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# Income Tax Buyouts and Income Tax Evasion 

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# Income Tax Buyouts and Income Tax Evasion 


#### Abstract

A tax buyout is a contract between tax authorities and a tax payer which reduces the marginal income tax rate in exchange for a lump-sum payment. While previous contributions have focussed on labour supply, we consider the interaction with tax evasion and show that a buyout can increase expected tax revenues. This will be the case if (1) the audit probability is constant and the penalty for evasion is a function of undeclared income or (2) the penalty depends on the amount of taxes evaded, and authorities use information about income generated by the decision about a tax buyout offer when setting audit probabilities. Since individuals will only utilise a tax buyout if they are better off, higher tax revenues imply that such contracts can be Pareto-improving.


JEL-Code: D820, H210, H240, H260.
Keywords: asymmetric information, revenues, self-selection, tax buyouts, tax evasion.

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## 1. Introduction

A tax buyout is a contract between tax authorities and an individual stating that the marginal tax rate will be reduced if the individual pays a fixed price, that is, makes a lump-sum tax payment. Since no individual is compelled to acquire the contract, signing it cannot make a tax payer worse off. In the case of income taxation, the decrease in the marginal income tax rate reduces labour supply distortions, thereby creating a larger tax base. This effect, together with the impact resulting from the lump-sum payment, can be sufficient to increase government revenues. Therefore, if the only distortionary consequences of income taxation are labour supply effects, offering a tax buyout can result in a Pareto-improvement (cf. Del Negro et al. 2010). However, taxation may not just affect the work-leisure trade-off. In particular, individuals can also respond by employing illegal means and attempt to evade taxes. Estimates of tax evasion and activities in the black economy range from about $8 \%$ to $30 \%$ of (official) GDP in the OECD with an average of almost $16 \%$ (Buehn and Schneider 2007). Consequently, even if third-party withholding restricts tax evasion activities, there appear to be ample opportunities to reduce the tax burden in an illegal manner. In this paper, we scrutinise this kind of response and enquire whether a tax buyout can also represent a Pareto-improvement if it affects tax evasion activities.

Our analysis generates the following main findings:
Firstly, if the penalty for evasion does not solely depend on the amount of taxes evaded but, for example, on undeclared income, a tax buyout can always constitute a Pareto-improvement. However, if the penalty is a function of the amount of taxes evaded, a buyout cannot be Pareto-improving for a given detection rate because the buyout reduces the progressivity of the tax system. This mitigates the incentives to evade, so that tax payers increase their payments, unless the penalty is proportional to the marginal tax rate and decreases in line with the gain from evasion. Since individuals will only accept a tax buyout offer if they become better off, higher expected tax payments are equivalent to a Pareto-improvement.

Secondly, if the penalty is proportional to the amount of taxes evaded, an individual's decision to accept a buyout offer is informative with regard to income. In particular, high-income individuals will benefit most from a lower marginal tax rate, while the fixed payment is the same for all tax payers. Therefore, only individuals with an income above a threshold level will benefit from a tax buyout. Accordingly, tax authorities can use the information resulting from the response to a buyout offer in order to adjust the audit probability if there is an inconsistency between this response and the tax declaration. Because a tax buyout reduces
income variability, risk-averse individuals raise their overall payments. Consequently, a tax buyout can also be Pareto-improving if the penalty is proportional to the amount of taxes evaded because it effectively offers a partial insurance against income variations resulting from tax evasion activities.

Although tax buyouts can be Pareto-improving, they do not constitute an encompassing element of income tax codes. However, in some countries particular provisions strongly resemble tax buyouts. In most Swiss cantons, for example, foreigners who work abroad may replace true income as tax basis by their (imputed) rent (OECD 2011, p. 46). This effectively substitutes a lump-sum payment for the (basically progressive) income tax. In the United Kingdom, a lump-sum payment allows non-domiciled residents to avoid the taxation of foreign income and capital gains which are not remitted to the UK (HM Treasury 2011). A final example stems from German law, which establishes lump-sum child benefits or a reduction in taxable income per child as alternatives.

In line with their limited empirical relevance, tax buyouts have not found much attention in the public finance literature. In an early contribution, Alesina and Weil (1992) show that it is possible to generate a Pareto-improvement if individuals are offered a set of linear income tax schemes from which they can select one, instead of facing a uniform linear tax system. Furthermore, in a calibration for the United States, Del Negro et al. (2010) predict that GDP rises by almost $1 \%$ owing to a tax buyout that reduces the marginal income tax rate by at most 5 percentage points and, thus, raises labour supply. However, the impact of tax buyouts on personal income tax evasion has not been considered so far.

Various contributions dealing with tax evasion also touch on issues relevant to the present analysis. Since a tax buyout mitigates the progressivity of a tax system, our investigation is related to analyses of the impact of tax progression on evasion (cf. Pencavel 1979, Koskela 1983a, b, Yitzhaki 1987, Goerke 2003). One relevant finding, already alluded to above, is that the impact of tax progression on evasion behaviour depends on the exact specification of the penalty function. However, these investigations do not allow individuals to choose between tax codes. Furthermore, for one setting, our analysis predicts that some income levels will never be declared, although the income distribution is continuous, and that there will be a concentration of tax payments at a certain cut-off level. Such theoretical predictions generally result from some cut-off property of the (optimal) audit rule which, for example, assigns a
higher probability of being audited for low tax base declarations. ${ }^{1}$ Finally, our contribution bears some similarities to investigations assuming that tax payers can make a fixed payment in order to avoid being audited (cf. Chu 1990 and Ueng and Yang 2001). In contrast to such a policy, a buyout does not allow tax payers to avoid audits; preserves the incentives to evade taxes, even for those who accept the offer; and does not result in the (counterfactual) prediction that only low-income individuals evade taxes. ${ }^{2}$

In the remainder of the paper we proceed as follows: in Section 2, we outline the model. Subsequently, in Section 3 we analyse the effects of a tax buyout in a setting with a constant detection probability and a general fine function. We show that a buyout can be Paretoimproving unless the penalty is a function of the amount of taxes evaded. In Section 4, we clarify that tax buyouts can also constitute a Pareto-improvement if the fine is proportional to taxes evaded. This can be the case if, additionally, tax authorities can suitably condition the audit probability on the income declaration which implicitly results from an individual's decision whether to use a buyout or not. Subsequently, in Section 5 we scrutinise how far the results obtained in Section 4 depend on the benchmark for the Pareto-comparison, namely a detection probability that is independent of income declarations. Section 6 summarises the analysis and provides a broader perspective. Some of the proofs are relegated to an appendix.

## 2. Model

The economy is populated by a large number of individuals. In order to clearly establish the incentive effects of a tax buyout in a world with evasion opportunities, we assume that the labour supply elasticity is zero and briefly comment on this simplification in the final section. Ex-ante, individuals differ only in their exogenous gross income $Y$, which may reflect disparities in productivity, ability, or preferences. Incomes are distributed across the interval $[\underline{Y}, \overline{\mathrm{Y}}]$ according to the distribution function $\mathrm{G}(\mathrm{Y})$ with density $\mathrm{g}(\mathrm{Y})$, assumed to be positive for all $\mathrm{Y} \in[\underline{Y}, \overline{\mathrm{Y}}]$. Furthermore, income is subject to a linear income tax at the rate $\mathrm{t}, \mathrm{t}>0 .{ }^{3}$

[^0]Tax authorities are unaware of an individual's true income and can (and will) directly obtain information about the correct value of Y and the tax burden only if they undertake an audit. Initially, the audit probability is given by $1-\mathrm{p}, 0<\mathrm{p}<1$. Moreover, the exact value of $\underline{\mathrm{Y}}$ is unknown to tax authorities. This assumption ensures that an individual characterised by a low income Y , such as $\mathrm{Y}=\underline{\mathrm{Y}}$, is also able to evade taxes without being penalised automatically. Tax payments in the absence of an audit are denoted by V . If $\mathrm{V}<\mathrm{Yt}$ holds and this violation of the tax code is detected, the individual will have to pay the full amount of taxes due Yt and, in addition, a fine.

In the majority of OECD countries (cf. OECD 2009, pp. 126 ff ), the fine depends on the amount of taxes evaded, such that $(\mathrm{Yt}-\mathrm{V}) \mathrm{f}$ holds, where f represents the marginal penalty rate. However, there may also be other determinants of the penalty and it is well known that some comparative static properties of the Allingham-Sandmo (1972) model depend on the exact specification of the penalty function. The alternative often considered is a penalty which varies with the amount of undeclared (taxable) income, Y - V/t. We pursue a general approach because of the relevance of the penalty for the impact of tax buyouts. Accordingly, the fine is given by $\left[\mathrm{Yt}^{1-\alpha}-\frac{\mathrm{V}}{\mathrm{t}^{\alpha}}\right] \mathrm{f}$, where $\alpha=0(=1)$ implies that the penalty is a function of the amount of taxes evaded (undeclared income). Finally, we assume that disposable income rises with gross income Y if evasion is observed, implying that $\mathrm{t}+\mathrm{ft}^{1}-\alpha<1$ holds.

Individuals exhibit von Neumann-Morgenstern preferences and the utility function $u$, which is continuous and differentiable, increases with disposable income at a decreasing rate, $\mathrm{u}^{\prime}>0>$ $u^{\prime \prime}$. Consequently, expected utility EU in the absence of a tax buyout can be expressed as:

$$
\begin{equation*}
\mathrm{EU}(\mathrm{~V} ; \mathrm{B}, \mathrm{r}=0)=\mathrm{pu}(\mathrm{Y}-\mathrm{V})+(1-\mathrm{p}) \mathrm{u}\left(\mathrm{Y}(1-\mathrm{t})-\left(\mathrm{Yt}{ }^{1-\alpha}-\frac{\mathrm{V}}{\mathrm{t}^{\alpha}}\right) \mathrm{f}\right) \tag{2.1}
\end{equation*}
$$

We can integrate a tax buyout into this setting and assume that the government allows an individual to pay a fixed amount $\mathrm{B}, \mathrm{B}>0$, in return for a reduction of the tax rate from t to t $r, t>r \geq 0$. Accordingly, the official tax burden shrinks to $Y(t-r)$ and the penalty in the case

[^1]of tax evasion declines to $(Y(t-r)-V) \widetilde{f}$, for a given payment $V$ and $\widetilde{f}:=\frac{f}{(t-r)^{\alpha}}$. Expected utility in the presence of a tax buyout is denoted by $\mathrm{EU}(\mathrm{V} ; \mathrm{B}, \mathrm{r}>0)$ :
\[

$$
\begin{equation*}
\mathrm{EU}(\mathrm{~V} ; \mathrm{B}, \mathrm{r}>0)=\mathrm{pu}(\mathrm{Y}-\mathrm{V}-\mathrm{B})+(1-\mathrm{p}) \mathrm{u}(\mathrm{Y}(1-(\mathrm{t}-\mathrm{r}))-(\mathrm{Y}(\mathrm{t}-\mathrm{r})-\mathrm{V}) \tilde{\mathrm{f}}-\mathrm{B}) \tag{2.2}
\end{equation*}
$$

\]

The timing of decisions is as follows: firstly, the government decides whether to offer a tax buyout or not. Subsequently, individuals get to know their income level Y, decide whether to accept the buyout offer or not and determine the amount of taxes V they pay voluntarily. Finally, audits take place and fines are imposed.

We start by considering the second and last decision, namely with respect to the magnitude of the tax payment. If the tax buyout is used, the first-order condition characterising optimal payments $V^{*}(Y, B, r)$ is given by:

$$
\begin{equation*}
\frac{\partial E U}{\partial V}=-p u^{\prime}(\underbrace{(Y-V-B}_{:=x^{\prime}})+(1-p) u^{\prime} \underbrace{(Y(1-(t-r))-(Y(t-r)-V) \widetilde{f}-B)}_{:=x} \widetilde{f}=0 \tag{2.3}
\end{equation*}
$$

In order to ensure that tax evasion takes place, $\mathrm{V}^{*} \leq \mathrm{Y}(\mathrm{t}-\mathrm{r})$, the derivative (2.3) has to be negative when evaluated at $x^{n}=x^{c}$. This implies that $p-(1-p) \widetilde{f}>0$ holds, a restriction which we subsequently impose, as it is commonly done (cf., f. e., Allingham and Sandmo 1972 and Yitzhaki 1974). Furthermore, we assume that optimal payments V* are greater than zero, that is, that the derivative of $(2.3)$ is positive when evaluated at $\mathrm{V}=0$. The second-order condition for a maximum of (2.2) is:

$$
\begin{equation*}
\frac{\partial^{2} \mathrm{EU}}{\partial \mathrm{~V}^{2}}:=\mathrm{EU}_{\mathrm{VV}}=\operatorname{pu}^{\prime \prime}\left(\mathrm{x}^{\mathrm{n}}\right)+(1-\mathrm{p}) \mathrm{u}^{\prime}\left(\mathrm{x}^{\mathrm{c}}\right) \widetilde{\mathrm{f}}^{2}<0 \tag{2.4}
\end{equation*}
$$

Given the above assumptions, all individuals attempt to evade taxes. This outcome could easily be avoided by introducing fixed costs of evasion which vary across individuals and prevent some tax payers from misreporting income. In the absence of such costs, there is a direct link between tax payments and true income. Therefore, evasion will only prevail if the tax authority's implicit knowledge of such activities does not automatically imply a punishment. Effectively, a detection probability $1-p$, such that $\frac{\widetilde{\mathrm{f}}}{1+\widetilde{\mathrm{f}}}<\mathrm{p}<1$, is tantamount

[^2]to the assumption that penalising tax evasion requires not only the authority's awareness of such activities, but also a verification of the exact amount of taxes evaded.

For later use, it is helpful to note the impact of a variation in the fixed tax payment $B$ and a reduction of the tax rate $\mathrm{t}-\mathrm{r}$ on the first-order condition (2.3).

$$
\begin{gather*}
\frac{\partial^{2} E U}{\partial V \partial B}=\operatorname{pu}^{\prime \prime}\left(x^{n}\right)-(1-p) u^{\prime \prime}\left(x^{c}\right) \widetilde{f}  \tag{2.5}\\
\frac{\partial^{2} E U}{\partial V \partial r}=(1-p) \widetilde{f}\left[u^{\prime \prime}\left(x^{c}\right)\left(Y(1+(1-\alpha) \widetilde{f})+\alpha V \frac{\widetilde{f}}{t-r}\right)+\frac{u^{\prime}\left(x^{c}\right) \alpha}{t-r}\right] \tag{2.6}
\end{gather*}
$$

A rise in the fixed payment B represents a pure income alteration, and its effect on optimal payments $V^{*}$ depends on how absolute risk aversion changes with income. If the penalty is a function of the amount of taxes evaded $(\alpha=0)$, an increase in $r$ is comparable to a reduction in the amount of taxes due in the event that tax evasion is detected, so that $\mathrm{V}^{*}$ declines (cf. Yitzhaki 1974). If the fine depends on undeclared income ( $\alpha=1$ ), there is an additional effect because the effective marginal tax rate declines. This mitigates the incentives to evade taxes (cf. Allingham and Sandmo 1972), as clarified by the last term in equation (2.6), making the overall impact of a tax rate change ambiguous. Observe, finally, that expected utility $\mathrm{EU}\left(\mathrm{V}^{*}(\mathrm{Y}), \mathrm{Y}\right)$ is increasing and strictly concave in gross income Y (see Appendix 7.1).

## 3. Constant Detection Probability

In this section, we consider one version of a buyout (whereas we briefly discuss alternatives in the final Section 6). In particular, the government offers a single tax buyout that combines a tax rate reduction by r and a fixed payment B , while the detection probability $1-\mathrm{p}$ is constant. Such a buyout introduces a regressive component into a previously linear tax system since the average tax rate $\mathrm{B} / \mathrm{Y}+(\mathrm{t}-\mathrm{r})$ declines with income. We will, first of all, investigate under what conditions an individual accepts such an offer. Secondly, we will analyse how expected tax payments of an individual change when that individual is indifferent between accepting and declining the offer of a buyout. Subsequently, we will compare the highest fixed payment $B$ an individual is willing to make in order to obtain a rate reduction $r$, with the payment which is required to hold expected tax payments constant. If the willingness to pay
exceeds the required payment, a tax buyout which does not affect expected utility will raise expected tax revenues and, hence, can be Pareto-improving. ${ }^{5}$

An individual will be indifferent between accepting and declining the offer of a tax buyout if expected utility EU does not change, that is if:

$$
\begin{equation*}
\mathrm{dEU}=\left(\frac{\partial \mathrm{EU}}{\partial \mathrm{~V}} \frac{\partial \mathrm{~V}}{\partial \mathrm{r}}+\frac{\partial \mathrm{EU}}{\partial \mathrm{r}}\right) \mathrm{dr}+\left(\frac{\partial \mathrm{EU}}{\partial \mathrm{~V}} \frac{\partial \mathrm{~V}}{\partial \mathrm{~B}}+\frac{\partial \mathrm{EU}}{\partial \mathrm{~B}}\right) \mathrm{dB}=0 \tag{3.1}
\end{equation*}
$$

Since tax payments are chosen optimally $(\partial \mathrm{EU} / \partial \mathrm{V}=0$; cf. equation (2.3)), we have $\partial \mathrm{EU} / \partial \mathrm{B}=$ $-\mathrm{pu}^{\prime}\left(\mathrm{x}^{\mathrm{n}}\right)-(1-\mathrm{p}) \mathrm{u}^{\prime}\left(\mathrm{x}^{\mathrm{c}}\right)=-(1-\mathrm{p}) \mathrm{u}^{\prime}\left(\mathrm{x}^{\mathrm{c}}\right)(1+\widetilde{\mathrm{f}})<0$ in accordance with the first-order condition (2.3). Furthermore, $\partial E U / \partial r$ is given by:

$$
\begin{equation*}
\frac{\partial \mathrm{EU}}{\partial \mathrm{r}}=(1-\mathrm{p}) \mathrm{u}^{\prime}\left(\mathrm{x}^{\mathrm{c}}\right)\left[\mathrm{Y}(1+(1-\alpha) \tilde{\mathrm{f}})+\alpha \mathrm{V}^{*}(\mathrm{Y}, \mathrm{~B}, \mathrm{r}) \frac{\widetilde{\mathrm{f}}}{\mathrm{t}-\mathrm{r}}\right] \tag{3.2}
\end{equation*}
$$

Therefore, the increase in B which is required to compensate a rise in $r$ in terms of expected utility, is defined by:

$$
\begin{equation*}
\frac{\mathrm{dB}}{\mathrm{dr}} \left\lvert\, \mathrm{dEU}=0=-\frac{\frac{\partial \mathrm{EU}}{\partial \mathrm{r}}}{\frac{\partial \mathrm{EU}}{\partial \mathrm{~B}}}=\frac{\mathrm{Y}(1+(1-\alpha) \widetilde{\mathrm{f}})}{1+\widetilde{\mathrm{f}}}+\alpha \mathrm{V}^{*}(\mathrm{Y}, \mathrm{~B}, \mathrm{r}) \frac{\widetilde{\mathrm{f}}}{(1+\widetilde{\mathrm{f}})(\mathrm{t}-\mathrm{r})}\right. \tag{3.3}
\end{equation*}
$$

We can summarise the result in

## Proposition 1:

Suppose a constant detection probability $1-\mathrm{p}$.
a) An individual with an income $Y^{c r i t}$ will be indifferent between accepting and rejecting a tax buyout offer if a decline in the tax rate from $t$ to $t-r$ is combined
with a fixed tax payment $B(1+\widetilde{f})=r\left[Y^{c r i t}(1+(1-\alpha) \widetilde{f})+\frac{\alpha V^{*}\left(Y^{\text {crit }}\right) \widetilde{f}}{t-r}\right]$.
b) For $\alpha=0$, all individuals with an income $\mathrm{Y}>(<) \mathrm{Y}^{\text {crit }}$ accept (decline) the buyout offer $\mathrm{B}=\mathrm{rYcrit}$.

[^3]Proof: Part a) of the Proposition follows from equation (3.3). Further, $\mathrm{dB} / \mathrm{dr}$ increases with Y for $\alpha=0$, but not necessarily for $\alpha \neq 0$, because $\mathrm{V}^{*}(\mathrm{Y}, \mathrm{B}, \mathrm{r})$ may decline with Y . This establishes the implicit restriction contained in part b$)$.

The intuition for Proposition 1 is as follows: suppose, initially, that the penalty depends on the amount of taxes evaded $(\alpha=0)$. A tax buyout consisting of a rate reduction by $r$ and a fixed payment $\mathrm{B}=\mathrm{rY}^{\text {crit }}$ induces an individual with income $\mathrm{Y}^{\text {crit }}$ to reduce the optimal payment $\mathrm{V}^{*}\left(\mathrm{Y}^{c r i t}, \mathrm{~B}, \mathrm{r}\right)$ by $\mathrm{r} \mathrm{Y}^{c r i t}=\mathrm{B}$. Hence, the sum of payments in each state of the world remains unaffected, that is, $\mathrm{Y}^{\operatorname{crit}_{t}(1+\mathrm{f})-\mathrm{Y}^{\text {crit }_{r}}+\mathrm{B}-\mathrm{f}\left(\mathrm{Y}^{\text {crit }_{r}}+\mathrm{V}^{*}\left(\mathrm{Y}^{\text {crit, }}, \mathrm{B}, \mathrm{r}\right)\right) \text { if evasion is detected }}$ and $\mathrm{V}^{*}\left(\mathrm{Y}^{c r i t}, \mathrm{~B}, \mathrm{r}\right)+\mathrm{B}$ otherwise,. In consequence, disposable incomes in both states of the world $x^{n}$ and $x^{c}$ and expected utility EU do not change. If income exceeds $Y$ crit, a given reduction in the tax rate will have a larger positive effect on disposable income than for $\mathrm{Y}^{\mathrm{crit}}$, whereas the costs in terms of a higher fixed payment are the same. ${ }^{6}$ Accordingly, any individual with an income $\mathrm{Y}>\mathrm{Y}^{\text {crit }}$ will benefit from accepting a buyout (for $\alpha=0$ ).

If the fine depends on undeclared income $(\alpha=1)$, an additional effect occurs since the effective marginal fine $\widetilde{f}=\frac{f}{t-r}$ rises with $r$. This implies that the gain in expected utility from a higher tax payment increases as well. Therefore, the buyout mitigates the incentives to evade taxes, ceteris paribus. As a further consequence, the willingness to pay for a tax buyout is not only a function of gross income Y (cf. equation (3.3)). Whether this willingness to pay is, ceteris paribus, greater than in a setting in which the fine depends on the amount of taxes evaded is analytically uncertain because of the adjustment in the optimal payment $\mathrm{V}^{*}$. Therefore, it is likely but not obvious that B, as defined in Proposition 1, rises with $\mathrm{Y}^{c r i t}$, given that $\mathrm{V}=\mathrm{V}^{*}\left(\mathrm{Y}^{c r i t}, \mathrm{~B}, \mathrm{r}\right)$.

Turning to the budgetary consequences of a tax buyout, we focus on expected tax payments S per individual:

$$
\begin{equation*}
\mathrm{S}:=\mathrm{B}+\mathrm{p} V^{*}(\mathrm{Y}, \mathrm{~B}, \mathrm{r})+(1-\mathrm{p})\left[\mathrm{Y}(\mathrm{t}-\mathrm{r})+\left(\mathrm{Y}(\mathrm{t}-\mathrm{r})-\mathrm{V}^{*}(\mathrm{Y}, \mathrm{~B}, \mathrm{r})\right) \widetilde{\mathrm{f}}\right] \tag{3.4}
\end{equation*}
$$

The required increase in the payment B , in order to compensate for a reduction in the tax rate by r and, hence, to retain expected tax payments S , can be determined by totally

[^4]differentiating equation (3.4), while using equations (2.4) to (2.6), the definition of $\widetilde{\mathrm{f}}$, $\widetilde{\mathrm{f}}=\frac{\mathrm{f}}{(\mathrm{t}-\mathrm{r})^{\alpha}}$, and $\mathrm{V}^{*}=\mathrm{V}^{*}(\mathrm{Y}, \mathrm{B}, \mathrm{r})$ for notational simplicity:
\[

$$
\begin{equation*}
\left.\frac{\mathrm{dB}}{\mathrm{dr}}\right|_{\mathrm{dS}} ^{\mathrm{S}}=0=-\frac{\frac{\partial \mathrm{S}}{\partial \mathrm{r}}}{\frac{\partial \mathrm{~S}}{\partial \mathrm{~B}}}=\frac{-\frac{\partial \mathrm{V}^{*}}{\partial \mathrm{r}}(\mathrm{p}-(1-\mathrm{p}) \tilde{\mathrm{f}})+(1-\mathrm{p})\left[\mathrm{Y}(1+(1-\alpha) \widetilde{\mathrm{f}})+\alpha \mathrm{V}^{*} \frac{\widetilde{\mathrm{f}}}{\mathrm{t}-\mathrm{r}}\right]}{1+\frac{\partial \mathrm{V}^{*}}{\partial \mathrm{~B}}(\mathrm{p}-(1-\mathrm{p}) \widetilde{\mathrm{f}})} \tag{3.5}
\end{equation*}
$$

\]

In Appendix (7.2) we show:

$$
\begin{equation*}
\left.\frac{\mathrm{dB}}{\mathrm{dr}}\right|_{\mathrm{dEU}=0}-\left.\frac{\mathrm{dB}}{\mathrm{dr}}\right|_{\mathrm{dS}=0}=(>) 0 \text { for } \alpha=(>) 0 \tag{3.6}
\end{equation*}
$$

This implies

## Proposition 2:

Suppose a constant detection probability $1-\mathrm{p}$. If a tax buyout leaves expected utility constant, expected tax payments per individual and, hence, government revenues will rise if the fine is a function of undeclared income $(\alpha=1)$ and will remain unchanged if it is levied on the amount of taxes evaded $(\alpha=0)$.

Proof: see Appendix 7.2.

To provide an intuition for Proposition 2, note that alterations in optimal tax payments $V^{*}$ due to changes in the lump-sum amount $B$ or the marginal tax rate $t-r$ have no first-order effect on expected utility. If the fine is a function of the amount of taxes evaded $(\alpha=0)$, then a rise in B and in $r$, such that expected utility remains constant, will result in adjustments in optimal payments $\mathrm{V}^{*}$ of such magnitude that the effective amount transferred by a tax payer to authorities in each state of the world is unaffected (see Proposition 1). In consequence, a tax reform which does not alter the tax payer's payoff, changes neither expected tax payments of this individual nor the resulting amount of tax revenues (for $\alpha=0$ ). However, if the fine is a function of undeclared income $(\alpha=1)$, a reduction in the marginal tax rate has a further consequence, namely that the effective marginal fine $\widetilde{\mathrm{f}}$ rises. This, in turn, increases the gain from a higher declaration (cf. equation (2.3)), and thus the optimal tax payment, ceteris paribus. The additional impact has no direct consequences with regard to expected utility EU because it occurs via an adjustment in the optimal payment V*. However, the change in V* enhances government revenues. Consequently, the increase in tax revenues resulting from a
given tax reform is more pronounced in a setting in which the fine depends on the amount of undeclared income than as if it is a function of taxes evaded.

We can conclude that a tax reform which raises the fixed payment B and reduces the marginal tax rate $t-r$ in such a manner that expected utility EU of a tax payer remains unaffected, induces this individual to increase expected tax payments if the fine depends - at least marginally $(0<\alpha \leq 1)$ - on the amount of undeclared income. ${ }^{7}$ Consequently, a tax buyout provides scope for a Pareto-improvement. Such an improvement would be realised if every tax payer could be induced to make a fixed payment, for a given reduction in the marginal rate, which just leaves his/ her expected utility unchanged. However, since tax authorities do not know the tax payers' true income level, they are not aware of the maximum willingness to pay and are, therefore, unable to tailor a buyout offer to an individual's income and, thus, willingness to pay. Therefore, a buyout $\mathrm{B}, \mathrm{B}=\mathrm{r} \mathrm{Y}^{c r i t}$, is likely to decrease expected utility for some individuals with an income $\mathrm{Y} \neq \mathrm{Y}^{\text {crit }}$, and may increase expected utility for others. While the former will not accept the tax buyout offer, the latter will utilise it, possibly also because expected tax payments decline. The overall budgetary consequences of a tax buyout will thus be uncertain and can surely be negative.

However, tax authorities can always offer a tax buyout which unambiguously constitutes a Pareto-improvement (for $\alpha>0$ ), as long as it is possible to ascertain the maximum willingness to pay for a buyout within the population. If the offer of a tax buyout is only accepted by the individual with the highest willingness to pay, expected utility of all other individuals will be unaffected. In addition, the individual with the maximum willingness will be (weakly) better off and expected tax revenues will rise. Inducing more individuals to accept the tax buyout offer by, for example reducing the fixed payment, will continue to induce a Paretoimprovement, as long as overall expected tax payments rise.

Although, therefore, a tax buyout can be Pareto-improving in the presence of a penalty which depends at least partially on undeclared income, such an improvement cannot occur if the fine is solely a function of the amount of taxes evaded $(\alpha=0)$. This is the case because, even if individuals can be induced to pay the maximal amount they are willing to give for a tax buyout, expected tax revenues will be unchanged (cf. equation (3.6)). This implies that a government which cannot extract the maximum willingness to pay incurs a loss in expected revenues. Those individuals who accept the offer tend to pay less than a fraction $r$ of their true

[^5]income, whereas those do not accept the offer for whom B exceeds r multiplied by their true income.

We can summarise our above considerations regarding Pareto-effects in a
Corollary (to Proposition) 2:
Suppose a constant detection probability $1-\mathrm{p}$. If the penalty does not depend entirely on (is solely a function of) the amount of taxes evaded, $0<\alpha \leq 1(\alpha=0)$, a tax buyout can always (can never) be a Pareto-improvement.

Since the penalty in the case of tax evasion being detected is predominantly related to the amount of taxes evaded in OECD countries (OECD, 2009, pp. 126 ff ), we will subsequently analyse, in relation to this fine structure, under what conditions a tax buyout may nevertheless make every individual (weakly) better off, without causing detrimental consequences to expected tax revenues.

## 4. Detection Probability Depends on Response to Tax Buyout Offer

Suppose now that the fine is a function solely of the amount of taxes evaded $(\alpha=0)$ and tax authorities are more sophisticated than presumed thus far. In Section 3 we have shown that accepting a tax buyout offer conveys information about the true income level (for $\alpha=0$ ). In particular, we know from Proposition 1b that setting B and r divides the population into two groups: low-income individuals for whom the tax buyout offer is unattractive, and individuals with an income $\mathrm{Y} \geq \mathrm{B} / \mathrm{r}$ who would (weakly) increase their expected utility by accepting the offer. If an individual with an income Y marginally above $\mathrm{B} / \mathrm{r}$ accepts the tax buyout offer and then pays $\mathrm{V}^{*}(\mathrm{Y}, \mathrm{B}, \mathrm{r})<\mathrm{B}(\mathrm{t}-\mathrm{r}) / \mathrm{r}<\mathrm{Y}(\mathrm{t}-\mathrm{r})$, tax authorities can immediately infer that the individual is evading taxes. ${ }^{8}$ Accordingly, the subsequent analysis is based on the assumption that an individual who signals that his or her income is greater than $B / r$ by accepting the tax buyout offer, but then implicitly declares an income of less than $\mathrm{B} / \mathrm{r}$, is detected and punished with probability one. ${ }^{9}$

[^6]Formally, for any tax buyout scheme $\mathrm{Y}_{1}=\mathrm{B} / \mathrm{r}$, the detection probability in the presence of such a response by tax authorities equals $1-p(a)$, where

$$
p(a)=\left\{\begin{array}{lll}
=0 & \text { if } a=1 & \text { and } V<Y_{1}(t-r)  \tag{4.1}\\
=p & \text { if } a=0 \text { or } a=1 \text { and } V \geq Y_{1}(t-r)
\end{array}\right.
$$

and $\mathrm{a}=1(=0)$ indicates that the individual has accepted (declined) the tax buyout offer. Given the impact on the detection probability, those individuals whose optimal unconstrained tax payments $\mathrm{V}^{*}(\mathrm{Y} \mid \mathrm{a}=1)$ exceed $\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})$ will certainly accept the tax buyout offer. Such individuals are characterised by an income level $\mathrm{Y} \geq \mathrm{Y}_{3}>\mathrm{Y}_{1}$, implicitly defined by $\mathrm{V}^{*}\left(\mathrm{Y}_{3} \mid \mathrm{a}=1\right)=\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})$. However, individuals with an income somewhat below $\mathrm{Y}_{3}$ also have an incentive to accept the tax buyout offer. In order to avoid being detected evading taxes with certainty, they have to make tax payments $\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})$ which are higher than would be optimal at their income level $\mathrm{Y}, \mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})>\mathrm{V}^{*}(\mathrm{Y} \mid \mathrm{a}=1)$. Therefore, paying $\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})$ instead of $\mathrm{V}^{*}(\mathrm{Y} \mid \mathrm{a}=1)$ is the price for the reduction in the marginal tax rate. Such excessive payments $\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})>\mathrm{V}^{*}(\mathrm{Y} \mid \mathrm{a}=1)$, in order to benefit from the tax buyout, are made by individuals whose income surpasses a level $\mathrm{Y}_{2}$, which is defined by:

$$
\begin{align*}
& \operatorname{EU}\left(\mathrm{Y}_{2} ; \mathrm{V} *\left(\mathrm{Y}_{2} \mid \mathrm{a}=0\right)\right) \\
&=\operatorname{pu}\left(\mathrm{Y}_{2}-\mathrm{V}^{*}\left(\mathrm{Y}_{2} \mid \mathrm{a}=0\right)\right)+(1-\mathrm{p}) \mathrm{u}\left(\mathrm{Y}_{2}(1-\mathrm{t}(1+\mathrm{f}))+\mathrm{V}^{*}\left(\mathrm{Y}_{2} \mid \mathrm{a}=0\right) \mathrm{f}\right) \\
&=\operatorname{pu}\left(\mathrm{Y}_{2}-\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})-\mathrm{B}\right)+(1-\mathrm{p}) \mathrm{u}\left(\mathrm{Y}_{2}(1-(\mathrm{t}-\mathrm{r})(1+\mathrm{f}))+\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r}) \mathrm{f}-\mathrm{B}\right) \\
&=\mathrm{EU}\left(\mathrm{Y}_{2} ; \mathrm{V}=\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r}) ; \mathrm{a}=1\right) \tag{4.2}
\end{align*}
$$

Therefore, we have four groups of individuals: for members of the first group, characterised by an income $\mathrm{Y}<\mathrm{Y}_{1}$, the tax buyout consisting of a fixed payment $\mathrm{B}, \mathrm{B}=\mathrm{Y}_{1} \mathrm{r}$, and a rate reduction $r$ is too expensive. They will not accept the offer, irrespective of whether the detection probability increases with the acceptance of an offer or not. The second group of individuals has an income $\mathrm{Y}, \mathrm{Y}_{1} \leq \mathrm{Y}<\mathrm{Y}_{2}$, and is deterred from accepting the tax buyout offer because of the increase in the detection probability which occurs if the implicit income declaration is inconsistent with the information conveyed by purchasing the reduction in the tax rate to $t-r$. The third group has an income $\mathrm{Y}, \mathrm{Y}_{2} \leq \mathrm{Y}<\mathrm{Y}_{3}$, accepts the tax buyout offer and has to make a payment $\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})$ in excess of the level $\mathrm{V}^{*}(\mathrm{Y} \mid \mathrm{a}=1)$ in order to avoid
detection. Consequently, in the remainder of the paper we distinguish these amounts by referring to $\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})$ as quasi-voluntary payment and to $\mathrm{V}^{*}(\mathrm{Y} \mid \mathrm{a}=1)$ as optimal unconstrained payment. Finally, the fourth group consists of those individuals with an income $\mathrm{Y}, \mathrm{Y} \geq \mathrm{Y}_{3}$, whose optimal unconstrained tax payment $\mathrm{V}^{*}(\mathrm{Y} \mid \mathrm{a}=1)$ weakly exceeds the quasi-voluntary amount $\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r}) .{ }^{10}$

## Graphical Illustration

In Figure 1, expected utility of individuals who do not accept the buyout offer is characterised by the line denoted by W (without buyout). The line labelled $\mathrm{B}^{\mathrm{u}}$ (buyout, unconstrained choice of V ) is based on the assumption of an unconstrained use of a buyout and an unchanged detection probability $1-\mathrm{p}$. It is dashed in its left part and continuous to the right of $\mathrm{Y}_{3}$. For all individuals with an income $\mathrm{Y}>\mathrm{Y}_{1}$ and a given detection probability $1-\mathrm{p}$, expected utility would be higher than if the buyout offer were not accepted. Therefore, to the right of $\mathrm{Y}_{1}, \mathrm{~B}^{\mathrm{u}}$ lies above the line W . However, an individual with an income level slightly greater than $\mathrm{Y}_{1}$ cannot make the optimal unconstrained payment $\mathrm{V}^{*}(\mathrm{Y} \mid \mathrm{a}=1)$ as defined by equation (2.3) because this would immediately reveal evasion activities. Therefore, to avoid detection, the individual has to make a quasi-voluntary payment $\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})>\mathrm{V}^{*}(\mathrm{Y} \mid \mathrm{a}=1)$. Since the gain from using the tax buyout is small for an individual with an income just above $\mathrm{Y}_{1}$, accepting the tax buyout offer and paying $\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})$ instead of $\mathrm{V}^{*}(\mathrm{Y} \mid \mathrm{a}=1)$ would clearly make this individual worse off. The higher the income of an individual, the more likely it is that the optimal unconstrained payment is greater than $\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})$. Therefore, the thick line $\mathrm{B}^{\mathrm{C}}$ (buyout, constrained choice of V ), which depicts expected utility of an individual who accepts the buyout offer but cannot make the preferred evasion decision, lies below the line W at an income $\mathrm{Y}_{1}$. For $\mathrm{Y}=\mathrm{Y}_{3}, \mathrm{~V}^{*}\left(\mathrm{Y}_{3} \mid \mathrm{a}=1\right)=\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})$ holds (by definition). In consequence, $B^{c}$ coincides with the line $B^{u}$ at $Y=Y_{3}$. To the left of $\mathrm{Y}_{3}$, the constraint binds and $\mathrm{B}^{\mathrm{C}}$ lies below $\mathrm{B}^{\mathrm{U}}$. Furthermore, the constraint is no longer relevant for income levels in excess of $\mathrm{Y}_{3}$, so that $\mathrm{B}^{\mathrm{u}}$ and $\mathrm{B}^{\mathrm{c}}$ also coincide to the right of $\mathrm{Y}_{3}$.

[^7]Figure 1: Expected Utility and Declaration Choices


In Appendix 7.3 we develop the features of the curves depicted in Figure 1 more rigorously, showing in particular that the intersection of $\mathrm{B}^{\mathrm{c}}$ and W must lie between incomes $\mathrm{Y}_{1}$ and $\mathrm{Y}_{3}$ and is unique. This implies that $\mathrm{Y}_{1}<\mathrm{Y}_{2}<\mathrm{Y}_{3}$ holds and defines the four groups of individuals previously mentioned. Individuals can only obtain expected utility as described by the line $\mathrm{B}^{\mathrm{u}}$ if optimal unconstrained tax payments exceed $\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})$. Hence, the dashed part of $\mathrm{B}^{\mathrm{u}}$ is not attainable and the highest continuous line in Figure 1 indicates maximum expected utility for any given income Y.

Note that the above reasoning implies that a 'missing middle' and bunching of (implicit) income declarations occur. In particular, there is no individual who voluntarily makes a payment V that stems from the interval $\left[\mathrm{V}^{*}\left(\mathrm{Y}_{2} \mid \mathrm{a}=0\right), \mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})[\right.$, because such an individual would be detected evading taxes with probability one. Instead, quasi-voluntary payments of all these individuals equal $\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r}) .{ }^{11}$ Figure 2 illustrates the relationship between income declaration choices for the case of constant absolute risk aversion (CARA).

[^8]Figure 2: Declaration Choices and Income


The upper (lower) line $\mathrm{V}^{*}(\mathrm{Y} \mid \mathrm{a}=0)\left(\mathrm{V}^{*}(\mathrm{a}=1)\right.$ ) in Figure 2 depicts optimal declarations for tax rates $\mathrm{t}(\mathrm{t}-\mathrm{r})$, assuming the detection probability to be given by $1-\mathrm{p}$. Since the fraction of income declared optimally is constant in a setting with linear income tax system and CARApreferences and rises with the tax rate (Yitzhaki 1974), the upper line $V^{*}(Y \mid a=0)$ is steeper than the lower one $\mathrm{V}^{*}(\mathrm{Y} \mid \mathrm{a}=1)$. All individuals with an income $\mathrm{Y} \leq \mathrm{Y}_{2}$ do not accept the buyout offer and pay $\mathrm{V}^{*}(\mathrm{Y} \mid \mathrm{a}=0)$, as defined by equation (2.3). All individuals characterised by an income weakly greater than $\mathrm{Y}_{3}$ pay $\mathrm{V}^{*}(\mathrm{Y} \mid \mathrm{a}=1)>\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})$, according to the definition of $\mathrm{Y}_{3}$. Finally, all individuals having an income $\mathrm{Y}, \mathrm{Y}_{2}<\mathrm{Y} \leq \mathrm{Y}_{3}$, utilise the buyout but pay more than the optimal unconstrained amount $\mathrm{V}^{*}(\mathrm{Y} \mid \mathrm{a}=1)$ because they would otherwise be punished with certainty. Accordingly, bunching of income declarations occurs at the income level $\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})$. In sum, the thick lines in Figure 2 depict tax payments as a function of income $Y$.

## Welfare Considerations

The tax buyout will represent a Pareto-improvement if expected government revenues T rise (see footnote 5). This is the case for two reasons: firstly, an individual will make use of the scheme only if he or she is better off. Secondly, the entire resources spent on monitoring individuals are constant. This is the case since every individual evades taxes and because $\mathrm{p}(\mathrm{a})$ $=\mathrm{p}$ holds in equilibrium, given that no individual will voluntarily pay an amount of taxes which immediately reveals that this person is a tax evader.

Expected tax revenues T are given by:

$$
\begin{align*}
\mathrm{T}= & \int_{\underline{Y}}^{\mathrm{Y}_{2}\left(\mathrm{Y}_{1}, \mathrm{r}\right)}[\underbrace{\left[\mathrm{pV}^{*}(\mathrm{Y} \mid \mathrm{a}=0)+(1-\mathrm{p})\left(\mathrm{Yt}(1+\mathrm{f})-\mathrm{V}^{*}(\mathrm{Y} \mid \mathrm{a}=0) \mathrm{f}\right)\right.}_{:=\mathrm{T}_{\mathrm{W}}(\mathrm{Y})}] \mathrm{dG}(\mathrm{Y}) \\
& \left.+\int_{:=\mathrm{T}_{\mathrm{B}^{\mathrm{c}}}\left(\mathrm{Y}, \mathrm{Y}_{1}, \mathrm{~B}\right)}^{\mathrm{Y}_{3}\left(\mathrm{Y}_{1}, \mathrm{r}\right)} \mathrm{Y}_{1}, \mathrm{r}\right) \\
& +\underbrace{\left[\mathrm{Y}\left(\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})+\mathrm{B}\right)+(1-\mathrm{p})\left(\mathrm{Y}(\mathrm{t}-\mathrm{r})(1+\mathrm{f})-\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r}) \mathrm{f}+\mathrm{B}\right)\right.}_{:=\mathrm{T}_{\mathrm{B}}(\mathrm{Y}, \mathrm{~B})}] \mathrm{dG}(\mathrm{Y})  \tag{4.3}\\
& \left.\int_{\mathrm{Y}_{3}\left(\mathrm{Y}_{1}, \mathrm{r}\right)}^{\left[\mathrm{p}\left(\mathrm{~V}^{*}(\mathrm{Y} \mid \mathrm{a}=1)+\mathrm{B}\right)+(1-\mathrm{p})\left(\mathrm{Y}(\mathrm{t}-\mathrm{r})(1+\mathrm{f})-\mathrm{V}^{*}(\mathrm{Y} \mid \mathrm{a}=1) \mathrm{f}+\mathrm{B}\right)\right.}\right] \mathrm{dG}(\mathrm{Y})
\end{align*}
$$

The first term in equation (4.3) describes expected tax payments by those individuals who do not accept the tax buyout offer. The remaining terms depict revenues from groups 3 and 4 who utilise the buyout and are either constrained in their choice of payments (to the quasivoluntary amount $\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})$ ) or can choose them optimally. The full characterisation of the government's optimal policy is based on the derivatives $\partial \mathrm{T} / \partial \mathrm{r}$ and $\partial \mathrm{T} / \partial \mathrm{Y}_{1}$, while taking into account that $\mathrm{B}=\mathrm{Y}_{1}$ r.

$$
\begin{gather*}
\frac{\partial \mathrm{T}}{\partial \mathrm{Y}_{1}}=\frac{\partial \mathrm{Y}_{2}}{\partial \mathrm{Y}_{1}}\left\{\mathrm{~T}_{\mathrm{W}}\left(\mathrm{Y}_{2}\right)-\mathrm{T}_{\mathrm{B}^{\mathrm{c}}}\left(\mathrm{Y}_{2}\right)\right\} \mathrm{g}\left(\mathrm{Y}_{2}\right)+\frac{\partial \mathrm{Y}_{3}}{\partial \mathrm{Y}_{1}}\left(\mathrm{~T}_{\mathrm{B}^{\mathrm{c}}}\left(\mathrm{Y}_{3}\right)-\mathrm{T}_{\mathrm{B}}\left(\mathrm{Y}_{3}\right)\right) \mathrm{g}\left(\mathrm{Y}_{3}\right) \\
+\left[\mathrm{G}\left(\mathrm{Y}_{3}\right)-\mathrm{G}\left(\mathrm{Y}_{2}\right)\right][(\mathrm{t}-\mathrm{r})(\mathrm{p}-(1-\mathrm{p}) \mathrm{f})]+\left(1-\mathrm{G}\left(\mathrm{Y}_{2}\right)\right) \mathrm{r}  \tag{4.4}\\
\frac{\partial \mathrm{~T}}{\partial \mathrm{r}}=\frac{\partial \mathrm{Y}_{2}}{\partial \mathrm{r}}\left(\mathrm{~T}_{\mathrm{W}}\left(\mathrm{Y}_{2}\right)-\mathrm{T}_{\mathrm{B}^{\mathrm{c}}}\left(\mathrm{Y}_{2}\right)\right) \mathrm{g}\left(\mathrm{Y}_{2}\right)+\frac{\partial \mathrm{Y}_{3}}{\partial \mathrm{r}}\left(\mathrm{~T}_{\mathrm{B}^{\mathrm{c}}}\left(\mathrm{Y}_{3}\right)-\mathrm{T}_{\mathrm{B}}\left(\mathrm{Y}_{3}\right)\right) \mathrm{g}\left(\mathrm{Y}_{3}\right) \\
-(1-\mathrm{p})(1+\mathrm{f}) \int_{\mathrm{Y}_{2}}^{\bar{Y}} \mathrm{YdG}(\mathrm{Y})+\left[\mathrm{G}\left(\mathrm{Y}_{3}\right)-\mathrm{G}\left(\mathrm{Y}_{2}\right)\right] \mathrm{Y}_{1}((1-\mathrm{p}) \mathrm{f}-\mathrm{p})+\left(1-\mathrm{G}\left(\mathrm{Y}_{2}\right)\right) \mathrm{Y}_{1} \tag{4.5}
\end{gather*}
$$

Since we are interested in the question of whether a tax buyout scheme can constitute a Pareto-improvement, we only need to determine whether the value of $Y_{1}$ which maximises $T$ exceeds $\underline{Y}$ and simultaneously implies that $\mathrm{Y}_{2}<\overline{\mathrm{Y}} .{ }^{12}$ These restrictions ensure that expected tax revenues are maximised if some but not all individuals accept a tax buyout offer.

[^9]The first term in (4.4) captures the impact of a rise in the number of individuals who do not use the tax buyout scheme because $\mathrm{Y}_{1}$ increases the threshold income $\mathrm{Y}_{2}$ (see Appendix 7.4 for the proof that $\partial \mathrm{Y}_{2} / \partial \mathrm{Y}_{1}>0$ ). This positive impact on $\mathrm{Y}_{2}$ occurs because the increase in $\mathrm{Y}_{1}$ requires an individual to make a higher payment $\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})$ in order to avoid being detected evading taxes if using the tax buyout. Therefore, the level of income $\mathrm{Y}_{2}$ at which individuals start utilising the buyout, but pay more taxes than they would optimally at a given detection probability, $\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})>\mathrm{V}\left(\mathrm{Y}_{2}{ }^{*} \mid \mathrm{a}=1\right)$, also goes up. To ascertain the budgetary consequences of this effect, it should be observed that a buyout reduces the penalty when evasion is detected because the relevant tax base declines from Yt to $\mathrm{Y}(\mathrm{t}-\mathrm{r})$. Individuals who accept the buyout offer are willing to pay an amount $B$ in both states of the world for this decrease in the penalty and, additionally, to increase payments from the optimal unconstrained level $\mathrm{V}^{*}(\mathrm{Y} \mid \mathrm{a}=1)$ to the quasi-voluntary amount $\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})$. Because a tax buyout reduces income variability, riskaverse individuals raise their overall payments as a compensation for the lower exposure to risk. This implies that expected tax payments $\mathrm{T}_{\mathrm{B}} \mathrm{c}(\mathrm{Y})$ of individuals who utilise the buyout, but can only make quasi-voluntary payments $\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})$, exceed expected tax payments $\mathrm{T}_{\mathrm{W}}(\mathrm{Y})$ if no buyout is used. Therefore, the term in curly brackets in (4.4), $\mathrm{T}_{\mathrm{W}}\left(\mathrm{Y}_{2}\right)-\mathrm{T}_{\mathrm{B}} \mathrm{c}\left(\mathrm{Y}_{2}\right)$, is negative (see Appendix 7.5 for the proof which relies on the existence of risk-aversion). In Figure 1, this negative impact on revenues is captured by the greater slope of the line $\mathrm{B}^{\mathrm{c}}$ than of W at an income $\mathrm{Y}_{2}$. Since a rise in $\mathrm{Y}_{1}$ raises the number of individuals who do not use the tax buyout scheme, given that $\partial \mathrm{Y}_{2} / \partial \mathrm{Y}_{1}>0$ applies, the overall budgetary impact as captured by the first term in (4.4) is negative.

The second term in equation (4.4) describes the change in expected tax revenues T because an individual no longer pays the optimal unconstrained amount, but instead makes the higher quasi-voluntary payment $\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})$. Since income $\mathrm{Y}_{3}$ is defined by $\mathrm{V}^{*}\left(\mathrm{Y}_{3} ; \mathrm{a}=1\right)=\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})$, quasi-voluntary and optimal unconstrained payments coincide at $\mathrm{Y}=\mathrm{Y}_{3}$, such that $\mathrm{T}_{\mathrm{B}} \mathrm{c}\left(\mathrm{Y}_{3}\right)$ $=\mathrm{T}_{\mathrm{B}}\left(\mathrm{Y}_{3}\right)=0$. In terms of Figure 1, the identical slopes of the lines $\mathrm{B}^{\mathrm{c}}$ and $\mathrm{B}^{\mathrm{u}}$ at $\mathrm{Y}=\mathrm{Y}_{3}$ indicate the irrelevance of (a marginal change in) Y for government revenues.

The third term expresses the increase in expected tax payments T because a rise in the income threshold $\mathrm{Y}_{1}$ forces all those (inframarginal) individuals who use the tax buyout scheme to

[^10]pay more and make quasi-voluntary tax payments in excess of the optimal unconstrained level. A change in $\mathrm{Y}_{1}$ alters the distance between the lines $\mathrm{B}^{\mathrm{c}}$ and $\mathrm{B}^{\mathrm{u}}$ for a given income. Finally, the fourth term in (4.4) depicts the effect of a rise in $\mathrm{Y}_{1}$ on the fixed payment $\mathrm{B}, \mathrm{B}=$ $Y_{1}$ r. In Figure 1, the rise in B would lead to a downward shift of the lines $B^{c}$ and $B^{u}$. If at least one individual utilises the tax buyout, the third and fourth terms in (4.4) will be positive.

In order to further interpret the derivative (4.4), assume initially that $\mathrm{Y}_{1}$ is chosen such that $Y_{2}$ equals $\bar{Y}$, so that $Y_{3}$ exceeds $\bar{Y}$. In consequence, $G\left(Y_{2}\right)=G\left(Y_{3}\right)=1$ and the third term in (4.4) is zero because there are no individuals who use the tax buyout and make quasivoluntary tax payments in excess of the optimal unconstrained level. Furthermore, the individual with the highest income $\overline{\mathrm{Y}}$ is indifferent between making an excessive payment and using the buyout on the one hand, and not accepting the offer on the other hand. Therefore, the term in curly brackets in the first term of (4.4) is less than zero for $\mathrm{Y}_{2}=\overline{\mathrm{Y}}<$ $\mathrm{Y}_{3}$, while $\partial \mathrm{Y}_{2} / \partial \mathrm{Y}_{1}>0$ holds. Accordingly, it is not optimal to set $\mathrm{Y}_{1}$ in such a manner that the tax buyout scheme will not be used. The reason is that the complete abolition of the buyout scheme has no positive impact on expected revenues, but instead a negative marginal effect since no individual can pay for the reduction in the effective penalty.

Assume next that $\mathrm{Y}_{2}<\underline{\mathrm{Y}} \leq \mathrm{Y}_{3}$. Therefore, every individual accepts the offer of a tax buyout and a fraction $\mathrm{G}\left(\mathrm{Y}_{3}\right)$ is unable to select payments optimally. In this case, a change in $\mathrm{Y}_{1}$ does not have an impact on expected tax revenues via its effect on $\mathrm{Y}_{2}$ because there is no individual who refrains from accepting the tax buyout offer. In consequence, only the third and fourth terms in (4.4) remain, where $\mathrm{Y}_{2}$ is replaced by $\underline{Y}$. Both are unambiguously positive for $\underline{Y}<Y_{3}$. Therefore, expected tax revenues cannot be maximised if all individuals use the buyout. The reason for this feature is that marginally raising the threshold $\mathrm{Y}_{1}$ causes no loss of revenues in terms of the number of individuals who do not accept the buyout offer, but increases payments by those who make use of it.

Assume, finally, that $\underline{Y}<\mathrm{Y}_{2}<\overline{\mathrm{Y}}$ holds, while $\mathrm{Y}_{3}$ may fall short of or exceed $\overline{\mathrm{Y}}$. In this case, the first term in (4.4) is negative and there is a detrimental revenue effect of raising the income threshold $\mathrm{Y}_{1}$. Since the marginal revenue gain, as captured by the third and fourth terms in (4.4) is positive for $\underline{\mathrm{Y}}<\mathrm{Y}_{2}<\overline{\mathrm{Y}}$, and becomes zero at $\mathrm{Y}_{2}=\overline{\mathrm{Y}}<\mathrm{Y}_{3}$, there is some level of income $\mathrm{Y}_{1}$ which maximises expected government revenues.

We can summarise these considerations as

## Proposition 3:

Assume a buyout scheme characterised by a general tax rate reduction $r$ and a fixed payment $B$ such that $Y_{1}=B / r$ holds. For a penalty function given by (4.1) and an income level $\mathrm{Y}_{2}\left(\mathrm{Y}_{1}\right), \underline{\mathrm{Y}}<\mathrm{Y}_{2}\left(\mathrm{Y}_{1}\right)<\overline{\mathrm{Y}}$, where $\mathrm{Y}_{2}$ is defined in equation (4.2), the tax buyout scheme represents a Pareto-improvement.

Proof: see above.

A tax buyout is utilised by two groups. The first consists of individuals with a sufficiently high income above $\mathrm{Y}_{3}$. These individuals make their optimal unconstrained tax payments and benefit from the reduction in the expected tax burden that results because the buyout entails a fixed payment $\mathrm{B}=\mathrm{Y}_{1} \mathrm{r}<\mathrm{Y}_{3} \mathrm{r}$. The participation of such high-income individuals reduces expected tax revenues. The second group benefiting from a buyout consists of individuals who would reveal that they had evaded taxes if they utilised the buyout and made the optimal unconstrained payment. Individuals with an income level close enough to $\mathrm{Y}_{3}\left(\mathrm{Y}_{2} \leq \mathrm{Y}<\mathrm{Y}_{3}\right)$ obtain a reduction in the tax rate which makes them willing to increase their payment to above the optimal unconstrained level. This restriction on tax payments which occurs, because utilising the tax buyout changes the audit probability for all individuals implicitly declaring an income $\mathrm{Y}, \mathrm{Y}_{2} \leq \mathrm{Y}<\mathrm{Y}_{3}$, cannot be fully compensated by an adjustment in evasion activities. Effectively, a tax buyout, combined with an adjustment in the probability of detection for inconsistent income declarations, reduces tax evasion opportunities and the variability of income. In consequence, a tax buyout reduces the exposure to risk. Since individuals are risk averse, they are willing to pay for the decline in income uncertainty. Therefore, expected revenues resulting from payments by members of this group can increase without making these individuals worse off. Introducing a buyout scheme in which only individuals who benefit from this quasi-insurance feature can participate is Pareto-improving as the revenuereducing impact described above does not (yet) occur.

## 5. Benchmark for Comparison

The reasoning in Section 4 with respect to the potential Pareto-improvement has relied on the assumption that the detection probability is fixed at the level $1-\mathrm{p}$ prior to the existence of a
tax buyout and changes to $1-p(a)$, as defined by (4.1), with its introduction. Accordingly, the tax buyout implies two alterations, namely a change of the tax structure and a variation of the detection probability. Therefore, it can be questioned whether it is the tax buyout per se which brings about the Pareto-improvement or whether this impact is due to the alteration of the detection probability for those individuals who do not use the tax buyout anyhow.

In order to analyse this issue, suppose that initially any individual whose tax payment implies that income is less than some level $\mathrm{Y}_{0}$ will be audited and detected with certainty. ${ }^{13}$ The detection probability in such a setting, in the absence of a buyout, is given by $1-\hat{\mathrm{p}}$, where:

$$
\hat{\mathrm{p}}=\left\{\begin{array}{lll}
=\mathrm{p} & \text { if } & \mathrm{V} \geq \mathrm{Y}_{0} \mathrm{t}  \tag{5.1}\\
=0 & \text { if } & \mathrm{V}<\mathrm{Y}_{0} \mathrm{t}
\end{array}\right.
$$

Therefore, individuals with an income less than $\mathrm{Y}_{0}$ will pay the amount of taxes Yt actually due in order to avoid a certain penalty payment.

If a buyout is introduced, the audit and detection probability will equal $1-\hat{p}(a)$, where $\hat{p}(a)$ can be defined as.

$$
\hat{\mathrm{p}}(\mathrm{a})=\left\{\begin{array}{lll}
=0 & \text { if } \mathrm{V}<\mathrm{Y}_{0} \mathrm{t} & \text { or } \mathrm{a}=1 \text { and } \mathrm{V}<\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})  \tag{5.2}\\
=\mathrm{p} & \text { if } \mathrm{a}=0 \text { and } \mathrm{V} \geq \mathrm{Y}_{0} \mathrm{t} & \text { or } \mathrm{a}=1 \text { and } \mathrm{V} \geq \mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})>\mathrm{Y}_{0} \mathrm{t}
\end{array}\right.
$$

Given the detection probability $1-\hat{\mathrm{p}}(\mathrm{a})$ and $\mathrm{Y}_{0} \leq \mathrm{Y}_{1}$, individuals with an income (weakly) below $\mathrm{Y}_{1}$ will not benefit from a tax buyout and expected tax revenues $\mathrm{T}^{0}$ equal: ${ }^{14}$

The first term in equation (5.3) describes payments by those individuals who neither evade taxes nor use the buyout. The remaining terms are defined in and explained below equation (4.3). It is straightforward to see that the derivative of (5.3) with respect to the income threshold $\mathrm{Y}_{1}$ is given by (4.4). Consequently, if the audit rule $\hat{\mathrm{p}}$ (a) only induces individuals

[^11]to behave honestly who would not make use of the tax buyout anyhow, the analysis of Section 4 will apply and introducing a tax buyout scheme will be a Pareto-improvement.

However, the income threshold $\mathrm{Y}_{0}$ may also exceed the level $\mathrm{Y}_{1}, \mathrm{Y}_{0}>\mathrm{Y}_{1}$. Expected tax revenues then amount to:

$$
\begin{array}{r}
\mathrm{T}^{0} \mid \mathrm{Y}_{0}>\mathrm{Y}_{1}{ }_{\underline{\mathrm{Y}}}^{=\mathrm{Y}_{1} \mathrm{YtdG}(\mathrm{Y})+\int_{\mathrm{Y}_{1}}^{\mathrm{Y}_{0}}(\mathrm{Y}(\mathrm{t}-\mathrm{r})+\mathrm{B}) \mathrm{dG}(\mathrm{Y})+\int_{\mathrm{Y}_{0}}^{\mathrm{Y}_{3}\left(\mathrm{Y}_{1}, \mathrm{r}\right)} \mathrm{T}_{\mathrm{B}^{\mathrm{c}}} \mathrm{dG}(\mathrm{Y})} \\
+\int_{\bar{Y}}^{\mathrm{Y}} \int_{\left.\mathrm{Y}_{1}, \mathrm{r}\right)}^{\mathrm{T}_{\mathrm{B}} \mathrm{dG}(\mathrm{Y})} \tag{5.4}
\end{array}
$$

The first term in equation (5.4) depicts tax payments by individuals who are honest and whose income is too low to use a tax buyout. The second term captures payments by honest individuals who accept the buyout offer. For people with an income above the level $\mathrm{Y}_{0}$, $\hat{p}(a)=p$ holds and they behave in the same way as described in Section 4. Accordingly, the derivative of (5.4) with respect to $Y_{1}$, taking into account $B=Y_{1} r$, is given by:

$$
\begin{gather*}
\frac{\partial T^{0}}{\partial Y_{1}}=\underbrace{\left.Y_{1} \operatorname{tg}\left(Y_{1}\right)-\left(Y_{1}(\mathrm{t}-\mathrm{r})+\mathrm{Y}_{1} \mathrm{r}\right)\right) \mathrm{g}\left(\mathrm{Y}_{1}\right)}_{=0}+\frac{\partial \mathrm{Y}_{3}}{\partial \mathrm{Y}_{1}} \underbrace{\left(\mathrm{~T}_{\mathrm{B}^{\mathrm{c}}}\left(\mathrm{Y}_{3}\right)-\mathrm{T}_{\mathrm{B}}\left(\mathrm{Y}_{3}\right)\right)}_{=0} \mathrm{~g}\left(\mathrm{Y}_{3}\right) \\
+\left[\mathrm{G}\left(\mathrm{Y}_{3}\right)-\mathrm{G}\left(\mathrm{Y}_{0}\right)\right][(\mathrm{t}-\mathrm{r})(\mathrm{p}-(1-\mathrm{p}) \mathrm{f})]+\left(1-\mathrm{G}\left(\mathrm{Y}_{0}\right)\right) \mathrm{r}>0 \tag{5.5}
\end{gather*}
$$

The positive sign of (5.5) implies that a buyout scheme which results in $\mathrm{Y}_{1}<\mathrm{Y}_{0}$ cannot maximise expected tax revenues $\mathrm{T}^{0}$. Raising the income threshold $\mathrm{Y}_{1}$ slightly has no negative revenue effect resulting from a change in the composition of the groups of tax payers. The reason is that making a tax buyout slightly less attractive to an individual who is indifferent between accepting a given buyout offer and declining it, irrespective of whether tax evasion occurs or not, will not alter the individual's (expected) tax payments. However, raising the income level which makes accepting a buyout attractive for tax evaders, that is augmenting their fixed payment for a given reduction in the marginal tax rate, implies that they will raise their quasi-voluntary payment. Consequently, expected tax revenues rise. Therefore, it is optimal for tax authorities to increase the income threshold $\mathrm{Y}_{1}$ until it exceeds the level $\mathrm{Y}_{0}$ because only then effectively excluding further individuals from utilising a buyout involves negative revenue effects.

We can conclude: introducing a tax buyout and punishing individuals whose tax payments are inconsistent with their behaviour with respect to a buyout, will also be Pareto-improving if the benchmark for a comparison is a world in which initially the detection probability is unity for all implicit income declaration below a threshold $\mathrm{Y}_{0}$. Therefore, the Pareto-improving feature of a tax buyout does not require the penalty rate to increase for all individuals who declare too low an income. Rather, the Pareto-improvement becomes feasible because the detection probability may rise for those who utilise the tax buyout.

## 6. Conclusions

A tax buyout constitutes an offer to tax payers to purchase a reduction in the marginal tax rate in exchange for a lump-sum payment. In the case of income taxation, a buyout reduces labour supply distortions and may raise tax revenues. However, the prediction that an income tax buyout will be Pareto-improving is based on the assumption that taxes due are actually paid. In this paper, we consider the possibility of tax evasion. Tax buyouts alter the incentives to evade and, therefore, the question arises whether the effects of buyouts on tax evasion strengthen or mitigate the potentially positive consequences via adjustments in labour supply.

We have shown that a tax buyout can be Pareto-improving in a setting in which the detection probability is constant and the penalty payment is a function of undeclared income or, more generally, not proportional to the amount of taxes evaded. A buyout makes the tax system regressive, mitigates evasion incentives and thereby enhances tax revenues. Since utilising a buyout is voluntary, higher tax payments result in a Pareto-improvement. However, in a world in which the fine is proportional to the level of tax evasion, the positive revenue effect cannot arise. The reason is that the reduction in the marginal tax rate also lowers the fine. In the unlikely event that tax authorities can induce all tax payers to make a lump-sum payment which leaves them indifferent between accepting and declining a tax buyout offer, there are no net budgetary effects of the buyout scheme; otherwise expected tax revenues will decline. We have further shown that if the penalty is a function of the amount of taxes evaded, by accepting a tax buyout an individual conveys information about the true level of income to tax authorities. Therefore, in an extension of the basic model, tax authorities are modelled as more sophisticated agents that adjust the audit probability in line with the information generated by an individual's response to the offer of buying out taxes. More specifically, the detection probability is assumed to rise to unity if the implicit income declaration and the information resulting from the acceptance of a tax buyout offer are inconsistent. In order to
avoid being detected evading taxes when using a buyout, some of the strictly risk-averse individuals are willing to raise their payments. Consequently, even if the penalty is a function of the amount of taxes evaded, a tax buyout can be a Pareto-improvement. We finally show that this prediction does not rely on the increase in the detection probability for those individuals who would never use a tax buyout.

The Pareto-improving potential of an income tax buyout has been derived in a setting in which all individuals are basically treated equally ex-ante. The buyouts investigated are, hence, unlikely to exhaust all gains from modifying the tax code. Suppose, therefore, that the government could offer more than one tax buyout scheme, say one for 'high'-income and one for 'low'-income individuals. These schemes would consist of low and high tax rate reductions and fixed payments. If the government can, furthermore, prevent high-income individuals from mimicking low-income ones, tax revenues may increase beyond the level ensured by a single buyout scheme. Consequently, allowing for more elaborate tax buyout schemes than analysed above can strengthen the conclusion captured by Corollary 2 and Proposition 3.

The distributional effects of the proposed scheme are ambiguous. Firstly, if the fine depends on the amount of undeclared income, the willingness to pay for a tax buyout may not be correlated systematically with gross income. Secondly, even if this is unambiguously the case, as in a model in which the penalty is a function of the amount of taxes evaded, absolute income and utility changes as well as absolute and relative alterations may differ. In the model looked at in Section 4, expected utility of low-income individuals is unaffected because these individuals do not accept the tax buyout offer. High-income individuals are better off, while those with an intermediate income who raise their expected tax payments experience a reduction in expected disposable income. However, high and middle income individuals benefit in terms of expected utility if they accept the buyout scheme. Accordingly, everyone is better off, with the exception of low-income individuals who experience a decline in relative but not in absolute terms.

Finally, the present analysis has assumed a given amount of labour supply. A labour supply response to a change in income taxation could clearly be added to the framework considered here. The interaction between a pure labour supply and a pure tax evasion effect could either strengthen or mitigate the consequences derived above. ${ }^{15}$ However, the basic insight that tax buyout schemes constitute a mechanism that reduces tax evasion activities would essentially

[^12]not be affected by such interaction effects. Therefore, we conclude that tax buyouts have an additional positive effect besides their labour supply impact. This suggests that their impact should be scrutinised more thoroughly and, foremost, also in other contexts than the one investigated above. Such analysis could help to ascertain whether tax buyouts should be added to the toolkit of tax policy and applied more widely than this is currently the case.

## 7. Appendix

### 7.1 Strict Concavity of Expected Utility EU in Gross Income Y

Using $\widetilde{\mathrm{f}}=\frac{\mathrm{f}}{(\mathrm{t}-\mathrm{r})^{\alpha}}$ and $\tilde{\mathrm{t}}:=1-(\mathrm{t}-\mathrm{r})(1+\widetilde{\mathrm{f}})>0$, the total derivatives of expected utility EU as defined in equation (2.2) with respect to income Y are found to be:

$$
\begin{align*}
& \frac{\mathrm{dEU}\left(\mathrm{Y} ; \mathrm{V}^{*}(\mathrm{Y})\right)}{\mathrm{dY}}=\frac{\partial \mathrm{EU}}{\partial \mathrm{Y}}+\frac{\partial \mathrm{EU}}{\partial \mathrm{~V}} \frac{\partial \mathrm{~V}^{*}}{\partial \mathrm{Y}}=\frac{\partial \mathrm{EU}}{\partial \mathrm{Y}}=\mathrm{pu}^{\prime}\left(\mathrm{x}^{\mathrm{n}}\right)+(1-\mathrm{p}) \mathrm{u}^{\prime}\left(\mathrm{x}^{\mathrm{c}}\right) \tilde{\mathrm{t}}>0  \tag{A.1}\\
& \frac{\mathrm{~d}^{2} \mathrm{EU}\left(\mathrm{Y} ; \mathrm{V}^{*}(\mathrm{Y})\right)}{\mathrm{d}(\mathrm{Y})^{2}}=\frac{\partial^{2} \mathrm{EU}}{\partial \mathrm{Y}^{2}}+\frac{\partial^{2} \mathrm{EU}}{\partial \mathrm{Y} \partial \mathrm{~V}} \frac{\partial \mathrm{~V}^{*}}{\partial \mathrm{Y}}=\frac{\partial^{2} \mathrm{EU}}{\partial \mathrm{Y}^{2}}-\frac{\partial^{2} \mathrm{EU}}{\partial \mathrm{Y} \partial \mathrm{~V}} \frac{\partial^{2} \mathrm{EU}}{\partial \mathrm{Y} \partial \mathrm{~V}}  \tag{A.2}\\
& \mathrm{EU} \mathrm{VV}
\end{align*},
$$

where $\mathrm{EU}_{\mathrm{VV}}$ is defined in (2.4). The other derivatives are given by:

$$
\begin{align*}
& \quad \frac{\partial^{2} \mathrm{EU}}{\partial \mathrm{Y}^{2}}=\mathrm{pu}^{\prime \prime}\left(\mathrm{x}^{\mathrm{n}}\right)+(1-\mathrm{p}) \mathrm{u}^{\prime \prime}\left(\mathrm{x}^{\mathrm{c}}\right) \widetilde{\mathrm{t}}^{2}<0  \tag{A.3}\\
& \frac{\partial^{2} \mathrm{EU}}{\partial \mathrm{Y} \partial \mathrm{~V}}=-\mathrm{pu} u^{\prime \prime}\left(\mathrm{x}^{\mathrm{n}}\right)+(1-\mathrm{p}) \mathrm{u}^{\prime \prime}\left(\mathrm{x}^{\mathrm{c}}\right) \widetilde{\mathrm{t}} \mathrm{f} \tag{A.4}
\end{align*}
$$

Substituting (2.4), (A.3), and (A.4) into (A.2) and simplifying, we obtain:

$$
\begin{equation*}
\frac{d^{2} E U\left(Y ; V^{*}(Y)\right)}{d(Y)^{2}}=\frac{p u^{\prime \prime}\left(x^{n}\right)(1-p) u^{\prime \prime}\left(x^{c}\right)(\widetilde{f}+\widetilde{t})^{2}}{\frac{\partial^{2} E U}{\partial V^{2}}}<0 \tag{A.5}
\end{equation*}
$$

Note that the signs of (A.1) and (A.5) are independent of the magnitudes of B and $r$. Therefore, expected utility EU increases with gross income Y at a decreasing rate, irrespective of whether a tax buyout is utilised or not.

### 7.2 Expected Utility and Expected Tax Payments

We have $\mathrm{EU}_{\mathrm{VV}}<0$ from the second-order condition, $\mathrm{t}>\mathrm{r}$, and $\partial \mathrm{S} / \partial \mathrm{B}>0$ because otherwise the government could reduce B , thereby raise expected payments per capita S and make individuals better off (see the derivation below equation 3.1). For $\Omega:=\mathrm{p}-(1-\mathrm{p}) \widetilde{\mathrm{f}}=1-(1-$ $\mathrm{p})(1+\widetilde{\mathrm{f}})>0$ (see the discussion below equation (2.3)), we can define a difference A :

$$
\begin{align*}
& A:=\left.\frac{d B}{d r}\right|_{d E U=0}-\left.\frac{d B}{d r}\right|_{d S=0}=\frac{Y(1+(1-\alpha) \widetilde{f})+\alpha V^{*} \frac{\tilde{f}}{t-r}}{1+\widetilde{f}}+\frac{\frac{\partial S}{\partial r}}{\frac{\partial S}{\partial B}} \\
& =\frac{Y(1+(1-\alpha) \widetilde{f})+\alpha V^{*} \frac{\widetilde{f}}{\mathrm{t}-\mathrm{r}}+\mathrm{Y}(1+(1-\alpha) \widetilde{\mathrm{f}}) \frac{\partial \mathrm{V}^{*}}{\partial \mathrm{~B}} \Omega+\frac{\partial \mathrm{V}^{*}}{\partial \mathrm{~B}} \Omega \alpha \mathrm{~V}^{*} \frac{\widetilde{\mathrm{f}}}{\mathrm{t}-\mathrm{r}}}{\frac{\partial \mathrm{~S}}{\partial \mathrm{~B}}(1+\widetilde{\mathrm{f}})} \\
& +\frac{\frac{\partial V^{*}}{\partial r} \Omega(1+\widetilde{\mathrm{f}})-(1-\mathrm{p})(1+\widetilde{\mathrm{f}})\left[\mathrm{Y}\left(1+(1-\alpha) \widetilde{\mathrm{f}}+\alpha \mathrm{V}^{*} \frac{\widetilde{\mathrm{f}}}{\mathrm{t}-\mathrm{r}}\right]\right.}{\frac{\partial \mathrm{S}}{\partial \mathrm{~B}}(1+\widetilde{\mathrm{f}})} \tag{A.6}
\end{align*}
$$

Collecting common terms yields:

$$
\begin{equation*}
A=\Omega \frac{Y(1+(1-\alpha) \widetilde{f})+\alpha V^{*} \frac{\widetilde{f}}{t-r}+Y(1+(1-\alpha) \widetilde{f}) \frac{\partial V^{*}}{\partial B}+\frac{\partial V^{*}}{\partial B} \alpha V \frac{\widetilde{f}}{t-r}+\frac{\partial V^{*}}{\partial r}(1+\widetilde{f})}{\frac{\partial S}{\partial B}(1+\widetilde{f})} \tag{A.7}
\end{equation*}
$$

Substituting in accordance with (2.4) to (2.6) and rearranging we obtain:

$$
\begin{align*}
& A \frac{\partial S}{\partial B} \frac{(1+\widetilde{f})}{\Omega}=\left[Y(1+(1-\alpha) \widetilde{f})+\alpha V^{*} \frac{\widetilde{f}}{t-r}\right] \frac{\mathrm{pu}^{\prime \prime}\left(x^{n}\right)+(1-p) u^{\prime \prime}\left(x^{c}\right) \tilde{f}^{2}}{E U_{V V}} \\
&-\left[Y(1+(1-\alpha) \widetilde{f})+\alpha V^{*} \frac{\widetilde{f}}{t-r}\right] \frac{p u^{\prime \prime}\left(x^{n}\right)-(1-p) u^{\prime \prime}\left(x^{c}\right) \widetilde{f}}{E U_{V V}} \\
&-\frac{(1-p) \tilde{f}}{E U_{V V}}\left[u^{\prime \prime}\left(x^{c}\right)\left(Y(1+(1-\alpha) \widetilde{\mathrm{f}})+\alpha V^{*} \frac{\widetilde{f}}{t-r}\right)+\frac{u^{\prime}\left(x^{c}\right) \alpha}{t-r}\right](1+\widetilde{f}) \tag{A.8}
\end{align*}
$$

Collecting common terms and simplifying, we find:

$$
\begin{equation*}
A=-\alpha \frac{(1-p) \Omega \widetilde{f} u^{\prime}\left(x^{c}\right)}{E U_{V V} \frac{\partial S}{\partial B}(t-r)} \geq 0 \tag{A.9}
\end{equation*}
$$

### 7.3 Derivation of Figure 1

Suppose $\alpha=0$. The definition of $\mathrm{Y}_{3}, \mathrm{~V}^{*}\left(\mathrm{Y}_{3} \mid \mathrm{a}=1\right)=\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})$, and the assumption of an interior solution imply that $\mathrm{V}^{*}\left(\mathrm{Y}_{3} \mid \mathrm{a}=1\right)<\mathrm{Y}_{3}(\mathrm{t}-\mathrm{r})$ and $\mathrm{Y}_{1}<\mathrm{Y}_{3}$ hold. The line $\mathrm{B}^{\mathrm{u}}$ is defined by $\operatorname{EU}\left(\mathrm{Y} ; \mathrm{V}^{*}(\mathrm{Y} \mid \mathrm{a}=1)\right.$ ), the line W by $\operatorname{EU}\left(\mathrm{Y} ; \mathrm{V}^{*}(\mathrm{Y} \mid \mathrm{a}=0)\right.$ ), and the line $\mathrm{B}^{\mathrm{c}}$ by $\operatorname{EU}\left(\mathrm{Y} ; \mathrm{V}=\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r}) ; \mathrm{a}=1\right)$. Expected utility levels $\mathrm{EU}\left(\mathrm{Y} ; \mathrm{V}^{*}(\mathrm{Y} \mid \mathrm{a}=0)\right)$ and $\mathrm{EU}\left(\mathrm{Y} ; \mathrm{V}^{*}(\mathrm{Y} \mid \mathrm{a}\right.$
$=1$ ) are the same at an income level $\mathrm{Y}_{1}=\mathrm{B} / \mathrm{r}$ according to Proposition 1a. Therefore, the lines $\mathrm{B}^{\mathrm{u}}$ and W cross at the income level $\mathrm{Y}_{1}$. Furthermore, $\mathrm{EU}\left(\mathrm{Y} ; \mathrm{V}^{*}(\mathrm{Y} \mid \mathrm{a}=0)\right)<\mathrm{EU}(\mathrm{Y}$; $\left.\mathrm{V}^{*}(\mathrm{Y} \mid \mathrm{a}=1)\right)$ at any income $\mathrm{Y}>\mathrm{Y}_{1}$ according to Proposition 1 b . This implies that the line $\mathrm{B}^{\mathrm{u}}$ lies above $W$ at $Y=Y_{3}$. Since $V^{*}\left(Y_{3} \mid a=1\right)=Y_{1}(t-r)$, we have $E U\left(Y ; V=Y_{1}(t-r) ; a=1\right)$ $=\mathrm{EU}\left(\mathrm{Y} ; \mathrm{V}^{*}(\mathrm{Y} \mid \mathrm{a}=1)\right)$ for $\mathrm{Y}=\mathrm{Y}_{3}$ and given levels of B and r . Hence, the lines $\mathrm{B}^{\mathrm{u}}$ and $\mathrm{B}^{\mathrm{c}}$ coincide at an income level $\mathrm{Y}_{3}$. Finally, $\mathrm{B}^{\mathrm{c}}$ lies below W at $\mathrm{Y}=\mathrm{Y}_{1}$ because quasi-voluntary payments exceed the optimal unconstrained level $\mathrm{V}^{*}\left(\mathrm{Y}_{1} \mid \mathrm{a}=0\right)$. Evaluating $\mathrm{EU}(\mathrm{Y} ; \mathrm{V}=$ $\left.\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r}) ; \mathrm{a}=1\right)$ at $\mathrm{Y}=\mathrm{Y}_{1}=\mathrm{B} / \mathrm{r}$, yields $\mathrm{EU}\left(\mathrm{Y} ; \mathrm{V}=\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r}) ; \mathrm{a}=1 ; \mathrm{B}=\mathrm{Y}_{1} \mathrm{r}\right)=\mathrm{pu}\left(\mathrm{Y}_{1}(1-\mathrm{t})\right)$ $+(1-\mathrm{p}) \mathrm{u}\left(\mathrm{Y}_{1}(1-\mathrm{t})\right)$. Expected utility $\mathrm{EU}\left(\mathrm{Y} ; \mathrm{V}^{*}(\mathrm{Y} \mid \mathrm{a}=0)\right)$ calculated at an income $\mathrm{Y}_{1}$ will equal $\operatorname{EU}\left(\mathrm{Y} ; \mathrm{V}=\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r}) ; \mathrm{a}=1 ; \mathrm{B}=\mathrm{Y}_{1} \mathrm{r}\right)$ if $\mathrm{V}^{*}\left(\mathrm{Y}_{1} \mid \mathrm{a}=1\right)=\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})$ holds. Since the optimal unconstrained payment $\mathrm{V}^{*}$ at an income $\mathrm{Y}_{1}$ is lower than $\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})$, given tax evasion, $\operatorname{EU}\left(\mathrm{Y} ; \mathrm{V}^{*}(\mathrm{Y} \mid \mathrm{a}=0)\right)>\mathrm{EU}\left(\mathrm{Y} ; \mathrm{V}=\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r}) ; \mathrm{a}=1 ; \mathrm{B}=\mathrm{Y}_{1} \mathrm{r}\right)$ holds at $\mathrm{Y}=\mathrm{Y}_{1}$.

To prove that the intersection of the lines $\mathrm{B}^{\mathrm{c}}$ and W is unique, implying that $\mathrm{Y}_{1}<\mathrm{Y}_{2}<\mathrm{Y}_{3}$, we furthermore have to show that all lines are increasing and strictly concave in Y. In Appendix 7.1 we have already done so for $\mathrm{B}^{\mathrm{u}}$ and W . Since, furthermore, $\mathrm{EU}\left(\mathrm{Y} ; \mathrm{V}=\mathrm{Y}_{1}(\mathrm{t}-\right.$ $\mathrm{r})$; $\mathrm{a}=1$ ) does not depend on Y via V , its derivative is simply the partial derivative of $\mathrm{EU}(\mathrm{Y}$; $\mathrm{V}^{*}(\mathrm{Y} \mid \mathrm{a}=0)$ ) with respect to $\mathrm{Y}, \partial \mathrm{EU} / \partial \mathrm{Y}$, as derived in $(\mathrm{A} .1)$ and (A.3). Therefore, the line $\mathrm{B}^{\mathrm{C}}$ is also increasing and strictly concave in Y .

### 7.4 Derivative $\partial \mathrm{Y}_{2} / \partial \mathrm{Y}_{1}$ in Equation (4.4)

The income $\mathrm{Y}_{2}$ is defined by (4.2), which, for ease of exposition, we will slightly rewrite:

$$
\begin{align*}
\mathrm{Z}:= & \mathrm{EU}\left(\mathrm{Y}_{2} ; \mathrm{V}^{*}\left(\mathrm{Y}_{2} \mid \mathrm{a}=0\right)\right)-\mathrm{EU}\left(\mathrm{Y}_{2} ; \mathrm{V}=\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r}) ; \mathrm{a}=1\right) \\
= & \operatorname{pu}\left(\mathrm{Y}_{2}-\mathrm{V}^{*}\left(\mathrm{Y}_{2} \mid \mathrm{a}=1\right)\right)+(1-\mathrm{p}) \mathrm{u}\left(\mathrm{Y}_{2}(1-\mathrm{t}(1+\mathrm{f}))+\mathrm{V}^{*}\left(\mathrm{Y}_{2} \mid \mathrm{a}=1\right) \mathrm{f}\right) \\
& -\operatorname{pu}\left(\mathrm{Y}_{2}-\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})-\mathrm{B}\right)-(1-\mathrm{p}) \mathrm{u}\left(\mathrm{Y}_{2}(1-(\mathrm{t}-\mathrm{r})(1+\mathrm{f}))+\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r}) \mathrm{f}-\mathrm{B}\right)=0 \tag{A.10}
\end{align*}
$$

The impact of a rise in $\mathrm{Y}_{1}$ on $\mathrm{Y}_{2}$ is given by $\frac{\partial \mathrm{Y}_{2}}{\partial \mathrm{Y}_{1}}=-\frac{\partial \mathrm{Z} / \partial \mathrm{Y}_{1}}{\partial \mathrm{Z} / \partial \mathrm{Y}_{2}}$. Furthermore, we have:
$\frac{\partial \mathrm{Z}}{\partial \mathrm{Y}_{1}}=(\mathrm{t}-\mathrm{r})\left[\mathrm{pu}^{\prime}\left(\mathrm{Y}_{2}-\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})-\mathrm{B}\right)-(1-\mathrm{p}) \mathrm{u}^{\prime}\left(\mathrm{Y}_{2}(1-(\mathrm{t}-\mathrm{r})(1+\mathrm{f}))+\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r}) \mathrm{f}-\mathrm{B}\right) \mathrm{f}\right]$

$$
+\left(\mathrm{pu}^{\prime}\left(\mathrm{Y}_{2}-\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})-\mathrm{B}\right)+(1-\mathrm{p}) \mathrm{u}^{\prime}\left(\mathrm{Y}_{2}(1-(\mathrm{t}-\mathrm{r})(1+\mathrm{f}))+\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r}) \mathrm{f}-\mathrm{B}\right)\right) \frac{\partial \mathrm{B}}{\partial \mathrm{Y}_{1}}(\mathrm{~A} .11)
$$

From the first-order condition (2.3) we know that:

$$
\begin{equation*}
-\mathrm{pu}^{\prime}\left(\mathrm{Y}_{2}-\mathrm{V}^{*}\left(\mathrm{Y}_{2}\right)-\mathrm{B}\right)+(1-\mathrm{p}) \mathrm{u}^{\prime}\left(\mathrm{Y}_{2}(1-(\mathrm{t}-\mathrm{r}))-\left(\mathrm{Y}(\mathrm{t}-\mathrm{r})-\mathrm{V}^{*}\left(\mathrm{Y}_{2}\right)\right) \mathrm{f}-\mathrm{B}\right) \mathrm{f}=0 \tag{A.12}
\end{equation*}
$$

At an income $\mathrm{Y}_{2}$, an individual cannot make the optimal payment $\mathrm{V}^{*}\left(\mathrm{Y}_{2} \mid \mathrm{a}=1\right)$ but has to pay more in order to avoid detection. Since expected utility is strictly concave in V (cf. (2.4)), the derivative (2.3) evaluated at $\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})>\mathrm{V}^{*}\left(\mathrm{Y}_{2} \mid \mathrm{a}=1\right)$ is negative. Therefore, the term in square brackets in (A.11) is positive and $\partial \mathrm{Z} / \partial \mathrm{Y}_{1}>0$ applies, irrespective of whether the repercussion of a change in $Y_{1}$ on $B$ is taken into account or not, since $B=Y_{1} r$.

To derive $\partial \mathrm{Z} / \partial \mathrm{Y}_{2}<0$, we note that the line $\mathrm{B}^{\mathrm{c}}$ crosses W only once in Figure 1 and does so from below (see Appendix 7.2). Therefore, the line $\mathrm{B}^{\mathrm{C}}$ has a larger slope than W at $\mathrm{Y}_{2}$, and $\partial \mathrm{Z} / \partial \mathrm{Y}_{2}<0$ must hold. Combining this result with $\partial \mathrm{Z} / \partial \mathrm{Y}_{1}>0$ implies that $\partial \mathrm{Y}_{2} / \partial \mathrm{Y}_{1}>0$.

### 7.5 Difference $\mathrm{T}_{\mathrm{W}}\left(\mathrm{Y}_{2}\right)-\mathrm{T}_{\mathrm{B}} \mathrm{c}\left(\mathrm{Y}_{2}\right)$ in Equation (4.4)

Note that $\mathrm{Y}_{2}$ will only exist if $\mathrm{V}^{*}\left(\mathrm{Y}_{2} \mid \mathrm{a}=1\right)<\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})$. If tax evasion remains undetected, the tax payment by an individual who accepts the tax buyout will be higher than by someone who declines the offer; implying that $\mathrm{Y}_{2}-\mathrm{V}^{*}\left(\mathrm{Y}_{2} \mid \mathrm{a}=1\right)>\mathrm{Y}_{2}-\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})-\mathrm{B}$ holds. Inspection of equation (4.2), which defines $\mathrm{Y}_{2}$, then clarifies that $\mathrm{Y}_{2}$ will only exist if the income when caught evading taxes is greater for someone who has accepted the tax buyout scheme than for an individual who has declined the offer. Therefore, $\mathrm{x}^{\mathrm{c}}(\mathrm{W})=\mathrm{Y}_{2}(1-\mathrm{t}(1+\mathrm{f}))$ $+\mathrm{V}^{*}\left(\mathrm{Y}_{2} \mid \mathrm{a}=1\right) \mathrm{f}<\mathrm{Y}_{2}(1-(\mathrm{t}+\mathrm{r})(1+\mathrm{f}))+\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r}) \mathrm{f}-\mathrm{B}=\mathrm{x}^{\mathrm{c}}(\mathrm{B})$ and $\mathrm{x}^{\mathrm{n}}(\mathrm{B})<\mathrm{x}^{\mathrm{n}}(\mathrm{W})$ hold, where $B(W)$ indicates that the buyout offer has been accepted (declined). Since $Y_{2}$ is defined such that $\mathrm{pu}\left(\mathrm{x}^{\mathrm{n}}(\mathrm{W})\right)+(1-\mathrm{p}) \mathrm{u}\left(\mathrm{x}^{\mathrm{c}}(\mathrm{W})\right)=\mathrm{pu}\left(\mathrm{x}^{\mathrm{n}}(\mathrm{B})\right)+(1-\mathrm{p}) \mathrm{u}\left(\mathrm{x}^{\mathrm{c}}(\mathrm{B})\right)$ holds, expected income $p x^{n}(B)+(1-p) x^{c}(B)$ must be less than expected income $p x^{n}(W)+(1-p) x^{c}(W)$, given strict risk aversion. Since expected income can also be expressed as $Y_{2}-T_{W}\left(Y_{2}\right)=Y_{2}-\left(p^{n}(W)\right.$ $\left.+(1-\mathrm{p}) \mathrm{x}^{\mathrm{c}}(\mathrm{W})\right)$ and $\mathrm{Y}_{2}-\mathrm{T}_{\mathrm{B}} \mathrm{c}\left(\mathrm{Y}_{2}\right)=\mathrm{Y}_{2}-\left(\mathrm{p} \mathrm{x}^{\mathrm{n}}(\mathrm{W})+(1-\mathrm{p}) \mathrm{x}^{\mathrm{c}}(\mathrm{W})\right)$, expected tax payments in this constrained case must exceed the payments made if the individual does not accept the tax buyout offer, implying that $\mathrm{T}_{\mathrm{W}}\left(\mathrm{Y}_{2}\right)-\mathrm{T}_{\mathrm{B}} \mathrm{c}\left(\mathrm{Y}_{2}\right)$ is negative.

### 7.6 A Simple Numerical Example

Willingness to pay for a tax buyout if the fine depends on undeclared income (Section 3)
Let utility be given by $u=\frac{x^{1-\delta}}{1-\delta}, 0<\delta$, where x is income and $\delta$ the constant Arrow-Pratt measures of relative risk aversion. The first-order condition (2.3) for a general fine function can be expressed as:

$$
\begin{equation*}
\widehat{\beta} x^{n}=x^{c} \tag{A.13}
\end{equation*}
$$

where $\hat{\beta}:=\left(\frac{p}{(1-p) \hat{f}}\right)^{-1 / \delta}$ is less than unity given the restriction on $p$ and $\hat{f}$, and $\hat{f}, x^{n}$ and $x^{c}$ are defined in the main text. Solving this linear equation for V yields:

$$
\begin{equation*}
V^{*}(Y, r, B)=\frac{Y(\hat{\beta}-(1-(t-r)(1+\hat{f})))+(1-\widehat{\beta}) B}{\hat{f}+\hat{\beta}} \tag{A.14}
\end{equation*}
$$

Plugging this value for the optimal voluntary payment into the definition of the critical income level $Y^{\text {crit }}$ (see Proposition 1), collecting common terms and simplifying, we obtain:

$$
\begin{equation*}
Y^{\text {crit }}=\frac{B}{r} \underbrace{\left.\frac{(1+\hat{f})(t-r)(\hat{f}+\hat{\beta})-\alpha \hat{f}(1-\widehat{\beta})}{(1+\hat{f})(t-r)(\hat{f}+\hat{\beta})-\alpha \hat{f}(1-\widehat{\beta})+\alpha \hat{f}(t-r)(1-\hat{\beta})}\right]}_{:=\frac{1}{M}}=\frac{B}{r M} \leq \frac{B}{r} \tag{A.15}
\end{equation*}
$$

Note that $\mathrm{M}>(=) 1$ holds for $\alpha>0(=0)$. Therefore, the required change in B , resulting from an increase in $r$, holding expected utility EU constant (cf. eq (3.3) of the main text), equals:

$$
\begin{equation*}
\frac{\mathrm{dB}}{\mathrm{dr}}{ }_{\mathrm{dEU}=0}=\mathrm{YM}>\mathrm{Y} \text { if } \alpha>0 \tag{A.16}
\end{equation*}
$$

From the definition of a balanced budget (cf. eq. (3.4)) we obtain upon substitution for $\mathrm{V}^{*}$ :

$$
\begin{equation*}
B=S-(p-(1-p) \hat{f}) \frac{Y(\hat{\beta}-(1-(t-r)(1+\hat{f})))+(1-\widehat{\beta}) B}{\hat{f}+\hat{\beta}}-(1-p) Y(t-r)(1+\hat{f}) \tag{A.17}
\end{equation*}
$$

Solving for B and taking the derivative with respect to r , we obtain:

$$
\begin{equation*}
\frac{\mathrm{dB}}{\mathrm{dr}}_{\mid \mathrm{dS}=0}=\mathrm{Y} \tag{A.18}
\end{equation*}
$$

To obtain an approximation of the magnitude of M and, therefore, of the difference $\frac{\mathrm{dB}}{\mathrm{dr}}{ }_{\mid \mathrm{dEU}=0}-\left.\frac{\mathrm{dB}}{\mathrm{dr}}\right|_{\mathrm{dS}=0}$ in equation (3.6), we note that M is increasing in $\alpha$. Accordingly, the
maximum difference will result for $\alpha=1$. We, furthermore, presume $\mathrm{p}=0.9, \mathrm{f}=1, \mathrm{t}=0.4$, $\mathrm{r}=0.05$ and $\delta=2$. Therefore, $\tilde{\mathrm{f}}=\frac{\mathrm{f}}{(\mathrm{t}-\mathrm{r})^{\alpha}}=1 / 0.35 \approx 2.86$, and $\hat{\beta}=\left(\frac{\mathrm{p}}{(1-\mathrm{p}) \hat{\mathrm{f}}}\right)^{-1 / \delta} \approx$ $(9 / 2.86)^{-0.5} \approx 0.5634$. These values for the parameters ensure an interior solution.

This implies:

$$
\begin{align*}
M & =\frac{(1+\hat{f})(t-r)(\hat{f}+\hat{\beta})-\alpha \hat{f}(1-\hat{\beta})+\alpha \hat{f}(t-r)(1-\hat{\beta})}{(1+\hat{f})(t-r)(\hat{f}+\hat{\beta})-\alpha \hat{f}(1-\hat{\beta})} \\
& =1+\frac{\hat{f}(t-r)(1-\hat{\beta})}{(1+\hat{f})(t-r)(\hat{f}+\hat{\beta})-\hat{f}(1-\hat{\beta})} \approx 1+\frac{0.437}{4.625-1.2486} \approx 1.1294 \tag{A.19}
\end{align*}
$$

Given the above parameter values, the willingness to pay for a tax buyout which entails a reduction in the marginal tax rate from $40 \%$ to $35 \%$, in order to hold constant expected utility, exceeds the required increase in the fixed payment B , in order to balance the budget, by about $13 \%$ if the fine is a function of the amount of undeclared income $(\alpha=1)$.

## Incomes which define the various groups of individuals (Section 4)

In Section 4, we assume $\alpha=0$, so that $\mathrm{f}=\hat{\mathrm{f}}$ and $\beta=\left(\frac{\mathrm{p}}{(1-\mathrm{p}) \mathrm{f}}\right)^{-1 / \delta}$. The maximum willingness to pay for a tax buyout in order to avoid detection is defined by $\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})$ $\mathrm{V}^{*}\left(\mathrm{Y}_{2} \mid \mathrm{a}=0\right)$. In order to calculate this difference, we assume the same parameter values as above and solve for the income level $\mathrm{Y}_{2}$ which makes individuals indifferent between using a buyout and refraining to do so. We first calculate:

$$
\begin{equation*}
\operatorname{EU}\left(\mathrm{Y}, \mathrm{~V}^{*}(\mathrm{Y}, \mid \mathrm{a}=0)\right)=\frac{\mathrm{p}}{1-\delta}\left[\mathrm{Y}-\mathrm{V}^{*}\right]^{1-\delta}+\frac{1-\mathrm{p}}{1-\delta}\left[\mathrm{Y}(1-\mathrm{t}(1+\mathrm{f}))+\mathrm{V}^{*} \mathrm{f}\right]^{1-\delta} \tag{A.20}
\end{equation*}
$$

Setting $\delta=2$, substituting for $\mathrm{V}^{*}$ in accordance with (A.14), and simplifying, we obtain:

$$
\begin{equation*}
\operatorname{EU}\left(\mathrm{Y}, \mathrm{~V}^{*}(\mathrm{Y}, \mid \mathrm{a}=0)\right)=-\frac{\mathrm{f}+\beta}{\mathrm{Y}(1-\mathrm{t})(1+\mathrm{f})}\left[\mathrm{p}+\frac{1-\mathrm{p}}{\beta}\right] \tag{A.21}
\end{equation*}
$$

The expected utility of making a quasi-voluntary payment $V=Y_{1}(t-r)$, in order to use the tax buyout $\mathrm{B}=\mathrm{Y}_{1} \mathrm{r}$, without being recognised evading taxes immediately, is given by $\mathrm{EU}(\mathrm{Y}$; $\left.\mathrm{V}=\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r}) ; \mathrm{a}=1 ; \mathrm{B}=\mathrm{rY} \mathrm{Y}_{1}\right)$. Using the functional form (A.13) in the definition of $E U$, we obtain:

$$
\begin{align*}
& E U\left(Y ; V=Y_{1}(t-r) ; a=1 ; B=Y_{1} r\right) \\
& =\frac{p\left[Y-Y_{1}(t-r)-r Y_{1}\right]^{1-\delta}}{1-\delta}+\frac{1-p}{1-\delta}\left[Y(1-(t-r)(1+f))+Y_{1}(t-r) f-Y_{1} r\right]^{1-\delta} \tag{A.22}
\end{align*}
$$

For $\delta=2$, we have:

$$
\begin{align*}
\operatorname{EU}(Y ; V & \left.=Y_{1}(t-r): a=1 ; B=Y_{1} r\right) \\
& =-\frac{p}{Y-Y_{1} t}-\frac{1-p}{Y(1-(t-r)(1+f))+Y_{1} t f-Y_{1} r(1+f)} \tag{A.23}
\end{align*}
$$

Setting (A.21) and (A.23) equal in order to solve for the income $Y_{2}=\mu Y_{1}$ for $\alpha=r=B=0$, yields:

$$
\begin{equation*}
\frac{\mathrm{f}+\beta}{\mu(1+\mathrm{f})(1-\mathrm{t})}\left(\mathrm{p}+\frac{1-\mathrm{p}}{\beta}\right)=\frac{\mathrm{p}}{\mu-\mathrm{t}}+\frac{1-\mathrm{p}}{\mu(1-(\mathrm{t}-\mathrm{r})(1+\mathrm{f}))+\mathrm{tf}-\mathrm{r}(1+\mathrm{f})} \tag{A.24}
\end{equation*}
$$

For $f=1, t=0.4, p=0.9, r=0.05$ and, hence, $\beta=0.333$, the value of $\mu$ which solves (A.24) is approximately $\mu=2.16$. Therefore, if the tax buyout defines a level of income $\mathrm{Y}_{1}$, individuals at an income level $\mathrm{Y}_{2}=\mu \mathrm{Y}_{1} \approx 2.16 \mathrm{Y}_{1}$ will be indifferent between making a quasi-voluntary payment $\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})=0.35 \mathrm{Y}_{1}$ and using the buyout, instead of making an optimal unconstrained payment $\mathrm{V}^{*}\left(\mathrm{Y}_{2}, \mathrm{r}, \mathrm{B}=0\right)=0.1 \mu \mathrm{Y}_{1}=0.216 \mathrm{Y}_{1}$ and refraining from using the buyout.

Note, finally, that the income level $\mathrm{Y}_{3}$ is defined by $\mathrm{V}^{*}\left(\xi \mathrm{Y}_{1}, \mathrm{a}=1\right)=\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r})$ for $\mathrm{B}=\mathrm{Y}_{1} \mathrm{r}=$ $0.1 \mathrm{Y}_{1}$ and $\alpha=0$ (cf. (A.14)) and, hence, given by:

$$
\begin{equation*}
\frac{\xi Y_{1}(\beta-(1-(\mathrm{t}-\mathrm{r})(1+\mathrm{f})))+(1-\beta) \xi r \mathrm{Y}_{1}}{\mathrm{f}+\beta}=\mathrm{Y}_{1}(\mathrm{t}-\mathrm{r}) \tag{A.25}
\end{equation*}
$$

Solving (A.25) for $\mathrm{f}=1, \mathrm{t}=0.4, \mathrm{r}=0.05$ and $\beta=0.333$, we obtain $\xi \approx 4.66$. Hence, a buyout which defines an income level $\mathrm{Y}_{1}$, will not affect evasion choices by individuals whose gross income exceeds the level $4.66 \mathrm{Y}_{1}$, given the above parameter values.

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[^0]:    ${ }^{1}$ See, for example, Bigio and Zilberman (2011), Carillo et al. (2011), and Tonin (2013). Models with a continuum of tax payers and cut-off rules for auditing are usually characterised by bunching (at the cut-off), but not necessarily by the absence of a whole interval of income declarations (see Franzoni 2009 for a survey). Similar features, that is, bunching and the absence of certain income declarations, will result if there is no evasion while taxation entails fixed administrative cost, such that it may be optimal to effectively exempt those with low (potential) tax bases from taxation (cf. Keen and Mintz 2004 and Dharmapala et al. 2011).
    ${ }^{2}$ Falkinger and Walther (1991), furthermore, analyse the impact of giving tax payers the option to reduce the marginal tax rate in exchange for a higher penalty and note some similarities to Chu's (1990) proposal.
    ${ }^{3}$ Alternatively, the initial tax system could be progressive and the official tax burden given by $(\mathrm{Y}-\mathrm{s}) \mathrm{t}$, where s , $\mathrm{Y}>\mathrm{s}>0$, represents the level of tax exemption. The findings derived below for a setting with $\mathrm{s}=0$ carry over to

[^1]:    a model in which $\mathrm{s}>0$ holds. This is the case because the level of taxable income, $\mathrm{Y}-\mathrm{s}$, does not qualitatively affect the merits of tax buyouts. A derivation of the respective results is available upon request.

[^2]:    ${ }^{4}$ Setting $t=r$ and $B>0$ is equivalent to the offer of lump-sum income taxation. A tax payer who accepts such a buyout can no longer evade taxes because $\mathrm{s} / \mathrm{he}$ has committed to paying the entire tax obligation B . Consequently, we focus on the case of $\mathrm{r}<\mathrm{t}$.

[^3]:    ${ }^{5}$ This assertion relies on the assumption that higher expected tax revenues make the government better off and that higher expected utility of individuals does not reduce the government's payoff. Consequently, we need no further restrictions on the specification of the government objective in order to establish the possibility that a tax buyout can be a Pareto improvement. I am grateful to an anonymous referee for the suggestion to make explicit this assumption which underlies the above argument with respect to a Pareto improvement.

[^4]:    ${ }^{6}$ For similar results, see Chu (1990) and Sleet (2010), who discusses the contribution by Del Negro et al. (2010). In Alesina and Weil (1992), there is an intermediate range of productivities for which the response to a tax buyout offer is ambiguous because marginal utilities from consumption and leisure change.

[^5]:    ${ }^{7}$ Appendix 7.6 contains a numerical example which allows us to calculate the willingness to pay for a tax buyout and to show for $\alpha=1$ that this willingness exceeds the balanced-budget increase in the fixed payment B.

[^6]:    ${ }^{8}$ The findings of this paper will basically be valid also if individuals do not differ in income but in another characteristic, such as the attitude towards risk, as long as their response to a tax buyout offer reveals information about the true tax base. To illustrate, suppose that people differ in the degree of risk aversion, so that the extent of evasion, given identical gross income, is monotonically related to risk attitudes. Since, moreover, the willingness to accept a tax buyout offer will be related to the degree of risk aversion, accepting or declining a buyout offer will be informative about evasion behaviour.
    ${ }^{9}$ We know that a reduction in tax rate from t to $\mathrm{t}-\mathrm{r}$ unambiguously reduces optimal tax payments for $\alpha=0$, since $\partial \mathrm{V}^{*} / \partial \mathrm{r}<0$ (cf. equations (2.4) and (2.6)). The change in $\mathrm{V}^{*}$ due to a rise in B is zero for a constant level of absolute risk aversion (cf. (2.5)). Therefore, the proposed tax reform can actually reduce optimal payments.

[^7]:    ${ }^{10}$ Appendix 7.6 provides a numerical example which enables us to compute explicitly the income levels $\mathrm{Y}_{2}$ and $\mathrm{Y}_{3}$ and the optimal unconstrained tax payment $\mathrm{V}^{*}$, relative to $\mathrm{Y}_{1}$.

[^8]:    ${ }^{11}$ This 'missing middle' of tax declarations is a further feature of a tax buyout, in addition to those mentioned in the Introduction, which distinguishes a buyout from the mechanism proposed by Chu (1990). If high-income individuals do not evade taxes, as in Chu (1990), there is no need to avoid certain income declarations in order to escape being detected evading taxes.

[^9]:    ${ }^{12}$ A complete and meaningful description of the tax buyout which maximises T, i. e. an explicit derivation of $\mathrm{B}^{*}$ and $r^{*}$, would require a more elaborate specification, for example, of payoffs and the distribution function G. Note, however, that the first term in the second line of (4.5) is deducted while the remaining terms in the second

[^10]:    line are positive for $\mathrm{Y}_{2}<\mathrm{Y}_{3}$. Accordingly, if tax authorities can set B and r and thereby $\mathrm{Y}_{1}$ and r , they are able to maximise expected tax revenues T .

[^11]:    ${ }^{13}$ Reinganum and Wilde (1985), for example, postulate an audit rule according to which an audit is certain if the income declaration falls below a predetermined level and is zero otherwise. I am extremely grateful to an anonymous referee for bringing the issue analysed in this section to my attention.
    ${ }^{14}$ Note that the same number of individuals will evade taxes in the presence and absence of a tax buyout, so that we can ignore auditing resources in the definition of the government's budgetary constraint.

[^12]:    ${ }^{15}$ Note that responses to tax rate changes in settings in which individuals can adjust labour supply and tax evasion choices at the intensive margin are generally ambiguous, unless the utility function is strongly separable in income and leisure (cf. Pencavel 1979 or Slemrod and Yitzhaki 2002).

