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Abstract

Until recently, government procurement bidding processes have generally favored domestic firms by awarding the contract to a domestic firm even if a foreign firm tenders a lower bid, so long as the difference between the two is sufficiently small. This has been replaced by an agreement abolishing this practice. However, the presence of other trade barriers, such as tariffs, can continue to disadvantage foreign firms. We analyze the bidding strategies in such a game and show that when domestic profits are valued, tariffs will be used to discriminate against foreign firms. Furthermore, we find that optimal tariffs can be more protectionist than the optimal price preference, resulting in lower expected domestic welfare and total surplus.

JEL-Code: F130, H570, F120.

Keywords: government procurement, tariffs, price preference.

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1 Introduction

Government procurement contracts are a significant part of many economies, often amounting to 15-20 percent of GDP (WTO, 2013). When seeking a provider for a government contract, it has been a long-standing tradition that the nature of the bidding favors domestic firms over foreign ones. For the most part, this has taken place via a system in which the contract is awarded to a foreign firm only if that firm's bid is sufficiently lower than the lowest bid tendered by a domestic firm, known as a price preference. For example, under the European Community regulations, the contract was awarded to a member firm so long as its bid was no more than three percent higher than the lowest non-member bid (Branco, 1994). This preferential procurement policy has been attributed to a government which values domestic firm profits more than foreign firms (McAfee and McMillan, 1989).[1] This notion was expanded upon by Branco (1994), who considers the optimal mechanism to implement this preference, and Miyagiwa (1991), who includes both public and private consumption. In 1996, this practice began to be dismantled by the Government Procurement Agreement, an international agreement in which signatories agree to non-discrimination, that is, a selection process by which foreign firms are treated no differently than their domestic competitors.[2] This, however, ensures equal treatment under the bidding process but does not eliminate other mechanisms by which foreign firms are treated differently than domestic firms, most notably trade policy. Recently, concerns have been expressed that due to increasing demands for protectionism, governments will resort to such methods to put foreigners at a disadvantage in procurement bidding. For example, within the European Union, there has been mounting pressure to restrict bidding to foreign firms that would use a sufficiently high level of domestic content when fulfilling a government contract (Economist, 2012). Although there is a literature examining procurement processes with a price preference offered to domestic firms, as yet, there is no analysis of competition under protection. That is the gap

[1] They also demonstrate that, by shifting bidding behavior, this can increase domestic welfare when foreign firms have cost advantages.

[2] See WTO (2013) for a detailed description of this agreement.

this paper fills.

We do so by considering an auction for a government contract in which two firms, one domestic and one foreign, tender bids to the domestic government. In contrast to the preferential procurement processes studied elsewhere, the contract is awarded to the firm with the lowest bid. A second difference is that we allow the domestic government to impose a tariff, either specific or ad valorem, on the foreign firm. These tariffs can also be thought of as the added cost to the foreign firm of meeting domestic content restrictions that would make it eligible to bid. We show that the imposition of a tariff does indeed imped the probability of the foreign firm winning the contract. Furthermore, it increases the range of bids that are submitted by the domestic firm.

When domestic profits are positively valued, the government will impose a positive tariff in equilibrium. Further, this optimal tariff (from the home government's perspective) is increasing in the value placed on domestic profits. When domestic profits and government surplus are equally valued, this results in a tariff which is, in expectation, more protectionist than the optimal price preference (as derived by Branco (1994)). This holds for both the specific and ad valorem tariffs. Comparing the two optimal tariffs, the specific tariff is particularly protectionist for low cost foreign firms whereas the ad valorem tariff tends to be especially protectionist for moderate cost foreigners, with both driving high cost foreign firms out of the market entirely. This is due to the fact that, whereas the absolute cost of the ad valorem tariff is increasing in the foreign firm's cost, the specific one is not. Since the optimal specific tariff is geared towards the "average" foreign firm, this then creates a greater burden for low cost foreign firms. Further, both tariff schemes lower welfare, measured as that in the domestic economy or the world as a whole, in comparison to the price preference welfare with the optimal ad valorem tariff being marginally welfare superior to the optimal specific tariff. This then is in line with the concern that the Government Procurement Agreement may be driving governments towards other, less efficient protectionist methods. In contrast, we show that when the weight the government places on domestic profits is low, the equilibrium under either tariff regime is one of free trade and social welfare is maximized. This then highlights the importance of favoritism in the equilibrium policy.

The paper proceeds as follows. In Section 2, we present the model and solve for the bidding strategies of firms. Section 3 describes the government's optimal tariff. Section 4 compares the equilibrium tariff with the equilibrium price preference scheme using the parameters in Branco (1994). Section 5 concludes.

2 The Model

The model has three players, a domestic government, a domestic firm, and a foreign firm. The government has a project, to which it attaches a value V, that it wishes to have completed.[3] To that end, it runs a first-price, sealed-bid auction in which the two firms simultaneously tender bids (b_d for the domestic firm and b_f for the foreign firm) which are the firm's price for which it will carry out the contract. The bidding results in the contract being awarded to the firm with the lowest bid. Note that this is in contrast to a policy with a price preference, in which the domestic firm can win the contract even if $b_d > b_f$ so long as this difference is less than some predetermined level. In the event of equal bids, the contract is awarded to the domestic firm. The timing of the model is as follows. In the first stage, the government sets a tariff which the foreign firm must pay if it wins the contract. We solve for two separate scenarios: When the government uses a specific tariff $\tau \geq 0$ and when it uses an ad valorem tariff $t \geq 0$. This can be thought of as a tariff in which the foreign firm must pay on inputs brought in from its own country. Alternatively, instead of a tariff, this could represent the additional cost to the foreign firm of negotiating domestic content regulations (as might occur if it is forced to use a domestic supplier rather than its preferred supplier located in another country).[4] The key distinction between these tariffs is that the specific tariff payment is the

[3]We assume that this is sufficiently large so that, in equilibrium, the government's value exceeds the winning equilibrium bid. We formalize this condition below.

[4]Further, if there are registration fees that the domestic firm, by virtue of its existence has already paid but the foreign firm has not, then the "tariff" could represent those additional fees the foreign firm must pay

same for any successful foreigner (i.e. it is a flat fee) where as the ad valorem tariff payment will vary across foreigners with different costs. In the second stage, the firms simultaneously submit bids. In the final stage, bids are opened, the contract is awarded, production takes place, and payoffs accrue. We solve the game via backwards induction. We begin by solving the general problem without any assumption on the type of tariff used, then analyze the solution under each tariff separately.

2.1 Firms

Prior to the commencement of the game, each firm $i = d, f$ obtains a cost c_i which is independently drawn from a uniform distribution with the support $[\underline{c}_d, \overline{c}_d]$ for the domestic firm and $[\underline{c}_f, \overline{c}_f]$ for the foreign firm. The bounds of the foreign cost distribution are functions of tariffs, as discussed momentarily. We assume that the two cost distributions differ only due to tariffs, i.e. that under free trade the cost distributions are identical. This assumption greatly simplifies the analysis, however we expect that the results generalize somewhat to asymmetric distributions even with a zero tariff.[5] Note that there are limits to the potential asymmetry. In particular, if $\overline{c}_d \leq 2\underline{c}_f - \overline{c}_f$, the Nash equilibrium must have the domestic firm always winning by bidding $\underline{c}_f$, i.e. the foreign firm is unable to compete.[6] As a consequence, we only consider cases wherein $\overline{c}_d > 2\underline{c}_f - \overline{c}_f$ since for tariffs where this fails, the domestic firm always wins.[7] While the distribution of costs are public knowledge, each firm's cost realization is private information. Define $b_i(c_i)$ as the bid function and $c_i(b)$ as the inverse bid function for firm i. As shown by Griesmer, et al. (1967) any non-trivial equilibrium (i.e. one in which both firms have a positive probability of winning) must be characterized by monotonic and differentiable bid functions and we therefore restrict our attention to this

after winning but before it can fulfill the contract. Under this latter interpretation, the tariff would not be discriminatory against the foreign firm in the strictest sense.

[5]This then shuts down the cost-driven motives for the price preference considered in McAfee and McMillan (1989).

[6]This is discussed in detail for first-price auctions by Kaplan and Zamir (2012) and Kaplan and Wettstein (2000).

[7]Note that, as we demonstrate below, since the optimal tariff is non-prohibitive this restriction has no consequences for our equilibrium analysis.

class of bid functions.

In the following analysis, we build on the results of Kaplan and Zamir (2012) who derive analytic solutions to an auction with uniform, but asymmetric, valuation distributions. We modify their analysis in order to fit it to the setting we consider. Before proceeding certain aspects of bidding behavior should be made clear. First, to rule out multiple equilibria, we assume that a firm with a zero probability of winning bids its cost. Second, in equilibrium, no bid greater than $\overline{c}_f$ will be tendered. The reason for this is that, should one firm do so, the other would be able to marginally undercut that bid and discretely increase its probability of winning while only marginally lowering its payoff. Thus, bids will be bounded from above.

We begin by considering the domestic firm. In the second stage of the game, firms take the tariff as given. The domestic firm's expected profit from tendering a bid of b_d is

$$E[\pi_d] = (b_d - c_d)\,\text{Pr}(\text{domestic firm wins}|\ c_d)$$

where the probability of it winning is the probability that it tenders the lower bid, i.e. that the foreign firm's cost leads it to tender a higher bid:

$$\big(1 - H_C[c_f(b_d)]\big) = 1 - \frac{c_f(b_d) - \underline{c}_f}{\overline{c}_f - \underline{c}_f} = \frac{\overline{c}_f - c_f(b_d)}{\overline{c}_f - \underline{c}_f}.$$

This results in expected domestic firm profits of:

$$E[\pi_d] = (b_d - c_d)\left[\frac{\overline{c}_f - c_f(b_d)}{\overline{c}_f - \underline{c}_f}\right]. \tag{1}$$

Maximizing expected profit, for an interior solution, the optimal inverse bidding strategy for firm d solves the first order condition:

$$\left[\frac{\overline{c}_f - c_f(b_d)}{\overline{c}_f - \underline{c}_f}\right] - (b_d - c_d(b_d))\frac{c_f'(b_d)}{\overline{c}_f - \underline{c}_f} = 0.$$

Defining the minimum and maximum bids which are tendered in equilibrium as $\underline{b}$ and $\overline{b}$ respectively, for any $b_d \in [\underline{b}, \overline{b}]$, we must have

$$[b - c_d(b)]c_f'(b) + c_f(b) = \overline{c}_f. \tag{2}$$

Similarly for the foreign firm, the probability that b_f is the winning bid is

$$\big(1 - H_C[c_d(b_f)]\big) = 1 - \frac{c_d(b_f) - \underline{c}_d}{\overline{c}_d - \underline{c}_d} = \frac{\overline{c}_d - c_d(b_f)}{\overline{c}_d - \underline{c}_d}.$$

Therefore, it has the following expected profit

$$E[\pi_f] = (b_f - c_f)\left[\frac{\overline{c}_d - c_d(b_f)}{\overline{c}_d - \underline{c}_d}\right]. \tag{3}$$

Maximizing expected profit, the optimal inverse bidding strategy for the foreign firm solves the following first order condition:

$$\left[\frac{\overline{c}_d - c_d(b_f)}{\overline{c}_d - \underline{c}_d}\right] - (b_f - c_f(b_f))\frac{c_d'(b_f)}{\overline{c}_d - \underline{c}_d} = 0.$$

Therefore, in equilibrium, because this must hold for any $b_f \in [\underline{b}, \overline{b}]$, we must have

$$[b - c_f(b)]c_d'(b) + c_d(b) = \overline{c}_d. \tag{4}$$

Thus, we have a system of two differential equations that define the equilibrium:

$$[b - c_d(b)]c_f'(b) + c_f(b) = \overline{c}_f, \tag{5}$$

$$[b - c_f(b)]c_d'(b) + c_d(b) = \overline{c}_d. \tag{6}$$

From this, three results follow. First, a foreign firm with a cost $\overline{c}_f$ will submit a bid equal to this cost, i.e. $c_f(\overline{c}_f) = \overline{c}_f$ which implies that $\overline{b} \leq \overline{c}_f$.[8] Second, as the bidding functions are monotone, the lowest bid tendered by a domestic firm must be tendered by a firm with cost $\underline{c}_d$. The analogous result must hold for the foreign firm. Third, these minimum bids must be the same for each firm (and are therefore equal to $\underline{b}$). If this is not the case, for example if the domestic minimum bid is lower than the minimum foreign bid, then the domestic firm tendering such a bid can raise its bid and increase profits without decreasing its probability of winning, implying that such a bid could not have been an equilibrium bid. Thus, $c_d(\underline{b}) = \underline{c}_d$ and $c_f(\underline{b}) = \underline{c}_f$.

Adding equations (5) and (6) together and rearranging yields:

$$c_d(b)c_f'(b) + c_f(b)c_d'(b) = c_f(b) + c_d(b) + c_f'(b)b + c_d'(b)b - (\overline{c}_f + \overline{c}_d) \tag{7}$$

or, recognizing that the right-hand side is the derivative of $\big(c_f(b) + c_d(b) - (\overline{c}_f + \overline{c}_d)\big)b$ with respect to b:

$$c_d(b)c_f'(b) + c_f(b)c_d'(b) = \Big[\big(c_f(b) + c_d(b) - (\overline{c}_f + \overline{c}_d)\big)b\Big]' \tag{8}$$

By integrating with respect to b, we obtain:

$$c_d(b) \cdot c_f(b) = [c_d(b) + c_f(b)] \cdot b - [\overline{c}_f + \overline{c}_d] \cdot b + \varsigma \tag{9}$$

where ς is the constant of integration. In order to determine ς, we require the maximum bid which is solved in our first lemma.

Lemma 1. *The upper bound of the bid functions, $\overline{b}$, is given by*

$$\overline{b} = \frac{\overline{c}_d + \overline{c}_f}{2}.$$

Proof. If the foreign firm has a cost greater than or equal to $c_f(\overline{b})$, it has no chance of winning and therefore bids its cost. Note that this implies that $c_f(\overline{b}) = \overline{b}$. This makes the

[8]Recall that firms with no chance of winning bid their costs.

probability of the domestic firm winning with a bid $\bar{b}$ equal to $\left[\frac{\bar{c}_f-\bar{b}}{\bar{c}_d-\underline{c}_d}\right]$. By definition of the inverse bid function, the domestic firm with cost $c_d(\bar{b})$ does not benefit from bidding more than $\bar{b}$, meaning that expected profits must be such that:

$$(\bar{b}-c_d(\bar{b}))\left[\frac{\bar{c}_f-\bar{b}}{\bar{c}_d-\underline{c}_d}\right] \geq (b-c_d(\bar{b}))\left[\frac{\bar{c}_f-b}{\bar{c}_d-\underline{c}_d}\right], \quad \forall\, b \geq \bar{b}.$$

This can be rewritten as:

$$-\left(\bar{b}-\left(\frac{c_d(\bar{b})+\bar{c}_f}{2}\right)\right)^2 \geq -\left(b-\left(\frac{c_d(\bar{b})+\bar{c}_f}{2}\right)\right)^2 \tag{10}$$

Since this has to hold for all $b \geq \bar{b}$, this requires that $\bar{b} \geq \frac{c_d(\bar{b})+\bar{c}_f}{2}$. Similarly, by definition of the maximum bid, the domestic firm with cost $c_d(\bar{b})$ cannot benefit from bidding below $\bar{b}$:

$$(\bar{b}-c_d(\bar{b}))\left[\frac{\bar{c}_f-\bar{b}}{\bar{c}_d-\underline{c}_d}\right] \geq (b-c_d(\bar{b}))\left[\frac{\bar{c}_f-c_f(b)}{\bar{c}_d-\underline{c}_d}\right], \quad \forall\, b \leq \bar{b}.$$

However since $c_f(b) \leq b$, we have

$$(\bar{b}-c_d(\bar{b}))\left[\bar{c}_f-\bar{b}\right] \geq (b-c_d(\bar{b}))\left[\bar{c}_f-b\right], \quad \forall\, b \leq \bar{b}.$$

This can happen only if $\bar{b} \leq \frac{c_d(\bar{b})+\bar{c}_f}{2}$. Combining these then implies that:

$$\bar{b} = \frac{\bar{c}_d+\bar{c}_f}{2}. \tag{11}$$

$\square$

Note that this is the upper bound of the bid function, not the maximum bid tendered by a foreign firm. Because $\bar{b}$ is the average of the two upper limits of the cost distributions, for a positive tariff it is strictly less than the highest foreign firm cost. This implies that for a positive tariff, a high-cost foreign firm will bid its cost which exceeds this level. Recalling that for both firms to be able to compete that $\overline{c}_d > 2\underline{c}_f - \overline{c}_f$, this maximum bid pins down the minimum value the government places on the project, implying that $V \geq \frac{\overline{c}_d+\tilde{c}_f}{2}$ where $\tilde{c}_f$ is the highest foreign cost inclusive of a tariff. Intuitively, this valuation implies that for any tariff in which both firms compete, the government will choose to accept the winning bid in equilibrium.

Returning to (9), we can use the maximum bid to solve for the constant of integration. Recalling that $c_f(\overline{b}) = \overline{b}$, evaluating (9) at $\overline{b}$ reduces to:

$$\varsigma = \left(\frac{\overline{c}_d + \overline{c}_f}{2}\right)^2. \tag{12}$$

This allows us to rewrite (9) as:

$$c_d(b) \cdot c_f(b) = [c_d(b) + c_f(b)] \cdot b - [\overline{c}_f + \overline{c}_d] \cdot b + \left(\frac{\overline{c}_d + \overline{c}_f}{2}\right)^2 \tag{13}$$

With this in hand, we can find the lower bound of the bid function, $\underline{b}$.

Lemma 2. *The lower bound of the bid function, $\underline{b}$ is given by*

$$\underline{b} = \frac{\left(\frac{\overline{c}_d+\overline{c}_f}{2}\right)^2 - \underline{c}_d \cdot \underline{c}_f}{(\overline{c}_d - \underline{c}_d) + (\overline{c}_f - \underline{c}_f)}. \tag{14}$$

Proof. Recall that the lowest cost firms both submit bids of $\underline{b}$, i.e. $c_d(\underline{b}) = \underline{c}_d$ and $c_f(\underline{b}) = \underline{c}_f$. Using this and evaluating (13) at $\underline{b}$, the above result is found. □

2.1.1 Equilibrium Bid Functions

In order to solve for our bid functions, we need to reduce our two differential equations to one. Using (13), we can solve for $c_f(b)$ in terms of $c_d(b)$

$$c_f(b) = \frac{c_d(b)b - [\overline{c}_f + \overline{c}_d]b + \left(\frac{\overline{c}_d+\overline{c}_f}{2}\right)^2}{c_d(b) - b} \tag{15}$$

Plugging this into (6) yields:

$$-\left[\left(\frac{\bar{c}_d+\bar{c}_f}{2}\right)-b\right]^2 c_d'(b)=\left[\bar{c}_d-c_d(b)\right]\left[c_d(b)-b\right]. \tag{16}$$

Rearranging this, we are left with the single differential equation

$$\big[(\bar{c}_d+\bar{c}_f)-2b\big]^2 c_d'(b)=4\big[c_d(b)-\bar{c}_d\big]\big[c_d(b)-b\big]. \tag{17}$$

Note that this solution not only satisfies the first order conditions but, as proven by Griesmer, et al. (1967), the second order conditions as well. We can now solve for the analytic solutions of the equilibrium bid functions.

Proposition 1. *The equilibrium inverse bid functions are given by*

$$c_d(b)=\bar{c}_d-\frac{(\bar{c}_f-\bar{c}_d)^2}{4(b-\bar{c}_f)+(2b-(\bar{c}_d+\bar{c}_f))\lambda_1\exp\left(\frac{\bar{c}_f-\bar{c}_d}{\bar{c}_d+\bar{c}_f-2b}\right)} \tag{18}$$

$$c_f(b)=\bar{c}_f-\frac{(\bar{c}_f-\bar{c}_d)^2}{4(b-\bar{c}_d)+(2b-(\bar{c}_d+\bar{c}_f))\lambda_2\exp\left(\frac{\bar{c}_d-\bar{c}_f}{\bar{c}_d+\bar{c}_f-2b}\right)} \tag{19}$$

where

$$\lambda_1=-\left[\frac{\exp\left(\frac{\bar{c}_d-\bar{c}_f}{2(\bar{b}-\underline{b})}\right)}{2(\bar{b}-\underline{b})}\right]\left[\frac{(\bar{c}_f-\bar{c}_d)^2}{\bar{c}_d-\underline{c}_d}+4(\bar{c}_f-\underline{b})\right]<0 \tag{20}$$

$$\lambda_2=-\left[\frac{\exp\left(\frac{\bar{c}_f-\bar{c}_d}{2(\bar{b}-\underline{b})}\right)}{2(\bar{b}-\underline{b})}\right]\left[\frac{(\bar{c}_f-\bar{c}_d)^2}{\bar{c}_f-\underline{c}_f}+4(\bar{c}_d-\underline{b})\right]<0. \tag{21}$$

Proof. First define $\alpha\equiv(\bar{c}_d+\bar{c}_f)-2\bar{c}_d=\bar{c}_f-\bar{c}_d$, $x\equiv b-\bar{c}_d$, and $D(x)$ such that

$$c_d(b)=\frac{\alpha^2}{D(x)}+\bar{c}_d \tag{22}$$

We then have $c_d'(x) = -\frac{\alpha^2}{D(x)^2}D'(x)$, and equation (17) becomes

$$
\begin{aligned}
-\frac{\alpha^2}{D(x)^2}D'(x)(\alpha-2x)^2 &= 4\left[\frac{\alpha^2}{D(x)} + \bar{c}_d - \bar{c}_d\right]\left[\frac{\alpha^2}{D(x)} + \bar{c}_d - b\right], \\
-\frac{\alpha^2}{D(x)^2}D'(x)(\alpha-2x)^2 &= 4\left[\frac{\alpha^2}{D(x)}\right]\left[\frac{\alpha^2}{D(x)} - x\right], \\
D'(x)(\alpha-2x)^2 &= 4\left[xD(x) - \alpha^2\right], \\
D'(x)(\alpha-2x)^2 &= 4xD(x) - 16x(\alpha-x) - 4(\alpha-2x)^2, \\
(D'(x)+4)(\alpha-2x)^2 &= 4x[D(x) - 4(\alpha-x)],
\end{aligned}
$$

Furthermore

$$
\begin{aligned}
\frac{D'(x)+4}{D(x)-4(\alpha-x)} &= \frac{4x}{(\alpha-2x)^2} \\
&= \frac{2\alpha}{(\alpha-2x)^2} - \frac{2}{\alpha-2x}
\end{aligned}
$$

By integrating both sides, we obtain

$$
\ln\big(D(x) - 4(\alpha-x)\big) = \frac{\alpha}{\alpha-2x} + \ln(\alpha-2x) + \ln\lambda_1,
$$

and taking the exponent of both sides yields

$$
D(x) = (\alpha-2x)\lambda_1 e^{\frac{\alpha}{\alpha-2x}} + 4(\alpha-x) \tag{23}
$$

where λ_1 is a constant of integration. The lower boundary condition $c_d(\underline{b}) = \underline{c}_d$ determines λ_1. When $b = \underline{b}$, we have $x = \underline{x} \equiv \underline{b} - \bar{c}_d$. From the definition, it follows that $D(\underline{x}) = \frac{\alpha^2}{\underline{c}_d - \bar{c}_d}$. Hence the boundary condition becomes

$$
\lambda_1 = \left[\frac{e^{-\frac{\alpha}{\alpha-2\underline{x}}}}{\alpha-2\underline{x}}\right]\left[\frac{\alpha^2}{\underline{c}_d - \bar{c}_d} - 4(\alpha-\underline{x})\right]
$$

which can be rewritten as (recall the definition of α and that $\overline{b} = \frac{\overline{c}_d+\overline{c}_f}{2}$)

$$\lambda_1 = -\left[\frac{\exp\left(\frac{\overline{c}_d-\overline{c}_f}{2(\overline{b}-\underline{b})}\right)}{2(\overline{b}-\underline{b})}\right]\left[\frac{(\overline{c}_f-\overline{c}_d)^2}{\overline{c}_d-\underline{c}_d}+4(\overline{c}_f-\underline{b})\right] < 0. \tag{24}$$

The analogous sequence of steps results in the foreign inverse bid function and λ_2. □

Up to this point, we have not discussed precisely how the foreign cost depends on the tariff. With these results in hand, we now do so by considering the specific and ad valorem tariffs in turn.

2.1.2 Specific Tariff

Given the assumption that the cost distributions differ only in tariffs, with a specific tariff the foreign cost distribution is simply the domestic one shifted upwards by τ. In order to simplify the analysis, without loss of generality, we restrict the domestic firm's cost distribution to the unit interval.[9] Therefore, in this section we analyze the case in which the government charges a specific tariff which results in a foreign cost $c_f \in [\tau, 1+\tau]$. Note that this implies that the tariff is such that $0 \leq \tau < 2$ and, by plugging in the minimum and maximum bids, that $\frac{-(4-\tau^2)}{4} \leq \eta_1 \equiv 2b-2-\tau \leq 0$ with strict equality at the minimum and maximum bids. Thus, the inverse bid functions can be simplified to:

$$c_d(b) = 1 - \frac{\tau^2}{4(b-1-\tau)+\eta_1\lambda_1\exp\left(\frac{-\tau}{\eta_1}\right)} \tag{25}$$

$$c_f(b) = 1 + \tau - \frac{\tau^2}{4(b-1)+\eta_1\lambda_2\exp\left(\frac{\tau}{\eta_1}\right)} \tag{26}$$

[9]With a specific tariff, the only implication of this range is for the minimum value of V. As will be discussed below, this assumption does, however, have additional implications in the ad valorem case. However, in order to compare our results to Branco (!994) who makes this assumption, we will continue to use it there as well.

where

$$\overline{b} = \frac{2+\tau}{2}, \qquad \underline{b} = \frac{(2+\tau)^2}{8},$$
$$\lambda_1 = -\left[\frac{2\big(2+\tau\big)}{2-\tau}\right] \exp\left(\frac{-4\tau}{4-\tau^2}\right), \text{ and} \qquad \lambda_2 = \frac{4}{\lambda_1}.$$

Further, notice that in both of these bid functions, the fraction terms are non-negative (since costs are bounded from above by the maximum costs for each type of firm). In addition, by using the maximum bid, it must be that $\eta_1 \leq 0$; with equality only at the maximum bid. In addition, as $\tau \to 0$, we have the symmetric case. Using L'Hôpital's Rule, we have

$$\lim_{\tau \to 0} c_d(b) = 2b - \overline{c}_d \tag{27}$$

$$\lim_{\tau \to 0} c_f(b) = 2b - \overline{c}_d. \tag{28}$$

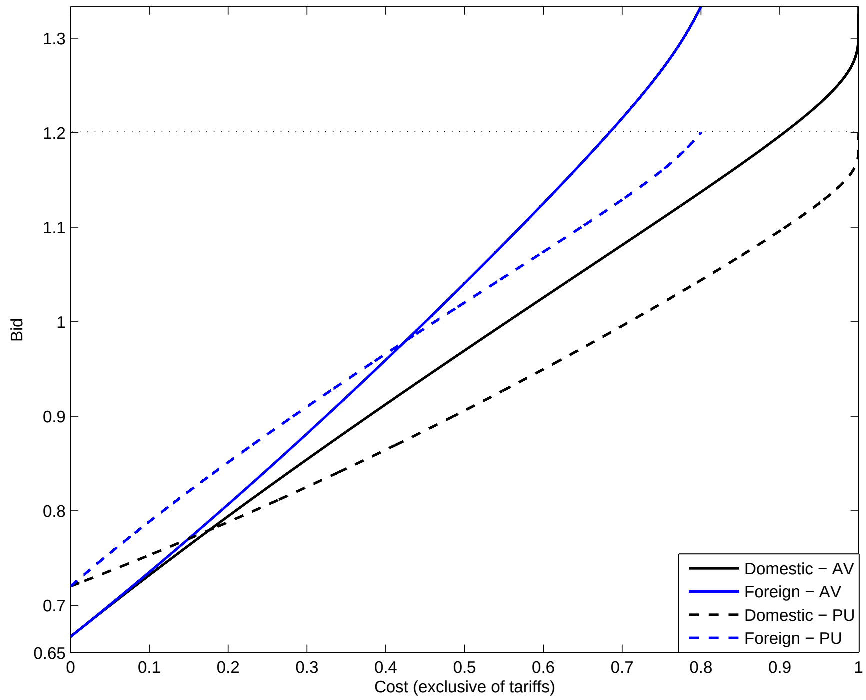
These will be useful for evaluating welfare under free trade. Also, note that in this case, the maximum bid is $\overline{c}_d = 1$.

In Figure 1, we illustrate the bid functions corresponding to these these inverse bid functions. We plot these as a function of firm costs where the foreign cost is exclusive of $\tau > 0$. For comparison and space issues, we also plot the bid functions corresponding the case in which the government charges an ad valorem tariff; the scenario we explore in more detail in the next section. This demonstrates some of the features of the bidding functions discussed above. First, the lowest cost domestic and foreign firms both submit the minimum bid. Second, the highest cost domestic firm submits the maximum bid. Third, for foreign firms with costs in excess of $\overline{b}$, they bid their cost and have a zero probability of winning. This is why the foreign bid function does not extend beyond $\overline{b}$. We analyze the characteristics of these bid functions in further in the following proposition and lemmas.

Proposition 2. *The domestic firm's bid is increasing in the specific tariff.*

Proof. Begin by considering a domestic firm with a cost $c_d(b) < \overline{c}_d$. Taking the derivative of

Figure 1: Bid Functions $\tau = t = 0.4$



(25) with respect to the tariff yields:

$$\frac{dc_d(b)}{d\tau} = (c_d(b) - 1)\left[\frac{2}{\tau} + \frac{4 + \eta_1\lambda_1 \exp\left(\frac{-\tau}{\eta_1}\right)\left[\xi - \frac{\tau}{\eta_1^2}\right]}{4(b - 1 - \tau) + \eta_1\lambda_1 \exp\left(\frac{-\tau}{\eta_1}\right)}\right] \tag{29}$$

where

$$\xi \equiv \left(\frac{8\tau^2}{(4-\tau^2)^2}\right) = \left(\frac{1}{[\eta_1(\underline{b})]^2}\right)\frac{\tau^2}{2}.$$

This makes the second term positive and, since $c_d(b) < \bar{c}_d = 1$, the inverse cost function is weakly decreasing in the tariff. This means that as the tariff rises, the domestic firm cost associated with a given bid declines. Put differently, as the bid function is monotone in costs, the bid associated with a given cost increases. Further, because a domestic firm with the maximum cost $\bar{c}_d$ bids the maximum bid, $\bar{b} = \frac{2+\tau}{2}$, such a firm's bid also rises. □

Thus, as the foreign firm is disadvantaged due to the tariff, the bid submitted by the domestic firm increases. Turning to the foreign firm's inverse bid function, we are able to identify two properties of the relationship between it and the tariff. The first shows that, like the domestic firm, the foreign inverse bid function is non-increasing in the tariff when beginning from free trade.

Lemma 3. *When domestic costs are distributed uniform on the unit interval, at a zero tariff, the foreign inverse bid function is non-increasing in the specific tariff.*

Proof. For when $\overline{c}_d = 1$ and $\underline{c}_d = 0$,

$$\lim_{\tau \to 0} \frac{dc_f(b)}{d\tau} = -\left[\frac{1}{3} + \frac{8(b-1)^3}{3}\right]$$

Recall that with a zero tariff, the foreign inverse bid function is $c_f(b) = 2b - \overline{c}_d$, thus bids will be between .5 and 1. Therefore this expression will range from 0 for the lowest cost foreign firm to $\frac{-1}{3}$ for the highest cost foreign firm. □

Beginning from free trade and introducing a tariff, for a given bid this means that the associated cost inclusive of the tariff falls. Alternatively, a firm with a given cost increases its bid. It is important to note, however, that it increases its bid by no more than one. This implies that, although it increases its bid in an effort to cover the rise in costs, only the highest cost foreign firm fully passes through the cost of the tariff. For firms with less than this cost, they choose to absorb part of the tariff increase in order to better their chances of winning the contract. This introduces a tension for the foreign firm between its desire to still win the contract (leading it to absorb the tariff) and its desire to maximize the profit of winning (leading it to pass through the tariff). Unlike the domestic firm where there is no such tradeoff, this tension prevents us from signing the change in the foreign inverse bid function with respect to a positive tariff for the full range of inverse costs. Nevertheless, we can show the following result.

Lemma 4. *The foreign firm's inverse bid function is decreasing in the specific tariff for bids* $b \in \left[\underline{b}, \hat{b}\right)$. *Further, for bids outside this range,* $\frac{dc_f(b)}{d\tau} < 1$.

Proof. Focusing on the lower bound of the bid and cost space for the foreign firm, as the tariff rises there are two things that change in response to a tariff, $\underline{b}$ and $\underline{c}_f$ (which, recall, is inclusive of the tariff). Specifically,

$$\frac{d\underline{b}}{d\tau} = \frac{2+\tau}{4} \text{ and}$$
$$\frac{d\underline{c}_f}{d\tau} = 1.$$

Recalling that the minimum bid is submitted by the lowest cost firm, these combine so that the inverse bid function moves such that: $\frac{dc_f(\underline{b})}{d\tau} = \frac{\tau}{2+\tau}$ which is between 0 and 1 for any positive tariff. Further, since $\tau \leq 2$, it follows that

$$\frac{d\underline{b}}{d\tau} \leq \frac{d\underline{c}_f}{d\tau}.$$

By continuity and the fact that $c_f(b)$ is monotonically increasing in b, then for bids close to $\underline{b}$, the same holds.

However, even if the foreign inverse bid function increases in the tariff, meaning that a firm with a given cost lowers its bid as the tariff increases, it will not absorb the full amount of the tariff increase. To see this, suppose that it instead lowers its bid by the same amount as the increase in the tariff. In this case, its bid would stay the same, meaning so too would its probability of winning. If that were true, the domestic firm would not change its bid either, implying no change in foreign's probability of winning. Recalling that the optimal bid balances the gain from a higher winning bid (the probability of winning) against the loss from a lower chance of winning (the bid less the cost times the change in the probability of winning) absorbing the full cost of the tariff would not affect the first but would lower the second. As such, that would not be profit maximizing for the foreign firm. Combining this fact with the monotonicity of the inverse bid function therefore implies that $\frac{dc_f(b)}{d\tau} < 1$ even if it is not negative. □

Thus for at least some range of costs, the foreign firm increases its bid as the tariff rises even when the tariff is positive. This is because for firms with low costs, the probability of winning is sufficiently large that they are willing to trade off a lower chance of winning with a higher payoff if it does win. One implication of this is that it implies an increase in the expected profits of the domestic firm.

Lemma 5. *Expected domestic profits are increasing in the specific tariff for all values of* c_d*.*

Proof. Inserting the domestic bid function into its first order condition, taking the derivative with respect to τ, using the envelope theorem, and using the result that $\frac{dc_f(b)}{d\tau} < 1$: $\frac{dE[\pi_d]}{d\tau} = (b_d(c_d) - c_d)\left[\frac{1 - \frac{dc_f(b_d(c_d))}{d\tau}}{\overline{c}_d - \underline{c}_d}\right] > 0$ i.e. given its cost, the domestic firm's expected profits rise following the tariff increase. □

2.1.3 Ad Valorem Tariff

In this section, we consider an ad valorem tariff instead of a specific tariff. Given that Proposition 1 did not rely on the mapping between tariffs and foreign costs, it is not surprising that the nature of the results are quite similar. Therefore in this section, we focus on the difference in bidding behavior between the ad valorem and specific tariffs. This primarily is about what happens when $t \to 1$ under our assumption that the lowest domestic cost is zero.

To maintain tractability, we assume the tariff is placed on the bid which we will define as t. Note that, since the bid represents the value of the foreign firm's activity, i.e. the value of its imports, this is not unreasonable. This changes the expected profit of the foreign firm to be equal to

$$E[\pi_f] = [(1-t)b_f - c_f]\left[\frac{\overline{c}_d - c_d(b_f)}{\overline{c}_d - \underline{c}_d}\right]. \tag{30}$$

This can be rewritten as

$$E[\pi_f] = (1-t)\left[b_f - \frac{c_f}{(1-t)}\right]\left[\frac{\overline{c}_d - c_d(b_f)}{\overline{c}_d - \underline{c}_d}\right] = (1-t)\left[b_f - C_f\right]\left[\frac{\overline{c}_d - c_d(b_f)}{\overline{c}_d - \underline{c}_d}\right] \tag{31}$$

where $C_f = \frac{c_f}{(1-t)}$. This transformation allows us to utilize the methods previously used in the specific tariff case, where the foreign cost is now $C_f \sim U\left[0, \frac{1}{1-t}\right]$. Since multiplying the expected profit of this transformed profit function by $(1-t)$ is simply a monotonic transformation, the general bidding strategies are the same as equations (18) and (19). Using the transformed bounds on the foreign cost distribution, we have

$$c_d(b) = 1 - \frac{\left(\frac{t}{1-t}\right)^2}{4\left(b - \frac{1}{(1-t)}\right) + \eta_2 \lambda_1 \exp\left(\frac{-t}{(1-t)\eta_2}\right)} \tag{32}$$

$$C_f(b) = \frac{1}{1-t} - \frac{\left(\frac{t}{1-t}\right)^2}{4(b-1) + \eta_2 \lambda_2 \exp\left(\frac{t}{(1-t)\eta_2}\right)} \tag{33}$$

where

$$\underline{b} = \frac{(2-t)}{4(1-t)}, \qquad \overline{b} = \frac{(2-t)}{2(1-t)},$$

$$\lambda_1 = -2\left[\frac{\exp\left(\frac{-2t}{(2-t)}\right)}{(1-t)}\right] < 0, \qquad \lambda_2 = \frac{4}{\lambda_1} < 0, \text{ and}$$

$$\eta_2 = 2b - \left(\frac{2-t}{1-t}\right).$$

Note that as $t \to 1$, foreign firm expected profits go to zero. However, as can be seen by the fact that $\overline{b} - \underline{b} = \frac{(2-t)}{4(1-t)}$, the bid space does not collapse. This is because the lowest exclusive-of-tariff cost foreign firm has a cost of zero. As a result, marking this up by $\frac{1}{1-t}$ still results in a zero cost, meaning that such a firm is not driven from the bidding game for a $t \leq 1$.[10] Given the continuity of the bid functions, foreign firms with a sufficiently small cost but greater than zero will still bid as long as $t < 1$. In fact, unlike in the specific case where $\overline{c}_F \to 0$ as we approached the prohibitive tariff, for the ad valorem case, we have $\overline{c}_f \to 0.5$ as we approach the prohibitive tariff; we show this formally in the appendix. This will have consequences when we analyze the home government's welfare at various tariff

[10]When $\underline{c}_d > 0$, however, there again is a prohibitive tariff since there will be a tariff for which $\overline{c}_d > \frac{2\underline{c}_d - \overline{c}_d}{1-t}$, i.e. in which even the highest cost domestic firm wins with certainty.

levels. Outside of this, however, the bid functions behave qualitatively similarly across the the two regimes as illustrated in Figure 1. One difference worth noting, however, is that the gap between the domestic and foreign bids widens faster for the specific tariff than the ad valorem tariff as the exclusive-of-tariff cost rises. This is because the size of the burden created by the ad valorem tariff is proportional to the foreign firm cost whereas the specific tariff is not. As a result, the tariff has less impact on low-cost foreign firms resulting in smaller differences in bidding behaviors.

3 The Government's Optimal Tariff

In this section we analyze the government's optimal tariff under the two tariff policies; specific and ad valorem. In both, the government sets the relevant tariff to maximize expected welfare, which is the sum of the value of the project, the expected payoff conditional on the domestic firm winning, and the expected payoff conditional on the foreign firm winning. In this, the government weights the domestic firm's profits by $\theta \in [0,1]$. Such weighting is comparable to McAfee and McMillan (1989). If $\theta = 0$, the profit of the domestic firm has no effect on the government's payoff and if $\theta = 1$, the domestic firm's profit fully enters the government's payoff (as it does in Branco (1994)).

3.1 Specific Tariff

With a specific tariff, conditional on a cost $\tilde{c}_d$ for the domestic firm, conditional expected welfare is:

$$\begin{aligned} W\left(\tilde{c}_d, \tau\right) = & \int_{\underline{c}_f}^{c_f(b_d(\tilde{c}_d))} \frac{1}{\overline{c}_f - \underline{c}_f} \left[V + \tau - b_f(c_f)\right] dc_f \\ & + \int_{c_f(b_d(\tilde{c}_d))}^{\overline{c}_f} \frac{1}{\overline{c}_f - \underline{c}_f} \left[V - b_d\left(\tilde{c}_d\right) + \theta\left(b_d(\tilde{c}_d) - \tilde{c}_d\right)\right] dc_f \end{aligned} \tag{34}$$

or, integrating across c_f:

$$W(\tilde{c}_d, \tau) = V + \frac{\overline{c}_f - c_f(b_d(\tilde{c}_d))}{\overline{c}_f - \underline{c}_f}((\theta - 1) b_d(\tilde{c}_d) - \theta \tilde{c}_d)$$
$$+ \left(\frac{c_f(b_d(\tilde{c}_d)) - \underline{c}_f}{\overline{c}_f - \underline{c}_f} \right) \tau - \frac{1}{\overline{c}_f - \underline{c}_f} \int_{\underline{c}_f}^{c_f(b_d(\tilde{c}_d))} b_f(c_f) dc_f. \tag{35}$$

To find expected welfare, it is then necessary to integrate across $\tilde{c}_d$, making expected welfare as a function of the tariff:

$$W(\tau) = \int_{\underline{c}_d}^{\overline{c}_d} \frac{1}{\overline{c}_d - \underline{c}_d} W(\tilde{c}_d, \tau) \, d\tilde{c}_d. \tag{36}$$

In order to compare our results to Branco's (1994) for the equilibrium price premium, from this point forward we assume that domestic costs are distributed on the unit interval. This allows us to rewrite expected welfare as:

$$W(\tau) = V + \int_0^1 \Bigg[[1 + \tau - c_f(b_d(\tilde{c}_d))] [(\theta - 1) b_d(\tilde{c}_d) - \theta \tilde{c}_d] + c_f(b_d(\tilde{c}_d)) \tau$$
$$- \tau^2 - \int_\tau^{c_f(b_d(\tilde{c}_d))} b_f(c_f) dc_f \Bigg] d\tilde{c}_d. \tag{37}$$

We can now state our next proposition.

Proposition 3. *The optimal specific tariff is non-negative and strictly positive whenever* $\theta >$ 0*, i.e. when domestic profits are positively valued. This optimal specific tariff is increasing in* θ*. In no case is the optimal specific tariff prohibitive.*

Proof. Taking the limit of the government's first order condition as $\tau \to 0$ we see that:

$$\lim_{\tau \to 0} \frac{dW}{d\tau} = \frac{3\theta}{10}. \tag{38}$$

Thus, the optimal tariff is positive as long as the government puts a strictly positive value on domestic firm profits in its welfare function; i.e. if $\theta > 0$.[11]

Furthermore, when $\tau = 0$, in which case the bid functions are linear function of costs, expected government welfare is $V + \frac{\theta}{6} - \frac{2}{3}$. When the tariff is prohibitive, (i.e. when $\tau = 2$ in the case of costs on the unit interval), expected government welfare is $W(2) = \left(V + \frac{3\theta - 4}{2}\right)$. Comparing the two, we see that:

$$W(0) - W(2) = \frac{4(1-\theta)}{3} \geq 0. \tag{39}$$

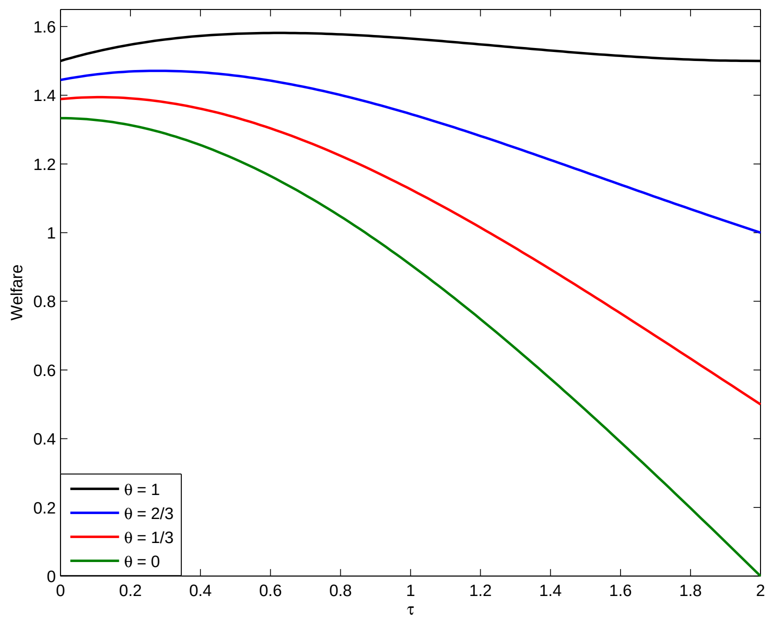
This implies that free trade never does worse than autarky and has strictly higher expected welfare whenever $\theta < 1$. Thus, combining these, when $\theta = 0$ a zero tariff is optimal and that when $\theta > 0$, the optimal tariff is between zero and 2, i.e. between free trade and autarky.

To recognize that the optimal tariff is increasing in θ, recognize that the weight on domestic profits enters the welfare function's first order condition only as $\theta \frac{dE(\pi_d)}{d\tau}$, that is, the weighted impact on expected domestic profits. Because these are increasing in the tariff for each cost, so too are ex-ante expected domestic profits, meaning that a higher θ increases this positive element in the government's first order condition, increasing the optimal tariff. □

This is illustrated using numerical analysis in Figure 2. Note that in this analysis, we have only considered non-negative tariff values to simplify discussion. If we were to permit negative tariffs, however, in equilibrium they would never be used. To recognize this, note that there is a discontinuity in the expected government welfare at a zero tariff which guarantees that only non-negative tariffs will be used. This is because when $\tau < 0$, it is no longer the case that the domestic firm has a (weak) cost advantage. Therefore the ranking of the two cost distributions reverse and the bid functions switch, i.e. the home firm bids according to (19) and the foreign firm bids according to (18). After making that adjustment, because the home government places no weight on the foreign firm's profits (similar to a $\theta = 0$), it has no incentive to subsidize the foreign firm, particularly as this

[11] The full derivation of equation (38) is available upon request.

Figure 2: Government Welfare



comes with the cost of a negative subsidy. Therefore, for values of θ below this cutoff, the optimal tariff is a corner solution and equal to zero.

As an alternative to the government welfare above where foreign profits are not valued, we could instead consider a social planner who seeks to maximize expected total surplus, that is the (unweighted) sum of expected government surplus, expected domestic profits, and expected foreign profits. It is straightforward to show that this is maximized by a zero tariff. This is because with a zero tariff, firms use the same bidding function, ensuring that the contract is awarded to the lowest cost firm. As the social planner is indifferent between surplus accruing to the government or either firm, minimizing the expected cost maximizes expected total surplus.

Lemma 6. *When $\theta = 0$ the equilibrium maximizes expected total surplus.*

3.2 Ad Valorem Tariff

With an ad valorem tariff, expected welfare is:

$$W(t) = V + \int_0^1 \Bigg[[1-(1-t)C_f(b_d(\tilde{c}_d))] [(\theta-1)\, b_d(\tilde{c}_d) - \theta\tilde{c}_d] - (1-t)^2 \int_0^{C_f(b_d(\tilde{c}_d))} b_f(C_f) dC_f \Bigg] d\tilde{c}_d. \tag{40}$$

As above, with free trade, expected welfare is

$$W(0) = V + \frac{(\theta-4)}{6}.$$

Unfortunately, for $t = 1$, expected welfare is not analytically calculable since this tariff is not prohibitive for the reasons described in Section 2.1.3. We can nevertheless express it as:[12]

$$\lim_{t\to 1} W(t) = V + \int_0^1 \left[\left[\frac{(\theta-1)\, c_d^{-1}(\tilde{b}) - \theta\tilde{c}_d}{1 + c_d^{-1}(\tilde{b})} \right] - \lim_{t\to 1} \left\{ (1-t)^2 \int_0^{C_f(b_d(\tilde{c}_d))} b_f(C_f) dC_f \right] d\tilde{c}_d \right\}$$

where

$$c_d(\tilde{b}) = 1 - \frac{1}{(1-\tilde{b}) \exp\left(\frac{2\tilde{b}}{(1-\tilde{b})}\right)}.$$

Similarly to the specific tariff case, we have the following proposition.

Proposition 4. *The optimal ad valorem tariff is non-negative and strictly positive whenever $\theta > 0$, i.e. when domestic profits are positively valued. This optimal ad valorem tariff is increasing in θ. In no case is the optimal ad valorem tariff equal to unity.*

Proof. Taking the limit of the government's first order condition as $t \to 0$ we see that:[13]

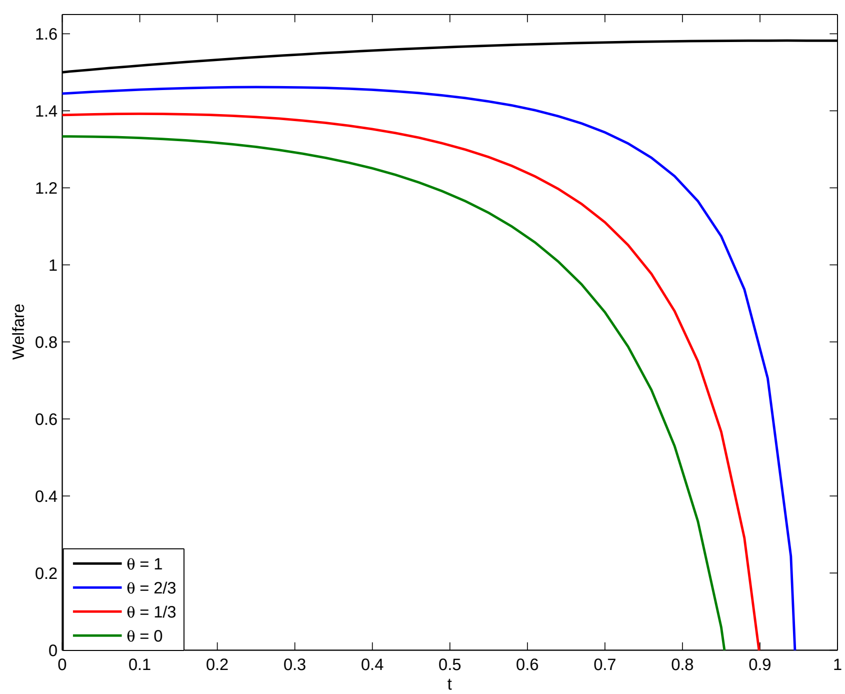
$$\lim_{t\to 0} \frac{dW}{dt} = \frac{11\theta}{60} > 0. \tag{41}$$

[12] See appendix for the derivation of $c_d(\tilde{b})$.
[13] The full derivation of equation (41) is available upon request.

Thus, the optimal ad valorem tariff is positive for any positive θ. Using numerical analysis, we find that when $\theta = 1$, the optimal tariff is approximately equal to $0.93353 < 1$. Also, recall (as shown in the appendix) that as $t \to 1$ the range of possible active foreign firms approaches $c_f \in [0, 0.5]$. Further, numeric analysis indicates that the optimal ad valorem tariff is monotonic in θ, ranging from 0 when $\theta = 0$ to this level when $\theta = 1$. Thus, no optimal ad valorem tariff is such that the government attempts to capture the entire foreign bid. □

Using numerical methods, we plot the welfare function as the tariff goes between 0 and 1 (free trade and prohibitive levels) in Figure 3. This illustrates the three features of the above proposition. Finally, note that since the optimal ad valorem tariff is zero when $\theta = 0$, identical to Lemma 6 the equilibrium ad valorem tariff maximizes total surplus in this special case.

Figure 3: Government Welfare



4 Relative Protectionism

A natural question is how the two tariff regimes compare not only two each other, but also to the price preference system. In particular, is there reason to be concerned that eliminating price preferences might lead governments to pursue second-best, less efficient methods of protection? To do so, we compare the equilibrium under the equilibrium specific tariff, the equilibrium ad valorem tariff, and the equilibrium price preference which is analyzed by Branco (1994). In his alternative auction, the winning firm is paid its bid, however, the winner need not be the lowest bid firm, since the domestic firm wins so long as the difference in bids does not exceed the price preference. In his paper, Branco derives, among other things, the optimal price preference under a sealed-bid first price auction when firm profits are valued the same as government surplus and the costs of both the domestic and foreign firm are distributed uniformly on the unit interval.[14] As shown in his paper, at the government's optimal price preference, equilibrium bids are:

$$b_d(c_d) = \frac{3 - c_d^2}{2(2 - c_d)} \tag{42}$$

and

$$b_f(c_f) = \max\left\{c_f, \frac{1 + 2c_f}{4}\right\} \tag{43}$$

with the domestic firm winning whenever

$$\frac{c_D(b_D) - c_F(b_F)}{c_F(b_f)} < 1. \tag{44}$$

To compare the three systems, we consider a measure of the level of protection as well as equilibrium expected welfare. As a measure of protection, we use the expected foreign firm profits. The higher this measure, the lower the level of ex-ante protection (i.e. ex-ante

[14] Among the other aspects of his model is that he allows for a deadweight loss to raising government revenues, something we do not consider. Thus, to draw the comparisons between his results and ours, set $\lambda = 0$ in his notation and $\theta = 1$ in ours.

to the determination of the foreign cost). Figure 4 provides this comparison by plotting the conditional expected foreign profit, where it is conditional on its cost c_f, across the three policies as a function of the exclusive-of-tariff foreign cost. Beginning with a low cost foreign firm, we see that both tariffs result in lower profits than the price premium. This is because a tariff is a cost to the firm.[15] Between the two tariffs, the specific tariff is more protectionist for these low cost foreigners than is the ad valorem tariff. This is because the cost of the ad valorem tariff is proportional to the bid. Since bids are monotonic in costs, low cost foreigners pay the least under the ad valorem tariff, making this less burdensome to them that the equilibrium "one size fits all" specific tariff. As the foreign cost rises, the ranking of the two tariffs reverses itself, precisely because the cost of the ad valorem tariff rises in the firm's cost whereas the cost of the specific tariff does not. Further, for a range of foreign costs around .5, the tariffs are less protectionist than the price premium. This range is markedly larger for the specific tariff (roughly exclusive-of-tariff costs in the range $(.47, .55)$) for the reason just discussed. Finally, high cost foreign firms have no probability of winning under any of the three equilibrium policies. Thus, conditional on the foreign cost, the ranking of protectionism varies with that cost. In order to gauge ex-ante protectionism, we calculate the unconditional expected foreign profit by integrating the probability weighted conditional expected foreign profits over the range of foreign costs exclusive of the tariff (i.e. $[0, 1]$). Note that this accounts for the different levels of protectionism for different foreign costs. The results of this are:

Table 1: Expected Foreign Profit Under Different Regimes

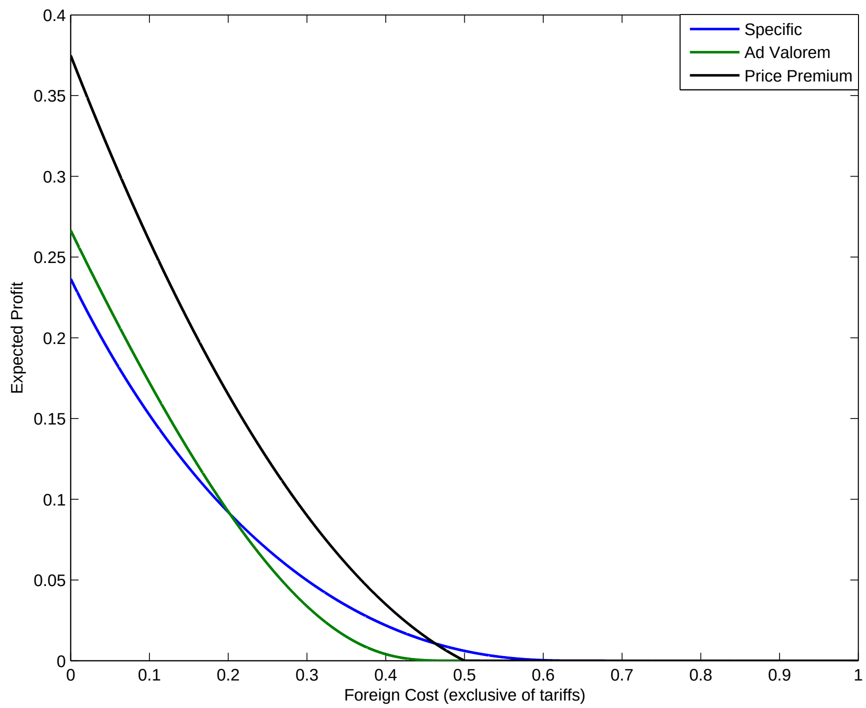
Specific Tariff	Ad Valorem Tariff	Price Premium	Free Trade
0.043253236	0.042724554	0.072916667	0.166666667

As can be seen, all three policies are ex-ante more protectionist than free trade. Further, both tariffs are ex-ante protectionist relative to the price premium, with the ad valorem tariff marginally more so. Although these results are driven by our assumptions, not the

[15]Recall that the ad valorem tariff is on the bid which is positive even when the exclusive-of-tariff cost, implying a payment from the firm to the home government.

least of which are those for the cost distributions, it does suggest that the recent shift in the approach towards foreign firms bidding for government contracts can be a step towards protectionism.

Figure 4: Expected Foreign Profit



Although it is tempting to equilibrate this increase in protectionism under tariffs with higher expected domestic welfare, it must be remembered that the bids tendered by foreigners differ across the three regimes and therefore so too does expected welfare. Calculating this numerically results in equilibrium expected welfare values of (where again, $\theta = 1$):

Table 2: Expected Welfare Under Different Regimes

	Specific Tariff	Ad Valorem Tariff	Price Premium	Free Trade
Optimal Tariff	0.62410	0.93353	–	–
Domestic Welfare	1.5814149	1.5821041	1.5833333	1.5
Global Welfare	1.6246682	1.6248286	1.6562500	1.6666667

From this, three things are seen. First, consistent with the above analytic results, domestic welfare is lowest with free trade whereas global welfare (the sum of domestic expected welfare and expected foreign profits) is highest with free trade. Second, the price premium is preferable to either tariff, whether considered at the domestic level where the policy is chosen or at the global level. This indicates that both tariffs are less desirable to the home government than the optimal price preference. This is due to the fact that, although tariff revenue is collected, foreign firms attempt to pass through a part of the tariff resulting in higher equilibrium bids and lowering expected welfare. Furthermore, since average foreign expected profits are lower under the two tariff regimes, this means that global welfare is also lower under the tariffs than under the price preference. As a result, it lends some credence to the concern that the Government Procurement Agreement may have shifted policy towards second-best methods of protection. Third, despite the fact that ad valorem tariffs are marginally more protectionist than specific tariffs, both domestic and global welfare are marginally greater for ad valorem tariffs. This is because ad valorem tariffs are less protectionist against low-cost foreigners, a feature which enhances expected efficiency in equilibrium.

5 Conclusion

The purpose of this paper has been to consider the role of trade barriers such as tariffs, on competition for government contracts. Comparable to the prior regime in which price preferences are used, tariffs act as a barrier for foreign firms and inhibit their ability to successfully bid for contracts. Further, similar to the results of McAfee and McMillan (1989) and Branco (1994), the domestic government will choose erect a barrier against foreign firms when domestic firm profits are positively valued. Despite this, however, these policies are not identical. In particular, we show that protectionism under either a specific or an ad valorem tariff is greater than that under the price preference. This results in a decline in both expected domestic welfare and expected total surplus. Therefore moving from the previous regime with price preferences need not result in the desired goal of an equal playing field for all firms; in fact, it can represent a step in the opposite direction.

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A Appendix

In this appendix we investigate the inverse bid functions under an ad valorem tariff, as $t \to 1$. First, we look at the foreign firm. Recall that

$$c_f(b) = (1-t)C_f(b) = 1 - \frac{t^2}{(1-t)\left[4(b-1) + \eta_2\lambda_2 \exp\left(\frac{t}{(1-t)\eta_2}\right)\right]} \tag{A-1}$$

When taking the limit as $t \to 1$, the difficulty is that the bid space becomes unbounded. Thus, we transform b into a monotonic function of $\tilde{b}$ where $\tilde{b} \in [0,1]$. Let

$$b = \underline{b} + \left[\overline{b} - \underline{b}\right]\tilde{b} = \frac{(2-t)}{4(1-t)}(1+\tilde{b}).$$

After this transformation, we can rewrite our foreign inverse bid function as

$$c_f(\tilde{b}) = 1 - \frac{t^2}{\left[(2-t)(1+\tilde{b}) - 4(1-t) - (2-t)(1-t)(\tilde{b}-1)\exp\left(\frac{-2t\tilde{b}}{(2-t)(1-\tilde{b})}\right)\right]}. \qquad \text{(A-2)}$$

Now taking the limit, we have

$$\lim_{t\to 1} c_f(\tilde{b}) = \left[\frac{\tilde{b}}{(1+\tilde{b})}\right] \in \left[0, \frac{1}{2}\right]. \qquad \text{(A-3)}$$

Now looking at the domestic firm, using the same transformation to the bid:

$$c_d(\tilde{b}) = 1 - \frac{t^2}{(2-t)\left[(1-t)\left((1+\tilde{b}) - \frac{4}{(2-t)}\right) - (\tilde{b}-1)\exp\left(\frac{2t\tilde{b}}{(2-t)(1-\tilde{b})}\right)\right]} \qquad \text{(A-4)}$$

Now taking the limit, we get

$$\lim_{t\to 1} c_d(\tilde{b}) = 1 - \frac{1}{(1-\tilde{b})\exp\left(\frac{2\tilde{b}}{(1-\tilde{b})}\right)}. \qquad \text{(A-5)}$$