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Abstract

We explore the benefits of intergenerational risk-sharing through both private funded pensions and via the public debt. We use a multi-period overlapping generations model with a PAYG pension pillar, a funded pension pillar and a government. Shocks are smoothed via the public debt and variations in the indexation of pension entitlements and the pension contribution rate, which both respond to funding ratio of the pension fund. The intensity of these adjustments increases when the funding ratio or the public debt ratio get closer to their boundaries. The best-performing pension arrangement is a hybrid funded scheme in which both contributions and entitlement indexation are deployed as stabilisation instruments. We find trade-offs between the optimal use of these instruments. We also find that entitlement indexation and the response of the tax rate to public debt movements are complements. We compare different taxation regimes and conclude that a regime in which pension benefits are taxed, while contributions are paid before taxes, is preferred to a regime in which contributions are paid after taxes, while benefits are untaxed.

JEL-Code: G230, H550, H630.

Keywords: intergenerational risk-sharing, pension funds, public debt, EET and TEE tax regimes, welfare.

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1 Introduction

Pension arrangements have moved to the top of the policymaking agenda over the past decade. Particular attention is given to the question how arrangements can be adapted to deal with the ongoing ageing of the population and the costs associated with it. Therefore, many countries have started to shift away from unfunded to funded pension arrangements. The latter often take the form of a Defined Contribution (DC) scheme. Moreover, other countries that already have a substantial funded pension pillar are now shifting away from Defined Benefit (DB) towards Defined Contribution. The consequence is that the extent to which individual risks can be shared is becoming more limited. In fact, an individual DC scheme in its purest form does not admit any risk sharing among its participants. With the decreasing capacity of pension arrangements to share risks among different cohorts the question also arises whether there exist alternative channels through which such risks can be shared.

In this paper, we investigate intergenerational risk sharing via both private funded pension arrangements and via the government debt. An important question is to what extent variations in the public debt can substitute for risk sharing via a DB funded pension arrangement. A priori we might expect that if intergenerational risk-sharing through the pension system is reduced, there is more need for intergenerational risk-sharing through fiscal policy. Unexpected bad shocks can be smoothed by limiting the increase in taxes and allow for a rise in the public debt. This way future generations will be forced to pay part of the bill. Vice versa, if a good shock hits pension fund participants.

We conduct our analysis in the context of an overlapping generations (OLG) model with a pay-as-you-go (PAYG) pension pillar, a funded pension pillar and a government. We assume a fluctuation band on both the funding ratio (the ratio of assets over liabilities) of the pension fund as well as on the public debt, while allowing for three margins of adjustment. These are the pension contribution rate, the indexation of pension entitlements and the adjustment of the taxes. The intensity of the various adjustments is allowed to increase when the funding ratio or the debt ratio get closer to the boundaries of a band we assumed to be imposed upon them.

We obtain a number of results. First, we observe that among the collective schemes, the hybrid scheme, which allows both contributions and indexation of pension entitlements to respond to funding ratio imbalances, performs better than the collective defined contribution (CDC) scheme, which holds contributions constant, and the DB scheme, which holds indexation constant. The advantage of the hybrid scheme is that by having both contributions and indexation respond to funding ratio imbalances, the volatility of consumption during working life and during retirement can be better balanced. Second, there are trade-offs concerning the parameters regulating the pension contribution and the indexation of entitlements. For a CDC scheme, it is optimal to have indexation respond strongly to deviations in the funding ratio from its target, while the opposite is found when the contribution reacts relatively strongly to such deviations. Further, for given indexation parameter, we find an internal optimum for the responsiveness of the contribution to the funding ratio. Finally, we observe that the degree of indexation of entitlements and the responsiveness of the tax rate to deviations of debt from its target are complements. Stronger indexation of pension rights implies larger movements in the retirement benefits, hence larger fluctuations in tax revenues, thereby necessitating stronger adjustments in the tax rate. Third, while the hybrid collective scheme dominates individual DC overall in terms of welfare, the latter scheme is associated with lower consumption volatility and on this account individual DC is the best-performing scheme. Hence, the opportunity to share risks across generations may actually come at the price of higher consumption volatility. Fourth, the degree of riskiness of the pension fund's asset portfolio affects the volatility of tax revenues under the EET scheme, thereby affecting the optimal tax adjustment parameter.

This paper connects to different strands of literature. First, it relates to the literature on intergenerational risk-sharing. There is already quite a substantial amount of work that studies intergenerational risk-sharing within a funded pension scheme. Examples are Teulings & Vries (2006); Gollier (2008) and Cui *et al.* (2011), who show how a well-designed pension fund improves welfare. By exploiting the benefits of intergenerational risk-sharing, more risk can be taken, which results in higher expected returns. As in this paper, these contributions use a multi-period OLG model with consumption equal to net income. By contrast, Draper *et al.* (2011) use a multi-period OLG model in which households can also save outside their pension fund. They show that a DB pension scheme is welfare improving in terms of risk-neutral valuation, but not in terms of market valuation. Therefore, in the absence of mandatory participation, the fund suffers from a commitment problem. In contrast to these contributions, the current paper allows for PAYG social security benefits as an additional source of retirement income, while, more importantly, it allows for the government's budget as an additional channel for intergenerational risk sharing. In particular, by only taxing retirement savings during the pay-out phase, future generations sharing in equity risks through variations in the public debt.

There is a fairly small literature that explores the combination of a PAYG public pensions and funded pensions. Examples are Matsen & Thøgersen (2004); Borsch-Supan *et al.* (2006) and Beetsma & Bovenberg (2009). Matsen & Thøgersen (2004) investigates the optimal split between a PAYG pillar and a DC funded pillar in the context of two-OLG model with wage income, the population size and the equity return as the risk factors. The current paper only considers equity risk. However, in contrast to Matsen & Thøgersen (2004), who evaluates welfare only in the second period of an individual's life, the current paper evaluates welfare of future generations over many periods, including the working years. Borsch-Supan *et al.* (2006) study the effects of ageing and pension reforms on international capital markets using an OLG model with multiple countries. They find that aggregate savings rates go up due to population ageing, which can be amplified by a pension reform in which the PAYG contributions are frozen and its benefits are reduced. Beetsma & Bovenberg (2009) also investigate a two-OLG model with a PAYG and a funded pillar. Human capital is a non-tradable asset. In a pure market economy the young possess too much human capital and too little financial capital. Hence, it is optimal for them to acquire more equity exposure. A pension fund can raise welfare by effectively making human capital tradable, thereby completing the asset markets, and by allowing the incoming young to engage in asset trading with the old generation. Hence, the pension fund yields the benefit of intergenerational risk-sharing. The authors show that in general only a DB fund can achieve optimal risk sharing. By contrast, the current paper shows that pure DB arrangements are unstable and, hence, a hybrid between DB and DC scheme is preferable. Under the hybrid scheme, both the pension contribution and the benefit are employed to absorb financial risk. Further, and importantly, none of the aforementioned articles consider the public debt as a separate channel for intergenerational risk sharing. Hence, the current paper contributes to the literature on multi-pillar pension schemes by allowing for public debt to fulfil this role.

This paper also connects to the literature on the taxation of pensions. Governments can stimulate pension savings through their tax policies. For example, in most OECD member countries (Whitehouse, 1999), we observe that pension savings are tax exempt. This paper considers the role of intergenerational risk-sharing through retirement income and public debt under two tax regimes. These are the "TEE regime", under which pension fund contributions are levied on after-tax income, while the accumulation and pay-out phases are tax exempt, and the "EET regime", under which contributions are levied on before-tax income, the pension wealth accumulation phase is tax exempt, while the benefits themselves are taxed. Most OECD member countries facilitate or even stimulate the accumulation of pension wealth by making pension contributions tax deductible (up to a certain limit) and taxing the pension benefits. Hence, these countries follow at least partly the EET regime. We will see that the potential for intergenerational risk-sharing depends substantially on which taxation regime prevails. Gordon & Varian (1988); Bohn (1999); Shiller (1999); Smetters (2006) and Ball & Mankiw (2007) show that a government holding equity or taxing capital returns can improve welfare. Whitehouse (1999) makes a case for both the TEE and the EET regime, as they tax either when contributions are paid or when benefits are obtained. This way, the consumption - savings decision during the accumulation phase is undistorted. In Huang (2008) no contributions are paid during accumulation and the marginal tax rates during work and retirement are identical, implying that the EET and TEE regimes are equivalent. However, Beetsma *et al.* (2011) highlight circumstances in which the equivalence breaks down. For example, the marginal tax rate during retirement is typically lower than during working life. Hence, pension savings are more attractive under the EET regime. Furthermore, the government also shares in the asset market risk under the EET regime, thereby affecting the risk-taking of the pension fund. Romaniuk (2013) analyses the optimal pension fund portfolio assuming that utility in retirement is maximized. The taxes levied under the TEE regime do not affect this optimization problem, while those under the EET regime do. Again, the equivalence between the two regimes breaks down. In contrast to Romaniuk (2013) we take the composition of the fund's investment portfolio as given, while focussing on the role of the various adjustment channels for intergenerational risk sharing and social welfare. In the context of our framework we show that taxing income after pension contributions have been paid raises social welfare, because the resulting additional investment in pension wealth earns the equity premium, while the reduction in future taxes by paying pension contributions on after-tax income effectively only earns the risk-free rate of return through a reduction in the public debt. This effect outweighs the higher consumption volatility under the EET relative to the TEE regime. Hence, for the various pension regime settings the EET regime welfare dominates the TEE regime.

The remainder of this paper is structured as follows: Section 2 lays out the model, while Section 3 presents the calibration. In Section 4 we discuss our social welfare criterion. The outcomes of the analysis are found in Section 5. Finally, in Section 6 we conclude the main text of this paper. Some technical details are found in the Appendix.

2 The model

The model features overlapping generations with identical agents in each generation. Each period a new generation of unity mass is born. During the first part of their life individuals work, while during the second part they are retired. Retirement benefits are provided by a first pillar that pays a pay-as-you-go (PAYG) social-security benefit and a second pillar formed by a pension fund. Labour supply is exogenous and normalised to unity at the individual level. Hence, the total amount of labour supplied by a working cohort is also unity. The only exogenous risk factor is the return on a risky asset referred to as equity. There are two assets, namely equity and a risk-free asset. Finally, the variables in our model are expressed in real terms.

2.1 Individuals

An individual lives for T^D periods in total. At the start of his life an individual born in period ν features utility

$$U_\nu = \sum_{t=\nu}^{\nu+T^D-1} \delta^{t-\nu} u(c_{t,\nu}), \tag{1}$$

where δ is the discount factor and $c_{t,\nu}$ is consumption. Period utility is given by the constant relative risk aversion (CRRA) function

$$u(c_{t,\nu}) = \frac{c_{t,\nu}^{1-\rho}}{1-\rho}. \tag{2}$$

Before retirement the individual receives each period an exogenous wage income of unity and he pays a social security tax, while after retirement he receives a pay-as-you-go (PAYG) social security benefit. The individual does not save voluntarily. All his savings are channelled to a pension fund. This assumption is not as unrealistic as it may seem, because in countries with large funded pension pillars, like the Netherlands, we tend to see relatively little free savings outside those pillars.

We consider two different regimes for the taxation of the pension income received from the funded pension pillar. Under the first regime pension contributions are paid after taxes have been levied on income and the pension benefits are untaxed. The accumulation of pension wealth is also untaxed. This regime is called the "tax-exempt-exempt" (TEE) regime. Under this regime, the individual's consumption profile is given by:

$$c_{t,\nu} = \begin{cases} 1 - (\lambda + p_t + \tau_{t,\nu}), & t-\nu \in \{0,...,T^R-1\} \quad \text{(working)} \\ \zeta + \pi_{t,\nu}, & t-\nu \in \{T^R,..,T^D-1\} \quad \text{(retired)} \end{cases}, \tag{3}$$

where λ is the social security tax, p_t is the pension contribution, $\tau_{t,\nu}$ a tax payment to the government, T^R is the number of working periods, ζ is the social-security benefit and $\pi_{t,\nu}$ is the pension benefit. Retirement thus takes place in period $\nu + T^R$. Under the other tax regime, the "exempt-exempt-tax" (EET) regime, the pension contribution is subtracted from income before taxes are paid, while the pension benefit is taxed. Again, the accumulation of pension wealth is untaxed. In this case,

$$c_{t,\nu} = \begin{cases} (1-p_t)(1-\tau_{t,\nu}) - \lambda, & t-\nu \in \{0,...,T^R-1\} \quad \text{(working)} \\ \zeta + (1-\tau_{t,\nu})\,\pi_{t,\nu}, & t-\nu \in \{T^R,..,T^D-1\} \quad \text{(retired)} \end{cases}, \tag{4}$$

Regarding the pension fund, we also distinguish two cases. The first is the case of an individual DC (IDC) fund, where the individual pays a fixed contribution ($p_t = \bar{p}$) during the working period and converts his pension assets into an annuity at retirement. The second case is that of a collective pension fund, where pension rights are indexed. The pension benefits under these pension plans are given by

$$\pi_{t,\nu} = \begin{cases} a_{t,\nu} & \text{(individual DC)} \\ (1+I_t)\,b_{t,\nu} & \text{(collective)} \end{cases}, \tag{5}$$

where $a_{t,\nu}$ is the annuity payment and $(1+I_t)b_{t,\nu}$ is the indexed pension benefit, where I_t is the rate of indexation, which is defined below.

2.2 Retirement arrangements

This subsection discusses the details of the retirement arrangements.

2.2.1 The first pillar

Because of the PAYG character of the first pillar, each period total contributions by the working cohorts equal total benefit payments to the retired:

$$\lambda T^R = \zeta(T^D - T^R). \tag{6}$$

2.2.2 The IDC second pillar

The second pillar consists of a pension fund. First, we consider the IDC arrangement and denote by $W_{t,\nu}$ the pension asset holdings at the beginning of period t in the IDC scheme. Individuals start with zero initial asset holdings, i.e. $W_{\nu,\nu} = 0$. Each period of their working life they add their pension contribution to these asset holdings, which are invested in risk-free debt and risky equity. Hence, the total asset holdings of the individual evolve as,

$$W_{t+1,\nu} = W_{t,\nu}(1 + r_t^w) + \bar{p}, \qquad t - \nu \in \{0, ..., T^R - 1\} \quad \text{(working)}, \tag{7}$$

where $\bar{p}$ is the contribution paid (at the end of the period) and r_t^w denotes the return on the asset portfolio, which is given by:

$$r_t^w = (1 - \omega^p)\, r^f + \omega^p r_t^e, \tag{8}$$

where ω^p is the fraction of the pension fund's assets invested in equity, r^f is the constant return on the risk-free debt and r_t^e is the return on equity. Each period in retirement, i.e. from the beginning of period $t - \nu = T^R$ and on, the individual converts his pension assets into an annuity:

$$\begin{aligned} W_{t,\nu} &= a_{t,\nu} \textstyle\sum_{j=0}^{\nu+T^D-t} \frac{1}{(1+E[r_t^w])^j} \\ \Rightarrow a_{t,\nu} &= W_{t,\nu} / \textstyle\sum_{j=0}^{\nu+T^D-t} \frac{1}{(1+E[r_t^w])^j} \end{aligned} \quad , \qquad t - \nu \in \{T^R, ..., T^D - 1\} \quad \text{(retired)}. \tag{9}$$

This is a variable annuity of the type considered in Feldstein & Ranguelova (1998, 2001) and Beetsma & Bucciol (2013). It differs from an annuity that pays out the same amount each period. The advantage of the variable annuity is that it allows the individual to take advantage of the equity premium.

2.2.3 The collective second pillar

Let us now turn to the collective pension fund. The advantage of the collective fund is that risks can be shared over many cohorts of participants. Through their contributions into the system, individuals accrue pension rights, $b_{t,\nu}$. At the start of the working life, accrued pension entitlements are zero, $b_{\nu,\nu} = 0$. Pension entitlements evolve as follows:

$$b_{t+1,\nu} = \begin{cases} (1 + I_t) b_{t,\nu} + \psi, & t - \nu \in \{1, ..., T^R\} \quad \text{(working)} \\ (1 + I_t) b_{t,\nu}, & t - \nu \in \{T^R + 1, ..., T^D\} \quad \text{(retired)} \end{cases} , \tag{10}$$

where I_t is the indexation rate and ψ is the accrual rate. The accrual is received at the end of period t so that it is not heightened up by the indexation in period t. Notice that all participants in the pension arrangement receive the same indexation.

The pension fund's assets A_{t+1} evolve as:

$$A_{t+1} = (1 + r_t^p) A_t + T^R p_t - (1 + I_t) \sum_{\nu=t-T^D+1}^{t-T^R} b_{t,\nu}, \tag{11}$$

where r_t^p is the return on the pension fund's asset portfolio. Hence, the new level of pension fund assets is equal to the old level multiplied by the gross portfolio return plus total contribution payments, minus total benefit payments. We assume some given starting level A_0 for the pension fund's assets. For convenience, we can set $A_0 = \bar{A}$, the target level of assets to be discussed below. Note that, while contributions are identical for all cohorts in a given period, this is not necessarily the case for the benefits. To facilitate the comparison with the case of the IDC system, we assume that the composition of the fund's portfolio is the same as that of the IDC portfolio. Hence, the return on the pension fund's portfolio is:

$$r_t^p = (1 - \omega^p) r^f + \omega^p r_t^e, \tag{12}$$

where ω^p is the fraction of the pension fund's assets invested in equity.

We evaluate the pension fund's liabilities according to the so-called "Accumulated Benefit Obligation" (ABO), which is the discounted sum of all future pension benefits, where its calculation is done under the assumption that the benefit level throughout the retirement period is equal to the *current* level of accrued entitlements. More specifically, this calculation ignores the further accrual of entitlements by current and future workers through future contributions and the future indexation of entitlements for any current and future participating cohorts. The question is what is the appropriate rate at which those benefits should be discounted? If they are risk-free, they should be discounted at the risk-free rate of interest. However, the indexation rate of the pension rights is stochastic, which makes the cash flows stochastic as well. Risk aversion would justify a marginally higher discount rate. Nevertheless, real-world pension arrangements, like the Dutch second pillar use the market risk-free rate to calculate pension liabilities. We use the risk-free rate to discount future pension benefits. Therefore, liabilities L_t are given by

$$L_t = \sum_{\nu=t-T^D+1}^{t-T^R} b_{t,\nu} \sum_{i=t-\nu}^{T^D-1} \left(\frac{1}{1+r^f} \right)^{i-(t-\nu)} + \sum_{\nu=t-T^R+1}^{t} b_{t,\nu} \sum_{i=T^R}^{T^D-1} \left(\frac{1}{1+r^f} \right)^{i-(t-\nu)}. \tag{13}$$

Hence, liabilities consist of a component based on the current and future benefit payments to the retired, the first term on the right-hand side of (13), and a component based on future benefit payments to current workers, the second term on the right-hand side. The first component takes the sum of a retired cohort's benefits discounted to time t and then it sums over all retired cohorts. The second component takes the sum of all benefit payments as of retirement discounted back to time t.

Rewriting (13) gives the recursive representation

$$L_{t+1} = \left(1 + r^f\right)(1 + I_t) L_t - \left(1 + r^f\right)(1 + I_t) \sum_{\nu=t-T^D+1}^{t-T^R} b_{t,\nu} + \psi \sum_{\nu=0}^{T^R-1} \sum_{i=0}^{T^D-T^R-1} \left(\frac{1}{1+r^f}\right)^{i+\nu} \tag{14}$$

The current liabilities consist of the present value of the previous liabilities corrected for indexation, minus the present value of the pension payouts in the previous period corrected for indexation (the second term on the right-hand side), plus the present value of newly accumulated pension entitlements through the accrual obtained by all working cohorts (the final term).

An important input for policy decisions is the so-called "funding ratio", defined as:

$$F_t = A_t / L_t.$$

The funding ratio is subject to a lower bound F^l and an upper bound F^u. In reality, boundaries on the funding ratio are frequently observed. In the context of the current model, we conjecture that in the absence of such boundaries it would be optimal to not have the fund's steering instruments react at all to the funding ratio. This way shocks are spread out over as many generations as possible. However, the funding ratio could then reach very low or very high values that are clearly unrealistic. When it is substantially below one, young cohorts could refuse to continue participating in the pension arrangement, because the contributions they have to make to restore the fund's financial position would far outweigh the benefit they perceive to obtain when they are themselves old (e.g, see Beetsma *et al.* (2012); Chen & Beetsma (2013)). By contrast, when the funding ratio is substantially above unity, old generations could put pressure on the fund's board to dismantle the fund and distribute its assets over the participants (possibly in proportion to the contributions that the various participating cohorts have made in the past), see, for example, Penalva & Bommel (2011) and Beetsma & Romp (2013). Alternatively, the government might want to tax some of the fund's reserves away.

The pension fund aims at achieving a target $\bar{F}$ for the funding ratio, with $\bar{F} = \frac{1}{2}(F^l + F^u)$, the average of the upper and lower bounds on the funding ratio. These bounds define a proportionality parameter $q_F = 1 - F^l/\bar{F}$ indicating the range over which the funding rate can fluctuate. Based on the funding ratio F_t, the pension fund applies its steering instruments, namely the pension contribution and the rate of indexation of the pension entitlements. In response to a deviation of the funding ratio from its target level $\bar{F}$, the pension contribution will be adjusted as follows

$$p_t = \left[1 + g_\alpha\left(F_t/\bar{F}\right)\right]\bar{p}, \quad g'_\alpha(.) \leq 0, \quad g_\alpha(1) = 0, \quad g'_\alpha(1) = -\alpha,$$

where $\bar{p}$ is a target level for the pension contribution (to be discussed below). For g_α we use the so-called tangent hyperbolic adjustment specification with $\alpha \geq 0$,

$$g_{\alpha}\left(F_t/\bar{F}\right) = -\alpha q_F \tanh^{-1}\left(\frac{F_t^* - \bar{F}}{q_F \bar{F}}\right),$$

where F_t^* is defined as follows

$$F_t^* = \begin{cases} \bar{F}\left(1 - 0.9q_F\right) & \text{for } F_t < \bar{F}\left(1 - 0.9q_F\right) \\ F_t & \text{for } F_t \in \left[\bar{F}\left(1 - 0.9q_F\right), \bar{F}\left(1 + 0.9q_F\right)\right] \\ \bar{F}\left(1 + 0.9q_F\right) & \text{for } F_t > \bar{F}\left(1 + 0.9q_F\right) \end{cases}.$$

Hence, if the funding ratio falls below its target ($F_t < \bar{F}$), then the pension contribution is raised, and vice versa.[1] To prevent the adjustment in the contribution rate reaching extreme values, the adjustment is kept constant as a function of F_t when F_t gets close to its boundaries, i.e. when $F_t < \bar{F}\left(1 - 0.9q_F\right)$ or $F_t > \bar{F}\left(1 + 0.9q_F\right)$. The reason is that the ensuing discrete-time simulation of the model could lead to values of the funding ratio so close to its boundaries that the adjustment in the contribution rate reaches totally unrealistic levels and produces very sharp movements of the funding ratio in the direction of the opposite boundary. If it were possible to simulate in continuous time this problem would be avoided, because the funding ratio would likely have been pushed back towards its long-run equilibrium value before it could get close to its boundaries. Moreover, the adjustment of the contribution rate would only be short-lived if the funding ratio reaches extreme values. Hence, the current specification ensures smooth adjustment policies for a model that is simulated only at discrete time intervals.

Likewise, the rate of indexation of accumulated rights is made a function of the actual funding ratio relative to its target level:

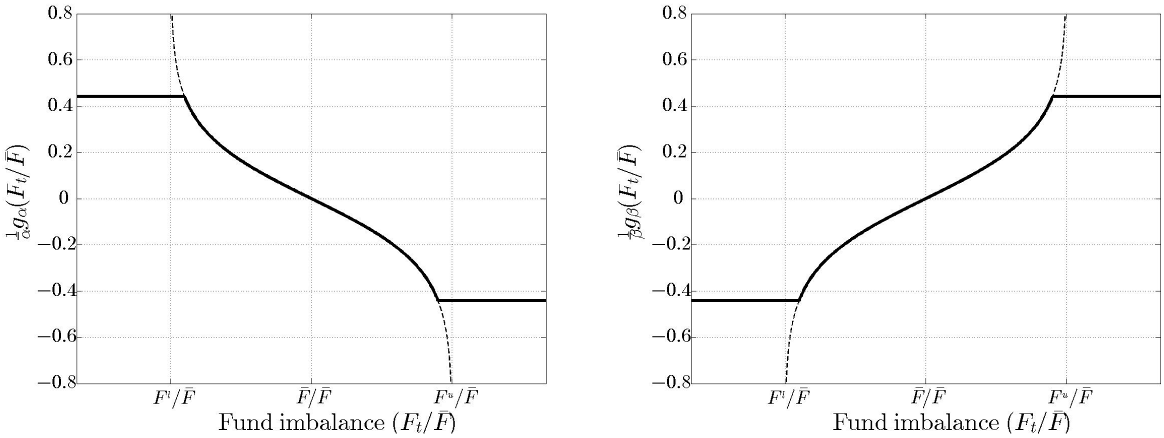
$$I_t = g_{\beta}\left(F_t/\bar{F}\right), \quad g_{\beta}'(.) \geq 0, \quad g_{\beta}(1) = 0, \quad g_{\beta}'(1) = \beta,$$

where for g_{β} we also use the tangent hyperbolic adjustment function with $\beta \geq 0$, now specified as:

$$g_{\beta}\left(F_t/\bar{F}\right) = \beta q_F \tanh^{-1}\left(\frac{F_t^* - \bar{F}}{q_F \bar{F}}\right)$$

In Figure 1, we graphically illustrate the policies of the pension fund as a function of the funding ratio. In the left panel we observe that when the funding ratio is below its target, the pension contribution is raised, while in the right panel we observe that the indexation of pension rights increases if the funding ratio improves. The further the funding ratio moves away from its target, the stronger the policy response. The vertical lines $F_t/\bar{F} = F^l/\bar{F}$

[1] The simplest possible adjustment policy would have been one that is linear in $F_t/\bar{F}$. However, such a specification leads to an unstable dynamic system of assets and liabilities. The Appendix contains a proof of this feature.



(a) adjustment policy pension contribution (b) indexation policy pension rights

Figure 1: Tangent hyperbolic policies of the pension fund.

and $F_t/\bar{F} = F^u/\bar{F}$ are the asymptotes of the tangent hyperbolic functions.

2.2.4 Consistency among the targets

To avoid a situation in which pension rights need to be systematically revised into one direction, the target levels for the pension contribution, the pension benefit and the funding ratio need to be consistent among themselves. Concretely, in the absence of shocks, and starting from a situation in which all variables are at their target levels, they should remain at their target levels in the next period. For convenience, we refer to this situation as the "steady state". Based on the zero indexation when the funding ratio is at its target, we have for the pension accrual:

$$\psi = \bar{b}/T^R,$$

hence,

$$b_{t,\nu} = \begin{cases} (t-\nu)\,\bar{b}/T^R, & t-\nu \in \{1,...,T^R\} \quad \text{(working)} \\ \bar{b}, & t-\nu \in \{T^R+1,...,T^D\} \quad \text{(retired)} \end{cases}. \tag{15}$$

The target benefit level $\bar{b}$ is a choice variable that determines the scale of the funded pension pillar.[2] We can substitute these expressions of $\bar{b}$ for $b_{t,\nu}$ into equation (13). This yields a "target level" for the liabilities $\bar{L}$. Given the target for the funding ratio, we obtain the target asset level as $\bar{A} = \bar{F}\bar{L}$. Then, using (11), we obtain $\bar{p}$ as:

$$\bar{A} = \bar{A}\,(1+\bar{r}^p) + T^R\bar{p} - \left(T^D - T^R\right)\bar{b},$$

where $\bar{r}^p$ is the mean of the net return on the pension portfolio. Hence,

$$\bar{p} = \frac{\left(T^D - T^R\right)\bar{b} - \bar{A}\bar{r}^p}{T^R}.$$

[2]Alternatively, one can fix the target contribution level $\bar{p}$ to set the size of the funded pension pillar.

We see that the target contribution is increasing in the length of the retirement period $T^D - T^R$ and the target benefit, but decreasing in the initial pension assets and the mean net return on the portfolio. Moreover, assuming that the numerator of this expression is positive, i.e. that not all the benefit payments can be financed out of the net return on the pension assets, the target contribution is decreasing in the length of the contribution period.[3]

2.3 The government

The government faces an exogenous and constant amount of primary spending $\bar{G} \geq 0$. Further, it starts off with a given initial debt level D_0. It places its debt on the international capital market. Assuming that it pays off the debt with certainty, it pays the risk-free interest rate on its debt. The dynamics of the debt D depend on the taxation regime. Under the TEE regime they evolve as:

$$D_{t+1} = D_t \left(1 + r^f\right) + \bar{G} - \sum_{\nu=t}^{t-(T^R-1)} \tau_{t,\nu}. \tag{16}$$

Hence, debt at the start of the next period is current debt multiplied by its gross return, plus primary government spending, minus total tax revenues, which is the number of contributing cohorts times the size of a cohort (unity) times the individual tax payment. The debt dynamics are slightly more complicated under the EET regime and evolve as:

$$D_{t+1} = D_t \left(1 + r^f\right) + \bar{G} - (1 - p_t) \sum_{\nu=t}^{t-(T^R-1)} \tau_{t,\nu} - \sum_{\nu=t-T^D+1}^{t-T^R} \tau_{t,\nu}\pi_{t,\nu}, \tag{17}$$

where total tax revenues are the result of taxing income after the pension contribution has been paid plus the taxation of the pension benefits received by the retired.

The government tries to limit the movements of the public debt by imposing both an upper bound D^u and a lower bound D^l on the debt. The upper bound resembles the ceiling that the EU Treaty in principle imposes on the public debt. Such a ceiling would prevent the debt from becoming unsustainable. In practice, the main concern is that debt becomes too high, while there seems to be little concern about debt becoming too low. However, this may be the consequence of the fact that debt levels have mostly been substantial in recent history. Yet, there are also disadvantages to low or negative debt. For example, financial markets would find it difficult to determine equilibrium interest rates if there is very little debt to be traded, while if debt even becomes negative, hence the government becomes a net creditor, the question is in which assets the government should invest. Moreover, being

[3] An alternative way of defining the target contribution is the "actuarially fair" contribution rate by taking the present value of the pension rights – see, for example, Cui *et al.* (2011). However, this contribution rate is higher, such that the pension fund creates a buffer when the funding ratio is close to its steady-state level. This is at the cost of the current working cohorts, while young and future generations benefit from these buffers.

a large creditor, the government may be held hostage in its policies by its debtors.[4] In line with these arguments, we assume that besides an upper bound there is also a lower bound on the public debt. The government aims at achieving a target level $\bar{D}$ on its debt, with $\bar{D} = \frac{1}{2}(D^l + D^u)$.

Bearing these considerations in mind, the tax rate is determined by:

$$\tau_{t,\nu} = \bar{\tau}\left[1 + g_\gamma\left(D_t/\bar{D}\right)\right], \quad g'_\gamma(.) \geq 0, \quad g_\gamma(1) = 0, \quad g'_\gamma(1) = \gamma, \tag{18}$$

where $\gamma \geq 0$ and $\bar{\tau}$ is the target tax rate given by

$$\bar{\tau} = \begin{cases} \left(r^f \bar{D} + \bar{G}\right)/T^R, & \text{if TEE} \\ \left(r^f \bar{D} + \bar{G}\right)/\left(T^R(1-\bar{p}) + \left(T^D - T^R\right)\bar{\pi}\right), & \text{if EET} \end{cases}.$$

The target tax rates differ between the TEE and EET regimes, as the government's tax revenues are different under the two regimes. Under the EET regime, the total tax revenues are the sum of the revenues of taxing wage income after the pension contributions have been paid, i.e. $T^R(1-\bar{p})$, and the revenues of taxing retirement income, i.e. $\left(T^D - T^R\right)\bar{\pi}$, while total tax revenues under the TEE regime are obtained by taxing the gross wages of all working generations T^R. Similar to the case of the pension fund, we focus on the tangent hyperbolic specification for debt stabilization,

$$g_\gamma\left(D_t/\bar{D}\right) = \gamma q_D \tanh^{-1}\left(\frac{D^*_t - \bar{D}}{q_D \bar{D}}\right),$$

with $q_D = 1 - D^l/\bar{D}$, a measure of how much government debt is allowed to fluctuate. This implies that the tax rate equals its target level if debt also equals its target level. Furthermore, to prevent the tax rate from achieving extreme values, we apply a cut-off to D^*_t when debt gets close to its boundaries. Hence, D^*_t is given by

$$D^*_t = \begin{cases} \bar{D}\left(1 - 0.9q_D\right) & \text{for } D_t < \bar{D}\left(1 - 0.9q_D\right) \\ D_t & \text{for } D_t \in \left[\bar{D}\left(1 - 0.9q_D\right), D\left(1 + 0.9q_D\right)\right] \\ \bar{D}\left(1 + 0.9q_D\right) & \text{for } D_t > \bar{D}\left(1 + 0.9q_D\right) \end{cases}.$$

Figure 2 graphically illustrates that when debt moves away from its target, the deviation of the tax rate from its target becomes larger. The vertical lines $D_t/\bar{D} = D^l/\bar{D}$ and $D_t/\bar{D} = D^u/\bar{D}$ are the asymptotes of the tangent hyperbolic function.

3 Calibration

We calibrate the individual life cycle as follows. Each generation starts working at the age of 25, retires at the age of 65 (hence, $T^R = 40$) and dies at the age of 85 (hence, $T^D = 60$).

[4] In a way, this is the case for China, which holds such substantial amounts of U.S. public debt, that, in order to avoid capital losses, it is forced to follow policies that do not unduly undermine the confidence in the financial strength of the U.S. government.

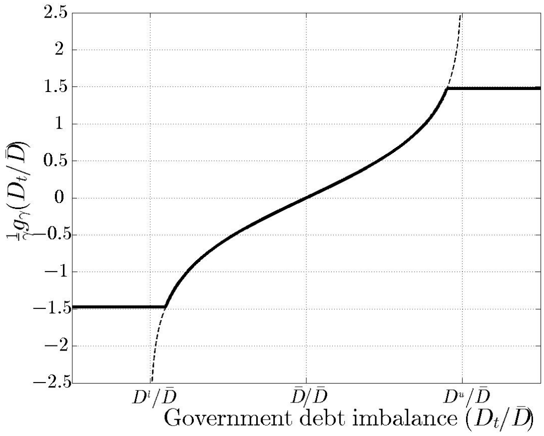


Figure 2: Tangent hyperbolic debt stabilization policies.

Hence, we follow each generation over a period of 60 years. We set the annual return on the risk-free asset at $r^f = 2\%$ and assume that the gross return on equity $1 + r_t^e$ is log-normally distributed with equity premium parameter $\mu^e = 3\%$ and volatility parameter $\sigma^e = 15\%$.[5] Hence, the expected gross return on equity is $\exp(r^f + \mu^e + \frac{1}{2}(\sigma^e)^2)$.

In view of the absence of capital as a production factor, we calculate GDP as aggregate labour income. Therefore, given that each cohort is of size unity and that labour income is unity, the GDP level is $1 * T^R$. The target debt level is set at $\bar{D} = 30\%$ of GDP and $q_D = 1$. Therefore, the lower and upper boundaries D^l and D^u on the debt correspond to 0% and 60% of GDP, respectively. Further, government spending is set constant at $\bar{G} = 33\frac{1}{3}\%$ of GDP. The social security benefit is set constant at $\zeta = 20\%$, which implies a social security tax of $\lambda = 10\%$, because the length of retirement is half the length of working life and no one dies prematurely.

Further, the target funding ratio is $\bar{F} = 100\%$ and $q_F = 0.3$, implying funding ratios between 70% and 130%. We calibrate the fraction of the pension fund's assets invested in equity as $\omega^p = 0.50$. The pension contribution and the accrual rate are calibrated such that the consumption levels are constant over life in the absence of shocks. With $\delta\left(1 + r^f\right) = 1$ this is the optimal time profile for consumption in the absence of shocks. Hence, under the TEE regime we calibrate such that $1 - (\lambda + \bar{p} + \bar{\tau}) = \zeta + \bar{\pi}$ and under the EET regime $(1 - \bar{p})(1 - \bar{\tau}) - \lambda = \zeta + (1 - \bar{\tau})\bar{\pi}$.

Table 1 summarises the calibration of the parameters. Further, with the above inputs we can calculate steady states for the various regimes. For the individual EET regime we have the steady-state variable annuity level $\bar{a} = 0.5104$ and $\bar{p} = 0.0693$ and $\bar{\tau} = 0.2861$. Hence, steady-state consumption is $(1 - \bar{p})(1 - \bar{\tau}) - \lambda = \zeta + (1 - \bar{\tau})\bar{a} = 0.5644$. For the individual TEE regime we find that $\bar{a} = 0.3175$, $\bar{p} = 0.0431$ and $\bar{\tau} = 0.3393$, hence steady-state consumption is $1 - (\lambda + \bar{p} + \bar{\tau}) = \zeta + \bar{a} = 0.5175$. For the collective EET regime we find that $\psi = 0.0141$, $\bar{A} = \bar{L} = 247.21$, $\bar{b} = 0.5640$, $\bar{p} = 0.0250$ and $\bar{\tau} = 0.2700$. Hence,

[5]Dimson *et al.* (2011) suggest an expected annual equity premium in the range of $3 - 3.5\%$. In line with this, we assume an equity premium in our model of $\mu^e = 3\%$.

Table 1: Calibration of parameters

Description	Symbol	Calibration
Return on risk-free asset	r^f	0.02
Equity premium parameter	μ^e	0.03
Equity volatility parameter	σ^e	0.15
Fraction invested in equity	ω^p	0.50
Subjective discount factor	δ	$\frac{1}{1+r^f}$
Relative risk aversion	ρ	5
Age of death	T^D	60
Retirement age	T^R	40
Target funding ratio	$\bar{F}$	1
Target debt	$\bar{D}$	30% of GDP
Range of funding ratio boundaries	q_F	0.3
Range of debt boundaries	q_D	1
Social security benefit	ζ	0.20
Government spending	$\bar{G}$	$33\frac{1}{3}$% of GDP
Accrual rate under collective EET	ψ	0.0141
Accrual rate under collective TEE	ψ	0.0086
Contribution rate under collective EET	$\bar{p}$	0.0250
Contribution rate under collective TEE	$\bar{p}$	0.0153
Contribution rate under EET individual DC	$\bar{p}$	0.0693
Contribution rate under TEE individual DC	$\bar{p}$	0.0431

steady-state consumption is $(1-\bar{p})(1-\bar{\tau})-\lambda=\zeta+(1-\bar{\tau})\,\bar{b}=0.6118$. Finally, for the collective TEE regime, we obtain $\psi=0.0086$, $\bar{A}=\bar{L}=151.36$, $\bar{b}=0.3453$, $\bar{p}=0.0153$ and $\bar{\tau}=0.3393$. Hence, steady-state consumption is $1-(\lambda+\bar{p}+\bar{\tau})=\zeta+\bar{b}=0.5453$.

4 Social welfare evaluation

We simulate $N=10,000$ paths for the equity returns and we assume that the economy is in its steady state at time $t=0$. There is hardly any adjustment close to time $t=0$. Hence, we evaluate the results after a "burn-in" period of 100 years, i.e. from time $t=100$ onward.[6] We evaluate the risk-sharing arrangements provided by the government and the pension fund in terms of social welfare. Social welfare evaluated at time $t=100$, SW, is the sum of the expected discounted utilities of future generations $\nu\geq 100$,

$$SW=\mathbb{E}_{t=100}\left(\sum_{\nu=100}^{\infty}\delta^{\nu-100}U_\nu\right). \tag{19}$$

The discounted utility of the generation born in period ν converges to zero as ν goes to infinity. Therefore, we simulate paths up to time $t=1000$ - utility flows obtained after that period will be negligible in their contribution to social welfare. To ease the welfare comparison of different arrangements, we calculate the certainty-equivalent consumption level

[6]Since we take a burn-in period of 100 years, the confidence intervals of all variables have converged, which takes about 30 to 40 periods.

CEC_{SW}, which is the constant consumption level over the lifetime of future generations $\nu \geq 100$ such that the social welfare level SW is achieved. Hence, CEC_{SW} is calculated from

$$SW = \sum_{\nu=100}^{\infty} \sum_{t=0}^{T^D-1} \delta^{t+\nu-100} u(CEC_{SW}). \tag{20}$$

The Appendix shows that

$$CEC_{SW} = \left(\frac{SW(1-\rho)(1-\delta)^2}{(1-\delta^{T^D})} \right)^{\frac{1}{1-\rho}}. \tag{21}$$

To compare the different regimes in more detail, for a given parameter setting $\phi = (\alpha, \beta, \gamma, q_F, q_D)$ we define a number of Sharpe ratios. Compared to the certainty-equivalent consumption level, they have the advantage of highlighting the effects of changes in expected consumption and the volatility of consumption in an intuitive measure if one changes the system parameter values. First, we define the following Sharpe ratio for consumption of generation ν in a given year $t \geq \nu$:

$$S\left(c_{t,\nu}\right) = \frac{\frac{1}{N}\sum_{n=1}^{N}\left(c_{t,\nu,n}\right)}{\sqrt{\frac{1}{N}\sum_{n=1}^{N}\left(c_{t,\nu,n} - \frac{1}{N}\sum_{n=1}^{N}\left(c_{t,\nu,n}\right)\right)^2}},$$

where the subscript n refers to the specific simulation run. Next, we define the Average Lifetime Sharpe Ratio (ALSR) for consumption as

$$ALSR\left(\phi\right) = \frac{1}{M+1-100} \sum_{\nu=100}^{M} \left(\frac{1}{T^D} \sum_{t=\nu}^{\nu+T^D-1} S\left(c_{t,\nu}\right) \right),$$

where we compute first the average Sharpe ratio of generation ν over it lifetime, which is the part between the brackets, after which we take the average over the generations $\nu = 100$ to $\nu = M \geq 100$. To deal with cases in which consumption is certain during working life, we also define the Average Retirement Sharpe Ratio (ARSR) for consumption as

$$ARSR\left(\phi\right) = \frac{1}{M+1-100} \sum_{\nu=100}^{M} \left(\frac{1}{T^D - T^R} \sum_{t=\nu+T^R}^{\nu+T^D-1} S\left(c_{t,\nu}\right) \right)$$

where the relevant average for a specific generation ν is not computed over its full lifetime, but only over its retirement period.

5 Outcomes

This section discusses the outcomes of our simulations. First, we discuss the results for the individual pension scheme. Second, we investigate some specific collective pension arrangements. Third, we consider general collective pension arrangements and explore the effects of varying the hyperbolic adjustment policies. Fourth, we derive socially-optimal combinations of risk-sharing parameters under different pension and taxation regimes.

5.1 IDC

As we abstract from wage uncertainty, there are no fluctuations in tax revenues under the TEE regime, implying constant government debt and thus constant tax rates. However, under the EET regime the government debt fluctuates, because the volatile annuity payments are also taxed. The intensity of the tax adjustment is determined by the parameter γ. For high values of γ, even a small deviation of the government debt from its target value results in substantial tax adjustments. For low values of γ, most of the adjustment takes place when the government debt is close to its boundaries. This also leads to a situation with highly volatile tax rates, because adjustments can be substantial when the debt boundaries are approached. For values of γ lower than 0.25, the intensity of the tax adjustment is so low that government debt attains unrealistic values. Hence, we only consider values $\gamma \geq 0.25$. Then, an interior optimum obtained at $\gamma = 0.73$. For the remainder of this paper, we take this as our benchmark value of the tax adjustment policy parameter.

Figure 3 shows the mean simulation paths and the 90% confidence intervals around those means for the IDC pension arrangement under both taxation regimes. The welfare levels in terms of certainty equivalent consumption are 0.5457 and 0.5049 under the EET and TEE regimes, respectively. The advantage of the EET regime is that individuals can gain from additional investment returns due to the tax savings during working life. The disadvantage of the EET regime is that, in contrast to the TEE regime, the tax rate fluctuates, leading to higher consumption volatility under EET than under TEE. However, the former effect dominates the latter, which explains that the certainty-equivalent consumption level is higher under EET than under TEE.

5.2 Collective pension arrangements

Recall that we have calibrated the accrual rates such that the steady state consumption level is constant over life. Table 2 summarizes the steady state values of our variables that were computed above.

5.2.1 Varying the boundaries on government debt and the funding ratio

The boundaries on the government debt and the funding ratio are important determinants for the adjustment policies, as strong adjustments are required close to the boundaries. First, we consider the EET regime, which is the relevant tax regime for most of the OECD

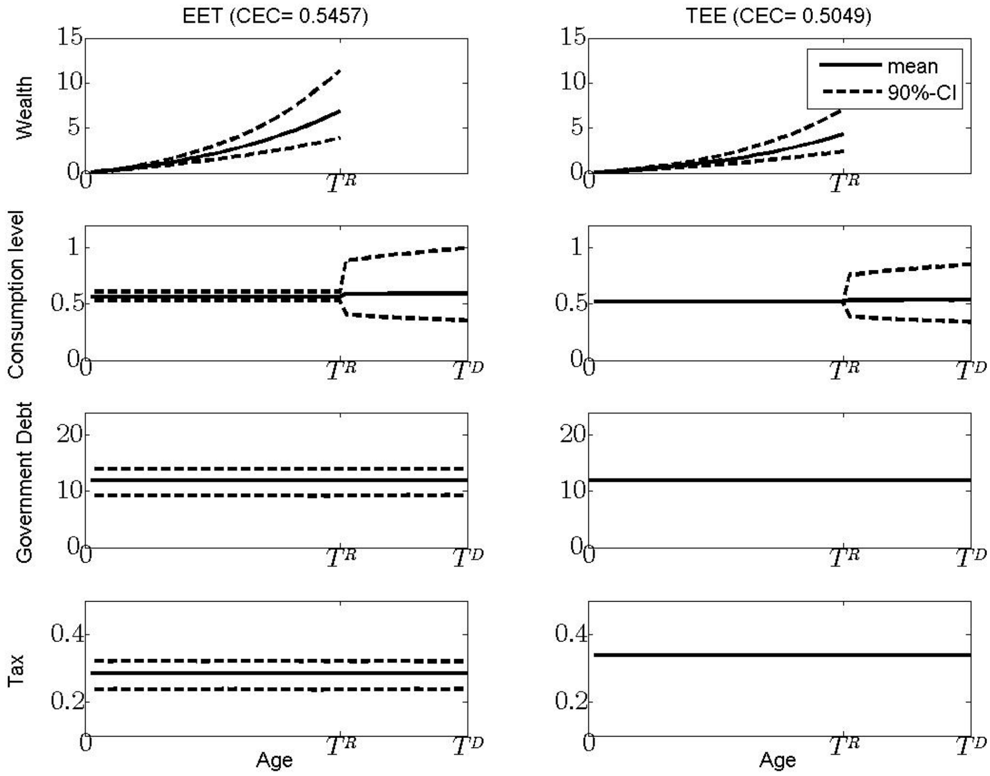


Figure 3: IDC pension scheme (EET: $\gamma = 0.73$)

Table 2: Steady-state values collective model

Description	Symbol	EET	TEE
Pension contribution	$\bar{p}$	0.0250	0.0153
Pension benefit	$\bar{b}$	0.5640	0.3453
Stabilization tax	$\bar{\tau}$	0.2700	0.3393
Consumption level	$\bar{c}$	0.6118	0.5453
Government debt	$\bar{D}$	12	12
Funding ratio	$\bar{F}$	1	1
Liabilities	$\bar{L}$	247.21	151.36
Assets	$\bar{A}$	247.21	151.36

member countries. The investment portfolio of the pension fund is the source of risk, which is eventually shared between the current and future participants though a variety of channels. Current participants absorb part of the risk through the fluctuations in indexation and/or their pension contribution, which also causes tax revenues to fluctuate. Part of the risk is absorbed in future periods and by future generations by letting the funding ratio and the public debt (under the EET regime) vary between their boundaries.

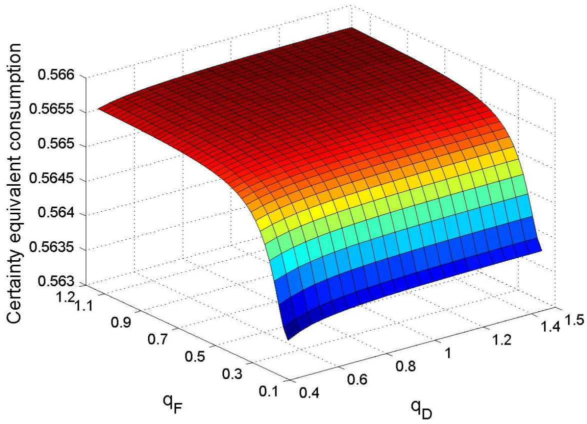
Figure 4(a) shows the welfare effects of varying the widths of the bands on the government debt and the funding ratio under the EET regime. Here, we assume $\alpha = 30$, $\beta = 0.5$ and $\gamma = 0.73$. However, we investigate the effects of other parameter settings in detail below. Welfare, as measured in terms of certainty equivalent consumption, rises if the bands on the funding ratio and the debt level become wider. A wider band on the funding ratio means that the indexation rate can be kept more stable, allowing for a more stable retirement income. Similarly, a wider band on debt means that the tax rate can be kept more stable, implying a more stable after-tax retirement income. In effect, wider bands on the funding ratio and the public debt level allow for more intergenerational risk-sharing. However, the welfare effects of widening these fluctuation margins are rather small. For $q_F = 20\%$ and $q_D = 50\%$, certainty equivalent consumption is 0.5641, while for $q_F = 120\%$ and $q_D = 150\%$, a substantial widening of both fluctuation margins, certainty-equivalent consumption rises to 0.5660.

While most OECD member countries tax retirement benefits according to the EET principle, some countries use a TEE regime.[7] In our TEE model, only investment risk affects the funding ratio and the consumption patterns of the participants. The government does not absorb any of the uncertainty and, therefore, the debt is stable. Hence, in this case the only relevant adjustment margins concern the contribution rate and the indexation rate. Panel (b) of Figure 4 shows that a widening of the band on the funding ratio raises social welfare under the TEE regime, because the scope for intergenerational risk-sharing is enhanced. Quantitatively the effect is again rather small. Raising q_F from 20% to 120% lifts certainty-equivalent consumption by about 0.0016.

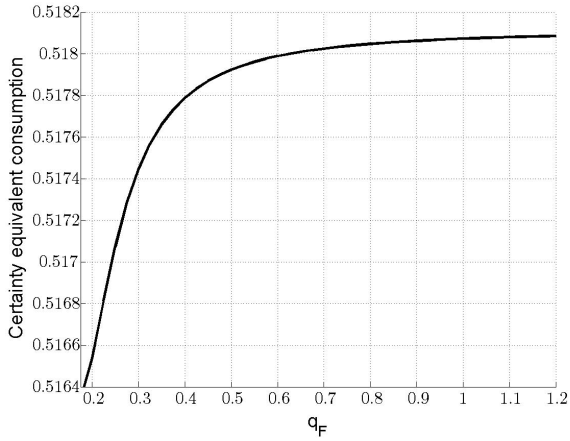
Panel (c) of Figure 4 shows that wider boundaries correspond to higher Sharpe ratios, in line with the welfare effects shown in panels (a) and (b). A question is what causes a widening of the bands on the funding ratio and the public debt to have only rather small welfare effects. Figure 5 shows the frequencies of the funding ratio and the government debt at time $t = 100$ based on $N = 10,000$ simulation runs. Clearly, a widening of the boundaries both on the funding ratio and on the public debt has only marginal effects on the frequency distributions, in line with the small welfare effects. The skewness in the frequency distributions is the result of the assumption of a log-normal distribution for the equity returns, which is positively skewed.

For most of the remainder of this section we assume that the boundaries on the funding ratio and government debt are given. Specifically, we set these boundaries again at the

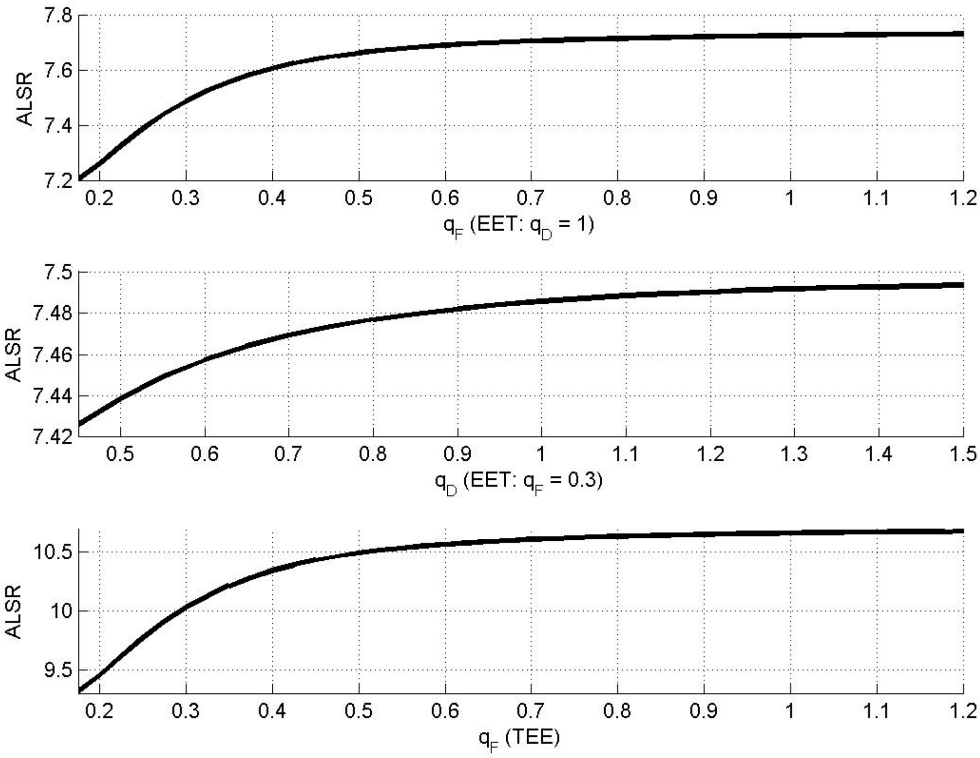
[7] Luxembourg, Hungary and Poland have a TEE regime for pension taxation. Germany used to have a TEE regime as well, but changed it to EET quite recently (Schonewille, 2007).



(a) EET regime($\alpha = 30, \beta = 0.5, \gamma = 0.73$)



(b) TEE regime($\alpha = 30, \beta = 0.5$)



(c) Average Lifetime Sharpe ratios of consumption($\alpha = 30, \beta = 0.5, \gamma = 0.73$)

Figure 4: Certainty equivalent consumption (panels (a) and (b)) and average lifetime Sharpe ratio of consumption (panel (c)) corresponding to different boundary settings.

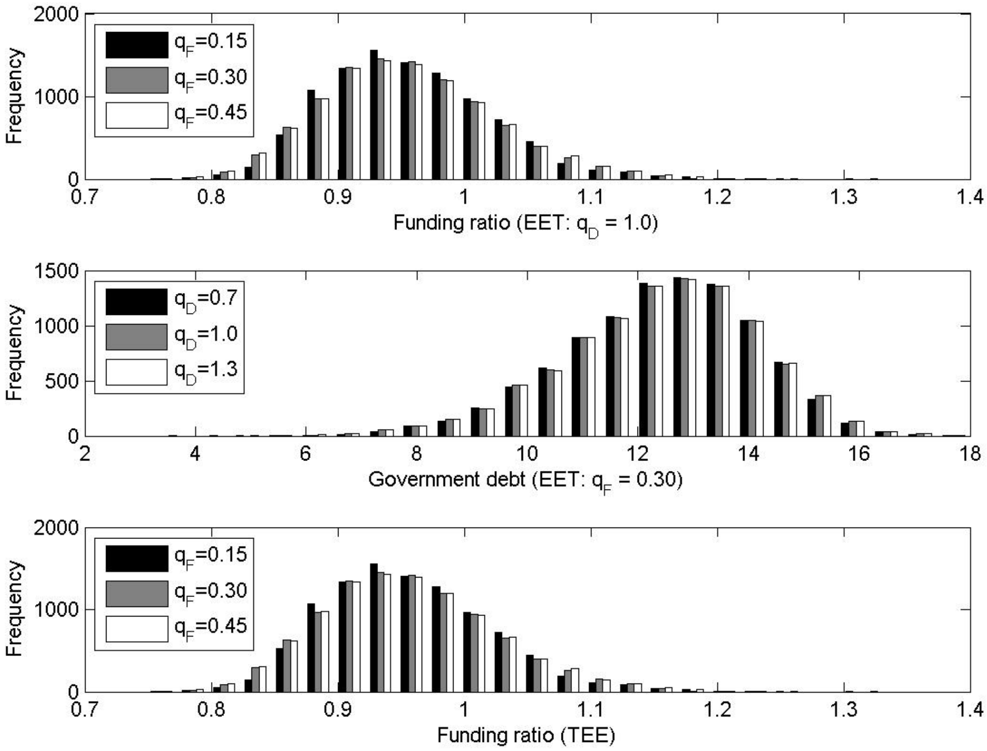


Figure 5: Frequencies (based on 10, 000 simulations) for funding ratio and government debt at time $t = 100$ corresponding to different boundary settings.

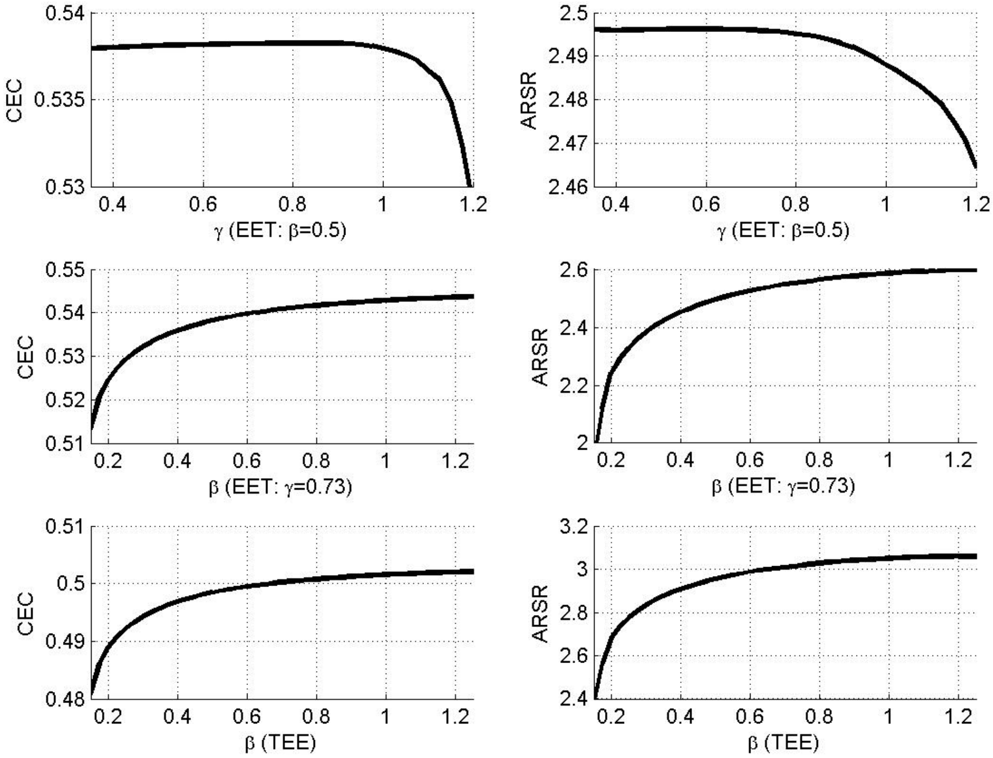


Figure 6: Certainty equivalent consumption (CEC) and Average Retirement Sharpe Ratio (ARSR) corresponding to different adjustment parameters, with $\alpha = 0$ (CDC).

benchmark calibration values corresponding to $q_F = 30\%$ and $q_D = 1$.

5.2.2 The collective defined contribution (CDC) scheme

The first arrangement we consider is a CDC scheme, which features a fixed contribution rate, i.e. $\alpha = 0$, and a variable payout. Imbalances in the funding ratio will be restored through indexation policy only. Under the TEE regime, due to the constant contribution during working life, consumption is also constant during working life and the average lifetime Sharpe ratio (ALSR) of consumption is not defined. Also, under the EET regime, we get very large values for the ALSR because of the low volatility of consumption during working life. Hence, for the CDC scheme we compare only the ARSRs.

In Figure 6, we depict the consequences of changing the indexation parameter β and the tax stabilization parameter γ. The left panels show the welfare levels in terms of CEC for different adjustment parameters γ and β, while the right panels show the corresponding ARSR of consumption. In the two top panels, we vary the tax stabilization parameter γ under the EET regime. We observe that a reduction in γ from a high level results in both higher welfare and a higher ARSR, confirming that less aggressive intervention via taxes when debt is not too close to its boundaries is beneficial, because it allows to better exploit intergenerational risk-sharing by allowing more fluctuation in the government debt.

The middle and bottom panels of Figure 6 show that an increase in the indexation parameter β under both the EET and TEE regime raises welfare as expressed in certainty-equivalent consumption and, in line with this, also results in a higher value of the ARSR. Low values of β imply that changes in indexation tend to kick in only when the funding ratio gets close to its boundaries, resulting in substantial correction, hence in relatively large changes in pension entitlements. Hence, an increase in β essentially smooths the adjustment in indexation as the funding ratio fluctuates and dampens the fluctuations in entitlements.[8]

5.2.3 The defined-benefit (DB) scheme

The second specific regime is the DB scheme, which is obtained by setting $\beta = 0$. In this case, fund imbalances are restored through adjustments in the contribution only, while the retirement benefits are constant, as there is no (risky) indexation. Funding ratios turn out to be unstable for $\alpha < 50$. That is, they explode for some simulation paths, because contribution adjustments are too small. This is no longer the case for $\alpha \geq 50$. However, now, the volatility of the pension contribution becomes large, thereby resulting in volatile consumption paths during working life. At $\alpha = 50$, social welfare is 0.4605 and 0.4777 under the EET and TEE regime, respectively. A higher value of α raises the volatility of the pension contribution. With this high value of α the higher volatility of the pension contribution under the EET regime outweighs the effect of the higher expected consumption under this regime and, hence, social welfare is lower under EET than under TEE. Actually, for sufficiently large values of α in some scenarios contributions need to be so large that consumption becomes negative. These results essentially confirm the view that the contribution level has lost its power as a steering instrument for most pension funds in the Netherlands and in particular for funds suffering from an ageing of their population of participants. For such pension funds, in the absence of other instruments the correction of deviations of the funding ratio from its target would require such large and detrimental (for the economy) changes in the pension contribution that it becomes practically impossible to stabilise the funding ratio.

5.2.4 The hybrid DB-DC scheme

The hybrid DB-DC pension scheme is obtained by setting $\alpha > 0$ and $\beta > 0$. The panels in the top row and the fourth row in Figure 7 show that compared to CDC ($\alpha = 0$) the hybrid regime can improve welfare for both EET and TEE, respectively. Hence, given β, the possibility to vary contributions in response to the funding ratio is beneficial. This

[8] This finding supports the so-called "Adjustment-mechanism Financial Shocks" (AFS), which is planned to be incorporated in the new Dutch pension contract. The mechanism allows for frequent, though relatively small, adjustments of the pension entitlements in response to shocks, instead of infrequent, but abrupt, adjustments. The maximum smoothing period of financial shocks under the AFS is ten years (Bovenberg *et al.*, 2012).

way the burden of adjustment in response to shocks can be better spread over the entire population than when $\alpha = 0$ at the same level of β. Under CDC the adjustment burden falls disproportionately on indexation and therefore on the retired, who hold most of the entitlements. Welfare is increasing in α when α is not too high and decreasing in α when α is relatively large. When α is large the burden of adjustment in response to shocks falls so disproportionately on the working cohorts that a further increase in α leads to a reduction in social welfare. Effectively the use of the fluctuation margin in the funding ratio for intergenerational risk-sharing becomes minimal. The social optimum under both taxation regimes is around $\alpha = 30$ for $\beta = 0.5$ and $\gamma = 0.73$ under the EET regime and also around $\alpha = 30$ for $\beta = 0.5$ under the TEE regime.

The right panels of Figure 7 show both the ALSR (solid line, left vertical axis) and the ARSR (dashed line, right vertical axis) of consumption. Not surprisingly, the ARSR is rising in α. The higher is α, the larger is the adjustment burden on the working generations and, hence, the more stable is consumption for the retired generations. The ALSR, which is based on the Sharpe ratios of both the retired generations and the working generations is decreasing in α. The increased uncertainty in consumption of the working generations as α increases is now also weighed and in fact dominates the reduced instability of the consumption of the elderly in the calculation of the ALSR. The trade-off between consumption volatility during working life and during retirement is optimal when α is around 30, as shown in the corresponding left panels of Figure 7.

In the presence of a substantial adjustment burden via the pension contribution, it is socially optimal to reduce the adjustment burden through indexation, as the second and fifth row in Figure 7 show. For $\alpha = 30$, the optimal value for β is around 0.27 and 0.19 in the cases of the EET regime and the TEE regime, respectively (see the left panels of the figure). Not surprisingly, the ARSR is falling with β, because the additional adjustment burden associated with an increase in β falls relatively heavily on the elderly. Similar as to the case under the CDC regime we observe that reducing γ is welfare enhancing – see the left panel of the third row of Figure 7. Also the Sharpe ratios improve as γ falls.

5.2.5 Hyperbolic adjustment policies

We now take a closer look at the hyperbolic adjustment policies and explore to what extent the various instruments can act as substitutes in producing intergenerational risk sharing. Figure 8 shows certainty equivalent consumption for different combinations of adjustment policies. The EET regime is considered in panels (a)-(d), where the TEE regime, under which the tax rate is stable, is considered in panel (e).

Holding pension contributions constant, Figure 8(a) shows social welfare for different combinations of the indexation and tax adjustment parameters. For low values of β fund imbalances are reduced only slowly and abrupt changes in pension benefit levels in the proximity of the boundaries on the funding rate are relatively frequent. Therefore, it is welfare improving to increase β in order to keep the funding ratios more stable. A reduction

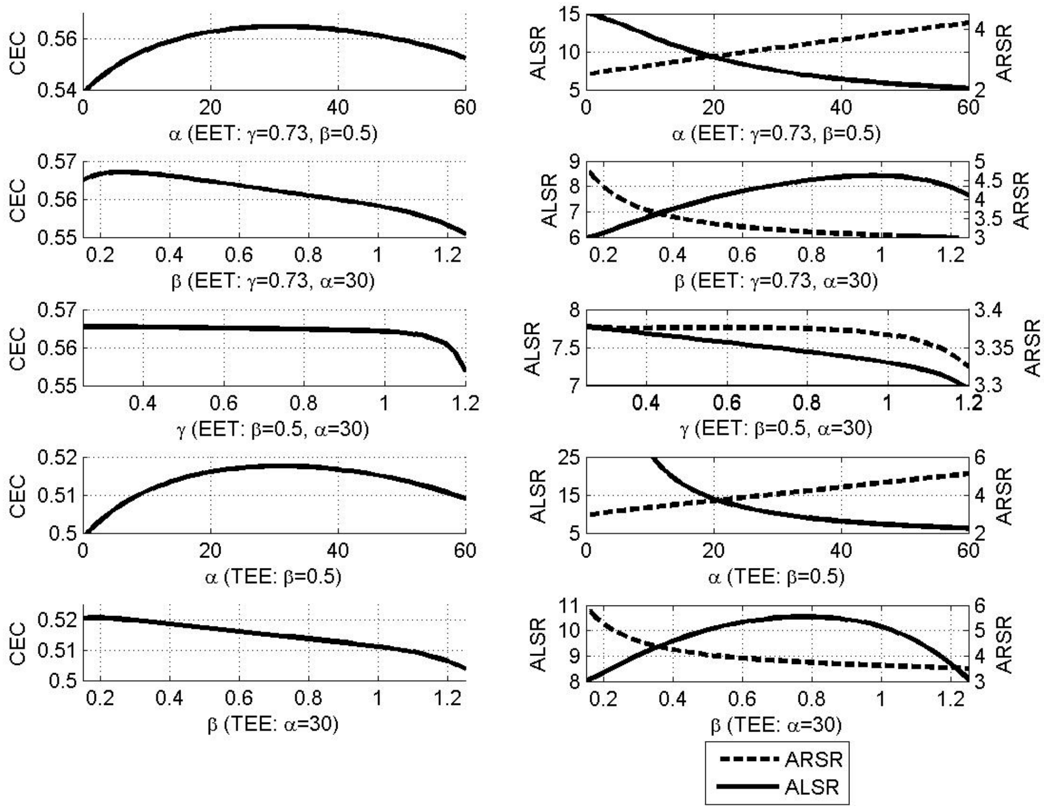
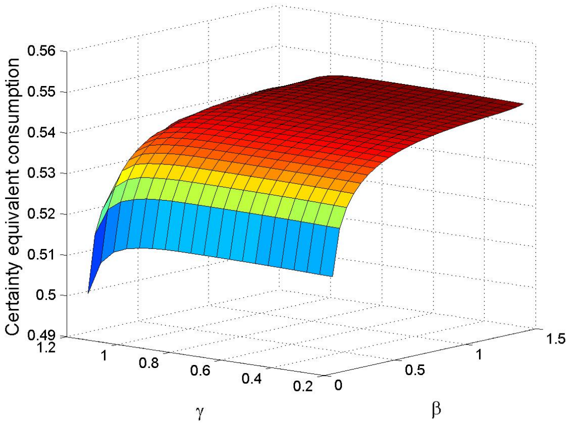
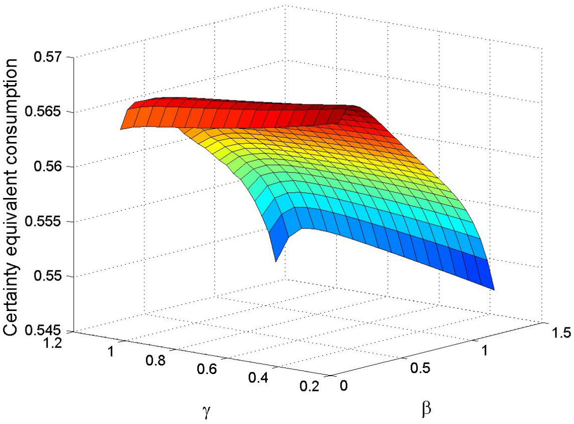


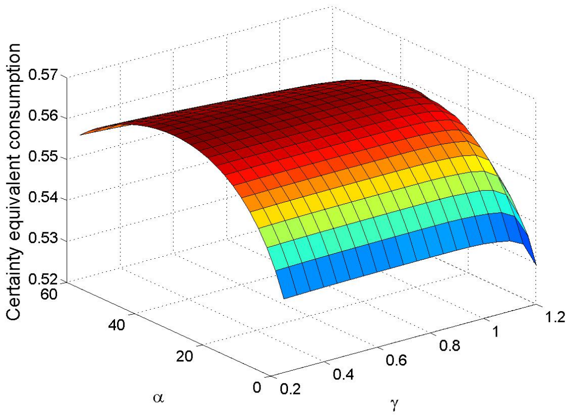
Figure 7: Certainty equivalent consumption (CEC), Average Lifetime Sharpe Ratio (ALSR) (solid line) and Average Retirement Sharpe Ratio (ARSR) (dashed line) of consumption corresponding to different adjustment parameters under the hybrid DB-DC pension arrangement.



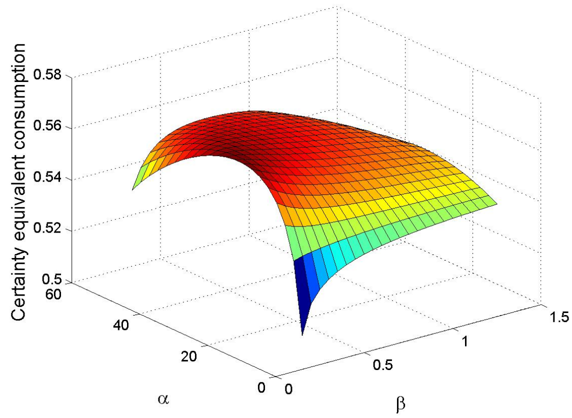
(a) EET regime (CDC): debt stabilization and indexation policy ($\alpha = 0$)



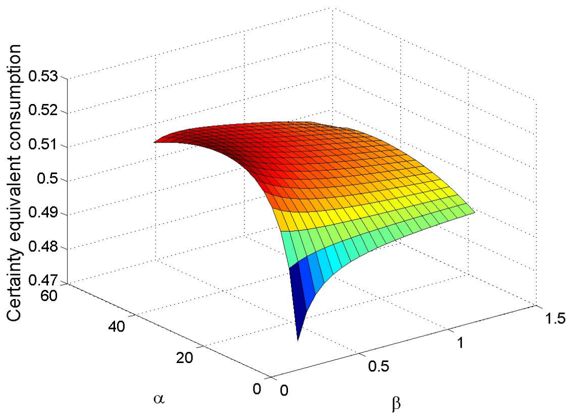
(b) EET regime (hybrid): debt stabilization and indexation policy ($\alpha = 30$)



(c) EET regime: contribution adjustment and debt stabilization ($\beta = 0.5$)



(d) EET regime: contribution adjustment and indexation policy ($\gamma = 0.73$)



(e) TEE regime: contribution adjustment and indexation policy

Figure 8: Certainty equivalent consumption corresponding to adjustment parameters.

in γ implies higher welfare, as a lower γ corresponds to less volatile taxes. For example, for $\beta = 0.15$ and $\gamma = 1.20$, $CEC = 0.4994$, while for $\beta = 1.50$ and $\gamma = 0.25$, $CEC = 0.5447$.

Figure 8(b) shows the welfare effects when we also employ contributions as an adjustment policy. We set $\alpha = 30$. The optimal tax adjustment parameter depends on the indexation policy. As we have seen earlier, large adjustments to indexation lead to volatile retirement income, which in turn causes tax revenues to be more volatile. Effectively, for higher β government debt absorbs more equity risk, which can be stabilized by increasing the intensity of tax rate adjustments. Hence, Figure 8(b) shows that for low β the optimal tax adjustment parameter γ is low, while for high β the optimal value of γ is also high. This indicates that the two risk sharing channels (indexation and debt policy) serve as complements. However, the differences in the welfare level between the scenarios are rather small. Moving from ($\beta = 1.25, \gamma = 1.10$) to $\beta = \gamma = 0.25$ lifts welfare by about 0.02.

We will now explore how social welfare reacts to changing combinations of the contribution and indexation parameters, holding the tax adjustment parameter constant, and to changing combinations of the contribution and tax adjustment parameters, holding the indexation parameter constant – see panels (c), (d) and (e) of Figure 8. We reach a number of interesting findings. First, for both a given indexation parameter and a given tax adjustment parameter, we see that the contribution adjustment parameter achieves an interior optimum - see panel (c) of Figure 8. For very low values of α, much of the necessary adjustment takes place when the funding ratio is close to its boundaries, in which case the adjustment in the contribution is quite large, thereby feeding a lot of fluctuation into disposable income of the workers. Similarly, for high values of α there is relatively strong intervention when the funding ratio deviates only by a small amount from its target level, again implying relatively strong fluctuations in disposable income. The optimal value of α trades off these two effects and is found in between these extremes. Second, as panel (d) of Figure 8 shows, if β increases, more and more of the adjustment takes place through changing the indexation rate, hence the effect of varying α becomes weaker. In other words, these adjustment policies act as substitutes. Furthermore, in line with our earlier discussions of the DB and the hybrid systems, when α is close to zero, an increase in β is welfare enhancing, while the opposite is true for larger values of α. These findings are qualitatively replicated in the case of the TEE regime - see panel (e) of Figure 8.

5.3 Social welfare comparison

In this section we calculate and compare social welfare under the different tax and pension regimes. First, we consider the CDC scheme, characterised by $\alpha = 0$. In Section 5.2.2 we already saw that in this case welfare can be raised by employing indexation policy, i.e. $\beta > 0$. However, when β becomes too large, the adjustment in indexation becomes so large that the funding ratio starts bouncing between the two boundaries from year to year. To avoid this, we restrict indexation policy to $\beta \leq 1.5$. It turns out that under both tax regimes welfare under CDC is maximised at the boundary of this constraint. Second, we

Table 3: Optimal designs and welfare levels under different pension schemes.

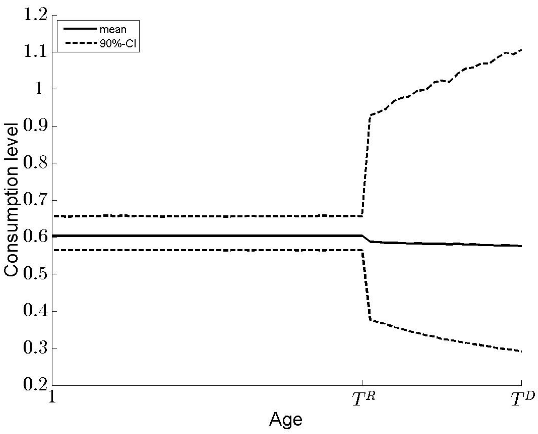
description	formula/symbol	IDC	CDC	DB	hybrid
EET regime ($\gamma = 0.73$)					
contribution adjustment	α	-	(0.0)	50.0	18.6
indexation adjustment	β	-	1.5	(0.0)	0.15
steady state consumption	$\bar{c}$	0.5644	0.6118	0.6118	0.6118
social welfare	CEC	0.5457	0.5448	0.4605	0.5698
welfare over individual scheme (in %)	$\frac{CEC-CEC^{ind}}{CEC^{ind}}$	0.00	-0.16	-15.61	4.42
deviation from steady state (in %)	$\frac{CEC-\bar{c}}{\bar{c}}$	-3.31	-10.94	-24.72	-6.86
corrected welfare level	CEC_{corr}	0.4989	0.4506	0.3663	0.4756
TEE regime					
contribution adjustment	α	-	(0.0)	50.0	20.4
indexation adjustment	β	-	1.5	(0.0)	0.15
steady state consumption	$\bar{c}$	0.5175	0.5453	0.5453	0.5453
social welfare	CEC	0.5049	0.5028	0.4777	0.5226
welfare over individual scheme (in %)	$\frac{CEC-CEC^{ind}}{CEC^{ind}}$	0.00	-0.42	-5.39	3.51
deviation from steady state (in %)	$\frac{CEC-\bar{c}}{\bar{c}}$	-2.44	-7.80	-12.40	-4.16
corrected welfare level	CEC_{corr}	0.5049	0.4750	0.4499	0.4948

consider the DB pension scheme. The pension benefits are fixed by setting $\beta = 0$. We limit ourselves to values of the contribution parameter of $\alpha \geq 50$, because lower values result in unstable funding ratios, as discussed in Section 5.2.3. This constraint on α is binding under the optimal DB pension scheme, i.e. $\alpha = 50$, for both tax regimes.

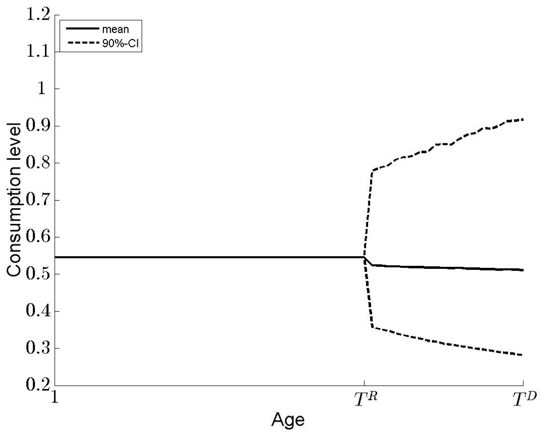
Third, we consider hybrid pension schemes, i.e. α and β are allowed to be positive at the same time. To avoid exploding funding ratios or funding ratios that bounce between their limits from year to year, we limit β to the interval $0.15 \leq \beta \leq 1.50$. The optimal combination of α and β under this restriction yields a value of β at its boundary 0.15, while $\alpha = 18.6$ under the EET regime and $\alpha = 20.4$ under the TEE regime.

So far, we have made welfare comparisons for given regimes when we vary the parameters for the fund contribution, the indexation of pension entitlements and taxation. However, we have not made any comparison across regimes. The socially-optimal parameter settings under the different pension regimes and the corresponding welfare levels in terms of certainty-equivalent consumption are shown in Table 3. The highest welfare level ($CEC = 0.5698$) is obtained under the hybrid collective pension scheme with EET and the lowest welfare level ($CEC = 0.4605$) is obtained under the DB scheme with EET. Furthermore, we observe that for the chosen adjustment parameter values the hybrid collective schemes outperform the individual schemes under both pension taxation regimes. This is in particular the case under the EET regime, for which the hybrid collective scheme produces a welfare gain of 0.0442 over the IDC scheme. However, the latter scheme outperforms the CDC and DB schemes. The DB scheme in particular exhibits low welfare levels. This is due to the large volatility of the pension contributions, which is needed to stabilise the funding ratio as discussed in Section 5.2.3.

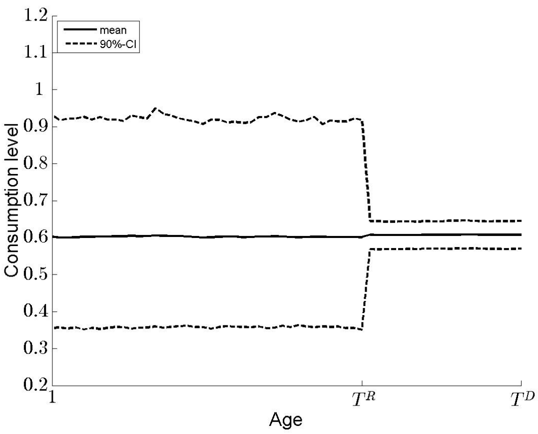
Notice that the comparison across the various regimes is affected by differences in the



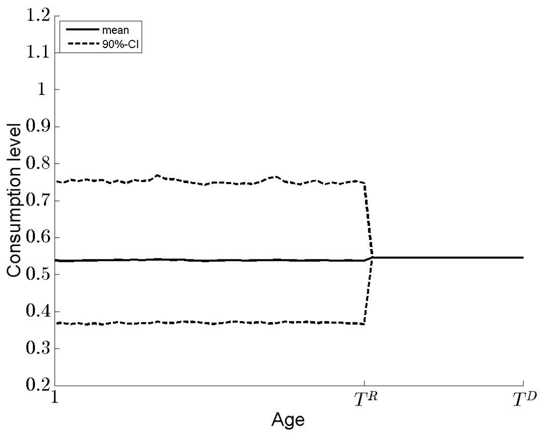
(a) CDC & EET ($\alpha = 0.0, \beta = 1.50, \gamma = 0.73$)



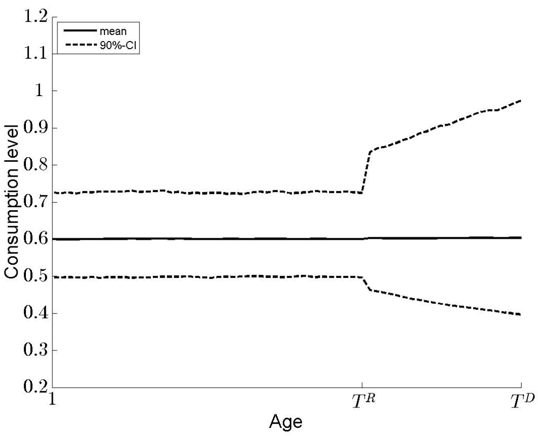
(b) CDC & TEE ($\alpha = 0.0, \beta = 1.50$)



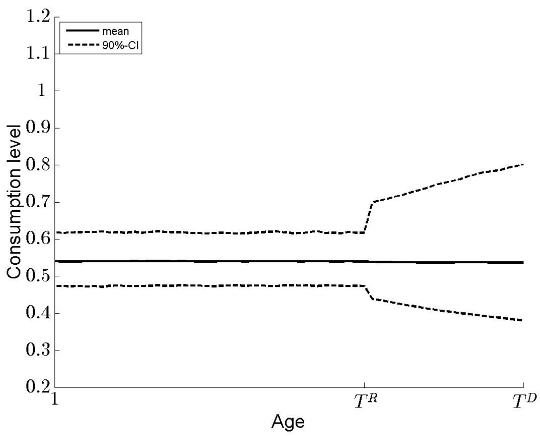
(c) DB & EET ($\alpha = 50.0, \beta = 0.0, \gamma = 0.73$)



(d) DB & TEE ($\alpha = 50.0, \beta = 0.0$)



(e) hybrid & EET ($\alpha = 18.6, \beta = 0.15, \gamma = 0.73$)



(f) hybrid & TEE ($\alpha = 20.4, \beta = 0.15$)

Figure 9: Lifetime consumption generation $\nu = 100$

steady-state consumption levels. Steady-state consumption is higher under the collective schemes than under the individual scheme. The reason is that the collective pension fund already has a substantial amount of assets when a participant enters, while wealth starts at zero under the individual pension scheme. The high level of pension fund assets at the moment of entry into the fund allows for a fast accrual of pension entitlements that exceeds the speed of pension wealth accrual under the individual scheme, especially for those who are still young. In addition, as already explained in Section 5.1, steady-state consumption levels are higher under the EET regime than under the TEE regime, because by postponing taxation individuals can gain from additional investment returns. However, as illustrated by Figure 9 for the collective schemes for the lifetime consumption of generation $\nu = 100$, the 90%-confidence bands on consumption are always wider under EET as well. Nevertheless, at the given degree of risk aversion, the advantage of the higher average consumption level under the EET regime dominates its disadvantage of more volatile retirement income.

To explore how the various regimes fare in terms of the risks that their participants run, we calculate the fractional deviation of certainty-equivalent consumption from steady-state consumption. Based on this criterion, the individual schemes would actually perform best, as certainty-equivalent consumption falls short of steady-state consumption by only 0.0331 and 0.0244 under the EET and TEE regimes, respectively. Interestingly, the percentage deviation from steady state is smaller under the TEE regime than under the EET regime.[9] An alternative way of comparing the performance in terms of participants' risks is the following. The lowest steady-state consumption level, which we refer to as $\bar{c}_{low}$, was obtained for the IDC pension plan under the TEE regime, i.e. $\bar{c}_{low} = 0.5175$. We now define a "corrected welfare measure" CEC_{corr}, which is obtained by subtracting from the original welfare measure a correction term equal to the difference between the steady state consumption $\bar{c}$ of the scheme under consideration and that of the scheme with the lowest steady-state level of consumption, $\bar{c}_{low}$. Concretely, the corrected welfare measure is $CEC_{corr} = CEC - (\bar{c} - \bar{c}_{low})$. Table 3 shows that on the basis of this corrected measure the individual schemes outperform the collective schemes, while the TEE regimes outperform the corresponding EET regimes.

This section suggests some potentially important policy conclusions. Overall, the collective pension scheme outperforms the individual scheme, but only when both the contribution and indexation adjustment policies can be simultaneously employed to share the risks. The same result was obtained by Cui *et al.* (2011). Further, for given adjustment parameters for the contribution and indexation, EET outperforms TEE. However, the relative performances of the various schemes may to a large extent be driven by differences in average consumption under the different schemes.

[9] This is reminiscent of the result of Gollier (2008) that intergenerational risk-sharing is welfare improving, as the expected return increases. However, intergenerational risk-sharing does not necessarily lead to reduced risk.

5.4 Pension fund portfolio risk and risk sharing through the public budget

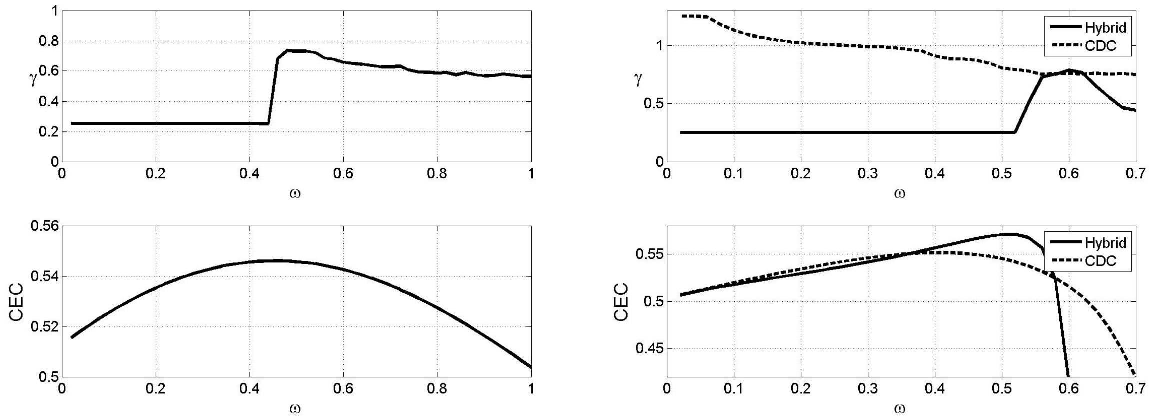
In this subsection we explore the relationship between the riskiness of the portfolio of the pension fund and the optimal choice of the tax smoothing parameter γ. Higher portfolio risk ω^p of the fund raises the volatility of tax revenues under the EET regime, implying larger fluctuations in the public debt. Because tax revenues and debt are constant under the TEE regime, here we confine ourselves to the EET regime. Again the optimal choice of γ involves a trade-off. If γ is low, the adjustment only takes place when the government debt is close to its boundaries, implying relatively infrequent, but rather abrupt adjustments, while if γ is high, small deviations of debt from its target already lead to substantial adjustment. The optimal tax adjustment policy trades off these two effects, resulting in an interior optimum for γ. However, again if γ is close to zero, due to the discreteness of the simulations, for some simulation runs government debt levels go far beyond their boundaries. To exclude such scenarios we impose that $\gamma \geq 0.25$.

5.4.1 IDC

Figure 10(a) shows the interaction of portfolio risk and the tax smoothing policy under the IDC pension scheme. We can distinguish two types of investment portfolios. First, for the relatively high-risk portfolios, $0.44 < \omega^p \leq 1$, the optimal tax stabilization parameter is an interior solution. For those relatively high-risk portfolios, a higher share of equity investment ω^p results in a lower optimal value of γ, because more pension fund risk would call for more risk sharing via the public debt so as to have all generations share in the risk. Second, for relatively low-risk portfolios, $0 \leq \omega^p \leq 0.44$, the need for increasing γ, in order to keep the public debt away from its boundaries, is dominated by the need for smoothing the tax rate. In this case the optimal tax stabilization parameter γ equals its lower bound, $\gamma = 0.25$.

5.4.2 Collective pension arrangements

We now turn to the collective pension arrangements – see Figure 10(b). In line with the analysis in Section 5.3, we set $\alpha = 0$ and $\beta = 1.5$ under the CDC pension scheme and $\alpha = 18.6$ and $\beta = 0.15$ under the hybrid pension scheme. Consider first the CDC scheme. The right top panel of Figure 10(b) shows that more pension fund risk would call for more risk sharing through the public debt, as the optimal γ decreases. Regarding the hybrid pension scheme, we can again distinguish two portfolio risk categories. For the low-risk portfolios, $0 \leq \omega^p \leq 0.52$, the optimal tax stabilization parameter γ equals its lower bound $\gamma = 0.25$, implying that it is welfare improving to provide as much tax smoothing as possible. For the high risk portfolios, $0.52 < \omega^p \leq 0.70$, the optimal tax stabilization parameter is an interior solution, similar to the case of the IDC pension scheme. We do not show the results for $\omega^p > 0.70$, as we obtain unrealistically large funding ratio imbalances.



(a) IDC pension arrangement

(b) Collective pension arrangements

Figure 10: Interaction portfolio risk and optimal tax smoothing.

6 Conclusion

This paper has studied intergenerational risk-sharing when funded pensions and public debt can be simultaneously employed for this purpose. We considered two possible instruments to stabilise pension funding ratios, namely the pension contribution and the indexation of pension rights. In addition we considered two possible tax regimes, EET and TEE. Under the former, pension contributions are paid before taxes and the pension accumulation phase is untaxed, while pension benefits are taxed. Under the TEE regime, the pension contribution is paid after taxes, while the other two phases are untaxed.

We obtained several noteworthy results from our analysis. First, comparing the tax regimes, under EET participants in a pension scheme are effectively able to save a larger proportion of income for their retirement than under TEE and, therefore, benefit more from the positive expected equity premium. However, this also leads to more volatile consumption paths due to tax policy aimed at stabilising public debt. From a welfare perspective, and regardless of whether we consider an individual or collective pension scheme, the former effect dominates the latter, hence EET outperforms TEE. Second, among the collective schemes, we observe that the hybrid scheme performs better than both the DB scheme and the CDC scheme. The advantage of the hybrid scheme is that by having both contributions and indexation respond to funding ratio imbalances, the volatility of consumption during working life and during retirement can be better balanced. Third, our findings suggest the existence of trade-offs concerning the parameters regulating the pension contribution and the indexation of entitlements. For a CDC scheme, it is optimal to have indexation respond strongly to deviations in the funding ratio from its target, while the opposite is found when the contribution reacts relatively strongly to such deviations. Further, for given indexation parameter, we find an internal optimum for the responsiveness of the contribution to the funding ratio. Finally, we observe that the degree of indexation of entitlements and the responsiveness of the tax rate to deviations of debt from its target are complements. A higher indexation of pension rights implies stronger movements in the retirement benefits, hence stronger fluctuations in tax revenues, thereby necessitating stronger adjustments in the tax rate. These findings may give us some useful leads for pension system design. For example, as part of the new pension contract in the Netherlands, it is agreed that the contribution rate will on average across the sectors no longer be increased, implying that this channel for sharing risks would be weakened. Alternatively, if one envisages to strengthen the role of contributions in facilitating intergenerational risk sharing, then this is an argument for raising the retirement age, such that for a given level of pension benefits the contribution can be reduced on average and more room is created for allowing it to fluctuate around this average. Fourth, while the hybrid collective scheme dominates the IDC overall in terms of welfare, the latter scheme is associated with lower consumption volatility and on this account IDC is the best-performing scheme. Hence, the opportunity to share risks across generations may actually come at the price of higher consumption volatility. Fifth, the degree of riskiness of the pension fund's asset portfolio affects the volatility of tax revenues under the EET scheme, thereby affecting the optimal tax adjustment parameter.

This paper can potentially be extended into a number of directions. First, our model could be extended with a third pension pillar based voluntary personal savings. This way, participants can make use of both intergenerational risk-sharing and attain their optimal life cycle investment strategy. However, this extension will make the simulations far more computationally intense and the question is whether it makes the model more realistic, because free savings in countries with substantial funded pension pillars tend to be quite low. Second, the model can be extended by considering flexible investment portfolios. Third, the model could be extended by making the labour supply endogenous. Fourth, future extensions could consider additional sources of risk, such risks in mortality, fertility and wage rates. The question is whether this will have consequences for the policy trade-offs that we have identified. After all, our policy adjustment parameters respond to deviations of the pension funding ratio and public debt from their targets, irrespective of the factors that cause them to move away from their targets. Finally, we have taken the size of the PAYG first pillar as given. It would be particularly interesting to explore the consequences of varying the size of this pillar when introducing wage and demographic risks, because a PAYG arrangement is relatively effective in sharing these risks. One would also expect that, with a larger PAYG pillar, the size of our second-pillar arrangements would need to be reduced to smooth consumption over the life cycle. This, in turn, would mean that the welfare differences among our second-pillar arrangements will become smaller.

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A Derivations

A.1 Asset-liability-dynamics with linear adjustment policies

Here we show that we have an unstable system of the pension fund's asset-liability-dynamics in case we have linear adjustment policies instead of the tangent hyperbolic functions. Suppose the adjustment policies are given by

$$
\begin{aligned}
g_\alpha\left(F_t/\bar{F}\right) &= \alpha(1 - F_t/\bar{F}) \\
g_\beta\left(F_t/\bar{F}\right) &= \beta\left(F_t/\bar{F} - 1\right)
\end{aligned}
$$

Then we can write the liabilities as follows

$$
\begin{aligned}
L_{t+1} &= \left(1 + r^f\right)(1 + I_t)\, L_t - \left(1 + r^f\right)(1 + I_t)\, \widetilde{B}_t + Q \\
\widetilde{B}_t &= \sum_{\nu = t - T^D + 1}^{t - T^R} b_{t,\nu} \\
Q &= \psi \sum_{\nu=0}^{T^R - 1} \sum_{i=0}^{T^D - T^R - 1} \left(\frac{1}{1 + r^f}\right)^{i+\nu} \\
\Leftrightarrow L_{t+1} &= \left(1 + r^f\right)(1 - \beta)\, L_t + \beta\left(1 + r^f\right)\frac{A_t}{\bar{F}} - \left(1 + r^f\right)(1 - \beta)\, \widetilde{B}_t - \beta\left(1 + r^f\right)\left(\frac{A_t}{L_t}\frac{\widetilde{B}_t}{\bar{F}}\right) + Q \\
\Leftrightarrow dL_{t+1} &= \frac{\beta}{\bar{F}}\left(1 + r^f\right)\left(\frac{L_t - \widetilde{B}_t}{L_t}\right) dA_t + \left(1 + r^f\right)\left(1 - \beta + \frac{\beta}{\bar{F}}\frac{A_t}{L_t}\frac{\widetilde{B}_t}{L_t}\right) dL_t
\end{aligned}
$$

and we can write the assets as follows

$$
\begin{aligned}
A_{t+1} &= (1 + r_t^p) A_t + P_t - (1 + I_t)\widetilde{B}_t \\
P_t &= T^R\left(1 + g_\alpha\left(F_t/\bar{F}\right)\right)\bar{p} \\
\Leftrightarrow A_{t+1} &= (1 + r_t^p) A_t + (1 + \alpha)\,\bar{P} - (1 - \beta)\,\widetilde{B}_t - \left(\alpha\bar{P} + \beta\widetilde{B}_t\right)\frac{A_t}{L_t}\frac{1}{\bar{F}} \\
\Leftrightarrow dA_{t+1} &= \left(1 + r_t^p - \left(\alpha\bar{P} + \beta B_t\right)\frac{1}{L_t}\frac{1}{\bar{F}}\right) dA_t + \left(\alpha\bar{P} + \beta B_t\right)\frac{A_t}{L_t}\frac{1}{L_t}\frac{1}{\bar{F}} dL_t.
\end{aligned}
$$

In the steady state, the following equations hold:

$$
\bar{F} = 1, A_t = \bar{A} = \bar{L} = L_t, \widetilde{B}_t = \bar{B}, r_t^p = \bar{r}^p,
$$

which can be used to write the dynamics in the steady state, yielding

$$\begin{aligned}
\Leftrightarrow \begin{bmatrix} dA_{t+1} \\ dL_{t+1} \end{bmatrix} &= \begin{bmatrix} 1+\bar{r}^p - \frac{\alpha \bar{P}+\beta \bar{B}}{\bar{L}} & \frac{\alpha \bar{P}+\beta \bar{B}}{\bar{L}} \\ \beta\left(1+r^f\right)\left(1-\frac{\bar{B}}{\bar{L}}\right) & \left(1+r^f\right)\left(1-\beta\left(1-\frac{\bar{B}}{\bar{L}}\right)\right) \end{bmatrix} \begin{bmatrix} dA_t \\ dL_t \end{bmatrix} \\
&= \begin{bmatrix} 1+\bar{r}^p-\theta & \theta \\ \eta & 1+r^f-\eta \end{bmatrix} \begin{bmatrix} dA_t \\ dL_t \end{bmatrix} \\
&= \Omega \begin{bmatrix} dA_t \\ dL_t \end{bmatrix} \quad \text{(with } \theta = \frac{\alpha \bar{P}+\beta \bar{B}}{\bar{L}} \geq 0 \text{ and } \eta{=}\beta\left(1+r^f\right)\left(1-\tfrac{\bar{B}}{\bar{L}}\right) {\geq} 0\text{).}
\end{aligned}$$

Then, the eigenvalues of Ω are denoted by

$$\begin{aligned}
\lambda_1 &= \frac{1}{2}\left(2+\bar{r}^p+r^f-\theta-\eta-\sqrt{(r^f-\bar{r}^p)^2+2(\theta-\eta)(r^f-\bar{r}^p)+(\eta+\theta)^2}\right) \\
\lambda_2 &= \frac{1}{2}\left(2+\bar{r}^p+r^f-\theta-\eta+\sqrt{(r^f-\bar{r}^p)^2+2(\theta-\eta)(r^f-\bar{r}^p)+(\eta+\theta)^2}\right)
\end{aligned}$$

Here we will discuss several properties of these eigenvalues. First, we have that $\lambda_2 > \lambda_1$, which is shown by:

$$\begin{aligned}
\min_{\theta} \lambda_2 - \lambda_1 &= \min_{\theta} \sqrt{(r^f-\bar{r}^p)^2+2(\theta-\eta)(r^f-\bar{r}^p)+(\eta+\theta)^2} \\
&= 2\sqrt{\eta(\bar{r}^p-r^f)} \qquad \text{(with } \theta^* = \bar{r}^p - r^f - \eta\text{)} \\
&\geq 0.
\end{aligned}$$

Second, the mininum value obtained for $\lambda_2 > 1$, as we have

$$\min_{\eta,\theta\geq 0} \lambda_2 = 1 + r^f > 1$$

This means that the pension system is unstable when we use linear adjustment policies, as there is always one eigenvalue greater than one. Hence, the asset-liability-dynamics do not converge after a shock occurs.

Furthermore, if we have $\lambda_1 = 1$, then

$$\begin{aligned}
\theta &= \bar{r}^p - \eta\frac{\bar{r}^p}{r^f} \\
\Leftrightarrow \beta &= \frac{r^f\left(\bar{r}^p\bar{L}-\alpha\bar{P}\right)}{(r^f-\bar{r}^p(1+r^f))\bar{B}+(1+r^f)\bar{r}^p\bar{L}} \\
\Leftrightarrow \lambda_2 &= 1+r^f+\beta\frac{1+r^f}{r^f}\left(\bar{r}^p-r^f\right)\left(1-\frac{\bar{B}}{\bar{L}}\right)
\end{aligned}$$

and if we have $\lambda_1 = 0$, then

$$\begin{aligned}
\theta &= 1 + \bar{r}^p - \eta \frac{1+\bar{r}^p}{1+r^f} \\
\Leftrightarrow \beta &= \frac{(1+\bar{r}^p)\bar{L} - \alpha \bar{P}}{(1+\bar{r}^p)\bar{L} - \bar{r}^p \bar{B}} \\
\Leftrightarrow \lambda_2 &= 1 + r^f + \beta \left(\bar{r}^p - r^f\right) \left(1 - \frac{\bar{B}}{\bar{L}}\right)
\end{aligned}$$

A.2 Certainty-equivalent consumption from social welfare function

$$\begin{aligned}
\sum_{\nu=100}^{\infty} \sum_{t=0}^{T^D-1} \delta^{t+\nu-100} u(CEC_{SW}) &= SW \\
\Leftrightarrow u(CEC_{SW}) \sum_{\nu=0}^{\infty} \delta^{\nu} \sum_{t=0}^{T^D-1} \delta^t &= SW \\
\Leftrightarrow u(CEC_{SW}) \frac{1}{1-\delta} \frac{1-\delta^{T^D}}{1-\delta} &= SW \\
\Leftrightarrow u(CEC_{SW}) &= SW(1-\delta)^2/(1-\delta^{T^D}) \\
\Leftrightarrow CEC_{SW} &= \left(\frac{SW(1-\rho)(1-\delta)^2}{(1-\delta^{T^D})}\right)^{\frac{1}{1-\rho}}
\end{aligned}$$