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## Abstract

Using an intertemporal model of saving and capital accumulation we demonstrate that it is impossible for any binding minimum wage to increase the after-tax incomes of workers if the production function is Cobb-Douglas with constant returns to scale, or if there are no differences in ability among workers.

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## 1. Introduction

The use of minimum wages as a way to reduce poverty and redistribute income has re-emerged forcefully in recent policy discussions. Critics of minimum-wage legislation focus usually on its disemployment effects and on whether it is an effective redistributive tool. While these issues are subject to intense discussion among economists, it is taken for granted by both sides of this debate that, following a minimum wage increase, the incomes of (at least some) workers that remain in employment will be higher (e.g. Card and Krueger, 1995; Saint-Paul, 2000; Neumark and Wascher, 2008).[1]

The objective of the present paper is to argue that this presumption is by no means guaranteed once we move away from static models and allow for capital accumulation. To this purpose we construct a model with two types of agents, i.e. workers and capitalists (e.g. Judd, 1985; Acemoglu, 2009, part VIII). The latter are a homogeneous group, do all the saving and own the capital stock, whereas the workers are differentiated according to their ability. Assuming a constant-returns-to-scale Cobb-Douglas production function, we demonstrate that the imposition of *any* binding minimum wage, in addition to generating unemployment amongst the least able workers, will also reduce the steady-state capital stock and reduce the after-tax incomes of employed workers. Our result also implies that the (joint) existence of economic profits and differences in ability among workers can allow workers above an ability threshold to increase their (after-tax) incomes through the imposition of a

[1] This effect is behind some political economy explanations regarding the unwillingness of policymakers to dismantle unemployment-generating labour-market legislation (e.g. Sobel, 1999; Saint-Paul, 2000; Adam and Moutos, 2011).

binding minimum wage; however, they can achieve this only at the expense of low-ability workers who become unemployed.

## 2. The Model

We consider a closed economy producing a single good under perfectly competitive conditions, and consisting of two sets of agents: workers and capitalists.

### 2.1 *The Perfectly Competitive Case*

#### *2.1.1 Workers*

There is a fixed number of workers in the economy (normalized to one). All worker-based households (*workers*, thereafter) are assumed to have identical preferences. However, workers differ in ability, as reflected in their endowment of effective number of labour units per unit of time (e.g. per hour, day, or year). We assume that all workers have the same endowment of *time units* at their disposal (which we also normalize to one), and that they supply inelastically their endowment of effective labour units.[2] The distribution of effective labour units (ability) among workers is described by the Pareto distribution. Letting $e$ denote the ability of a worker, the Pareto distribution is defined over the interval $e \geq b$, and its CDF is

$$F(e) = 1 - (b/e)^a, \quad a > 1, b > 0, \tag{1}$$

Parameter $b$ stands for the lowest ability in the population of workers, and parameter $a$ determines the shape of the distribution (higher values of $a$ imply greater equality in the distribution of ability). The mean of the Pareto distribution is equal to,

$$\mu = ab/(a-1). \tag{2}$$

Workers (denoted by the superscript *L*) have preferences of the form,

[2] Introducing the possibility of an endogenous determination of the time devoted to work by workers of different ability would be an interesting extension of the model if one wished to study further how minimum wages affect the distribution of hours worked and utility among workers of different ability.

$$U_i^L = \sum_{t=0}^{\infty} \beta^t ln(C_{t,i}^L) , \tag{3}$$

where 0<β<1 is the discount factor, and $C_{t,i}^L$ stands for worker's *i* consumption at time *t*. Workers' incomes are equal to their labour earnings, which are equal to worker's ability times the wage per *effective* unit of labour, $e_i w_t$. As a result, the consumption of workers evolves according to:

$$C_{t,i}^L = e_i w_t. \tag{4}$$

### *2.1.2 Capitalists*

There is a fixed number, $N$, of identical capitalists in the economy. In contrast to workers, they do not directly participate in production, but hold shares in various firms and receive as dividends the firms' profits. For simplicity, we assume that the number of capitalists is equal to the number of firms. Their preferences[3] are similar to workers', i.e.

$$U^K = \sum_{t=0}^{\infty} \beta^t ln(C_t^K) , \tag{5}$$

whereas their budget constraint is,

$$C_t^K + K_{t+1}^K - (1-\delta)K_t^K = \Pi_t + r_t K_t^K. \tag{6}$$

In equation (5), $C_t^K$ stands for the consumption of each capitalist, and in equation (6), $K_t^K, r_t K_t^K$, and $\Pi_t$ stand for the capital stock, capital income, and profits accruing to each capitalist. Each capitalist solves the following programme:

$$\max_{C_t^K, K_{t+1}^K} \mathcal{L} = \sum_{t=0}^{\infty} \beta^t \{lnC_t^K + \lambda_t(\Pi_t + r_t K_t^K - C_t^K - K_{t+1}^K + (1-\delta)K_t^K)\}$$

The resulting first-order conditions are:

$$\lambda_t = 1/C_t^K, \tag{7a}$$
$$\lambda_t = \beta\lambda_{t+1}(r_{t+1} + 1 - \delta). \tag{7b}$$

[3] For convenience we drop the subscript pertaining to each capitalist since they are identical.

Combining equations (7a) and (7b) we get:

$$C_{t+1}^{K} = \beta(r_{t+1} + 1 - \delta)C_{t}^{K} \,. \tag{8}$$

Equation (8) summarizes the optimal consumption path for the capitalists, and (implicitly), along with their budget constraint, the supply of capital in the economy.

### *2.1.3 Firms*

Firms' technology of converting inputs into output is a Cobb-Douglas one,

$$Y_t = \left(K_t^f\right)^{\gamma}\left(L_t^f\right)^{\eta}, \;\; \gamma, \eta < 1 \tag{9}$$

where $Y_t$ denotes output, $K_t^f$ is the capital stock used by the firm, $L_t^f$ is the number of *effective* units of labour used by the firm. Profit maximization implies,

$$w_t = \eta\left(K_t^f\right)^{\gamma}\left(L_t^f\right)^{\eta-1} \tag{10}$$

$$r_t = \gamma\left(K_t^f\right)^{\gamma-1}\left(L_t^f\right)^{\eta} \;\; . \tag{11}$$

As a result, the profits accruing to each entrepreneur are:

$$\Pi_t = (1 - \eta - \gamma)Y_t. \tag{12}$$

### *2.1.4 Factor Market Equilibrium*

The aggregate supply of effective units of labour of all workers is,

$$L^s = \int_b^{\infty} e\left\{\alpha \frac{b^{\alpha}}{e^{\alpha+1}}\right\} de = \frac{\alpha b}{\alpha-1}, \tag{13}$$

i.e., it is just equal to the mean units of effective labour (since the number of workers is equal to one). Labour market equilibrium obtains when the aggregate demand for labour by the $N^K$ firms is equal to aggregate labour supply, i.e. when,

$$N^K L_t^f = \frac{\alpha b}{\alpha-1}. \tag{14}$$

Similarly, equilibrium in the capital market obtains when the total supply of capital – as provided by the capitalists – is equal to the demand for capital by firms, i.e. when

$$K_t^K = K_t^f. \tag{15}$$

### 2.1.5 General Equilibrium

The dynamic behavior of the model is described by equations (4), (6), (8), (9), (10), (11), (12), (14), and (15), which in long-run equilibrium collapse to the following system (for ease of exposition we drop the time subscripts, and the superscripts distinguishing between capitalists and firms, since each firm is owned by a single capitalist, e.g. $L_t^f = L$ ):

$$C_i^L = e_i w \tag{LR1}$$

$$1 = \beta(r + 1 - \delta) \tag{LR2}$$

$$C^K + \delta K = \Pi + rK \tag{LR3}$$

$$Y = L^\eta K^\gamma \tag{LR4}$$

$$\Pi = (1 - \eta - \gamma)Y \tag{LR5}$$

$$N^K L = \frac{\alpha b}{\alpha - 1} \tag{LR6}$$

$$w = \eta K^\gamma L^{\eta-1} \tag{LR7}$$

$$r = \gamma K^{\gamma-1} L^\eta \tag{LR8}$$

Equations (LR2)-(LR8) determine the long-run equilibrium values of *w, r, K, L, Y, Π,* and $C^K$.[4] We note that once the value of the wage rate is found we can determine the entire distribution of workers' consumption through equation (LR1).

## 2.2 Minimum Wages

We now assume the existence of a government-imposed minimum wage per unit of labour *time* (e.g. per hour) equal to $y$, which is the minimum amount that an

[4] In fact, the entire system can be solved recursively in the long-run: once we find *r* from (LR2), and *L* from (LR6), (LR8) solves for solves for *K*, then (LR7) for $w$, (LR4) for $Y$, and so on.

employer must pay in order to employ one person. This minimum wage per unit of time must be distinguished from the wage rate per *effective* unit of labour, which will be market-determined (i.e. as in the previous section).

### *2.2.1 Labour Market*

The minimum wage constraint implies that firms will not be willing to employ workers whose level of ability (i.e. number of efficient units of labour per unit of time) is such that $y > e_i \varpi_t$, where $\varpi_t$ stands for the market-determined wage rate per effective unit of labour in the presence of the minimum-wage (per unit of time) constraint at time $t$.[5] In order to avoid confusion in what follows we shall refer to the exogenously set, $y$, simply as the minimum wage, in order to differentiate it from the minimum wage *rate* , $\varpi_t$, and the competitive wage *rate*, $w$, both of which are endogenously determined. Let $\varepsilon_t$ denote the level of ability for which it holds that:

$$y = \varepsilon_t \varpi_t \, . \tag{16}$$

It follows that only workers with $e_i \geq \varepsilon_t$ will be employed by firms, and that the individual with ability $\varepsilon_t$ will just earn the minimum wage, $y$. Workers with ability smaller than $\varepsilon_t$ will be unemployed, thus the unemployment rate will be:

$$u_t = 1 - \left\{\frac{b}{\varepsilon_t}\right\}^{\alpha} \tag{17}$$

The total number of effective units of labour possessed (and supplied) by those individuals with $e_i \geq \varepsilon_t$ is,

$$L^s = \int_{\varepsilon}^{\infty} e\left\{\alpha \frac{b^{\alpha}}{e^{\alpha+1}}\right\} de = \frac{\alpha}{\alpha-1}\left\{\frac{b}{\varepsilon_t}\right\}^{\alpha}. \tag{18}$$

We can thus describe the condition describing equilibrium in the labour market (i.e. the analogue of equation (14) as:

$$N^K L_t^f = \frac{\alpha}{\alpha-1}\left\{\frac{b}{\varepsilon_t}\right\}^{\alpha}. \tag{19}$$

[5] We assume that the minimum wage per unit of time is such that $y > b\varpi_t$.

A simple comparison of equations (14) and (19) reveals that –ceteris paribus- a binding minimum wage constraint, which implies that $b < \varepsilon$, will be associated with a higher wage rate per effective unit of labour than in its absence ($\varpi > w$), due to the reduction in the aggregate effective units of labour supply caused by the exclusion of the lowest-ability workers from employment.

### *2.2.2 Government*

In addition to setting (and enforcing) the minimum wage constraint, the government is assumed to levy a comprehensive income tax ($\tau$) on all sources of income (with the exception of unemployment benefits[6]), in order to finance benefits for the low-ability workers that are unemployed. We assume that the level of the unemployment benefit is a fixed proportion of the minimum wage, i.e. it is equal to $\phi y$ $(0 < \phi < 1)$. Equation (20), i.e. the government budget constraint, just states that the net payments to the unemployed are equal to the total tax receipts:

$$\phi y u_t = \tau_t Y_t. \tag{20}$$

We assume that $\tau_t$ adjusts in every period so as to keep the budget in balance.

### *2.2.3 General Equilibrium*

The existence of taxes implies that equations (4), (6), and (8) must be modified to:

$$C_{t,i}^{L} = (1-\tau)e_i\varpi_t \tag{4a}$$

$$C_t^K + K_{t+1}^K - (1-\delta)K_t^K = (1-\tau)(\Pi_t + r_t K_t^K). \tag{6a}$$

$$C_{t+1}^K = \beta((1-\tau)r_{t+1} + 1 - \delta)C_t^K \tag{8a}$$

These three equations along with equations (16) – (20) describe the dynamic evolution of the system, whose long-run equilibrium is described by the following equations:

[6] None of our results hinges on this assumption.

$$C_i^L = e_i\varpi \tag{LR1a}$$

$$1 = \beta((1-\tau)r + 1 - \delta) \tag{LR2a}$$

$$C^K + \delta K = (1-\tau)(\Pi + rK) \tag{LR3a}$$

$$Y = L^\eta K^\gamma \tag{LR4a}$$

$$\Pi = (1-\eta-\gamma)Y \tag{LR5a}$$

$$N^K L = \frac{\alpha}{\alpha-1}\left\{\frac{b}{\varepsilon_t}\right\}^\alpha \tag{LR6a}$$

$$\varpi = \eta K^\gamma L^{\eta-1} \tag{LR7a}$$

$$r = \gamma K^{\gamma-1} L^\eta \tag{LR8a}$$

$$u = 1 - \left\{\frac{b}{\varepsilon}\right\}^\alpha \tag{LR9}$$

$$y = \varepsilon\varpi \tag{LR10}$$

$$(1-\tau)\phi y u = \tau Y \tag{LR11}$$

These equations determine the long-run values of $\varpi$, $\varepsilon$, *r*, *u*, *K*, *L*, *Y*, *Π*, $C^K$, $C_i^L$, and $\tau$. We note that unlike the perfectly competitive (PC) case, the system is no longer recursive, since equation (LR2a) does not uniquely solve for *r.* Nevertheless, we can draw some useful results by comparing the PC with the minimum wage (MW) case.

### *3. Comparison*

Using equations (LR2), (LR6), (LR7), and (LR8) we find that the PC wage rate is:

$$w = \frac{\eta\gamma^{1/(1-\gamma)}[\frac{b\alpha}{(\alpha-1)N^K}]^{(\eta+\gamma-1)/(1-\gamma)}}{[\delta-1+(1/\beta)]^{1/1-\gamma}} \tag{21}$$

Similar manipulations for the MW case yield:

$$\varpi = \frac{\eta((1-\tau)\gamma)^{\gamma/(1-\gamma)}[\frac{b^\alpha\alpha}{(\alpha-1)N^K}]^{(\eta+\gamma-1)/(1-\gamma)}\varepsilon^{(\eta+\gamma-1)(1-\alpha)/(1-\gamma)}}{[\delta-1+(1/\beta)]^{1/1-\gamma}} \tag{22}$$

We wish to enquire whether the workers which retain their jobs after the imposition of the MW have higher after-tax incomes than in the PC case. This will be the case if the after-tax wage rate (per effective unit of labour) in the MW case is larger than the PC wage rate, i.e. if

$$(1-\tau)\varpi > w. \tag{23}$$

Using equations (21) and (22), inequality (23) can be written as

$$1-\tau > [b/\varepsilon]^{(\alpha-1)(1-\eta-\gamma)}. \tag{24}$$

Since both $\tau$ and $\varepsilon$ are endogenous (i.e. they depend, among other things, on the size of the imposed minimum wage, $y$), it is impossible to assess, in general, whether inequality (24) is satisfied.[7] For this reason, we now examine the consequences of adopting two assumptions widely employed in macroeconomics.

Consider, first, the case that firms make no profits, which obtains if there are constant returns to scale ($\eta+\gamma=1$). The required condition now becomes $1-\tau>1$, which is impossible.[8]

The same result obtains if $\varepsilon \to b$, which would arise if $\alpha \to \infty$, i.e. if there is complete equality in the distribution of ability among workers. Thus, we can state that:

*Proposition*: *With Cobb-Douglas technology, the imposition of any binding minimum wage will lead to a decrease in the after-tax incomes of the employed workers in the long run, if either*

*(a) There are constant returns to scale, or*

*(b) All workers have the same ability.*

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[7] Numerical simulations indicate that it is possible to find a binding minimum wage which succeeds in increasing the after-tax incomes of employed workers but at the expense of workers of low ability who are forced into unemployment.

[8] Note that even if the unemployed were left without social support, or taxes were imposed on capitalists only ($\tau=0$), it would still be impossible to increase the income of workers remaining in employment.

The reason that employed workers may not become worse off in the presence of profits is that the capital stock will be higher than in their absence. This is because the ability to tax profits (as well) reduces by less the incentives for capital accumulation than if the full burden of taxation falls on worker and capital income alone – since taxes on profits are less distortive than on capital. In the case of complete equality in ability among workers (i.e. a homogeneous labour force), the MW regime decreases the aggregate units of effective labour used in proportion to the fall in the number of persons employed, thus it leads to a larger drop in output and in the income accruing to employed members – since the (pre-tax) share of output accruing to workers is fixed - out of which the taxes to support the unemployed must also be paid.

**3. Conclusion**

Our demonstration that it is impossible for *any* binding minimum wage to increase the after-tax incomes of workers if either the production function is Cobb-Douglas, or if there are no differences in ability among workers, is a direct application of the Chamley–Judd result.[9] This result states that any linear tax on capital will reduce the aggregate income received by workers by more than the revenue raised by the tax. By introducing heterogeneity in ability among workers, the present paper has shown that not only aggregate workers' income will decline following the imposition of a minimum wage, but also the incomes of workers remaining in employment as long as they are called to share in the cost of financing social welfare benefits for the less able who become unemployed. Nevertheless, the paper suggests that the joint

[9] See, Chamley (1986) and Judd (1985).

existence of decreasing returns to scale and worker heterogeneity can allow a binding minimum wage to increase the incomes of workers who remain employed, but only at the expense of those who become unemployed (see Knabe and Schöb (2009) for an empirical analysis of the latter effect in the German context).

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