right-most term in equation (1), results from receiving, at rate λ_0, a job offer (w, δ) randomly drawn from $\int\int \mathrm{d}\hat{F}(w|\delta)\mathrm{d}H(\delta)$. An accepted job offer (w, δ) will improve the worker's lifetime expected value by $V(w, \delta) - V_0$. Therefore, an offer will be accepted if and only if this term is strictly positive.[11] Similarly, the flow value for an employed worker at a δ-firm earning a wage w is:

$$rV(w, \delta) = w + \lambda_1 \int\int \max\{V(w', \delta') - V(w, \delta), 0\}\mathrm{d}\hat{F}(w'|\delta')\mathrm{d}H(\delta') + \delta(V_0 - V(w, \delta)). \tag{2}$$

Therefore, the value of holding this job is given by the flow wage w plus the expected capital gain of moving to a firm offering a higher employment value. Outside offers are received at rate λ_1, randomly drawn from $\int\int \mathrm{d}\hat{F}(w|\delta)\mathrm{d}H(\delta)$. Offers are accepted if and only if $V(w', \delta') - V(w, \delta)$ is strictly positive. The worker also faces an expected capital *loss* from becoming unemployed, represented by the term $\delta(V_0 - V(w, \delta))$.

If offers arrive from firms with layoff risks identical to the current firm, the lowest wage that triggers a job-to-job transition follows naturally from equation (2). Since $V(w, \delta)$ increases with w, wages above the current wage are acceptable and those below are not. Differently, if offers originate from firms with different levels of job security, we start by looking how workers rank different combinations of wages and layoff risks. Consider a worker's current employment in a firm with (w_1, δ_1), and consider an alternative job offer from a firm with (w_2, δ_2): the worker is indifferent when $V(w_2, \delta_2) = V(w_1, \delta_1)$; as equation (2) indicates, this occurs precisely when:

$$w_2 = w_1 + (\delta_2 - \delta_1)(V(w_1, \delta_1) - V_0). \tag{3}$$

Then, if the worker, currently with $V(w_1, \delta_1)$, is offered a wage larger than w_2 in a new firm with layoff risk δ_2, he will accept the new job, rejecting it otherwise. Thus, to move to a riskier firm, the worker needs 'compensation' in the form of a wage increase; this increase must cover the additional amount of risk taken on, multiplied by the

[11] In the case of a tie, the worker will remain in his current job.

cost of the layoff ($V(w,\delta)-V_0$). We will return to discuss the 'value' of job security in more detail. Let us first use equation (3) to completely characterize the value $V(w,\delta)$ associated with employment in any δ-firm at wage w. The reservation property and the established indifference condition above, lead to

$$rV(w,\delta) = w + \lambda_1 \int \Big(\int\limits_{w+(\delta'-\delta)(V(w,\delta)-V_0)} \big(V(w',\delta') - V(w,\delta)\big) \mathrm{d}\hat{F}(w'|\delta') \Big) \mathrm{d}H(\delta') + \delta(V_0 - V(w,\delta)). \quad (4)$$

This implies that (given the indifference at the optimal acceptance choices),[12]

$$\frac{\partial V(w,\delta)}{\partial w} = \frac{1}{r+\delta+\lambda_1 \int \Big(1-\hat{F}\big(w+(\delta'-\delta)(V(w,\delta)-V_0)|\delta'\big)\Big)\mathrm{d}H(\delta')}. \quad (5)$$

To characterize $V(w,\delta)$ along the δ-dimension, we can similarly find

$$\frac{\partial V(w,\delta)}{\partial \delta} = -\frac{V(w,\delta)-V_0}{r+\delta+\lambda_1 \int \Big(1-\hat{F}\big(w+(\delta'-\delta)(V(w,\delta)-V_0)|\delta'\big)\Big)\mathrm{d}H(\delta')}. \quad (6)$$

Together with the relevant initial conditions, equations (5) and (6) form a partial differential equation that we can solve to fully characterize $V(w,\delta)$. This leads to the results stated in Lemma 1, where, for notational ease in its last line (and for use in the subsequent analysis of the firms' decisions), we define $w(V,\delta)$ as the wage in a firm with layoff risk δ that implies a life-time expected value V to the worker. In other words, $w(V,\delta)$ is the inverse of $V(w,\delta)$ in w, given δ.

Lemma 1. *(a) Given the reservation wage out of unemployment R_0 and wage distributions $\hat{F}(w|\delta)$, the value function of employed workers $V(w,\delta)$ is the solution to the partial differential equation, defined by* (5) *and* (6)*, with initial conditions for every δ defined by*

$$w(V_0,\delta) = R_0, \text{ and } V_0 = \frac{\lambda_0 R_0 - \lambda_1 b}{r(\lambda_0-\lambda_1)} \quad (7)$$

(b) In equilibrium, R_0 satisfies

$$R_0 = b + (\lambda_0 - \lambda_1)\int_{V_0} (V-V_0)dF(V) \quad (8)$$

with

$$F(V) = \int \hat{F}(w(V,\delta)|\delta)dH(\delta). \quad (9)$$

[12] Equation (5) has a familiar appearance to the differential equation that captures the relation between values and wages in the standard Burdett and Mortensen setup (where there is only one value of δ for all firms); there it is given by $\mathrm{d}V(w)/\mathrm{d}w = 1/(r+\delta+\lambda_1(1-F(w)))$. Notice, however, that in equation (5) the derivative $\mathrm{d}V(w)/\mathrm{d}w$ is a function of *both* w and $V(w,\delta)$, instead of only the former, while $V(w,\delta)$ is precisely the endogenous object that we are after. Even with this additional dimension, equation (5), as a differential equation, remains standard and tractable.

We have relegated all proofs to the appendix. Lemma 1 shows that, given the reservation wage out of unemployment, R_0, it is possible to solve directly (in one iteration) for all values as a function of the associated wage and the firm-level layoff risk $V(w, \delta)$. In the second part of the lemma, R_0 is found as the solution of the fixed point problem in (8), given the set of per-firm-type posting stategies $\hat{F}(w|\delta)$ for every type δ and the distribution of firm types $H(\delta)$. Notice that we can proceed to the firms' maximization and resulting distributions, leaving R_0 implicit for the time being. Then, we find the *equilibrium* R_0 as the fixed point of a mapping with value distribution $F(V)$ incorporating all other equilibrium relations.

In the process of deriving Lemma 1, we have quantified the wage increase needed to offset a discrete increase in unemployment risk, $w_2 - w_1 = (\delta_2 - \delta_1)(V(w_1, \delta_1) - V_0)$. This difference is directly related to the concept of the *marginal willingness to pay* (MWP), employed in hedonic estimations of the value of job amenities, including job security. Therefore, the MWP for job security is given by the derivative of the workers' indifference curve in (w, δ)-space, which here is the derivative of equation (3) with respect to δ

$$MWP = \left.\frac{dw}{d\delta}\right|_{V \text{constant}} = -\frac{\frac{\partial V(w,\delta)}{\partial \delta}}{\frac{\partial V(w,\delta)}{\partial w}} \quad = \quad V(w, \delta) - V_0. \tag{10}$$

Looking more closely at equation (10), note that at the reservation wage out of unemployment, the marginal willingness to pay for job security is zero.[13] We see intuitively that if one is indifferent between being in state A or B and there are no transition costs, whether and how frequently one transits from one to the other becomes irrelevant. Thus, no compensating wages are required to hire out of unemployment even when $R_0 > b$. As a result, the reservation wage out of unemployment is identical for all firm types. This is why the initial condition in the partial differential equation, R_0, in Lemma 1, is invariant with δ in equation (7). By contrast, at employment values strictly above the value of unemployment, a worker is willing to give up some of his wage in exchange for an increase in job security. The amount of wage the worker is willing to give up for the same difference in job security increases as the worker's value in his current job increases. Thus, there is an intuitive, but critical complementarity between the attractiveness of a job, i.e. the *rent* a worker receives in a job, and how he values job security. In terms of wages: since the value of the job increases with the wage paid, and more strongly so the higher the wage level is, the MWP can be easily shown to be increasing and convex in the wage paid, ceteris paribus.[14]

Moreover, the worker's MWP for job security depends not only on the wage of the firm in question, but also on the extent of frictions in the labor market as well as on competing firms' types and offered wages, captured in the joint distribution of firms' job security and posted wages. These dimensions are subsumed in the rent of the worker's employment in his current firm, $V(w, \delta) - V_0$. In order to explicitly present the MWP's components,

[13] See also Burdett and Mortensen 1980.

[14] The derivative of the marginal willingness to pay with respect to w equals $\partial V(w, \delta)/\partial w > 0$ in equation (5). The second derivative $\partial^2 V(w, \delta)/(\partial w \partial w)$ is also positive.

we rewrite $V(w,\delta)-V_0$, substituting $V(w,\delta)$ and V_0 by the expressions obtained in equations (1) and (2),

$$MWP=\frac{1}{r}\Bigg(\Big(w-b\Big)-\underbrace{\lambda_0(1-F(V(w,\delta)))(V(w,\delta)-V_0)}_{=(\bigstar)}-\underbrace{\lambda_0\int_{V_0}^{V(w,\delta)}(V-V_0)dF(V)}_{=(\bigstar\bigstar)}$$
$$-\underbrace{(\lambda_1-\lambda_0)\Big(\int_{V(w,\delta)}(V-V(w,\delta))dF(V)\Big)}_{=(\bigstar\bigstar\bigstar)}-\delta(V(w,\delta)-V_0)\Bigg). \tag{11}$$

In equation (11), the term $w-b$ captures that the value of job security strictly increases with the wage and decreases with the benefit flow in unemployment b. When a worker loses his job and becomes unemployed, the adverse shock to life-time income is mitigated when he can re-enter employment quickly, particularly when the new job is at least as good as the old one. The probability that the displaced worker finds such employment depends on both the arrival rate λ_0 of offers while unemployed and the measure of firms that provide a value at least as good as the old job, i.e. $1-F(V(w,\delta))$. This probability is captured by ($\bigstar$) in equation (11). A laid-off worker might also find employment inferior in value to his previous job; this allows him to at least partially recover the lost income (in term ($\bigstar\bigstar$) of equation (11)). This reduction in income loss also decreases his willingness to pay for job security. The term ($\bigstar\bigstar\bigstar$) accounts for the fact that an employed worker occassionally receives offers allowing a move to an even better firm. The rate of meeting these ‘desirable’ firms might increase or decrease upon being laid off, depending on whether the arrival rate of offers in unemployment, λ_0, is larger or smaller, respectively, than the arrival rate of offers when employed, λ_1. This will increase or decrease, respectively, the value of job security, while the magnitude of the impact depends on the measure of firms offering these better positions relative to $V(w,\delta)$. Finally, as shown by the term $\delta(V(w,\delta)-V_0)$ in equation (11), the willingness to trade off wages for an incremental change in job security also depends on the baseline level of unemployment risk in the worker's existing job, δ; the same (absolute) change in job security in a very safe job has a more profound effect on the worker's value than it would at a job which would likely end soon anyway.

As mentioned in the introduction, our valuation of job security differs from the standard valuation methods, which assume that job security is a generic amenity, entering directly in the worker's instantaneous utility. Below, we will draw some interesting contrasts between the assumptions and implications of these methods, and those of job security in our setting. To do so, we will take a step back and first consider the valuation of firm-level job security when using conventional methods, before returning to highlight differences with our setting.

The perhaps most-used method of valuing amenities is based on hedonic wage regressions, first introduced by Thaler and Rosen (1976), in which the wage as dependent variable is regressed on amenity measures and controls, e.g. for worker quality. In its most straightforward interpretation, the estimated relationship is thought to map out identical workers' indifference curve between wages and amenities. To link the regression coefficient of an amenity measure to the marginal willingness to pay for it, a static utility function is posited that is linear and *additively separable* in wages and amenities.[15] However, the coefficients in these hedonic wage regressions, estimated in the cross section, are problematic; as pointed out already by Brown (1980), they are often statistically insignificant, and in many case have an unexpected sign. Even after controlling for observable and unobservable worker heterogeneity[16] in a panel data setup, support for compensating differentials remained weak.[17](See e.g. Bonhomme and Jolivet (2009), who discuss this in detail, both with regard to their data findings and the literature.)

Hwang et al. (1998), as well as Lang and Majumdar (2004), show that when search frictions matter in the labor market, the empirical cross-sectional relationship between wages and amenities does structurally deviate from the underlying MWP of workers. For example, firms that are better at producing amenities, ceteris paribus, will choose to provide higher overall utility to workers. If all else is indeed equal, the 'marginal willingness to pay' as found in hedonic wage regressions is biased downward. While Hwang et al. confine themselves to simulation exercises, Bonhomme and Jolivet (2009) estimate a partial-equilibrium version of Hwang et al. (1998) model on a set of amenities that includes subjectively reported job security, and they find that the model implies a high marginal willingness to pay, even though the cross-sectional relationship is very different, often with wages increasing in job security.[18]

In Bonhomme and Jolivet (2009), Hwang et al. (1998), Lang and Majumdar (2004), and Sullivan and To (2013), among others, all job amenities enter the workers' static (or instantaneous) utility function in a non-pecuniary way. In their setup, the valuation of an amenity is, in a sense, taken to be *fundamental*, an invariant aspect of the environment. In addition, in most models, the utility function of wages and amenities keeps the same additive and linear form as the standard hedonic wage functions. The valuation of job security, therefore, is

[15]To make this explicit, let the static utility be $v(w, a)$, where w is wages and a is the vector of amenities. To be able to construct a 'hedonic wage function' which rationalizes the regression under the assumption that all firms offer the same utility, i.e. $v(w, a) = \bar{v}$, we must be able to isolate w on one side of equation, which means that w enters this utility additively separable. Moreover, given these assumptions, observing combinations of wages and amenities (w, a) and (w', a') allows us to calculate the average willingness to pay; to let this pin down the *marginal* willingness to pay as well, one assumes that the marginal willingness is constant, hence the utility resulting utility function is $v(w, a) = w + \gamma a$.

[16]According to Gronberg and Reed (1994), much like unobserved productivity on the worker side, unobserved productivity on the firm side induces spurious correlation between the disturbance term and right-hand side job variables that biases the respective coefficients in the opposite direction of their true sign.

[17]According to Cahuc et. al. (2013), in order to deal with unobserved heterogeneity, a common identification strategy consists of following workers who change jobs, identifying the compensating differences on the basis of the relation between the wages these persons earn and the jobs they hold. The benefit of this strategy is that it controls for unobservable time-invariant individual characteristics through fixed effects. The main issues with this approach that are hard to overcome are the endogeneity of job switch – in particular given that are job characteristics that are unobservable to the econometrician – and the fact that changes in remuneration observed when jobs are changed reflect both differences in job quality and perhaps differences in compensation.

[18]Sullivan and To (2013) structurally estimate a similar partial-equilibrium version of Hwang et al. (1998). Differently from Bonhomme and Jolivet (2009), they aggregate all the amenities in one job-specific non-wage utility flow, derived by the combination of all available amenities. Their estimates show that non-wage job characteristics play an important role in determining job mobility, the value of jobs to workers, and the gains from job search. In particular, according to their estimates non-wage utility accounts for approximately one-half of the total gains from job mobility.

an unknown that needs to be estimated from the wages or workers' quit behavior in their current jobs as a function of those jobs' professed layoff risk. Furthermore, the value of job security in this framework is independent of the job's wage, as is required in standard hedonic wage analyses (and analyses building on these). In other words, it assumes that job security is equally valued across the wage distribution.[19]

In contrast, in our setting job security enters differently in the worker's utility. It is *instrumental*: workers care about increasing their job security, not because it is instantaneously enjoyable, but because it implies that the realized stream of future incomes – what they actually care about – will likely be larger.[20] The valuation of job security, therefore, is not an unknown that needs to be estimated from the wages or workers' quit behavior in their current jobs as a function of those jobs' professed layoff risk. Instead it can be inferred by comparing streams of income realizations when the worker keeps his job to those when he loses his job.[21] In other words, incorporating labor market dynamics more fully (as we have done here in an equilibrium labor market model) informs us about the valuation of job security itself.

Furthermore, our approach draws attention to how the shape of the valuation of job security is molded by labor market conditions. We show that a worker values job security more when he has a lot to lose from becoming unemployed. Notice that large losses occur precisely when the worker is displaced from a high-wage, otherwise relatively secure job, whose equivalent would be hard to find in the labor market. The difficulty in finding these high-pay, secure jobs highlights (as did equation (11)) the impact of labor market frictions on the valuation of job security. This direct effect of labor market frictions in the MWP is missing from the *fundamental* approach that puts job security directly in the instantaneous utility function.

This absence of a direct effect of frictions leads to different results when the degree of frictions is varied. Hwang et al. (1998) and Bonhomme and Jolivet (2009) discuss at length how a reduction in frictions can bring the estimated coefficient of the cross-sectional hedonic regression closer to the MWP. Their argument is based on the fact that the estimated coeffients are affected by frictions, while MWP is invariant to frictions in their setup.[22] While this applies to the generic amenity that is consumed on the spot, job security is different. As we showed, labor market frictions directly affect MWP. In particular, a change in frictions also changes the ease

[19] A clear example of this is Sullivan and To (2013), where the MWP is constant.

[20] Other amenities, such as a company car, or sport or child care facilities, are clearly well-captured by a contemporaneous *fundamental* utility flow. Job security differs because it affects future outcomes, not today's consumption. It would interact with these amenities in the same way as it interacts with income in our setting . In general, the worker would compare the expected stream of *utility* and how it is affected by job security.

[21] Confining oneself to wage and turnover data within a job spell to estimate amenity valuation is more difficult in the case of job security, as observed transitions into unemployment imply that quits will not be observed for those spells. Bonhomme and Jolivet avoid this by the additional use of self-reported data on the worker's *satisfaction level* with respect to job security". However, the mapping from an answer to this question to objective job security measurement is not without difficulties. Bonhomme and Jolivet (2009) deal with these by considering the survey answer to map to a binary (threshold) job security measure and incorporating controls for the unobservable individual-level heterogeneity in answering the survey question.)

[22] Although interestingly, Bonhomme and Jolivet (2009) find that local changes around the estimated extent of frictions will not radically alter the inferred difference between the marginal willingness to pay and the estimated coefficient of the hedonic wage regression, which remains substantial.

with which a laid-off worker can make up his losses in new employment. Therefore, there are two counteracting effects when frictions decrease: firms will be forced to offer more similar utility; simultaneously, workers will consider job security a less important dimension of jobs, and will have a reduced MWP for it. We will study the overall impact that changes in frictions have on equilibrium outcomes in section 3.

Finally, the fact that in some jobs the worker values job security more than in other jobs is not only an important feature behind workers' choices; it also reverberates through the choices that firms make in competition. In the sections to come, we will see that the impact of job security on firm behavior comes in a number of ways. First, because the complementarity between wages and job security affects worker's lifetime utility, job security may provide an advantage in recruitment. Moreover, job security affects the firms' profit maximization also directly. A more secure firm's reduced outflow rate into unemployment prolongs the expected time its workers contribute to the firm's life-time profit. This longer expected job relation affects the wages the firm will optimally choose (with an eye on recruitment and retention). Finally, as some firms send more of their workers back to unemployment, these firms employ fewer workers for other firms to poach. All these issues affect the firms' decision making and all would be missed if job security were placed only in the workers' utility .

Overall, the workers' endogenously heterogeneous valuation of job security affects individual firms' behavior in turn, making the valuation of job security a true equilibrium object. In section 2.2, we consider the firms' profit maximization, taking into account the endogenous valuation of job security. In section 2.3, we show how firms' optimal wage setting and their job security affects the way workers are distributed over firms. Finally, we put the two sides of the market together, to fully characterize the equilibrium in section 2.4.

2.2 The Firm's Problem and Labor Market Equilibrium

Firms face the problem of picking the wage that maximizes their steady state profits. If a firm pays a higher wage, ceteris paribus, workers will value being employed in this firm more relative to other firms. As a result, workers employed in other firms are more likely to move into the firm in question whenever they receive this firm's wage offer. Similarly, when the firm's own workers receive alternative offers themselves, they are more likely to reject those and stay with the firm. Therefore, with higher wages facilitating recruitment and retention, the firm employs more workers in the steady state. While this force pushes overall profit upwards, it comes at the cost of earning less profit per worker, $p - w$, which pushes profits downwards. At the optimal wage, the firm balances those two counteracting dimensions, maximizing *total* steady-state profits. As we will see below, this balance works out differently for firms with different degrees of job security.

In order to explicitly find the profit-maximizing wage for each firm, we first look at how steady state firm sizes vary by the value provided to the worker and the firm's layoff rate[23]. Keep in mind that, in steady state, a firm's labor inflows must be completely balanced by its outflows. Since we are dealing with measures of firms and workers, we first look at the inflow and outflow measures of workers in a positive measure of firms, then take

[23]Notice that a firm's layoff rate impacts directly its size. Even if safe and risky firms offer the same value to their employees, risky firms lose workers at a faster rate and are consequently smaller.

the appropriate limit to isolate the firm size, $l(V, \delta)$.

Given the value of an employed worker in a δ-firm at wage w, $V(w, \delta)$, we can define $\tilde{F}(V, \delta)$ as the joint distribution of firm-level unemployment risk δ and firm-offered *value* V. We focus on values V rather than wages, because worker mobility and therefore worker flows are directly dictated by these values. Notice that the distribution $\tilde{F}(V, \delta)$ is constructed by combining the distributions of equilibrium wages offered by each firm-type δ, $\hat{F}(w|\delta)$, the translation of these wages into values $w(V, \delta)$[24], and the distribution of the firms' layoff risk δ, $H(\delta)$. Formally, $\tilde{F}(V, \delta)$ equals $\int_{\delta' \leq \delta} \hat{F}(w(V, \delta'))dH(\delta')$.

Therefore, the steady-state measure of workers[25] who are employed at values weakly below V, in firms with a layoff risk rate weakly below δ, must satisfy[26]

$$\int_{\delta' \leq \delta, V_0 \leq V' \leq V} \left(\delta' + \lambda_1 \int_{\tilde{V} > V} d\tilde{F}(\tilde{V}, \tilde{\delta}) + \lambda_1 \int_{\substack{\hat{\delta} > \delta, \\ V \geq \hat{V} > V'}} d\tilde{F}(\hat{V}, \hat{\delta}) \right) d\tilde{G}(V', \delta')(m-u) =$$
$$\int_{\delta' \leq \delta, V_0 \leq V' \leq V} \left(\lambda_0 u + \lambda_1 \int_{\breve{\delta} > \delta, \breve{V} < V'} d\tilde{G}(\breve{V}, \breve{\delta})(m-u) \right) d\tilde{F}(V', \delta'). \tag{12}$$

where $\tilde{G}(V, \delta)$ is the distribution of employees across firms.

The left-hand side captures the outflow, consisting of (in order of appearance) the outflow to unemployment (δ'), to firms with a higher value $\tilde{V} > V$, and to riskier firms $\hat{\delta} > \delta$ which offer values better than the worker's current value V', but weakly lower than V. The right-hand side accounts for the inflows, first from unemployment ($\lambda_0 u$), and second, from riskier firms $\breve{\delta} > \delta$ that offer values $\breve{V}$ below the new firm's V'.

Notice that the distribution of employees across firms, $\tilde{G}(V, \delta)$ is pinned down by the steady-state equality of inflows and outflows in (12), together with the joint firm-type offer distribution $\tilde{F}(V, \delta)$. It is easy to show that $\tilde{G}(V, \delta)$ is absolutely continuous with respect to $\tilde{F}(V, \delta)$; if a subset $A \in \mathbb{R}^2$ has probability $\int_A d\tilde{F}(V, \delta)$ equal to zero, then the LHS of (12), adapted to integrate only over the set A, equals zero; since $\delta > 0$, it must be that $\int_A d\tilde{G}(V, \delta) = 0$ as well. Then, by the Radon-Nikodym theorem, a function $l(V, \delta)$ exists such that $(m-u)\tilde{G}(V, \delta) = \int_{V_0}^{V} \int_{\underline{\delta}}^{\delta} l(V', \delta')d\tilde{F}(V', \delta')$; roughly, $l(V, \delta)$ corresponds to the measure of employed workers divided by the measure of firms, as both get very small, and we take this as the firm's size, in Lemma 2.

Lemma 2. *The size of a firm posting a wage that induces worker's value of V depends on the aggregate distribution of offered values, $F(V)$, the aggregate distribution of employed workers over values, $G(V)$, and on the firm's own δ,*

$$l(V, \delta) = \frac{\lambda_0 u + \lambda_1 G^-(V)(m-u)}{\lambda_1(1 - F^+(V)) + \delta}, \tag{13}$$

where $G^-(V) = \int_{V' < V} d\tilde{G}(V', \delta')$, $F^+(V) = \int_{V' \leq V} d\tilde{F}(V', \delta')$. (Note these are integrated over the entire set of δ).

[24]Notice that this function is the inverse of $V(w, \delta)$, for a given δ.

[25]Steady-state calculations based on standard random matching. For more details, see Podczeck and Puzzello, 2011

[26]Note that, since mass can be concentrated at a single (V, δ), we are explicit whether the boundaries are included.

Likewise for unemployment,

$$\lambda_0 u \int_{V \geq V_0} d\tilde{F}(V,\delta) = (m-u) \int \delta d\tilde{G}(V,\delta) \tag{14}$$

The fact that the size of a firm will be affected by both the value (or wage) offered to the worker and its own unemployment risk stands in contrast to BM and to Bontemps et al. (1999). Although their models allow many sources of heterogeneity on the firm and worker sides, their equilibria keep the property that the firm size depends only on the wage. As a direct consequence of the dependence of firm size on both value V and unemployment risk δ in our model, the distribution function of workers across values – $G(V) = (m-u)^{-1} \int_{V' \leq V} l(V', \delta') d\tilde{F}(V', \delta')$ – depends also on the types of the firms that offer each value V. Consider a subset of values which are offered predominantly by riskier firms: these firms send workers into unemployment at a faster rate. If these firms offer high values, they will attract many workers, who will be subjected to a high unemployment risk, generating a large inflow into unemployment. On the other hand, if these risky firms offer low values, only a smaller subset of employed workers will be subject to this increased risk. Hence, the average unemployment risk in the labor market depends on the joint distribution of firm types and values. Likewise, a firm's recruitment inflow from other firms depends not only on the number of firms that offer less attractive employment, but also on the layoff risk at these firms. If competitors offering worse job values are predominantly risky firms, their firm size is relatively small compared to a case in which safer firms offer these values. As a result, there are fewer *employed* workers that a firm can poach by offering a higher value. Since *unemployed* workers accept any wage offer above R_0, the relative benefit of offering a high wage is lower in this case. In equilibrium, we have to take these dependencies between the distribution of firm layoff risk and the distribution of posted wages into account.

Formally then, a firm with layoff rate δ chooses w to maximize $(p-w)l(V(w,\delta),\delta)$. Combining Lemma 1 and Lemma 2, we can derive that safer firms will offer better values, and the rank of the firm in the value distribution corresponds to the ranking with respect to job security.

Proposition 1 (Ranking Property)**.** *Suppose in equilibrium a riskier firm (with layoff risk δ_h) and a safer firm (with layoff risk δ_l, with $\delta_l < \delta_h$) offer wages of w_s and w_r, respectively. Then, we must have $V(w_s, \delta_l) \geq V(w_r, \delta_h)$.*

Below we prove proposition 1 without reference to the shape of $H(\delta)$, and therefore the result holds whether this distribution is discrete, continuous, or a mixture. As a proof strategy, we look at both the relative gains in the steady state number of workers in firm and the relative losses in profit-per-worker when offering a higher value.

First, consider how much larger a firm will be in steady state when offering a larger V_h, as opposed to V_l

$$\frac{l(V_h,\delta)}{l(V_l,\delta)} = \frac{\lambda_0 u + \lambda_1 G^-(V_h)(m-u)}{\lambda_0 u + \lambda_1 G^-(V_l)(m-u)} \times \frac{\lambda_1(1-F^+(V_l)) + \delta}{\lambda_1(1-F^+(V_h)) + \delta}. \tag{15}$$

Notice that for an increase of offered value from V_l to V_h, firm size increases by a relatively larger amount when the firm is safer (i.e. $\delta_l < \delta_h$.) As the *inflows* into a firm depend only on the value offered, here V_h or V_l, the first term on the right-hand side is the same for any δ-type of firm. *Outflows*, on the other hand, depend on the unemployment risk. The flow of workers lost to other firms is a relatively more important component of overall outflows in safer firms, $\frac{\lambda_1(1-F^+(V_l))+\delta_l}{\lambda_1(1-F^+(V_h))+\delta_l} \geq \frac{\lambda_1(1-F^+(V_l))+\delta_h}{\lambda_1(1-F^+(V_h))+\delta_h}$, for $\delta_l < \delta_h$. Hence a change in this component has a larger effect on firm size for safer firms. As a result, safer firms gain relatively more in firm size from offering a higher value,

$$V_h \geq V_l \Longrightarrow \frac{l(V_h,\delta_l)}{l(V_l,\delta_l)} \geq \frac{l(V_h,\delta_h)}{l(V_l,\delta_h)}, \tag{16}$$

and the converse holds in strict inequalities. Put differently, for a given increase in value V, the accompanying relative increase in expected job duration for the firm's workers is higher in a safer firm. As a result, safer firms care more about worker retention than riskier firms. Thus, job security does not only enter the valuation of workers, it also enters the firm's objective function directly.

Second, consider the profit flow per worker for a δ-type firm providing a value V to its workers, $p - w(V,\delta)$. The *relative* loss of profit when offering the worker a higher value V_h instead of a lower value V_l is given by $\frac{w(V_h,\delta)-w(V_l,\delta)}{p-w(V_l,\delta)}$. From the worker's indifference in 'compensating wages' (equation (3)), we know that when two firms provide the same worker's value $V > V_0$, the safer firm can do so while paying a lower wage. Once $w(V,\delta_l) < w(V,\delta_h)$, safer firms make a higher profit per worker at value V. Moreover, equation (5) has told us that a change of value in riskier firms requires a larger change in wage than in safer firms; for $V_h > V_l$, we have

$$\forall\, V,\ \left.\frac{dV(w,\delta_h)}{dw}\right|_{V(w,\delta_h)=V} < \left.\frac{dV(w,\delta_l)}{dw}\right|_{V(w,\delta_l)=V} \Longrightarrow w(V_h,\delta_h) - w(V_l,\delta_h) > w(V_h,\delta_l) - w(V_l,\delta_l). \tag{17}$$

Then, the relative loss of profit from offering higher values is larger for risky firms for two reasons: first, because they need to raise wages more (in the numerator) and second, because their initial profits at baseline value $V_l > V_0$ (in the denominator) are lower. Therefore, even if the required change in wages is the same for the safe firm and risky firm, it leads to a larger relative profit loss for the risky firm,

$$V_h > V_l \geq V_0 \iff \frac{w(V_h,\delta_h) - w(V_l,\delta_h)}{p - w(V_l,\delta_h)} > \frac{w(V_h,\delta_l) - w(V_l,\delta_l)}{p - w(V_l,\delta_l)}. \tag{18}$$

$$\iff \frac{p - w(V_h,\delta_h)}{p - w(V_l,\delta_h)} < \frac{p - w(V_h,\delta_l)}{p - w(V_l,\delta_l)}. \tag{19}$$

Now consider the optimal value V_r for a riskier firm, and the optimal value V_s for a safer firm. The riskier firm makes more profits at V_r than at V_s, while the reverse is true for the safer firm. This means that

$$\frac{(p - w(V_s,\delta_l))l(V_s,\delta_l)}{(p - w(V_r,\delta_l))l(V_r,\delta_l)} \geq \frac{(p - w(V_s,\delta_h))l(V_s,\delta_h)}{(p - w(V_r,\delta_h))l(V_r,\delta)}. \tag{20}$$

But according to equation (19) and the converse of equation (15), it follows from the optimality conditions for δ_h and δ_l-firms in inequality (20) that it must be that $V_s \geq V_r$, proving proposition 1.

Overall, intuitively, a safer firm gains relatively more in firm size from offering a higher value, while giving up strictly less (in relative terms) in profit per worker. That is, a safer firm will gain strictly more in overall profit (or lose less) than the riskier firm when offering a higher value. Then, if V_r is optimal for the riskier firm, the safer firm will strictly gain when offering $V_s \geq V_r$.

In the case that amenities enter additively in the utility and some firms are more efficient at providing amenities, these firms also offer higher values in equilibrium, ceteris paribus as shown by Hwang et al. 1998. However, in this case, the effect comes entirely through the profit per worker, once productivity is adjusted for differences in the cost of providing amenities. For the same increase in job value, the effect on firm size is the same irrespective to how efficient the firm is at providing amenities. In our case, there are two additional forces that work crucially in the same direction. First, for the safer firm, the relative gain in firm size is larger when raising the value, in equation (15). Second, that firm needs to raise wages less (in absolute terms) to provide the additional value, in equation (17).

While Proposition 1 establishes that safer firms offer better values, we cannot yet make inferences about the *actual wages* posted, since the higher job security of safer firms by itself might (more than) deliver the higher value required in equilibrium. Fortunately, we will be able to explicitly link firm types to wages in a tractable way in the next sections. Towards that end, let us first define the state state equilibrium properly, and then take several steps to characterize this equilibrium.

Definition 1. *The* ***steady state equilibrium*** *in this labor market consists of distributions $\hat{F}(w|\delta)$, $\tilde{F}(V,\delta)$, $\tilde{G}(V,\delta)$, $F(V), G(V)$; an unemployment rate u; a value function $V(w,\delta)$ for employed workers, and a value V_0 and reservation wage R_0 for unemployed workers, such that*

1. *workers' utility maximization: optimal mobility decisions result in a value function $V(w,\delta)$ for employed workers, and a reservation wage R_0, with associated value V_0, according to eqs.* (5)-(9)*, given $\hat{F}(w|\delta)$ and $H(\delta)$.*
2. *Firms' profit maximization: given $F(V)$, $G(V)$ and $V(w,\delta)$, for each δ, $\exists\, \pi$ such that $\forall\, w \in supp\ \hat{F}(w|\delta)$, it holds that $\pi = (p-w)l(V(w,\delta),\delta)$ and $\forall\, w \notin supp\ \hat{F}(w|\delta)$, $\pi \geq (p-w)l(V(w,\delta),\delta)$, where $l(V,\delta)$ is given by* (13)*.*
3. *steady state distributions follow from individual decisions aggregated up. For firms: $\tilde{F}(V,\delta)$ is derived from $\hat{F}(w|\delta)$ and $H(\delta)$ using $V(w,\delta)$. For workers: $\tilde{G}(V,\delta)$, and u follow from the steady state labor market flow accounting in* (12)-(14)*. 'Aggregate' value distributions $F(V)$ and $G(V)$ follow from $\int_{V'\leq V} d\tilde{F}(V,\delta)$, and $\int_{V'\leq V} d\tilde{G}(V,\delta)$.*

Adapting the proofs in BM and Bontemps et al. to incorporate the heterogeneity in δ, we can show that $F(V)$ is a continuous and strictly increasing distribution function, and so is $G(V)$. The intuition for these results follows the aforementioned papers: mass points or intermediate intervals where no firms offer wages in the distribution of offered wages allow discrete gains in firm size or profit per worker through deviations whose cost can be made arbitrarily small.

Proposition 2. *In equilibrium, the distribution of posted values, $F(V)$, has the following properties: (i) The support of the distribution is a connected set, (ii) there are no mass points in $F(V)$, and (iii) the lowest value offered is V_0, i.e. $F(V_0) = 0$. Properties (i)-(iii) likewise hold for $G(V)$, the distribution of employed workers across job values, which is derived from $F(V)$ and* (12)*.*

Proposition 2 implies that $F(V)$ and $G(V)$ are strictly increasing, continuous functions between V_0 (with $F(V_0) = G(V_0) = 0$) and some $\bar{V}$ (with $F(\bar{V}) = G(\bar{V}) = 1$). Combining propositions 1 and 2, it is easy to see that the conditional distribution of firm types that offer value V, $\hat{F}^{-1}(\delta|V)$, has all probability mass concentrated at a unique δ. Conversely, if $H(\delta)$ has a continuous probability density, it also follows that each δ posts a unique value. This does not imply that an *actual* wage w is offered by at most one δ-type of firm; overlaps in the actual wage distribution (with concomitant wage cuts in transitions) are possible, as we show below.

One of the strengths of the results in Propositions 1 and 2 is that, although now workers and firms are affected by two dimensions, wages and job security, and job security's valuation is endogenous, the model is quite tractable. We can solve for the firm size distribution and subsequently the equilibrium wage distributions (almost) as easily and as explicitly as in BM. We turn to this now.

2.3 Equilibrium Firm Sizes

Employed workers move to different firms when these offer a higher value of employment. Thus, for worker mobility, the absolute level of worker's value is not relevant; what matters is how this value ranks among competing firms' values (and to the value of unemployment V_0). In addition, the 'ranking property' in Proposition 1 tells us that, in equilibrium, offered values are strictly decreasing in layoff risk. Thus, we can create a firm rank in terms of offered worker values that has a one-to-one relationship with the firm's layoff risk. It follows that this firm rank captures both the workers' job-to-job and the job-to-unemployment mobility. As a result, we can solve for equilibrium firm sizes as a function of rank only, without any further reference to equilibrium wages paid or value levels.

Formally, we define firm rank z as $z = F(V)$. Since the *riskiest* firm posts wages that generate a workers' value of V_0 and $F(V_0) = 0$, the *riskiest* firm has rank $z = 0$. Then, in Proposition 2, we show that $F(V)$ is continuous and strictly increasing, so $V(z) = F^{-1}(z)$ exists and is unique. We can also define the firm's layoff risk as a function of *equilibrium* firm rank in the value distribution, $\delta(z)$; in equilibrium this is the layoff risk associated with the z^{th} firm, starting from the *riskiest* firm[27]. Similarly, we define $G^z(z) = G(V(z))$ as the proportion of employed workers at firms ranked z or lower. We use the absolute continuity of $F(V)$ and the fact that the distribution of types offering value V, $\hat{F}^{-1}(\delta|V)$, concentrates all mass at a unique δ[28] to express an intuitive correspondence between firm size and rank as follows:

$$\begin{aligned} G^z(z)(m-u) = G(V)(m-u) &= \int_{V' \leq V} l(V,\delta) dF(V,\delta) \\ &= \int_0^z \frac{\lambda_0 u + \lambda_1 G^z(z')(m-u)}{\lambda_1(1-z') + \delta(z')} dz' \end{aligned} \tag{21}$$

where $z = F(V)$.

[27]To formally deal with mass points in $H(\delta)$, define $\overline{H}(\delta)$ as the closed graph of $1 - H(\delta)$, then let $\delta(z) \overset{def}{=} \max\{\delta | conv(\overline{H}(\delta)) = z\}$.Taking the maximum here is without loss of generality for our results, since alternative assumptions at points where the convex closure of $H(\delta)$ is an interval, would change δ only for a zero measure of firms.

[28]Both properties follow from proposition 2

The intuition of the firm size is straightforward: the steady state firm size is given by the ratio of worker inflows to outflows, now functions of ranking z only.[29] Rank z determines layoff risk through equilibrium mapping $\delta(z)$, while the proportion of employed workers below rank z, $G^z(z)$, is implicitly defined by equation (21) itself. From the last line in (21), it follows that the individual firm size $l(z)$ is given by

$$l(z) = \frac{dG^z(z)}{dz}(m-u) = \frac{\lambda_0 u + \lambda_1 G^z(z)(m-u)}{\lambda_1(1-z)+\delta(z)}. \tag{22}$$

Comparing this to firm size in (13), $l(V,\delta) = (\lambda_0 u + \lambda_1 G(V)(m-u))/(\lambda_1(1-F(V))+\delta)$, using the continuity of $F(V)$ and $G(V)$, we see that a reformulation of firm size in terms of firm rank z is as intuitive as before. Inflows consist of a flow from unemployment $\lambda_0 u$, and a flow from lower-ranked firms $\lambda_1 G^z(z)(m-u)$, whose workers will move to the firm whenever they receive an offer, which occurs at rate λ_1. The outflow rate is determined by the amount of higher-ranked firms $(1-z)$, the rate at which workers meet these firms, λ_1, and the firm's own layoff rate, $\delta(z)$[30].

The implicit relationship between z and $G^z(z)$ in Equation (21) is the solution to differential equation

$$\frac{dG^z(z)}{dz} = \frac{\frac{\lambda_0 u}{m-u} + \lambda_1 G^z(z)}{\lambda(1-z)+\delta(z)} \tag{23}$$

with initial condition $G^z(0) = 0$, which follows directly from (22). Note that there is no reference to another equilibrium object in this formulation of the distribution but $G^z(z)$ itself. Given this, we can now solve *explicitly* for firm size $l(z)$ and distribution $G^z(z)$, which is done in the next lemma.

Lemma 3. *The cumulative distribution of employed worker at firms with rank lower than z, $G^z(z)$, and equilibrium firm size $l(z)$ and measure of unemployed, are given by: $u = \frac{m\int_0^1 \delta(z)g(z)dz}{\lambda_0 + \int_0^1 \delta(z)g(z)dz} = (\lambda_0)^{-1}\int_0^1 \delta(z)l(z)dz$, and*

$$G^z(z) = \frac{\lambda_0 u}{\lambda_1(m-u)}\left(e^{\int_0^z \frac{\lambda_1}{\lambda_1(1-z')+\delta(z')}dz'} - 1\right), \tag{24}$$

$$l(z) = \frac{\lambda_0 u}{\lambda_1(1-z)+\delta(z)} e^{\int_0^z \frac{\lambda_1}{\lambda_1(1-z)+\delta(z')}dz'}. \tag{25}$$

[29] To see the substitutions needed to yield the result in equation (21), consider

$$\begin{aligned}\int_{V'\leq V} l(V,\delta)dF(V,\delta) &= \int_{V_0}^{V}\int_{\delta} \frac{\lambda_0 u + \lambda_1 G(V')(m-u)}{\lambda_1(1-F(V'))+\delta} d\hat{F}(\delta|V')dF(V') \\ &= \int_{V_0}^{V} \frac{\lambda_0 u + \lambda_1 G(V')(m-u)}{\lambda_1(1-F(V'))+\delta(F^{-1}(V'))} dF(V') \\ &= \int_0^z \frac{\lambda_0 u + \lambda_1 G^z(z')(m-u)}{\lambda_1(1-z')+\delta(z')} dz' = \int_0^z l(z')dz'\end{aligned}$$

where in the first step, we split up $dF(V,\delta)$ into $dF(\delta|V)dF(V)$, after which proposition 2 tells us that $F(\delta|V)$ all mass concentrated at one and only one δ. In the next line, we substitute this into the function $\delta(z)$ (which uses the type-ranking of proposition 1 and the strict monotonicity of $F(V)$ of proposition 2), leaving us with an integration over V'. Finally, the last line incorporates the change of variable to $z = F(V)$.

[30] Proposition 2 tells us that $F(V)$ is strictly increasing and continuous on a connected support of values V, and hence the mapping $z = F(V)$ is invertible, continuous and strictly increasing as well, allowing us to change the variable to z in employment distribution function $G^z(z)$ in (21), and in firm size $l(z)$ in (22) (using the steps in footnote 29, where Proposition 1 allows us to construct the mapping $\delta(z)$)

The dependence of the size of the zth-ranked firm on the unemployment risks of all lower-ranked firms is explicit in the integral term in the exponent. Solving firm size as a function of the firm rank provides some advantages. First, it yields a clean and simple expression for firm size. Second, it allows for general probability distributions of firm types through $\delta(z)$, since it relies on a minimal set of relevant parameters. Consequently, the formulation in Lemma 3 can deal with discrete distributions as easily as with continuous distributions or any mixture of the two. We believe that our approach is applicable for any situation in which a firm-specific factor affects the firm size separately from the workers' preferences. Our procedure works as long as one can establish a mapping between this firm factor and the equilibrium rank in the distribution of offered values, as Proposition 1 does for unemployment risk[31].

2.4 Equilibrium Wage Distributions

In the previous section, we derived equilibrium firm size as a function of firm rank in the value distribution. In this section, we link these firm sizes to firms' wage-setting. Instead of formulating the firm's problem as choosing the optimal wage (which in equilibrium implies a certain rank $z = F(V(w, \delta))$), we use the equivalent formulation of the firm choosing a rank z (which in equilibrium implies a wage w and an associated value V). Firm optimization in equilibrium requires that a firm ranked z in job security does not want to deviate to another rank z' in the value distribution. Therefore, the first-order condition from profit maximization pins down how values change with firm rank in equilibrium for each firm. This condition, however, depends on the wage w *and* on the value V at a given firm rank. Hence, we need to keep track of both wages and values as a function of firm rank; this introduces a second condition that needs to be satisfied in equilibrium. This additional condition makes our setting different from the standard BM model, where one equation is sufficient to characterize equilibrium wages (given the initial condition R_0). Here, we show that the resulting system of two ordinary differential equations pins down all the equilibrium components (given initial conditions R_0 and V_0). The system also establishes the existence and uniqueness of the labor market equilibrium.

We can set up the firm's profit maximization problem such that the firm chooses the rank z it wants to occupy in the firm distribution, given the equilibrium objects $V(z) = F^{-1}(z)$, $G^z(z)$, $l(z)$ and $w(V, \delta)$ (the inverse of

[31] We can be more explicit in the case of a pure discrete and a purely continuous distribution. For a continuous probability density $h(\delta)$ with $H'(\delta) = h(\delta) > 0$, $\delta(z)$ is differentiable everywhere with $\delta'(z) > 0$, and with the appropriate change of variable , this results in

$$l(\delta) = \frac{\lambda_0 u}{\lambda_1 + \bar{\delta}} e^{\int_{\delta}^{\bar{\delta}} -\left(\frac{2\lambda_1 h(\delta)+1}{\lambda_1(1-H(\delta))+\delta}\right) d\tilde{\delta}}.$$

In case of a discrete distribution $h(\delta_j), j = 1, \ldots, J$, with $\sum_j h(\delta_j) = 1$ and $\bar{\delta} = \delta_1 > \ldots > \underline{\delta} = \delta_J$, Lemma 3 tells us that the mass of workers in δ_i firms, $v(\delta_i)$ can be derived from (24), using $e^{\int_a^b \frac{\lambda_1}{\lambda_1(1-z)+\delta} dz} = \frac{\lambda_1(1-z_a)+\delta}{\lambda_1(1-z_b)+\delta}$. Suppose that $\sum_{i=1}^{j-1} h(\delta_i) < z < 1 - \sum_{i=j+1}^{J} h(\delta_i)$ for some j. Then from (24),

$$G(z) = \frac{\lambda_0 u}{\lambda_1(m-u)} \left(\frac{\lambda_1(1 - \sum_{i=j}^{J} h(\delta_i)) + \delta_j}{\lambda_1(1-z) + \delta_j} \prod_{i=1}^{j-1} \frac{\lambda_1(1 - \sum_{h=i}^{J} h(\delta_i)) + \delta_i}{\lambda_1(1 - \sum_{h=i+1}^{J} h(\delta_i)) + \delta_i} - 1 \right)$$

$V(w, \delta)$).[32] In particular, firm optimization in equilibrium requires that no firm of rank z, with layoff risk $\delta(z)$, strictly prefers to offer a value $V(z')$ associated with a different rank z' in firm distribution $F(V)$, compared to the value, $V(z)$, associated with rank z. Formally, this means that a z-rank firm profit maximization problem can be written as:

$$\max_{z'}(p - w(V(z'), \delta(z)))l^d(z', \delta(z)), \tag{26}$$

where $w(V(z'), \delta(z))$ is the wage the z^{th} riskiest firm, with its immutable unemployment risk $\delta(z)$, must pay to provide value $V(z')$. The term $l^d(z', \delta(z))$ is the steady-state firm size of a *deviating* z-type firm which offers its workers a value $V(z')$ instead. Then the rank-z firm will have exactly the same worker inflows as well as worker *outflow rate to other jobs* as a rank-z' firm. However, the rank-z firm cannot do anything to change its own outflow rate *to unemployment*, $\delta(z)$. This means that the outflow rate of a z-firm deviating to $V(z')$, relative to the outflow rate of the z'-firm itself, is larger by a factor $\frac{\lambda_1(1-z')+\delta(z)}{\lambda_1(1-z')+\delta(z')}$, which implies that the size of the deviating firm is the size of the z'th-most risky firm divided by this factor. Therefore, the deviating firm's size is:

$$l^d(z', \delta(z)) = \frac{\lambda_1(1 - z') + \delta(z')}{\lambda_1(1 - z') + \delta(z)} l(z') = \frac{\lambda_0 u}{\lambda_1(1 - z') + \delta(z)} e^{\int_0^{z'} \frac{\lambda_1}{\lambda_1(1-\tilde{z})+\delta(\tilde{z})} d\tilde{z}}. \tag{27}$$

The firm size upon deviating in equation (27) can be formally derived using the firm size as a function of values in equation (13) in conjunction with Lemmas 2 and 3. From equation (27), we see that $l^d(z', \delta(z))$ is differentiable in z'. The first-order condition of profit in equation (26) with respect to z', evaluated at the equilibrium choice, $z' = z$ is:

$$(p - w(V(z), \delta(z))) \left. \frac{\partial l^d(z', \delta(z))}{\partial z'} \right|_{z'=z} - \left(\frac{\partial w(V(z), \delta(z))}{\partial V(z)} \frac{dV(z)}{dz} \right) l^d(z, \delta(z)) = 0 \tag{28}$$

(Second-order conditions establishing that this indeed maximizes profit are established in theorem 1 below.) This can be rewritten as:

$$\frac{\frac{\partial w(V(z),\delta(z))}{\partial V(z)} \frac{dV(z)}{dz}}{(p - w(V(z), \delta(z)))} = \frac{\left. \frac{\partial l^d(z', \delta(z))}{\partial z'} \right|_{z'=z}}{l(z, \delta(z))}. \tag{29}$$

On the right-hand side is the relative gain in firm size when the 'mimicked' rank z' is incrementally increased. It is profit-maximizing for the z^{th} riskiest firm to be ranked z^{th} in the firm value distribution $F(V)$ when the relative gain in firm size from marginally increasing rank at rank z is offset exactly by the relative profit loss per worker on the left-hand side. Rearranging first-order condition (29), we find that this is satisfied when:

[32] Using these functions, the resulting optimization problem is equivalent to firm optimization with respect to value V or wage w, taking as given the corresponding functions $F(V), G(V), l(V, \delta)$. There is one additional aspect that needs to be taken care of: deviations to values that are not part of the support of $F(V)$, which by proposition 2 means only values below V_0, or above $V(1)$. This is easily verified: $V < V(0)$ implies zero profit, since no workers will ever find it optimal to accept employment; $V > V(1)$ implies less profit per worker than $V(1)$ with a firm size equal to the firm size when offering $V(1)$. Hence if a firm does not want to deviate to $V(1)$, it does not want to deviate to $V > V(1)$.

$$\frac{dV(z)}{dz} = (p - w(V(z), \delta(z))) \frac{\left.\frac{\partial l^d(z', \delta(z))}{\partial z'}\right|_{z'=z}}{l(z, \delta(z))} \frac{\partial w(V(z), \delta(z))}{\partial V(z)} \qquad \Longleftrightarrow$$

$$\frac{dV(z)}{dz} = (p - w(V(z), \delta(z))) \frac{2\lambda_1}{\lambda_1(1-z) + \delta(z)} \frac{1}{r + \delta(z) + \lambda_1(1-z)}. \tag{30}$$

Thus, equation (30) is a differential equation capturing how much the value needs to change with firm rank z to guarantee that rank-z firms indeed find it profit-maximizing to offer $V(z)$. It is iluminating to delve briefly into the difference between firms at the bottom of the job security distribution and those higher up. Safer firms have a higher term $2\lambda_1/(\lambda_1(1-z) + \delta(z))$ in equation (30), which can be seen from the derivative of this term with respect to layoff risk δ, which is $-\frac{2\lambda_1}{(\lambda_1(1-z')+\delta)^2} < 0$.[33] Safer firms thus compete more aggressively, ceteris paribus; raising $dV(z)/dz$ on the left-hand side of equation (30). A higher $dV(z)/dz$ will be a force that also pushes up wages as a function of firm rank z. A higher $dV(z)/dz$ is, however, not the entire story for wages, as the change in unemployment risk $\delta(z)$ must also be taken into account.

Note that equation (30) incorporates the wage function $w(V(z), \delta(z))$. This mapping depends on the *distribution* of values offered by firms and the distribution of unemployment risk across firms; hence it cannot be replaced by an explicit solution at this stage. However, the *change in wage* as a function of firm rank z can be established more easily, leading to another first-order differential equation, which combined with equation (30) gives rise to a system of two differential equations crucial to characterizing the equilibrium.

The pattern of wages offered by differently ranked firms depends not only on how values change with firm rank in $V(z)$, but also on how higher values can be delivered, partially or wholly, by the increased job security that a higher-ranked firm provides. As we move through the ranks of firms, the value offered by firms changes with the job security they provide, while at any z, the wage must be given by $w(V(z), \delta(z))$. This means that at any differentiable $\delta(z)$, we have[34]:

$$\frac{dw(V(z), \delta(z))}{dz} = \frac{\partial w(V(z), \delta(z))}{\partial V(z)} \frac{dV(z)}{dz} + \frac{\partial w(V(z), \delta(z))}{\partial \delta(z)} \delta'(z). \tag{31}$$

We can decompose the equilibrium wage change with firm rank into two components. The first is a competition effect coming from the firm's wage setting first-order condition (29), $\frac{\partial w(V(z),\delta(z))}{\partial V(z)} \frac{dV(z)}{dz}$. The second effect comes from the distribution of firm types in the labor market, with $\delta'(z)$, derived from $H(\delta)$, capturing how fast job security increases with firm rank. Substituting (29) into the last expression yields:

$$\frac{dw(z)}{dz} = (p - w(z)) \frac{2\lambda_1}{\lambda_1(1-z) + \delta(z)} + \delta'(z)(V(w(z), \delta(z)) - V_0). \tag{32}$$

[33]This term is the cross-derivative $\frac{\partial^2 \ln l(z', \delta)}{\partial z' \partial \delta} = -\frac{2\lambda_1}{(\lambda_1(1-z')+\delta)^2}$, which is the differential analogue (now in terms of rank z) to condition (15).

[34]Since it is derived from the cumulative density function $H(\delta)$ the function $\delta(z)$ is differentiable a.e. With abuse of notation, we use $\delta'(z)$ as a function that is defined everywhere, and consistent with the derivative of $\delta(z)$ almost everywhere. Moreover, note that $V(z)$ is continuous everywhere; if there is a discontinuity in $\delta(z)$ at z, 'compensating wage' indifference (3) will tell us the size of the discrete drop in wages at this z (as stated more explicitly in theorem 1).

While the firm's optimization pins down $\frac{dV(z)}{dz}$, the increased job security of the higher ranked firm could itself deliver part of the increased value V. If workers value job security greatly (i.e. $V(w(z),\delta(z))-V_0$ is high) or the higher-ranked firms' job security is significantly higher (i.e. $\delta'(z)$ is far below zero), an increase in job security can even lead to wage cuts. However, the gains of holding onto workers are also larger for firms with high job security, and hence these safer firms will compete more fiercely, as argued in equation (29). It is therefore uncertain whether wages paid will rise or fall with firm-level job security, even though worker values are strictly increasing with job security. One can see that $\delta'(z)$ might play an important role here, scaling $V(w(z),\delta(z))-V_0$, and therefore the strength of the forces. In the next section, we will study the conditions under which wage cuts in exchange for job security occur.

Let us finish this section by putting all the pieces of the equilibrium together: we can find $(w(z),V(z))$ as the solution of a system of *two* differential equations: one derived from (28) which tells us $\frac{\partial V(z)}{\partial z}$; the other derived from (32) and (3), which tells us $\frac{\partial w(z)}{dz}$. Both equations are functions of parameters, distributions and wages $w^s(z), w(z)$ themselves. The solution to this system of differential equations $\{V(z), w(z)\}$, in combination with the appropriate initial conditions fully characterizes the equilibrium. Moreover, we are able to establish the existence and uniqueness of this solution, and therefore of the labor market equilibrium, as presented in Definition 1. Theorem 1 below collects all these results.

Theorem 1 (Existence, Uniqueness, Characterization)**.** *Consider functions $\{w(z),V(z)\}$, and $R_0\in\mathbb{R}$ (and the associated $V_0=\frac{\lambda_0 R_0-\lambda_1 b}{r(\lambda_0-\lambda_1)}$), such that $w(z),V(z)$ are a solution to the system of two ODEs, represented below, for all z at which $\delta(z)$ is continuous:*

$$\frac{dV(z)}{dz}=(p-w(z))\frac{2\lambda_1}{\lambda_1(1-z)+\delta(z)}\frac{1}{r+\delta(z)+\lambda_1(1-z)} \tag{33}$$

$$\frac{dw(z)}{dz}=(p-w(z))\frac{2\lambda_1}{\lambda_1(1-z)+\delta(z)}+\delta'(z)(V(z)-V_0), \tag{34}$$

and, in the case of a jump discontinuity at every $\widehat{z}$ such that $\lim_{z\uparrow\widehat{z}}\delta(z)>\delta(\widehat{z})$, $w(z)$ will jump down according to

$$w(z)=\lim_{z\uparrow\hat{z}}w(\widehat{z})-\Big(\delta(z)-\lim_{z\uparrow\widehat{z}}\delta(\widehat{z})\Big)(V(z)-V_0), \tag{35}$$

$$V(z)=\lim_{z\uparrow\widehat{z}}V(\widehat{z}) \tag{36}$$

under initial conditions $w(0)=R_0$, and $V(0)=V_0$, where R_0 additionally satisfies

$$R_0=b+(\lambda_0-\lambda_1)\int_0^1(V(z)-V_0)dz=b+(\lambda_0-\lambda_1)\int_0^1(1-z)\frac{dV(z)}{dz}dz \tag{37}$$

Denote the inverse of $V(z)$ as $F(V)$. This distribution, and $G(V(z))=G^z(z)$, value functions $V(w,\delta)$, and u, $\hat{F}(w|\delta)$, $G(V,\delta)$,$F(V,\delta)$, all constructed from $\{w(z),\tilde{V}(z),R_0\}$, are the functions associated with the steady state equilibrium in the environment; this steady state exists and is unique.

In our model, it is necessary to follow a path different from BM and Bontemps et al. towards characterizing the equilibrium wage distribution. In this environment, neither wages nor values alone are sufficient to characterize the equilibrium. How much competition drives up the values offered to workers depends on wages through the instantaneous profit flows $p - w$ and also upon the job security of the firm in question. On the other hand, how wages co-move with job security depends on the valuation of job security, which takes into account the cost in value and likelihood of job loss. However, not all elements of equilibrium depend on both wages and values; firm size depends only on the rank of the firm and its associated job security. Exploiting this exception, we are able to solve for equilibrium firm sizes first and then find the wages and value distributions that must arise with them.

To see the effects of our different equilibrium characterization, it may be useful to compare the case with heterogeneous unemployment risk to the standard case in BM and Bontemps et al. ($\delta(z) = \tilde{\delta}, \ \forall\ z$). In the absence of heterogeneity in $\delta(z)$, the two equations (33) pre-multiplied by dw/dV and (34) are identical to each other; the differential equation $w'(z) = (p - w(z))(2\lambda_1/(\lambda_1(1-z)+\delta))$ with initial condition $w(0) = R_0$ has solution

$$\frac{p - w(z)}{p - R_0} = \left(\frac{\lambda_1(1-z)+\delta}{\lambda_1+\delta}\right)^2 \quad \Longrightarrow \quad F(w) = \frac{\delta+\lambda_1}{\lambda_1}\left(1 - \left(\frac{p-w}{p-R_0}\right)^{0.5}\right), \tag{38}$$

which is precisely the wage distribution in BM.[35]

Note that one way of showing existence and uniqueness in Burdett-Mortensen model would be to combine the left expression in (38) with (37), showing that $T(R_0)$ is linear in R_0, while $T(p) = b$, and $T(r) < r$, for a small enough r. The proof of Theorem 1 relies on this method. We can rescale the differential equation (34) by $p - w$ and show that the term $\frac{V(z)-V_0}{p-w(z)}$ is independent of R_0. Therefore the term that captures the division of overall rent between firm and worker is a function of parameters, unemployment risk distribution $H(\delta)$, and firm rank z. Concretely, the strategy of the proof is to show that – even though the problem has two dimensions – there still exists a term $\mathtt{A}(z)$ which again depends only on parameters, $H(\delta)$, and firm rank, such that $p - w(z) = (p - R_0)\mathtt{A}(z)$. From this the continuity of $T(R_0)$ in R_0, needed for the existence proof, can be established, as well as the uniqueness of the equilibrium.

3 Wages and Transition Hazards

In the previous section, we derived equations that characterize how wages, workers' values, and firm riskiness are linked in equilibrium. In this section, we look more specifically at the labor market outcomes implied by the characterization.

[35]Exploiting the ranking property inherent in BM-type models allows one to incorporate more heterogeneity in standard models. Here, we deal with firm heterogeneity that cannot be incorporated straightforwardly in the standard model, where e.g. there is a simple, unique one-dimensional mapping between wages and worker's values. Moscarini and Postel-Vinay (2010) exploit a similar ranking property to deal with time-varying aggregate productivity, an otherwise notoriously difficult problem.

First, since safer jobs are more attractive jobs, workers in safe jobs are much less likely to transition to unemployment or another job. Thus, our model produces an unemployment hazard that in the aggregate declines with firm tenure, as it does in the data.[36]. We will refer to this particular transition rate as the *unemployment hazard*. We formally present this outcome in the following result (once again, all proofs are relegated to the appendix),

Result 1. *The transition rate into unemployment as a function of tenure is decreasing.*

Next, we consider the relationship between the layoff risk a worker faces and the wage he receives. If wages are increasing with the firm job security, there is in some sense a strong failure of compensating wage differentials. Not only do riskier firms offer lower employment values (as established in Proposition 1), but in fact they offer values so much lower that in addition to a higher unemployment risk they actually pay lower wages. For jobs at the bottom of the wage distribution, we obtain the following result without restrictions on parameters or on the firm risk distribution.

Result 2. *The lowest wage, R_0, is paid by the firm with the highest unemployment risk. There exists a nontrivial interval of wages $[R_0, \hat{w}]$ where job security increases with wages*

Under typical conditions, spelled out next, this interval spans a large part of the wage distribution, while wage cuts may also occur higher up in the wage distribution. For analytic simplicity and to be consistent with steady state profit maximization, we let $r \to 0$ and consider the standard case of a distribution of firm unemployment risk with a differentiable density function $h(\delta)$.

Result 3. *In equilibrium the relation between wages and job security depends on the firm distribution of unemployment risk in the following way:*

1. *Wages increase with job security (i.e. $\frac{dw}{dz} > 0$ at $\tilde{z}$, where $\tilde{z} = 1 - H(\delta)$), whenever the density is constant or increasing in job security (equivalently, decreasing in unemployment risk δ, i.e. $h'(\delta) \leq 0$)*
2. *Wage cuts for increased job security will occur if*

$$\frac{h(\delta)}{\delta + \lambda_1(1 - H(\delta))} < \int_{\delta}^{\bar{\delta}} \frac{h(\tilde{\delta})}{(\tilde{\delta} + \lambda_1(1 - H(\tilde{\delta})))^2} d\tilde{\delta} \tag{39}$$

The first point highlights that in the distribution of job security $(1 - \delta)$, being the mirror image of the distribution for unemployment risk δ, wage cuts will not occur at a firm with lower job security than the mode of the distribution. Moreover, in general wage cuts can only occur when the density is strictly falling in job security. For example, for a uniform distribution of job security, there are no wage cuts at all in the labor market equilibrium. The second point tells us that wage cuts occur if equation (39) holds. Note however, that this requires a λ_1 small enough, in conjunction with a term $h(\delta)/\delta$ which is decreasing fast enough in job security. This could occur, e.g., when densities $h(\delta)$ have a tail with sufficient kurtosis. In particular, for λ_1 close enough

[36] See e.g. Menzio et al. 2012.

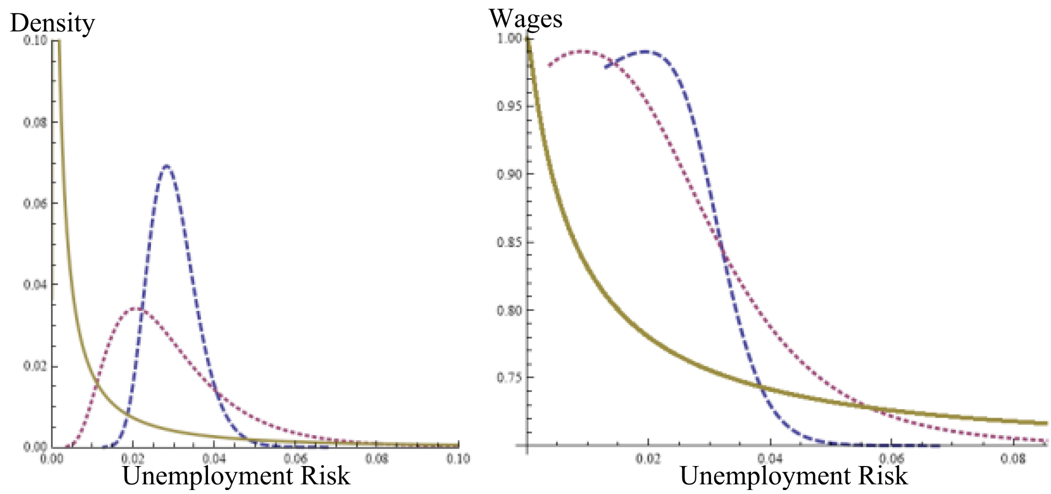

Figure 1: Left panel: log-normal p.d.f.s of δ, with standard deviation 0.03 (dashed), 0.2 (dotted), 2.0 (safe). Right panel: wage as a function of unemployment risk (x-axis) ($\lambda_1 = 0.23, r = 0.0025, R_0 = 0.7, p = 1$)

to zero, and a distribution of unemployment risk with $\frac{h(\delta)}{\delta} < \int_{\delta}^{\bar{\delta}} \frac{1}{\tilde{\delta}} \frac{h(\tilde{\delta})}{\tilde{\delta}} d\tilde{\delta}$ over an interval of δ, wage cuts are present.[37]

In figure 1, we have drawn, in the left panel, the density of log-normal distributions over unemployment risk. In the right panel, wages are drawn as a function of the underlying unemployment risk, for an empirically reasonable set of parameters.[38] On the x-axis in the right-hand panel is unemployment risk, and therefore job mobility occurs from right to left. Hence, wage cuts can occur at those unemployment risks δ in the right panel where the graph is upward-sloping. However, a wage cut only occurs when the unemployment risk in the previous job is higher, but not too different. (For substantially different unemployment risks, previous wages would be further to the right in the graph, and hence lower). Wages must increase with unemployment risk wherever the graph is decreasing. Note that the job security distribution with almost completely decreasing density does not generate any wage cuts, but for the other two distributions, with the clear left tails, some wage cuts occur when moving from relatively safe firms to the very safest firms. At the lowest wages, which come with significantly higher unemployment risk, no wage cuts are observed. At these wages, accepted largely by the previously unemployed, we see a complete absence of compensating wage differentials. In general, the complementarity between wages and job security works both on the firm's and worker's sides, and it can push actual wages either up or down, depending on the distribution. We think that this is a nice illustration of the value of studying unemployment risk and wage setting in a full-fledged labor market equilibrium environment, as the

[37] In case of a discrete distribution, the existence of wage cuts will follow directly from condition (35) in theorem 1. We discussed this type of wage cuts extensively in a previous version of the paper. This case however is intuitively close to the case where $h(\delta)$ is very close to zero on an interval, in this case (39) implies that for $h(\delta)$ small enough over an interval, wage cuts will also occur.

[38] Note that the proof of theorem 1 establishes that the shape of the function that links wages to unemployment risk does not depend on R_0, b, λ_0.

willingness to take wage cuts for safety could be offset by increased competition among firms.

We can study the case where we let search frictions for both unemployed and employed workers disappear in the limit, to see how the competitive limit looks when we let all frictions go to zero. We find that the equilibrium is well-behaved, in the sense that it converges to the perfectly competitive outcome when all frictions disappear.

Result 4. *Let $\lambda_0 > \lambda_1$, $\lambda_1 \to \infty, \lambda_0 \to \infty$, while keeping $\frac{\lambda_1}{\lambda_0} = \alpha < 1$ constant. Then $w(z) \to p$ for all z.*

To see this, observe that for reservation wage out of unemployment, R_0, the following holds

$$\frac{R_0 - b}{p - R_0} = (\alpha - 1)\int_0^1 \frac{\lambda_1(1-z)}{\underline{\delta} + \lambda(1-z)} \frac{\underline{\delta} + \lambda_1(1-z)}{\delta(z) + \lambda_1(1-z)} \frac{2\lambda_1}{\delta(z) + \lambda_1(1-z)} \frac{p - w(z)}{p - R_0} dz \tag{40}$$

$$\geq (\alpha - 1)\int_0^1 \frac{2\lambda_1^2(1-z)}{(\bar{\delta} + \lambda(1-z))^2} dz = -2\frac{\lambda_1}{\lambda_1 + \bar{\delta}} + 2\log\frac{\bar{\delta} + \lambda_1}{\bar{\delta}} \tag{41}$$

As we let λ_1 (and $\lambda_0 = \lambda_1/\alpha$ with it) go to infinity in (40), it follows that $R_0 \to p$, as the RHS goes to infinity. Since $R_0 \leq w(z) < p$, it follows that all wages approach the perfectly competitive wage p, $w(z) \to p$, for every z.

The degree of competition between firms is linked to parameter λ_1: an increase in this parameter makes it easier for higher ranked firms to poach workers from lower-ranked firms, raising firm competition. Keeping λ_0 constant, one might think that an increase in competition among firms would lead them to offer less dispersed employment values in equilibrium, tracing out the workers' marginal rate of substitution between job security and wages closely. The latter turns out not to be true: the increased competition among firms in fact pushes wages towards being *increasing* in job security. For any continuously differentiable distribution $H(\delta)$ (with full support on a connected bounded interval), we can find a finite $\tilde{\lambda}_1$ such that above this $\tilde{\lambda}_1$, for any λ_0 and b, wages will be increasing in job security throughout the entire distribution.[39]

Result 5. *If $\lambda_1 > h'(\delta)/(h(\delta))^2$, wages will be increasing with job security at δ, for any λ_0, b. If $h'(\delta)/(h(\delta))^2$ is bounded from above, there exists $\tilde{\lambda}_1$ such that for all $\lambda_1 > \tilde{\lambda}_1$ wages are increasing with job security for all δ, for any b, λ_0.*

As the labor market gets more competitive, the scope for wage cuts disappears. Note that this bound holds for *any* $b < p$ and λ_0. When λ_1 becomes large enough, the increased competition among firms drives up wages with job security throughout the *entire* wage distribution. At the same time, workers keep experiencing a strictly nonzero loss of lifetime utility when becoming unemployed. Keeping λ_0 constant means that the cost of becoming unemployed stays bounded away from zero, even as λ_1 becomes very large. (As $\lambda_1 \to \infty$, the cost of job loss goes to $V_1(\underline{\delta}) - V_0 = \frac{p-b}{r+\lambda_0+\underline{\delta}}$.). To prove this, we rely on the ranking property, which holds for every λ_1, meaning that firm ranking is preserved even after any limit is taken with respect to λ_1 (and λ_0). Thus, we can easily calculate firm sizes as a function of the rank of the firm as we approach the limit without

[39]Since the bound $\tilde{\lambda_1}$ in Result 5 will be shown to be uniform in λ_0, we also know that as the limit is approached in Result 4, with both λ_0 and λ_1 going to infinity, wages will be increasing in job security for all δ, when $\lambda_1 > \tilde{\lambda}_1$.

having to recalculate the wage distribution. In turn, firm profit maximizing decisions are still easily characterized, following Theorem 1, even as we move towards the limit, $\lambda_1 \to \infty$.

As we saw in Result 4, when the economy approaches the competitive limit, no compensating wages are paid. In this case, where search frictions also disappear in the limit for the unemployed, job security ceases to be a payoff-relevant dimension for workers. This is intuitive because, apart from the loss of 'search capital,' there is no additional cost to unemployment.[40] Decreasing λ_1 and λ_0 means that search frictions become more important. As a consequence, job security becomes quite important for workers and firms, while competition among firms becomes more limited. This reduction in competition allows for wage dispersion as well as for the possibility of wage cuts even with increasing job values. Thus while wage cuts seem closely related to the notion of compensating wages paid in competitive settings, cuts are actually associated with a low degree of competition among firms in our environment. Though ironic, the result is intuitive: a low λ_1 means that climbing up the job ladder is a slow process in which gains are lost upon becoming unemployed; therefore, at a lower λ_1, workers value job security more, ceteris paribus. At the same time, a lower λ_1 reduces the competition among firms, diminishing the relative gains of a higher value ranking for firms. As a result, higher ranked firms do not increase the values offered as much. This, however, does not mean that Result 5 immediately follows from the intuition; as firm competition is reduced due to market frictions, firms can use the additional market power to reduce worker values offered. This reduction has a second order negative impact on the value of job security that could, in theory, more than offset the direct positive (ceteris paribus) effect of the decrease in λ_1 on the workers' valuation of job security. However, Result 5, using the explicit equilibrium characterization, shows that this is not the case. As λ_1 goes down, the value of job security increases, allowing firms with high job security to offer a lower wage while still delivering higher values due to their increased job security alone. As a result, wage cuts become possible in equilibrium at low levels of λ_1.

4 Discussion

We have established that for a substantial portion of (standard) wage distributions, wages increase with job security, and we have thus provided a foundation for the observed lack of compensating wage differentials for job security. Our theory emphasizes that, in particular in the lower part of the wage distribution, the forces that push towards the positive correlation between wages and job security are strong. A further implication is that the unemployed are the predominant takers of the riskiest low pay-jobs, with the consequence that unemployed workers are particularly vulnerable for 'no-pay/low-pay'-cycles in their subsequent labor market outcomes. When treating job security like any other amenity, the locus of wage cuts appears less clear: wage cuts, as well as the positive correlation between job security and wages, may occur at any part of the job security distribution. In particular, the lowest wage does not need to occur in the least preferred (and riskiest) firm. In contrast, the endogenous complementarity in our setting closely ties the low-pay to the no-pay in workers' histories.

[40] If a transition into unemployment comes with an explicit cost, then in the limiting economy only the low turnover firms would survive.

On-the-job search plays a key role in the model and the resulting patterns. By allowing (imperfect) competition among firms, on-the-job search creates heterogeneity in the worker's rent as well as in the expected match termination rate due to employer-to-employer transitions.[41] There is an interesting interaction between firm competition and the job destruction distribution across firms. In particular, workers care more about job security, ceteris paribus, when the expected match duration is longer (due to lower outflow rate to other jobs), and firms care relatively more about retention when the expected match duration is longer (due to a lower separation rate into unemployment). This complementarity is missed in models that consider only unemployment-to-employment flows and vice versa.

The randomness of the search technology, on the other hand, does not appear to be essential for our results. In Appendix C, we show in a *directed* search setting with job-to-job mobility, adapted from Delacroix and Shi (2006), that safer firms offer higher values than riskier firms. At work are the same forces as before, including the aforementioned complementarity between outflows to other jobs and outflows to unemployment. Because of the job-to-job mobility, workers are again heterogeneous in the lifetime value of their current employment. In higher-value jobs they optimally direct their search towards jobs with even better conditions – but these come with a lower matching probability. As before, since safer firms care more about worker retention, they have an additional incentive to offer the high values that attract employed workers. Unemployed workers care the least of all workers about the unemployment risk of the jobs, and they target employment in risky firms. It follows that in a direct search setting, mobility patterns would look similar to our setting, to the extent that unemployed workers will likely end up in the most risky firms and long-term employed workers will only switch to safe firms.

Finally, the assumptions that firms commit to the posted wages, while useful to simplify the exposition, does not seem restrictive. Let us consider the case where firms cannot commit to wages ex ante. Given the result in Coles (2001), one would expect a steady state labor market equilibrium with $r \to 0$ that looks precisely like the one in our paper. This equilibrium would be supported by workers who punish any firm deviation from the announced wage by quitting as if they were employed by the least-valued firm, which offers the reservation wage out of unemployment.

Our theory emphasizes that ex ante known heterogeneity in unemployment risk in firms is consistent with a number of labor market observations, among which repeat-unemployment patterns and the decline of separations into unemployment as a function of tenure in the firm. The latter two patterns can also arise when workers differ in productivity and job attachment, with less productive workers being more unattached, as well as when the quality of job matches is only slowly revealed. These theories of sorting of heterogeneous workers across firms and theories of learning about the job match, relative to ours, have very different implications for the distribution

[41] There are other ways in which competition among firms can be introduced in models with search. Firm competition is present in models of non-sequential search such as the noisy search model of Burdett and Judd (1982) and the non-dynamic models of Butters (1977). Most of our qualitative results, as the lack of compensating differentials, would be preserved in a frictional model without on-the-job search but with some form of stochastic direct firm competition. However, the distinctive behavior of employed and unemployed workers while facing a job offer would be lost. One important drawback in this case is that it would not allow us to discuss the apparent unemployment scarring effect that we observe in a on-the-job search framework. Similarly, we would also lose the interaction, over the endogenous duration of a match, of worker poaching among firms and the layoff risk.

of income risk over the labor market. In our theory a worker who has recently lost his job will find that the jobs taken out of unemployment are likely to end in unemployment again. As a result, the uncertainty about lifetime income is amplified, with, for the same worker, high-income secure employment as one possible outcome and low-income unstable employment as another, with the transition from the former situation to the latter in the case of an unexpected job loss. With a match quality that is learned (in part) over the course of the employment relationship, a transition to a new firm itself triggers an increase in uncertainty, which could be avoided by staying put in a match with a known quality, in contrast to our model where job-to-job transitions improve job security.[42] With worker heterogeneity, a high-quality, stable worker would not face the same prospect of repeat-unemployment and low wages that a typical unemployed worker experiences.[43]

Given these different implications, it would be of great interest to further distinguish quantitatively among the sources of patterns of persistent low pay and repeated unemployment among unemployed workers. This is not necessarily an easy task: as we argued above, repeated unemployment spells and low-wage employment spells for a subset of individuals can arise not only due to worker heterogeneity, but can also occur because workers who have become unemployed will accept jobs in differently-looking matches, in different firms, than do long-employed workers. In the latter cases, the jobs that unemployed workers take, will more often be ones with lower wages and a higher risk of becoming unemployed again.

Empirically, there does appear to be an important role to play for firm and job match heterogeneity in workers' unemployment outcomes. In order to corroborate this claim and isolate the role of unobservable worker heterogeneity, one can look at the employment histories of individual workers. If worker heterogeneity is important for unemployment patterns, then the entire labor market history of a worker before a current unemployment spell is informative for future labor market outcomes. In contrast, if only firm and match quality matter, past matches with firms would become irrelevant when a worker becomes unemployed, and hence previous labor market history should not predict future labor market outcomes for the unemployed. In general, after controlling for observable and unobservable worker heterogeneity, along these lines, the empirical literature typically finds that the the causal effect of an unemployment spell on subsequent employment outcomes to be substantial (see e.g. Arulampalam et al. (2000) and Böheim and Taylor (2002)).

One can attempt to further distinguish between the role of ex-ante known firm heterogeneity and the uncertainty about the quality of a new match. The latter can explain an unemployment hazard that could potentially first increase and subsequently decrease with tenure *within a firm*. In contrast, ex-ante known differences among firms in unemployment risk, as captured in our model, could shift up or down the firm-specific unemployment

[42] In the case of learning, a worker who remains at his old firm would also miss out on the upside of the realization of match quality at the new firm.

[43] A correlation between wages and job security could occur because low-ability workers are also unstable workers: they *prefer* not to stay with the same employer for long (Salop and Salop 1976). Alternatively, their skills are less job specific, making them more mobile (Neal 1998), or they are repeatedly screened out during a lower-wage probationary period (Wang and Weiss 1998). Sorting could also behind the worker-specific unemployment risk, with low-ability workers could also sort into risky firms (Evans and Leighton 1989, which could be modeled in a frictional labor market e.g. by extending Albrecht and Vroman (2002) with on-the-job search and firm heterogeneity in layoff rates). See also Carrillo-Tudela and Kaas (2011)

hazard *uniformly at all tenures* across different firms. Longitudinal matched-employer-employee data would simultaneously allow for estimation of worker, firm, and tenure-within-firm effects in the unemployment hazard.[44] A potentially important role for ex-ante known firm differences in unemployment risk is already suggested by findings that observable firm characteristics correlate with the unemployment hazard after controlling for tenure effects (e.g. Winter-Ebmer 2001).[45]

Overall, labor market frictions make unemployment risk matter; in the face of frictions and the ex ante known heterogeneity in unemployment risk, the firms' endogenous wage setting becomes an important dimension, which does not lead to immediately obvious outcomes. It appears to us that, maybe because of a strong intuition about the need for compensating differentials, in more policy-oriented analyses, firm heterogeneity in unemployment risk has not been considered on the same level as other explanations for the persistence in low wages and repeated entry of the same worker into unemployment. This paper, however, suggests that, in the labor market equilibrium, wages can rise with job security even though all workers are identical, and all differences in firms' conditions are fully known in advance, leading precisely to patterns of persistence in low pay and unemployment incidence.

5 Conclusion

In this paper, we have presented a model with homogeneous workers and search frictions in which, in equilibrium, wages do not compensate for differences in unemployment risk. Therefore workers move whenever they have the chance from risky companies to more stable firms, which are also larger. We are able to characterize the joint distribution of equilibrium wages and job security in a very tractable way, making this model amenable to further extensions and estimation. Theoretically, we find that wages increase with job security for the lowest wages; this pattern can extend over a significant part of the wage distribution. While safer firms can offer lower wages while still attracting more workers, the increased job duration makes a worker more valuable to the firm, raising the firm's incentive to prevent worker mobility to other firms, by offering higher wages. At low and

[44] One could perhaps further link the firm-average outflow rate into unemployment with the patterns that occur as function of worker's tenure. For example, it would be interesting to see how the firm-specific *slope* of the unemployment outflow rate varies as a function of job tenure relates to the firm-average *level* of unemployment risk. To our knowledge, this is not an dimension of the data that has received substantial attention, and neither have clear theoretical predictions been derived. Jovanovic (1984) combines the Jovanovic (1979) learning model with Burdett's (1978) partial equilibrium on-the-job search model, and concludes [p.117]: "The job-to-unemployment hazard is hard to characterize. The only result is that it must initially increase." In general, learning models set in frictional labor markets most often abstract from the notion of multi-worker firms which differ in an ex-ante known quality component. Furthermore, in richer models, strategic wage setting is also often abstracted from, by tying wages one-for-one to productivity by assumption (a notable exception to the latter is Moscarini (2005)).

[45] A further way to distinguish gradual learning about match quality from ex ante known firm-level differences in job security is to look at the worker mobility following a wage cut. In our model, when workers move to firm taking a wage cut, this firm will have a lower unemployment risk, and a lower total separation risk (incl. separations to other firms). In the case of uncertain match quality, workers take a wage cut to move to a match with *more* variance. After such a move, separation and even unemployment risk can increase. This way of inference is hampered by the tendency of wages to increase in job security, which we find to be strong in our model.

average wages, the benefit of reducing labor turnover outweights the loss of per worker profit by offering higher wages. However, at high wages, depending on the distribution of unemployemnt risk across firms, wage cuts can occur.

In our model, a willingness to pay for job security does not arise from some deep discomfort in the utility function, but rather from the loss of life-time discounted income. As a result, workers at different positions in the job ladder value job security differently, and a complementarity between the job's rent and security arise endogenously. Taking into account the firms' incentives in their wage setting, a potentially large discrepancy can result between the workers' willingness to pay for job security (which is different for workers in different firms), and the overall cross-sectional correlation between wages and job security, which may very well be positive.

The model also generates an unemployment hazard rate that is declining with tenure, as in the data, while in the standard Burdett and Mortensen (1998) model it is counterfactually constant. We thus show that unemployment scarring in terms of wages *and* risk of repeated job losses arises in equilibrium. However, differently from previous models, the unemployment scarring pattern is neither a consequence from a decline in workers' (perceived) productivity when they become unemployed nor a manifestation of a selection effect on workers, being fundamentally driven by firm heterogeneity.

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APPENDIX

A Proofs

Proof of Lemma 1 What remains to be done is to fill in the few gaps that were not taken care of in the main text. First, note that $V(w,\delta)$ exists as the fixed point of the functional mapping $T:\mathcal{C}\to\mathcal{C}$

$$TV(w,\delta)=\frac{1}{r+\delta+\lambda_1}\Big(w+\lambda_1\int\int\max\{V(w',\delta'),V(w,\delta),V_0\}d\hat{F}(w|\delta)dH(\delta)+\delta V_0\Big). \tag{42}$$

It further follows straightforwardly from the above equation that $V(w,\delta)$ is continuous, increasing in w, and decreasing in δ when $V(w,\delta)\geq V_0$. Given that the support of $H(\delta)$ and $\hat{F}(w|\delta)$ is bounded by assumption, $V(w,\delta)$ is bounded as well. Then, since $V(w,\delta)$ is monotone, continuous and bounded, it is also a.e. differentiable with respect to w (Kolmogorov and Fomin (1975), 31.2, th. 6); similarly, it is a.e. differentiable with respect to δ. At those points, using the right-hand side of equation (2) we find $\partial V(w,\delta)/\partial w$ in (5). Again, similarly, we find $\partial V(w,\delta)/\partial\delta$ in (6). From equations (2) or (42), in particular the integration on the right-hand side of these equations, it follows that $V(w,\delta)$ is in fact absolutely continuous (Kolmogorov and Fomin (1975), 33.2 Th. 5), and therefore, the derivatives in (5) and (6), together with the initial conditions characterize $V(w,\delta)$. (cf. Kolmogorov and Fomin, 33.2 Th. 6). At a zero measure set of points $V(w,\delta)$ is not differentiable; in our formulation, we use (5)-(6) at those points, without affecting the solution $V(w,\delta)$.

The results for R_0 and V_0 in (7) follow from the compensating differential equation (3), which now implies a reservation wage from unemployment R_0 that is unaffected by δ at value $V(w,\delta)=V_0$. Moreover, substituting out the double integral term in (1), using $V(R_0,\delta)=V_0$ in equation (2), yields V_0 as a function of R_0 (or vice versa). As $V(w,\delta)$ is strictly increasing in w above V_0, we can invert it (keeping δ fixed); hence $w(V,\delta)$ exists, is continuous, strictly increasing, and a.e. differentiable. Changing the variable of integration yields (8).

Proof of Lemma 2 In this proof, we show that the appropriate ratio of limits of a sequence of sets agrees with (13) a.e. (with respect to $F(V,\delta)$). (We do not have to worry about firm sizes at a set of measure zero of firms for overall outcomes: anything that happens on a set of firms of measure zero will not affect the choices or utility and profit attained by workers and other firms.) First, we can define

$$\begin{aligned}I(\delta,V)\stackrel{def}{=}&\int_{\delta'\leq\delta,V_0\leq V'\leq V}\left(\delta'+\lambda_1\int_{\tilde{V}>V}dF(\tilde{V},\tilde{\delta})+\lambda_1\int_{\substack{\tilde{\delta}>\delta,\\ V\geq\tilde{V}>V'}}dF(\tilde{V},\tilde{\delta})\right)dG(V',\delta')(m-u)\\&-\int_{\delta'\leq\delta,V_0\leq V'\leq V}\left(\lambda_0u+\lambda_1\int_{\tilde{\delta}>\delta,\tilde{V}<V'}dG(\tilde{V},\tilde{\delta})(m-u)\right)dF(V',\delta')\end{aligned} \tag{43}$$

Then, for $\delta''>\delta'$ and $V''>V'$, we have $I(\delta'',V'')-I(\delta',V'')-I(\delta'',V')+I(\delta',V')=0$, because in steady state $I(\delta,V)=0$ for every (δ,V). After some tedious algebra, in which we drop the flow-terms that cancel each other out, add up the remaining flows where possible, but split the integral such that in one set the upper bound is not included and in the other set the value to integrate over is a singleton $\{V''\}$; this results in

$$\begin{aligned}
&\int_{\substack{\delta'<\delta\leq\delta''\\ V'<V<V''}} \left(\delta + \lambda_1 \int_{\tilde V > V''} dF(\tilde V,\tilde\delta) + \lambda_1 \int_{\substack{\tilde\delta\notin(\delta',\delta'']\\ V<\tilde V\leq V''}} dF(\tilde V,\tilde\delta)\right) dG(V,\delta)\\
&\quad + \int_{\substack{\delta'<\delta\leq\delta''\\ V=V''}} \left(\delta + \lambda_1 \int_{\tilde V > V''} dF(\tilde V,\tilde\delta)\right) dG(V,\delta)\\
&\quad = \int_{\substack{\delta'<\delta\leq\delta''\\ V'<V<V''}} \left(\lambda_0 u + \lambda_1 \int_{\tilde V \leq V'} dG(\tilde V,\tilde\delta) + \lambda_1 \int_{\substack{\tilde\delta\notin(\delta',\delta'']\\ V'\leq\tilde V<V}} dG(\tilde V,\tilde\delta)\right) dF(V,\delta)\\
&\quad + \int_{\substack{\delta'<\delta\leq\delta''\\ V=V''}} \left(\lambda_0 u + \lambda_1 \int_{\tilde V \leq V'} dG(\tilde V,\tilde\delta)\right) dF(V,\delta) \qquad (44)
\end{aligned}$$

Now, we can take the limit as $\delta' \to \delta''$ and $V' \to V''$. There are two cases: (i) $\int_{V=V'',\text{all }\delta} dF(V,\delta) = 0$, and (ii) $\int_{V=V'',\text{all }\delta} dF(V,\delta) > 0$. In case (i), the terms on the second and fourth line equal zero, while the rightmost terms in the integral on the first and third line are equal in value to an integral that has a strict upper or lower bound on values, i.e. $\lambda_1 \int_{\substack{\tilde\delta\notin(\delta',\delta'']\\ V<\tilde V\leq V''}} dF(\tilde V,\tilde\delta) = \lambda_1 \int_{\substack{\tilde\delta\notin(\delta',\delta'']\\ V<\tilde V<V''}} dF(\tilde V,\tilde\delta) \overset{def}{=} T(V,\delta')$. Moreover, T is continuous at (δ'',V''), with $T(\delta'',V'') = 0$, so we have

$$(\delta + \lambda_1(1 - F(V''))\frac{\lim_{\substack{\delta'\to\delta''\\ V'\to V''}} \int_{\substack{\delta'<\delta<\delta''\\ V'<V\leq V''}} dG(V,\delta)}{\lim_{\substack{\delta'\to\delta''\\ V'\to V''}} \int_{\substack{\delta'<\delta<\delta''\\ V'<V\leq V''}} dF(V,\delta)} = (\delta + \lambda_1(1 - F(V''))l(V'',\delta) = \lambda_0 u + \lambda G(V''),$$

using that $\frac{dG(V'',\delta)}{dF(V'',\delta)} = l(V'',\delta)$, and that $G(V''), F(V'')$ are continuous at V''. Rearranging yields (13).

For case (ii), we can first take the limit $V' \to V''$ on both sides of the equation. The terms on the first and third line go to zero. If the second and fourth line are zero as well, we are dealing a set $\{(V,\delta)|V = V'', \delta \in (\delta',\delta'']\}$ that is of measure zero in F, which wlog for the aggregate patterns, we can ignore. Suppose therefore that $B(\delta',\delta'') \overset{def}{=} \{(V,\delta)|V = V'', \delta \in (\delta',\delta'']\}$ is of positive measure. Then in the limit as $V' \to V''$ (44) reduces to

$$\int_{\substack{\delta'<\delta\leq\delta''\\ V=V''}} \left(\delta + \lambda_1 \int_{\tilde V > V''} dF(\tilde V,\tilde\delta)\right) dG(V,\delta) = \int_{\substack{\delta'<\delta\leq\delta''\\ V=V''}} \left(\lambda_0 u + \lambda_1 \int_{\tilde V < V''} dG(\tilde V,\tilde\delta)\right) dF(V,\delta) \qquad (45)$$

Consider now the limit as $\delta' \to \delta''$ while $B(\delta',\delta'')$ stays of positive measure (if it becomes of zero measure, we can ignore it, wlog). The terms between brackets inside the integrals stay constant, and hence can be taken outside the integrals. Dividing both sides by $\int_{\substack{\delta'<\delta\leq\delta''\\ V=V''}} dF(V,\delta)$, and taking the limit wrt δ, we have

$$(\delta + \lambda_1(1 - F^+(V''))\frac{\lim_{\delta'\to\delta''} \int_{\substack{\delta'<\delta\leq\delta''\\ V'=V''}} dG(V,\delta)}{\lim_{\delta'\to\delta''} \int_{\substack{\delta'<\delta\leq\delta''\\ V'=V''}} dF(V,\delta)} = (\delta + \lambda_1(1 - F^+(V''))l(V'',\delta) = \lambda_0 u + \lambda G^-(V''),$$

where $(1 - F^+(V'') = \int_{\tilde V > V''} dF(\tilde V,\tilde\delta)$ and $G^-(V) = \int_{\tilde V < V''} dG(\tilde V,\tilde\delta)$. $\square$

Proof of proposition 1 To spell out the last step in the proof, consider

$$\frac{(p - w(V_s,\delta_l))l(V_s,\delta_l)}{(p - w(V_r,\delta_l))l(V_r,\delta_l)} \geq \frac{(p - w(V_s,\delta_h))l(V_s,\delta_h)}{(p - w(V_r,\delta_h))l(V_r,\delta)} \qquad (46)$$

This implies that either

$$\frac{(p - w(V_s, \delta_l))}{(p - w(V_r, \delta_l))} \geq \frac{(p - w(V_s, \delta_h))}{(p - w(V_r, \delta_h))}, \tag{47}$$

in which case it must be that $V_s \geq V_r$ by equation (19); and/or

$$\frac{l(V_s, \delta_l)}{l(V_r, \delta_l)} \geq \frac{l(V_s, \delta_h)}{l(V_r, \delta)}, \tag{48}$$

in which case it again must be that $V_s \geq V_r$ by the converse to equation (15). Since these two cases cover all possibilities (using that all terms are positive), it must be that $V_s \geq V_r$. $\square$

Proof of proposition 2. First, the same argument that established that $\tilde{G}(V, \delta)$ is absolutely continuous with respect to $\tilde{F}(V, \delta)$ can be made to establish that $\tilde{F}(V, \delta)$ is absolutely continuous with respect to $\tilde{G}(V, \delta)$, which then necessarily implies that each property (i)-(iii) applies to $F(V)$, if and only if it applies to $G(V)$.

To establish property (ii), towards a contradiction, consider a V which is offered by a strictly positive mass of firms, with δ_1 as the infimum of those types offering this value. The expected profit gain for this type when offering a wage that implies a value $V + \varepsilon$ is greater than

$$\frac{d\pi}{d\varepsilon} > -(r + \delta + \lambda_1)\, \varepsilon\, l(V, \delta_1) + (p - w(V, \delta)) \frac{\lambda_1 G^-(V + \varepsilon) - \lambda_1 G^-(V)}{\lambda_1(1 - F^+(V)) + \delta_1}, \tag{49}$$

where the first term follows from the fact that $dw(V, \delta)/dV < r + \delta + \lambda(1 - F^+(V))$ almost everywhere. The change in firm size (from offering a value ε higher than V) is larger than the left-most term. Note that a mass point at V implies that, there exists a $\eta > 0$, such that for all $\varepsilon > 0$, $G^-(V + \varepsilon) - G^-(V) \geq \frac{\eta(\lambda_1 + \bar{\delta})}{\lambda_1} > 0$. Then, as $\varepsilon \to 0$, $\frac{d\pi}{d\varepsilon} \geq (p - w(V, \delta))\eta > 0$ and hence a strictly profitable deviation exists to offering for firms who offer V, to offer $V + \varepsilon$ with ε small enough. Moreover, this applies to a non-zero measure of agents, since it holds *all* firms offering V: the argument above does not depend on the size of δ.

Next, (towards contradiction) consider the case where the support of $F(V)$ is not connected. Let $\underline{V}$ be the minimum value in the support of $F(V)$, and $\bar{w}^s$ the corresponding maximum value. Then, there exist $\underline{V} < V_1 < V_2 < \bar{V}$ such that $F(V_1) = F(V_2)$, and $G(V_1) = G(V_2)$ by the continuity of $F(V)$, $G(V)$. Consider a firm posting at V_2, this firm can keep the same firm size but make strictly more profit when deviating to V_1. If $\underline{V} > V_0$, the same argument applies: the firm offering $\underline{V}$ can deviate to V_0, which does not affect his firm size, but strictly raises the profit per worker. $\square$

Proof of theorem 1 There are three steps in this proof. First, one can show that the equilibrium objects $F(V), G(V), V(w, \delta), \hat{F}(w|\delta), R_0$ constructed from $V(z), w(z)$ satisfy workers' and firms' optimization. This is straightforward, with this and the derivation in the paper, we have established that a steady state equilibrium corresponds to $\{V(z), w(z), V_0\}$ and vice versa.[46] Second, we establish that the second-order conditions are also satisfied whenever the first-order conditions hold. Finally, we show that the existence of the equilibrium is guaranteed, and its uniqueness.

[46]For completeness, this in appendix B.

Pseudoconcavity of the firm's problem Secondly, we have to check that the first-order conditions indeed pick the maximum in the firm's problem, at any point where $\delta(z)$ is continuous and differentiable. We can verify that the problem is pseudo-concave, using $dV(z)/dz$ in equation (34) and the firms's first-order condition (28) by showing the derivative of the first-order condition below is negative when (29) holds.

$$\frac{d}{dz'}\left((p - w(V(z'), \delta(z)))\frac{\partial l^d(z', \delta(z))}{\partial z'} - \left(\frac{\partial w(V(z'), \delta(z))}{\partial V(z')}\frac{dV(z')}{dz'}\right) l^d(z', \delta(z))\right) \tag{50}$$

Evaluated at a point where the first order condition is equal to zero ($z = z'$), this has the same sign as

$$\frac{\partial}{\partial z'}\left((p - w(V(z'), \delta(z)))\frac{2\lambda_1}{\lambda_1(1-z') + \delta(z)}\right) - \frac{\partial}{\partial z'}\left(\frac{\partial w(V(z'), \delta(z))}{\partial V(z')}\frac{\partial V(z')}{\partial z'}\right) \tag{51}$$

Evaluating the above term at $z = z'$, the second term equals $\frac{\partial}{\partial z'}\left((p - w(V(z'), \delta(z)))\frac{2\lambda_1}{\lambda_1(1-z')+\delta(z')}\right)$, which differs from the first, left-most, term only by $\delta(z)$ vs. $\delta(z')$ in the denominator. Therefore, the only term that does not cancel out between the first and second term of second-order condition (51) is

$$(p - w(V(z), \delta(z))\frac{\delta'(z)}{(\lambda(1-z) + \delta(z))^2} < 0, \tag{52}$$

which is negative, since $\delta'(z) < 0$.

Existence and uniqueness of the fixed point R_0. Finally, we have to show existence and uniqueness of the reservation wage $R_0 = w(0)$, and associated $V_0 = \frac{\lambda_0 R_0 - \lambda_1 b}{r(\lambda_0 - \lambda_1)} = V(0)$, satisfying (33), (34), (37). From this, the existence and uniqueness of the steady state equilibrium then follows. Index $\frac{dV(z;R_0)}{dz}$, $\frac{dw(z;R_0)}{dz}$ by initial condition R_0; then (37) is the solution to the following fixed point

$$R_0 = T(R_0), \text{ where } T(R_0) = b + (\lambda_0 - \lambda_1)\int_0^1 (1-z)\frac{dV(z;R_0)}{dz}dz \tag{53}$$

Note that $\frac{dV(z)}{dz}$ depends implicitly on the reservation only through $(p - w(z))$.

Manipulating (33) and (34), we can define $x(z) = \frac{V(z)-V_0}{p-w(z)}$ and find that the system of two equations $dV(z)/dz$ and $dw(z)/dz$ can equivalently be written as two equations of which one differential equation only takes itself as argument,

$$\frac{d(p-w(z))}{dz} = -(p - w(z))\left(\frac{2\lambda_1}{\lambda_1(1-z) + \delta(z)} + \delta'(z)x(z)\right) \tag{54}$$

$$\frac{dx(z)}{dz} = \frac{2\lambda_1}{\lambda_1(1-z) + \delta(z)}\frac{1}{r + \lambda_1(1-z) + \delta(z)} + \left(\frac{2\lambda_1}{\delta(z) + \lambda_1(1-z)}\right)x(z) + \delta'(z)x(z)^2. \tag{55}$$

Note that $p - w(0) = p - R_0$, and $x(0) = 0$. Note that $\frac{d(p-w(z;R_0))}{dR_0}$, by standard FODE theory, is continuous in R_0, and we will see this derived below as well. Consider first the interval $[0, \tilde{z}]$ on which $\delta(z)$ is continuous. On this interval, $x(z)$ does not depend on R_0. We can rewrite (54) to get

$$\frac{\frac{d(p-w(z))}{dz}}{(p - w(z))} = -\left(\frac{2\lambda_1}{\delta(z) + \lambda_1(1-z)} + \delta'(z)x(z)\right); \tag{56}$$

Integrating over z yields

$$p - w(z) = e^{-\int_0^z \left(\frac{2\lambda_1}{\delta(z)+\lambda_1(1-z)} + \delta'(z)x(z)\right)dz}(p - R_0), \tag{57}$$

where the exponential term does not depend on R_0. It follows immediately that

$$\frac{d(p - w(z))}{dR_0} = -e^{-\int_0^z \left(\frac{2\lambda_1}{\delta(z)+\lambda_1(1-z)} + \delta'(z)x(z)\right)dz} < 0.$$

To generalize this to general distributions $H(\delta)$, consider next a point where $\delta(z)$ is discontinuous: this a point where $\delta(z)$ drops discretely. We want to show that the properties of $\frac{d(p-w(z))}{dR_0}$, $\frac{dx(z)}{dR_0}$ are preserved. Consider first $x(z)$, from (35), which in turn comes from the worker's indifference curve in equation (3),

$$(p - w(\check{z})) = \lim_{z\uparrow\check{z}}(p - w(z)) - (\delta(\check{z}) - \lim_{z\uparrow\check{z}}\delta(z))(V(z) - V_0) \tag{58}$$

To shorten notation, let, for a generic function $y(z)$, the limit $\lim_{z\uparrow\check{z}} y(z)$ be denoted by $y_L(z)$. Then we can rewrite the above equation (58) as

$$\big(x(z)\big)^{-1} = \big(x_L(z)\big)^{-1} - (\delta(z) - \delta_L(z)) \iff \tag{59}$$

$$x(z) = \frac{x_L(z)}{x_L(z) - (\delta(z) - \delta_L(z))} < x_L(z). \tag{60}$$

Hence, if $dx_L(z; R_0)/dR_0 = 0$, it follows that $dx(z; R_0)/dR_0 = 0$.

Thus, the irresponsiveness of $\frac{dx(z)}{dR_0}$ is also preserved whenever $\delta(z)$ drops discretely. Let $Z = \{\zeta_i\}$ be the countable set of ranks z at which $\delta(z)$ drops discretely; define additionally $\zeta_0 = 0$. Then, letting $\bar{\zeta}(z) = \sup\{\zeta \in Z | \zeta < z\}$

$$p - w(z) = (p - R_0)\left(\prod_{\{i|\zeta_i\in Z,\zeta_i<z\}} e^{-\int_{\zeta_{i-1}}^{\zeta_i}\left(\frac{2\lambda_1}{\delta(z')+\lambda_1(1-z')} + \delta'(z')x(z')\right)dz'}\left(\frac{x_L(\zeta_i)}{x_L(\zeta_i) - (\delta(\zeta_i) - \delta_L(\zeta_i))}\right)\right.$$
$$\left. \cdot\ e^{-\int_{\bar{\zeta}(z)}^{z}\left(\frac{2\lambda_1}{\delta(z')+\lambda_1(1-z')} + \delta'(z')x(z')\right)}\right) \iff \tag{61}$$

$$p - w(z) = (p - R_0)\mathtt{A}(z), \tag{62}$$

summarizing the entire bracketed term, which only depends on firm rank z and fundamentals but not on R_0, in equation (61)into term $\mathtt{A}(z)$ in (62). [47] From this it immediately follows that $\frac{d(p-w(z;R_0))}{dR_0} = -\mathtt{A}(z) < 0$. Moreover the $T(R_0)$ mapping becomes

$$T(R_0) = b + (p - R_0)(\lambda_0 - \lambda_1)\int_0^1 \frac{2\lambda_1(1 - z')}{\lambda_1(1 - z') + \delta(z')}\frac{1}{r + \delta(z') + \lambda_1(1 - z')}\mathtt{A}(z')dz' \tag{63}$$

[47] A similar result holds true in the standard Burdett and Mortensen model, where

$$p - w(z) = (p - R_0)\left(\frac{\lambda_1(1 - z) + \delta}{\lambda_1 + \delta}\right)^2,$$

however, here we have take care of the heterogeneity in δ, and the resulting influence on the wages (with wage cuts etc.). Notice that if we set $\delta'(z) = 0$ and hence $\Delta(z) = 0$ and $x(z) = 0\ \forall z$, the Burdett and Mortensen result in fact follows. The observation that a similar property is preserved in our more complicated setting is encouraging for the wider applicability of the BM-type wage posting framework, e.g. when incorporating further heterogeneity.

Denoting the term post-multiplying $(p - R_0)$ by $\mathtt{B}$, we find

$$R_0 = \frac{(b/p + \mathtt{B})}{1 + \mathtt{B}} p, \tag{64}$$

which for any $b \leq p$ gives the reservation wage R_0. This establishes the existence, and the uniqueness of the equilibrium reservation wage of the unemployed, and given the existence and uniqueness of the firm posting and workers' value function given the reservation wage R_0, it establishes the overall existence and uniqueness of the equilibrium|. $\square$

Proof of Result 1 The inflow into employment $\lambda_0 u + \lambda G(z) = l(z)(\delta(z) + \lambda(1 - z))$, the probability that an inflow at time t survives until $t + \tau$ is $e^{(\delta(z)+\lambda_1(1-z))\tau}$; thus the number of workers in the z^{th} firm who have been around τ periods $t_{eu}(z, \tau) \overset{def}{=} l(z)(\delta(z) + \lambda(1 - z)e^{-(\delta(z)+\lambda(1-z))\tau}$. Then, the derivative of the empirical hazard rate with respect to tenure is

$$d \ln \left(\frac{\int_0^1 \delta(z) t_{eu}(z, \tau) dz}{\int_0^1 t_{eu}(z, \tau) dz} \right) \Big/ d\tau = -\int_0^1 \frac{(\delta(z) - \delta^{ave}) t_{eu}(z, \tau)\ (\delta(z) + \lambda_1(1 - z)) dz}{\int \delta^{ave} t_{eu}(z', \tau) dz'} < 0 \tag{65}$$

The derivative $dt_{eu}(z, \tau)/d\tau = -t_{eu}(z, \tau)(\delta(z)+\lambda_1(1-z))$. Define $\delta^{ave} \int t_{eu}(z', \tau) dz' = \int \delta(z') t_{eu}(z', \tau) dz'$. Then $\int_0^1 (\delta(z) - \delta^{ave}) t_{eu}(z, \tau) dz$ equals zero; since $\delta(z) - \delta^{ave}$ and $(\delta + \lambda_1(1 - z))$ are both decreasing, the latter one strictly, the integral term in (65) is positive, establishing the result.

Proof of Result 2 We have to make sure not only that $w(z)$ is increasing on an interval $[0, \tilde{z}]$ itself, but also that there exists an interval $[0, \check{z}]$ where additionally for no further $z > \check{z}$ we have that $w(z) < w(\tilde{z})$. By theorem 1, locally, for z close 0, we have $w(z)$ strictly increasing, while by proposition 2 $V(z)$ strictly increasing everywhere. Then immediately there exists $\tilde{z} > 0$ such that $w(z)$ is strictly increasing for all $0 < z < \tilde{z}$. Now, towards a contradiction, suppose that there does not exist a $\check{z} > 0$ such that for all $z > \tilde{z}$, it holds that $w(z) > w(\check{z})$. Then, there must exist a sequence $\{z_n\}$ with $z_n > \check{z}$, such that $w(z_n) \to R_0$. By (3), then also $V(z_n) \to V_0$. But then, there exists an n such that $V(z_n) < V(\check{z})$, contradicting the ranking property/the strict monotonicity of $V(z)$.

Proof of Result 3 Note that since $dw(0)/dz > 0$, at the the $\check{z}$ from which onwards an *interval* of wage cuts occurs, both $dw(\check{z})/dz = 0$ and $d^2 w^*(\check{z})/(dzdz) < 0$, i.e. $dw^*(z)/dz$ cuts 0 from above, at $\check{z}$. Note that a point at which first and second derivative are zero will not translate in any wage cuts, or strict decrease of wage in job security. The second derivative at $\check{z}$ equals

$$\begin{aligned} \frac{d^2 w^*}{dzdz} = &-\frac{dw^*}{dz} \frac{2\lambda_1}{\lambda_1(1 - z) + \delta(z)} + (p - w^*(z)) \frac{2\lambda_1(\lambda_1 - \delta'(z))}{(\lambda_1(1 - z) + \delta(z))^2} + \delta''(V(w^s(z), \underline{\delta}) - V_0) \\ &+ \delta'(z) \frac{dV(z)}{dz}. \end{aligned} \tag{66}$$

Note that after substituting in $dV(z)/dz$ from (33), the terms with $\delta'(z)$ cancel out. Evaluated at a point where $dw/dz = 0$, and letting $r \to 0$, this turns into

$$\left.\frac{d^2w^*}{dzdz}\right|_{\frac{dw^*}{dz}=0} = (p - w^*(z))\frac{2\lambda_1^2}{(\lambda_1(1-z)+\delta(z))^2} + \delta''(V(w^s(z),\underline{\delta}) - V_0) \tag{67}$$

This can be smaller than zero only if $\delta''(z) < 0$, which in turn occurs if and only if $h'(\delta) > 0$, since $\delta'(z) = -1/h(\delta)$ and $\delta''(z) = \frac{h'(\delta)}{h(\delta)^2}\delta'(z)$.

The second point follows from (34) being negative. Substituting in $V(z) - V_0 = \int_0^z dV(z)/dzdz$ into $dw(z)/dz$ in equation (34), a change of the integrating variable ($dz = -h(\delta)d\delta$), this can be written equivalently as

$$(p - w(1 - H(\delta)))\frac{2\lambda_1}{\delta + \lambda_1(1 - H(\delta))} < \frac{1}{h(\delta)}\int_\delta^{\bar{\delta}} \frac{2\lambda_1}{(\tilde{\delta} + \lambda(1 - H(\tilde{\delta})))^2}(p - w(1 - H(\tilde{\delta})))h(\tilde{\delta})d\tilde{\delta}$$

Since $p - w((1 - H(\tilde{\delta}))) > p - w(1 - H(\delta))$ for $\tilde{\delta} > \delta$ if no other wage cuts with increased job security have occurred, the above equation will be negative whenever (39) holds. (In the other case, if wage cuts have occurred lower in the value distribution (at lower z), then point 2. holds trivially.)

Proof of Result 4 is in the main text

Proof of Result 5 We can show this by establishing that $\frac{d^2w(\tilde{z})}{dwdw} < 0$, and $\frac{dw(\tilde{z})}{dz} = 0$ cannot occur for $\lambda_1 > \frac{h'(\delta)}{(h(\delta))^2}$.Note that the existence of wage cuts implies a z such that $dw(z)/dz = 0$, $\frac{d^2w(z)}{dzdz} \leq 0$. This implies that at that z, from (34) and (67),

$$(p - w(z))\frac{2(\lambda_1)^2}{\delta(z) + \lambda_1(1-z)} < -\delta''(z)\int_0^z \frac{dV(z')}{dz'}dz \tag{68}$$

$$(p - w(z))\frac{2\lambda_1}{\delta(z) + \lambda_1(1-z)} = -\delta'(z)\int_0^z \frac{dV(z')}{dz'}dz \tag{69}$$

Dividing the RHS of (68) by the RHS of (69), and similarly for the LHS, this yields

$$\lambda_1 < \delta''(z)/\delta'(z) = \frac{h'(\delta)}{(h(\delta))^2}$$

as a necessary condition. Hence if $\lambda_1 > \frac{h'(\delta)}{(h(\delta))^2}$, we do not satisfy the necessary condition, and therefore can rule out $dw(z)/dz < 0$ at $z = 1 - H(\delta)$.

ADDITIONAL MATERIAL (NOT FOR PUBLICATION)

B Theorem 1, verification

Theorem 1: Direct Verification in terms of Original Equilibrium Objects In the paper, we derived $\{V(z), w(z), R_0\}$ satisfying (33), (34), (37) from the equilibrium conditions. Here, we outline the reverse argument: any set $\{V(z), w(z), R_0\}$ satisfying (33), (34) and (37) corresponds to a steady state equilibrium with the equilibrium objects listed in definition 1.

The reverse construction of equilibrium objects, such as $\hat{F}(w|\delta)$ etc., is very straightforward, and we will mention it here for the sake of completeness. We construct (i) the aggregate conditions and distributions, then show that these are consistent with the orginal (ii) firm's optimization, and (iii) the original worker's optimization. First invert $V(z)$, to construct $F(V)$, $G(V) = G^z(F^{-1}(V))$, and $V(w, \delta)$ implicitly from $V(z), w(z), \delta(z)$ for those wages that are actually posted on the equilibrium path. Then we can construct $\hat{F}(w|\delta)$ from $\delta(z), w(z)$, using that if $\delta(z)$ is strictly decreasing at z, then $F(w(z)|\delta(z)) = 1$, while if $\delta(z)$ is constant on a compact interval, then we isolate this interval, rescale it to [0,1] and invert the re-scalled $w(z)$ to find $\hat{F}(w|\delta)$. $\tilde{F}(V, \delta), \tilde{G}(V, \delta)$ can be constructed straightforwardly from $F(\hat{w}|\delta)$ and $H(\delta)$, or from $V(z)$ and $\delta(z)$ directly by the monotonicity of the latter functions. Similarly for $\tilde{G}(V, \delta)$.

Now we have to show that these distribution are consistent with optimal decision making, (ii)-(iii). Given $F(V)$, $V(w, \delta)$, where for off-equilibrium (w, δ) we use the same partial derivatives as in lemma 1, to derive the entire function $V(w, \delta)$. For worker optimization and firm optimization, consider that firms choose w in the profit maximization, which leads to the following first order condition

$$(p - w)\frac{\partial l(V(w, \delta), \delta)}{\partial w} - l(V(w, \delta), \delta) = 0, \tag{70}$$

which can be rewritten as

$$1 = (p - w)\frac{\frac{\partial l(V(w,\delta),\delta)}{\partial V}}{l(w^s, \delta)}\frac{dV(w, \delta)}{dw} = 0, \iff \frac{\frac{\partial w}{\partial V(w,\delta)}}{p - w(V(z), \delta(z))} = \frac{2\lambda_1 F'(V)}{\delta + \lambda(1 - F(V))} \tag{71}$$

Using $F'(V) = (dV(z)/dz)^{-1}$, we can verify that (28) and (31) combined with (33) and (34) imply the equivalence of (71) and (28). In other words, given the constructed distributions, firms indeed maximize their profit when taking the original optimization problem with respect to posted wage w to post. Moreover, if $\{V(z), w(z), R_0\}$ does not satisfy (33), then the first order condition of wages (71) is likewise violated.

Finally, consider the worker's optimization. This is fully captured in $V(w, \delta)$ and R_0. Value function $V(w, \delta)$ in turn is characterized by partial derivatives (5) and (6). Function list $\{V(z), w(z), \delta(z)\}$ defines points of $V(w, \delta)$ that occur in equilibrium as a parametric equation of z. For a generic 'parametric equation' it has to hold that $dV/dz = \partial V/\partial w \cdot dw/dz + \partial V/\partial\delta \cdot d\delta/dz$. The equivalence of this and (34) after the appropriate substitutions for distributions using dw/dz from (33) follows. Finally, the equivalence of R_0 in (37), and R_0 defined in (8), follows directly from a change of the variable of integration.

Hence the existence and uniqueness of the equilibrium $\{V(z), w(z), R_0\}$ maps into the existence and uniqueness of the equilibrium as laid out in definition 1.

C A model of Directed Search, on-the-job Search and heterogeneous layoff risks across firms.

In this appendix, we show that our results are robust to the introduction of directed search, instead of random search. In order to do that, we extend the directed search model with on-the-job search presented by Delacroix and Shi (2006) to an environment in which different firms post vacancies with different layoff risks. In order to avoid discussions about the different costs to entry, we do not consider entry conditions, although the model can be extended to include that. Therefore, apart from these two deviations, we will follow Delacroix and Shi's set up.

Model

Consider a labor market that lasts forever in continuous time. There is a measure m of homogeneous, risk-neutral and infinitely lived workers. An employed worker at a firm offering a wage rate $w > 0$ produces a flow of output, $y > 0$, and receives a income flow of $w > 0$. An unemployed worker enjoys the unemployment benefit, $b > 0$. The unemployment rate u is endogenous. We consider that there is a measure of firms M. Firms are identical up to the layoff risk of the offered jobs. In this sense, there are 2 types of firm: type H, which offer jobs with layoff risk δ_H, and type L, which offer jobs with layoff risk $\delta_L < \delta_H$. The proportion of type H firms is given by $\gamma_H \in (0, 1)$. We assume that each firm has one job to offer, and the cost of posting a vacancy is $C > 0$ per period, irrespective of the job's layoff risk profile. Both firms and workers are infinitely-lived, risk-neutral, and discount the future at rate $r > 0$.

Each employed worker receives an exogenous job destruction shock at poisson rate Poisson $\delta_i > 0$, where i indicates the type of firm that the worker is in, $i \in \{H, L\}$. The worker also receives a job application opportunity at a Poisson rate $\lambda_1 > 0$ if employed and $\lambda_0 > 0$ if unemployed. A job application opportunirty enables the worker to apply to other jobs. Unemployed workers can receive a job application opportunity shock with probability $\lambda_0 \geq \lambda_1$. We assume that $\lambda_j < 1,\ j \in \{0, 1\}$. Firms are assumed to commit to the contracts, but workers can quit a job at any time. In particular, a firm cannot respond to the employee's outside offers. Finally, firms will post wage levels rather than contracts.

There is a potentially infinite number of submarkets indexed by the offer value x. Each submarket x has a tightness $\theta(x)$, wich is the ratio of applicants to vacancies in that submarket. The total number os matches in submarket x is given by a linearly homogeneous matching function $\mathcal{M}\left(N(x), \frac{N(x)}{\theta(x)}\right)$, where $N(x)$ is the number of applicants in the submarket. In submarket x, a vacancy is filled at the Poisson rate $q(x) \equiv \mathcal{M}(\theta(x), 1)$ and an applicant obtains an offer at rate $p(x) = \mathcal{M}\left(1, \frac{1}{\theta(x)}\right)$. Notice that we are assuming here that the firm layoff risk does not play a role in its ability to match. However, notice that to provide the same value x, a firm with a high layoff risk will need to pay a higher wage, as it will be clear in future. Denote the fraction of high layoff risk firms posting in submarket x as $\gamma_H(x)$. We assume that workers incur a fee $S > 0$ to enter any submarket. If a vacancy is not filled, the firm must incur the vancancy cost again in order to recruit.

In equilibrium, $q(x)$ is increasing and $p(x)$ is decreasing in x. Thus, search is directed in the sense that agents face a trade-off between offers and matching rates when choosing which submarket to enter. Although the function $\mathcal{M}$ is exogenous, the functions $q(\cdot)$, $p(\cdot)$, and $\theta(\cdot)$ are equilibrium objects. Then, we can eliminate θ from the expressions for p and q to express $p(x) = M(q(x))$. Because $M(q)$ inherits all essential properties of the function $\mathcal{M}$, we can take $M(\cdot)$ as a primitive of the model and refers to it as the matching function.

We focus on stationary equilibria where the set of offered contracts and the functions $q(x)$ and $p(x)$ are time invariant. Moreover, we focus on an equilibrium in which $p(\cdot)$ satisfies:

$$\begin{aligned}
&(i)\;\; p\left(\overline{V}\right) = 0; \\
&(ii)\;\; p(x) \text{ is bounded, continous and concave for all } x; \qquad\qquad (A.1) \\
&(iii)\;\; p(x) \text{ is strictly decreasing and continously differentiable for all } x < \overline{V}
\end{aligned}$$

REMARK: This assumptions are not necessary for most of the intuition and results in the paper. It is just to get a nice and concave $p(\cdot)$;

We first characterize individual's decision under any arbitrary p function that satisfies $(A.1)$, then verify that an equilibrium satisfying $(A.1)$ exists.

Worker's Optimal Search Decision

Notice that a worker's search decision is twofold, once he receives an opportunity to search. First, he needs to decide if he will take the opportunity or not. Second, once he decides to pay the fee $S > 0$, which submarket he should enter. Let's start with the latter. Refer to a worker's value, V, as the worker's state or type. If the worker searches in submarket x, he obtains the offer x at rate $p(x)$, which yields the gain $(x - V)$. The expected gain from search in submarket x is $p(x)(x - V)$. The optimal search decision x solves:

$$S(V(t)) = \max_{x \in \left[V(t), \overline{V}\right]} p(x)(x - V) \qquad (A.2)$$

Denote the solution as $x = F(V)$.

Lemma 4. *Assume* (2.2). *Then* $F\left(\overline{V}\right) = \overline{V}$. *For all* $V < \overline{V}$, *the following results hold:*

(i) $F(V)$ *is interior, strictly increasing in* V, *and satisfies:*

$$V = F(V) + \frac{p(F(V))}{p'(F(V))}. \qquad (3.2)$$

(ii) $F(V)$ *is unique for each* V *and continuous in* V;

(iii) $S(V)$ *is differentiable with* $S'(V) = -p(F(V)) < 0$;

(iv) $F(V_2) - F(V_1) \leq \frac{1}{2}(V_2 - V_1)$, $\forall V_2 \geq V_1$;

(v) If $p''(\cdot)$ exists, then $F'(V)$ and $S''(V)$ exist, with $0 < F'(V) \leq \frac{1}{2}$.

Proof. See Shi (2009) $\square$

Now, let's consider the first decision. Notice that a worker will decide to search if and only if $S(V) \geq S$. Since $S(V)$ is strictly decreasing, this implies that there is a cut off on V, V^*, where workers stop searching if $V > V^*$. But then, no firm has an incentive to offer a wage above $V^* \Rightarrow V^* = \overline{V}$.

Value Functions of Workers and Firms

Employed worker at a job of current value V value function in a firm with layoff risk δ_i is given by:

$$rV = w(\delta_i, V) + \lambda_1 \{I_V \times [p(F(V))[F(V) - V] - S]\} + \delta_i [V_U - V] \tag{72}$$

where V_U is the value function for an unemployed worker and I_V is an indicator function that is equal to 1 if $S(V) \geq S$. Then, based on this expression, we can see that the salaries offered by firms with different levels of job insecurity must differ according to the following expression:

$$w(\delta_H, V) = w(\delta_L, V) + (\delta_H - \delta_L)(V - V_U)$$

Then, the value function for an unemployed worker is given by:

$$rV_U = b + \lambda_0 \{p(F(V_U))[F(V_U) - V_U] - S\}$$

Now, consider the value of a firm that has layoff risk δ_i and has a filled job that offers the worker a value V. Then, we have:

$$rJ_f(\delta_i, V) = y - w(\delta_i, V) + (\delta_i + \lambda_1 I_V p(F(V)))\left[\overline{J}_v(\delta_i) - J_f(\delta_i, V)\right]$$

where $\overline{J}_v(\delta_i)$ is the value function for a firm with no employees that needs to decide if it opens a vacancy or not.

Then, the value for a firm that opened a vacancy at submarket V and has layoff risk δ_i is:

$$rJ_v(\delta_i, V) = -C + q(V)\left[J_f(\delta_i, V) - \overline{J}_v(\delta_i)\right]$$

Then, a firm chooses to enter a market V in order to maximize:

$$q(V)\left[J_f(\delta_i, V) - \overline{J}_v(\delta_i)\right]$$

Notice if two values $-\ V_A$ and $V_B\ -$ are posted by firms of the same type i, we must:

$$q(V_A)\left[J_f(\delta_i, V_A) - \overline{J}_v(\delta_i)\right] = q(V_B)\left[J_f(\delta_i, V_B) - \overline{J}_v(\delta_i)\right]$$

Finally, notice that:

$$\overline{J}_v(\delta_i) = \max_V \frac{1}{r}\left\{-C + q(V)\left[J_f(\delta_i, V) - \overline{J}_v(\delta_i)\right]\right\}$$

Before we continue, let's prove an auxiliary lemma that will help us latter:

Lemma 2: The wage function $w(\delta, V)$, for $V < V^*$, has the following properties:

(i) $\frac{dw(\delta,V)}{d\delta} = V - V_U > 0$;

(ii) $\frac{dw(\delta,V)}{dV} > 0$;

(iii) $\frac{d^2w(\delta,V)}{dVd\delta} > 0$.

Proof: From (1), we have:

$$rV - \lambda_1\{I_V \times [p(F(V))[F(V) - V] - S]\} + \delta_i[V - V_U] = w(\delta_i, V)$$

Assuming $V < V^*$, such that $I_V = 1$, we have:

$$w(\delta_i, V) = rV - \lambda_1 p(F(V))[F(V) - V] + \delta_i[V - V_U] + \lambda_1 S$$

Then, we have:

$$\frac{dw(\delta, V)}{d\delta} = V - V_U > 0$$

$$\frac{dw(\delta_i, V)}{dV} = r + \delta_i - \lambda_1 p'(F(V))F'(V)[F(V) - V] - \lambda_1 p(F(V))\left[F'(V) - 1\right] > 0$$

since $p'(\cdot) < 0$ and $0 < F'(V) \leq \frac{1}{2}$. Finally:

$$\frac{d^2w(\delta_i, V)}{dVd\delta} = 1 > 0.$$

■

Theorem 2. *Suppose that V_L and V_H are profit maximizing values offered in equilibrium by resp. a solid and a risky company. Then, if $q'(V) < 0$, we must have $V_L \geq V_H$.*

Proof. Let's consider two levels of values posted by firms V_A and V_B, and assume that $V_A > V_B$. Let's assume that a L chooses to post V_B and H chooses V_A. This implies that:

$$q(V_B)\left[J_f(\delta_L, V_B) - \overline{J}_v(\delta_L)\right] \geq q(V_A)\left[J_f(\delta_L, V_A) - \overline{J}_v(\delta_L)\right]$$

and

$$q(V_A)\left[J_f(\delta_H, V_A) - \overline{J}_v(\delta_H)\right] \geq q(V_B)\left[J_f(\delta_H, V_B) - \overline{J}_v(\delta_H)\right]$$

Then:

$$\begin{array}{ccc} \dfrac{q\left(V_{B}\right)\left[J_{f}\left(\delta_{L},V_{B}\right)-\overline{J}_{v}\left(\delta_{L}\right)\right]}{q\left(V_{B}\right)\left[J_{f}\left(\delta_{H},V_{B}\right)-\overline{J}_{v}\left(\delta_{H}\right)\right]} & \geq & \dfrac{q\left(V_{A}\right)\left[J_{f}\left(\delta_{L},V_{A}\right)-\overline{J}_{v}\left(\delta_{L}\right)\right]}{q\left(V_{A}\right)\left[J_{f}\left(\delta_{H},V_{A}\right)-\overline{J}_{v}\left(\delta_{H}\right)\right]} \\ & \Downarrow & \\ \dfrac{\left[J_{f}\left(\delta_{L},V_{B}\right)-\overline{J}_{v}\left(\delta_{L}\right)\right]}{\left[J_{f}\left(\delta_{H},V_{B}\right)-\overline{J}_{v}\left(\delta_{H}\right)\right]} & \geq & \dfrac{\left[J_{f}\left(\delta_{L},V_{A}\right)-\overline{J}_{v}\left(\delta_{L}\right)\right]}{\left[J_{f}\left(\delta_{H},V_{A}\right)-\overline{J}_{v}\left(\delta_{H}\right)\right]} \end{array}$$

Then, from the expression for $J_f\left(w,\sigma_j\right)$, we have that:

$$rJ_{f}\left(\delta_{i},V\right)-rJ_{v}\left(\delta_{i}\right)=y-w\left(\delta_{i},V\right)+\left(\delta_{i}+\lambda_{1}I_{V}p\left(F\left(V\right)\right)\right)\left[\overline{J}_{v}\left(\delta_{i}\right)-J_{f}\left(\delta_{i},V\right)\right]-rJ_{v}\left(\delta_{i}\right)$$

$$\left[J_{f}\left(\delta_{i},V\right)-\overline{J}_{v}\left(\delta_{i}\right)\right]=\frac{y-w\left(\delta_{i},V\right)-r\overline{J}_{v}\left(\delta_{i}\right)}{r+\delta_{i}+\lambda_{1}I_{V}p\left(F\left(V\right)\right)}$$

Substituting it back, we have:

$$\begin{array}{ccc} \dfrac{\left[J_{f}\left(\delta_{L},V_{B}\right)-\overline{J}_{v}\left(\delta_{L}\right)\right]}{\left[J_{f}\left(\delta_{H},V_{B}\right)-\overline{J}_{v}\left(\delta_{H}\right)\right]} & \geq & \dfrac{\left[J_{f}\left(\delta_{L},V_{A}\right)-\overline{J}_{v}\left(\delta_{L}\right)\right]}{\left[J_{f}\left(\delta_{H},V_{A}\right)-\overline{J}_{v}\left(\delta_{H}\right)\right]} \\ & \Downarrow & \\ \left\{\begin{array}{c} \frac{y-w\left(\delta_{L},V_{B}\right)-rJ_{v}\left(\delta_{L}\right)}{y-w\left(\delta_{H},V_{B}\right)-rJ_{v}\left(\delta_{H}\right)}\times \\ \times\frac{r+\delta_{H}+\lambda_{1}I_{V_{B}}p\left(F\left(V_{B}\right)\right)}{r+\delta_{L}+\lambda_{1}I_{V_{B}}p\left(F\left(V_{B}\right)\right)} \end{array}\right\} & \geq & \left\{\begin{array}{c} \frac{y-w\left(\delta_{L},V_{A}\right)-rJ_{v}\left(\delta_{L}\right)}{y-w\left(\delta_{H},V_{A}\right)-rJ_{v}\left(\delta_{H}\right)}\times \\ \times\frac{r+\delta_{H}+\lambda_{1}I_{V_{A}}p\left(F\left(V_{A}\right)\right)}{r+\delta_{L}+\lambda_{1}I_{V_{A}}p\left(F\left(V_{A}\right)\right)} \end{array}\right\} \end{array} \qquad (\bigstar)$$

Before we continue, let's prove the following lemma:

Lemma: $\frac{d\overline{J}_v(\delta)}{d\delta}<0$.

Proof: Let's consider $V^{\bigstar}$ such that $V^{\bigstar}\in\arg\max_V \frac{1}{r}\left\{-C+q\left(V\right)J_f\left(\delta_i,V\right)+\left[1-q\left(V\right)\right]\overline{J}_v\left(\delta_i\right)\right\}$. Therefore,

$$\overline{J}_{v}\left(\delta\right)=\frac{1}{r}\left\{-C+q\left(V^{\bigstar}\right)\left[J_{f}\left(\delta,V^{\bigstar}\right)-\overline{J}_{v}\left(\delta\right)\right]\right\}$$

Rearranging, we have:

$$\overline{J}_{v}\left(\delta\right)=\frac{q\left(V^{\bigstar}\right)J_{f}\left(\delta,V^{\bigstar}\right)-C}{r+q\left(V^{\bigstar}\right)}$$

Therefore, how the value of a vacancy reacts to changes in δ depends on how the value of a filled vacancy changes with δ. Notice that:

$$rJ_{f}\left(\delta,V^{\bigstar}\right)=y-w\left(\delta,V\right)+\left(\delta+\lambda_{1}I_{V^{\bigstar}}p\left(F\left(V^{\bigstar}\right)\right)\right)\left[\overline{J}_{v}\left(\delta\right)-J_{f}\left(\delta,V^{\bigstar}\right)\right]$$

Substituting $\overline{J}_v\left(\delta\right)$, we have:

$$rJ_{f}\left(\delta,V^{\bigstar}\right)=\frac{\left(r+q\left(V^{\bigstar}\right)\right)\left[y-w\left(\delta,V\right)\right]-\left(\delta+\lambda_{1}I_{V^{\bigstar}}p\left(F\left(V^{\bigstar}\right)\right)\right)C}{r+q\left(V^{\bigstar}\right)+\delta+\lambda_{1}I_{V^{\bigstar}}p\left(F\left(V^{\bigstar}\right)\right)}$$

Then:

$$\frac{dJ_{f}\left(\delta,V^{\bigstar}\right)}{d\delta}=\frac{\left\{\begin{array}{c}\left(-\left(r+q\left(V^{\bigstar}\right)\right)\frac{dw(\delta,V)}{d\delta}-C\right)\times\left(r+q\left(V^{\bigstar}\right)+\delta+\lambda_{1}I_{V^{\bigstar}}p\left(F\left(V^{\bigstar}\right)\right)\right)\\ -\left(\left(r+q\left(V^{\bigstar}\right)\right)\left[y-w\left(\delta,V\right)\right]-\left(\delta+\lambda_{1}I_{V^{\bigstar}}p\left(F\left(V^{\bigstar}\right)\right)\right)C\right)\end{array}\right\}}{r\left[\left(r+q\left(V^{\bigstar}\right)\right)\left[y-w\left(\delta,V\right)\right]-\left(\delta+\lambda_{1}I_{V^{\bigstar}}p\left(F\left(V^{\bigstar}\right)\right)\right)C\right]^{2}}$$

Since,

$$\frac{dw(\delta, V)}{d\delta} = V - V_U > 0$$

Therefore:

$$\frac{dJ_f\left(\delta, V^{\bigstar}\right)}{d\delta} < 0.$$

Then:

$$\frac{d\overline{J}_v(\delta)}{d\delta} < 0.$$

□

Therefore, the value of a vacancy decreases as the layoff risk increases. Then $\overline{J}_v(\delta_L) > \overline{J}_v(\delta_H)$.

Now,

$$\begin{aligned}
\frac{\partial\left(\frac{y-w(\delta_L,V)-rJ_v(\delta_L)}{y-w(\delta_H,V)-rJ_v(\delta_H)}\right)}{\partial V} &= \frac{\left\{\begin{array}{c} -\frac{dw(\delta_L,V)}{dV}\left[y - w(\delta_H, V) - rJ_v(\delta_H)\right] \\ +\frac{dw(\delta_H,V)}{dV}\left[y - w(\delta_L, V) - rJ_v(\delta_L)\right] \end{array}\right\}}{\left[y - w(\delta_H, V) - rJ_v(\delta_H)\right]^2} \\
&= \frac{\left\{\begin{array}{c} +(\delta_H - \delta_L)\left[y - w(\delta_L, V) + \frac{dw(\delta_L,V)}{dV}(V - V_U)\right] \\ +r\left[\frac{dw(\delta_L,V)}{dV}J_v(\delta_H) - \frac{dw(\delta_H,V)}{dV}J_v(\delta_L)\right] \end{array}\right\}}{\left[y - w(\delta_H, V) - rJ_v(\delta_H)\right]^2} \\
&> \frac{\left\{+(\delta_H - \delta_L)\left[y - w(\delta_L, V) + \frac{dw(\delta_L,V)}{dV}(V - V_U) - rJ_v(\delta_L)\right]\right\}}{\left[y - w(\delta_H, V) - rJ_v(\delta_H)\right]^2} > 0
\end{aligned}$$

Finally:

$$\begin{aligned}
\frac{d\left(\frac{r+\delta_H+\lambda_1 p(F(V))}{r+\delta_L+\lambda_1 p(F(V))}\right)}{dV} &= \frac{\left\{\begin{array}{c} \lambda_1 p'(F(V))F'(V)\left[r + \delta_L + \lambda_1 p(F(V))\right] \\ -\lambda_1 p'(F(V))F'(V)\left[r + \delta_H + \lambda_1 p(F(V))\right] \end{array}\right\}}{\left[r + \delta_L + \lambda_1 p(F(V))\right]^2} \\
&= \frac{-\lambda_1 p'(F(V))F'(V)(\delta_H - \delta_L)}{\left[r + \delta_L + \lambda_1 p(F(V))\right]^2} > 0
\end{aligned}$$

Therefore, since $V_A > V_B$, we have:

$$\frac{y - w(\delta_L, V_A) - rJ_v(\delta_L)}{y - w(\delta_H, V_A) - rJ_v(\delta_H)} > \frac{y - w(\delta_L, V_B) - rJ_v(\delta_L)}{y - w(\delta_H, V_B) - rJ_v(\delta_H)} > 0$$

and

$$\frac{r + \delta_H + \lambda_1 p(F(V_A))}{r + \delta_L + \lambda_1 p(F(V_A))} > \frac{r + \delta_H + \lambda_1 p(F(V_B))}{r + \delta_L + \lambda_1 p(F(V_B))} > 0$$

But this implies that:

$$\left\{\begin{array}{c} \frac{y-w(\delta_L,V_A)-rJ_v(\delta_L)}{y-w(\delta_H,V_A)-rJ_v(\delta_H)}\times \\ \times\frac{r+\delta_H+\lambda_1 I_{V_A}p(F(V_A))}{r+\delta_L+\lambda_1 I_{V_A}p(F(V_A))} \end{array}\right\} \geq \left\{\begin{array}{c} \frac{y-w(\delta_L,V_B)-rJ_v(\delta_L)}{y-w(\delta_H,V_B)-rJ_v(\delta_H)}\times \\ \times\frac{r+\delta_H+\lambda_1 I_{V_B}p(F(V_B))}{r+\delta_L+\lambda_1 I_{V_B}p(F(V_B))} \end{array}\right\}$$

which contradicts the inequality in $(\bigstar)$. □

Therefore, if high layoff and low layoff risk open vacancies at submarket x, only low layoff risk firms offer jobs at values higher than x and only high layoff risk firms offer jobs at values lower than x.

Definition: *An equilibrium consists of a set of offers $\mathcal{V}$, a hiring rate function $q(\cdot)$, an employment function $p(\cdot)$, an application strategy $F(\cdot)$, a value function $J(\cdot)$, wage functions $w(\cdot;\delta_i), \forall i \in \{H, L\}$, a distribution of employed workers over values $G(\cdot)$* and a fraction of employed workers n that satisfy the following requirements:

(i) Given $p(\cdot)$, $F(V)$ solves $(A.2)$;

(ii) Given $F(\cdot)$ and $p(\cdot)$, each offer $x \in \mathcal{V}$ maximizes $\overline{J}_v(\delta_i)$ for some $i \in \{H, L\}$;

(iii) Expected profit of recruiting is the same for all type i firms, $i \in \{H, L\}$.

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D Putting a foundation under the heterogeneous unemployment risk

Model

Following the knowledge hierarchy literature (see Garicano (2000) and Garicano and Rossi-Hansberg (2006), for example), we assume a firm is an organization that handles a flow of production tasks. These tasks can be scaled, so that any worker can be fully occupied with her own flow. Each task requires specialized knowledge to be completed. If a task is successfully completed, it results in a flow output of p. If the worker fails to complete a task, the flow output is 0 (or otherwise low enough to make it worthless to keep a worker). We assume that there is a change in tasks faced by a given worker at a Poisson rate $\zeta \in (0, \infty)$. The new task is drawn [48] from a distribution $H(\cdot)$ with support $[\underline{z}, \overline{z}]$. In order to simplify exposition, we follow Garicano and Rossi-Hansberg (2006) and assume that the p.d.f. is strictly decreasing, i.e., $h'(\cdot) < 0$. Therefore, based on the distribution's characteristics, problems are ranked from the most to the least common ones, and the most common problems require the least amount of knowledge. We would like to emphasize that our results do not depend on this assumption.

We consider that there is no team production, in the sense that each worker in the firm either solves a task by herself or not at all. We also assume that workers cannot be reallocated to a different task. Suppose that workers have the knowledge necessary to solve the tasks in a set $A = [\underline{z}, z_0]$, where $z_0 < \overline{z}$.[49] Notice that, if the worker cannot solve a problem, given task persistence and a bad enough productivity in this case, the optimal decision by the firm as well as the worker it is to destroy the job match. Therefore, a worker is dismissed with probability $\zeta [1 - H(z_0)]$.

Following Cremer (1993) and Cohen and Bacdayan (1994), we assume that a firm is a set of knowledge and management practices that are shared by workers in an organization, boosting these workers' ability to solve problems. In this sense, we consider that a firm is able to provide the worker the knowledge to solve problems in the range $[z_0, z_1]$, for example. Then, the probability a worker is dismissed becomes $\zeta [1 - H(z_1)]$. We assume that this additional knowledge is tacit and cannot be transferred across firms, being part of the firm's total factor productivity.[50]

For example, let's assume that there are two types of firms, that are identical on every aspect but their levels of institutional knowledge, i.e., knowledge embodied in their management practices and procedures. Therefore, let's consider type s firms that provide workers with knowledge over the interval $[z_0, z_s]$ and type d firms that provide workers with knowledge over the interval $[z_0, z_d]$ with $z_d < z_s$. While workers from firm s will be unable to solve a problem and be dismissed with probability $\zeta [1 - H(z_s)]$, workers in a type d firm will be

[48] Therefore, as in Mortensen and Pissarides (1994), we have persistence in task-specific shocks.

[49] According to Garicano and Rossi-Hansberg (2006), given the assumption $h'(\cdot) < 0$ and a convex cost of education, an optimal knowledge set would be a subset of $[\underline{z}, \overline{z}]$ starting at $\underline{z}$.

[50] There is clear evidence in the literature that managerial practices survive changes in top managerial team (Bloom and Van Reenen, 2007) as well as major firm restructures, as spin offs - Cronqvist, Low, and Nilsson (2009) find that spin-off firms have practices closer to their parent companies than to its industry peers even after over long periods of time as well as after changes in the managerial team.

dismissed with probability $\zeta\left[1 - H\left(z_d\right)\right]$. Therefore, workers in d firms are more likely to be laid off - They face an additional $\zeta\left[H\left(z_s\right) - H\left(z_d\right)\right]$ probability of being dismissed. Therefore, these differences in managerial practices and or codefied knowledge that can be accessed by workers generate differences in job destruction rates among firms.

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