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Is Emission Intensity or Output U-shaped in the Strictness of Environmental Policy?

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Abstract

In a model where firms face a continuous choice of how much to invest in environmental innovation, we show that an ever stricter environmental policy does not always lead to ever cleaner production methods and ever lower production of polluting goods. It does so when the abatement technology is end-of-pipe. When the abatement technology is integrated however, either emission intensity or output is U-shaped in the strictness of policy. If the emission intensity is U-shaped, it will reach its lowest value at the point where the Marginal Abatement Cost curves intersect. These results hold with emission taxation (whether firms are price-takers or they interact strategically on the output market) as well as in the social optimum.

JEL-Code: L130, Q550, Q580.

Keywords: environmental innovation, environmental taxation, oligopoly, marginal abatement costs.

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1 Introduction

Environmental policy gives polluting firms an incentive to find cleaner ways of producing. There is a large literature on the effect of environmental policy on innovation (see e.g. Requate (2005) for an overview) starting from Kneese and Schultze (1978). One question that has received relatively little attention is: When environmental policy becomes stricter and stricter, will firms invest more and more in environmental R&D to reduce their emission-to-output ratio? Our immediate intuition might suggest that this should be the case. However, making production cleaner is only one of two ways in which firms can respond to stricter environmental policy. The other way is to reduce output. This in turn reduces firms' incentives to use cleaner production methods. If a firm will produce very little because of a very strict environmental policy, it also has little incentive to invest in environmental R&D. This suggests that when output is decreasing in the strictness of environmental policy, the emissions-to-output ratio might be a U-shaped function of strictness. However, the effects might conceivably also be reversed: When an ever stricter environmental policy prompts a firm to invest more and more in environmental R&D, production might eventually become so clean that it starts to increase again.

In the present paper we study the effects of the strictness of environmental policy on output and on the incentives to install cleaner technology. We start with a very general formulation of the firm's environmental technology, with end-of-pipe and integrated technology as two specific cases. With end-of-pipe technology, where a firm can reduce its absolute emission level by a certain amount, we find that emission intensity as well as output is decreasing throughout in strictness. With integrated technology, where a firm can reduce its emissions-to-output ratio to a certain level, either output or emission intensity is U-shaped in strictness.

As Perino and Requate (2012) have shown, the question of whether a stricter environmental policy induces more investment in environmental R&D is linked with the recent literature on pivoting Marginal Abatement Costs (MAC) curves. Traditionally it has been assumed that environmental innovation reduces MAC at any level of emissions. In these models, a stricter environmental policy leads to more environmental innovation (e.g.

Downing and White, 1986; Milliman and Prince, 1989; Jung et al., 1996; Requate and Unold, 2003). Recently, a number of papers (Amir et al., 2010; Baker et al., 2008; Baumann et al., 2008; Bréchet and Jouvet, 2008) have shown that environmental innovation may not cause the whole MAC curve to shift downward. Indeed, Amir et al. (2010), Baumann et al. (2008) and Bréchet and Jouvet (2008) show that a decrease in the marginal emission intensity of "dirty" inputs leads to a clockwise rotation or pivoting of the MAC curve: it is lower for higher emission levels, but higher for lower emission levels.[5] While these three papers take the output response of a firm into account, they do so in a very simplified manner. They only consider one firm faced with a constant output price. In our paper we endogenize the output market, which makes it much more difficult to define a MAC function.

While the literature that introduced pivoting MAC curves took environmental innovation as exogenous, we examine how this affects the incentives for innovation. On this subject, Perino and Requate (2012) have shown that when the MAC curve pivots clockwise, adoption of a clean technology is U-shaped in the strictness of environmental policy.[6] The authors assume that there is a continuum of small firms that can choose between two technologies and they do not model the output market explicitly.

Bréchet and Meunier's (2012) analysis of an integrated technology does consider the output market. Otherwise their model is similar to Perino and Requate (2012), with firms choosing between two technologies. Bréchet and Meunier (2012) also find that adoption of a clean technology is U-shaped in the strictness of environmental policy.

There is an earlier literature that explicitly takes the output market into account.[7] Ulph (1997) sets up a free-entry Cournot duopoly model with constant marginal cost of production and an integrated abatement technology, treating the environmental tax rate as an exogenous variable. Ulph (1997) derives conditions under which an increase in the tax rate reduces output. He finds that the effect of the tax rate on R&D spending is am-

[5] Baker et al. (2010) find that the MAC curve pivots by assuming that Total Abatement Cost decreases for all levels of abatement except for complete abatement of all emissions.

[6] Endres and Friehe (2011) examine the effects of environmental liability law on the incentives to diffuse advanced abatement technology that reduces MAC everywhere or pivots the MAC curve clockwise.

[7] A related research strand focuses on environmental innovation in the context of international trade, showing the ambiguous effect of domestic emission taxation (Ulph and Ulph, 1996; Simpson and Bradford, 1993; Feess and Muehlheusser, 2002).

biguous. Katsoulacos and Xepapadeas (1996) set up a Cournot oligopoly with technology spillovers, where the government taxes emissions and subsidizes R&D. The authors' conclusion that the effect of the tax rate on output is ambiguous is in accordance with Ulph's (1997) findings. Katsoulacos and Xepapadeas (1996) further find that R&D spending is increasing in the tax rate.

The model in our paper is similar to Perino and Requate (2012) and Bréchet and Meunier (2012), however there are some crucial differences. Most importantly, whereas Perino and Requate (2012) and Bréchet and Meunier (2012) model the firm's choice between two technologies, we model the technology choice as continuous. This explains why we find that either emission intensity or output is U-shaped in strictness, while Perino and Requate (2012) and Bréchet and Meunier (2012) find that emission intensity is always U-shaped and output is decreasing in the strictness of the policy.

Furthermore, unlike Perino and Requate (2012) we explicitly take the output market into account. With Cournot competition, emission taxation does not implement the social optimum. However, we find that in the social optimum as well, either output or emission intensity is U-shaped in strictness. This shows that the U-shapes are not due to some kind of policy or market failure.

Finally, unlike Perino and Requate (2012) and Bréchet and Meunier (2012) we take the interaction between individual firms' MAC curves into account. The literature until now has assumed that a firm's MAC curve only depends on its own choice of technology. However, with integrated technology and output market interactions, one firm's technology choice affect the other firms' MAC curves. When one firm chooses a cleaner technology, this reduces its effective tax rate on output. This firm will produce more, which depresses the product price for all other firms, shifting their MAC curves downward. Because of the interaction between firms' individual MACs, we find it useful to work with a more aggregate concept of the MAC. With emission taxation, we define the Aggregate Marginal Abatement Cost for the whole industry. In the social optimum, the Social Marginal Abatement Cost includes the whole industry as well as the consumer surplus.

The rest of this paper is organized as follows. In Section 2, we introduce our model. We analyze emission taxation in Section 3 and the welfare optimum in Section 4. Since we cannot obtain clear-cut results for the integrated technology in general, we analyze this technology in more detail with specific functional forms deriving MAC curves in Section 5. The concluding Section 6 discusses the implications for policy and empirical work.

2 The model

There are n identical firms producing a homogeneous good. We shall mainly focus on imperfect (Cournot) competition between firms, but we will also discuss results for perfect competition and for a perfectly elastic market demand function. Firm i, $i = 1, \cdots, n$, producing q_i faces the inverse demand function $P(Q)$, where $Q \equiv \sum_{i=1}^{n} q_i$, $P' \leq 0$ and:[8]

$$P'(Q) + P''(Q)q_i \leq 0 \tag{1}$$

$$\lim_{q \to 0} \; [P'(nq) + P''(nq)q] \text{ is finite} \tag{2}$$

Production is polluting. Firm i's total emissions e_i are given by:

$$e_i = \varepsilon_i q_i \tag{3}$$

where $\varepsilon_i \in [0, 1]$ is the emissions-to-output ratio, which depends on the abatement technology that the firm installs. If the firm does not spend anything on abatement, $\varepsilon_i = 1$.

The firm's cost function is $C(q_i, \varepsilon_i)$ with $C_q(0, \varepsilon_i) = 0$ and $C_q > 0$ for $q_i > 0$; $C_\varepsilon(q_i, 1) = 0$, $C_\varepsilon < 0$ for $\varepsilon_i \in [0, 1)$; $C_{q\varepsilon} \leq 0$, $C_{qq} > 0$, $C_{\varepsilon\varepsilon} > 0$ and:

$$\lim_{\varepsilon_i \to 0} \; C_{\varepsilon\varepsilon}(q_i, \varepsilon_i) \text{ is finite} \tag{4}$$

Our assumption $C_\varepsilon(q_i, 1) = 0$ implies that a firm will reduce ε_i even when environmental policy is very lenient (McKitrick, 1999). $C_{q\varepsilon} \leq 0$ means that reducing the emission-to-output ratio becomes more expensive (or at least not cheaper) as output rises.

Finally we impose:

$$\lim_{q \to 0} \; [P'(nq) - C_{qq}(q, \varepsilon)] \text{ is finite} \tag{5}$$

In addition to this general formulation of a firm's cost function, we shall consider two specific abatement technologies: integrated and end-of-pipe. With integrated technology

[8] Gaudet and Salant (1991) introduced condition (1) to ensure uniqueness of the Cournot equilibrium.

(previously analyzed by Ulph, 1997), when a firm wants to reduce its emission-to-output ratio to ε_i, it has to spend an amount $F(\varepsilon_i)$ which does not depend on the output level. The cost function is then:

$$C(q_i, \varepsilon_i) = k(q_i) + F(\varepsilon_i) \tag{6}$$

with $k'(0) = 0$ and $k' > 0$ for $q_i > 0$; $k'' > 0$; $F'(1) = 0$ and $F'(\varepsilon_i) < 0$ for $\varepsilon_i \in [0, 1)$; $F'' > 0$. Note that this cost function features $C_{q\varepsilon}(q_i, \varepsilon_i) = 0$. Examples of integrated abatement technologies are those which allow firms to be more energy efficient in their production processes. In the steel and iron industry, one of the largest industrial sources of CO_2 emissions, examples of such technologies are coke dry quenching and top pressure recovery turbines (Carpenter, 2012).

With end-of-pipe technology, when a firm wants to reduce its emissions by the absolute amount r_i, it has to spend an amount $V(r_i)$ which does not depend on the output level. The cost function in this case is:

$$C(q_i, \varepsilon_i) = k(q_i) + V(r_i) \tag{7}$$

where:

$$e_i = q_i - r_i, \qquad r_i = q_i(1 - \varepsilon_i) \tag{8}$$

with $k'(0) = 0$ and $k' > 0$ for $q_i > 0$; $k'' > 0$; $V'(0) = 0$ and $V'(r_i) > 0$ for $r_i \in (0, q_i]$; $V'' > 0$. Note that this cost function features $C_{q\varepsilon}(q_i, \varepsilon_i) = -V' - r_i V'' < 0$. Montero (2002) has previously modelled end-of-pipe technology in a Cournot duopoly in order to analyze the environmental R&D incentives of different environmental policy instruments.

An example of an end-of-pipe technology is Carbon Capture and Storage (CCS). Herzog (2011) estimates the cost of capturing, transporting, injecting and monitoring at $60-65 per metric ton of CO_2, independent of the amount of fossil fuel combusted. Lafforgue et al. (2008) and Amigues et al. (2013) model CCS as an end-of-pipe technology.

Endres and Friehe (2013) also examine end-of-pipe and integrated abatement technologies, in the different context of environmental liability law. While our definition of integrated technology is equivalent to theirs, we collapse their two variables for the end-of-pipe technology (technology improvement and abatement) into one. Endres and Friehe (2013) assume that the polluting firm can apply integrated and end-of-pipe technologies simultaneously, a scenario that we capture in our general formulation. We also analyze the two technologies separately.

The environmental damage caused by pollution is given by $D(\beta, E)$, where $E \equiv \sum_{i=1}^{n} e_i$, $D(\beta, 0) = D_E(\beta, 0) = 0$, $D_E > 0$ and $D_{EE} > 0$ for $E > 0$; $D(0, E) = D_\beta(0, E) = 0$, $D_\beta > 0$ and $D_{\beta E} > 0$ for $E > 0$. Thus total and marginal damage are increasing in the environmental damage parameter β. This parameter measures the severity of the environmental problem or the strength of the policy maker's preference for the environment. In the next two sections, we will determine how ε_i and q_i respond to a change in β.

We will focus our attention on symmetric equilibria only. When analyzing emission taxation under imperfect competition, we assume that β is high enough to guarantee a positive tax rate ($t > 0$).[9]

Firm i's profits Π_i can be written as:

$$\Pi_i = \pi(q_i, \varepsilon_i) - te_i = P(Q)q_i - C(q_i, \varepsilon_i) - t\varepsilon_i q_i \tag{9}$$

where $\pi(q_i, \varepsilon_i)$ denotes the firm's operating profits, i.e. its profits net of the tax payment.

The policy maker or regulator's objective is to maximize welfare, which is the sum of consumer surplus and operating profits, minus environmental damage. It can also be written as the utility from the good (the area below the inverse demand function) minus production cost and environmental damage, i.e.:

$$W(q_1, \cdots, q_n; \varepsilon_1, \cdots, \varepsilon_n; \beta) = \int_0^Q P(Y)dY - \sum_{i=1}^{n} C(q_i, \varepsilon_i) - D(\beta, E) \tag{10}$$

3 Emission taxation

With emission taxation, the regulator sets the tax rate in stage one. In stage two, all firms simultaneously choose their emission-to-output ratio ε_i and its output level q_i.

In stage two, each firm i maximizes its profits Π_i in (9). The first order conditions with respect to q_i and ε_i respectively are:

$$P + P'(Q)q_i - C_q - t\varepsilon_i = 0 \tag{11}$$

$$-C_\varepsilon - tq_i = 0 \tag{12}$$

[9] For small β, the policy maker will want to set $t < 0$, to induce the imperfectly competitive firms to produce more.

When product market demand is perfectly elastic $[P'(Q) = 0]$ and under perfect competition where each firm takes the product price as given, (11) turns into:

$$P - C_q - t\varepsilon_i = 0 \tag{13}$$

Returning to the general case, if there is no environmental policy ($t = 0$), then $C_\varepsilon = 0$ from (12). In this case $\varepsilon = 1$ and we denote the profit-maximizing output level by $\bar{q}_t$, which from (11) is implicitly defined by:

$$P(n\bar{q}_t) + P'(n\bar{q}_t)\bar{q}_t - C_q(\bar{q}_t, 1) = 0 \tag{14}$$

The second order condition for profit maximization is that the matrix

$$\mathbf{\Pi}_{xx} \equiv \begin{pmatrix} \pi_{qq} & \pi_{q\varepsilon} - t \\ \pi_{q\varepsilon} - t & \pi_{\varepsilon\varepsilon} \end{pmatrix} = \begin{pmatrix} 2P' + P''q_i - C_{qq} & -C_{q\varepsilon} - t \\ -C_{q\varepsilon} - t & -C_{\varepsilon\varepsilon} \end{pmatrix} \tag{15}$$

is negative semidefinite. We shall make the slightly stronger assumption that $\mathbf{\Pi}_{xx}$ is negative definite. This implies that $\mathbf{h}\mathbf{\Pi}_{xx}\mathbf{h}' < 0$ for all vectors $\mathbf{h}$ and the determinant is positive:

$$-C_{\varepsilon\varepsilon}\left[2P' + P''q_i - C_{qq}\right] - (t + C_{q\varepsilon})^2 > 0 \tag{16}$$

Under perfect competition, (16) becomes:

$$C_{\varepsilon\varepsilon}C_{qq} - (t + C_{q\varepsilon})^2 > 0 \tag{17}$$

Totally differentiating (11) and (12) with respect to t yields:

$$\left[(n+1)P' + nP''q_i - C_{qq}\right]\frac{dq_i}{dt} - C_{q\varepsilon}\frac{d\varepsilon_i}{dt} = \varepsilon_i + t\frac{d\varepsilon_i}{dt} \tag{18}$$

$$-C_{\varepsilon\varepsilon}\frac{d\varepsilon_i}{dt} - C_{\varepsilon q}\frac{dq_i}{dt} = q_i + t\frac{dq_i}{dt} \tag{19}$$

Solving for dq_i/dt and $d\varepsilon_i/dt$ we find:

$$\frac{dq_i}{dt} = \frac{-\varepsilon_i C_{\varepsilon\varepsilon} + q_i(t + C_{q\varepsilon})}{-\left[(n+1)P' + nP''q_i - C_{qq}\right]C_{\varepsilon\varepsilon} - (t + C_{q\varepsilon})^2} \tag{20}$$

$$\frac{d\varepsilon_i}{dt} = \frac{q_i\left[(n+1)P' + nP''q_i - C_{qq}\right] + \varepsilon_i(t + C_{q\varepsilon})}{-C_{\varepsilon\varepsilon}\left[(n+1)P' + nP''q_i - C_{qq}\right] - (t + C_{q\varepsilon})^2} \tag{21}$$

Since the denominator is positive by (1) and (16), the sign of (20) and (21), as well as of (22) below, is the sign of the numerator on the RHS. We cannot sign dq_i/dt and $d\varepsilon_i/dt$

unambiguously, but we can show that emissions are decreasing in the tax rate. From (20) and (21):

$$\frac{de_i}{dt} = \varepsilon_i \frac{dq_i}{dt} + q_i \frac{d\varepsilon_i}{dt} = \frac{-\varepsilon_i^2 C_{\varepsilon\varepsilon} + 2\varepsilon_i q_i (t + C_{q\varepsilon}) + q_i^2 \left[(n+1)P' + nP'' q_i - C_{qq} \right]}{-C_{\varepsilon\varepsilon} \left[(n+1)P' + nP'' q_i - C_{qq} \right] - (t + C_{q\varepsilon})^2} < 0 \quad (22)$$

The sign follows from the fact that using (15), the numerator can be written as:

$$\begin{pmatrix} q_i & \varepsilon_i \end{pmatrix} \mathbf{\Pi}_{xx} \begin{pmatrix} q_i \\ \varepsilon_i \end{pmatrix} + (n-1) q_i^2 (P' + P'' q_i) < 0$$

where the inequality follows from (1) and setting $\mathbf{h} = \begin{pmatrix} q_i & \varepsilon_i \end{pmatrix}$ in $\mathbf{h} \mathbf{\Pi}_{xx} \mathbf{h}' < 0$ because $\mathbf{\Pi}_{xx}$ in (15) is negative definite.

In stage one, the regulator sets the tax rate that maximizes welfare. In the symmetric case where $q_i = q$ and $\varepsilon_i = \varepsilon$ for all $i = 1, \cdots, n$, welfare (10) can be written as:

$$W(q, \varepsilon, \beta) = \int_0^{nq} P(Y) dY - nC(q, \varepsilon) - D(\beta, n\varepsilon q) \quad (23)$$

Writing welfare as a function of t and β, the first order condition with respect to β is:

$$\frac{\partial W(t, \beta)}{\partial t} = P \frac{dQ}{dt} - nC_q \frac{dq}{dt} - nC_\varepsilon \frac{d\varepsilon}{dt} - D_E \frac{dE}{dt} = 0 \quad (24)$$

Using the implicit function theorem we find:

$$\frac{dt}{d\beta} = -\frac{\partial^2 W / \partial\beta \partial t}{\partial^2 W / \partial t^2} = D_{\beta E} \frac{dE/dt}{\partial^2 W / \partial t^2} > 0 \quad (25)$$

The second equality follows from (24). The inequality follows from $D_{\beta E} > 0$, $dE/dt < 0$ by (22) and $\partial^2 W / \partial t^2 < 0$ as the SOC for welfare maximization. Thus an increase in β will prompt the regulator to raise the tax rate. This means that the signs of $dq/d\beta$ and $d\varepsilon/d\beta$ are the signs of dq/dt and $d\varepsilon/dt$ respectively.

Returning to (20) and (21) and setting the tax rate very low (close to zero), the numerators on the RHS become respectively:

$$-\varepsilon_i C_{\varepsilon\varepsilon} + q_i C_{q\varepsilon} < 0$$

$$q_i \left[(n+1)P' + nP'' q_i - C_{qq} \right] + \varepsilon_i C_{q\varepsilon} < 0$$

The second inequality follows from (1), $P' \leq 0$ and $C_{qq} > 0$. Thus when the tax rate is very low, both q_i and ε_i are decreasing in t. In order to make more definite statements on the effect of t on q_i and ε_i, let us look at the two special cases of environmental technology introduced in Section 2: end-of-pipe and integrated abatement technology.

3.1 End-of-pipe technology

With end-of-pipe technology (7), firm i's profit function (9) becomes:

$$\Pi_i = \pi(q_i, \varepsilon_i) - te_i = P(Q)q_i - k(q_i) - V(r_i) - t(q_i - r_i)$$

The first order conditions with respect to q_i and r_i are, respectively:

$$P + P'q_i - k' = t \tag{26}$$

$$V' = t \tag{27}$$

We see that the FOC (26) for output q_i does not feature abatement r_i. Thus if firm i changes its abatement technology r_i (for instance because $V(r_i)$ falls), this will not affect its output level. This is because the firm's effective tax rate t on output is not affected by its abatement technology.

It is straightforward but instructive for comparison with the integrated technology to interpret this result in terms of Marginal Abatement Costs (MAC). We follow the approach by Amir et al. (2010), Baumann et al. (2008) and Bréchet and Jouvet (2008) who build upon McKitrick's (1999) definition of the MAC function keeping abatement technology (here r_i) constant.[10] Substituting (8) into (9), we write firm i's operating profits as a function of its emissions, its abatement technology and all other firms' total output Q_{-i}:

$$\pi(e_i, r_i, Q_{-i}) = P(Q_{-i} + e_i + r_i)(e_i + r_i) - k(e_i + r_i) - V(r_i)$$

Marginal abatement cost, defined for a given level of r_i, is then:

$$MAC(e_i, r_i, Q_{-i}) = \frac{\partial \pi(e_i, r_i, Q_{-i})}{\partial e_i} = P + P'(e_i + r_i) - k' \tag{28}$$

Note that firm i's MAC depends on the other firms' output choices, but not on their abatement technology, since (as we have seen) a firm's choice of abatement technology does not affect its output choice.

[10] An alternative definition of MAC keeps firm i's output constant at the level q_i^* implicitly defined by (26) and sets $MAC(e_i) = \partial\pi(q_i^*, r_i)/\partial r_i = -V'(r_i)$. However, this alternative definition is not compatible with the definition used in the "pivoting MAC" literature.

The MAC curve can be drawn as a function of e_i for given levels of r_i and Q_{-i}. Let us now examine the horizontal shift in the MAC curve when r_i changes. That is, we wish to determine how e_i changes with r_i for a given level of MAC. Setting the total differential of (28) with respect to r_i equal to zero yields:

$$\frac{dMAC(e_i, r_i, Q_{-i})}{dr_i} = [2P' + P''(e_i + r_i) - k''] \left[\frac{de_i}{dr_i} + 1\right] = 0$$

The first term between square brackets is negative by (1), $P' \leq 0$ and $k'' < 0$. Thus we have $de_i/dr_i = -1$: a cleaner end-of-pipe technology shifts the whole MAC curve to the left in parallel fashion.

Returning to (26) and (27), totally differentiating them with respect to t yields:

$$\frac{dq_i}{dt} = \frac{1}{(n+1)P' + nP''q_i - k''} < 0 \quad (29)$$

$$\frac{dr_i}{dt} = \frac{1}{V''} > 0 \quad (30)$$

The inequalities follow from the second order conditions and (1). For the emissions-to-output ratio $\varepsilon_i = 1 - \frac{r_i}{q_i}$, we find:

$$\frac{d\varepsilon_i}{dt} = \frac{1}{q_i^2}\left(r_i \frac{dq_i}{dt} - q_i \frac{dr_i}{dt}\right) < 0$$

The inequality follows from (29) and (30). Thus with an end-of-pipe technology, both q_i and ε_i are monotonically decreasing in t. Given that the optimal tax rate is increasing in β, as shown in (25), we can state that both q_i and ε_i are decreasing in β.

3.2 Integrated technology

With integrated technology (6), firm i's first order conditions (11) and (12) in stage two are, respectively:

$$P + P'(Q)q_i - k'(q_i) - t\varepsilon_i = 0 \quad (31)$$

$$-F'(\varepsilon_i) - tq_i = 0 \quad (32)$$

In contrast to (26) for end-of-pipe technology, the FOC (31) for output under integrated technology does feature the abatement technology parameter (here ε_i). A cleaner technology decreases the effective tax rate $t\varepsilon_i$ on output and will thus prompt the firm to produce more.

Since $C_{q\varepsilon} = 0$ with integrated technology, equations (20) and (21) become:

$$\frac{dq_i}{dt} = \frac{-\varepsilon_i F'' + q_i t}{[k'' - (n+1)P' - nP''q_i] F'' - t^2} \tag{33}$$

$$\frac{d\varepsilon_i}{dt} = \frac{q_i [(n+1)P' + nP''q_i - k''] + \varepsilon_i t}{[k'' - (n+1)P' - nP''q_i] F'' - t^2} \tag{34}$$

The denominator is positive, as in (20) and (21). The signs of (33) and (34) are thus the signs of the numerators on the respective RHSs. Substituting (32) into (33), $dq_i/dt < 0$ if and only if:

$$-\varepsilon_i F'' - F' < 0 \tag{35}$$

According to Ulph (1997), $dq_i/dt < 0$ if and only if:

$$\varepsilon(F)\varepsilon''(F) - [\varepsilon'(F)]^2 > 0 \tag{36}$$

This is equivalent to our condition (35).[11]

When environmental policy is very strict, emissions are very low: $e_i = \varepsilon_i q_i \to 0$. This means that either ε_i or q_i, or both, approach zero. Let us first see what happens when ε_i approaches zero. From (33) and by (4):

$$\frac{dq_i}{dt} = \frac{q_i t}{[k'' - (n+1)P' - nP''q_i] F'' - t^2} > 0$$

Thus while ε_i is decreasing towards zero for ever stricter environmental policy, q_i is increasing. Indeed from (31) with $\varepsilon_i \to 0$, q_i approaches the output level $\bar{q}_t$ without environmental policy, as defined by (14).

By (2) and (5), when q_i goes to zero, (34) becomes:

$$\frac{d\varepsilon_i}{dt} = \frac{\varepsilon_i t}{[k'' - (n+1)P' - nP''q_i] F'' - t^2} > 0$$

Thus while q_i is decreasing towards zero for ever stricter environmental policy, ε_i is increasing. Indeed from (32) with $q_i \to 0$, ε_i approaches unity again: the firm does not spend anything on reducing the emissions-to-output ratio.

[11] The proof is as follows. Differentiating both sides of the equation $F(\varepsilon[F]) = F$ with respect to F yields $F'(\varepsilon[F])\varepsilon'[F] = 1$. Differentiating both sides of this equation with respect to F again yields $F''(\varepsilon')^2 + F'\varepsilon'' = 0$. Substituting $F' = 1/\varepsilon'$ and $F'' = -F'\varepsilon''/(\varepsilon')^2 = -\varepsilon''/(\varepsilon')^3$ into (35) and multiplying by $(\varepsilon')^3 < 0$ yields (36).

In Section 5, we will explore the behavior of the emission-to-output ratio and output for any tax rate, determining the conditions under which either is non-monotonic, for a specific integrated technology.

3.3 Summary

To summarize the results from this section, we state:[12]

Proposition 1 *With emission taxation:*

1. *The optimal tax rate t is increasing in the environmental damage parameter β;*
2. *Emissions E are decreasing in t, and consequently in β;*
3. *When t is very low, both output q and emission intensity ε are decreasing in t, and consequently in β;*
4. *With integrated technology, when t is very high so that E is close to zero:*
   (a) *When ε is close to zero, q is increasing in t, and consequently in β.*
   (b) *When q is close to zero, ε is increasing in t, and consequently in β.*
5. *With end-of-pipe technology, q is decreasing and emission reduction r is increasing in t, so that ε is decreasing in t. This implies that q and ε are decreasing in β, while r is increasing in β.*

4 The welfare optimum

In this section, we will show that the non-monotonic behaviour of the emission-to-output ratio or output is not due to market failure. To that purpose, we will solve the welfare optimum w, where the regulator chooses ε_i and q_i, $i = 1, \cdots, n$, to maximize welfare. As

[12]The results for integrated technology seem to contradict Ulph (1997) and Boom and Dijkstra (2009). Ulph (1997, p. 49) lists integrated technology cost functions where q is constant or increasing monotonically in t. Boom and Dijkstra (2009) find that output is monotonically decreasing in the strictness of environmental policy. We explain these differences in Appendix B.

firms are symmetric, we focus on the symmetric outcome where $q_i = q$ and $\varepsilon_i = \varepsilon$ for all $i = 1, \cdots, n$. The first order conditions are, from (23):

$$W_q = n\left[P - C_q - \varepsilon D_E\right] = 0 \tag{37}$$

$$W_\varepsilon = -n\left[C_\varepsilon + qD_E\right] = 0 \tag{38}$$

Comparing (37) and (38) to (11) and (12), we see that the regulator cannot implement the welfare optimum with imperfect competition. However, when $P' = 0$ and under perfect competition, (11) turns into (13). Now the regulator can implement the welfare optimum by setting the emission tax rate $t = D_E$. With imperfect competition, setting $t = D_E$ does not implement the welfare optimum, because the firms would produce too little. The regulator would need an additional policy instrument to implement the welfare optimum in this case.

If there is no environmental damage ($\beta = 0$ so that $D_E = 0$), then from (37) and (38), $C_\varepsilon = 0$ so that $\varepsilon = 1$ and the welfare-maximizing output level is $\bar{q}_w$, implicitly defined by:

$$P(n\bar{q}_w) - C_q(\bar{q}_w, 1) = 0 \tag{39}$$

The second order condition for welfare maximization is that the matrix

$$\mathbf{W}_{xx} \equiv \begin{pmatrix} W_{qq} & W_{q\varepsilon} \\ W_{q\varepsilon} & W_{\varepsilon\varepsilon} \end{pmatrix} = n \begin{pmatrix} nP' - C_{qq} - n\varepsilon^2 D_{EE} & -C_{q\varepsilon} - D_E - n\varepsilon q D_{EE} \\ -C_{q\varepsilon} - D_E - n\varepsilon q D_{EE} & -C_{\varepsilon\varepsilon} - nq^2 D_{EE} \end{pmatrix} \tag{40}$$

is negative semidefinite. We shall make the slightly stronger assumption that $\mathbf{W}_{xx}$ is negative definite. This implies that $\mathbf{h}\mathbf{W}_{xx}\mathbf{h}' < 0$ for all vectors $\mathbf{h}$ and the determinant is positive, so that:

$$\begin{aligned} \Delta_w &\equiv \frac{W_{qq}W_{\varepsilon\varepsilon} - W_{q\varepsilon}^2}{n^2} \\ &= -\left[nP' - C_{qq} - n\varepsilon^2 D_{EE}\right]\left[C_{\varepsilon\varepsilon} + nq^2 D_{EE}\right] - \left(C_{q\varepsilon} + D_E + n\varepsilon q D_{EE}\right)^2 > 0 \end{aligned} \tag{41}$$

Totally differentiating (37) and (38) with respect to β yields:

$$W_{qq}\frac{dq}{d\beta} + W_{q\varepsilon}\frac{d\varepsilon}{d\beta} = n\varepsilon D_{\beta E} \tag{42}$$

$$W_{\varepsilon\varepsilon}\frac{d\varepsilon}{d\beta} + W_{q\varepsilon}\frac{dq}{d\beta} = nqD_{\beta E} \tag{43}$$

with the second derivatives of W given by (40). Solving (42) and (43) using (40) and (41) yields:

$$\frac{dq}{d\beta} = \frac{nD_{\beta E}\left[\varepsilon W_{\varepsilon\varepsilon} - qW_{q\varepsilon}\right]}{W_{qq}W_{\varepsilon\varepsilon} - W_{q\varepsilon}^2} = \frac{D_{\beta E}}{\Delta_w}\left[q\left(C_{q\varepsilon} + D_E\right) - \varepsilon C_{\varepsilon\varepsilon}\right] \tag{44}$$

$$\frac{d\varepsilon}{d\beta} = \frac{nD_{\beta E}\left[qW_{qq} - \varepsilon W_{q\varepsilon}\right]}{W_{qq}W_{\varepsilon\varepsilon} - W_{q\varepsilon}^2} = \frac{D_{\beta E}}{\Delta_w}\left[q\left(nP' - C_{qq}\right) + \varepsilon\left(C_{q\varepsilon} + D_E\right)\right] \tag{45}$$

Note that $D_{\beta E} > 0$, and $\Delta_w > 0$ by (41). Thus the sign of (44) and (45) as well as of (46) and (47) below is the sign of the term in square brackets. We cannot sign (44) and (45) unambiguously, but we can use them to show that emissions are decreasing in β:

$$\frac{dE}{d\beta} = n\varepsilon\frac{dq}{d\beta} + nq\frac{d\varepsilon}{d\beta} = \frac{D_{\beta E}}{\Delta_w}\left[q^2W_{qq} - 2\varepsilon qW_{q\varepsilon} + \varepsilon^2W_{\varepsilon\varepsilon}\right] < 0 \tag{46}$$

The term in square brackets is negative from setting $\mathbf{h} = \begin{pmatrix} q & -\varepsilon \end{pmatrix}$ in $\mathbf{hW}_{xx}\mathbf{h}' < 0$ which holds by negative definiteness of $\mathbf{W}_{xx}$ in (40).

We can also show that marginal environmental damage $MD \equiv D_E$ is increasing in β:

$$\frac{dD_E}{d\beta} = D_{\beta E} + D_{EE}\frac{dE}{d\beta} = \frac{D_{\beta E}}{\Delta_w}\left[\left\{C_{\varepsilon\varepsilon}C_{qq} - \left(C_{q\varepsilon} + D_E\right)^2\right\} - nP'C_{qq}\right] > 0 \tag{47}$$

The second equality follows from (41) and (46). The term in square brackets is positive, because $P' \leq 0$, $C_{qq} > 0$ and the term in curly brackets is positive. The latter term is second order condition (17) for perfect competition which implements the welfare optimum with $t = D_E$.

Returning to (44) and (45) and setting environmental damage very low (D_E close to zero), the terms in square brackets on the RHS are respectively:

$$-\varepsilon C_{\varepsilon\varepsilon} + qC_{q\varepsilon} < 0$$

$$q\left(nP' - C_{qq}\right) + \varepsilon C_{q\varepsilon} < 0$$

Thus with low environmental damage, both q and ε decrease as the damage becomes more serious.

When environmental damage is very high, emissions are very low: $E = n\varepsilon q \to 0$. When ε is close to zero, the term in square brackets on the RHS of (44) is $q\left(C_{q\varepsilon} + D_E\right)$ by (4). We cannot sign this for the general case $C_{q\varepsilon} \leq 0$, but for integrated technology ($C_{q\varepsilon} = 0$) this is positive, so that output is increasing in the severity of environmental damage as ε falls to a very low level. Indeed, from (37) with $\varepsilon \to 0$, q approaches the output level $\bar{q}_w$ without environmental damage, as defined by (39).

When q is close to zero, the term in square brackets on the RHS of (45) is $\varepsilon\left(C_{q\varepsilon}+D_E\right)$ by (5). Again, we cannot sign this for the general case $C_{q\varepsilon} \leq 0$, but for integrated technology ($C_{q\varepsilon}=0$) this is positive, so that the emissions-to-output ratio is increasing in the severity of environmental damage as output falls to a very low level. Indeed, from (38) with $q \to 0$, ε approaches unity again: the firm does not spend anything on reducing its emission intensity.

As with emission taxation, we will conduct further analysis in Section 5 for a specific integrated technology to establish the behavior of these two variables for any level of environmental damage, and the conditions under which either is non-monotonic.

Finally, let us solve for the social optimum when the environmental technology is end-of-pipe (7). The first order conditions for welfare maximization are, from (23):

$$\begin{aligned} W_q &= n\left[P-k'-D_E\right]=0 \\ W_r &= n\left[D_E-V'\right]=0 \end{aligned}$$

Totally differentiating with respect to β yields:

$$W_{qq}\frac{dq}{d\beta}+W_{qr}\frac{dr}{d\beta} = nD_{\beta E} \tag{48}$$

$$W_{rr}\frac{dr}{d\beta}+W_{qr}\frac{dq}{d\beta} = -nD_{\beta E} \tag{49}$$

with:

$$W_{qq} = n\left[nP'-k''-nD_{EE}\right]<0 \tag{50}$$

$$W_{rr} = -n\left[nD_{EE}+V''\right]<0 \tag{51}$$

$$W_{qr} = n^2 D_{EE}>0 \tag{52}$$

Solving (48) and (49) yields, using (50) to (52):

$$\frac{dq}{d\beta} = \frac{nd'\left[W_{rr}+W_{qr}\right]}{W_{qq}W_{rr}-W_{qr}^2} = \frac{-D_{\beta E}V''}{\left[nP'-k''-nD_{EE}\right]\left[nD_{EE}+V''\right]-\left[nD_{EE}\right]^2}<0 \tag{53}$$

$$\frac{dr}{d\beta} = \frac{-nd'\left[W_{qq}+W_{qr}\right]}{W_{qq}W_{rr}-W_{qr}^2} = \frac{D_{\beta E}\left[k''-nP'\right]}{\left[nP'-k''-nD_{EE}\right]\left[nD_{EE}+V''\right]-\left[nD_{EE}\right]^2}>0 \tag{54}$$

The denominator on the RHS of both expressions is positive, because this is a SOC for welfare maximization, analogous to (41).

For the emissions-to-output ratio $\varepsilon = 1 - \frac{r}{q}$, we find:

$$\frac{d\varepsilon}{d\beta} = \frac{1}{q^2}\left(r\frac{dq}{d\beta} - q\frac{dr}{d\beta}\right) < 0$$

The inequality follows from (53) and (54). Thus with end-of-pipe technology, both q and ε are monotonically decreasing in β.

Summarizing our findings, we have:

Proposition 2 *1. Emission taxation implements the welfare optimum if $P' = 0$ or with perfect competition.*

In the welfare optimum:

2. *Emissions E are decreasing and marginal damage D_E is increasing in the environmental damage parameter β;*

3. *When β is very low, both output q and emission intensity ε are decreasing in β;*

4. *With integrated technology, when β is very high so that E is close to zero:*

   (a) When ε is close to zero, q is increasing in β.

   (b) When q is close to zero, ε is increasing in β.

5. *With end-of-pipe technology, q is decreasing and emission reduction r is increasing in β, so that ε is decreasing in β.*

5 Integrated technology: Example

While we have obtained definite answers for the effect of stricter environmental policy on output and emission intensity for the end-of-pipe technology (Propositions 1.5 and 2.5), we can only derive partial results for the integrated technology. For any technology, we know that when policy is very lenient, both output q and emission intensity ε are decreasing in the strictness of environmental policy (Propositions 1.3 and 2.3). With integrated technology and very strict policy, q is increasing in strictness if ε is low, and ε is increasing in strictness if q is low (Propositions 1.4 and 2.4). In order to find out which of these two scenarios will occur, and what happens for intermediate levels of strictness, we have to place more restrictions on the cost function. In this section, we will assume that each firm i's cost function is given by the following specification of the general integrated-technology cost function which satisifies all the restrictions imposed upon (6):

$$C(q_i, \varepsilon_i) = \frac{c}{2}q_i^2 + \frac{\gamma}{2}(1-\varepsilon_i)^2, \qquad c, \gamma > 0 \tag{55}$$

We do not need to specify the damage function $D(\beta, E)$. As long as the damage function satisfies all the conditions that we have imposed upon it (directly in Section 2 and as a part of the welfare function in Sections 3 and 4), we can apply Propositions 1.1 and 2.2 stating that the emission tax rate t and marginal damage $MD \equiv D_E$ respectively are increasing in the environmental damage parameter β. This means we only need to analyze q and ε as functions of t or MD, because the derivatives of q and ε with respect to t and MD have the same sign as their derivatives with respect to β.

We shall investigate the outcome with a constant product price (where emission taxation implements the welfare optimum by Proposition 2.1) in subsection 5.1. We then move on to taxation (subsection 5.2) and the welfare optimum (subsection 5.3) when the product price is decreasing in total production Q according to:

$$P = a - Q \tag{56}$$

This function satisfies conditions (1), (2) and (5), the latter in combination with cost function (55). The outcomes of these three scenarios are very similar. We provide the general solution to all three scenarios, along with the formal proof, in Appendix A.

5.1 Constant product price: Taxation and welfare optimum

We start with the case p where the product price P is constant, i.e. demand is perfectly elastic. As we know from Proposition 2.1, the regulator can implement the welfare optimum in this case by setting the tax rate t equal to marginal damage.

While the constant P scenario may seem rather unrealistic, it is worth analyzing for the following reasons. First, it is the easiest to analyze, because there are no interactions between firms through the output market. Secondly, it maintains the standard assumption in the literature[13] that a firm's MAC curve only depends on its own technology choice. The only way to maintain this assumption with integrated technology is to assume that P is constant, as Amir et al. (2010), Baumann et al. (2008) and Bréchet and Jouvet (2008) have done explicitly. Finally, this scenario serves to highlight the crucial difference between our model and Perino and Requate (2012). Perino and Requate (2012) do not model the output market explicitly, but assume that a firm's MAC only depends on its own technology choice. Thus they implicitly assume constant P. They find that emission intensity is always U-shaped in the strictness of environmental policy. In our model with constant P, emission intensity can be decreasing throughout in strictness. This difference in outcome is therefore not due to our explicit modelling of the output market, but because we assume a continuous choice of technology while Perino and Requate (2012) assume a discrete choice.[14]

Since there are no interactions between firms, we can focus on the behaviour of a single firm that sets output q and emission intensity ε facing the constant product price P and an emission tax rate t. Its emissions are given by (3) and its cost function by (55). The firm maximizes its profits Π, consisting of operating profits minus the tax bill:

$$\max\ \pi(q,\varepsilon) - te = Pq - \frac{c}{2}q^2 - \frac{\gamma}{2}(1-\varepsilon)^2 - t\varepsilon q \tag{57}$$

The first order conditions are, with respect to q and ε respectively:

$$P - cq - t\varepsilon = 0 \tag{58}$$

$$\gamma(1-\varepsilon) - tq = 0 \tag{59}$$

Solving for q and ε yields:[15]

$$q = \frac{\gamma(P-t)}{c\gamma - t^2}, \qquad \varepsilon = \frac{c\gamma - Pt}{c\gamma - t^2}, \qquad e = \frac{\gamma\,(P-t)\,(c\gamma - Pt)}{\left(c\gamma - t^2\right)^2} \tag{60}$$

[13] For instance Downing and White (1986), Jung et al. (1996), Milliman and Prince (1989), Requate and Unold (2003), among many others.

[14] A comparison with Bréchet and Meunier (2012) further serves to make this point. Bréchet and Meunier (2012) model the output market explicitly (as we do), but assume a discrete choice of technology (like Perino and Requate, 2012). They find, like Perino and Requate (2012), that emission intensity is always U-shaped in strictness.

[15] Substuting (A2) into (A7) in Appendix A shows that the numerators in (60) are positive.

When $t = 0$, the firm's output, emissions and operating profits are given by:

$$q = \bar{q}_p \equiv \frac{P}{c}, \qquad e = \bar{q}_p, \qquad \pi = \bar{\pi}_p \equiv \frac{P^2}{2c} \tag{61}$$

Applying Lemma 1 and Proposition 3 from Appendix A, we see that when $\gamma < P^2/c$, emission intensity is monotonically decreasing in t and output is U-shaped in t, with the turning point at $\varepsilon = \frac{1}{2}$. When output is very clean (to be precise: when the emissions-to-output ratio is below half the no-regulation level), it can increase again with the tax rate while becoming ever cleaner.

If $\gamma > P^2/c$, output is monotonically decreasing in t and emission intensity is U-shaped in t, with the turning point at

$$\tilde{q}_p \equiv \frac{\bar{q}_p}{2} = \frac{P}{2c} \tag{62}$$

Thus ε decreases until the point where output is so low that it is no longer worthwhile to invest in cleaner production. From (62), this point is where output is at half its no-regulation level of $\bar{q}_p$.[16]

Intuitively, when $\gamma < P^2/c$, production and abatement costs are low relative to the product price. As t keeps increasing, the firm is keen to take advantage of its low emission intensity to let output increase again. This means that the firm has to keep reducing its emission intensity as t rises, but it is happy to do so as abatement is relatively cheap. When $\gamma > P^2/c$, production and abatement costs are relatively high compared to the product price. Then the firm does not want to produce too much or spend too much on abatement. Thus as t keeps increasing, the firm keeps decreasing its output. When output is getting very low, the firm can increase its emission intensity again, reducing its abatement cost as well.

Let us now interpret this result in terms of Marginal Abatement Costs (MAC). We follow the approach by Amir et al. (2010), Baumann et al. (2008) and Bréchet and Jouvet (2008) who build upon McKitrick's (1999) definition of the MAC function keeping the emission intensity (ε in our model) constant. Substituting (3) into (57), operating profits can be written as a function of emissions and emission intensity:

$$\pi(e, \varepsilon) = \frac{Pe}{\varepsilon} - \frac{c}{2}\left(\frac{e}{\varepsilon}\right)^2 - \frac{\gamma}{2}(1-\varepsilon)^2$$

[16] Due to space constraints, we omit discussion of the knife-edge case (here: $\gamma = P^2/c$) here and in the following two subsections. We briefly discuss the knife-edge case in Appendix A.

Marginal abatement costs, defined for a given level of ε, are then:

$$MAC(e,\varepsilon) \equiv \frac{\partial \pi(e,\varepsilon)}{\partial e} = \frac{P}{\varepsilon} - \frac{ce}{\varepsilon^2} \tag{63}$$

The firm sets $MAC = t$. When $t = 0$, the firm sets $MAC = 0$, so that $q = \bar{q}_p$ and $\pi = \bar{\pi}_p - \frac{\gamma}{2}(1-\varepsilon)^2$, with $\bar{q}_p$ and $\bar{\pi}_p$ given by (61). Unless $\varepsilon = 0$, $e = 0$ can only be achieved by setting $q = 0$ which implies $\pi = -\frac{\gamma}{2}(1-\varepsilon)^2$. We can now determine the effect of decreasing ε on the MAC curve. A decrease in ε shifts the horizontal intercept $\bar{e}_p = \varepsilon\bar{q}_p$ where $MAC = 0$ to the left. The area under the MAC curve must remain the same, because it is the difference $\bar{\pi}_p$ in profits between $MAC = 0$ and $e = q = 0$.[17] This means that the vertical intercept $MAC(0)$ has to move up according to:

$$MAC(0,\varepsilon) = \frac{2\bar{\pi}_p}{\bar{e}_p} = \frac{P}{\varepsilon}$$

Figure 1 shows MAC curves for different levels of ε when $P = c = 1$.[18] The lower is ε, the further to the left is the point where $MAC = 0$, the higher is $MAC(0)$ and the steeper is the curve. This means that any MAC curve for a given ε value intersects every other MAC curve.

Note the big difference in the effect of cleaner technology on the MAC curve between end-of-pipe and integrated technology. As we have seen in subsection 3.1, cleaner end-of-pipe technology shifts the whole MAC curve inward in parallel fashion without changing its slope. With an integrated technology, cleaner technology not only changes the slope of the MAC curve, but actually raises MAC for low emission levels.

Intuitively, a decrease in emission intensity ε has two effects on marginal abatement costs MAC as a function of emissions e. First, a lower ε means that output q has to be reduced further to achieve a given emission reduction. This effect raises MAC and is dominant for low levels of e. Secondly, a lower ε means that a given level of e is achieved with a higher q. With increasing marginal production costs, the profit margin on the last unit of output, which has to be given up in order to reduce e, is lower when ε is lower and q is higher. This second effect reduces MAC and is dominant for high levels of e.

[17] Perino and Requate (2012) do not impose the constraint that the area under the MAC curve is the same for the two technologies that they consider. However, this constraint follows necessarily from our definition of the integrated technology.

[18] It only shows the MAC curves for $MAC \leq P = 1$, since by Proposition 3 and (A2), $e = 0$ is achieved for $MAC = t = \gamma c/P = \gamma$ when $\gamma < P^2/c = 1$ and for $MAC = t = P = 1$ when $\gamma > P^2/c = 1$.

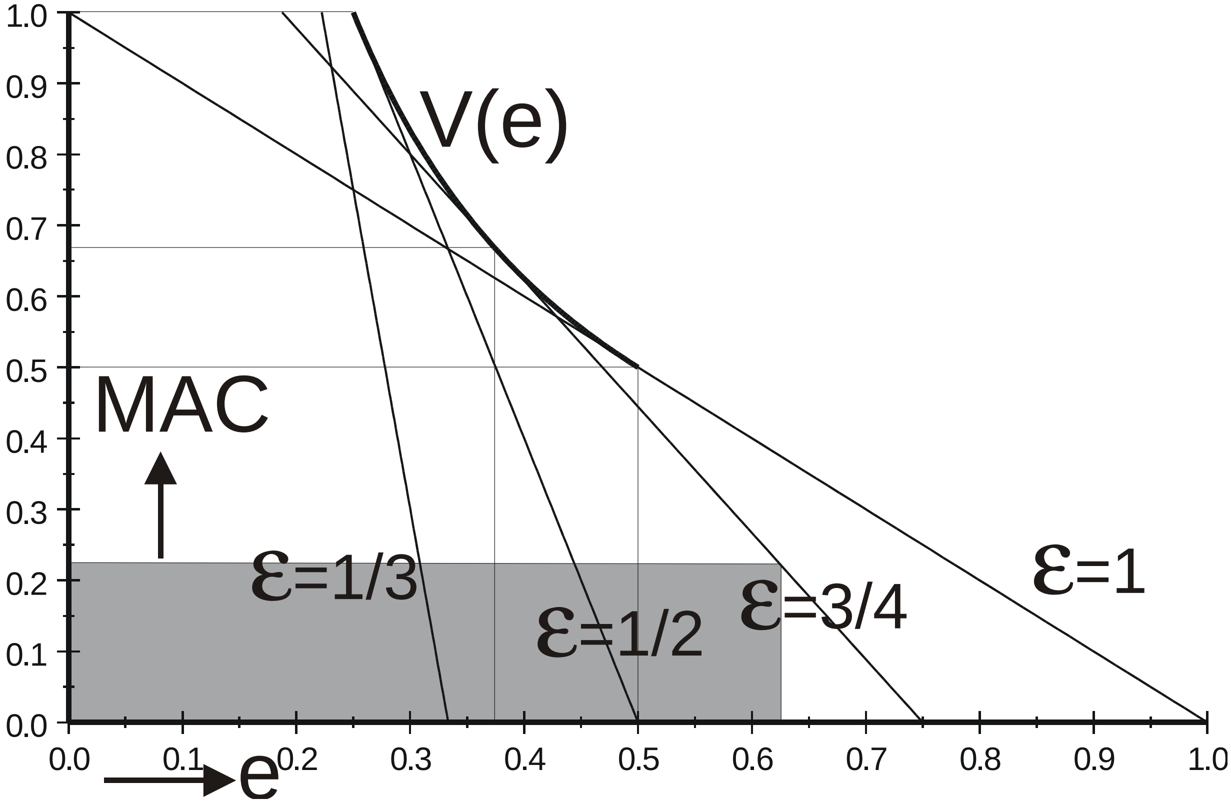

Figure 1: Marginal Abatement Cost (MAC) curves for different values of emission intensity ε ($P = c = 1$).

When ε falls marginally, the MAC curve pivots clockwise around its middle point, so that the area underneath remains constant at $\bar{\pi}_p$.[19] Since $MAC = 0$ at $e = \bar{e}_p$, the pivot point is at:

$$e = \tilde{e}_p \equiv \frac{P\varepsilon}{2c} = \frac{1}{2}\bar{e}_p = \varepsilon\tilde{q}_p \tag{64}$$

The pivot point is thus at $q = \tilde{q}_p$, defined in (62) as the output level where ε reaches the bottom of its U-shaped curve. Substituting (64) back into (63) to eliminate ε, we can find the curve that connects all these pivot points, which is the envelope curve $V(e)$ that gives the maximum value of MAC for a given level of e:

$$V(e) = \frac{P^2}{4ce} \tag{65}$$

Figure 1 shows the envelope curve $V(e)$ for $P = c = 1$.

[19]Formally, the pivot point is found by setting $\partial MAC/\partial\varepsilon = 0$ in (63).

Let us now examine with the aid of Figure 1 why ε reaches the bottom of its U-shaped curve at the pivot point of the MAC curve. When t rises from $t = 0$, the firm starts reducing e and ε, crossing successive MAC curves on its way. Since the total tax payment is $t\varepsilon q$, the marginal benefit to the firm of decreasing ε is tq. Equation (59) shows that the higher is tq, the higher the firm will set the marginal cost of reducing ε and thus the further it will reduce ε. Figure 1 illustrates how tq changes at the point where the firm crosses the MAC curve for $\varepsilon = \frac{3}{4}$. Substituting $P = c = 1$ and $\varepsilon = \frac{3}{4}$ into (63), this curve is given by:

$$MAC\left(e, \frac{3}{4}\right) = \frac{4}{3} - \frac{9}{16}e$$

The point $(e, MAC) = \left(\frac{5}{8}, \frac{2}{9}\right)$ on the curve features $q = \frac{5}{6}$, which by (59) is the solution for $\gamma = \frac{20}{27}$. Then te is the shaded rectangle under the curve with area $\frac{5}{36}$. Given $\varepsilon = \frac{3}{4}$, this area is proportional to the marginal benefits $tq = \frac{10}{36}$ of reducing ε. If the firm responded to an increase in t by reducing e while keeping $\varepsilon = \frac{3}{4}$, the rectangle under the $\varepsilon = \frac{3}{4}$ curve would grow larger. Thus te and the marginal benefit tq of reducing ε would rise. This means that the firm will respond by reducing ε below $\frac{3}{4}$. Looking at all the points where the firm crosses the $\varepsilon = \frac{3}{4}$ curve as t rises from 0 to $\frac{2}{3}$, we see that the associated γ also rises and the rectangle under the $\varepsilon = \frac{3}{4}$ curve keeps increasing, pushing the firm to reduce ε further. At $t = \frac{2}{3}$, halfway up the $\varepsilon = \frac{3}{4}$ curve, the rectangle under the curve is at its maximum size. At this point, where $q = \tilde{q}_p = \frac{1}{2}$ and $e = \frac{3}{8}$, which by (59) will happen for $\gamma = \frac{4}{3}$, the firm will not change ε when t rises. For $t > \frac{2}{3}$, the rectangle under the $\varepsilon = \frac{3}{4}$ curve decreases as t rises (with γ also decreasing), prompting the firm to raise ε again. Thus $\varepsilon = \frac{3}{4}$ is the minimum emission intensity for $q = \tilde{q}_p$. As we have seen above, this is also the pivot point for the $\varepsilon = \frac{3}{4}$ curve.[20]

Figure 2 shows emissions e in (60) as a function of the tax rate for $P = c = 1$ and different values of γ (note that the axes are interchanged compared to Figure 1). When $\gamma > \frac{P^2}{c} = 1$, q is monotonically decreasing in t and as we have just illustrated, ε is U-shaped in t, reaching its minimum at $q = \tilde{q}_p$ given by (62). In Figure 2, the point where ε reaches its minimum is where the emissions curve touches the $q = \tilde{q}_p$ curve. This curve

[20] We can use an analogous method to illustrate why the lowest output level occurs at $\varepsilon = \frac{1}{2}$. This involves defining $MAC_q \equiv \partial\pi(e, q)/\partial e = \frac{\gamma}{q}(1 - \frac{e}{q})$, fixing γ and drawing the linear MAC_q curves as a function of e for different values of q.

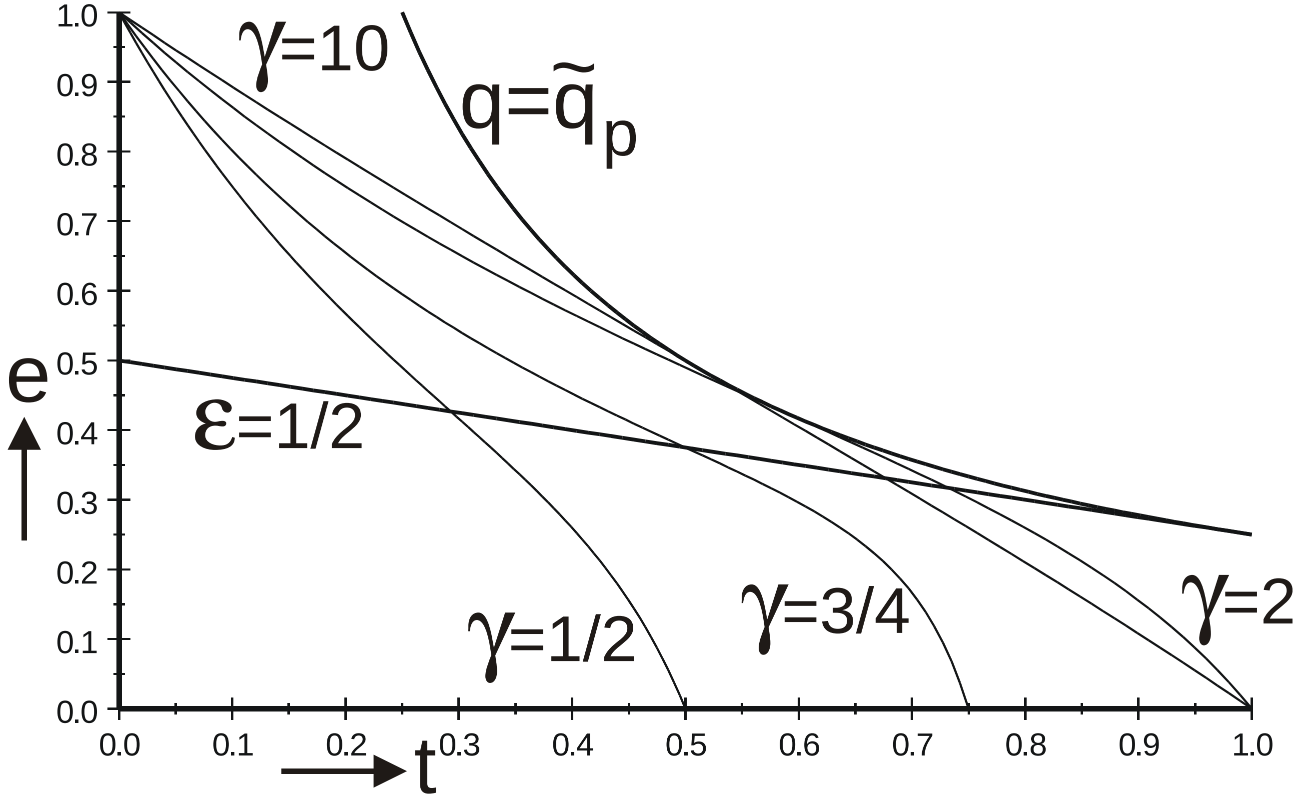

Figure 2: Emissions as a function of the tax rate for different values of γ ($P = c = 1$).

is the inverse of the $V(e)$ curve in (65) and Figure 1.

When $\gamma < \frac{P^2}{c} = 1$, ε is monotonically decreasing in t and q is U-shaped in t, reaching its minimum at $\varepsilon = \frac{1}{2}$. Solving $\varepsilon = \frac{1}{2}$ for γ and substituting this into the expression for e in (60), we find that the point where q reaches its minimum is given by $e = (2P - t)/4c$. This is the curve "$\varepsilon = \frac{1}{2}$" in Figure 2. The emission curves for $\gamma < P^2/c = 1$ feature decreasing q above the $\varepsilon = \frac{1}{2}$ curve and increasing q below it.[21]

5.2 Decreasing price: Taxation

In the taxation scenario, denoted by subscript t, there are n firms facing inverse demand function (56). Substituting (55) and (56) into (9), profits can be written as:

$$\Pi_i = \pi(q_i, \varepsilon_i) - te_i = (a - Q)q_i - \frac{c}{2}q_i^2 - \frac{\gamma}{2}(1 - \varepsilon_i)^2 - t\varepsilon_i q_i \tag{66}$$

[21]The emission curves for $\gamma > 1$ also intersect the $\varepsilon = \frac{1}{2}$ curve, but this is irrelevant for $\gamma > 1$.

Firms choose q_i and ε_i simultaneously. The FOCs for maximization are, respectively:

$$a - (2+c)q_i - Q_{-i} - t\varepsilon_i = 0 \tag{67}$$

$$\gamma(1-\varepsilon_i) - tq_i = 0 \tag{68}$$

Substituting (55) and (56), second order condition (16) becomes:

$$\gamma(2+c) - t^2 > 0 \tag{69}$$

Solving (67) and (68), the symmetric equilibrium solutions are given by:[22]

$$q = \frac{\gamma(a-t)}{\gamma(1+c+n)-t^2}, \quad \varepsilon = \frac{\gamma(1+c+n)-ta}{\gamma(1+c+n)-t^2}, \quad E = \frac{n\gamma(a-t)\left[\gamma(1+c+n)-ta\right]}{\left[\gamma(1+c+n)-t^2\right]^2} \tag{70}$$

Without environmental policy, $t=0$ so that:

$$q = \bar{q}_t \equiv \frac{a}{1+c+n}, \qquad e = \bar{q}_t, \qquad \pi = \bar{\pi}_t \equiv \frac{a^2\,(2+c)}{2\,(c+n+1)^2} \tag{71}$$

Figure 3 illustrates the symmetric equilibrium, where (67) becomes:

$$a - \frac{(n+1)Q}{n} = \frac{cQ}{n} + t\varepsilon \tag{72}$$

On the LHS is each firm's equilibrium marginal revenue (MR^* in Figure 3) where all firms produce the same amount $q = Q/n$. On the RHS of (72) is the sum of the industry's aggregate marginal production costs cq ($AMPC$ in Figure 3) and the effective tax rate $\tau \equiv t\varepsilon$ on output. When $t=0$, (72) holds at point Z in Figure 3 so that each firm sets $q = \bar{q}_t$ as given by (71), total production is $\bar{Q}_t = n\bar{q}_t$ and the product price is $\bar{P}_t = P(\bar{Q}_t)$ by (56). Let us assume that for a given emission intensity level ε' (not shown in the figure), the regulator sets the emission tax rate at t' so that the effective tax rate on output is $\tau' \equiv t'\varepsilon'$ and the industry produces Q' as shown in Figure 3. Thus τ creates a wedge between MR^* and $AMPC$. As τ rises continuously from zero to a to reduce output per firm from $\bar{q}_t$ to zero, the continuum of wedges fills the whole area $OaZ = \frac{1}{2}a\bar{Q}_t$ between MR^* and $AMPC$.

Defining

$$\gamma_t \equiv \frac{a^2}{1+n+c} \tag{73}$$

[22]The numerators are positive by (69).

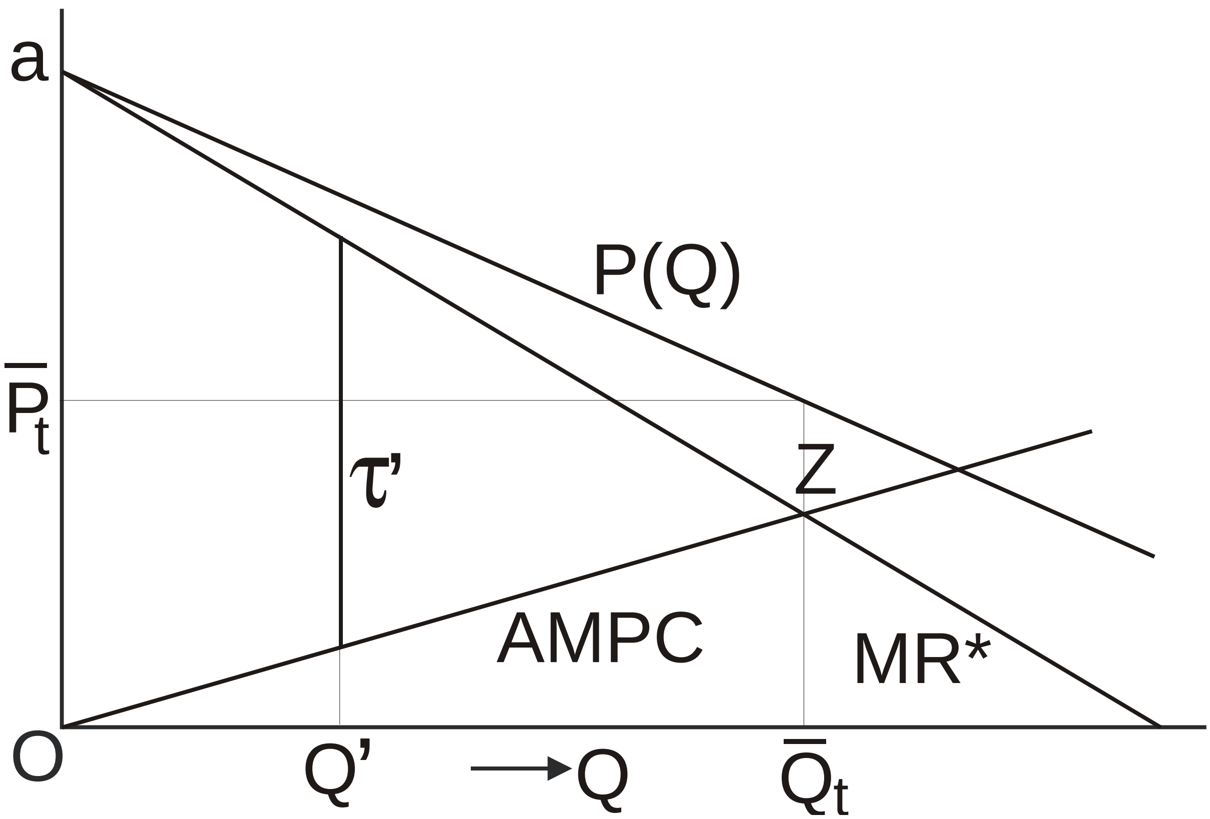

Figure 3: Symmetric market equilibrium with emission taxation

and applying Lemma 1 and Proposition 3 from Appendix A, we see that when $\gamma < \gamma_t$, emission intensity is monotonically decreasing in t and output is U-shaped in t, with the turning point at $\varepsilon = \frac{1}{2}$. When output is very clean (to be precise: when the emissions-to-output ratio is below half the no-regulation level), output can increase again with the tax rate in while output is becoming even cleaner.

If $\gamma > \gamma_t$, output is monotonically decreasing in t and emission intensity is U-shaped in t, with the turning point at

$$\tilde{q}_t \equiv \frac{a}{2(1+n+c)} = \frac{\bar{q}_t}{2} \tag{74}$$

Thus ε decreases until the point where output is so low that it is no longer worthwhile to invest in cleaner production. This occurs when output is at half its no-regulation level of $\bar{q}_t$, given by (71).

The intuition behind γ_t in (73) being the critical value of γ is as follows. When $\gamma < \gamma_t$, production costs are low. Abatement costs are relatively low as well, for two reasons.

First, the cost of reducing emission intensity to a certain level is low, because γ is low. Secondly, since the number n of firms is low, each firm has a relatively high production level. This raises the benefit of investing in a reduction of the emission intensity of output. As t keeps increasing, firms are keen to take advantage of their low emission intensity to let output increase again. This means that firms have to keep reducing their emission intensity as t rises, but they are happy to do so as abatement is relatively cheap. When $\gamma > \gamma_t$, production and abatement costs are high. Then firms do not want to produce too much or spend too much on abatement. Thus as t keeps increasing, firms keep decreasing their output. When output is getting very low, firms can increase their emission intensity again, reducing their abatement cost as well.

Let us now interpret these results in terms of marginal abatement cost (MAC) curves. Substituting (3) into (66), firm i's profits can be written as a function of emissions, emission intensity and the aggregate output Q_{-i} of all other firms:

$$\pi(e_i, \varepsilon_i, Q_{-i}) = \left(a - Q_{-i} - \frac{e_i}{\varepsilon_i}\right)\frac{e_i}{\varepsilon_i} - \frac{c}{2}\left(\frac{e_i}{\varepsilon_i}\right)^2 - \frac{\gamma}{2}(1-\varepsilon_i)^2$$

Firm i's marginal abatement costs, defined for a given level of ε_i, are then:

$$MAC(e_i, \varepsilon_i, Q_{-i}) \equiv \frac{\partial \pi(e_i, \varepsilon_i, Q_{-i})}{\partial e_i} = \left(a - Q_{-i} - 2\frac{e_i}{\varepsilon_i}\right)\frac{1}{\varepsilon_i} - \frac{ce_i}{\varepsilon_i^2} \tag{75}$$

Unlike in (63) previously with constant product price, firm i's MAC depends on the choice of q_j by all other firms $j \neq i$, and therefore indirectly on their choice of abatement technology $\varepsilon_j \equiv e_j/q_j$. This is also in contrast with firm i's MAC (28) under end-of-pipe technology, which does not depend on the abatement technology r_j chosen by the other firms.

Since this dependence of an individual firm's MAC on other firms' abatement decisions limits its usefulness, we will instead make use of the aggregate marginal abatement cost $AMAC$ for the whole industry in a symmetric equilibrium where $q_j = q$ and $\varepsilon_j = \varepsilon$ (and thus $e_j = e$ and $E = ne$) for all $j = 1, \cdots, n$. From (75):

$$AMAC(E, \varepsilon) \equiv nMAC\left(\frac{E}{n}, \varepsilon, \frac{(n-1)E}{n\varepsilon}\right) = \frac{a}{\varepsilon} - \frac{(1+n+c)E}{n\varepsilon^2} \tag{76}$$

When $t = 0$, each firm sets $AMAC = 0$, so that $Q = \bar{Q}_t \equiv n\bar{q}_t$ as defined by (71). Unless $\varepsilon = 0$, $e = 0$ can only be achieved by setting $Q = 0$. A decrease in ε shifts the

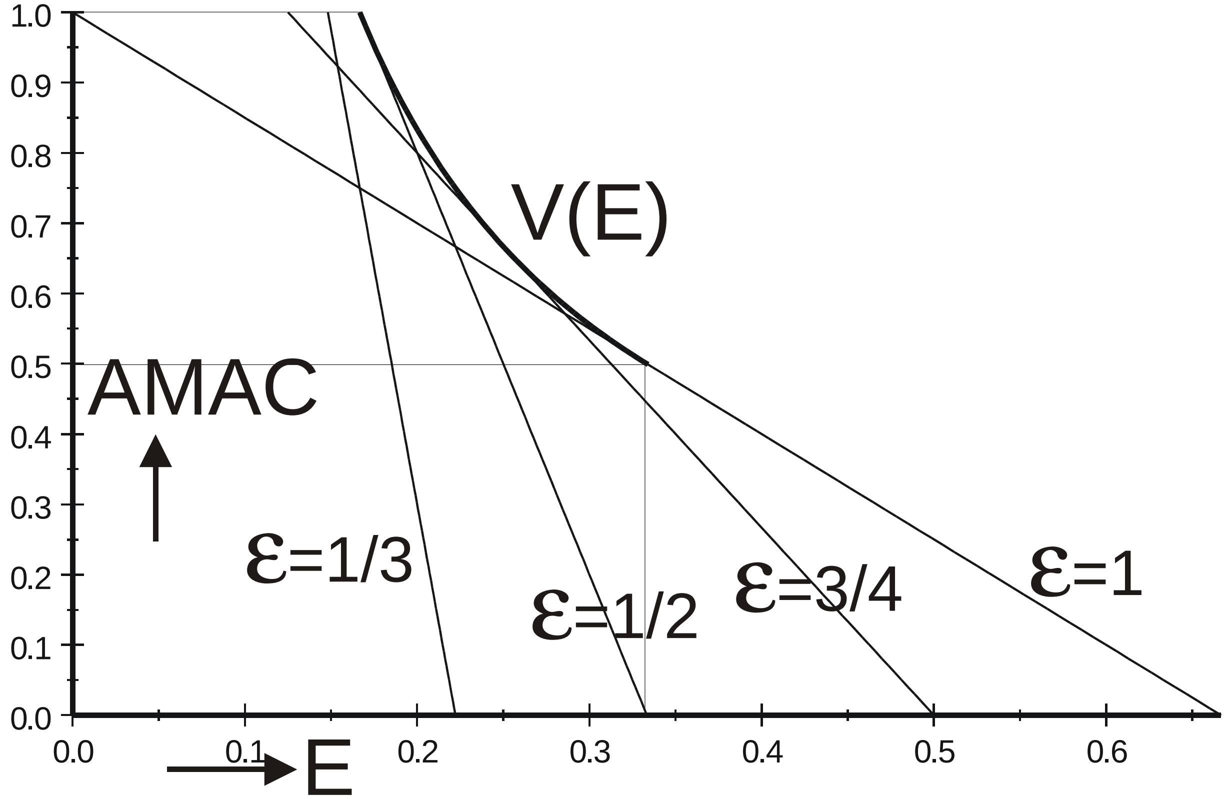

Figure 4: Aggregate Marginal Abatement Cost ($AMAC$) curves for different values of emission intensity ε ($a = c = 1,\ n = 4$).

point $\bar{E}_t \equiv \varepsilon\bar{Q}_t$ where $AMAC = 0$ to the left. The area under the $AMAC$ curve must remain the same, because it is the area $OaZ = \frac{1}{2}a\bar{Q}_t$ filled by the wedges of τ in Figure 3, as discussed above. This means that $AMAC(0, \varepsilon)$ has to move up according to:

$$AMAC(0, \varepsilon) = \frac{a}{\varepsilon}$$

Figure 4 shows $AMAC$ curves for different levels of ε when $P = c = 1,\ n = 4$, so that γ_t in (73) equals $\frac{1}{6}$.

When ε falls marginally, the $AMAC$ curve pivots clockwise around its middle point, so that the area underneath remains constant at $\frac{1}{2}a\bar{Q}_t$. Since $AMAC = 0$ at $E = \bar{E}_t$, the pivot point is at:

$$E = \tilde{E}_t \equiv \frac{na\varepsilon}{2(1+n+c)} = \frac{1}{2}\bar{E}_t = \varepsilon\tilde{Q}_t \tag{77}$$

The pivot point is thus where $Q = \tilde{Q}_t = n\tilde{q}_t$ as defined by (74). Substituting (77) back into (63) to eliminate ε, the curve that connects all these pivot points is the envelope

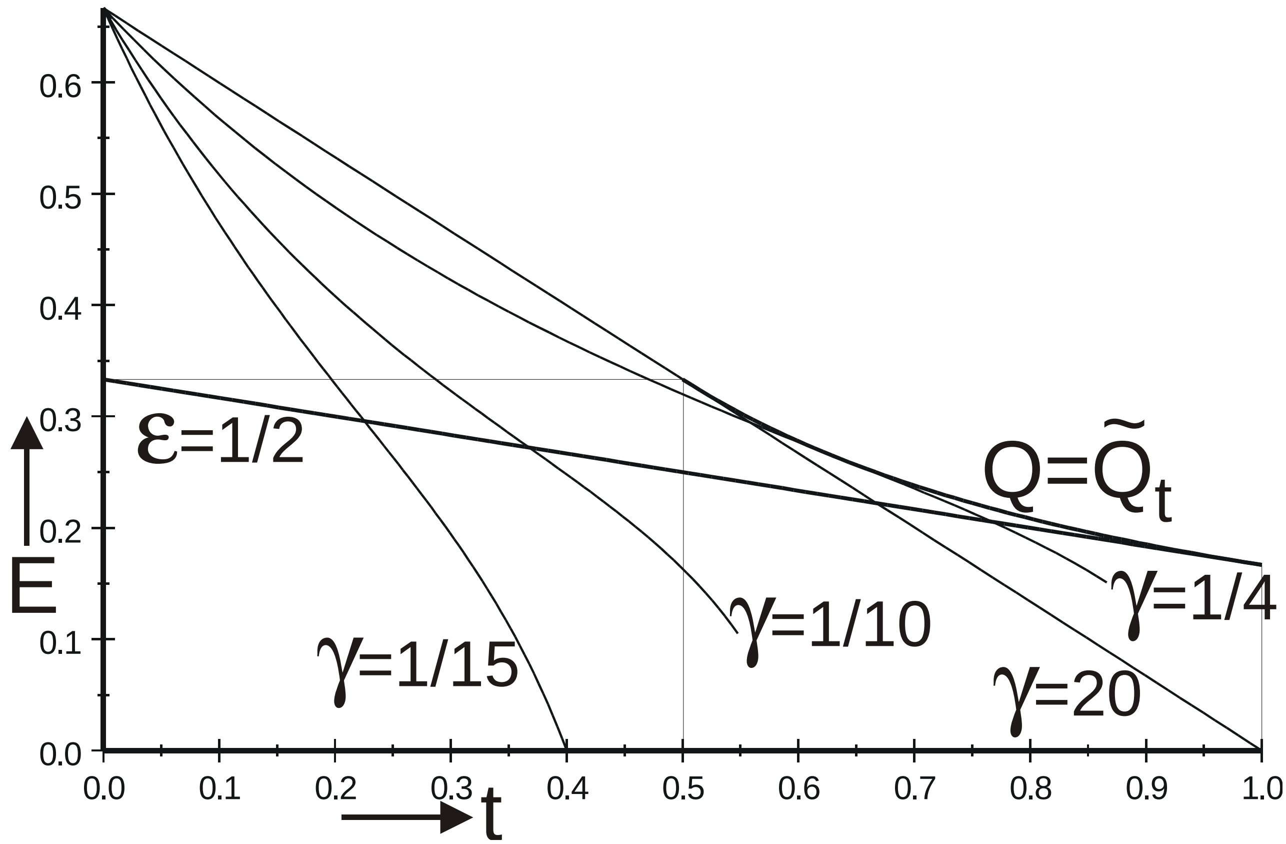

Figure 5: Emissions as a function of the tax rate for different γ values ($a = c = 1,\ n = 4$).

curve $V(E)$ that gives the maximum value of $AMAC$ for a given level of E:

$$V(E) = \frac{na^2}{4E(1+n+c)^2} \tag{78}$$

Figure 4 shows the envelope curve $V(E)$ for $P = c = 1,\ n = 4$.

Figure 5 shows total emissions E from (70) as a function of the tax rate for $P = c = 1,\ n = 4$ and different values of γ (with the axes interchanged compared to Figure 4). For the γ values of $\frac{1}{10}$ and $\frac{1}{4}$, second order condition (69) does not hold when the tax rate is very high, so that (70) results in negative profits. The curves for these γ values are only shown for t values where profits are positive. For the γ values of $\frac{1}{15}$ and 20, however, profits are positive throughout.

When $\gamma > \gamma_t = \frac{1}{6}$, q is monotonically decreasing in t and ε is U-shaped in t, reaching its minimum at $q = \tilde{q}_t$ given by (74). As we have seen above, this is where the $AMAC$

curves cross.[23] In Figure 5, the point where ε reaches its minimum is where the emissions curve touches the $Q = \tilde{Q}_t$ curve. The $Q = \tilde{Q}_t$ curve is the inverse of the $V(E)$ curve in (78) and Figure 4.

When $\gamma < \gamma_t = \frac{1}{6}$, ε is monotonically decreasing in t and q is U-shaped in t, reaching its minimum at $\varepsilon = \frac{1}{2}$. Solving $\varepsilon = \frac{1}{2}$ for γ and substituting this into the expression for E in (70), we find that the point where q reaches its minimum is given by:

$$E = \frac{n(2a-t)}{4(1+n+c)}$$

This is the curve "$\varepsilon = \frac{1}{2}$" in Figure 5. The emission curves for $\gamma < \gamma_t$ feature decreasing q above the $\varepsilon = \frac{1}{2}$ curve and increasing q below it.[24]

5.3 Decreasing price: Welfare optimum

In this subsection we investigate the welfare optimum w, where the regulator chooses both ε_i and q_i, $i = 1, \cdots, n$, to maximize social welfare. As firms are symmetric, we focus on the symmetric equilibrium of the game; that is, where $q_i = q$ and $\varepsilon_i = \varepsilon$ for all $i = 1, \cdots, n$. In symmetry, welfare (23) can be written as $W = nw$ where, from (55) and (56):

$$w = \left(a - \frac{n}{2}q\right) q - \frac{c}{2}q^2 - \frac{1}{2}\gamma(1-\varepsilon)^2 - D(\beta, E) \tag{79}$$

The first order conditions are:

$$\frac{\partial w}{\partial q} = a - nq - cq - \varepsilon MD = 0 \tag{80}$$

$$\frac{\partial w}{\partial \varepsilon} = \gamma(1-\varepsilon) - qMD = 0 \tag{81}$$

with marginal damage $MD \equiv D_E(\beta, E)$.

Without environmental damage ($\beta = 0$ so $D = MD = 0$), the welfare optimum is:

$$Q = \overline{Q}_w \equiv \frac{na}{n+c}, \qquad E = \overline{Q}_w, \qquad W = \overline{W} \equiv \frac{na^2}{2(n+c)} \tag{82}$$

Solving (80) for ε, Q and E as functions of MD yields:[25]

$$Q = \frac{n\gamma\,(a - MD)}{\gamma(n+c) - MD^2}, \qquad \varepsilon = \frac{\gamma(n+c) - aMD}{\gamma(n+c) - MD^2}, \qquad E = \frac{n\gamma\,[a - MD]\,[\gamma(n+c) - aMD]}{[\gamma(n+c) - MD^2]^2} \tag{83}$$

[23] The intuition behind this result is analogous to the explanation given in subsection 5.1.
[24] The curves for $\gamma > 1/6$ also intersect the $\varepsilon = \frac{1}{2}$ curve, but this is irrelevant for $\gamma > 1/6$.
[25] Substituting (A4) into (A7) in Appendix A shows that the numerators in (83) are positive.

Applying Lemma 1 and Proposition 3 from Appendix A, we see that when $\gamma < a^2/(n+c)$, emission intensity is monotonically decreasing in MD and output is U-shaped in MD, with the turning point at $\varepsilon = \frac{1}{2}$. Initially, output is declining in marginal damage. When output is very clean however (to be precise: when the emissions-to-output ratio is below half the no-regulation level), it can increase again with marginal damage while becoming even cleaner.

When $\gamma > a^2/(n+c)$, output is monotonically decreasing in MD and emission intensity is U-shaped in MD, with the turning point at

$$\tilde{Q}_w \equiv \frac{\overline{Q}_w}{2} = \frac{na}{2(n+c)} \tag{84}$$

Thus ε decreases until the point where output is so low that it is no longer worthwhile to invest in cleaner production. This point is where output is at half its no-regulation level of $\overline{Q}_w$.

The significance of the comparison between γ and $a^2/(n+c)$ can be explained as follows. When a is high, demand is high, so that the regulator does not want to reduce output by too much and is anxious to increase it again if possible. When γ and n are high, the cost of reducing emission intensity per firm γ and for all firms n is high. Then the regulator does not want to spend too much on reducing emission intensity and is happy to increase emission intensity again if possible. Finally when c is high, production is costly, again making emission reduction more efficient than increasing output from the social welfare point of view.

When interpreting this result in terms of marginal abatement costs, it is useful to define the social marginal abatement cost ($SMAC$) as a function of total emissions $E = ne$ divided equally among all firms. Substituting (3) into (79), welfare $W = nw$ can be written as a function of total emissions and emission intensity:

$$W(E, \varepsilon) = PB - \frac{n\gamma}{2}(1-\varepsilon)^2 - D(\beta, E) \tag{85}$$

with pollution benefits PB the difference between the utility and the production cost of output:

$$PB \equiv \left(a - \frac{E}{2\varepsilon}\right)\frac{E}{\varepsilon} - \frac{c}{2n}\left(\frac{E}{\varepsilon}\right)^2 \tag{86}$$

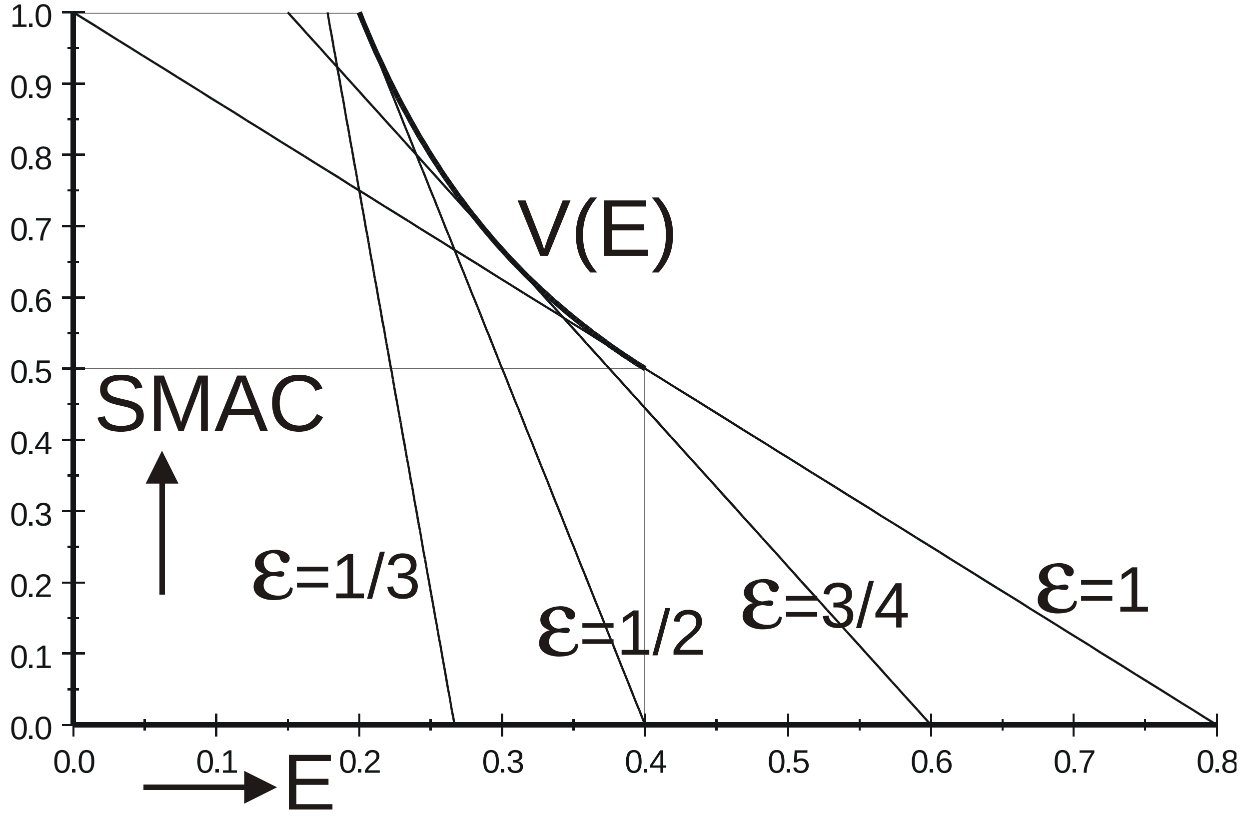

Figure 6: Social Marginal Abatement Cost (SMAC) curves for different values of emission intensity ε ($a = c = 1, n = 4$).

Maximizing (85) with respect to E shows that social marginal abatement cost[26] ($SMAC$) should equal marginal damage (MD):

$$SMAC \equiv \frac{dPB}{dE} = \frac{a}{\varepsilon} - \frac{(n+c)E}{n\varepsilon^2} = MD \tag{87}$$

When there is no environmental damage ($\beta = 0$ so that $MD = 0$), the welfare optimum has $SMAC = 0$ in (87), so that $Q = \overline{Q}_w$, $E = \overline{E}_w \equiv \varepsilon\overline{Q}_w$ and $PB = \overline{PB} \equiv \overline{W}$, with $\overline{Q}_w$ and $\overline{W}$ given by (82) and PB by (86). Unless $\varepsilon = 0$, $E = 0$ can only be achieved by setting $Q = 0$ which implies $PB = 0$. A decrease in ε shifts the point $\overline{E}_w \equiv \varepsilon\overline{Q}_w$ where $SMAC = 0$ to the left. The area under the $SMAC$ curve must remain the same, because it is the difference $\overline{PB}$ in pollution benefits between $SMAC = 0$ and $E = Q = 0$. This

[26] Whereas marginal abatement costs are usually defined in terms of a single firm's profits, our definition of social marginal abatement costs emcompasses all the firms' profits as well as the consumer surplus.

means that by (82), $SMAC(0)$ has to move up according to:

$$SMAC(0) = \frac{2\overline{W}}{\overline{E}_w} = \frac{a}{\varepsilon}$$

Figure 6 shows $SMAC$ curves for different levels of ε when $a = c = 1,\ n = 4$.

When ε falls marginally, the $SMAC$ curve pivots clockwise around its middle point, so that the area underneath remains constant at $\overline{PB}$. Since $SMAC = 0$ at $E = \bar{E}_w$, the pivot point is at:

$$E = \tilde{E}_w \equiv \frac{na\varepsilon}{2(c+n)} = \frac{1}{2}\bar{E}_w = \varepsilon\tilde{Q}_w \tag{88}$$

The pivot point is thus where $Q = \tilde{Q}_w$ as defined by (82). Substituting (88) back into (87) to eliminate ε, the curve that connects all these pivot points for different ε values is the envelope curve $V(E)$ that gives the maximum value of $SMAC$ for a given level of E:

$$V(E) = \frac{na^2}{4E\,(c+n)} \tag{89}$$

Figure 6 shows the envelope curve $V(E)$ for $a = c = 1,\ n = 4$.

Figure 7 shows emissions (83) as a function of marginal damage MD in the optimum for $a = c = 1, n = 4$ and different values of γ (note that the axes are interchanged compared to Figure 6). When $\gamma > a^2/(n+c) = 1/5$, Q is monotonically decreasing in MD and ε is U-shaped in MD, reaching its minimum at $Q = \tilde{Q}_w$ given by (62). As we have seen above, this is where the $SMAC$ curves cross.[27] In Figure 7, the point where ε reaches its minimum is where the emissions curve touches the $Q = \tilde{Q}_w$ curve. This curve is the inverse of the $V(E)$ curve in (89) and Figure 6.

When $\gamma < a^2/(n+c) = 1/5$, ε is monotonically decreasing in MD and Q is U-shaped in MD, reaching its minimum at $\varepsilon = \frac{1}{2}$. Solving $\varepsilon = \frac{1}{2}$ for γ and substituting this into the expression for E in (83), we find that the point where Q reaches its minimum is given by:

$$E = \frac{n\,(2a - MD)}{4(n+c)}$$

This is the curve "$\varepsilon = \frac{1}{2}$" in Figure 7. The emission curves for $\gamma < a^2/(n+c) = 1/5$ feature decreasing Q above the $\varepsilon = \frac{1}{2}$ curve and increasing Q below it.[28]

[27] The intuition behind this result is analogous to the explanation given in subsection 5.1.
[28] The curves for $\gamma > 1/5$ also intersect the $\varepsilon = \frac{1}{2}$ curve, but this is irrelevant for $\gamma > 1/5$.

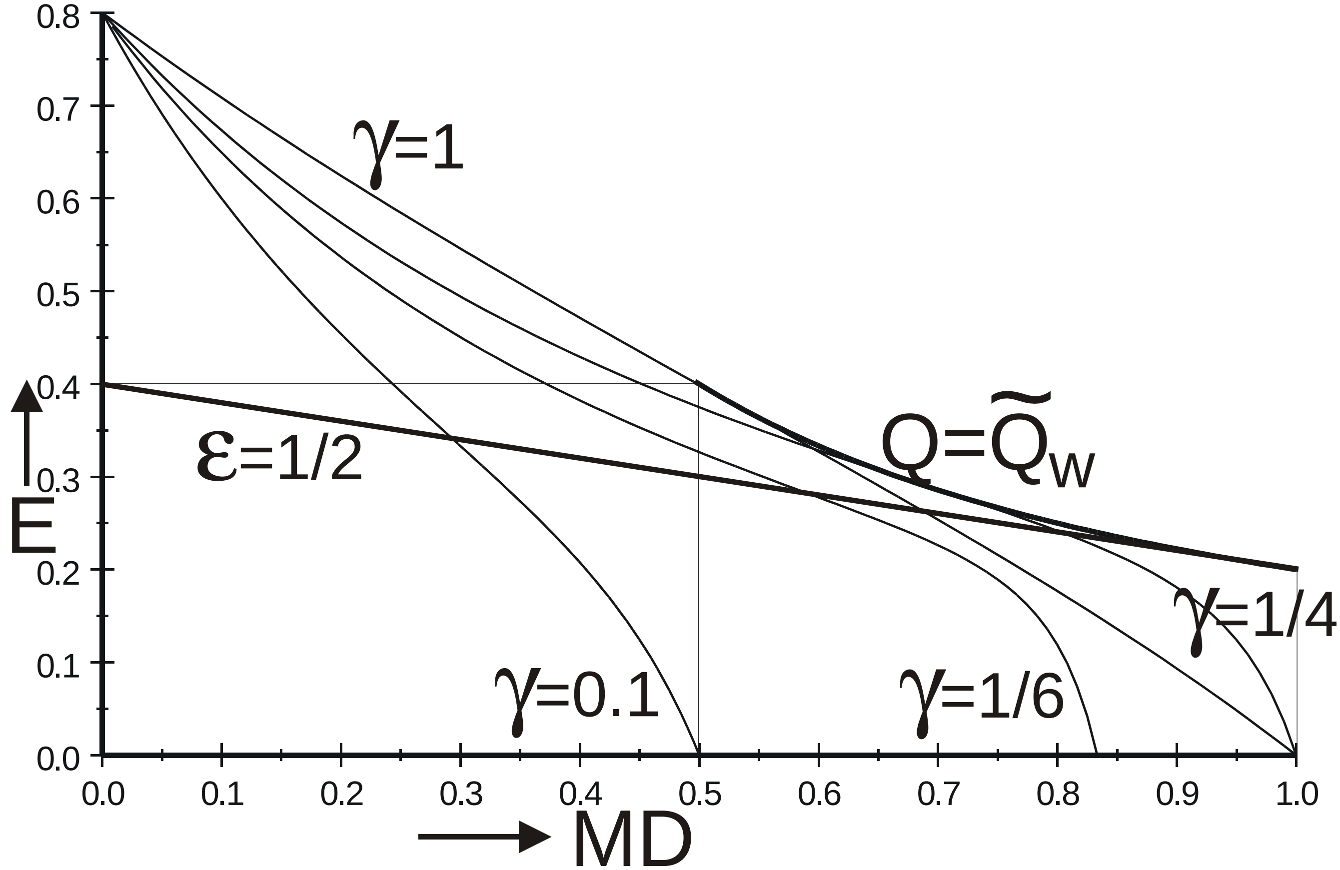

Figure 7: Industry emissions as a function of marginal damage in the welfare optimum for different values of γ ($a = c = 1, n = 4$).

6 Conclusion

Does an increasingly strict environmental policy spur on the polluting industry to invest more and more in finding cleaner ways to produce? The answer might seem obvious, but it is not once we take the output market into account. When a stricter environmental policy leads to a reduction in output, investment in reducing the emissions-to-output ratio becomes less profitable. Conversely, when a stricter environmental policy leads to very clean production methods, it may be possible to increase output again.

We find that with an integrated abatement technology (where a firm can invest in reducing the emission intensity to a certain level), either output or the emissions-to-output ratio is a U-shaped function of the environmental damage parameter. This happens with emission taxation as well as in the welfare optimum. Thus if we see polluting output increasing or production methods becoming less clean as environmental policy becomes stricter, this is not necessarily a sign that the policy is ineffective (or even counterproductive) or misguided.

This issue is linked with recent findings in the literature that cleaner technology can pivot the Marginal Abatement Cost (MAC) curve clockwise, with the new MAC curve intersecting the old one. We find that when emission intensity is U-shaped, the turning point occurs where the MAC curves cross.

When the product price is constant, the definition of MAC is relatively straightforward: It is a firm's decrease in profits from reducing emissions by reducing output. When the product price is decreasing in total output, however, a firm's profits and thus its MAC depend on the output and abatement decisions of the other firms. In this setting, we define the industry's Aggregate MAC for the case where all firms set the same emission intensity and output levels. For the welfare-maximizing outcome, the relevant concept is the Social MAC, which includes the changes in the industry's profits and in the consumer surplus.

Although it may be optimal for environmental policy, especially for greenhouse gases, to become ever stricter over time, it is likely that policy makers are unable to credibly commit to this. An alternative could be to stimulate environmental R&D, reducing the future cost of stricter environmental policy by reducing marginal abatement costs (Abrego and Perroni, 2002; Golombek et al., 2010). However, as we have seen, environmental R&D into integrated technologies does not reduce the MAC curve for all emission levels, but pivots it clockwise. Indeed, it is not clear if we need ever cleaner production methods if we want to reduce emissions further and further. This limits the usefulness of environmental R&D subsidies in overcoming the commitment problem.

Ulph and Ulph (2013) have recently found that an environmental R&D subsidy for an integrated technology can be useful in dealing with a different commitment problem. A government that faces uncertainty about the environmental preferences of a future government may want the firm to adopt a cleaner production technology. Applying the analysis of the present paper, we know that a cleaner production technology comes with a steeper MAC curve. Thus with the cleaner technology in place, the future government will implement an emission level that is closer to the current government's preferred level.

Our findings also have implications for empirical research. We find that the cleanliness of production is a far from perfect indicator of the strictness of the environmental policy. It may well be the case that stricter environmental policy will lead to less clean production. This has implications for empirical studies which have used the emission-to-output ratio as a proxy for the stringency of environmental policy. For instance, List and Co (2000) use the ratio of pollution abatement operating expenditures to value added as one of the measures of US state environmental regulation. Ederington et al. (2005) take the ratio of pollution abatement costs to total costs of materials as their measure of stringency of US (federal) environmental regulation.

In future empirical work, it would be interesting to examine whether abatement technology for a specific pollutant and industry can be described as an integrated technology. In this case, further investigation could reveal whether stricter environmental policy would lead, or perhaps has already led, to a U-shaped response in output or emission intensity. Emission intensity is more likely to be U-shaped if production and abatement costs are high, the number of firms is high and the size of the market is small.

References

[1] Abrego, L. and C. Perroni (2002), Investment subsidies and time-consistent environmental policy, Oxford Economic Papers 54: 617-635.

[2] Amigues, J-P., G. Lafforgue and M. Moreaux (2013), Optimal timing of CCS policies with heterogeneous energy consumption sectors, Environmental and Resource Economics (forthcoming).

[3] Amir, R., M. Germain and V. van Steenberghe (2008), On the impact of innovation on the Marginal Abatement Cost curve, Journal of Public Economic Theory 10: 985-1010.

[4] Baker, E., L. Clarke and E. Shittu (2008), Technical change and the marginal cost of abatement, Energy Economics 30: 2799-2816.

[5] Bauman, Y., M. Lee and K. Seeley (2008), Does technological innovation really reduce Marginal Abatement Costs? Some theory, algebraic evidence, and policy implications, Environmental and Resource Economics 40: 507-527.

[6] Boom, J-T. and B.R. Dijkstra (2009), Permit trading and credit trading: A comparison of cap-based and rate-based emissions trading under perfect and imperfect competition, Environmental and Resource Economics 44: 107-136.

[7] Bréchet, T. and P-A. Jouvet (2008), Environmental innovation and the cost of pollution abatement revisited, Ecological Economics 65: 262-265.

[8] Bréchet, T. and G. Meunier (2012), Are clean technology and environmental quality conflicting policy goals?, ECORE discussion paper 2012/20.

[9] Carpenter, A. (2012), CO2 abatement in the iron and steel industry, IEA Clean Coal Centre, Profiles, 12/1.

[10] Downing, P.B. and L.W. White (1986), Innovation in pollution control, Journal of Environmental Economics and Management 13: 18-29.

[11] Ederington, J., A. Levinson and J. Minier (2005), Footloose and pollution-free, Review of Economics and Statistics 87: 92-99.

[12] Endres, A. and T. Friehe (2011), Incentives to diffuse advanced abatement technology under environmental liability law, Journal of Environmental Economics and Management 62: 30-40.

[13] Endres, A. and T. Friehe (2013), The monopolistic polluter under environmental liability law: Incentives for abatement and R&D, Social Choice and Welfare 40: 753-770.

[14] Feess, E. and G. Muehlheusser (2002), Strategic environmental policy, clean technologies and the learning curve, Environmental and Resource Economics 23: 149-166.

[15] Gaudet, G. and S.W. Salant (1991), Uniqueness of Cournot equilibrium: New results from old methods, Review of Economic Studies 58: 399-404.

[16] Golombek, R., M. Greaker and M. Hoel (2010), Carbon taxes and innovation without commitment, BE Journal of Economic Analysis and Policy 10(1, Topics), Article 32.

[17] Herzog, H.H. (2011), Scaling up carbon dioxide capture and storage: From megatons to gigatons, Energy Economics 33: 597-604.

[18] Jung, C.H., K. Krutilla and R. Boyd (1996), Incentives for advanced pollution abatement technology at the industry level: An evaluation of policy alternatives, Journal of Environmental Economics and Management 30: 95-111.

[19] Katsoulacos, Y. and A. Xepapadeas (1996), Environmental innovation, spillovers and optimal policy rules, in: C. Carraro, Y. Katsoulacos, A. Xepapadeas (eds), Environmental policy and market structure, Kluwer, Dordrecht, 143-150.

[20] Kneese, A.V. and C.L. Schultze (1978), Pollution, Prices and Public Policy, The Brookings Institute, Washington DC, 1978.

[21] Lafforgue, G., B. Magné and M. Moreaux (2008), Energy substitutions, climate change and carbon sinks, Ecological Economics 67: 589-597.

[22] List, J. and C.Y. Co (2000), The effects of environmental regulations on Foreign Direct Investment, Journal of Environmental Economics and Management 40: 1-20.

[23] McKitrick, R. (1999), A derivation of the marginal abatement cost curve, Journal of Environmental Economics and Management 37: 306-314.

[24] Milliman, S.R. and R. Prince (1989), Firm incentives to promote technological change in pollution control, Journal of Environmental Economics and Management 17: 247-265.

[25] Montero, J-P. (2002), Permits, standards, and technology innovation, Journal of Environmental Economics and Management 44: 23-44.

[26] Perino, G. and T. Requate (2012), Does more stringent environmental regulation induce or reduce technology adoption? When the rate of technology adoption is inverted U-shaped, Journal of Environmental Economics and Management 64: 456-467.

[27] Requate, T. (2005), Dynamic incentives by environmental policy instruments – a survey, Ecological Economics 54: 175-195.

[28] Requate, T. and W. Unold (2003), Environmental policy incentives to adopt advanced abatement technology: Will the true ranking please stand up?, European Economic Review 47: 125-146.

[29] Simpson, R.D. and R.L. Bradford (1996), Taxing variable cost: Environmental regulation as industrial policy, Journal of Environmental Economics and Management 30: 282-300.

[30] Ulph, D. (1997), Environmental policy and technological innovation, in: C. Carraro, D. Siniscalco (eds), New directions in the economic theory of the environment, Cambridge University Press, Cambridge, 43-68.

[31] Ulph, D and A. Ulph (1996), Trade, strategic innovation and strategic environmental policy-a general analysis, in: C. Carraro, Y. Katsoulacos, A. Xepapadeas (eds), Environmental policy and market structure, Kluwer, Dordrecht, 181-208.

[32] Ulph, A. and D. Ulph (2013), Optimal climate change policies when governments cannot commit, Environmental and Resource Economics 56: 161-176.

A Appendix A: Integrated technology example

Lemma 1 *Let each firm's cost function be given by (55) and its emissions by (3). Then in scenario p with constant product price, in the welfare optimum w, and under emission taxation t, the general solution has the form:*

$$q = \frac{\gamma(\Lambda - T)}{\gamma\Theta - T^2}, \qquad \varepsilon = \frac{\gamma\Theta - \Lambda T}{\gamma\Theta - T^2}, \qquad E = \frac{n\gamma(\Lambda - T)(\gamma\Theta - \Lambda T)}{(\gamma\Theta - T^2)^2} \tag{A1}$$

where:

$$T_p = t, \qquad \Lambda_p = P, \qquad \Theta_p = c \tag{A2}$$

$$T_t = t, \qquad \Lambda_t = a, \qquad \Theta_t = 1 + n + c \tag{A3}$$

$$T_w = MD, \qquad \Lambda_w = a, \qquad \Theta_w = n + c \tag{A4}$$

so that $T = 0$ implies:

$$q = \bar{q} \equiv \frac{\Lambda}{\Theta} \tag{A5}$$

Proof. Equation (A1) follows from substituting (A2) into (60) with constant product price p, (A3) into (70) with emission taxation t, and (A4) into (83) in the welfare optimum w. ■

We can now state:[29]

Proposition 3 *In the general solution:*

1. *If $\gamma < \Lambda^2/\Theta$, then $d\varepsilon/dT < 0$ for all $T, e > 0$ and $dq/dT \begin{smallmatrix} < \\ = \\ > \end{smallmatrix} 0$ for $\varepsilon \begin{smallmatrix} > \\ = \\ < \end{smallmatrix} \frac{1}{2}$. When $T = \gamma\Theta/\Lambda$, $e = 0$ with $\varepsilon = 0$ and $q = \bar{q}$ given by (A5).*

2. *If $\gamma > \Lambda^2/\Theta$, then $dq/dT < 0$ for all $T, e > 0$ and $d\varepsilon/dT \begin{smallmatrix} < \\ = \\ > \end{smallmatrix} 0$ for $q \begin{smallmatrix} > \\ = \\ < \end{smallmatrix} \tilde{q}$ with:*

$$\tilde{q} \equiv \frac{\Lambda}{2\Theta} \tag{A6}$$

When $T = \Lambda$, $e = 0$ with $q = 0$ and $\varepsilon = 1$.

[29] Due to space constraints, we omit the formal analysis of the knife-edge case $\gamma = \Lambda^2/\Theta$. In this case, $dq/dT < 0$ and $d\varepsilon/dT < 0$ for low T values until $\varepsilon = \frac{1}{2}$ and $q = \tilde{q}$ given by (A6). For higher T values there are two solutions, one with $dq/dT < 0$ and $d\varepsilon/dT > 0$, and one with $dq/dT > 0$ and $d\varepsilon/dT < 0$.

3. Whether $\gamma < \Lambda^2/\Theta$ or $\gamma > \Lambda^2/\Theta$:

$$\Theta\gamma - T^2 > 0 \tag{A7}$$

Proof. Differentiating q and ε in (A1) with respect to T yields:

$$\frac{dq}{dT} = \frac{\gamma\left[T(2\Lambda - T) - \gamma\Theta\right]}{\left(\gamma\Theta - T^2\right)^2} = \frac{\gamma\left[1 - 2\varepsilon\right]}{\gamma\Theta - T^2} \tag{A8}$$

$$\frac{d\varepsilon}{dT} = \frac{2\Theta\gamma T - \Lambda(\gamma\Theta + T^2)}{\left(\gamma\Theta - T^2\right)^2} = \frac{2\Theta\left(\tilde{q} - q\right)}{\gamma\Theta - T^2} \tag{A9}$$

The second equality in (A8) follows from (A1). The second equality in (A9) follows from (A1) and (A6).

Emissions drop to zero either because $q = 0$, which from (A1) happens at $T = \Lambda$, or because $\varepsilon = 0$, which from (A1) happens at $T = \gamma\Theta/\Lambda$.

1. If $\gamma < \Lambda^2/\Theta$, $e = 0$ when $T = \gamma\Theta/\Lambda$, so that by (A1), $\varepsilon = 0$ and $q = \bar{q}$ given by (A5). For the numerator of the fraction in the middle of (A9), $\Lambda^2 > \gamma\Theta$ implies:

$$2\Theta\gamma T - \Lambda(\gamma\Theta + T^2) < -\sqrt{\Theta\gamma}\left(T - \sqrt{\Theta\gamma}\right)^2 < 0$$

   With ε decreasing monotonically from 1 to zero, (A8) implies that $dq/dT \begin{smallmatrix} < \\ = \\ > \end{smallmatrix} 0$ for $\varepsilon \begin{smallmatrix} > \\ = \\ < \end{smallmatrix} \frac{1}{2}$.

2. If $\gamma > \Lambda^2/\Theta$, $e = 0$ when $T = \Lambda$, so that $q = 0$ and $\varepsilon = 1$ by (A1). For the term in square brackets in the middle of (A8), $\Lambda^2 < \gamma\Theta$ implies:

$$T(2\Lambda - T) - \gamma\Theta < -(T - \Lambda)^2 < 0$$

   With q in (A1) decreasing monotonically from $\bar{q} > \tilde{q}$ (by (A5) and (A6)) to zero, (A9) implies that $d\varepsilon/dT \begin{smallmatrix} < \\ = \\ > \end{smallmatrix} 0$ for $q \begin{smallmatrix} > \\ = \\ < \end{smallmatrix} \tilde{q}$.

3. Condition (A7) is always met for $\gamma < \Lambda^2/\Theta$ since

$$\gamma\Theta - T^2 > \gamma\Theta - \left(\frac{\gamma\Theta}{\Lambda}\right)^2 = \gamma\Theta\left(1 - \frac{\gamma\Theta}{\Lambda^2}\right) > 0$$

   Condition (A7) is also met for $\gamma > \Lambda^2/\Theta$ since $\gamma\Theta > \Lambda^2 > T^2$.

■

B Appendix B: Comparison with Ulph (1997) and Boom and Dijkstra (2009)

In this appendix we reconcile our findings for emission taxation with integrated technology in subsection 3.2 with Ulph (1997) and Boom and Dijkstra (2009).

We find that with integrated technology, output q is decreasing in the emission tax rate t for low values of t (Proposition 1.3) and can be increasing in t for high t (Proposition 1.4). By contrast, Ulph (1997, p. 49) lists integrated technology cost functions where q is constant or increasing monotonically in t.

According to Ulph (1997), q is constant when $\varepsilon(F) = \varepsilon_0 e^{-\alpha F}$, or inverting the function and normalizing $\varepsilon_0 = 1$:

$$F(\varepsilon) = -\frac{1}{\alpha}\ln\varepsilon, \qquad F'(\varepsilon) = -\frac{1}{\alpha\varepsilon} \tag{B1}$$

Substituting (B1) into (31) and (32), we find that the two FOCs are satisfied with equality if and only if q is constant at $q^* < \bar{q}_t$ given by:

$$P(nq^*) + P'(nq^*)q^* - k'(q^*) - \frac{1}{\alpha q^*} = 0 \tag{B2}$$

so that ε is given by:

$$\frac{1}{\alpha\varepsilon} - tq^* = 0 \tag{B3}$$

However since $\varepsilon \leq 1$, (B2) and (B3) can only be satisfied for:

$$t \geq t^* \equiv \frac{1}{\alpha q^*} \tag{B4}$$

and $E \leq nq^*$. The regulator can achieve a total emission level between nq^* and $n\bar{q}_t$ by setting $t < t^*$, to which the firm will respond by not abating, so that $\varepsilon = 1$ and (B3) does not hold (McKitrick, 1999) and setting q according to:

$$P + P'(Q)q - k'(q) - t = 0$$

We do not allow for integrated technology cost function (B1) in our model, because it features $F'(1) = -1/\alpha < 0$ which violates our assumption $F'(1) = 0$.

According to Ulph (1997), q is increasing in t when $\varepsilon(F) = \varepsilon_0(1 - \frac{1}{2}\alpha F)^2$, or inverting the function and normalizing $\varepsilon_0 = 1$:

$$F(\varepsilon) = \frac{2\,(1 - \sqrt{\varepsilon})}{\alpha}$$

This function also features $F'(1) = -1/\alpha < 0$ which again violates our assumption $F'(1) = 0$.

Then for $0 < t < t^*$, with t^* again defined by (B4) and (B2), the firm will respond to a higher tax rate by decreasing its output and keeping ε at 1. For $t > t^*$, the firm will reduce ε and raise q.

Boom and Dijkstra (2009) do not model emission taxation directly, but their version of permit trading has the same effects on output and emissions as emission taxation. The authors find that with permit trading under perfect (Proposition 2.1) and imperfect (Proposition 7.1) competition, output is monotonically decreasing in the strictness of environmental policy. We find that with integrated technology, output can be increasing in strictness for high levels of t (Proposition 1.4). We shall now see that the reason for this difference is that Boom and Dijkstra's (2009) cost function does not include integrated technology as defined here in Section 2 by $C_{\varepsilon q}(q, \varepsilon) = 0$.

Boom and Dijkstra (2009) use the cost function $C(q, e)$ which we shall write here as $K(q, e) = K(q, \varepsilon q)$ in order to avoid confusion with our own cost function $C(q, \varepsilon)$. Differentiating both cost functions with respect to ε we find:

$$C_\varepsilon(q, \varepsilon) = qK_e$$

Differentiating both sides with respect to q yields:

$$C_{\varepsilon q}(q, \varepsilon) = K_e + qK_{eq} + eK_{ee}$$

With integrated technology, the RHS should equal zero. However, this is not possible in Boom and Dijkstra's (2009, p. 111) model, because they impose $K_e < 0$ and $qK_{eq} + eK_{ee} < 0$.