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Abstract

We show that once interfamily exchanges are considered, Becker's rotten kids mechanism has some remarkable implications that have gone hitherto unnoticed. Specifically, we establish that Cornes and Silva's (1999) result of efficiency in the contribution game amongst siblings extends to a setting where the contributors (spouses) belong to different families. More strikingly still, the mechanism does not just have consequences for *efficiency* but it may have dramatic *redistributive* implications. In particular, we show that the rotten kids mechanism combined with a contribution game to a household public good may lead to an astonishing equalization of consumptions between the spouses and their parents, even when their parents original wealth levels are quite different. We consider two families, each consisting of a parent and an adult child, who are "linked" by the young spouses. Children contribute part of their time to a household (couple) public good and provide attention to their respective parents "in exchange" for a bequest. Spouses behave towards their respective parents like Becker's rotten kids; they are purely selfish and anticipate that their altruistic parents will leave them a bequest. The most striking results obtain when wages are equal and when parent's initial wealth levels are not too different. For very large wealth differences the mechanism has been supplemented by a (mandatory) transfer that brings them back into the relevant range. When wages differ but are similar the outcome will be near efficient (and near egalitarian).

JEL-Code: D130, D610, D640.

Keywords: altruism, private provision of public good, subgame perfect equilibrium, family aid.

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1 Introduction

Becker's (1974; 1991) "rotten kids theorem" has by now become one of the cornerstones of family economics. In his seminal paper Becker presents the challenging idea that intergenerational exchanges within a family may be efficient even when the children are purely selfish and the altruistic parents lack the power to commit to a reward scheme that might provide the children with the proper incentives to behave according to the "common good". The extensive subsequent literature has both qualified and extended this result.[1]

The probably most prominent qualification is due to Bergstrom (1989) who shows that the result rests on a certain number of restrictive assumptions (single good, interior solution, etc.). However, none of these seriously undermines Becker's basic insight. While the outcome may not be efficient under realistic assumptions, the fundamental mechanism continues to be at work and spontaneously yields some "cooperative" behavior in a world which is otherwise biased towards totally selfish conduct.[2]

Amongst the various extensions, one of the most remarkable ones is Cornes and Silva (1999) who show that the rotten kids theorem holds in a world with a private *and* a public good. The siblings non-cooperatively contribute to the family public good. By transferring the private good after the children have chosen their contributions to the public good, the benevolent parent achieves fulfillment of the Samuelson condition. In other words, the rotten kid mechanism may even be an effective way to achieve efficient contributions to (household) public goods in a non-cooperative world (where Nash equilibria are otherwise typically not efficient).[3]

So far this literature has essentially concentrated on the exchanges within a single family.[4] We show that once interfamily exchanges are considered the rotten kids mech-

[1]See, Laferrère and Wolff (2006) for an overview.

[2]For instance, in a recent paper Cremer and Roeder (2013) show that when there are several goods, including family aid (and long-term care services in general) the outcome is likely to be inefficient. Still, the rotten kid mechanism is at work and ensures that a positive level of aid is provided as long as the bequest motive is operative.

[3]Efficiency is, however, only guaranteed if the solution to the kids problem is interior, that is, if all children make contributions to the family public good. Chiappori and Werning (2002) provide examples when this is or is not the case.

[4]A notable exception is Cornes, Itaya and Tanaka (2012) who consider two families and different scenarios of contributors to a (general) public good. They focus on Warr's (1982) neutrality result and show that it continues to hold in their setting. This result says that lump-sum redistributions between participants in a Nash game of private provision of a public good are allocatively neutral when all participants make positive contributions and have the same productivity in producing the public good.

anism has some remarkable implication that have gone hitherto unnoticed. Specifically, we establish that Cornes and Silva's result of efficiency in the contribution game amongst siblings extends to a setting where the contributors (spouses) belong to different families. More strikingly still, the mechanism does not just have consequences for *efficiency* but it may have dramatic *redistributive* implications. In particular, we show that the rotten kids mechanism combined with a contribution game to a household public good may lead to an astonishing equalization of consumptions between the spouses and their parents, even when their parent's original wealth levels are quite different.

We consider a setting with two families each consisting of a retired parent and an adult child who are "linked" by the young spouses. Children contribute part of their time to a household (couple) public good like child care or other domestic duties. Additionally, they provide attention (or caregiving services) to their respective parents "in exchange" for a bequest. Spouses behave towards their respective parents like Becker's rotten kids; they are purely selfish and anticipate that their altruistic parents will leave them a bequest. Parents cannot commit to a rule linking this bequest to the amount of attention provided by the child. In other words, a threat to, say, disinherit (other otherwise punish) the child who does not provide some specified level of attention is not credible, because children anticipate that the estate and its allocation will be determined by the altruistic parent.

We start by determining the set of Pareto-efficient allocations which are used as a benchmark. Not surprisingly, the levels of aid are set to equalize marginal cost (the child's wage) to the marginal benefits incurred by the parent. The optimal provision of the family public good satisfies the Samuelson rule. When children differ in wages, Pareto-efficiency requires that only the lower-wage spouse contributes to the household public good. When children have equal wages, only the total provision of the household public good is uniquely defined and any allocation of this total level between the individual spouses is equally efficient.

We then study the (subgame perfect) Nash equilibrium that occurs when parents and children play a two-stage game, the timing of which reflects the rotten kids approach. First, the children (spouses) choose simultaneously and non-cooperatively the time spend with their parents, and their contribution to the family public good. Second, the parents set (simultaneously and non-cooperatively) the bequest left to their respective child.

This equilibrium turns out to have a number of interesting properties some of which are rather surprising. Levels of family aid are always efficient; this is perfectly in line with the rotten kids specification and unsurprising. The most stunning results arise when wages are equal. Unless parents wealth levels are very different we then obtain a (unique) interior equilibrium where both spouses contribute to the public good. This equilibrium is efficient (the Samuelson condition is satisfied), which is otherwise typically not the case in non-cooperative contribution games; see Bergstrom *et al.* (1986). More surprisingly still, it always corresponds to the utilitarian (equal individual weights) Pareto-efficient allocation. Consequently, consumption levels are equalized within and across families, in spite of the fact that the spouses have parents with different wealth levels. Both properties arise because a rotten kid like mechanism is at work under which spouses' contributions are effectively subsidized through adjustments in the bequests. This is reminiscent of the results obtained by Cornes and Silva (1999) within a single family setting. The striking feature of our results is that this property extends to a setting where the contributors have different parents (they are spouses rather than siblings). In addition, the rotten kids mechanism proofs not only to promote efficiency but also to spontaneously achieve a "perfect" redistribution between the spouses and also between their respective parents. In other words, the initial wealth differences are spontaneously washed out by the interplay of contributions, aid and bequests.

These results occur when the contribution equilibrium is interior, which in turn is the case when the difference in parents' wealth does not exceed a certain threshold. The level of this threshold increases with the significance of the expenditure on the household public good; when these expenditures are sufficiently large, the contribution can neutralize initial wealth difference.

When wealth differences are large, there will be a (unique) corner equilibrium where only the spouse with the richest parents contributes. This equilibrium is no longer efficient and consumption levels are not equalized between parents. We also show that in this case some *ex ante* redistribution (at stage 0) between families can restore efficiency. Interestingly, to accomplish this it is not necessary to fully equalize wealth levels. The redistribution must just bring them within the range that yields an interior equilibrium. The contribution game then takes care of the rest, achieving efficiency and perfect equalization of consumption levels. We return to the case where the equilibrium corresponds to the utilitarian allocation.

The results are more complex in the case where the spouses differ in wage. While Pareto-efficiency requires that only the lower wage spouse contributes to the public good, the equilibrium can yield any pattern of contributions. Depending on the wealth and wage heterogeneity we can have an interior or a corner solution, with either of the spouses (even the high wage one) as sole contributor. This equilibrium is (almost) never efficient, even when the solution is of the right type. However, when spouses' wages are not exactly equal but sufficiently similar the solution will be interior and close to the utilitarian allocation.[5] In any event, whatever the wage differential, efficiency can once again be reestablished with a transfer in stage 0, but unlike in the previous case there is no longer a whole range of possible transfers but only a single level which does the job.

Our paper proceeds as follows. Section 2 sets up the model. Section 3 determines the Pareto-efficient allocations while Section 4 analyzes the *laissez-faire* solution. Section 5 shows how the Pareto-efficient solution can be implemented when the *laissez-faire* is not Pareto-efficient. Section 6 concludes and an appendix contains most of the proofs.

2 The model

We consider two families $i = 1, 2$ each consisting of one parent (superscript 'p') and one child (superscript 'c'). Parents are altruistic while children are purely selfish. The young constitute a couple who non-cooperatively produces a household public good, G, like housework. The production of this household public good is linear and costs $g_i \in [0, \tau]$ units of time. The total amount of time available is τ. Children may also spend some time $a_i \in [0, \tau]$ with their (own) parents providing them simply with attention or with aid in case of illness or dependency. The (monetary) value of this time for their parents is given by $h(a_i)$ with $h' > 0, h'' < 0$. The residual time $\tau - g_i - a_i$ is spend on the labor market for which the child receives the wage rate w_i. Parents own a wealth of x_i, and may leave a bequest $b_i \geq 0$ to their child. Wages of the children as well as wealth of the parents may differ between families implying $w_1 \lesseqgtr w_2$ and $x_1 \leq x_2$. Both

[5] Provided that parents' wealth differences are not too large.

generations derive utility from consumption of a numeraire commodity, while the young couple additionally enjoys consumption of the household public good. The altruistic parent maximizes the welfare function $W_i^p = U_i^p + U_i^c$. The parent's "own" utility (not including the altruistic element) is given by

$$U_i^p = u(x_i + h(a_i) - b_i) \quad \forall\ i,$$

while the utility of the child is represented by

$$U_i^c = u(w_i(\tau - g_i - a_i) + b_i) + \varphi(G) \quad \forall\ i.$$

The utility functions satisfy $u', \varphi' > 0$ and $u'', \varphi'' < 0$ and we have $G = g_1 + g_2$. Both families are perfectly informed about each other's characteristics, which allows us to focus on the efficiency and distributional issues. The timing of the game is as follows: first, the children (spouses) choose simultaneously and non-cooperatively the time spend with their parents, a_i, and their contribution to the family public good, g_i. Second, the parents set (simultaneously and non-cooperatively) the bequest, b_i, left to their respective child. To determine the subgame perfect Nash equilibrium we solve this game by backward induction. Before we turn our attention to the *laissez-faire* solution, we will study the Pareto-efficient allocations which provide a benchmark against which we can compare the Nash equilibrium outcome.

3 Pareto-efficient allocations

Denoting consumption levels of the parents by m_i and of the children by d_i, Pareto-efficient allocations solve the following maximization problem[6]

$$
\begin{aligned}
\max_{m_1,m_2,d_1,d_2,a_1,a_2,g_1,g_2} \quad & \mathcal{W} = \sum_{i=1}^{2} \Big\{ \pi_i^p u(m_i) + \pi_i^c \big[u(d_i) + \varphi(G) \big] \Big\} \\
\text{s.t.} \quad & \sum_{i=1}^{2} \big\{ w_i(\tau - g_i - a_i) + x_i + h(a_i) \big\} \geq \sum_{i=1}^{2} \big\{ m_i + d_i \big\} \\
& G = \sum_{i=1}^{2} g_i, \quad \text{and} \quad a_i + g_i \leq \tau \quad \forall\ i.
\end{aligned} \tag{1}
$$

where $\pi_i^c, \pi_i^p \in (0,1)$ denote the weights attached to the child's and parent's utility of family $i = 1, 2$. They are normalized to sum up to one:

$$\sum_{i=1}^{2} \big\{ \pi_i^c + \pi_i^p \big\} = 1.$$

Solving this problem for a given vector of weights yields a specific Pareto-efficient allocation and the full set of efficient allocations can be described by varying the weights. Denoting $\mathcal{L}$ the Lagrangian expression associated with problem (1), the first-order conditions (FOCs) are given by

$$\frac{\partial \mathcal{L}}{\partial m_i} = \pi_i^p u'(m_i) - \mu = 0 \quad \forall\ i, \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial d_i} = \pi_i^c u'(d_i) - \mu = 0 \quad \forall\ i, \tag{3}$$

$$\frac{\partial \mathcal{L}}{\partial a_i} = \mu \pi_i^c (h'(a_i) - w_i) = 0 \quad \Rightarrow \quad h'(a_i) = w_i \quad \forall\ i, \tag{4}$$

$$\frac{\partial \mathcal{L}}{\partial g_i} = -\mu w_i + (\pi_1^c + \pi_2^c)\varphi'(G) \leq 0 \quad \forall\ i, \tag{5}$$

where μ is the Lagrangian multiplier with respect to the resource constraint. Equations (2) and (3) state that the (weighted) marginal utilities between and across families should be equalized. Equation (4) shows that attention should be chosen such that its marginal benefit to the parent is equal to the marginal costs of its provision. It shows that the level of a_i is the same in all Pareto-efficient allocations (it does not depend on

[6]Throughout the paper, we assume that the time constraint $a_i + g_i \leq \tau$ will be never binding.

the weights). Equation (5) determines the Pareto-efficient public good contributions for both spouses; it can be easily verified that $g_1 > 0$ and $g_2 = 0$ if $w_1 < w_2$. In words, it is efficient that only the spouse with the lower wage rate (production costs) contributes to the family public good. Conditions (2), (3) and (5) can be simplified to

$$\min\{w_1, w_2\} = \frac{\varphi'(G)}{u'(d_1)} + \frac{\varphi'(G)}{u'(d_2)} \tag{6}$$

which is the Samuelson rule, stating that the sum of the marginal rates of substitution between the public and the private good must be equal to the marginal costs of production. When children have equal wages ($w_1 = w_2$), G is uniquely defined (for a given set of weights) by (6) along with the FOCs (2)–(4), but individual contributions can take any values satisfying $g_1 + g_2 = G$.[7]

We denote the utilitarian solution that arises with equal weights ($\pi_1^c = \pi_2^c = \pi_1^p = \pi_2^p = 1/4$) with the superscript ^{e}. It is given by

$$u'(m_1^e) = u'(d_1^e) = u'(m_2^e) = u'(d_2^e), \tag{7}$$

$$h'(a_i^e) = w_i \quad \forall\, i, \tag{8}$$

$$\min\{w_1, w_2\} = 2\frac{\varphi'(G^e)}{u'(d_1^e)}. \tag{9}$$

Note that the level of G^e is unique for a *given* total level of wealth in society $(x_1 + x_2)$. When either x_i or w_i changes so does the optimal G^e.[8] Observe that while g_1^e and g_2^e are not uniquely defined when wages are equal, they are well defined when wages differ. Specifically when $w_i < w_j$ $(i, j = 1, 2)$ we have $g_i^e = G^e$ and $g_j^e = 0$.

The following sections show that an equilibrium of the two-stage game will satisfy conditions (7)–(9) when children have the same wage rate, $w_1 = w_2$, while parents' wealth levels may differ but within a limited range. In other words, in these cases the

[7]The level of G will (in general) vary accross Pareto-efficient allocations.

[8]For equal wages ($w_1 = w_2$) G^e is determined by

$$-w_i u'\left(\frac{x_1 + x_2 + 2w_i(\tau - a_i^e) + 2h(a_i^e) - w_i G^e}{4}\right) + 2\varphi'(G^e) = 0.$$

Differentiating yields

$$\frac{\mathrm{d}G^e}{\mathrm{d}x_i} = \frac{\frac{w_i}{4}u''(d_i^e)}{\frac{w_i^2}{4}u''(d_i^e) + 2\varphi''(G^e)} = \frac{1}{w_i + \frac{8\varphi''(G^e)}{w_i u''(d_i^e)}} > 0.$$

laissez-faire equilibrium corresponds to the utilitarian optimum. On the other hand, when children differ in wages the contribution equilibrium is in general inefficient. However, efficiency of the *laissez-faire* solution and its coincidence with the utilitarian allocation can be reestablished through an appropriate lump-sum transfer between parents.

4 *Laissez-faire* solution

As usual in two-stage games, we begin by analyzing the second stage. The parent solves the following optimization problem

$$\max_{b_i} \quad U_i^p = u(x_i + h(a_i) - b_i) + u(w_i(\tau - g_i - a_i) + b_i) + \varphi(G) \quad \text{s.t.} \quad b_i \geq 0 \quad \forall\, i.$$

The FOC with respect to bequests is given by

$$\frac{\partial U_i^p}{\partial b_i} = -u'(m_i) + u'(d_i) \leq 0 \quad \forall\, i. \tag{10}$$

That is, bequests in both families are chosen so that consumption levels between the parent and the child are equalized. We assume throughout the paper that the bequest motive is operative so that b_i^* is given by an interior solution and (10) holds as equality.[9] Denote $b_i^* \equiv b_i(g_i, a_i)$ the optimal bequest level. Differentiating this expression shows that the derivatives of bequests with respect to public good investments and attention are as follows

$$\frac{\partial b_i^*}{\partial g_i} = \frac{u''(d_i) w_i}{u''(m_i) + u''(d_i)} = \frac{w_i}{2} > 0 \quad \forall\, i, \tag{11}$$

$$\frac{\partial b_i^*}{\partial a_i} = \frac{u''(m_i)h'(a_i) + u''(d_i) w_i}{u''(m_i) + u''(d_i)} = \frac{h'(a_i) + w_i}{2} > 0 \quad \forall\, i. \tag{12}$$

When the child increases his contributions to the family public good, the parent compensates the child by half of his forgone wage income, w_i. Additionally, when the child increases his attention to the parent, the bequest increases by half of the parent's return,

[9] Recall that bequests are restricted to be nonnegative, and one obtains from (10)

$$b_i > 0 \quad \Longleftrightarrow \quad x_i + h(a_i) > w_i(\tau - a_i - g_i) \quad \forall\, i.$$

In words, the net resources of the parents (including the monetary value of informal aid, if any) must be larger than that of the children otherwise the bequest motive is not operative.

$h'(a_i)$, plus by half of the child's forgone wage income, w_i.

At stage 1, child i's problem is

$$\max_{a_i, g_i} \quad U_i^c = u(w_i(\tau - a_i - g_i) + b_i^*) + \varphi(G) \quad \text{s.t.} \quad g_i \geq 0 \quad \forall\ i.$$

A non-negativity constraint is imposed on g_i because a corner solution is possible. When choosing the attention to the parent and investments in the (own) family public good, the child takes into consideration the adjustments in bequests and takes the spouse's contributions $g_{\text{-}i}$ as given

$$\frac{\partial U_i^c}{\partial a_i} = u'(d_i)\left(-w_i + \frac{\partial b_i^*}{\partial a_i}\right) = 0 \quad \forall\ i, \tag{13}$$

$$\frac{\partial U_i^c}{\partial g_i} = u'(d_i)\left(-w_i + \frac{\partial b_i^*}{\partial g_i}\right) + \varphi'(G) \leq 0 \quad \forall\ i. \tag{14}$$

With equations (11) and (12), the above first-order conditions can be written as

$$-w_i + \frac{h'(a_i) + w_i}{2} = 0 \quad \Rightarrow \quad h'(a_i^*) = w_i \quad \forall\ i, \tag{15}$$

$$-u'(d_i)\frac{w_i}{2} + \varphi'(G) \leq 0 \quad \Rightarrow \quad 2\varphi'(G) \leq u'(d_i)w_i \quad \forall\ i. \tag{16}$$

Equation (15) directly determines a_i^*; the spouse's level of attention $a_{\text{-}i}^*$ is of no relevance and there is effectively no strategic interaction on this variable. Substituting this level of attention into equation (16) and taking into account the constraint $g_i \geq 0$, we can solve for the spouses' best response functions for the contributions to the family public good $\widetilde{g}_1(g_2)$ and $\widetilde{g}_2(g_1)$. The Nash equilibrium levels of contributions (g_1^*, g_2^*) are defined in the usual way by the mutual best reply conditions $g_1^* = \widetilde{g}_1(g_2^*)$ and $g_2^* = \widetilde{g}_2(g_1^*)$. Existence of this equilibrium is easily established.[10] The total equilibrium amount of the family public good produced by the couple is then given by $G^* = g_1^* + g_2^*$.

Two distinct types of equilibria are possible; an interior solution, that is, one in which both spouses contribute to the household public good and a corner solution in which only one of the spouses contributes. For future reference note that with (10) an

[10]Strategy spaces are compact sets and each player's utility is continuous and quasi-concave in his own strategic variable.

interior Nash equilibrium satisfies

$$u'(d_1^*)w_1 = 2\varphi'(G^*) \quad \Leftrightarrow \quad u'\left(\frac{(\tau - a_1^* - g_1^*)w_1 + h(a_1^*) + x_1}{2}\right) w_1 = 2\varphi'(G^*), \tag{17}$$

$$u'(d_2^*)w_2 = 2\varphi'(G^*) \quad \Leftrightarrow \quad u'\left(\frac{(\tau - a_2^* - g_2^*)w_2 + h(a_2^*) + x_2}{2}\right) w_2 = 2\varphi'(G^*). \tag{18}$$

We shall now examine the properties of the Nash equilibrium and analyze the efficiency of the induced allocation. We start with the case where children have identical wages and then consider the case where wages differ.

4.1 Identical children

Assume children are equally productive in the labor market, $w_1 = w_2 \equiv w$. Recall that subscript 2 is used for families with higher wealth ($x_1 \leq x_2$). To simplify notation, we fix x_2 at some arbitrary level and then study the Nash equilibrium and its properties as a function of x_1. Observe that as long as $\widetilde{g}_1$ is an interior solution for which (16) holds as equality, we have

$$\frac{\partial \widetilde{g}_1}{\partial x_1} = \frac{u''(d_1)w}{u''(d_1)\frac{w_1^2}{2} + 2\varphi''(G)} > 0. \tag{19}$$

Thus, for a given level of x_2, the best response of spouse 1 to any level of g_2 decreases as x_1 becomes smaller. Consequently, we expect that the equilibrium moves from the interior one to the corner solution when spouse 1's wealth falls. This conjecture is confirmed in the following proposition which is established in the Appendix. It shows that the equilibrium is interior when wealth levels are not too different, while a corner solution may arise when x_1 is sufficiently small.

Proposition 1 *The Nash equilibrium is unique and an interior solution ($g_1^* > 0; g_2^* > 0$) if $x_1 > \widehat{x}_1$, while a corner solution ($g_1^* = 0; g_2^* > 0$) arises if $x_1 \leq \widehat{x}_1$, where $\widehat{x}_1 \equiv x_2 - \widetilde{g}_2(0)w_2$.*

To get an intuitive understanding of this proposition, consider equation (16) defining the best responses for $w_1 = w_2$. Assume that spouse 2 contributes $\widetilde{g}_2(0)$, *i.e.*, her best response to $g_1 = 0$. Equations (17) and (18) then show that $(0, \widetilde{g}_2(0))$ is an interior equilibrium if x_1 is at exactly the level which yields equal consumption levels (including the respective bequests) across spouses, $d_1 = d_2$ for these respective contributions.

With equal wages $a_1^* = a_2^*$ so that $d_1^* = d_2^*$ occurs when $x_1 = x_2 - \widetilde{g}_2(0)w_2$. In words, the wealth difference $x_2 - x_1$ corresponds to the costs of the spouses contributions: $\widetilde{g}_2(0)w_2 - 0w_1$. Taking into account (19) it is plain that for a level of wealth smaller then $\widehat{x}_1$ the best (interior) response of spouse 1 to $\widetilde{g}_2(0)$ is negative, which along with the non-negativity constraint brings us to a corner solution. Conversely, when $x_1 > \widehat{x}_1$ the poorer spouse wants to contribute a positive amount as response to $\widetilde{g}_2(0)$ and we get an interior equilibrium.

We now turn to the study of the properties of the equilibrium. It will turn out that they crucially depend on the type of equilibrium, interior or corner, and thus ultimately on the wealth difference between the spouses' parents; see Proposition 1.

Let us start with the special case where parents have equal wealth $x_1 = x_2 \equiv x$ (in which case we necessarily have an interior solution). It can be easily verified that the subgame-perfect equilibrium of the two-stage game coincides with the Pareto-efficiency conditions (7)–(9) for equal weights; marginal utilities are equalized within and across families, and time is optimally allocated to the parent and to the production of the family public good. In other words, the *laissez-faire* solution corresponds to the utilitarian optimum. Via an adjustment in bequests, the old not only induce the efficient amount of attention from their children, but they also achieve that the young couple produces the efficient amount of their family public good.

The intuition behind this outcome is as follows. The positive bequest equalizes consumption levels (between parents and children and between spouses) within each family. Since due to the adjustment in bequests, the child bears only half of the costs of higher attention but also receives half of its return, he opts for the efficient amount of $a_i^* \equiv a^e$. This resembles Becker's (1974; 1991) famous rotten kid theorem. However, in our setting also public good investments within the young generation are efficient. Again via the adjustment in bequests the child effectively bears only half of the costs of higher public good investments. Since each child equalizes his own marginal costs of investments with his own marginal benefits, the tradeoff by equation (16) becomes effectively the efficient one. Recall that from (17)–(18) we have $u'(d_1^*) = u'(d_2^*)$. Consequently, both spouses have the same marginal benefit of the public good.[11] In other words, public good investments by each spouse are chosen such that the Samuelson rule,

[11] So that the social benefit is exactly twice the individual benefit.

equation (6), is satisfied implying $G^* = G^e$.

To see this, note that for equal wages equations (17) and (18) imply

$$d_1^* = d_2^* \quad \Leftrightarrow \quad (\tau - g_1^*)w + x_1 = (\tau - g_2^*)w + x_2. \tag{20}$$

With $x_1 = x_2$ and $G^* = g_1^* + g_2^*$ we have $g_1^* = g_2^* = G^*/2$. That is, both spouses equally contribute to the family public good.

Interestingly, this result also holds when parents differ in their wealth levels, $x_1 < x_2$, as long as the difference is not too large so that the solution continues to be interior for both g_1 and g_2. In this case, the spouse who expects the higher bequest (spouse 2) contributes more to the family public good than the one with the lower bequest. More precisely, the contributions to the family public good by spouse 2 are chosen so that consumption levels between the couple are equalized and the *laissez-faire* allocation again coincides with the (utilitarian) Pareto-efficient solution. If, however, the difference in parent's wealth is strong, such that $x_1 \leq \widehat{x}_1 = x_2 - w\widetilde{g}_2(0)$, the spouse who expects the lower bequests (spouse 1) contributes nothing to the household public good; we have a corner solution and condition (6) is no longer satisfied (because the two spouses no longer have the same willingness to pay for the public good. The *laissez-faire* allocation then not only implies an inefficient level of the family public good, but also unequal consumption levels within the couple and thus across families. However, even in that case the rotten kids mechanism continues to be at work and enhances the provision of the household public good.[12] Similarly, since only the spouse with the richest parents contributes to the family public good (of which half is effectively paid by his parents) it continues to mitigate wealth differences. The following proposition summarizes our results.

Proposition 2 *The laissez-faire solution (subgame perfect equilibrium) of the two stage game with two families consisting of altruistic parents and selfish children (the latter constituting a couple who non-cooperatively produces a household public good) is Pareto-efficient if the children have the same wage rates,* $w_1 = w_2 \equiv w$, *and the parents' wealth is such that* $x_1 \geq \widehat{x}_1 \equiv x_2 - w\widetilde{g}_2(0)$ *where* $\widetilde{g}_2(0)$ *is the best-response of spouse 2 to* $g_1 = 0$.

[12] This follows because the term $\partial b_i^*/\partial g_i$ appears in equation (14). In words, the adjustment in bequests is formally equivalent to a subsidy on contributions which is well known to enhance provision (recall that individual contributions are strategic substitutes).

Specifically, for an operative bequest motive in both families $i = 1, 2$

(i) attention provided by the child satisfies $h'(a_i) = w_i \ \forall i$,

(ii) consumption levels between and across families are equalized,

(iii) public good investments by the children satisfy the Samuelson rule, and

(iv) the spouse with the richer parents provides more of the family public good.

For $x_1 < \widehat{x}_1$, *the subgame perfect equilibrium is not Pareto-efficient, the time allocation within families continues to be efficient, but the time devoted to the household public good is no longer interior but at a corner and the Samuelson rule is not satisfied.*

4.2 Heterogenous children

When children differ in wages $w_1 < w_2$ the pattern of equilibria that can arise is more complex. We can have *(i)* a corner equilibrium with only the *lower* wage spouse contributing, *(ii)* an interior solution with both spouses contributing and even *(iii)* a corner equilibrium with only the *higher* wage spouse contributing. Roughly speaking, one can expect the interior solution to arise when wage and parents' wealth are not too different. Equilibrium *(iii)* can be expected if wages are not too different and the high wage spouse has much richer parents. In all other cases, equilibrium *(i)* can be anticipated. A precise characterization of the parameter values yielding the different type of equilibria is tedious and not necessary for the issues we are dealing with. We shall thus restrict ourselves to presenting an example illustrating that the different cases can indeed arise.

Example 1 *Assume the following functional forms for utility* $u(d) = 4 \ln d$, $\varphi(G) = \frac{1}{2} \ln G$ *and* $h(a) = 4\sqrt{a} - 2$. *Additionally assume* $w_1 = 1 < w_2 = 2$, $x_2 = 20$ *and the total amount of time available is* $\tau = 8$. *With equation (15), we have for the optimal attention*

$$h'(a_i^*) = 2(a_i^*)^{-1/2} = w_i \quad \Rightarrow \quad a_1^* = 4, \quad a_2^* = 1$$

implying $h(a_1^*) = 6$ *and* $h(a_2^*) = 2$. *With our functional forms for utility equation (16) amounts to*

$$(\tau - a_i^* - g_i^*)w_i + h(a_i^*) + x_i \leq 8w_iG.$$

*With the above parameters, we can write the optimal response function for spouse-*1 *and*

2 as

$$(8-4-g_1)+6+x_1 \leq 8G, \tag{21}$$

$$(8-1-g_2)2+2+20 \leq 16G. \tag{22}$$

For $x_1 = 15$ we have a corner equilibrium with only the lower wage spouse contributing: $g_1^* = 3$ and $g_2^* = 0$; case (i). For $x_1 = 6\frac{5}{6}$ both spouses contribute: $g_1^* = \frac{3}{2}$ and $g_2^* = \frac{2}{3}$; case (ii). For $x_1 = 3$ we have a corner equilibrium with only the higher wage spouse contributing: $g_1^* = 0$ and $g_2^* = 2$; case (iii).

From our perspective, the interesting feature is that equilibria of types *(ii)* and *(iii)* are *never* efficient: the spouse with the higher time cost contributes at least partly to the public good production. As to type *(i)* equilibria, they are in general inefficient. The equilibrium is efficient (and corresponds to the utilitarian optimum) only when

$$d_1^e = \frac{(\tau - a_1^e - g_1^e)w_1 + h(a_1^e) + x_1}{2} = d_2^e = \frac{(\tau - a_2^* - g_2^e)w_2 + h(a_2^e) + x_2}{2}.$$

Since g_1^e and g_2^e are uniquely defined in the unequal wage case this can occur only "by coincidence"; see Subsection 5.2 for further details. Inefficiency arises for exactly the same reasons as in the corner solution case with identical wages considered in the previous subsection. Marginal utilities between spouses are no longer equalized; see equations (17) and (18). Thus, the Samuelson condition is not satisfied in the Nash equilibrium and the allocation in the *laissez-faire* is not Pareto-efficient. Notice however, that the levels of attention continue to be at their efficient levels (we have $a_i^* = a_i^e$).

Finally, the case where wages differ but are sufficiently close deserves some attention. Since the best-response functions are continues in wages the equilibrium allocation will also be a continuous function of both wages.[13] Consequently, when w_1 is sufficiently close to w_2 and when wealth differences are not "too large" the equilibrium will be interior and it will be "almost" or "near" efficient and utilitarian. To be more precise as w_1 tends to w_2 the outcome will tend to the one described in Subsection 4.1. While this result is rather trivial from a theoretical perspective it is quite important for the

[13] This requires some additional technical conditions, but since our best-reponse functions are "well-behaved" it is plain that the continuity applies in our setting.

practical implications of our analysis. In reality the case where wages are *exactly* equal may be very rare, but under suitable mating patterns wages may often be close enough so that the notion of near efficiency applies and has relevant implications.

The next section studies those cases where the *laissez-faire* solution is inefficient and shows how the efficient solution can be implemented through lump-sum transfers across families.

5 Implementation of the efficient solution

Assume now that some public authority can put in place policies *before* the game between children and parents takes place.

5.1 Corner solution with identical children

We have shown in Subsection 4.1 that with identical children the equilibrium is inefficient when it corresponds to a corner solution and $x_1 < \widehat{x}_1$. This in turn occurs (for any given level of x_2) when the wealth difference between parents is sufficiently significant. This problem can be overcome if wealth is redistributed (at stage 0, before the game is played) to bring wealth differences within the range that yields an interior solution. We then know from Proposition 3 that this will induce an equilibrium which corresponds to the utilitarian solution.

The result is formally stated in the following proposition (which is established in the Appendix).

Proposition 3 *Assume that children have equal wages $w_1 = w_2 \equiv w$, but the parent's wealth difference is such that $x_1 < \widehat{x}_1$, then the utilitarian Pareto-efficient solution can be implemented by a lump-sum transfer T from high- to low-wealth families, given by*

$$T \in \left[\frac{x_2 - x_1 - wG^e}{2}, \frac{x_2 - x_1 + wG^e}{2} \right].$$

Observe that G^e while being the utilitarian public public good level for the initial wealth levels x_1 and x_2 it is of course also the optimal level for the after transfer wealth levels (only total wealth matters for Pareto-efficiency). The fact that the transfer can take any value in the above interval resembles Warr's (1983) neutrality result. As long as the transfer induces an interior solution, income redistribution is irrelevant in the presence of a privately provided public good. One can of course set $T = (x_2 - x_1)/2$ to make (after transfer) wealth levels equal, but this is not necessary.

5.2 Different wages

Now we must design a transfer scheme that ensures that the equilibrium is such that (only) the low wage individual contributes *and* that spouses' (equilibrium) consumption levels are equal. Recall that this latter condition ensures that both spouses have the same willingness to pay for the public good, which in turn will ensure that the Samuelson condition, equation (6), holds. To understand why the sole contributor then provides the Pareto-efficient level recall that his contribution is subsidized through the extra bequest so that he only bears half of its cost; see expression (11). And with consumption levels equalized between spouses his private benefits are precisely equal to half of the social benefits. The following proposition, established in the appendix states the required level of transfer which is equal to half the difference in "total income" between both families (evaluated at the optimal solution).

Proposition 4 *If parents differ in wealth, $x_1 < x_2$ and children in wages, $w_1 \lesseqgtr w_2$, the Pareto-efficient allocation with equal weights can be decentralized by a lump-sum transfer from high- to low-income families. This transfer is simply half the income difference between both families and given by*

$$T = \frac{x_2 - x_1 + (\tau - a_2^e - g_2^e)w_2 - (\tau - a_1^e - g_1^e)w_1 + h(a_2^e) - h(a_1^e)}{2}. \qquad (23)$$

Observe that in this expression one of the g_i^e's (the one associated with the higher wages spouse) is always equal to zero. Intuitively, with this transfer, we achieve $d_1^* = d_2^*$ (which is necessary for the Nash equilibrium to satisfy the Samuelson condition) but for $w_j < w_i$ $(i, j = 1, 2)$ also implies $w_j u'(d_j^*) < w_i u'(d_i^*)$ which from equation (16) ensures that only the low wage spouse (type j) will contribute.

6 Concluding remarks

The main point we have made is, that when applied to an interfamily setting (where families are "linked" by young spouses), the rotten kids mechanism may take care of both efficiency and redistribution (between the spouses' respective families). When spouses have equal wages it will yield an efficient outcome *and* wash out parents' wealth differences (as long as they are not too large). For larger wealth differences the mechanism would have to be supplemented by a (mandatory) transfer scheme which brings the discrepancies back within the relevant range. Interestingly the mechanism continues to be effective (though less "perfect") when spouses' wages are not exactly equal but sufficiently similar. The outcome will then be close to the utilitarian allocation. This remark is crucial when it come to asses the practical implications of our result. In reality it is of course unlikely that spouses have *exactly* the same wages. Still, assortative mating is commonly observed and cases where spouses have sufficiently similar wages are not uncommon; see *e.g.* Schwartz and Mare (2005).

More generally, the mating pattern is crucial for assessing the implications of our results. In particular, when mating occurs mainly according to the spouses' wages this may have positive implications both for efficiency and redistribution. It may then contribute to eliminate wealth differences. However, when the dominant factor is the parent's wealth, mating behavior may be neither good for efficiency nor for redistribution.

In any event one has to keep in mind that the extent of redistribution achieved through this channel is limited (to families "linked" by marriage). Consequently, while it can eliminate some wealth differences, it cannot be considered as a substitute for a well designed redistributive policy (which can be more or less egalitarian according to the society's preferences). Still, this aspect adds to the various efficiency enhancing properties of the rotten kids mechanism which have been mentioned in the literature.

Appendix

Proof of Proposition 1

Assume $w_1 = w_2 = w$ and consider a given level of $x_2 > 0$ (and continue to assume without loss of generality that $x_1 \leq x_2$). From equation (16) we can see that a corner solution, $(g_1^* = 0, g_2^* > 0)$, prevails if

$$2\varphi'(G^*) = 2\varphi'(\widetilde{g}_2(0)) < u'(d_1^*)w, \tag{24}$$

where $G^* = g_2^* = \widetilde{g}_2(0)$. For $x_1 = x_2$, we have

$$u'(d_1^*)w = u'\left(\frac{(\tau - a_1^*)w + x_1}{2}\right)w < u'\left(\frac{(\tau - a_2^* - g_2^*)w + x_2}{2}\right)w = 2\varphi'(g_2^*),$$

so that condition (24) does *not* hold. Since $u'(d_1^*)$ increases as x_1 decreases there exists at most one $\widehat{x}_1$ defined by $\widehat{x}_1 = x_2 - g_2^* w_2$ (yielding $d_1^* = d_2^*$) with $g_2^* = \widetilde{g}_2(0)$ and $g_1^* = 0$ for which (24) holds as equality. When $x_1 < \widehat{x}_1$, there exist then a corner solution (with only type 2 contributing). And since $\widetilde{g}_2(g_1)$ is decreasing it is plain that there cannot also be an interior equilibrium (which would require $d_1^* = d_2^*$). When $x_1 > \widehat{x}_1$, condition (24) is violated and the equilibrium can only be interior. Observe that for $x_1 = \widehat{x}_1$ we have $g_1^* = 0$ and $g_2^* > 0$ but these levels also satisfy the conditions for an interior solution (the constraint that $g_1 \geq 0$ hold with equality but is not binding). This is where the "transition" between corner and interior solution occurs.

To complete the proof it remains to show that an interior equilibrium is unique. Observe that the slopes the reaction functions are (in absolute values) smaller than one. Substituting (15) into equation (16) and differentiating yields

$$\frac{\mathrm{d}g_i}{\mathrm{d}g_{\text{-}i}} = -\frac{2\varphi''(G)}{u''(d_i)\frac{w_i}{2} + 2\varphi''(G)} \in (-1, 0).$$

This means that the best-reply map is a contraction which immediately implies uniqueness; see Vives (2001), pages 47–48.

Proof of Proposition 3

To determine the optimal transfers, (T_1, T_2), (the ones that implement the utilitarian Pareto efficient solution) we have to revisit the different stages of the game. In stage 2, parents leave a bequest to their children. This bequest is chosen so as to equalize consumption between the parent and the child,

$$m_i = d_i = \frac{(\tau - a_i - g_i)w + h(a_i) + x_i + T_i}{2} \quad \forall\ i.$$

Note that as long as bequests are interior, it is irrelevant whether the lump sum transfer is paid by the children or by the parent.[14] With T_i set so that $T_1 = -T_2 \equiv T$, if follows from equations (17) and (18) that the best-response functions of spouses 1 and 2 are implicitly defined by

$$u'\left(\frac{(\tau - a_1^* - g_1^*)w_1 + h(a_1^*) + x_1 + T}{2}\right) w_1 = 2\varphi'(G^*), \tag{25}$$

$$u'\left(\frac{(\tau - a_2^* - g_2^*)w_2 + h(a_2^*) + x_2 - T}{2}\right) w_2 = 2\varphi'(G^*). \tag{26}$$

The transfer must be chosen such that an interior solution for both g_1^* and g_2^* is guaranteed. At an interior solution, we have $d_1^* = d_2^*$, implying

$$\frac{(\tau - a_1^* - g_1^*)w_1 + h(a_1^*) + x_1 + T}{2} = \frac{(\tau - a_2^* - g_2^*)w_2 + h(a_2^*) + x_2 - T}{2}.$$

Since $w_1 = w_2 \equiv w$, we have $a_1^* = a_2^*$ and the above equation reduces to

$$x_1 + T - g_1^* w = x_2 - T - g_2^* w.$$

At an interior solution, $(g_1^*, g_2^*) \in (0,1) \times (0,1)$, the overall public good production, $g_1^* + g_2^*$, is uniquely determined by G^e. That is, we can write

$$T = \frac{x_2 - x_1 + (2g_1^* - G^e)w}{2}.$$

Since $g_1^* \in (0, G^e)$ the optimal transfer is in the interval as stated in Proposition 2.

Proof of Proposition 4

The transfer across families must be chosen such that $d_1^* = d_2^*$, then from equations (17) and (18) it can be seen that only the spouse with the lower wage rate (spouse i) contributes to the family public good implying $g_i^* \equiv G^e$ and $g_j^* = 0$ $(i, j = 1, 2;\ i \neq j)$. The transfer T must thus be chosen such that

$$\frac{(\tau - a_1^* - g_1^*)w_1 + h(a_1^*) + x_1 + T}{2} = \frac{(\tau - a_2^* - g_2^*)w_2 + h(a_2^*) + x_2 - T}{2}.$$

[14] With operative bequests, Ricardian equivalence holds for the transfers.

Solving for T yields expression (23) in Proposition 4.

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