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Optimal Environmental Policy with Network Effects: Will Pigovian Taxation Lead to Excess Inertia?

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Abstract

We study a dynamic model with two competing durable goods; one dirty, the other clean. Due to network effects a consumer who adopts the dirty good today will increase the incentive future consumers have to adopt the dirty good. Thus, a consumer who chooses the dirty good, in a sense causes more pollution than just his own. This "externality multiplier effect" may warrant a dirty good tax in excess of the Pigovian tax. If this is not acknowledged, the market may stay with the dirty good even if it is socially beneficial to shift to the clean good.

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1 Introduction

The solution to an environmental problem often involves replacing an old dirty technology with a new clean technology. According to Barrett (1999), for instance, technological innovation was crucial for the success of the Montreal protocol protecting the ozone layer. The solution to the climate change problem is far more complex. GHG emissions must be abated across several sectors in society requiring innovation and diffusion of a number of new technologies. The question then arises; can we rely on a standard, flat carbon tax to induce innovation of the new, clean technologies, and will the new technologies diffuse to the socially efficient extent?

According to several authors the answer could be no: The market entry of carbon free technologies suffers from *excess inertia*, and a carbon tax equal to the social cost of carbon may not be sufficient to induce the necessary technological shift. Acemoglu et al (2012), Chakravorty, Leach and Moreaux (2011), and Greaker and Heggedal (2010) all argue that this may be the case for carbon free technologies.

Many mechanisms may lead to such situations of excess inertia. Acemoglu et al (2012) describe a process of market-driven directed research in which dirty technologies steadily improve and clean technologies are not developed further. Chakravorty, Leach and Moreaux (2011) find that fossil fuel resource owners have an incentive to slow learning in the alternative emissions-free technology by increasing their own extraction. In this paper, we will focus on *network effects* as a potential source for excess inertia, and ask whether a welfare-dominant clean technology will diffuse efficiently when there are network effects, and the clean technology is competing against an incumbent dirty technology.

Positive network effects arise if one agent's adoption of a good (a) benefits other adopters of the good; and (b) increases others' incentive to adopt it (Farrell and Klemperer, 2007). The literature so far has failed to agree on whether the market outcome will be efficient when network effects are present. For concreteness, let there be two technologies, where one is welfare-dominating the other. When players have complete information about each other's payoffs, the adoption problem is a coordination problem. In the absence of any installed base of users for any technology, an uncoordinated adoption process will lead to the efficient outcome (Farrell and Saloner, 1985). If players have private information about their own payoffs, or if there is an installed base of users of the inferior technology, you may get under-adoption, or too slow adoption of the superior technology (Farrell and Saloner, 1986). This is defined as *excess inertia*. It may arise because early adopters carry the burden of paving the way for later adopters, and may not find it worthwhile if they are uncertain about whether others will join them in the future, or if they instead can join an already matured (although ultimately inferior) network.

This result of under-adoption will also be the case in the presence of environmental externalities, and thus makes it relevant to the environmental regulator. Our first research question is thus: *Should environmental policy be adjusted when there are network effects?* In order to answer this, we study a dynamic model with two competing technologies, where both feature network effects. One technology is *clean*, the other is *dirty* and exerts an environmental externality. We find that; *yes*, optimal environmental policies should take into account the network effect, for instance through making the tax rate depend on the market share of the clean technology.

At first glance, this result may not look surprising. In the absence of other instruments than the environmental policy, one might suspect that environmental policy is being used to correct for both the network externality and the environmental externality requiring a tax rate above the Pigovian rate. However, the explanation is more complex. In a dynamic model the network effect will interact with the environmental externality. That is, a consumer who adopts a dirty good will make the dirty good more attractive to others in the future, and *ceteris paribus*, cause more future pollution than just his own. With the standard Pigovian tax, the consumer would only face a tax equal to the social costs of his own emissions, and not of those he might induce through the network effects. In other words, even if the network effects were internalized, the optimal emission tax should depart from the Pigovian tax.

Liebowitz and Margolis (1994) argue that the definition of inefficiency is that the benefits of an unrealized outcome must exceed its costs. If so, these benefits can be exploited by private agents with profit motives. Hence, they argue, inefficient outcomes due to network effects will rarely be observed. As far as we can tell, this Coasian argument will only hold under strict assumptions about transaction costs and property rights. But if it holds, it should in principle hold even if a network technology exerts a negative environmental externality, as long as the dirty technology faces a tax corresponding to the social cost of emissions.

To study how private agents may coordinate the network adoption process, define a *technology sponsor* to be a monopolist supplier of a network good. Such a sponsor could, in principle, play the role envisioned by Liebowitz and Margolis, as he can supply the network good at a loss today, in order to reap future benefits when the network matures. Sponsors were first introduced by Katz and Shapiro (1986), and they found that having a sponsor is crucial for the market development of a new technology. In particular, they find that if the superior technology has a sponsor, it will dominate the market. But conversely, if only the inferior technology is sponsored, it could also end up as the dominant technology.

Our second research question is then: *Does the need for policy adjustment depend on the existence of sponsors?* We find that; *yes*, the need to adjust the Pigovian tax depends heavily on the assumed market structure. When there are technology sponsors, the government effectively engages in a regulation game, which can result in taxes that are more extreme than in the competitive scenario.

Finally, we ask: *May a failure to internalize the network effects lead to excess inertia?* To answer this, we simulate a numerical version of the model to compare counterfactual industry evolutions. We find that a failure to internalize the network effects in the taxation may lead to excess inertia, *even* in the case where only the clean technology is sponsored. Due to the interaction between the network effects and the environmental externality, the social gain from clean adoption exceeds the private gain, even under Pigovian taxation. The clean sponsor thus settles for a too low market share, from a social point of view. In this case the government can improve on social welfare by subjecting the dirty technology to an emission tax in excess of the social cost of emissions.

This result is counter to the results of Katz and Shapiro (1986). In their model, the two technologies only differ with respect to their network sizes, while in our model, the technologies differ with respect to both network size and product characteristics. Since products are differentiated, firms may settle for a socially inefficient low market share, and only harvest the willingness to pay of the most eager consumers. Thus, in our model, an inferior technology may dominate the market even if only the superior technology has a sponsor, as long as the inferior technology starts out with a market leadership.

We also find examples of the opposite effect: Cases exist where it is efficient not to induce the technological shift, even though the clean technology is superior, yet the shift will take place under the naive Pigovian tax. The reason is that the structure of the network effects could impose too high costs on those consumers who will (temporarily) be stranded in the dirty network, making the shift not worthwhile. Network effects may thus make the technological shift more or less beneficial than what is dictated by the environmental externality alone. This will be taken into account by the optimal tax, but not by the Pigovian tax, which might induce consumers to coordinate on a welfare-decreasing technological shift.

Although Liebowitz and Margolis (1994) might be right in principle, it is hard to envision real-world economic institutions that can support such an efficient outcome. More specifically, Segal (1999) studies contracting under externalities, and outline sufficient conditions for when a network sponsor may contract with the adopters to achieve the efficient adoption. He defines a two-stage game in which the sponsor first extends offers to all buyers, and then the buyers simultaneously decide whether they want to adopt or not. He finds that if the sponsor makes public offers, and can commit to them, then as long as there are only network effects present for this current good, then the principal can achieve efficient adoption.

Ochs and Park (2010) extend the analysis in Farrell and Saloner (1986), and find that if consumers can time their entry endogenously (so that the most eager consumers move first) and entry decisions are irreversible (so that no network ever declines in size), then as the discount factor tends to one, and the size of the population of adopters tends to infinity, any coordination problem found by Farrell and Saloner (1986) vanishes, and the equilibrium becomes efficient. This illustrates how hard it would be to envision efficiency in real-world network markets: there is competition between different network goods; there are strategic suppliers; these suppliers are unable to commit to future price offers; adoption timing cannot always be completely determined by the adopters; and durables do wear out. All of these features are included in our model.

Network effects is only briefly covered in the environmental economics literature although they may be present for many types of clean goods that compete with dirty goods: Advanced virtual meeting-equipment versus air travel, hydrogen cars being dependent on hydrogen filling stations[1], carbon capture at power plants and industries requiring pipeline transport service etc. Greaker and Heggedal (2010) build an explicit model of the relationship between the market share of hydrogen cars and the density of hydrogen filling stations, and show that this could lead to multiple equilibria. However, unlike this paper, they only look at a static game. The transport market is also treated by Sartzetakis and Tsigaris (2005). In their model prices follow exogenously given rules, and the government does not set taxes optimally.

All our examples of environmental goods where network effects are important, are instances in which the network effects play out over the lifetime of the durable good. The important thing for consumers is not the size of the network at the time of purchase, but the size at every instance throughout the lifetime of their durable. This complicates the analysis, as we require consumers to be forward-looking and have rational

[1] Nicholas and Ogden (2009) report from a survey, demonstrating a strong relationship between the willingness to pay for a hydrogen car and the availability of hydrogen filling stations. The same type of interdependency would most likely also be the case for electric cars and the network of fast-charging stations.

expectations with regard to the market development. With this in mind, we construct a dynamic model of sequential adoption, based on Cabral (2011). There is a fixed number N of consumers, who each own either the dirty or the clean network good. The goods are durable, but their lifetime follows a Poisson process. When a good wears out, the consumer has to make an adoption decision: To adopt either the clean or the dirty good. The goods are horizontally differentiated, and the consumer's preferences are private information. The goods are supplied either by a monopolist or competitively, and suppliers engage in a dynamic price competition. The government sets taxes to maximize discounted social welfare.

The paper proceeds as follows: in Section 2 we lay out the model, while in Section 3 we derive the main results. In Section 4 we simulate the model numerically, and in Section 5 we conclude.

2 Model primitives

Following the original model of Cabral, we will have discrete timing with two competing networks and a fixed number N of consumers. The networks will be indexed by $k = c$ for clean and d for dirty. For each network there is an access price the consumer has to pay to join the network. These prices are set by the firms, and can be thought of as prices for some durable goods that grant the consumer access to the network in question. Denote these prices p_c and p_d, respectively.

The government will set two different taxes, one tax t on the purchase of the dirty durable, and one tax τ on the use of the dirty good. We will study markov-perfect equilibria (MPEs). The setup will be time-homogeneous, hence we suppress all time subscripts. The only payoff-relevant variables will be the network sizes, denoted n_c and n_d. We assume that the market is fully covered, so that all consumers own a good.

2.1 The consumers

At the beginning of each period, there are $N - 1$ consumers present in the market. One consumer arrives, and is confronted with the prices and taxes. Subject to these, he has to choose which network he wants to enter. After he makes his choice, there is an intermediate stage, the aftermarket stage, in which the durable goods are being put to use. At this stage all consumers in network k each enjoy some aftermarket benefits $\lambda(n_k)$, common to all consumers and weakly increasing in the network size n_k.[2] At the end of the period, with uniform probability, one random consumer is chosen to exit the market.

Due to this random exit, an entering consumer neither knows for how many periods he will enjoy the aftermarket benefits nor how large the network is going to be in the future. We therefore introduce the function $u_k(n_k)$ which is the expected present value (EPV) of entering network k at size n_k. That is, it is the expected discounted sum of the aftermarket benefits $\lambda(n_k)$ over all the future periods the consumer expects to be in the market.

In addition to the aftermarket benefits that are common to all consumers, each consumer draws two idiosyncratic, private utility components at birth. The components, $\{\zeta_c, \zeta_d\} \in \mathbb{R}^2$, determine the technology-specific utility he enjoys from joining either of the networks. The total expected net benefit of joining network k at size n_k today, B_k, is then given by:

$$B_k = \begin{cases} \zeta_c + u_c(n_c + 1) - p_c(n_c), & \text{if clean network} \\ \zeta_d + u_d(n_d + 1) - p_d(n_d) - t(n_d), & \text{if dirty network.} \end{cases}$$

We assume that the values of ζ_k are sufficiently high such that the consumer always chooses one of the networks. Since the market is then completely covered, we can restrict our attention to the distribution of the difference between the two utility parameters $\xi_c \equiv \zeta_c - \zeta_d$. As we assume that the ζ_k are $i.i.d$, ξ_c has expected value equal to zero.

The consumer who is indifferent between the two networks will have: $B_c = B_d$, or $\xi_c = x(n_c)$, where the latter is given by:

[2] For instance, the function $\lambda(n_k)$ can be seen as the reduced form of the explicit network model in Greaker and Heggedal (2010).

$$x(n_c) = p_c(n_c) - p_d(n_d) - t(n_d) - u_c(n_c + 1) + u_d(n_d + 1). \tag{1}$$

That is, $x(n_c)$ indicates the position along the real line of the consumer who is indifferent between the two goods when the clean network has size n_c, and prices and taxes are as given. Now, assuming that ξ_c is normally distributed with cdf $\Phi(\cdot)$ and density $\phi(\cdot)$, we derive the probability that a newborn consumer chooses the clean network:

$$\begin{aligned} q_c(n_c) &= Pr\left[\xi_c \geq x(n_c)\right] = 1 - Pr\left[\xi_c < x(n_c)\right] \\ &= 1 - \Phi\left[x(n_c)\right], \end{aligned} \tag{2}$$

and the probability of choosing the polluting network is:

$$\begin{aligned} q_d(n_d) &= Pr\left[\xi_c < x(n_c)\right] \\ &= \Phi\left[x(n_c)\right]. \end{aligned} \tag{3}$$

The taxes levied on the dirty network introduce asymmetries, such that $q_c(a) \neq q_d(a)$ in equilibrium, but the probabilities are related through $q_c(a) + q_d(N - 1 - a) \equiv 1$. From these expressions, we can see that the probability that firm k makes the next sale is, ceteris paribus, continuously and monotonically decreasing in p_k, $\partial q_k(n_k)/\partial p_k < 0$.

Given a sequence of taxes and prices, we now have the law of motion for the network shares. Given that every consumer has the same probability of being chosen to leave the market, the EPV of future network benefits does not depend on how long a consumer has been present. We can therefore define $u_k(n_k)$ recursively in the following way (first for the dirty network):

$$u_d(n_d) = \lambda(n_d) - \tau(n_d) + \frac{1}{N} \cdot 0 + \delta \frac{n_c}{N} q_d(n_d) u(n_d+1) \\ + \delta \left[\frac{n_c}{N} q_c(n_c - 1) + \frac{n_d - 1}{N} q_d(n_d - 1) \right] u_d(n_d) \\ + \delta \frac{n_d - 1}{N} q_c(n_c) u(n_d - 1) \tag{4}$$

Each period you enjoy the aftermarket benefit as a function of the market share, and consumers in the dirty network also pay a tax $\tau(\cdot)$ every period for the use of their good. At the end of each period, there is a probability $1/N$ that you are the one who dies, after which you get zero by assumption. If you are not chosen to exit, there are three possibilities: your network increases, decreases or remains at the same size. There is only one possible way your network can increase in size: with a probability of n_c/N someone in the clean network exits, and with probability $q_d(n_d)$ the arriving consumer opts for the dirty network, and the network size increases one step. There are two events that may reproduce the current state the next period; that is when one of the networks experience exit and the arriving consumer chooses to join that same network. And finally your network may decrease by one step if someone other than you dies, and the next consumer chooses the clean network.

For a consumer present in the clean network, we get the following value:

$$u_c(n_c) = \lambda(n_c) + \frac{1}{N} \cdot 0 + \delta \frac{n_d}{N} q_c(n_c) u(n_c+1) + \\ \delta \left[\frac{n_d}{N} q_d(n_d - 1) + \frac{n_c - 1}{N} q_c(n_c - 1) \right] u_c(n_c) \\ + \delta \frac{n_c - 1}{N} q_d(n_d) u(n_c - 1) \tag{5}$$

Note that there is no use tax $\tau(\cdot)$ in (5).

To gain some intuition on these expressions, we can consider the case with a constant use tax and zero network benefits e.g. $\lambda(\cdot) = 0$.

Equation (4) then collapses to $u_d = -\tau(1-\delta\frac{N-1}{N})^{-1}$, i.e. the expected net present value of the future outlays on the use tax, while (5) collapses to 0. Note that the discount factor is augmented with the factor $\frac{N-1}{N}$, that is the probability that the consumer will stay alive. Further, with constant access prices, the marginal consumer is given by: $x_c = p_c - p_d - t - \tau(1-\delta\frac{N-1}{N})^{-1}$. Hence, only consumers with $\zeta_c - \zeta_d \equiv \xi_c < p_c - p_d - t - \tau(1-\delta\frac{N-1}{N})^{-1}$ will choose the dirty network.

2.2 Firms

Firms derive revenue equal to the entry price p_k every time a new consumer enters their technology.[3] Costs are normalized to zero, and hence profits are equal to revenue. Remember that the utility a consumer gets from a technology depends on the number of consumers already using the technology. Hence, expected revenue for a given p_k will depend positively on the size of the network.

The value functions of the firms are evaluated before the firms set their prices and the arriving consumer makes his choice. The total number of consumers who *currently* are in the market is therefore $N-1$. For a network of technology $k = c, d$ we have:

$$(6) \qquad \begin{aligned} v_k(n_k) = {} & q_k(n_k)\left(p_k(n_k) + \delta\frac{n_{-k}}{N}v_k(n_k+1) + \delta\frac{n_k+1}{N}v_k(n_k)\right) \\ & + (1-q_k(n_k))\left(\delta\frac{n_{-k}+1}{N}v_k(n_k) + \delta\frac{n_k}{N}v_k(n_k-1)\right) \end{aligned}$$

where $n_k + n_{-k} = N-1 \Rightarrow n_{-k} = N-1-n_k$. The first line above is the event that the newborn consumer chooses network k when it's size is n_k. It happens with probability $q_k(n_k)$, decreasing in p_k. Then, network k sells a unit at value $p_k(n_k)$ and it's network size increases to n_k+1. In the next period there are two possibilities: either the other network has experienced exit (with probability n_{-k}/N); or someone in network k has

[3] Cabral (2011) firms also enjoy aftermarket benefits depending on the size of their networks. For simplicity, we disregard these here. This can for instance be the case if the complementary services are supplied from a sector separate from the two technology owners.

exited (with probability $(n_k + 1)/N$). The network size at the beginning of the next period is updated accordingly. The second line is the event that the arriving consumer chooses the other network. In that case there is a higher probability that the other network experiences an exit, and vice versa.

Note that (6) is compatible with different market configurations, for instance, I) both technologies are sponsored, or II) only the clean technology is sponsored, while the dirty technology is supplied by several firms.[4]

2.3 The government

Environmental damages from the polluting network accrues according to γn_d, where γ is a parameter and n_d is the number of consumers present in the polluting network today. This is a reasonable representation of environmental costs as long as a) the emissions from the network in question is only a part of the total emissions, and b) the use intensity is exogenous to the agents once they have joined the dirty network.

We equip the government with two instruments: a purchase tax $t(n_d)$ levied at the time of purchase, and a flow tax $\tau(n_d)$ levied each period on all consumers present in the polluting network, and thus affecting the the expected present value of entering network d.

In addition to the environmental damage function, the public welfare function is assumed to be utilitarian, it is the unweighted sum of profits and consumer utility. We are thus lead to the following value function evaluated before the consumer chooses a network:

[4] The first configuration corresponds to the setup used by Cabral. In the transport market application the durables could be either a fossil fuel car or a hydrogen (electric) car which both provide a tranportation service, the quality of which depends on the density of refueling stations. Further, the inventor owning the patent on the premium fuel cell (rechargeable battery) can through her pricing of the patent set the access price for the clean network. For the dirty network, we can either assume that current car companies act as a cartel using their pricing to keep consumers in the dirty technology, or we may have that only the green network has a sponsor.

$$
\begin{aligned}
g(n_c) = & \\
& q_c(n_c) \cdot \Big\{ \mathbb{E}\left[\zeta_c | \xi_c > x(n_c)\right] + (n_c+1)\lambda(n_c+1) - p_c(n_c) \\
& + n_d\left[\lambda(n_d) - \tau(n_d)\right] - \gamma n_d + \tau(n_d) n_d + p_c(n_c) \\
& + \delta\left[\frac{n_c+1}{N} g(n_c) + \frac{n_d}{N} g(n_c+1)\right] \Big\} \\
& + (1 - q_c(n_c)) \cdot \Big\{ \mathbb{E}\left[\zeta_d | \xi_c < x(n_c)\right] + (n_d+1)\left[\lambda(n_d+1) - \tau(n_d+1)\right] \\
& - p_d(n_d) - t(n_d) + n_c\lambda(n_c) - \gamma(n_d+1) + \tau(n_d+1)(n_d+1) \\
& + p_d(n_d) + t(n_d) + \delta\left[\frac{n_c}{N} g(n_c - 1) + \frac{n_d+1}{N} g(n_c)\right] \Big\}
\end{aligned}
\tag{7}
$$

The welfare measure is the expected value of two scenarios. First the case that the newborn consumer chooses the clean network. This happens with probability q_c. The value is then the expected idiosyncratic utility of the consumer, conditional on him choosing clean. Then we subtract the price he pays, we add the government tax revenue from the use of the dirty good and the consumers' network benefits, net of any flow tax paid. Further, we add the price revenue the clean network made from selling. Finally, we add the expected continuation value, conditional on the clean network having been chosen today. Then the same exercise is repeated in the event the dirty network is chosen. The only difference is that we now also have to take into account the purchase tax $t(n_d)$ levied on the consumer.

3 Solving the model

The timing of the game in every period is as follows: First, the government sets taxes. Second, firms observe the current taxes, and then they compete in prices. Finally, the consumer makes his choice, observing the prices and the current taxes. As we do not find it reasonable that the government can commit to future tax rates, we will only allow a stagewise leadership. We are thus considering Markov-perfect equilibria in which the government acts as a stagewise Stackelberg leader.

What does this mean in practice? Our interpretation is that first the government announces a markovian tax rule, and then the firms announce a markovian price rule, both specifying their respective optimal policy in every state. If both the industry and the current government believe the rules will be followed in all future periods, then it is optimal for the current government to follow it, too. This holds true in all periods, so the announced markovian strategies will indeed be followed.

To implement the equilibrium, we solve a set of dynamic programming problems by backwards induction. The consumer's choice problem is already solved by (4) and (5), which for given prices and taxes constitute a system of $2N$ equations with $2N$ unknowns, e.g. $u_c(1)...u_c(N)$ and $u_d(1)...u_d(N)$. We also have everything we need to solve the firms' problems for given taxes. Lastly we solve the government's problem, taking into account the response functions of the firms and the consumers. But first, we derive the first-best allocation.

3.1 First best

We start out by simplifying the expression for social welfare. First, the conditional, expected idiosyncratic utility of the consumer for the two possible outcomes can be written $\mu+\sigma^2\phi(x)$.[5] Second, note that all taxes and prices paid are just transfers, and do not affect welfare directly under the additive welfare measure. This reduces eq. (7) to

$$\begin{aligned}(8)\quad g(n_c) = {} & \mu + \sigma^2\phi(x) + q_c(n_c)\left[n_d\lambda(n_d) + (n_c+1)\lambda(n_c+1) - \gamma n_d\right] \\ & +(1-q_c(n_c))\left[(n_d+1)\lambda(n_d+1) + n_c\lambda(n_c) - \gamma(n_d+1)\right] \\ & +q_c(n_c)\delta\left[\frac{n_c+1}{N}g(n_c) + \frac{n_d}{N}g(n_c+1)\right] \\ & +(1-q_c(n_c))\delta\left[\frac{n_c}{N}g(n_c-1) + \frac{n_d+1}{N}g(n_c)\right]\end{aligned}$$

[5]The expression is derived in the Appendix. Note that the parameter μ is the expected value of the individual shocks ζ_k, σ^2 is the variance of the distribution of ξ_c.

What is the first-best allocation, that is, which network should the newborn consumer join, given the currently observed market shares? Since consumers are born with stochastic taste parameters, this amounts to choosing the state-dependent cut-off value $x(n_c)$ for the taste parameter, which divides the pool of potential newborn consumers into clean adopters and dirty adopters. Some consumers might be born with strong preferences in favor of the clean technology, while others will favor the dirty technology. The first-best allocation trades off these idiosyncratic preferences against the environmental damage and the network effects - current and future.

The first-best allocation is thus the policy rule $x^*(n_c)$ defined by

$$x^*(n_c) = \underset{x}{\text{argmax}}\ \ g(n_c).$$

To solve it, we differentiate wrt. x, and get:

$$\begin{aligned}\frac{\partial g(n_c)}{\partial x(n_c)} =& \sigma^2\phi'(x) - \phi(x)\left[n_d\lambda(n_d) + (n_c+1)\lambda(n_c+1) - \gamma n_d\right] \\ &-\phi(x)\delta\left[\frac{n_c+1}{N}g(n_c) + \frac{n_d}{N}g(n_c+1)\right] \\ &+\phi(x)\left[(n_d+1)\lambda(n_d+1) + n_c\lambda(n_c) - \gamma(n_d+1)\right] \\ &+\phi(x)\delta\left[\frac{n_c}{N}g(n_c-1) + \frac{n_d+1}{N}g(n_c)\right]\end{aligned}$$

Setting $\frac{\partial g(n_c)}{\partial x(n_c)} = 0$, using that $\sigma^2\phi'(x) = -x(n_c)\phi(x)$ and that $\phi(x)$ is bounded away from zero, and rearranging, we obtain:

$$\begin{aligned}(9)\quad x^*(n_c) =& \Delta_d(n_d) - \Delta_c(n_c) - \gamma \\ &+\delta\left[\frac{n_c}{N}g(n_c-1) + \frac{n_d}{N}g(n_c) - \frac{n_d}{N}g(n_c+1) - \frac{n_c}{N}g(n_c)\right]\end{aligned}$$

where $\Delta_d(n_d) = (n_d+1)\lambda(n_d+1) - n_d\lambda(n_d)$ and $\Delta_c(n_c) = (n_c+1)\lambda(n_c+1) - n_c\lambda(n_c)$, and the second-order condition is satisfied.[6]

The term $\Delta_d(n_d)$ is the gain in current network benefits if the consumer chooses dirty, while the term $\Delta_c(n_c)$ is the gain in current network

[6] See the Appendix for a derivation of the second-order condition.

benefits if the consumer chooses clean. For a concave λ-function we have: $\Delta_d(n_d) - \Delta_c(n_c) \leq 0$ for $n_c \leq n_d$. The next term is the additional environmental damage in the current period from the consumer choosing dirty. This is always negative.

The last term in (9) represents the change in continuation value following the consumer's choosing the dirty technology. When the consumer chooses dirty, the state in the next period is either $n_c - 1$ if a clean consumer dies or n_c if a dirty consumer dies. Whereas if the consumer had chosen clean, the state in the next period would have been either $n_c + 1$ in case a dirty consumer dies or n_c in case a clean consumer dies. Note that the choice of the dirty technology increases the expected future size of the dirty network. Furthermore, if we are in a situation in which $g(n_c - 1) < g(n_c) < g(n_c + 1)$, the change in continuation value for a dirty choice will be negative.

What does it mean that the left hand side of (9) is negative? Remember that a consumer will only choose the dirty good if he has sufficiently strong preferences for it e.g. if $\xi_c = \zeta_c - \zeta_d < x(n_c)$. Thus, if the optimal $x^*(n_c)$ is negative, only consumers with $\zeta_d > \zeta_c$ chooses the dirty good. In other words, only if the personal gain to the consumer of choosing the dirty good over the clean good exceeds the social costs of that same choice, should he be induced to make the choice.

The continuation values include all future network benefits, and all future environmental damages. Note that the future environmental damages will depend on the current choice in two ways: I) a consumer who chooses dirty now will pollute in the future as long as he is in the market, and II) he makes the dirty good more attractive to future consumers through the increased network benefit of the dirty good. This can most easily be seen if we simplify (9) in the following manner: Assume that $g(\cdot)$, Δ_k and q_k are linear, $g(n_c - 1) - g(n_c - 2) = g(n_c) - g(n_c - 1) = g(n_c + 1) - g(n_c)$, $\Delta_d(n_d + 1) = \Delta_d(n_d)$, $\Delta_c(n_c) = \Delta_c(n_c - 1)$, and $q_c(n_c + 1) - q_c(n_c) = q_c(n_c) - q_c(n_c - 1)$, which seems reasonable for N large. Implementing our assumptions in (8), and using that $q_d(n_d) = 1 - q_c(n_c)$, we obtain:

$$(g(n_c) - g(n_c - 1)) = -\left[\Delta_d(n_d) - \Delta_c(n_c) - \gamma\right] \frac{1 + q_d(n_d+1) - q_d(n_d)}{1 - \left[1 + q_d(n_d+1) - q_d(n_d)\right]\delta \frac{N-1}{N}}$$

which can be inserted into (9) to get:

$$x^*(n_c) = \frac{\Delta_d(n_d) - \Delta_c(n_c)}{1 - \left[1 + q_d(n_d+1) - q_d(n_d)\right]\delta \frac{N-1}{N}} - \frac{\gamma}{1 - \left[1 + q_d(n_d+1) - q_d(n_d)\right]\delta \frac{N-1}{N}} \tag{10}$$

The first term in (10) is the present value of the loss/gain in the flow of network benefits from a dirty choice today. If $n_c \leq n_d$ and the $\lambda(\cdot)$ function is concave, the term will be negative. The second term in (10) is the present value of the environmental costs from a dirty choice today. This is always negative. Let us compare the second term in (10) with the environmental damage resulting *only* from the consumer choosing the dirty product without taking into account any network effects. This present value is given by:

$$\sum_{t=0}^{\infty} \left(\frac{N-1}{N}\delta\right)^t \gamma = \frac{\gamma}{1 - \delta\frac{N-1}{N}} \tag{11}$$

where $\frac{N-1}{N}$ is the probability that the consumer stays alive in the next period. Comparing the two present values:

$$\frac{\gamma}{1 - \left[1 + q_d(n_d+1) - q_d(n_d)\right]\delta \frac{N-1}{N}} > \frac{\gamma}{1 - \delta\frac{N-1}{N}} \text{ if } q_d(n_d+1) > q_d(n_d)$$

Thus, as long as a consumer choosing dirty today increases the probability of a future consumer choosing dirty, we have:

Proposition 1 *There exists an externality multiplier effect which may warrant a more stringent environmental policy in case of network goods.*

With a more stringent policy we here mean that the cutoff value for when a consumer should choose dirty is set lower e.g. the difference $\zeta_d - \zeta_c$ has to be higher, so that fewer consumers should join the dirty network. Finally, note that this first-best allocation is independent of the underlying market structure. We now proceed to derive the MPE, starting out with the optimal pricing of firms.

3.2 Pricing monopolistic case

To derive the optimal price setting of the firms as a function of the given taxes when they act as monopolists, we maximize the firm value functions (6) with respect to $p_k(n_k)$. In solving this, the firms take the sequence of taxes, $\tau(n_d)$ and $t(n_d)$, as given, and the demand functions they face are the state- and price-dependent choice probabilities $q_k(n_k)$ of consumers. We can think of the firms deriving price rules for every state n_k. The first-order conditions with respect to price are:

$$q_k(n_k) + \frac{\partial q_k(n_k)}{\partial p_k(n_k)} p_k(n_k) + \frac{\partial q_k(n_k)}{\partial p_k(n_k)} \delta \left[\frac{n_{-k}}{N} v_k(n_k+1) + \frac{n_k - n_{-k}}{N} v_k(n_k) - \frac{n_k}{N} v_k(n_k-1) \right] = 0,$$

which can be rewritten to

$$\text{(12)} \qquad \frac{p_k(n_k;t,\tau) + w_k(n_k;t,\tau)}{p_k(n_k;t,\tau)} = \frac{q_k(n_k)}{-q_k'(n_k) p_k(n_k;t,\tau)} = \frac{1}{\epsilon_k}.$$

where $w_k(n_k;t,\tau) \equiv \delta \left[\frac{n_{-k}}{N} v_k(n_k+1) + \frac{n_k - n_{-k}}{N} v_k(n_k) - \frac{n_k}{N} v_k(n_k-1) \right]$. Equations (12) gives us $2N$ equations which can be used to solve for the pricing rules of the two firms, e.g. $p_c(0), p_c(1)...p_c(N-1)$ and $p_d(0), p_d(1)...p_d(N-1)$.

The function $w_k(n_k;t,\tau)$ has in the literature been dubbed the *discounted prize* of winning a sale, and it is the present value of the future excess revenue that stems from the current sale. It is thus very similar to the change in continuation value experienced by the social planner in (9). By rewriting the expression, we see that it is positive if $v_k(n_k+1) > v_k(n_k) > v_k(n_k-1)$. Clearly, a technology sponsor may benefit from a larger network, since *ceteris paribus*, the larger the network, the larger is the probability that the new consumer will choose the sponsor's network.

As Cabral pointed out, for a textbook monopoly we would have $(p - mc)/p = 1/\epsilon$ where p is the price and mc is the marginal cost of the monopoly. Thus, in (12) the discounted prize of the sale plays the role of a negative marginal cost in the firm's pricing problem. In other words, instead of experiencing a marginal cost associated with a sale, the firm experiences a more or less favorable distribution over the continuation values $v_k(\cdot)$. Hence, a technology sponsor would only be willing to forego profits today by setting a lower price, to the extent that the discounted prize is positive.

3.3 Pricing competitive case

When more firms are selling a durable good of the same type, we assume that these durables are perfect substitutes. Moreover, we assume that firms are symmetric with zero marginal cost. Hence price equal to zero in every period is clearly a symmetric equilibrium. With respect to sales in the current period, each firm has an incentive to undercut the other firms, as long as prices are positive. Since this incentive is present in all future periods as well, no firm has an incentive to set a negative price, since it cannot reap the benefits of a larger network later on. This gives us the following pricing function for a competitive network:

$$p_k(n_k) = 0, \ \forall \ n_k. \tag{13}$$

As mentioned, we study two competitive cases: Either $p_k(n_k) = 0$ for both clean and dirty, or only for the dirty network. Note that in the cases in which only the clean technology has a sponsor, the pricing rule for the clean technology, (12), must still hold. The only difference is that we must insert $p_d = 0$ into the probability function $q_c(p_c)$.

Finally, when prices are zero in all periods, the firm values $v_k(n_k)$ must be zero for all n_k.

3.4 Setting the optimal taxes

The problem now facing the government is a dynamic programming problem subject to two functional constraints on the prices. We write the dynamic programming problem of the government as follows:

$$
\text{(14)} \qquad g^*(n_c) = \max_{t(n_d),\tau(n_d+1)} g(n_c), \qquad \text{s.t. } p_k = f_k(n_k; t, \tau), \ \ k = c, d.
$$

where $g(n_c)$ is defined in (8).

Through the constraint $p_k = f_k(n_k, t, \tau), \ \ k = c, d$, the government takes into account the optimal price responses of the firms (equations (12) or (13)). Note, however, from (8) that neither the prices nor the taxes enter the value function directly. They only enter indirectly through $x(n_c)$.

For a formal solution to this problem, see the appendix. Here we note that both the entry tax $t(n_d)$ and the use tax $\tau(n_d)$ affect $x(n_c)$. They do not work in the same manner, but they achieve the same goal, and implement $x^*(n_c)$ in the exact same way: by making one network or the other more or less attractive *ex ante*. The equivalence arises because there are no distortions from taxes: the market is assumed to be covered, and the use of the goods once they are acquired is perfectly inelastic. The result is therefore that it does not matter whether the government taxes entry to, or participation in, the dirty network. In the following we will focus on the entry tax, since in our model, both the environmental externality and the network effects are directly linked to the choice of technology.[7]

Proposition 2 *The government can implement the first-best allocation.*

Since the prices and taxes only enter the welfare expression through the indifference parameter x, we find a first-order condition for the pur-

[7] If the use intensity was not fixed, the use tax would introduce a distortion different from the entry tax. The government would then likely use both.

chase tax, t, of the following form:

$$(15)\quad 0 = \frac{dx}{dt} \cdot \phi(x) \cdot \big[\Delta_d(n_d) - \Delta_c(n_c) + \delta \frac{n_c}{N}\left[g(n_c-1) - g(n_c)\right] + \delta \frac{n_d}{N}\left[g(n_c) - g(n_c+1)\right] - x^*(n_c)\big].$$

We can see that (9) is a solution to this.[8] This means that even in this dynamic taxation game, when we have restricted the government to choose from only time-consistent policies, the first best can be implemented. Since taxes are non-distortionary, the government can credibly promise to set whatever future taxes are needed to implement the first best. If high taxes were very costly (for instance if consumers then would choose some outside good and disappear from the market in question), one could imagine that the government's threat to set high taxes in the future was not credible, and that it therefore might struggle to implement the first best today. This does not happen here.

For the same reason, it is easy to see that (9) will solve (15) for any configuration of sponsors. This leads to the next proposition:

Proposition 3 *The implemented real allocation between the clean and dirty network does not depend on the configuration of sponsors.*

The expression for the optimal tax rate $t(\cdot)$ can, unfortunately, not be derived in closed form. The tax must be set such that the post-tax price competition implements the above value of $x(\cdot)$, but that depends on solving for the fixed point in (9), on the form $x(n_c) = G(x(n_c))$. The function $G(\cdot)$ involves the cumulative distribution function of the taste parameters, and the implicitly defined value function. However, by studying our simplified example in (9), we can expand the expression for x from (1), and hence decompose the optimal entry tax in the following manner:

[8] See the Appendix for a complete derivation of this result.

$$
\begin{aligned}
(16) \qquad t(n_d) = & \frac{\gamma}{1-\delta\frac{N-1}{N}} \\
& + \frac{[q_d(n_d+1)-q_d(n_d)]\,\gamma}{1-[1+q_d(n_d+1)-q_d(n_d)]\,\delta\frac{N-1}{N}} \frac{\delta\frac{N-1}{N}}{1-\delta\frac{N-1}{N}} \\
& + \frac{\Delta_c(n_c)-\Delta_d(n_d)}{1-[1+q_d(n_d+1)-q_d(n_d)]\,\delta\frac{N-1}{N}} \\
& + [p_c(n_c)-p_d(n_d)] + [u_d(n_d+1)-u_c(n_c+1)]
\end{aligned}
$$

The first term in (16) is the Pigovian entry tax.[9] Note that this term is constant. The second term in (16) is what we have coined the externality multiplier effect e.g. the additional environmental cost of a newborn consumer choosing dirty and making the dirty network more attractive. This term is positive as long as $q_d(n_d+1) > q_d(n_d)$ e.g. a dirty choice today increases the probability of a dirty choice tomorrow.

We already know the third term from (10). It is the present value of the loss/gain in the flow of total network benefits from the newborn consumers choice of the dirty good today. The sign on this term will depend on the form of the $\lambda(\cdot)$ function and the state n_c.

Lastly, the two last brackets can be coined *correction* terms. The first bracket corrects the tax for the monopolistic pricing of the two firms. For instance, if the clean producer uses a low price to attract the newborn consumer to her network, the tax should be lower. This term is zero when there are more firms supplying both technologies. The second bracket is the difference in *private* network benefits between the two networks. Note that if this term is positive, the tax is set higher. This is necessary since the optimal $x^*(n_c)$ only depends on the three first terms in (16).

It is obvious from (16) that the optimal tax cannot be constant when there are network effects. Even in the competitive case in which p_c and p_d are equal to zero, the tax will have a different numerical value for every n_c. Summing up, we have:

[9] In the appendix, we solve for the optimal entry tax without network effects and without market power, and show that this entry tax equals $\gamma/(1-\delta\frac{N-1}{N})$.

Proposition 4 *For all configurations of sponsors, the optimal entry tax departs from the Pigovian entry tax. Even if the pricing of the monopolistic producers fully internalizes the network effects, the optimal tax will be higher than the Pigovian tax as long as a dirty choice today increases the probability of a dirty choice tomorrow.*

By internalizing the network effects we imply setting $p_c(n_c)$ and $p_d(n_d)$ such that:

$$p_c(n_c)-p_d(n_d) = \frac{\Delta_d(n_d) - \Delta_c(n_c)}{1 - [1 + q_c(n_c) - q_c(n_c - 1)]\,\delta_G \frac{N-1}{N}} + u_c(n_c+1) - u_d(n_d+1)$$

The optimal tax will then only consist of the two first terms in (16). Clearly, the proposition also holds for the case in which the government uses a separate instrument to internalize the network effect.

Finally, it follows that:

Corollary 5 *The optimal tax rule will depend on the configuration of sponsors.*

Since the prices p_c and p_d enter directly in (16), this must be the case. In order to say more about how the tax will depart from the Pigovian tax, we need to be more explicit about the network effects. Thus, in the next section we simulate a numerical version of the model.

4 Numerical simulations

We have chosen to concentrate on the competitive case and the clean technology sponsor case. The results with two technology sponsors are very similar to the results for the only clean technology sponsor case.

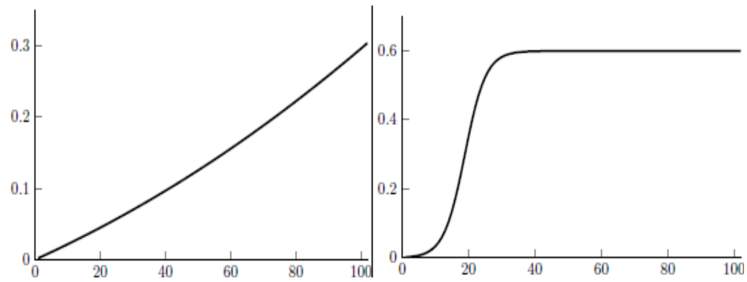
The model is not complicated to solve numerically. We have a fixed point problem for all n_c, and we can simply iterate to fix the optimal $x(n_c)$. When the optimal $x(n_c)$ is determined, we have the consumer values. The prices are determined as the solution to the first-order conditions that implements the pre-determined $x(n_c)$, thereby determining firm values.

We represent the network effect $\lambda(n_k)$ by a logistic curve:

$$\lambda(n_k) = \frac{\psi}{1 + e^{\alpha - \beta \frac{n_k}{N}}} \tag{17}$$

where ψ, α and β are parameters. For illustrative purposes, we consider two distinctly different forms for the network benefit function (17): One tends to give excess momentum, while the other tends to give excess inertia. The two forms are shown in Figure 1.

Figure 1 "The network benefit "



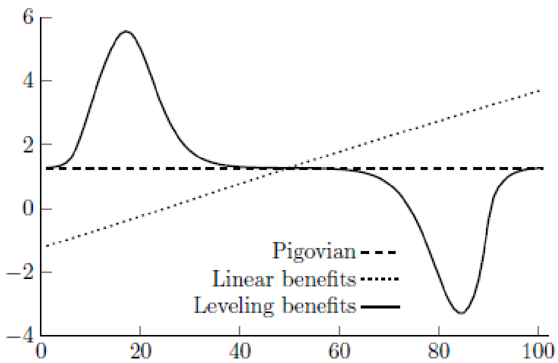
The market share of the clean technology is shown on the x-axis, while the y-axis shows the value of the network benefit to each consumer. In "linear" benefit case (left part of Figure 3) we have $\psi = 2$, $\alpha = 2$ and $\beta = 1$. The network benefit is then convex in the whole interval $n_k \in [0, 100]$. This implies that the regulator would like the market to choose one of the technologies. Moreover, we have $\Delta_k(n_k) \geq \Delta_{-k}(n_{-k})$ for $n_k \geq n_{-k}$.

In the leveling benefit case (right part of Figure 3) we have $\psi = 0.6$, $\alpha = 6$ and $\beta = 32$. The network benefit is as important as in the first case, but it starts to level off at about $n_k \approx 30$. This is consistent with the explicit model of the market for clean cars in Greaker and Heggedahl (2010). The government may then choose to keep both technologies in the market without incurring a loss in total network benefits. Moreover, after the market has "taken off" at around 20 users of a technology, the $\lambda(\cdot)$ is concave implying that $\Delta_k(n_k) \leq \Delta_{-k}(n_{-k})$ for $n_k \geq n_{-k}$.

4.1 Optimal policy with no technology sponsors

We compare the optimal entry tax for the two cases with the Pigovian entry tax, i.e. $t^{pig} = \gamma/(1 - \delta\frac{N-1}{N})$. We have used $\delta = 0.85$, and $\gamma = 0.2$. Simulating the model we get the following optimal entry tax rule:

Figure 2 "Optimal entry tax rule with no sponsors"



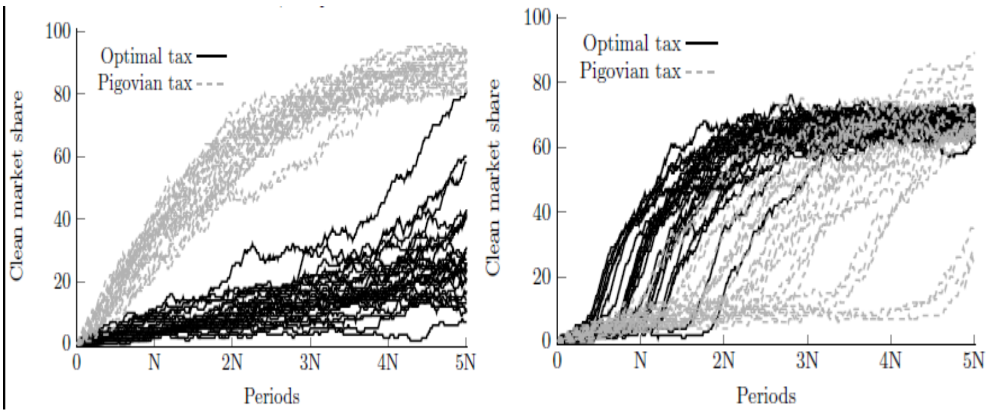
In the linear case the network benefits dominate the environmental damages including the externality multiplier. As long as $n_c < n_d$, $\Delta_d(n_d) > \Delta_c(n_c)$, and the government would like the market to stay dirty. This can be seen directly from the left part of Figure 2. The higher the market share of the dirty good, the lower the entry tax for the dirty good. However, when the market share of the clean technology exceeds $\backsim 50$, $\Delta_d(n_d) < \Delta_c(n_c)$, and there is gain in total network benefits from consumers choosing the clean good. Consequently, the government sets an entry tax in excess of the Pigovian rate, and goes for the clean network.

In leveling case there is no loss in total network benefits if a newborn consumer chooses clean when the market is dominated by the dirty technology e.g. $\Delta_c(n_c) > \Delta_d(n_d)$ for $n_c < n_d$. Consequently, the government sets an entry tax in excess of the Pigovian rate already from the introduction of the clean technology. For market shares of around 20, the entry tax is nearly five times the Pigovian rate, and it includes a significant externality multiplier effect (see Subsection 4.3). On the other hand, when the clean technology has reached a high market share, additional clean consumers will impose a significant loss on the existing dirty consumers. This is reflected in the negative tax rates for the leveling case towards the right end of Figure 2.

Looking at the industry dynamics also gives some insight into the optimal tax rules. In the competitive situation, without any intervention by the government, the market will for both cases over time move towards a $50-50$ split independent of the intial situation. The reason is that from time to time consumers with a high preference for one of the technologies are borne, and will choose that technology independent of the network sizes.

We start in a situation in which the market is dominated by the dirty technology. The grey lines are the situation with the Pigovian entry tax, and the black lines are the development with the optimal entry tax:

Figure 3 "Industry dynamics competitive situation"

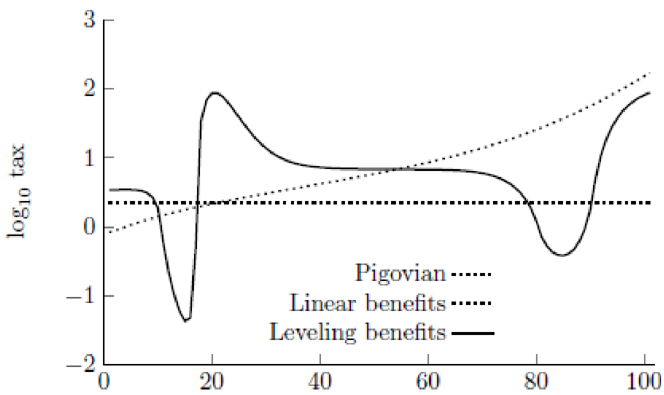


For linear case we note that the effect of the optimal policy is to slow the transition to the clean technology. Thus, due to the loss in total network benefits from consumers choosing clean when the dirty network dominates, we have excess momentum with just a Pigovian tax. In leveling case the policy has the opposite effect: It speeds up the transition to the clean technology, and we have excess inertia without the optimal tax.

4.2 Optimal policy with a clean technology sponsor

Remember that the government implements the same probability of choosing the clean good for all configurations of sponsors. Hence, any difference in the optimal entry tax rates between Figure 4 and 6 is due to the pricing of the clean technology supplier, see (16). The price of the clean technology supplier must satisfy (12). Since we start off with only the dirty technology in place, the clean sponsor could use its price to increase its network. From the optimal entry tax rule, we see that this to some extent happens for low market shares:

Figure 4 "Optimal entry tax rule with a clean technology sponsors"



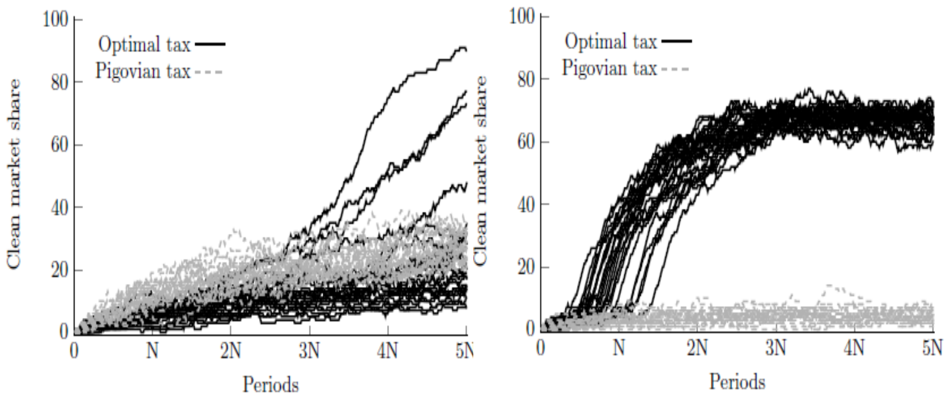
First, note that the variation in the tax is much higher compared to the competitive situation, which makes it necessary to use a log-cale on the Y-axis. This is a result of the price responses by the clean technology sponsor; as the government rises its tax, the clean technology sponsor sometimes responds by increasing her price etc.

In the linear case the government no longer subsidizes the dirty technology by setting a negative entry tax. This is not necessary since the clean technology sponsor sets a positive price and the price on the dirty technology is zero. Moreover, in order avoid excess inertia when the market share of the clean good has reached ∽ 50, the government has to increase its tax manyfold compared to the competitive situation. Again this is explained by the price response by the clean technology sponsor.

In the leveling case the optimal entry tax rule also has a similar shape as in the competitive situation. Note, however, the dip in the entry tax at a clean market share of $\backsim 10$. The clean technology sponsor at this point sets a very low price in order to get the market for the clean technology to take off. On the other hand, at once the clean technology has taken off, the clean sponsor starts to harvest the market. Consequently, the entry tax on the dirty good has to be higher than in the no technology sponsor case as long as the market share of the clean good is between $\backsim 20$ and $\backsim 80$.

In neither of the two cases the clean technology sponsor and the government agree with respect to the optimal diffusion of the clean technology as can be seen from the next figure.

Figure 5 "Industry dynamics situation with a clean technology sponsor"



The linear case in Figure 5 should be compared to the linear case in Figure 3. Due to pricing of the clean sponsor the Pigovian tax does not tip the market in Figure 5.[10]

The leveling case with a clean sponsor is maybe the most striking case. Without the optimal entry tax, the market for the clean technology may not develop. We can see from the figure that the market share stays

[10] This is also the case with two sponsors, but not so pronounced. Figures can be obtained from the authors upon request.

low for up to $5 * N$ periods even if the government has introduced a Pigovian tax. The clean technology sponsor tends to settle with a far too low market share, and rather earn a lot on the most eager consumers.

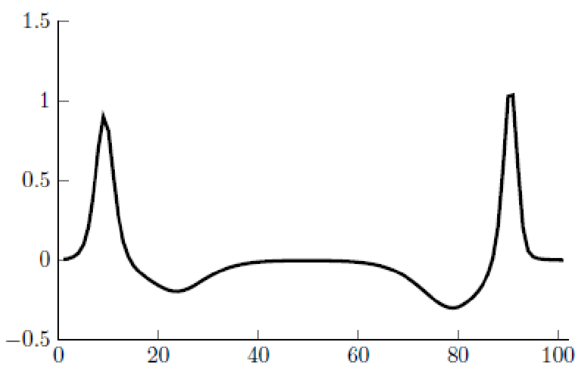
4.3 The externality multiplier

In (16) we decompose the optimal dirty network entry tax, and identify an externality multiplier effect. This effect can be numerically calculated by the following formula:

Full tax - Network tax for $\gamma = 0$ - Pigovian tax

The Pigovian tax is 1.26 in our case. Network tax for $\gamma = 0$ has to be found by simulating the model. The externality multiplier is very small for the linear case indicating that the probabilities are not very sensitive to the market shares. However, for the leveling case, it is significant. In Figure 6 we report the externality multiplier effect without technology sponsors.

Figure 6 "The externality multiplier effect"



Note first, that the externality multiplier effect has its highest value around the two tipping points - the market shares at which the single consumer choosing dirty influences most future consumers. Note next, that the multiplier effect may have a value almost equal to the Pigovian tax, indicating its importance.

5 Conclusion

As far as we know this is the first paper that treats environmental policy when there are network effects in a dynamic model with optimizing firms and consumers. We have found that governments should intervene with a tax that no longer equals the social cost of emissions independent of whether the green technology has a sponsor or not. Maybe not so surprising, the optimal tax takes into account both the network effect, and the mark-up pricing of the potential technology sponsors. However, even if the government succeeds to internalize the network effect and deals with the monopolistic pricing of the potential technology sponsors by separate measures, the optimal tax rate should still depart from the Pigovian tax rate. The reason is that there exists an externality multiplier effect which makes the network effect either increase or decrease the environmental externality through the future choices of the consumers.

Another striking finding is that non-intervention may lead to excess inertia as hypothesized by a number of earlier papers mentioned above. In our simulations, a Pigovian tax might not be enough to move a way from the dirty technology. In the clean-technology-sponsor case this result is most pronounced: In stead of pricing low in order to obtain a high market share, the clean technology sponsor earns higher profit on the customers willing to pay a high price for their product. Consequently, the government sets an entry tax far above the Pigovian rate. Note that in our model, the government could also have used a high entry subsidy for the clean network.

Excess inertia may also happen with competitive suppliers. On the other hand, without knowing more about the nature of the network effect, we cannot conclude by advising governments to always support any clean network from the start.

Is it likely that the government sets taxes and the firms set prices each time a consumer arrives? No, but the interpretation is that both the government and the firms set a rule. Such rule setting is for instance the case in Norway with respect to electric cars. The government has given them various kinds of subsidies, and have stated that these subsidies will gradually disappear as the market share of electric cars pick up.

Cabral (2011) finds that for given value functions, there is a unique equilibrium in prices. But the price you set today of course determines part of the value function, so the proof is not complete (a fact he acknowledges). It is the same type of result that we have. If you fix the value functions $g(\cdot)$, then there is a unique optimal solution.

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A The conditional expected private utility

We want an expression for $\mathbb{E}(X|X - Z > a)$, where X, Z : *i.i.d.* $\sim N(\mu_X, \sigma_X)$. The distribution of $(X|X - Z > a)$ is called *skew normal* The expectation is derived in Birnbaum (1950). Relabel $Y \equiv X - Z$, and we have

$$\mathbb{E}(X|Y > a) = \mu_X + \rho_{X,Y}\sigma_X \frac{f_Y(\frac{a-\mu_Y}{\sigma_Y})}{1 - F_Y(\frac{a-\mu_Y}{\sigma_Y})}$$

where $f_Y(\cdot)$ is standard normal. Then we replace the standard normal with the $\phi(\cdot)$-distribution $Y \sim N(0, \sigma_Y)$, which gives us

$$\mathbb{E}(X|Y > a) = \mu_X + \rho_{X,Y}\sigma_X\sigma_Y \frac{\phi(a)}{1 - \Phi(a)}$$

Now

$$\rho_{X,Y} = \frac{cov(X,Y)}{\sigma_X \sigma_Y}$$

which means that $\rho_{X,Y} \cdot \sigma_X \cdot \sigma_Y = cov(X,Y)$, and we have that $cov(X,Y) = cov(X, X-Z) = var(X) - cov(X,Z) = var(X) = \sigma_X^2$. Thus we get

$$\mathbb{E}(X|Y > a) = \mu_X + \sigma_X^2 \frac{\phi(a)}{1-\Phi(a)}$$

Our in our notation:

$$\mathbb{E}(\zeta_c|\xi_c > x(n_c)) = \mu_\zeta + \sigma_\zeta^2 \frac{\phi(x(n_c))}{1-\Phi(x(n_c))}$$

similarily

$$\mathbb{E}(\zeta_d|\xi_c < x(n_c)) = \mu_\zeta + \sigma_\zeta^2 \frac{\phi(x(n_c))}{\Phi(x(n_c))}$$

In our government value function, we want so sum

$$\begin{aligned} q_c(n_c)\mathbb{E}(\zeta_c|\xi_c > x(n_c)) + [1 - q_c(n_c)]\,\mathbb{E}(\zeta_d|\xi_c < x(n_c)) \\ = [1-\Phi(x(n_c))]\left[\mu_\zeta + \sigma_\zeta^2 \frac{\phi(x(n_c))}{1-\Phi(x(n_c))}\right] + \Phi(x(n_c))\left[\mu_\zeta + \sigma_\zeta^2 \frac{\phi(x(n_c))}{\Phi(x(n_c))}\right] \\ = \mu_\zeta + 2\sigma_\zeta^2\phi(x(n_c)) \\ = \mu_\zeta + \sigma_{\xi_c}^2\phi(x(n_c)) \end{aligned}$$

which is what we use in (8).

B Deriving the first-order conditions

The government's dynamic programming problem gives the following first-order conditions:

$$\text{FOC } t(n_d)\text{: } 0 = \sigma_{\xi_c}^2 \phi'(x(n_c))\frac{dx(n_c)}{dt(n_d)} + \phi(x(n_c))\frac{dx(n_c)}{dt(n_d)}\left[\Lambda^d(n_d) - \Lambda^c(n_c)\right]$$

$$\text{FOC } \tau(n_d+1)\text{: } 0 = \sigma_{\xi_c}^2 \phi'(x(n_c))\frac{dx(n_c)}{d\tau(n_d+1)} + \phi(x(n_c))\frac{dx(n_c)}{d\tau(n_d+1)}\left[\Lambda^d(n_d) - \Lambda^c(n_c)\right]$$

where $\Lambda^c(n_c) = n_d\lambda(n_d) + (n_c+1)\lambda(n_c+1) - \gamma n_d + \delta\frac{n_c+1}{N}g(n_c) + \delta\frac{n_d}{N}g(n_c+1)$ and $\Lambda^d(n_d) = (n_d+1)\lambda(n_d+1) + n_c\lambda(n_c) - \gamma(n_d+1) + \delta\frac{n_c}{N}g(n_c-1) + \delta\frac{n_d+1}{N}g(n_c)$. Thus, the functions $\Lambda^k(n_k)$ is the current networks benefits subtracted the environmental costs, and the continuation values, given that the consumer chooses technology k.

If we restrict our attention to the entry tax $t(n_d)$, we get

$$\left[\sigma_{\xi_c}^2 \phi'(x(n_c)) + \phi(x(n_c))(\Lambda^d(n_d) - \Lambda^c(n_c))\right] \cdot \frac{dx(n_c)}{dt(n_d)} = 0 \tag{18}$$

and when $\phi(\cdot)$ is the normal density, we have that

$$\phi'(x(n_c)) = -\frac{x(n_c)}{\sigma^2}\phi(x(n_c)) \tag{19}$$

Rewriting (18) we obtain

$$\left[(\Lambda^d(n_d) - \Lambda^c(n_c)) - x(n_c)\right] \cdot \frac{dx(n_c)}{dt(n_d)} \cdot \phi(x(n_c)) = 0$$

We must then show that $\frac{dx(n_c)}{dt(n_d)} \neq 0$. Using the the definition of $x(n_c)$ (1) and the optimal price (response function) of the firms (12), we have that

$$\begin{aligned} \frac{dx(n_c)}{dt(n_d)} &= \frac{\partial p_c(n_c)}{\partial t(n_d)} - \frac{\partial p_d(n_d)}{\partial t(n_d)} - 1 \\ &= 1 - \frac{q_c(n_c)}{-q_c'(n_c)}\frac{x(n_c)}{\sigma^2} - \left(-1 - \frac{q_d(n_d)}{-q_d'(n_d)}\frac{x(n_c)}{\sigma^2}\right) - 1 \\ &= 1 - \frac{x(n_c)}{\sigma^2}\left(\frac{q_c(n_c)}{-q_c'(n_c)} - \frac{q_d(n_d)}{-q_d'(n_d)}\right) = 1 - \frac{x(n_c)}{\sigma^2}\frac{1-2\Phi(x(n_c))}{\phi(x(n_c))} \end{aligned} \tag{20}$$

We first look at the case with two sponsors. Assume $\frac{dx(n_c)}{dt(n_d)} = 0$. This implies:

$$x(n_c) = \sigma^2 \frac{\phi(x(n_c))}{1 - 2\Phi(x(n_c))} \tag{21}$$

If we take the derivative of the right hand side (RHS), we get

$$\frac{\partial \text{RHS}}{\partial x} = \frac{\phi'(x)\left[1-2\Phi(x)\right]+2\phi(x)^2}{\left[1-2\Phi(x)\right]^2}\begin{cases} >0 & \text{if } x<0 \\ \text{not defined} & \text{if } x=0 \\ >0 & \text{if } x>0 \end{cases}$$

and as we have that RHS$(-\infty) = 0$, RHS$(x \nearrow 0) = +\infty$, while RHS$(x \swarrow 0) = -\infty$ and RHS$(+\infty) = 0$, we can see that (21) can never be satisfied.

In the competitive case we have $\frac{\partial p_c(n_c)}{\partial t(n_d)} = \frac{\partial p_d(n_d)}{\partial t(n_d)} = 0$. Hence, $\frac{dx(n_c)}{dt(n_d)} = -1$. In the case with only a clean sponsor, we have

$$\frac{dx(n_c)}{dt(n_d)} = \frac{\partial p_c(n_c)}{\partial t(n_d)} - 1 = \frac{1-\Phi\left[x(n_c)\right]}{-\phi(n_c)}\frac{x(n_c)}{\sigma^2}$$

which is zero only if $x(n_c) = 0$. This implies that the arriving consumer will choose either of the networks with equal probability for any network size and for any entry price the clean producer might set. This solution cannot be an optimum given that the dirty network pollutes. Hence, we conclude that the first-order condition requires:

$$x(n_c) = \Lambda^d(n_d) - \Lambda^c(n_c).$$

We can now turn to the second-order condition. We differentiate (18) to get

$$
\begin{aligned}
&\sigma^2\phi''(x)\left(\frac{dx}{dt}\right)^2 + \sigma^2\phi'(x)\frac{d^2x}{dt^2} \\
&+ \left(\Lambda^d(n_d) - \Lambda^c(n_c)\right)\cdot\left[\phi'(x)\left(\frac{dx}{dt}\right)^2 + \phi(x)\frac{d^2x}{dt^2}\right] \\
&= \sigma^2\left(\frac{dx}{dt}\right)^2\left(\frac{-\phi(x)}{\sigma^2} - \frac{x}{\sigma^2}\phi'(x)\right) - x\phi(x)\frac{d^2x}{dt^2} \\
&+ \left(\Lambda^d(n_d) - \Lambda^c(n_c)\right)\cdot\left[-\frac{x}{\sigma^2}\phi(x)\left(\frac{dx}{dt}\right)^2 + \phi(x)\frac{d^2x}{dt^2}\right] \\
&= \left(\frac{dx}{dt}\right)\left(\frac{-x}{\sigma^2}\right)\overbrace{\left[\sigma^2\phi'(x)\frac{dx}{dt} + \left(\Lambda^d(n_d) - \Lambda^c(n_c)\right)\phi(x)\frac{dx}{dt}\right]}^{=0} \\
&- \phi(x)\left(\frac{dx}{dt}\right)^2 + \phi(x)\frac{d^2x}{dt^2}\left[\underbrace{\Lambda^d(n_d) - \Lambda^c(n_c) - x}\right]_{=0} \\
&= -\phi(x)\left(\frac{dx}{dt}\right)^2 < 0
\end{aligned}
$$

We have a globally defined function, everywhere differentiable in x with only one stationary point, and this point is a local max. Hence it is also a global max. The problem might be that $g(\cdot)$ is not continuous in t if firms respond very non-linearly to taxes, even if it is continuous in x. Luckily, we have that $x(n_c)$ is differentiable in t, hence it is also continuous.

If this was an ordinary optimization problem, we would conclude that there is one unique solution, it is the global maximum of the welfare function, and we have found it! If this holds true also in this game, then given Cabral's uniqueness results, we have a unique equilibrium in our game. Unfortunately, we don't think this will hold in this game. The problem is that when the change the tax, we also change the value function, so we would need some other result to claim uniqueness.

C The Pigovian entry tax

In the numerical simulations, we use the Pigovian entry tax rate as a benchmark. This rate can be found by looking at the outcome of the model when all network effects are absent. The choice probabilities of the consumers will then only depend on the current prices and taxes, and not on the future expected sizes of the networks. The firm continuation values will therefore also be independent of the network sizes e.g. firms cannot increase the probability of a future sale by increasing their current network.

Assume there exists an equilibrium in constant prices and taxes. Then, the firms' value functions will be constant across states, and equal to $v_k(n_k) = \frac{1}{1-\delta} q_k p_k$. The optimal prices are then given by:

$$p_c(t,\tau) = \frac{1-\Phi(x_c)}{\phi(x_c)},\ p_d(t,\tau) = \frac{\Phi(x_c)}{\phi(x_c)} \tag{22}$$

where

$$x_c = p_c - p_d - t - \tau(1 - \delta\frac{N-1}{N})^{-1} \tag{23}$$

We note from (22) and (23) that if the two taxes are kept constant, prices must also be kept constant partly confirming that we are on the right track. When everything is constant and the environmental damage is linear, we can 'guess and verify' a linear government value function:

$$g(n_c) = \frac{1}{1-\delta}\left[\sigma^2\phi(x) - N\cdot\gamma + q_c(x)\frac{\gamma}{1-\delta\frac{N-1}{N}}\right] + \frac{\gamma}{1-\delta\frac{N-1}{N}} n_c.$$

Solving for the optimal $x(n_c)$, we find that the government implements the following:

$$\bar{x}(n_c) = \frac{-\gamma}{1-\delta\frac{N-1}{N}},\ \forall n_c$$

where the right-hand side is the present value of expected environmental damages of joining the dirty network. Each consumer pollutes to a marginal damage of d every period, and the discount rate is augmented to take into account that the consumer will die with probability $1/N$

each period. This expression is a constant, confirming that we have an equilibrium of the model.

From (23) we note that the two tax instruments are "perfect substitutes". Hence, the government needs only one of the taxes. To find the entry tax, we use that firms set prices according to (22), and we can calculate the entry tax to be:

$$\bar{t} = -\bar{x} + p_c - p_d = \frac{\gamma}{1 - \delta \frac{N-1}{N}} + \frac{1 - 2\Phi(\bar{x})}{\phi(\bar{x})}.$$

Since the firms have market power, their prices differ from the marginal costs, and so the tax differs from the environmental damage by the constant term $\frac{1-2\Phi(\bar{x})}{\phi(\bar{x})}$. If prices were set to marginal costs, then this discrepancy would disappear, and the tax would equal the damages.

In the simulations we use $\bar{t} = \frac{d}{1-\delta \frac{N-1}{N}}$ for all cases. Hence, our Pigovian tax rate does not adjust for market power.