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Teams and Tournaments in Relational Contracts

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Abstract

We analyze relational contracts for a set of agents when either (a) only aggregate output or (b) individual outputs are observable. A team incentive scheme, where each agent is paid a bonus for aggregate output above a threshold, is optimal in case (a). The team's efficiency may increase considerably with size if outputs are negatively correlated. Under (b) a tournament scheme with a threshold is optimal, where the threshold, for correlated outputs, depends on an agent's relative performance. The two cases reveal that it may be optimal to organize production as a team where only aggregate output is observable.

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Keywords: relational contracts, teams, tournaments.

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1 Introduction

An increasing number of firms tie compensation to their workers' performance, but the way firms arrange their incentive programs varies to a large extent (see Lemieux et al., 2009 and Bloom and Van Reenen, 2010). Some firms rely on team incentives in which bonuses are tied to the joint output of a team of workers. Other firms rely on tournament schemes in which workers compete against each other for bonuses or other rewards. Furthermore, many firms combine both tournaments and team incentive schemes.

An important reason for this variation in how firms provide incentives to their employees may be attributed to technological differences. First, it is a matter of observability. Some firms only observe the aggregate output from teams of workers, while other firms may be able to get an exact measure of each individual's output. Second, it is a matter of technological or stochastic dependence between the workers. Some workers' outputs are positively correlated, such as sales agents who are exposed to the same business cycles. In other situations, workers' outputs are negatively correlated, for instance when specialists with different expertise meet different sets of demand from customers or superiors.

In this paper we study how these issues affect optimal incentive design. In contrast to previous literature, we focus on repeated game relational contracts. A relational contract includes variables that are hard to verify by a third party, such as the quality of a service or the value of a performance. As a result, the contract cannot be enforced by a court of law and needs to be self-enforcing. We study how observability and technological/stochastic dependence between workers affect the conditions for implementing self-enforcing relational contracts, and furthermore, what the optimal relational contract looks like in different situations.

In particular, we analyze and compare optimal relational contracts between a principal and a set of agents when (a) only aggregate output can be observed, and (b) individual outputs can be observed. We first show that the optimal contract under (a) is a team incentive scheme where each agent is paid a maximal bonus for aggregate output above a threshold and a minimal (no) bonus otherwise. This parallels Levin's (2003) characterization for the single-agent case. We then show that the team's efficiency decreases with its size (number of agents, n) when outputs are non-negatively correlated, but that efficiency may increase considerably with size if outputs are negatively correlated. The latter effect arises because including more agents in the team leads to reduced variance for the team's output, and hence a more precise performance measure. This is beneficial not because a more precise measure reduces risk (since all agents are risk neutral by assumption), but because it strengthens, for any given bonus level, the incentives for each team member to provide effort. For sufficiently small variance it turns out that the standard first order approach (used by e.g. Levin, 2003) is not valid, but we show that a threshold bonus is nevertheless optimal, and we characterize its properties.[1] The analysis demonstrates that the added effort incentives coming from negative correlation and hence more precise performance measurement can be quite valuable. At a broader level, our results indicate that diversity and heterogeneity among team members can yield considerable efficiency improvements (see Horwitz and Horwitz, 2007, for a meta-analytic review documenting positive effects from team diversity).

Another benefit of having many agents in the team is demonstrated in a setting where agents have ex post bargaining power over the values they have created. In such a setting, a team of agents can also create values in case the relational contract breaks down. Due to the well known free-rider problem, this outside value decreases in the number of agents. However, the weaker outside option strengthens the relational contract and thereby allows for a higher bonus and thus cet. par higher effort. In other words, the $1/n$ free-rider problem might be a blessing in relational contracts.

In case (b), where individual output is observable, Levin (2002) has shown that for independent outputs the optimal relational contract entails a stark RPE scheme (relative performance evaluation); a form of a tournament, where at most one agent is paid a (maximal) bonus. We point out that the efficiency of this tournament scheme increases with the number of agents, and hence becomes progressively better compared to a team when the number of independent agents increases. Then we extend the analysis to correlated variables, and show, for a parametric (normal) distribution, that the

[1] In fact, the analysis reveals that in the single-agent case, MLRP alone ensures that a threshold bonus is optimal.

optimal contract is an RPE scheme with a threshold, where the threshold depends on an agent's relative performance, and where the conditions for an agent to obtain the (single) bonus are then stricter for negatively compared to positively correlated outputs. The efficiency of this tournament contract is shown to improve with higher correlation (both positive and negative).

We finally point out that if the firm can initially choose between organizations (technologies) that allow for either (a) only aggregate output or (b) individual outputs to be observed, and a subsequent reorganization is costly, then the firm may choose (a), i.e. organize production as a team. Thus, even if alternatives (a) and (b) are equally costly to set up initially (e.g. in terms of output measurement investments), the team alternative may yield a higher subsequent surplus. This occurs because relational contract constraints may be affected in a way to favor the team alternative. We show that although production efficiency in both alternatives increases with more negatively correlated outputs, the team alternative is more likely to be superior under such conditions.

Related literature: The closest related paper is the above mentioned Levin (2002). He considers a multilateral relational contract between a principal and n agents, and shows among other things that the stark RPE (tournament) scheme is optimal. Unlike Levin, we also consider the case where only aggregate output is observable. Moreover, we extend Levin's characterization to correlated outputs. Our paper is also related to the few papers considering team incentives in relational contracts, like Kvaløy and Olsen (2006, 2008), Rayo (2007) and Baldenius and Glover (2010). But in these papers individual outputs are observable, and so they do not consider how both observability and stochastic dependence between agents affect the optimal contract.[2]

Previous literature on incentive provision to multiple agents has mainly focused on risk sharing issues and the scope for cooperation. The informativeness principle (Holmström, 1979, 1982) states that an incentive contract should be based on all variables that provide information about the agents' actions. Stochastic and/or technological dependencies between agents then

[2]Seminal contributions to the (formal) literature on relational contracting include Klein and Leffler (1981), Shapiro and Stiglitz (1984), Bull (1987) and MacLeod and Malcomson (1989).

typically call for "peer-dependent" incentive schemes such as teams or tournaments. By tying compensation to an agent's relative performance, the principal can filter out common noise and thereby expose them to less risk (see Holmström, 1982; and Mookherjee, 1984).[3] And by tying compensation to the joint performance of a team of agents, the principal can exploit complementarities between the agents' efforts and foster cooperation, see Holmström and Milgrom, 1990; Itoh (1991, 1992) and Macho-Stadler and Perez-Castrillo, 1993).[4]Our paper shows that stochastic dependence between agents is highly important for incentive design even in the absence of risk considerations, and that team incentives may be optimal even without classical team effects such as complementarities in production, peer pressure or peer monitoring.

Our paper is also related to a recent literature on endogenous formation of teams. While there is a large agency literature that studies optimal incentives for teams[5], there are only a few papers that explore how and why firms may only hold a team of agents accountable for their joint output, even if individual accountability is technologically feasible. Mukherjee and Vasconcelos (2011) and Corts (2007) show that team production might help mitigate multitask problems, while Bar-Isaac (2007) show that teams consisting of juniors and seniors can restore the reputation concerns of seniors. We show that firms may use team production (team accountability) as a commitment device. By deliberately choosing team assignment instead of individual assignment, the firm makes it more costly to breach the relational contract. But we also show that there is a limit to how many agents the firm should hold accountable. The optimal team size depends both on the agents' ex post bargaining power and on the type of dependence between the agents.

Finally, our paper is related to a literature on asset ownership and bargaining

[3]See also Lazear and Rosen (1981), Nalebuff and Stiglitz (1983) and Green and Stokey (1983) for analyses of RPE's special form, rank-order tournaments.

[4]In addition, team incentives can provide implicit incentives not to shirk (or exert low effort), since shirking may have social costs (as in Kandel and Lazear, 1992), or induce other agents to shirk (as in Che and Yoo, 2001).

[5]Economists studying teams with unobservable individual ouputs, beginning with Alchian and Demsetz (1972), have mainly focused on the free-rider problem, in particular under what conditions the first-best outcome will be achieved, or what parameters affect the relative efficiency of teamwork. Influential papers include Holmstrom (1982) Rasmusen (1987), McAfee and McMillan (1991) and Legros and Matthews (1993).

power in relational contracts, such as Baker, Gibbons and Murphy (2002), Halonen (2002) and Kvaløy and Olsen (2012). A central point here is that agents' bargaining power may negatively affect the scope for relational contracting. In particular Halonen (2002) shows that agents may consider joint ownership of assets (similar to team production) in order to reduce outside options and thereby strengthen the relational contract between them. However, Halonen does not consider a principal-multiagent incentive problem like we do.

The rest of the paper is organized as follows. Section 2 presents the model and analyses team incentives, given that only total output can be observed. Section 3 deals with the case where individual outputs can be observed, and Section 4 contains a comparative analysis of the two cases. The last section concludes.

2 Model

We analyze an ongoing economic relationship between a principal and n (symmetric) agents. All parties are risk neutral. Each period, each agent i exerts effort e_i incurring a private cost $c(e_i)$. Costs are strictly increasing and convex in effort, i.e., $c'(e_i) > 0$, $c''(e_i) > 0$ and $c(0) = c'(0) = 0$. Each agent's effort generates a stochastic output x_i, with marginal density $f(x_i, e_i)$. Expected outputs are given by $\bar{x}(e_i) = E(x_i|\, e_i) = \int x_i f(x_i, e_i) dx_i$ and total surplus per agent is $W(e_i) = \bar{x}(e_i) - c(e_i)$. First best is then achieved when $\bar{x}'(e_i^{FB}) - c'(e_i^{FB}) = 0$. Outputs are stochastically independent (given efforts) across time.

The parties cannot contract on effort provision. We assume that effort e_i is hidden and only observed by agent i. With respect to output, we consider two cases: Either individual outputs x_i are observable (IO), or only total output $y = \Sigma x_i$ is observable. In both cases, we assume that outputs are non-verifiable by a third party. Hence, the parties cannot write a legally enforceable contract on output provision, but have to rely on self-enforcing relational contracts.

2.1 Team: only total output observed

We first consider the case where individual output is unobservable, and hence the parties can only contract on total output provision. Each period, the principal and the agents then face the following contracting situation. First, the principal offers a contract saying that agent i receives a non-contingent fixed salary α_i plus a bonus $\beta_i(y)$, $i = 1...n$ conditional on total output $y = \Sigma x_i$ from the n agents.[6] Second, the agents simultaneously choose efforts, and value realization $y = \Sigma x_i$ is revealed. Third, the parties observe y and the fixed salary α_i is paid. Then the parties choose whether or not to honor the contingent bonus contract $\beta_i(y)$.

Conditional on efforts, agent i's expected wage in the contract is then $w_i = E(\beta_i(y)|\, e_1...e_n) + \alpha_i$, while the principal expects $\Pi = E(y|\, e_1...e_n) - \Sigma w_i = \Sigma_i E(x_i|\, e_i) - \Sigma w_i$. If the contract is expected to be honored, agent i chooses effort e_i to maximize his payoff, i.e.

$$e_i = \arg\max_{e_i'} \left(E(\beta_i(y)|\, e_i', e_{-i}) - c(e_i') \right) \tag{IC}$$

If the contract is not honored, the parties instead bargain over the realized values. Given a realization y, we assume that they agree on a spot price ηy, where $\eta < 1$ is the agents' share.[7] The parameter η can be interpreted as an index of the agents' total hold-up power.

In a one shot relationship, the parties have no incentives to honor the bonus contract, and so they have to rely on spot contracting. The expected spot price is then $\eta E(y|\, e_1...e_n) = \eta \Sigma \bar{x}(e_i)$. Agent i thus chooses spot effort e_i^s according to $\frac{1}{n}\eta \bar{x}'(e_i^s) - c'(e_i^s) = 0$, and so the expected spot price can be written $S = \eta \bar{y}(e_i^s)$, while the principal's expected spot profit is given by $\pi_s = (1-\eta)\bar{y}(e_i^s)$.

Now consider the repeated game. Like Levin (2002) we consider a multilateral punishment structure where any deviation by the principal triggers

[6] We thus assume stationary contracts, which have been shown to be optimal in settings like this (Levin 2002, 2003).

[7] More specifically, we may assume that the spot price is determined by Nash bargaining. If the agents are able to attain θy, $\theta \in [0,1]$ in an alternative market, then in Nash bargaining the agents will receive θy plus a share σ of the surplus from trade, so the spot price will be $S = \theta y + \sigma(y - \theta y) = \eta y$ where $\eta = \sigma + \theta(1-\sigma)$.

punishment from all agents. The principal honors the contract only if all agents honored the contract in the previous period. The agents honor the contract only if the principal honored the contract with all agents in the previous period. Thus, if the principal reneges on the relational contract, all agents insist on spot contracting forever after. And vice versa: if one (or all) of the agents renege, the principal insists on spot contracting forever after.[8] A natural explanation for this is that the agents interpret a unilateral contract breach (i.e. the principal deviates from the contract with only one or some of the agents) as evidence that the principal is not trustworthy (see discussion in Bewley, 1999 Levin, 2002).

Now, (given that (IC) holds) the principal will honor the contract with all agents $i = 1, 2, ..., n$ if

$$-\Sigma_i \beta_i(y) + \frac{\delta}{1-\delta}\Pi \geq -\eta y + \frac{\delta}{1-\delta}\pi_s \tag{EP}$$

where δ is a common discount factor. The LHS of the inequality shows the principal's expected present value from honoring the contract, which involves paying out the promised bonuses and then receiving the expected value from relational contracting in all future periods. The RHS shows the expected present value from reneging, which involves spot trading of the realized outputs, and then receiving the expected value associated with spot trading in all future periods.

Agent i will honor the contract if

$$\beta_i(y) + \frac{\delta}{1-\delta}(w_i - c(e_i)) \geq \frac{1}{n}\eta y + \frac{\delta}{1-\delta}(\frac{1}{n}S - c(e_i^s)) \tag{EA}$$

where similarly the LHS shows the agent's expected present value from honoring the contract, while the RHS shows the expected present value from reneging.

Recall the definition $W(e_i) = E(x_i|\, e_i) - c(e_i)$ as the total surplus associated with agent i, and define 'modified' bonuses as follows:

$$b_i(y) = \beta_i(y) - \frac{1}{n}\eta y. \tag{1}$$

[8] See Miller and Watson (2013) on alternative strategies and "disagreement play" in repeated games.

Following established procedures (e.g. Levin 2002) we obtain the following:

Lemma 1 *For given efforts $e = (e_1...e_n)$ there is a wage scheme that satisfies (IC,EP,EA) and hence implements e, iff there are bonuses β and fixed salaries α with $b_i(y) = \beta_i(y) - \frac{1}{n}\eta y \geq 0$, such that (IC) and condition (EC) below holds:*

$$\Sigma_i b_i(y) \leq \frac{\delta}{1-\delta}\Sigma_i(W(e_i) - W(e_i^s)). \tag{EC}$$

To see sufficiency, set the fixed wages α such that each agent's payoff in the contract equals his spot payoff, i.e. $\alpha_i + E(\beta_i(y)|\, e) - c(e_i) = \frac{1}{n}S - c(e_i^s) \equiv u^s$. Then EA holds since $\beta_i(y) - \frac{1}{n}\eta y \geq 0$. Moreover, the principal's payoff in the contract will be $\Pi = \Sigma_i(W(e_i) - u^s) = \Sigma_i(W(e_i) - W(e_i^s)) + \pi_s$, i.e. the surplus generated by the contract plus her spot profits. Then EC and (1) imply that EP holds. Necessity follows by standard arguments.

Unless otherwise noted, we will follow the standard assumption in the literature and assume that the first order approach (FOA) is valid, and hence that each agent's optimal effort choice is given by the first-order condition (FOC):

$$\frac{\partial}{\partial e_i}E(\beta_i(y)|\, e_1...e_n) - c'(e_i) = 0$$

It is convenient to use the 'modified' (net) bonuses b_i when analyzing the contract. Since $Ey = \Sigma_j \bar{x}(e_j)$, the FOC can then be written

$$\frac{\partial}{\partial e_i}E(b_i(y)|\, e_1...e_n) + \frac{1}{n}\eta\bar{x}'(e_i) = c'(e_i) \tag{2}$$

Given that FOA is valid, the agents' optimal choices are characterized by the condition (2), which we will refer to as a 'modified' IC constraint. We will further assume that the 'monotone likelihood ratio property' (MLRP) holds for aggregate output y in the following sense: its density is assumed to be of the form $g(y; l(e_1...e_n))$ with $l_{e_i}(e_1...e_n) > 0$, and such that $\frac{g_l(y,l)}{g(y,l)}$ is increasing in y.

The optimal contract now maximizes total surplus ($\Sigma_i W(e_i) = \Sigma_i(E(x_i|\, e_i) - c(e_i))$) subject to EC and the 'modified' IC constraint (2). Then we have the following:

Proposition 1 *The optimal symmetric scheme pays a maximal bonus to* *each agent for output above a threshold ($y > y_0$) and no bonus otherwise. The threshold is given by $\frac{g_l(y_0,l(e))}{g(y_0,l(e))} = 0$. For $l(e_1...e_n) = \Sigma_i e_i$ no asymmetric scheme can be optimal.*

The maximal symmetric bonus is by EC $b_i(y) = b(y) = \frac{\delta}{1-\delta}(W(e_i) - W(e_i^s))$ when efforts e_i are equal for all i. This result parallels that of Levin (2003) for the single agent case. The threshold property comes from the fact that incentives should be maximal (minimal) where the likelihood ratio is positive (negative). Since this ratio is monotone increasing, there is a threshold y_0 where it shifts from being negative to positive, and hence incentives should optimally shift from being minimal to maximal at that point.

2.2 Team size and efficiency

We will now study team size and efficiency. To see how size (i.e. number of agents in the team) affects efficiency, note from Proposition 1 that the IC constraint (2) can now be written

$$c'(e_i) = b \int_{y>y_0} g_i(y; e_1...e_n)dy + \frac{1}{n}\eta\bar{x}'(e_i)$$

where g_i denotes partial derivative of the density wrt e_i, and hence that the optimal solution $e_i = e_i^*$ (the maximal effort per agent that can be implemented) is given by

$$\frac{c'(e_i^*) - \frac{1}{n}\eta\bar{x}'(e_i^*)}{\int_{y>y_0} g_i(y; e^*)dy} = b = \frac{\delta}{1-\delta}(W(e_i^*) - W(e_i^s(n))) \qquad (3)$$

The first equality shows the required bonus (per agent) to implement effort e_i^* (from the IC constraint). The second equality shows the feasible (maximal) bonus. When n increases, a single agent's marginal influence on his expected bonus payment (i.e. $b\int_{y>y_0} g_i(y; e^*)dy$) will be affected. If this marginal influence is reduced (as it typically will be for independent outputs), a larger bonus is required to maintain effort incentives (the first equality). A higher bonus is also required because the 'automatic incentive' ($\frac{1}{n}\eta\bar{x}'(e_i)$) is reduced when n increases. But a higher bonus is also feasible (the second equality) because the outside spot value $W(e_i^s(n))$ is decreasing in n. Which of these effects dominates will determine whether effort (per agent) will increase or decrease when the number of agents increases.

It is of particular interest to analyse teams with stochastic dependencies among the individual team members' contributions to total output. To make this analytically tractable we will *assume that outputs are (multi)normally distributed and correlated.* Given this assumption, and (by symmetry) each x_i being $N(e_i, s^2)$, then total output $y = \Sigma x_i$ is also normal with expectation $Ey = \Sigma e_i$ and variance

$$s_n^2 = var(y) = \Sigma_i var(x_i) + \Sigma_{i \neq j} cov(x_i, x_j) = ns^2 + s^2 \Sigma_{i \neq j} corr(x_i, x_j)$$

It follows from the form of the normal density that the likelihood ratio is linear and given by $\frac{g_i(y, e_1 \ldots e_n)}{g(y, e_1 \ldots e_n)} = (y - \Sigma e_i)/s_n$. As shown above, the optimal bonus is maximal (minimal) for outcomes where the likelihood ratio is positive (negative), and hence has a threshold $y_0 = \Sigma e_i^*$ in equilibrium. Applying the normal distribution, it then follows (as shown below, see (7)) that the marginal return to effort for each agent in equilibrium is given by

$$b \int_{y > y_0} g_i(y; e^*) dy = b/(M s_n), \qquad M = \sqrt{2\pi} \tag{4}$$

Since by assumption now $\bar{x}(e_i) = Ex_i = e_i$, the IC condition (2) for each agent's (symmetric) equilibrium effort is therefore $c'(e_i) - \frac{1}{n}\eta = \frac{b}{s_n}\frac{1}{M}$. It then follows from (3) that the maximal effort per agent that can be sustained, is now given by

$$\left(c'(e_i^*) - \frac{1}{n}\eta \right) s_n M = b = \frac{\delta}{1-\delta}(W(e_i^*) - W(e_i^s(n)) \tag{5}$$

Consider now variation in team size. In line with the discussion above, a higher n has here three specific effects:

1. It reduces the outside spot value and thereby allows for a higher bonus, and thus cet. par for higher effort.

2. It reduces the 'automatic incentive' $\frac{1}{n}\eta$ and thereby cet. par the effort.

3. It affects the variance s_n^2 of the performance measure ($y = \Sigma x_i$)

If all agents' outputs are fully symmetric in the sense that all correlations as well as all variances are equal across agents, i.e. $var(x_i) = s^2$ and $corr(x_i, x_j) = \rho$ for all i, j, then the variance in total output will be

$$s_n^2 = ns^2 + s^2\Sigma_{i\neq j}corr(x_i, x_j) = ns^2(1 + \rho(n-1))$$

If $\rho \geq 0$ the variance will increase with n and the third effect discussed above is detrimental for efficiency. Optimal n should therefore be smaller with larger ρ. Moreover, the standard deviation of total output (s_n) increases rapidly with n when $\rho \geq 0$ (at least of order $\sqrt{n}$), while all other terms in the relation (5) stay bounded, hence the effort per agent that can be sustained will then decrease rapidly with n. Large teams are therefore very inefficient if all agents' outputs are non-negatively correlated.

For negative correlations the situation is quite different. If $\rho < 0$ one can in principle reduce the variance to (almost) zero by including sufficiently many agents. The model then indicates that adding more and more agents to the team is beneficial, at least as long as $1 + \rho(n-1) > 0$ and the conditions for FOA to be valid are fulfilled. (We show below that for this to be the case, the variance of the performance measure, here s_n^2, cannot be too small.)

Note that assuming symmetric pairwise negative correlations among n stochastic variables only makes sense if the sum has non-negative variance, and hence $1 + \rho(n-1) \geq 0$.[9] Given $\rho < 0$, there can thus only be a maximum number n of such variables (agents). And given $n > 2$, we must have $\rho > -\frac{1}{n-1}$.

Note also that for given negative $\rho > -\frac{1}{2}$, the variance is first increasing, then decreasing in n (it is maximal for $n = \frac{1}{2}(1 - \frac{1}{\rho})$). Hence the optimal team size in this setting is either very small ($n = 2$) or 'very large' (includes all).

Proposition 2 *For symmetric agents, efficiency decreases rapidly with size if outputs are non-negatively correlated. For symmetric agents with negatively correlated outputs, efficiency first decreases (for n > 2) and then increases with increasing team size, hence efficiency is maximal either for a small or for a large team.*

[9] Indeed, $1 + \rho(n-1) > 0$ is the condition for the covariance matrix to be positive definit, and hence for the multinormal model to be well specified.

The assumption of equal pairwise correlations among all involved agents is admittedly somewhat special, but illustrates in a simple way the forces at play when the team size varies. In reality there might be positive as well as negative correlations among agents. A procedure to pick agents for least variance would then be for each n, to pick those n that yield the smallest variance. Then compare across n, weighting the three effects discussed above.

2.3 When is the first order approach (FOA) valid?

We will now examine under what conditions the FOA is valid for the model analyzed in the previous section. Thus consider y normally distributed with expectation $Ey = \Sigma e_i$ and a variance that will be denoted by $s^2 = var(y)$ in this section (to simplify notation). As already noted, this distribution satisfies MLRP. Each agent is offered a 'gross' bonus $\beta(y) = b(y) + \mu y$, where $\mu = \eta/n < 1/2$, and $b(y)$ is the net bonus with threshold at y_0.

Given that the principal seeks to implement effort e_i^* from each agent this way, the optimal threshold is $y_0 = \Sigma e_i^*$. Agent i's expected payoff, given own effort e_i and efforts $e_j^* = e_i^*$ from the other agents, is then

$$
\begin{aligned}
& b\Pr(y > y_0 |\, e_i) + \mu E(y|\, e_i) - c(e_i) \\
&= b\Pr(y - \Sigma_{j\neq i} e_j^* - e_i > e_i^* - e_i) + \mu e_i + \gamma' - c(e_i) \\
&= b(1 - H(e_i^* - e_i)) + \mu e_i + \gamma' - c(e_i)
\end{aligned}
$$

where $H()$ is the CDF for an $N(0, s^2)$ distribution and $\gamma' = \mu\Sigma_{j\neq i} e_j^*$. The FOC for the agent's choice is

$$bh(e_i^* - e_i) + \mu - c'(e_i) = 0 \tag{6}$$

where $h()$ is the density; $h() = H'()$. The FOA is valid if the agent's optimal choice is e_i^* and is given by this first-order condition, i.e. if

$$bh(0) + \mu - c'(e_i^*) = 0 \tag{7}$$

and no other effort $e_i \neq e_i^*$ yields a higher payoff for the agent. We note in passing that $h(0) = 1/\sqrt{2\pi var(y)}$, verifying the formula (4) above.

Due to the shape of the normal density, the agent's payoff is generally not concave. The second derivative is $-bh'(e_i^* - e_i) - c''(e_i)$, where $h'(e_i^* - e_i) < 0$ for $e_i < e_i^*$. The payoff is locally concave at $e_i = e_i^*$ (since $h'(0) = 0$), hence e_i^* is a local maximum, but there may be other local maxima (other solutions to FOC) for $e_i < e_i^*$. The situation is illustrated in Figure 1, which depicts the agent's marginal revenue $(bh(e_i^* - e_i) + \mu)$ and marginal cost for two values of the variance $s^2 = var(y)$ (and for $\mu = 0$). If the variance is sufficiently small there is a local maximum at some $e_i < e_i^*$ (satisfying the FOC), and the figure indicates (comparing areas under MC and MR) that this local maximum dominates that at e_i^*.

(See Figure 1 in the appendix)

This indicates that the FOA is valid here only if the variance of the performance measure (y) is not too small, and is confirmed in the following proposition.

Proposition 3 *The first order approach is valid if the variance of output s^2 is sufficiently large, but not valid if s^2 is sufficiently small.*

Remark. For the case of iso-elastic costs ($c(e) = ke^m$, $m \geq 2$) one can show that, for $\mu = 0$ FOA is valid if $\frac{e_i^*}{s} < K = K_0\sqrt{m-1}$, where $K_0 \approx 2.216$. For $m = 2$ FOA is valid if $\frac{e_i^* - \mu/2k}{s} < K_0$. These are conditions that ensure a unique solution to the agent's FOC, and are hence sufficient, but not necessary conditions for FOA to be valid.

2.4 A threshold bonus is optimal

We saw in Section 2.2 that for negatively correlated agents, the variance in the performance measure y could be made quite small by including many agents in the team. And we saw that this was beneficial for incentives and consequently for efficiency as long as the analysis building on FOA was valid. But for sufficiently small variance FOA is not valid, so this immediately raises the question of what a team can achieve under such circumstances. In the following we will show that a threshold bonus is always optimal for the team model considered in this paper, and moreover characterize its properties.

Mainly to simplify notation, we consider here the case $\eta = 0$, so that the EC constraint for symmetric efforts is $0 \leq b(y) \leq \frac{\delta}{1-\delta}W(e_i)$. To provide incentives, the bonus cannot be maximal for all outputs y, hence the expected bonus payment for an agent must be less than the maximal bonus, i.e. $E(b(y)|\, e_i, e_{-i}) < \frac{\delta}{1-\delta}W(e_i)$. On the other hand, the agent's expected payoff from exerting effort must be non-negative; $E(b(y)|\, e_i, e_{-i}) - c(e_i) \geq E(b(y)|\, e_i = 0, e_{-i}) \geq 0$, so in any symmetric equilibrium we must have $c(e_i) < \frac{\delta}{1-\delta}W(e_i)$. It follows from this that the effort e_u^* and associated surplus $W(e_u^*)$ defined by

$$c(e_u^*) = \frac{\delta}{1-\delta}W(e_u^*) \tag{8}$$

constitute upper bounds for, respectively, the effort and surplus (per agent) that can be achieved in a relational contract.[10] Note also that this upper bound can be achieved if there is no uncertainty, i.e. if (team) effort can be observed without noise; namely by paying the maximal bonus $b = c(e_u^*)$ to each agent conditional on total effort being at least ne_u^*.

We will now show that the optimal bonus is a threshold bonus which induces effort that converges to the upper bound as the variance in the performance measure goes to zero. The scheme is a simple modification of the threshold bonus scheme identified in Proposition 1, and consists of a relaxation of the threshold combined with an increase of the bonus relative to the latter scheme.

To show this let, for any (symmetric) bonus $b = b(y)$, agent i's performance-related payoff (utility) be denoted

$$u(b; e) = u(b; e_i, e_{-i}) = \int b(y)f(y, e)dy - c(e_i)$$

As a first step we show that if a non-threshold and a threshold bonus yield the same payoff to the agent (for given effort), the latter bonus yields the strongest marginal incentives.

Lemma 2 *If MLRP holds, we have:*

[10] We assume that δ is small enough so that e_u^* is below first best effort.

i) If $\tilde{b}(y)$ is not a threshold bonus, then for a threshold scheme $b_h(y)$ with $u(\tilde{b};e) = u(b_h;e)$ it holds: $u_{e_i}(b_h;e) > u_{e_i}(\tilde{b};e)$.
ii) If $\tilde{b}(y)$ is not a threshold bonus, and e^* is the associated equilibrium, then there is a threshold scheme $b_h(y)$ with $u(\tilde{b};e^*) = u(b_h;e^*)$, $u_{e_i}(b_h;e^*) > u_{e_i}(\tilde{b};e^*) = 0$ and $u(b_h;e_i,e^*_{-i}) \leq u(b_h;e^*_i,e^*_{-i})$ for all $e_i < e^*_i$.

Remark Note that *in a single-agent case, statement (ii) in the lemma implies that a threshold scheme must be optimal whenever MLRP holds.* For should some other scheme be optimal, then (ii) shows that there is a threshold scheme that will induce higher effort by the agent ($e_i > e^*_i$). This means that the assumptions typically invoked to ensure validity of FOA, such as convexity of the distribution function (CDF) in addition to MLRP (as in e.g. Levin 2003), are really not necessary in the context of relational contracting between two risk neutral individuals.[11]

Using this lemma, we can show that a threshold bonus will be optimal in the team model with normally distributed output considered in this paper.

Proposition 4 *For the team model with normally distributed output, the optimal symmetric bonus is a threshold bonus.*

When FOA is valid, the optimal threshold is the output at which the likelihood ratio is zero, which is the output $y_0 = \Sigma e^*_i$ in the normally distributed case. The problem with this scheme is that for sufficiently small s the agent's payoff is non concave. In particular, for $c'()$ convex ($c''' \geq 0$), the payoff has two local maxima[12], at e^*_i and at $e^0_i < e^*_i$, respectively, and e^0_i then gives the highest payoff for small s, so the agent will deviate from the supposed equilibrium effort e^*_i. The critical s is where the two local maxima yield the same payoff; i.e. $b(1 - H(0;s)) - c(e^*_i) = b(1 - H(e^*_i - e^0_i;s)) - c(e^0_i)$, where we as above have $\Pr(y > y_0 | e_i, e^*_{-i}) = 1 - H(e^*_i - e_i;s)$ and $H(\cdot;s)$ is the CDF for an $N(0,s^2)$ variable. In addition they both satisfy FOC, so $bh(e^*_i - e^0_i;s) = c'(e^0_i)$ and $bh(0;s) = c'(e^*_i)$.

[11] This result is in some respects similar to results in Poblete and Spulber 2012, showing that simpler assumptions than CDF and MLRP are sufficient for a debt-type contract to be optimal in the static principal-agent model under risk neutrality and limited liability.
[12] It follows from the shape of the density $h()$ that for $c'()$ convex ($c''' \geq 0$), the FOC (6) for effort can yield at most two local maxima.

For s below this critical level, the agent's payoff is higher at e_i^0. Now, this can be rectified by setting a lower threshold $y_0' < y_0 = ne_i^*$, i.e. making it easier to obtain the bonus, and at the same time increase the bonus level. For $y_0' = y_0 - \tau$ we have

$$\Pr(y > y_0' | \, e_{-i}^*, e_i) = 1 - H(e_i^* - e_i - \tau; s)$$

We can then choose τ and the bonus b such that e_i^* satisfies FOC and yields a payoff at least as high as the other local maximum e_i^0, i.e. such that we have

$$b(1 - H(-\tau; s)) - c(e_i^*) \geq b(1 - H(e_i^* - e_i^0 - \tau; s)) - c(e_i^0) \tag{9}$$

and

$$bh(-\tau; s) - c'(e_i^*) = 0 = bh(e_i^* - e_i^0 - \tau; s) - c'(e_i^0) \tag{10}$$

The smaller τ is, the smaller is the required bonus to satisfy FOC for e_i^*. The minimal such τ yields equality between the payoffs. Now, this scheme can at most allow a bonus

$$b \leq \frac{\delta}{1-\delta} W(e_i^*) \tag{11}$$

Hence, we see that the highest effort e_i that can be implemented by this scheme is the effort e_i^* defined by the conditions (9 - 11), where all hold with equality. Our next result shows that this is indeed the optimal scheme for s below the critical level where FOA ceases to be valid.

Proposition 5 *Given convex marginal costs ($c''' \geq 0$), there is a critical $s_c > 0$ for the standard deviation of output such that for $s \geq s_c$ FOA is valid and the optimal threshold y_0 is the output at which the likelihood ratio is zero, thus $y_0 = ne_i^*$. For $s < s_c$ the optimal threshold is an output $y_0' = ne_i^* - \tau$ (at which the likelihood ratio is negative), and the optimal scheme is given by (9 - 11) with all relations holding with equality. Effort e_i^* is strictly higher when s is lower, and $e_i^* \to e_u^*$ as $s \to 0$.*

It may be noted that for the set of variances $s^2 = var(y)$ sufficiently large to make FOA valid, the largest effort per agent that can be implemented must satisfy $2c(e_i^*) \leq \frac{\delta}{1-\delta} W(e_i^*)$, and hence be considerably smaller than the upper bound e_u^* defined in (8). This is so because the agent obtains the bonus

(b) with probability $\frac{1}{2}$ in equilibrium in the FOA scheme, hence we must have $b\frac{1}{2} \geq c(e_i^*)$ in that setting. This illustrates that a more precise performance measure can yield considerable benefits in relational contracting. The benefits are not associated with risk reduction (since all agents are risk neutral by assumption), nor with sharper competition, since in the team setting there is none. The benefits arise because a more precise measure strengthens individual incentives for effort, for a given bonus level. Since the bonuses in the relational contract are discretionary and hence must be kept within bounds, the added effort incentives coming from a more precise performance measure are valuable. And the value added may be considerable, as we have seen.

Thus far in this subsection we have taken the output variance ($s^2 = var(y)$) as an exogenous parameter. In Section 2.2 we pointed out that this variance can be substantially reduced if a team can be put together, consisting of several agents whose individual outputs are negatively correlated. As we have now illustrated, this may be of considerable value for the participants in the relational contract.

3 Individual outputs observed

Consider now the case where individual outputs are observable. The principal can then offer a bonus contract $\beta_i(x_1...x_n)$, to each agent $i = 1...n$, conditional on all individual outputs. Now, if the contract is expected to be honored, agent i's expected wage is then, for given efforts, $w_i = E(\beta_i(x_1...x_n)|\, e_1...e_n) + \alpha_i$, while the principal expects $\Sigma\bar{x}(e_i) - \Sigma w_i$. The agent then chooses effort

$$e_i = \arg\max_{e_i'} \left(E(\beta_i(x_1...x_n)|\, e_i', e_{-i}) - c(e_i') \right) \tag{12}$$

As in the case where individual output is unobservable, we assume that if the contract is not honored, the parties instead bargain over the realized values. But now the principal agrees on a spot price ηx_i with each individual agent. In a one shot relationship, the parties still have no incentives to honor the bonus contract, and so they have to rely on spot contracting. Expected spot price is then $S = \eta\bar{x}(e_i^s)$. Agent i thus chooses spot effort e_i^s according to $\eta\bar{x}'(e_i^s) - c'(e_i^s) = 0$, while the principal's expected spot profit is given by $\pi_s = (1-\eta)\bar{x}(e_i^s)$. Note here that spot effort is higher than in the team case since the marginal revenue from effort $\eta\bar{x}'(e_i^s)$ is not divided by n.

In a repeated relationship, we still assume that the principal honors the contract only if all agents honored the contract in the previous period, and that the agents honor the contract only if the principal honored the contract with all agents in the previous period.

Now, (given that the IC condition (12) holds) the principal will honor the contract with all agents $i = 1, 2, ..., n$ if

$$-\Sigma_i \beta_i(x_1...x_n) + \frac{\delta}{1-\delta}\Pi \geq -\Sigma_i \eta x_i + \frac{\delta}{1-\delta}\left[n\pi_s\right] \tag{13}$$

Agent i will honor the contract if

$$\beta_i(x_1...x_n) + \frac{\delta}{1-\delta}(w_i - c(e_i)) \geq \eta x_i + \frac{\delta}{1-\delta}(S - c(e_i^s)) \tag{14}$$

These enforcement constraints are stricter than in the team case where individual output is not observable. The reason is that the spot surplus is higher, and so the long-term costs from deviating from the relational contract are lower. This in turn may make it possible to implement higher effort under team incentives, as will be discussed later.

Define 'modified' (net) bonuses: $b_i(x_1...x_n) = \beta_i(x_1...x_n) - \eta x_i$. It is then straightforward to show (as in the previous case where only $y = \Sigma_i x_i$ is observed) that we have:

Lemma 3 *For given efforts $e = (e_1...e_n)$ there is a wage scheme that satisfies (12),(14)-(13) and hence implements e, iff there are bonuses β and fixed salaries α with $b_i(x_1...x_n) = \beta_i(x_1...x_n) - \eta x_i \geq 0$, such that (12) and condition (15) below hold:*

$$\Sigma_i b_i(x_1...x_n) \leq \frac{\delta}{1-\delta}\left(\Sigma_i W(e_i) - nW(e_i^s)\right) \tag{15}$$

Here $W()$ denotes as before surplus per agent; $W(e_i) = E(x_i|\, e_i) - c(e_i)$.

Assuming that FOA is valid, we can replace the IC constraint (12) with the first-order condition:

$$\frac{\partial}{\partial e_i}(E(b_i(x_1...x_n)|\, e_1...e_n) + \eta\bar{x}'(e_i) = c'(e_i) \tag{16}$$

The optimal contract then maximizes total surplus ($\Sigma_i W(e_i)$) subject to (15) and (16). Unless otherwise noted, all results here assume that FOA holds.

3.1 Independent outputs

Consider first independent outputs. This was analyzed by Levin (2002), who showed that the optimal contract is RPE with a bonus paid to at most one agent, namely the agent whose outcome yields the highest likelihood ratio. Moreover, the bonus is paid to this agent only if the likelihood ratio is positive. Given symmetric agents and strictly increasing likelihood ratios, this means that the agent with the largest output wins the bonus, but provided that his output exceeds some threshold x_0 (where the likelihood ratio $\frac{f_{e_i}(x_i;e_i)}{f(x_i;e_i)}$ is positive for $x_i > x_0$).

We will now use this result to analyze how the efficiency of this scheme varies with the number of agents (for independent outputs). The next section considers correlated outputs.

With n agents, agent i's probability of winning the bonus b, given own output $x_i = x > x_0$, and given symmetric efforts e_j from all others is now $\Pr(\max_j x_j < x) = F(x;e_j)^{n-1}$. Hence the expected bonus payment to agent i is $b\int_{x_0}^{\infty} F(x_i;e_j)^{n-1} f(x_i;e_i)dx_i$, and for symmetric efforts the IC condition (16) takes the form:

$$b\int_{x_0}^{\infty} F(x_i;e_i)^{n-1} f_{e_i}(x_i;e_i)dx_i + \eta\bar{x}'(e_i) = c'(e_i) \tag{17}$$

In passing, it is worth noting that the integral here extends only over values of x_i where $f_{e_i}(x_i;e_i) > 0$. In a standard tournament, where agent i would obtain a bonus when he had the largest output, the integral would extend over all values of x_i. The payment scheme here, which we may call a modified tournament, thus provides stronger incentives (for a given bonus b) than a standard tournament scheme.

The optimal RPE bonus is maximal, i.e. $b = \frac{\delta}{1-\delta}(\Sigma_i W(e_i) - nW(e_i^s))$, where $W(e_i)$ is total surplus (for agent i) and $W(e_i^s)$ is the outside spot surplus per agent. Hence, from (17) we have, in symmetric equilibrium

$$\frac{c'(e_i) - \eta \bar{x}'(e_i)}{\int_{x_0}^{\infty} F(x; e_i)^{n-1} f_{e_i}(x; e_i) dx} = b = \frac{\delta}{1-\delta} n(W(e_i) - W(e_i^s)) \qquad (18)$$

Consider now variations in the number of agents. Higher n increases the competition to obtain the bonus (the probability of winning is reduced), so the bonus must be increased to maintain effort; this is captured by the first equality in (18). The second equality shows how much the bonus can be increased; namely by the increased total surplus. The question is then whether the latter is sufficient to compensate for the reduced probability of winning.

The answer is affirmative, and the reason is essentially that while the surplus on the RHS increases proportionally with n, the marginal probability (in the denominator) on the LHS decreases less rapidly, so that $n \int_{x_0}^{\infty} F(x; e_i)^{n-1} f_{e_i}(x; e_i) dx$ increases with n. This allows a higher effort (per agent) to be implemented, so we have:

Proposition 6 *For observable and independent individual outputs, effort per agent in the RPE scheme (the modified tournament) increases with the number of agents.*

When individual output measures are available, and these outputs are independent, we thus see that efficiency in the (modified) tournament is improved by including more agents. This is in sharp contrast to efficiency in a team for independent outputs: as we saw above the team efficiency rapidly decreases under such conditions.

3.2 Correlated outputs

Consider now correlated outputs. For tractability reasons we will then again consider normal distributions, and as before limit attention to symmetric agents. A convenient feature of the multinormal distribution is that likelihood ratios are linear functions of the variables, and this simplifies comparisons of such ratios for these variables.

So assume now $x = (x_1 ... x_n)$ multinormal with $Ex_i = e_i$, $var(x_i) = s^2$ and (identical) correlations $corr(x_i, x_j) = \rho$. From the form of the multinormal distribution (see the appendix) the likelihood ratio for x_i is then

$$\frac{f_{e_i}(x|\, e_1 ... e_n)}{f(x|\, e_1 ... e_n)} = k_1(x_i - e_i) + k_2 \Sigma_{i \neq j}(x_j - e_j) \tag{19}$$

with $k_1 = \frac{1+(n-2)\rho}{(1+(n-1)\rho)(1-\rho)s^2} > 0$ and $k_2 = \frac{-\rho}{(1+(n-1)\rho)(1-\rho)s^2}$. Note that $k_1 - k_2 = \frac{1}{(1-\rho)s^2} > 0$

As we show in the appendix, for symmetric agents the optimal symmetric scheme pays a maximal bonus to the agent with the highest likelihood ratio, provided this ratio is positive, and no bonus to the other agents. From symmetry (including symmetric efforts in equilibrium; $e_i = e^*$ all i) the agent with the highest output has the highest likelihood ratio, and this ratio is positive iff[13]

$$x_i > e^* + \frac{\rho}{(n-2)\rho + 1} \Sigma_{j \neq i}(x_j - e^*) = E(x_i|\, x_{-i}) \tag{20}$$

This condition says that agent i's performance must exceed his expected performance, conditional on the performance of all other agents. Thus we have:

Proposition 7 *The optimal symmetric scheme pays a maximal bonus to the agent (say i) with the highest output, provided this output satisfies* $x_i > E(x_i|\, x_{-i})$.

For $n = 2$ agents we now have that agent 1 gets the bonus if and only if he has the highest output ($x_1 > x_2$) and $x_1 - e^* > \rho(x_2 - e^*)$. This is illustrated in Figures 2a and 2b for $\rho = \frac{1}{2}$ (left) and $\rho = -\frac{1}{2}$(right). Agent 1 is to get the bonus for outcomes to the right of the broken line.

[13] In this section it is convenient to let e^* be a scalar and denote the symmetric equilibrium effort level.

(See Figures 2a and 2b in the appendix)

In both cases the agent with the highest output gets the bonus if both of them have outputs that are above average $(x_1, x_2 > Ex_i = e^*)$. If agent 2 has below average output $(x_2 < Ex_i = e^*)$ the requirement for agent 1 to get the bonus is less strict when there is positive correlation than when there is negative correlation. In the latter case, agent 1 must have an output well above average to obtain the bonus, and more so the worse is the output for agent 2. Under negative (positive) correlation, a bad performance by agent 2 raises (lowers) the expected *conditional* performance of agent 1, and thus raises (lowers) the requirement –the hurdle (threshold)– for agent1 to get the bonus.[14]

Having characterized the optimal scheme, we will now consider its incentive properties. To make the analysis tractable, we restrict attention to $n = 2$ agents. Consider then agent 1's incentives in this scheme, with 'reference point' (equilibrium) $e_1^* = e_2^*$. His probability of obtaining the bonus is

$$\Pr(x_1 > \max[x_2, e_1^* + \rho(x_2 - e_2^*)] |\, e_1 e_2^*) \equiv \Pr(B) = \int_{x \in B} f(x |\, e_1 e_2^*) \quad (21)$$

So the marginal gain from effort is $\int_B f_{e_1}(x|\, e_1 e_2^*)$ and in symmetric equilibrium $e_1^* = e_2^* = e^*$ we will then have (given FOA valid)

$$b \int_B f_{e_i}(x|\, e^*, e^*) + \eta - c'(e^*) = 0$$

An interesting question is then: For given effort e^* to be implemented, how do marginal incentives vary with correlation ρ ? E.g. do these marginal incentives become stronger when ρ increases, implying that a lower bonus is required to implement the same effort? We should bear in mind that this is an RPE scheme and that such schemes generally work well both for positive and negative correlations in other settings. Perhaps not surprisingly a similar property turns out to be true here.

Proposition 8 *For correlated variables and $n = 2$, the agent's FOC for*

[14] To illustrate these points, if $\rho = .5$, and agent 2 has output 10% below expected $(x_2/e^* = .9)$, agent 1 can only win if his output is no more than 5% below expected. But if $\rho = -.5$, agent 1 must perform at least 5% *better* than expected in order to be eligible for the bonus (if in addition he wins).

(symmetric) equilibrium effort is

$$b\frac{1}{\sqrt{2\pi}s}\frac{1}{2}\left(\frac{1}{\sqrt{1-\rho^2}}+\frac{1}{\sqrt{1-\rho}}\frac{1}{\sqrt{2}}\right)+\eta=c'(e^*) \tag{22}$$

The marginal incentive in FOC (i.e. the expression on the LHS) is increasing in ρ for $\rho > \rho_0 \approx -0.236$ and decreasing in ρ for $\rho < \rho_0$. Hence, implementing a given effort requires a lower (higher) bonus when the correlation ρ increases for $\rho > \rho_0$ (for $\rho < \rho_0$).

This is illustrated in Figure 3, which depicts the marginal incentive as a function of ρ for the RPE scheme and for a standard tournament (dashed line).

(See Figure 3 in the appendix)

As a function of ρ, the marginal incentive (MI) for effort is thus U-shaped in the optimal scheme, which again is a modified tournament. In comparison, in a standard tournament the MI is monotone increasing in ρ (as shown by the dotted line; this MI is given by $\frac{d}{de_1}\Pr(x_1 > x_2) = \frac{1}{\sqrt{2\pi}s_d}$, where $s_d = \sqrt{2(1-\rho)}s$ is the standard deviation of $x_1 - x_2$, and the formula follows from the normal distribution). In comparison the modified tournament yields higher MI for effort for every ρ (which allows a higher effort to be implemented with the same bonus), and the MI is high both for strongly positive correlated and for strongly negative correlated outputs.

The latter property is caused by the specific criteria to obtain the bonus in the modified tournament, cfr the figures depicted above. In a standard tournament agent 1 wins and gets a bonus if $x_1 > x_2$, while in the modified tournament he gets a bonus only if $x_1 > x_2$ and $x_1 - e^* > \rho(x_2 - e^*)$. So the probability of obtaining the bonus is (all else equal) higher in a standard tournament, but the marginal effect of own effort on the probability (the marginal incentive MI), is higher in the modified tournament.

3.3 The validity of FOA

So far we have assumed FOA to be valid; this issue will now be examined more closely for the RPE scheme derived above. The question is then whether, for given symmetric efforts $e_1^* = e_2^*$ to be implemented by the modified tournament scheme, these efforts are indeed optimal choices for the respective agents.

In the appendix we show that the marginal gain to effort for agent 1 in the modified tournament scheme can be written as

$$\frac{b}{s}\Gamma(\frac{e_1 - e_1^*}{s}; \rho) + \eta - c'(e_1)$$

where $\Gamma(a; \rho)$ is a bell-shaped function defined there (see (35)). The FOC for $e_1 = e_1^*$ to be optimal (as stated in Proposition 8 above) can thus be written as $\frac{b}{s}\Gamma(0; \rho) + \frac{1}{2}\eta - c'(e_1^*) = 0$, and the local second order condition takes the form $\frac{b}{s}\Gamma_a(0; \rho)\frac{1}{s} - c''(e_1^*) \leq 0$. Since $\Gamma_a(0; \rho)$ turns out to be positive, these conditions imply that the standard deviation s cannot be too small. This is thus a necessary requirement for FOA to be valid in this setting. Moreover, we can also see that a 'large' s is sufficient for FOC to have a unique solution, and hence sufficient for FOA to be valid. More specifically we have the following result.

Proposition 9 *For given effort $e_i^* \leq e_i^{FB}$, a necessary condition for FOA to be valid is that*

$$\frac{e_i^*}{s} \leq \frac{m'}{1 - \eta/c'(e_i^*)}\sqrt{\pi}(\sqrt{2} + \sqrt{1+\rho})$$

where m' is the (local) elasticity of the marginal cost function; $m' = e_i^* \frac{c''(e_i^*)}{c'(e_i^*)}$. Moreover, there is $s' > 0$ such that FOA is valid for $s > s'$.

4 Teams or tournaments?

If individual outputs are observable, the principal may of course choose to base any discretionary bonuses only on aggregate output. Hence, if the relational contract constraints are unaffected by such a choice, the principal cannot do better with a scheme of the latter type. The RPE scheme based on individual outputs will then always be optimal. It follows that, if there is a choice between two equally costly technologies allowing for observation of, respectively, individual or aggregate output, the technology allowing individual output to be observed will be chosen. A (modified) tournament will then dominate a team.

However, if a chosen technology is costly to modify later on, the picture is no longer so clear. The reason is that relational contract constraints may be affected, in the sense that the respective outside options associated with spot trading will be different under the two technologies. Due to the free rider problem, spot trading is less efficient when only team output can be observed. Hence, if a team setup is chosen initially, then if the relational contract should break down, either a costly reorganization to individual output measurements and subsequent spot trading will take place, or (if reorganization costs are sufficiently high) spot trading based on team output will be the way the parties proceed. In any case the spot surplus will be smaller if the team organization was chosen initially. This implies in turn that the relational contract constraints are affected by the initial choice, and then it is no longer so clear that the team organization will be inferior. We will now examine this issue.

In the following we will assume that a reorganization of the team is so costly (relative to its benefits) that it will not take place if contract breakdown and subsequent spot trading should occur. The issue to be considered is then whether the surplus generated by the relational contract for the team (analyzed in Section 2) may dominate the surplus under the relational RPE contract based on individual outputs (analyzed in Section 3). Now, for each contract there will be a critical magnitude of the discount factor, say δ^{FB}, such that the contract generates the first-best surplus for $\delta \geq \delta^{FB}$, but not so for $\delta < \delta^{FB}$. A relatively simple way to compare the contracts is to compare their respective critical factors. The contract with the lower δ^{FB} will, for a range of $\delta' s$ exceeding the lower δ^{FB} strictly dominate the other.

For independent outputs, we know that the efficiency of the RPE tournament scheme improves with increasing number of agents, while the team's efficiency rapidly decreases with more agents. The team can thus only dominate if the number of agents is relatively small. In fact, for iso-elastic costs and independent outputs, the optimal team size (with respect to efficiency) typically turns out to be quite small ($n = 2$ or $n = 3$, depending on the magnitude of the elasticity and the magnitude of η). For quadratic costs (elasticity 2) we have the following.

Proposition 10 *For independent, normal outputs and quadratic effort costs we have: the optimal team size (in the sense of having the lowest critical δ^{FB}) is $n = 2$ if $\eta < \eta_0 \approx 0.805$, and $n = 3$ if $\eta > \eta_0$. Moreover, for $n = 2$ agents, the team dominates the RPE tournament (in the same sense) if and only if $\eta > \eta_1 \approx 0.739$.*

Consider next correlated outputs. For negatively correlated outputs, we know that the optimal team size may be large (Section 2.2), and that the efficiency of the team may be quite high. It thus seems reasonable to conjecture that, under such conditions, a team may dominate the RPE tournament even for n large. The analysis of this issue is hindered, however, by the optimal RPE scheme being difficult to analyze for correlated outputs and arbitrary $n > 2$. So we must at this stage confine the analysis to a comparison of the two schemes for $n = 2$ when outputs are correlated.

From the previous analysis we know that the RPE tournament has high efficiency both for strongly positive and strongly negative correlation. Since the team's efficiency is decreasing in ρ, it is thus to be expected that the tournament will tend to dominate for positive ρ. However, for negative ρ both schemes become more efficient with stronger (negative) correlation, hence it is not so clear what will happen there. It turns out that the team's efficiency improves relatively more for strongly negative ρ, as shown in the following proposition.

Proposition 11 *For correlated multinormal outputs, quadratic costs and $n = 2$ we have: A team dominates the RPE tournament in the sense of having a lower δ^{FB} iff $\eta > \eta_0(\rho)$, where $\eta_0(\rho)$ is increasing in ρ with $\eta_0(\rho) \to 0$ as $\rho \to -1$, $\eta_0(\rho) \to 1$ as $\rho \to 1$, and $\rho_0(0) \approx 0.739$. This holds irrespective of the magnitude of s (the standard deviation for individual output), but for each ρ, s must be sufficiently large so that FOA is valid.*

The function $\eta_0(\rho)$ is depicted in Figure 4. The RPE tournament has highest critical δ^{FB} above the curve, and is hence dominated by the team there. For high ρ the RPE tournament does comparatively better in the sense that the parameter set for which it is dominated is smaller. For strong negative correlations the opposite occurs: the team dominates there even for comparatively small parameter η.

(See Figure 4 in the appendix)

5 Concluding remarks

Both team work and tournament incentives are common features of modern organizations. According to Lawler (2001), 72 percent of Fortune 1000 companies make use of work teams, defined as groups of employees with shared goals or objectives, and in most firms of a certain size workers (explicitly or implicitly) compete for bonuses, wage raises or for better paid positions.

While the literature on tournaments has mainly been developed by economists, research on team work and team incentives has been multidisciplinary. There is a large literature within management and organizational behavior investigating team composition, team compensation, team leadership, and so forth, but this literature is mainly empirical, and the theoretical literature is conceptual rather than formal.

The economics literature on team organization is, in comparison, rather small. Theory has mainly focused on how the well-known free-rider problem can be solved or mitigated, while questions related to team size and team composition have remained unanswered, or not even asked. Moreover, endogenous formation of teams, in which firms deliberately choose to hold a team of workers accountable for their joint output, is not well understood.

Our paper contributes to the team literature by deriving testable theoretical predictions on team incentives, team size, team composition and team formation. We have done so by analyzing optimal self-enforcing (relational) contracts between a principal and a set of agents where only aggregate output can be observed. We have then considered how the efficiency of the contract is affected by variations in the number of agents and in the correlations between the agents. Finally, we have compared with a situation where individual output is observable.

First, we showed that the optimal team contract entails an incentive scheme in which each agent is paid a maximal bonus for aggregate output above a threshold and a minimal bonus otherwise. We then considered optimal team size. To the extent this is studied in the formal literature, the standard result is that more agents increases the free-rider problem and thus weakens incentives and effort. In our model, this is not necessarily the case. More agents in a team have three effects: First, it reduces the marginal incentive effect of a given bonus, which is the standard $1/n$ free-rider problem. Second, it also reduces the team's outside option. This strengthens the relational contract and thus allows for higher-powered incentives and thus higher effort. This positive effect of more agents is particularly strong if the agents' ex post bargaining power is high. Finally, it affects the variance of the performance measure. For positive correlations between the agents' outputs, the variance increases, while for negative correlations the variance is reduced. The latter is beneficial for the team because it increases the marginal incentives for each team member to provide effort.

Our model thus predicts that teamwork is more robust and more efficient when the team has high (ex post) bargaining power and when the team members' outputs are negatively correlated. The former implies that teamwork is more efficient (or prevalent) when the team is in a position to hold up values and sell their products in an alternative market. This is typically the case in human capital-intensive industries where groups of employees can potentially walk away with ideas, clients, innovations, etc.

The latter - negative correlations - relates to questions concerning optimal team composition. In the management literature a central question is whether teams should be homogenous or heterogeneous with respect to tasks (functional expertise, education, organizational tenure) as well as bio-demographic characteristics (age, gender, ethnicity). One can conjecture that negative correlations are more associated with heterogeneous teams than homogenous teams, and also more associated with task-related diversity than with bio-demographic diversity. There is no reason to believe that e.g. men and women's outputs are negatively correlated. However, workers with different functional expertise may be differently exposed to common shocks, or meet different sets of demands from customers or superiors. This can give rise to negative output correlations.

Interestingly, a comprehensive meta-study by Horwitz and Horwitz (2007), investigating 35 papers on the topic, finds no relationship between bio-demographic diversity and performance, but a strong positive relationship between team performance and task-related diversity. An explanation is that task-related diversity creates positive complementarity effects. We point to an alternative explanation, namely that diversity may create negative correlations that reduce variance and thereby increase marginal incentives for effort. The team members "must step forward when others fail". Diversity and heterogeneity among team members can thus yield considerable efficiency improvements.[15]

We have also compared with a situation where individual output is observable. For a parametric (normal) distribution, we have shown that the optimal contract is an RPE (relative performance evaluation) scheme; a form of a tournament, where the conditions for an agent to obtain the (single) bonus are stricter for negatively compared to positively correlated outputs. The efficiency of the RPE contract is shown to increase with the number of agents, and to improve with higher correlation (both positive and negative).

Now, if the firm can initially choose between organizations that allow for (a) only aggregate output or (b) individual outputs to be observed, we show that the firm may choose (a), i.e. to organize production as a team. Thus, even if alternatives (a) and (b) are equally costly to set up initially, the team alternative may yield a higher subsequent surplus.

There are two reasons for this. One is that teams create worse outside options. This is particularly the case under high ex post bargaining power. When individual outputs are observable, high bargaining power creates quite efficient spot contracts, while under team production the free-rider problem dampens the efficiency of the spot contract. Hence, since worse outside options strengthen the relational contract, higher bargaining power favors the team alternative.

[15] Hamilton et al (2003) provide one of a very few empirical studies on teams within the economics literature. They find that more heterogeneous teams (with respect to ability) are more productive (average ability held constant).

Second, negative correlations are even more beneficial for the relational team contract than for the relational RPE contract. That is, although efficiency in both alternatives increases with more negatively correlated outputs, the team alternative is more likely to be superior under such conditions. Hence, according to our model, team work is not only more robust and efficient under high bargaining power and negatively correlated outputs. The likelihood for firms to deliberately choose the team alternative, even if individual output is observable, is also higher under these conditions.

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APPENDIX

Proof of Proposition 1. Maximizing total surplus ($\Sigma_i W(e_i) \equiv \Sigma_i(E(x_i|\,e_i) - c(e_i)))$ subject to EC and the 'modified' IC constraint (2) yields

$$\mu_i g_l(y;l) l_{e_i}(e_1 ... e_n) - \lambda(y) \leq 0, \qquad b_i(y) \geq 0,$$

where the inequalities hold with complementary slackness, and $\mu_i > 0$, $\lambda(y) \geq 0$ are Lagrange multipliers.

For $y > y_0$ we have $g_l(y;l) > 0$ and hence $\lambda(y) > 0$, implying that EC is binding and at least one bonus is positive. In a symmetric solution the bonuses will thus all be equal and maximal for $y > y_0$. On the other hand, for $y < y_0$ we have $g_l(y;l) < 0$ by MLRP and hence $b_i(y) = 0$ for all i.

Finally suppose $l(e_1 ... e_n) = \Sigma_i e_i$, and assume the solution is asymmetric; say that $e_i < e_j$. Let $b_0 = (b_i + b_j)/2$ and consider

$$\int b_0(y) g_l(y; l(e_1...e_n))dy = \frac{1}{2}\int b_i(y) g_l(y; l(e_1...e_n))dy + \frac{1}{2}\int b_j(y) g_l(y; l(e_1...e_n))dy$$

$$= \frac{1}{2}c'(e_i) + \frac{1}{2}c'(e_j) \geq c'(\tfrac{e_i + e_j}{2})$$

Hence the bonus $b_0(y)$ to each of i and j is feasible and would induce effort at least $\frac{e_i+e_j}{2} = e_0$ from each. Thus a slightly lower bonus to each is feasible and will induce effort e_0 from each. This yields higher value since the objective is concave.

Proof of Proposition 3. It is obvious from the shape of $h()$ that the FOC for effort has a single solution for s sufficiently large, and hence that FOA is then valid. So consider s small.

If FOA is valid, the agent's optimal payoff is $b\frac{1}{2} + \mu e_i^* - c(e_i^*) + \gamma'$. This must be no less than the payoff for $e_i = e_i^s$, which strictly exceeds $\mu e_i^s - c(e_i^s) + \gamma'$, thus we have $0 < \mu e_i^s - c(e_i^s) - (\mu e_i^* - c(e_i^*)) < b\frac{1}{2} \leq \frac{\delta}{1-\delta}(W(e_i^*) - W(e_i^s))\frac{1}{2}$, where the first inequality follows from $e_i^s = \arg\max(\mu e_i - c(e_i))$. There is a critical $\delta^F > 0$ such that these inequalities do not hold for $e_i^* = e_i^{FB}$ and $\delta < \delta^F$, hence first best effort can not be obtained for $\delta < \delta^F$. Given such a δ, if FOA is valid for all $s > 0$, then $b \to 0$ as $s \to 0$ (since $h(0) \sim \frac{1}{s}$), and hence, since EC binds, $e_i^* \to e_i^s$. But this is a contradiction, since when FOA is valid, effort e_i^* should increase when s is reduced. This is so because if bonus b_s implements effort e_i^* for some $s > 0$, then b_s implements (by FOC) a higher effort for $s' < s$, yielding slack in EC, and hence room for a higher bonus to increase effort further. This shows that FOA cannot be valid for all $s > 0$.

Proof of Lemma 2. For given e, admissible bonuses satisfy $0 \leq b(y) \leq \frac{\delta}{1-\delta} W(e_i) \equiv B$. Let y_0 be the hurdle (threshold) for $b_h(y)$. Then

$$0 = u(b_h; e) - u(\tilde{b}; e) = \int_{\underline{y}}^{y_0} (-\tilde{b}(y)) f(y, e) + \int_{y_0}^{\bar{y}} (B - \tilde{b}(y)) f(y, e) \qquad (23)$$

This yields

$$u_{e_i}(b_h; e) - u_{e_i}(\tilde{b}; e) = \int_{\underline{y}}^{y_0} (-\tilde{b}(y)) \frac{f_{e_i}(y,e)}{f(y,e)} f(y, e) dy + \int_{y_0}^{\bar{y}} (B - \tilde{b}(y)) \frac{f_{e_i}(y,e)}{f(y,e)} f(y, e) dy$$

$$> \frac{f_{e_i}(y_1,e)}{f(y_1,e)} \left[\int_{\underline{y}}^{y_0} (-\tilde{b}(y)) f(y, e) dy + \int_{y_0}^{\bar{y}} (B - \tilde{b}(y)) f(y, e) dy \right] = 0$$

where the inequality follows from MLRP, and the last equality from $u(\tilde{b}; e) = u(b_h; e)$. This proves statement (i)

(ii) Note that, for given $e = e^*$ the RHS of (23) is decreasing in y_0, and hence there is a y_0 satisfying the equation. Let $b_h(y)$ be the associated hurdle scheme. To simplify notation, write $u(\tilde{b}; e) = \tilde{u}(e)$ and $u(b_h; e) = u(e)$. Then from (i) we now have $\tilde{u}(e^*) = u(e^*)$ and $u_{e_i}(e^*) > \tilde{u}_{e_i}(e^*)$, where $\tilde{u}_{e_i}(e^*) = 0$ since e^* is an equilibrium for bonus $\tilde{b}(y)$.

Now assume, to get a contradiction, that there is $e_i' < e_i^*$ with $u(e_i', e_{-i}^*) > u(e_i^*, e_{-i}^*)$. Then for $\xi(e_i) = u(e_i, e_{-i}^*) - \tilde{u}(e_i, e_{-i}^*)$ we have $\xi(e_i') > \xi(e_i^*) = 0$ and $\xi'(e_i^*) = u_{e_i}(e_i^*, e_{-i}^*) > 0$. Hence by continuty there must be some $e_i'' \in (e_i', e_i^*)$ such that $\xi(e_i'') = 0$ and $\xi'(e_i'') \leq 0$. At e_i'' we thus have $u(e_i'', e_{-i}^*) = \tilde{u}(e_i'', e_{-i}^*)$ and $u_{e_i}(e_i'', e_{-i}^*) \leq u_{e_i}(e_i'', e_{-i}^*)$. But this contradicts statement (i) in the lemma. This proves (ii) and thus the lemma.

Proof of Proposition 4. Suppose the optimal bonus $\tilde{b}(y)$ is not a hurdle (threshold) bonus, and let $e^* > 0$ be the associated efforts. So $u_{e_i}(\tilde{b}; e^*) = 0$ by FOC. Let $b = \frac{\delta}{1-\delta} W(e_i^*)$, and let b_h be a symmetric hurdle scheme (with $0 \leq b_h(y) \leq b$), with the same utility as $\tilde{b}$; i.e. $u(\tilde{b}; e^*) = u(b_h; e^*)$, and hence $u_{e_i}(b_h; e^*) > u_{e_i}(\tilde{b}; e^*) = 0$ by Lemma 2. Let y_0 be the threshold for b_h. The idea of the proof is to modify this threshold (to $y_0 - \tau_0$) such that e^* gets to be an equilibrium for the modified threshold bonus

To show this, note that for a bonus with threshold $y_0' = y_0 - \tau$ an agent's expected bonus payment is $b\Pr(y > y_0' | e)$, and that for $y \sim N(\Sigma e_i, s^2)$ the agent's expected payoff (excluding the fixed salary) can be written, for $e_{-i} = e_{-i}^*$ as

$$u(\tau, e_i, e_{-i}^*) = b(1 - H(y_0^* - \tau - e_i)) - c(e_i), \qquad y_0^* = y_0 - (n-1)e_i^*,$$

where $H()$ is the CDF for $N(0, s^2)$. For $\tau = 0$ the threshold is that of b_h (i.e. y_0) and we have by Lemma 2

$$u(0, e_i, e_{-i}^*) \leq u(0, e_i^*, e_{-i}^*) \quad \text{for all} \quad e_i < e_i^*, \tag{24}$$

and $0 < u_{e_i}(0, e_i^*, e_{-i}^*) = bh(y_0^* - e_i^*) - c'(e_i^*)$, where $h() = H'()$ is the normal density. Now define $\tau_0 > 0$ such that

$$u_{e_i}(\tau_0, e_i^*, e_{-i}^*) = h(y_0^* - \tau_0 - e_i^*) - c'(e_i^*) = 0 \text{ and } \quad y_0^* - \tau_0 - e_i^* < 0 \tag{25}$$

This is feasible because by the shape of $h()$, if $h(x) > C > 0$, then there is $\tau_0 > 0$ such that $h(x - \tau_0) = C$ and $x - \tau_0 < 0$. Note that this implies $h(y_0^* - \tau_0 - e_i) < h(y_0^* - \tau_0 - e_i^*)$ and thus $u_{e_i}(\tau_0, e_i, e_{-i}^*) < 0$ for $e_i > e_i^*$. No deviation to $e_i > e_i^*$ can therefore be profitable.

Next, if $2(y_0^* - \tau_0) > e_i^*$ define $e_i' \in (0, e_i^*)$ by

$$y_0^* - \tau_0 - e_i' = -(y_0^* - \tau_0 - e_i^*) > 0 \tag{26}$$

and note that this implies (by the shape of $h()$):

$$h(y_0^* - \tau_0 - e_i) > h(y_0^* - \tau_0 - e_i^*) \quad \text{for} \;\; e_i \in (e_i', e_i^*) \tag{27}$$

This in turn implies, since $h(y_0^* - \tau_0 - e_i^*) = c'(e_i^*) > c'(e_i)$ for $e_i < e_i^*$, that we have $u_{e_i}(\tau_0, e_i, e_{-i}^*) > u_{e_i}(\tau_0, e_i^*, e_{-i}^*) = 0$ and hence

$$u(\tau_0, e_i, e_{-i}^*) < u(\tau_0, e_i^*, e_{-i}^*) \quad \text{for} \quad e_i \in [e_i', e_i^*) \tag{28}$$

If $2(y_0^* - \tau_0) \leq e_i^*$ define $e_i' = 0$, and it is then straightforward to see that (27) and hence (28) holds for that case as well. In that case the proof is then complete since (28) implies that no deviations to $e_i < e_i^*$ can be profitable.

For the case $e_i' > 0$, define, for $e_i < e_i'$ and $\tau \in [0, \tau_0]$ the payoff difference

$$\Delta(\tau, e_i) = u(\tau, e_i^*, e_{-i}^*) - u(\tau, e_i, e_{-i}^*)$$

By (24) we know that for $\tau = 0$ we have $\Delta(0, e_i) \geq 0$ for all $e_i \leq e_i' < e_i^*$. Let now $\tau \in (0, \tau_0)$, and consider

$$\tfrac{\partial \Delta(\tau, e_i)}{\partial \tau} = bh(y_0^* - \tau - e_i^*) - bh(y_0^* - \tau - e_i)$$

For $\tau < \tau_0$ and $e_i < e_i'$ we have $y_0^* - \tau - e_i > y_0^* - \tau_0 - e_i' > 0$ (see (26)) and hence $h(y_0^* - \tau - e_i) < h(y_0^* - \tau_0 - e_i')$. Thus we have

$$\begin{aligned} \tfrac{\partial \Delta(\tau, e_i)}{\partial \tau} \tfrac{1}{b} &> h(y_0^* - \tau - e_i^*) - h(y_0^* - \tau - e_i') \\ &= h(y_0^* - \tau_0 - e_i^* + (\tau_0 - \tau)) - h(y_0^* - \tau_0 - e_i' + (\tau_0 - \tau)) \end{aligned}$$

Note that by (26) the last difference can be written as $h(-x + z) - h(x + z)$ with $x, z > 0$, and this difference is thus positive (by the shape of $h()$). Since $\tfrac{\partial \Delta(\tau, e_i)}{\partial \tau} > 0$ we then have, for $e_i \leq e_i'$:

$$\Delta(\tau_0, e_i) = u(\tau_0, e_i^*, e_{-i}^*) - u(\tau_0, e_i, e_{-i}^*) > u(0, e_i^*, e_{-i}^*) - u(0, e_i, e_{-i}^*)$$

It now follows from (24) that $u(\tau_0, e_i^*, e_{-i}^*) > u(\tau_0, e_i, e_{-i}^*)$ for $e_i \leq e_i'$, This completes the proof that e^* is a (symmetric) equilibrium for the modified bonus with threshold $y_0 - \tau_0$.

Proof of Proposition 5. We have $H(x; s) = \Phi(\frac{x}{s})$, and $h(x; s) = \phi(\frac{x}{s})\frac{1}{s}$ where $\Phi()$ is the N(0,1) CDF and $\phi()$ its density. The relations (9 - 11) can then be written as

$$b(1 - \Phi(\frac{-\tau}{s})) - c(e_i^*) \geq b(1 - \Phi(\frac{e_i^* - e_i^0 - \tau}{s})) - c(e_i^0) \tag{29}$$

$$b\phi(\frac{-\tau}{s})\frac{1}{s} - c'(e_i^*) = 0 = b\phi(\frac{e_i^* - e_i^0 - \tau}{s})\frac{1}{s} - c'(e_i^0) \tag{30}$$

$$b \leq \frac{\delta}{1 - \delta} W(e_i^*) \tag{31}$$

For $c''' \geq 0$, so $c'(e_i)$ is convex, there can at most be two local maxima (e_i^* and e_i^0) for the agent's payoff. Note that for the minimal $s = s_c$ for which the FOA is valid, all relations (9 - 11) hold with equality, and $\tau = 0$. Denote the associated effort and bonus by $e_i^* = e_c^*$ and $b = b_c$, respectively. For $s < s_c$ the optimal threshold must be some $y_0' \neq ne_i^*$, thus $y_0' = ne_i^* - \tau$, $\tau \neq 0$. We show below that $\tau > 0$, as assumed in the text, is optimal.

First we show that *for an optimal* $\tau > 0$ *all constraints must bind.* To see this, define Δ as the difference in payoffs between e_i^* and e_i^0, i.e. from (29);

$$\Delta = b(\Phi(\frac{e_i^* - e_i^0 - \tau}{s}) - \Phi(\frac{-\tau}{s})) - (c(e_i^*) - c(e_i^0)), \tag{32}$$

and note that Δ is increasing in b and in τ. This is so because (by the envelope property) $\frac{d\Delta}{db} = \Phi(\frac{e_i^*-e_i^0-\tau}{s}) - \Phi(\frac{-\tau}{s}) > 0$ and $\frac{d\Delta}{d\tau}s = c'(e_i^*) - c'(e_i^0) > 0$. But then, if the EC constraint (31) does not bind, we can increase b without violating the payoff constraint (29), since $\frac{d\Delta}{db} > 0$. The higher bonus will induce higher effort e_i^* (by FOC), hence EC must bind in optimum.

If the payoff constraint (29) does not bind, then by reducing τ, keeping b fixed, effort e_i^* will increase (by FOC), and the EC constraint (31) will be relaxed. The payoff constraint (29) must therefore also bind in optimum.

Now we show that $\tau < 0$ *cannot be optimal.* Suppose it is, i.e. that for some $s < s_c$ a hurdle $y_0' = y_0 - \tau'$ with $\tau' < 0$ is optimal. The optimal bonus b and effort e_i^* must satisfy FOC. Note that the FOC for e_i^* will also be satisfied for $\tau'' = -\tau' > 0$, because $\phi(\frac{-\tau}{s}) = \phi(\frac{\tau}{s})$ Then, since $\frac{d\Delta}{d\tau} > 0$, the payoff difference Δ will be strictly higher for $\tau = \tau'' > 0$ than for $\tau' < 0$. But then e_i^* is a strict optimum for the agent ($\Delta > 0$) for $\tau = \tau'' > 0$, and in such a case it is, as we have seen above, possible to implement an even higher effort by, say, increasing the bonus somewhat. A hurdle $y_0' = y_0 - \tau'$ with $\tau' < 0$ can thus not be optimal.

We now show that *effort* e_i^* *is higher when* s *is lower.* To this end fix $s_a < s_c$, and let the optimal effort, bonus and hurdle parameter be $e_i^* = e_a^*$, $b = b_a$ and $\tau = \tau_a$, respectively. Then $\Delta = 0$ and EC (31) binds. We first show that for $s < s_a$ effort $e_i^* = e_a^*$ can be implemented with $b = b_a$, and a suitable choice of τ. Indeed, fix $e_i^* = e_a^*$ and $b = b_a$, and let $\tau(s)$ and $e_i^0(s)$ be defined by the FOCs (30) for e_i^* and e_i^0, respectively. For $s = s_a$ we have $\tau = \tau_a$ and all relations hold with equality. We show below (see (33)) that the payoff difference $\Delta = \Delta(\tau(s), e_i^0(s))$ satisfies $\frac{d\Delta}{ds} < 0$ (keeping $e_i^* = e_a^*$ and $b = b_a$ fixed). This implies that $e_i^* = e_a^*$ can be implemented with $b = b_a$ and $\tau = \tau(s)$ when $s < s_a$, and that the associated payoff difference is then strictly positive ($\Delta > 0$). But in such a case we can, as shown above, implement a strictly higher effort $e_i^* > e_a^*$. This shows that for $s < s_a$ optimal effort is $e_i^* > e_a^*$, as was to be shown.

Finally we show that *in the limit we have* $e_i^* \to e_u^*$ as $s \to 0$. For suppose that (at least along a subsequence) $e_i^* \to e_l^* < e_u^*$ as $s \to 0$. Note that we then must have $\frac{\tau}{s} \to \infty$ as $s \to 0$. For if not, then $b \to 0$ by FOC for e_i^* in (30), which implies a negative payoff at e_i^*. For the same reason we must also have $\frac{e_i^* - e_i^0 - \tau}{s} \to \infty$. Then we must have $e_i^0 \to e_l^0 = 0$ as $s \to 0$, for otherwise the payoff at e_i^0 would converge to $-c(e_l^0) < 0$. This is impossible, since the payoff at e_i^0 exceeds that at $e_i = 0$, and hence must be non-negative.

Taking limits in the first relation (29) with equality, we then get $\lim b \cdot 1 - c(e_l^*) = 0$, and hence from the last equation (for b) that $c(e_l^*) = \frac{\delta}{1-\delta} W(e_l^*)$. This cannot hold for $e_l^* < e_u^*$, hence we must have $e_l^* = e_u^*$.

It remains to prove $\frac{d\Delta}{ds} < 0$, where Δ is given by (32), $\tau = \tau(s)$ and $e_i^0 = e_i^0(s)$ are given by the FOCs in (30), and b and e_i^* are kept fixed ($e_i^* = e_a^*, b = b_a$). In fact, we will show that

$$\frac{d\Delta}{ds} = (c'(e_i^*) - c'(e_i^0))(-\frac{s}{\tau}) - c'(e_i^0)\frac{e_i^* - e_i^0}{s} < 0 \tag{33}$$

To this end, using the FOC (30) we find, for the payoff at e_i^0:

$$\frac{d}{ds}\left(b(1 - \Phi(\frac{e_i^* - e_i^0 - \tau}{s})) - c(e_i^0)\right) = c'(e_i^0)\left(\frac{d\tau}{ds} + \frac{e_i^* - e_i^0 - \tau}{s}\right)$$

Similarly, for the payoff at e_i^* we find

$$\frac{d}{ds}\left(b(1 - \Phi(\frac{-\tau}{s})) - c(e_i^*)\right) = c'(e_i^*)(\frac{d\tau}{ds} - \frac{\tau}{s})$$

Hence

$$\frac{d\Delta}{ds} = (c'(e_i^*) - c'(e_i^0))(\frac{d\tau}{ds} - \frac{\tau}{s}) - c'(e_i^0)\frac{e_i^* - e_i^0}{s}$$

From the FOCs (30) and the fact that $\phi'(z) = -z\phi(z)$ we obtain by differentiation $(\frac{d\tau}{ds} - \frac{\tau}{s}) = -\frac{s}{\tau}$. This proves (33), and thus completes the proof.

Proof of Proposition 6. From (18) we have

$$\frac{c'(e_i) - \eta \bar{x}'(e_i)}{n \int_{x_0}^{\infty} F(x; e_i)^{n-1} f_{e_i}(x; e_i) dx} = \frac{b(n)}{n} = \frac{\delta}{1-\delta}(W(e_i) - W(e_s))$$

Consider

$$s(n) = n \int_{x_0}^{\infty} F(x; e_i)^{n-1} f_e(x; e_i) dx = \int_{x_0}^{\infty} \frac{d}{dx}(F(x; e_i)^n) \frac{f_{e_i}(x; e_i)}{f(x; e_i)} dx$$

Letting $h(x) = \frac{f_{e_i}(x;e_i)}{f(x;e_i)}$ here denote the likelihood ratio, we have, integrating by parts

$$s(n) = \int_{x_0}^{\infty} \frac{d}{dx}(F(x; e_i)^n - 1) h(x) dx = h(x_0) + \int_{x_0}^{\infty} (1 - F(x; e_i)^n) h'(x) dx$$

where $h(x_0) = 0$ by definition of x_0. Given MLRP we have $h'(x) > 0$ and hence we see that $s(n)$ *is increasing in* n.

This implies that $\frac{c'(e_i) - \eta \bar{x}'(e_i)}{s(n, e_i)}$ shifts down with n, and hence that effort per agent (e_i) increases.

Proof of Proposition 7. Maximizing total surplus ($\Sigma_i W(e_i) = \Sigma_i (E(x_i | e_i) - c(e_i))$) subject to (15) and (16) yields

$$\mu_i f_{e_i}(x; e) - \lambda(x) \leq 0, \qquad b_i(x) \geq 0,$$

where the inequlities hold with complementary slackness (and e, x are vectors). If two agents are paid a positive bonus, then $\mu_i f_{e_i}(x; e) = \lambda(x) = \mu_j f_{e_j}(x; e)$, so their weighted likelihood ratios must be equal; $\mu_i \frac{f_{e_i}(x;e)}{f(x;e)} = \mu_j \frac{f_{e_j}(x;e)}{f(x;e)}$. But this can only occur for a set of measure zero, hence at most one agent is paid a bonus (almost surely).

If $f_{e_i}(x; e) < 0$ then $b_i(x) = 0$. If $f_{e_i}(x; e) > 0$ then $\lambda(x) > 0$, and agent i is paid the bonus ($b_i(x) > 0$) if and only if he has the largest weighted likelihood ratio. Also, the bonus is maximal since EC is binding.

In a symmetric solution the weights (multipliers) μ_i will be equal, and hence the agent with the largest likelihood ratio will get the bonus, provided this ratio is positive.

Now consider variables with identical variances and identical correlations ($corr(x_i, x_j) = \rho$ all $i \neq j$.). The multinormal density has the form

$$C \exp(-\tfrac{1}{2}(x-e)'\Sigma^{-1}(x-e))$$

where Σ is the covariance matrix. Under our assumptions we have $\Sigma = s^2 R$, where the correlation matrix R can be written as

$$R = (1-\rho)I + \rho J,$$

where each element of J is $J_{ik} = 1$. Note that $J^2 = nJ$. It is straightforward to verify that

$$R^{-1} = \frac{1}{(1+(n-1)\rho)(1-\rho)} Q, \quad \text{where} \quad Q = (1+(n-1)\rho)I - \rho J \quad (34)$$

Note that the matrix Q has elements $(1+(n-2)\rho)$ on the diagonal, and $-\rho$ off the diagonal.

From the formula for R^{-1} and the definitions of k_1, k_2 in the text it follows that the quadratic form in the multinormal density can be written

$$-\tfrac{1}{2}(x-e)'\Sigma^{-1}(x-e) = -\tfrac{1}{2}\left(k_1 \Sigma_i z_i^2 + k_2 \Sigma_{i \neq j} z_i z_j\right), \qquad z_i = x_i - e_i$$

Differentiation of the density wrt e_i then yields the formula (19) for the likelihood ratio in the text.

From the formula (19) it follows that agent $i's$ likelihood ratio is positive iff the inequality in (20) holds. We now verify the last equality in (20), i.e. the validity of the expression for $E(x_i|\, x_{-i})$. To this end note that for the normal distribution the conditional expectation of, say x_1 can be written

$$E(x_1|\, x_{-1}) = E(x_1) + \Sigma_{12}\Sigma_{22}^{-1}(x_{-1} - Ex_{-1}),$$

where $\Sigma_{12} = s^2(\rho, ..., \rho)$ is the $(n-1)-$dimensional vecor of covariances $cov(x_1, x_j)$, $j > 1$, and Σ_{22} is the covariance matrix for $x_{-1} = (x_2...x_n)'$. It follows from (34) that $s^2\Sigma_{22}^{-1}$ has the same form as R^{-1}, with n replaced by $n-1$. Hence $\Sigma_{12}\Sigma_{22}^{-1} = (\rho...\rho)R_{n-1}^{-1}$, and each element of this $(1 \times n)$ matrix is, from (34):

$$((\rho...\rho)R_{n-1}^{-1})_i = \tfrac{\rho}{(1+(n-2)\rho)(1-\rho)}((1+(n-3)\rho) - (n-2)\rho) = \tfrac{\rho}{(1+(n-2)\rho)}$$

This verifies the last equality in (20).

Proof of Proposition 8. We will show that for $e_1^* = e_2^*$ the marginal gain from effort is

$$\int_B f_{e_1}(x|\, e_1 e_2^*) = \tfrac{1}{s}\Gamma(\tfrac{e_1 - e_1^*}{s}; \rho)$$

where $\Gamma(a;\rho)$ is defined by

$$\Gamma(a;\rho) = \frac{1}{\sqrt{1-\rho^2}}\frac{1}{2}\phi(\frac{a}{\sqrt{1-\rho^2}}) + \frac{1}{\sqrt{1-\rho}}\frac{1}{\sqrt{2}}\phi(\frac{a}{\sqrt{2}\sqrt{1-\rho}})(1-\Phi(\frac{-a}{\sqrt{1+\rho}\sqrt{2}})), \tag{35}$$

and where $\phi(z)$ is the standard normal density and $\Phi(z)$ its CDF. The agent's FOC then takes the form $\frac{b}{s}\Gamma(0;\rho) + \eta - c'(e_1^*) = 0$, which is precisely the formula (22) stated in the proposition.

The normal density depends on (vector) x via a quadratic form in $x - e$,, hence it satisfies $f_{e_i}(x;e)dx = -f_{x_i}(x;e)$. Taking account of the definition of the set B of outcomes (the set where agent 1 is paid a bonus) in (21) , we thus have

$$\begin{aligned} \int_B f_{e_1}(x|\, e_1 e_2^*) &= -\left(\int_{-\infty}^{e_2^*} dx_2 \int_{e_1^*+\rho(x_2-e_2^*)}^{\infty} dx_1 + \int_{e_2^*}^{\infty} dx_2 \int_{x_2}^{\infty} dx_1\right) f_{x_1}(x|\, e_1 e_2^*) \\ &= -\left(\int_{-\infty}^{e_2^*} dx_2\, [f(x|\, e_1 e_2^*)]_{x_1=e_1^*+\rho(x_2-e_2^*)}^{x_1=\infty} + \int_{e_2^*}^{\infty} dx_2\, [f(x|\, e_1 e_2^*)]_{x_1=x_2}^{x_1=\infty}\right) \end{aligned}$$

where

$$f(x|\, e_1 e_2^*) = k \exp\left(-\frac{(x_1-e_1)^2 + (x_2-e_2^*)^2 - 2\rho(x_1-e_1)(x_2-e_2^*)}{2(1-\rho^2)s^2}\right), \qquad k = \frac{1}{2\pi s^2\sqrt{1-\rho^2}}$$

Straightforward computations then yield

$$\begin{aligned} \int_B f_{e_1}(x|\, e_1 e_2^*) &= k\frac{1}{2}\sqrt{2\pi} s \exp\left(-\frac{(e_1^*-e_1)^2}{2(1-\rho^2)s^2}\right) \\ &\quad + k\int_0^\infty \exp\left(-\frac{z_2^2(1-\rho^2) + (z_2(1-\rho) + (e_2^* - e_1))^2}{2(1-\rho^2)s^2}\right) \end{aligned} \tag{36}$$

Further computations show that the last integral can be written as

$$k\int_{\frac{e_2^*-e_1}{s\sqrt{1+\rho}\sqrt{2}}}^{\infty} e^{-\frac{z^2}{2}} dz \frac{s\sqrt{1+\rho}}{\sqrt{2}} \exp\left(-\frac{(e_2^*-e_1)^2}{4(1-\rho)s^2}\right) \tag{37}$$

Setting $e_2^* = e_1^*$ in (36)-(37), using $\phi(z) = \frac{1}{\sqrt{2\pi}}e^{-z^2/2}$ and the definition of k above then verifies the formula (35). This completes the proof.

Proof of Proposition 9. Given that the marginal gain to effort for agent 1 in the modified tournament scheme can be written as $\frac{b}{s}\Gamma(\frac{e_1-e_1^*}{s};\rho)+\eta-c'(e_1)$, where $\Gamma(a;\rho)$ is given by (35), the FOC for $e_1=e_1^*$ to be optimal is $\frac{b}{s}\Gamma(0;\rho)+\eta-c'(e_1^*)=0$, and the local second order condition (SOC) is $\frac{b}{s}\Gamma_a(0;\rho)\frac{1}{s}-c''(e_1^*)\leq 0$. Since $\phi'(0)=0$ we see from (35) that

$$\Gamma_a(0;\rho)=\frac{1}{\sqrt{1-\rho^2}}\frac{1}{2}\phi(0)^2=\frac{1}{\sqrt{1-\rho^2}}\frac{1}{4\pi}$$

Since $\Phi(0)=\frac{1}{2}$ and $\phi(0)=1/\sqrt{2\pi}$, this in turn implies

$$\frac{\Gamma_a(0;\rho)}{\Gamma(0;\rho)}=\frac{\frac{1}{\sqrt{1-\rho^2}}\frac{1}{2}\phi(0)}{\frac{1}{\sqrt{1-\rho^2}}\frac{1}{2}+\frac{1}{\sqrt{1-\rho}}\frac{1}{\sqrt{2}}\frac{1}{2}}=\frac{\phi(0)}{1+\frac{\sqrt{1-\rho^2}}{\sqrt{1-\rho}}\frac{1}{\sqrt{2}}}=\frac{1/\sqrt{\pi}}{\sqrt{2}+\sqrt{1+\rho}}$$

From the FOC we have $\frac{b}{s}\Gamma(0;\rho)=c'(e_1^*)-\eta$, hence the SOC can be written

$$c''(e_1^*)\geq\frac{b}{s}\Gamma_a(0;\rho)\frac{1}{s}=\frac{c'(e_1^*)-\eta}{\Gamma(0;\rho)}\Gamma_a(0;\rho)\frac{1}{s}=\frac{c'(e_1^*)-\eta}{s}\frac{1/\sqrt{\pi}}{\sqrt{2}+\sqrt{1+\rho}}$$

This can be rearranged to yield the formula stated in the proposition.

We will now show that for s sufficiently large, the FOC has a single solution ($e_1=e_1^*$), which then must be a maximum, since the local SOC holds strictly for s large. To get a contradiction, suppose that, for every $s'>0$ there is $s>s'$ such that FOC has a solution $e_1=e_1(s)\neq e_1^*$, i.e. such that

$$\frac{b}{s}\Gamma(\frac{e_1-e_1^*}{s};\rho)+\eta-c'(e_1)=0=\frac{b}{s}\Gamma(0;\rho)+\eta-c'(e_1^*),$$

implying

$$\Gamma(\frac{e_1-e_1^*}{s};\rho)/\Gamma(0;\rho)=\frac{c'(e_1)-\eta}{c'(e_1^*)-\eta}$$

Then letting $s\to\infty$ (if necessary along a subsequence) we see that $e_1(s)\to e_1^*$. Hence $a(s)=\frac{e_1(s)\to e_1^*}{s}\to 0$, and the last equation above yields

$$\frac{1}{a(s)}(\frac{\Gamma(a(s);\rho)}{\Gamma(0;\rho)}-1)\frac{1}{s}=\frac{c'(e_1)-c'(e_1^*)}{e_1-e_1^*}\frac{1}{c'(e_1^*)-\eta}$$

Letting now $s\to\infty$, the LHS behaves like $\frac{\Gamma_a(0;\rho)}{\Gamma(0;\rho)}\frac{1}{s}$ and hence converges to zero, while the RHS converges to $\frac{c''(e_1^*)}{c'(e_1^*)-\eta}$. This yields a contradiction and thus completes the proof.

Proof of Proposition 10. It follows from (5) that the critical discount factor to implement first best effort e^{FB} is for a team with n independent agents given by

$$\left(c'(e^{FB}) - \eta/n\right) s_n M = (W(e^{FB}) - W_s(n))\delta^{FB}/(1-\delta^{FB}) \qquad (38)$$

Consider now quadratic costs: $c(e) = \frac{k}{2}e^2$, with associated surplus per agent $W(e) = e - \frac{k}{2}e^2$. First-best effort is then $e^{FB} = \frac{1}{k}$, with surplus $W(e^{FB}) = \frac{1}{2k}$. Spot effort is given by $\eta/n = c'(e_s) = ke_s$, with spot surplus

$$W_s(n) = W(e_s) = \left(\tfrac{\eta}{nk}\right) - \tfrac{k}{2}\left(\tfrac{\eta}{nk}\right)^2 = \tfrac{1}{2kn^2}\eta\,(2n-\eta)$$

Substituting this into (38), taking account of $c'(e^{FB}) = 1$ and $s_n = s\sqrt{n}$ we find

$$\frac{\delta^{FB}}{1-\delta^{FB}} = \frac{(1-\eta/n)M}{W(e^{FB}) - W_s(n)} s_n = \frac{(1-\eta/n)\sqrt{n}M}{1-\eta\,(2n-\eta)\,/n^2} 2ks \qquad (39)$$

The derivative of the last expression w.r.t. n is positive iff $n > 3\eta$. Hence, if $3\eta \le 2$, i.e. $\eta \le \frac{2}{3}$, then the critical discount rate δ^{FB} is increasing for $n \ge 2$, meaning that teams with $n > 2$ do worse than teams with $n = 2$ with respect to achieving FB.

The critical discount rate δ^{FB} is always increasing for $n > 3$ (since $\eta < 1$), hence teams with $n > 3$ will always do worse than teams with $n = 3$ regarding achieving FB.

Comparing the expressions in (39) for $n = 2$ and $n = 3$ we find that the former is smaller iff $\sqrt{6}\frac{2\eta-6}{9\eta-18} < 1$ i.e. $\eta < \eta_0 \approx 0.805$. This proves the first part of the proposition. The second part (comparison with the RPE tournament) follows from the proof for Proposition 11 below.

Proof of Proposition 11. Effort in the modified tournament is for given bonus given by the FOC (22). The EC conditon requires $b \le \frac{\delta}{1-\delta}n(W(e_i) - W_s)$, hence effort is given by (when $s = \sqrt{v}$ and $n = 2$)

$$\tfrac{\delta}{1-\delta}(W(e_i) - W_s) = \tfrac{b}{2} = \frac{c'(e_i)-\eta}{\frac{1}{\sqrt{2\pi}s}\frac{1}{2}\left(\frac{1}{\sqrt{1-\rho^2}} + \frac{1}{\sqrt{1-\rho}}\frac{1}{\sqrt{2}}\right)2} \equiv \tfrac{c'(e_i)-\eta}{r(\rho)}\sqrt{\pi v}$$

where $r(\rho)$ is defined by the identity.

Consider next the relational team contract. The maximal effort per agent that can be sustained in the team is given by (5), where now $M = \sqrt{2\pi}$ and $s_2^2 = 2v(1+\rho)$, and hence (5) is

$$\left(c'(e_i) - \tfrac{1}{n}\eta\right)\sqrt{2\pi}\sqrt{2v(1+\rho)} = b = \tfrac{\delta}{1-\delta}(W(e_i) - W_s(n))$$

Compare now critical $\delta' s$ to implement FB. They are given by the following conditions, for the tournament and the team, respectively

$$\tfrac{\delta}{1-\delta} = \tfrac{c'(e^{FB})-\eta}{W(e^{FB})-W_s}\tfrac{\sqrt{v\pi}}{r(\rho)}$$

$$\tfrac{\delta}{1-\delta} = \tfrac{c'(e^{FB})-\eta/2}{W(e^{FB})-W_s(2)}\sqrt{2\pi}\sqrt{2v(1+\rho)}$$

For quadratic costs we have (as in the proof above) $W(e^{FB}) = \frac{1}{2k}$, and spot surpluses in the team and the tournament given by, respectively $W(e_s(2)) = \frac{1}{8k}\eta\,(4-\eta)$ and $W_s = \frac{1}{2k}\eta\,(2-\eta)$.

Computing the ratio of the critical $\delta' s$ to implement FB, we then find that the tournament has the highest one iff

$$\tfrac{1}{2}\tfrac{2-\eta}{1-\eta}\tfrac{\sqrt{2}-\sqrt{\rho+1}}{\sqrt{1-\rho}} > 1, \qquad i.e. \qquad \eta > \tfrac{2\left(\sqrt{1-\rho}+\sqrt{\rho+1}-\sqrt{2}\right)}{2\sqrt{1-\rho}+\sqrt{\rho+1}-\sqrt{2}} \equiv \eta_0(\rho)$$

This completes the proof, since $\eta_0(\rho)$ has a positive derivative.

FIGURES

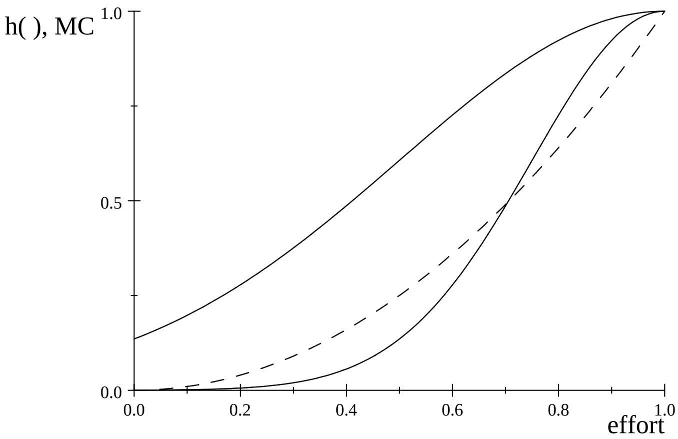

Figure 1. Illustration of FOC

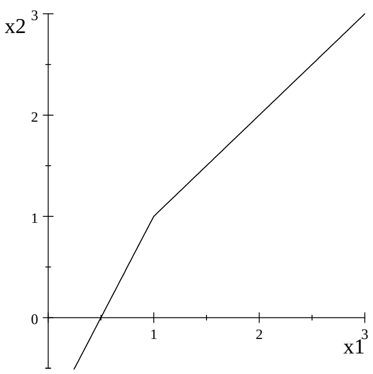

Figure 2a

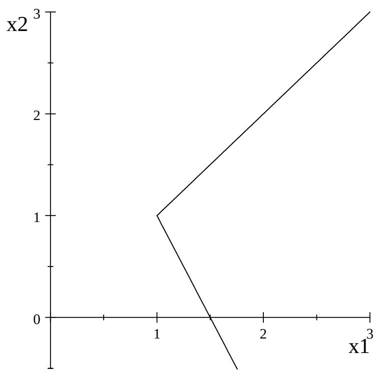

Figure 2b

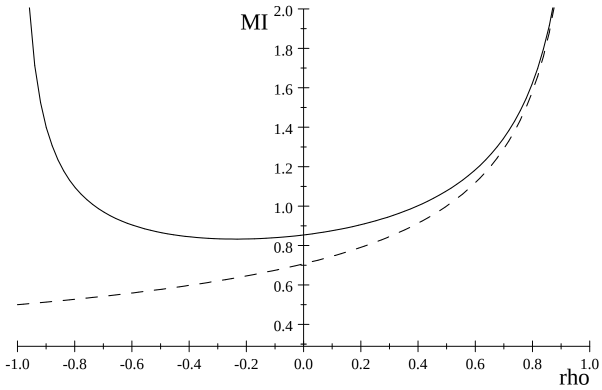

Figure 3. Marginal incentives as function of ρ

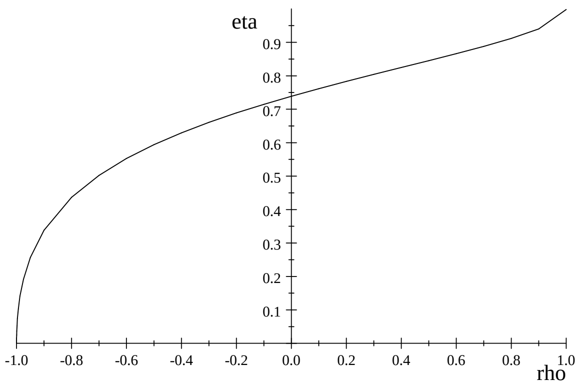

Figure 4. Illustration for Proposition 11