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Politically Sustainable Probabilistic Minority Targeting

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Abstract

We show that a transfer targeting a minority of the population is sustained by majority voting, however small the minority targeted, when the probability to receive the transfer is decreasing and concave in income. We apply our framework to the French social housing program and obtain that empirically observed departures from these assumptions are small enough that a majority of French voters should support a positive size of this program. We also provide a sufficient condition on this probability function under which more targeting results in a lower equilibrium size of the transfer system.

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1   Introduction

Social transfers targeted on the basis of income represent a sizeable component of public spending in all democratic countries. For instance, in 2007, almost 11% of public spending in OECD countries took the form of means-tested programs (Adema et al., 2011). Although targeting benefits towards those who need them most seems like an obvious proposition (Sen, 1992), the political economy literature has stressed the existence of a "paradox of redistribution", where increased targeting towards the poor hurts them because the erosion of popular support generated by this targeting results in a smaller program. Gelbach and Pritchett (2002) present several cases where "more for the poor means less for the poor" and show that given a choice of uniform and targeted transfers, the targeted regime may actually decrease the poors' welfare.[1]

An extreme case of this paradox obtains when too much targeting results in the absence of majority support for the program. From a political economy perspective, it is indeed difficult to explain why a majority of voters would support transfers targeted towards a minority. De Donder and Hindriks (1998) study a pure redistribution model where a monetary benefit, financed by a proportional income tax paid by all, is targeted towards agents with an income lower than a threshold level. They obtain that the transfer needs to be targeted towards significantly more than one half of the voting population to be supported by a majority of voters.[2] Moene and Wallerstein (2001) introduce an insurance motive for the transfer, beyond pure redistribution, by assuming that future income is uncertain, but they obtain a similar result: in their numerical simulations, the political support for a targeted system disappears when less than two-thirds of the population receive the transfer.

In this short paper, we propose a simple way to reconcile minority targeting and majority voting, by assuming that agents see the attribution of the benefit as a random process. For instance, it is well known (Cornia and Stewart (1995), Swaminathan and Misra (2001)) that purely means-tested programs make errors (of inclusion and of exclusion) that add a random component to the attribution process. More generally, attribution procedures are often complex and based on several criteria beyond income. If the way these criteria are weighted is unclear, or if agents do not know the joint distribution of these criteria in the population, they then consider the attribution process as a random event.

We build a simple model where agents differ in their exogenous income and pay a proportional income tax which finances a lump sum transfer served to a fraction α of the population. Each individual faces a probability p of receiving the transfer, which depends on her income y and on α.[3] Agents vote over the size of the program, measured by the tax rate which finances it. We first show that, if p is increasing in α, and decreasing and concave in y, then there is a unique majority voting equilibrium with a strictly positive size of the program, however small the fraction of agents targeted. We then give a simple

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[1]Korpi and Palme (1998) have been very influential in establishing this paradox. More recently, Marx et al. (2013) contend that this paradox "no longer holds as a robust empirical generalisation".

[2]Cardak et al. (2013) study a similar model where agents vote first over the size of the program and then over the extent of means-testing. They obtain that the majority chosen means-testing level is determined by the median income voter, so that minority targeting can not be an equilibrium.

[3]So, unlike Moene and Wallerstein (2001), we assume uncertainty as to the attribution of the benefit rather than as to future pre-tax income.

and intuitive condition on p (that its elasticity to α for the median income individual is at least equal to one) which guarantees that an increase in targeting (i.e., a decrease in α) results in a lower majority voting equilibrium size of the program (a necessary ingredient for the "paradox of redistribution" to occur). Finally, we apply our framework to a French social program targeted towards a minority– social housing. Using survey data, we show that empirically observed departures from our assumptions that p is decreasing and concave in income are small enough that a majority of French voters should support a positive size of this social housing program.

2 The model

A continuum of agents of mass one differ in exogenous income y, which is distributed over the interval $[0, y_{\max}]$ according to the (non degenerated) cdf F, with mean $E(y) = \mu$ and median y_{med}, and $y_{med} < \mu$. Each agent faces a probability of receiving a transfer described by a continuous function $p(y, \alpha) \in [0, 1]$ depending on her income y and on the (exogenous) fraction $\alpha \in [0, 1]$ of the population receiving the transfer.[4] By the law of large numbers, we then have

$$\int_0^{y_{\max}} p(y, \alpha) dF(y) = \alpha.$$

We make the following assumption:

Assumption 1 *For all $y \in [0, y_{\max}]$, the probability function $p(y, \alpha)$ is (i) strictly increasing in α, and (ii) strictly decreasing and (iii) concave in y.*

Assumption 1 (i) requires that an increase in the overall fraction of the population receiving the transfer translates into an increase in the probability of receiving the transfer for all individuals. Assumption 1 (ii) formalizes that the transfer is targeted towards lower income agents, in the sense that higher income individuals have a lower probability of receiving the transfer.

Denote by γ the income level such that $p(\gamma, \alpha) = \alpha$. We obtain:

Lemma 1 *For any $\alpha \in (0, 1)$ and under Assumptions 1 (ii) and (iii), γ exists and is such that $\gamma \geq \mu$.*

Proof. By Assumption 1 (iii), Jensen's inequality implies that $E(p(y, \alpha)) = \alpha \leq p(\mu)$. By Assumption 1 (ii), we obtain that $\gamma \geq \mu$, and that it is unique. ∎

The threshold γ divides the population into two groups: people endowed with an income lower than γ, who receive the transfer with a probability higher than α, and people with an income higher than γ, who receive the transfer with a probability lower than α. A corollary to this Lemma is that both the median and mean income individuals have a larger-than-average probability of receiving the transfer: $p(y_{med}, \alpha) > p(\mu, \alpha) \geq \alpha$.

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[4]We model a monetary transfer rather than the public provision of a private good for simplicity, but the reasoning developed in this paper applies to the latter case as well.

We move to the political determination of the transfer size as a function of the fraction α of agents receiving the transfer.

All individuals have the same preferences represented by an increasing and concave function $u(.)$ of consumption. They all pay a proportional tax t on their income that is used to finance the lump sum transfer received with probability $p(y, \alpha)$. The indirect utility function is then[5]

$$U(y; t, \alpha) = (1 - p)u(c_u) + pu(c_l),$$

where

$$c_u = (1 - t)y$$

is the consumption level if unlucky (no transfer), while

$$c_l = (1 - t)y + \frac{t\mu}{\alpha}$$

is the consumption level if lucky, with the amount of the transfer obtained from the government budget constraint together with the law of large numbers.

We look at the individuals' most-preferred value of t for a given exogenous value of α. The FOC with respect to t is

$$\frac{\partial U(y; t, \alpha)}{\partial t} = pu'(c_l) \left[\frac{\mu}{\alpha} - y \right] - (1 - p)u'(c_u)y, \tag{1}$$

where the first term measures the marginal benefit from increasing t (a larger consumption level in case the transfer is received, provided that y is not so large that the agent contributes more than the lump sum transfer received) while the second term reflects its marginal cost (a lower consumption level if unlucky). From Lemma 1, we have that

$$y_{med} < \mu \leq \gamma.$$

Proposition 1 makes use of the following assumption.[6]

Assumption 2 *The coefficient of relative risk aversion is lower than one:*

$$-\frac{xu''(x)}{u'(x)} < 1.$$

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[5]We simplify notation by not reporting the arguments of p unless there is a risk of confusion.

[6]Karagyozova and P. Siegelman (2012) survey the empirical literature on relative risk aversion. They report very large ranges for empirically plausible individual values of the coefficient of relative risk aversion: from [0.35, 1] for Hansen and Singleton (1983) to [0.029, 680] for Halek and Eisenhauer (2001). Holt and Laury (2002) estimate that two thirds of respondents in their study have a value of the coefficient between 0.15 and 0.93. Assumption 2 then seems reasonable.

Proposition 1 *Under Assumptions 1 and 2, the majority chosen (or Condorcet winning) tax level, denoted by t^V, is strictly positive, for all $\alpha > 0$.*

Proof. We denote by $t^*(y)$ individual y's most-preferred tax level. It is easy to see from the FOC (1) that the SOC holds, so that individual preferences are single-peaked in t and we can apply the median voter theorem to obtain that $t^V = med(t^*(y))$. We then show that $dt^*(y)/dy < 0$ under Assumption 2, so that $med(t^*(y)) = t^*(y_{med})$. Applying the implicit function theorem to equation (1), we obtain that

$$
\begin{aligned}
\frac{\partial t^*(y)}{\partial y} \quad &\overset{s}{=} \quad \frac{\partial^2 U(y;t,\alpha)}{\partial t \partial y} \\
&= \quad \frac{\mu}{\alpha}\left[\frac{\partial p}{\partial y}u'(c_l) + pu''(c_l)(1-t)\right] \\
&\quad -y\frac{\partial p}{\partial y}\left[u'(c_l) - u'(c_u)\right] \\
&\quad -p\left[u'(c_l) + y(1-t)u''(c_l)\right] \\
&\quad -(1-p)\left[u'(c_u) + y(1-t)u''(c_u)\right]
\end{aligned}
$$

< 0 since $u'(c_l)+y(1-t)u''(c_l) > 0$ and $u'(c_u)+y(1-t)u''(c_u) > 0$ follow from Assumption 2.

Finally, we obtain that $t^*(\mu) \geq 0$, since

$$
\frac{\partial U(\mu;t,\alpha)}{\partial t} = u'(c_l)\mu\frac{p}{\alpha} - \mu[pu'(c_l) + (1-p)u'(c_u)] \geq 0
$$

when $t = 0$ since $p(\mu) \geq \alpha$ (since $\mu \leq \gamma$ by Lemma 1, together with Assumption 1 (ii)), which in turn implies that $t^*(y_{med}) > 0$ (since $y_{med} < \mu$). ■

The proof of this result consists in two stages: first, we obtain that the most-preferred value of t decreases with y (so that the median income agent is decisive), and then we show that this agent favors a positive value of the tax rate. Higher income agents favor less taxation because of both a lower marginal benefit of taxation (because of both a lower probability of receiving the transfer and a lower marginal utility in case the transfer is received) and a higher marginal cost of taxation (because this cost is proportional to income, and because marginal utility does not decrease too fast under Assumption 2). As for why the median income agent prefers a positive tax rate, it is due to the inequality $y_{med} < \mu \leq \gamma$, which guarantees that even the richer-than-median average income favors a scheme for which he has a larger-than-average probability of receiving the transfer. This analysis is valid whatever the value of $\alpha > 0$.[7]

Proposition 1 then shows that it is possible to support under majority voting a scheme that targets a transfer to a minority of the population, however small this minority, when the probability of receiving the transfer is decreasing and concave in income and when the relative risk aversion is not too large.

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[7] The introduction of distortionary taxation, for instance in the form of endogenous labor supply, would not change these results, provided that the distortions when $t = 0$ are not too large.

We now tackle the impact of the proportion of the population receiving the transfer (α) on the majority chosen value of the tax rate, t^V. This impact is far from obvious, since more targeting (i.e., a decrease in α) decreases the probability to receive the transfer while increasing its value (for a given tax rate). Our second proposition makes use of the following assumption:

Assumption 3 *The elasticity of the probability of receiving the transfer to the targeting level is at least equal to one for the median income individual:*

$$\frac{\partial p(y_{med}, \alpha)}{\partial \alpha} \geq \frac{p(y_{med}, \alpha)}{\alpha}.$$

Proposition 2 *Under Assumptions 1, 2 and 3, more targeting (i.e., a lower exogenous α) results in a lower majority voting equilibrium size of the system:*

$$\frac{\partial t^V}{\partial \alpha} > 0.$$

Proof. By Proposition 1, $t^V = t^*(y_{med})$. Applying the implicit function theorem to the FOC (1) when $y = y_{med}$, we obtain that

$$
\begin{aligned}
\frac{\partial t^V}{\partial \alpha} &= \frac{\partial t^*(y_{med})}{\partial \alpha} \stackrel{s}{=} \frac{\partial^2 U(y_{med}; t, \alpha)}{\partial t \partial \alpha} \\
&= \frac{\partial p(y_{med}, \alpha)}{\partial \alpha} u'(c_l) \frac{\mu}{\alpha} \\
&\quad + p(y_{med}, \alpha) \left[-u''(c_l) t \mu \alpha^{-2} \frac{\mu}{\alpha} - u'(c_l) \mu \alpha^{-2} \right] \\
&\quad - y \left[\frac{\partial p(y_{med}, \alpha)}{\partial \alpha} \left(u'(c_l) - u'(c_u) \right) - p(y_{med}, \alpha) u''(c_l) t \mu \alpha^{-2} - \right] \\
&= u'(c_l) \frac{\mu}{\alpha} \left[\frac{\partial p(y_{med}, \alpha)}{\partial \alpha} - \frac{p(y_{med}, \alpha)}{\alpha} \right] \\
&\quad + p(y_{med}, \alpha) u''(c_l) t \mu \alpha^{-2} \left[y_{med} - \frac{\mu}{\alpha} \right] \\
&\quad - y \frac{\partial p(y_{med}, \alpha)}{\partial \alpha} \left(u'(c_l) - u'(c_u) \right)
\end{aligned}
$$

> 0 under Assumption 3. ∎

The intuition for Proposition 3 is that the decisive median income agent most-prefers a smaller transfer program (a necessary condition for the paradox of redistribution to occur) when more targeting decreases her own probability to receive the transfer sufficiently fast.

3   Empirical application - social housing in France

We now check whether probabilistic targeting may explain the political sustainability of a specific targeted aid system, namely social housing in France. A minority of the

population in France[8] benefits from social housing, which is heavily subsidized.[9] Demand for social housing exceeds supply, and there is uncertainty as to whether a candidate will obtain social housing (within a reasonable delay),[10] so that the access to social housing can be seen as a probabilistic event. In order to estimate this probability function, we make use of the "Enquête Logement" run by the French statistical Institute (INSEE) in 1996. This survey contains extensive information on a French representative sample of 42,694 households, including whether they occupy subsidized social housing ("Habitation à Loyer Modéré", or HLM) and the total yearly income of the household. We obtain that 8,780 out of the 42,694 households do occupy a HLM in the 2006 database, which means that $\alpha = 0.206$.[11] We construct the probability of obtaining social housing as a function of income by computing, for each income decile,[12] the proportion of households in this decile who currently benefit from social housing. We report our results on Figure 1. Figure 2 then fits a curve that goes through the ten (income, proportion) pairs, using the middle income amount inside each decile.

Figure 2 shows that the assumption that the function p is decreasing and concave holds for most but not all income levels. More precisely, the fitted curve is concave except for the bottom three deciles of the income distribution, and is decreasing in income except for the top two deciles.[13] Since the assumptions that p is decreasing and concave are sufficient but not necessary for the majority support of a targeted system (Proposition 1), we now look at whether the departures from these assumptions remain consistent with the existence of this program.

Insert Figures 1 and 2 around here

The marginal utility with respect to t, given by (1), when $t = 0$, is

$$\frac{\partial U(y; t, \alpha)}{\partial t} \Big|_{t=0} = u'(y) \left[p(y, \alpha) \frac{\mu}{\alpha} - y \right]. \tag{2}$$

Since utility is concave over t, whatever the shape of the function p, a positive marginal utility with respect to t at $t = 0$ implies that the most-preferred value of t is positive.

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[8]In 2011, 5.192 million out of the 28.2 million (main) residences in France consisted of social housing. This represents 44% of the rental market, and slightly below 20% of the total housing market for main residences.

[9]According to Trannoy and Wasmer (2013), social housing represents the equivalent of a transfer (the gap between market and actual rent) varying from 500€ to 1,500€ per household per year, depending on the characteristics of the house(hold).

[10]A recent report sponsored by the office of the Prime Minister (France Stratégie, 2014) mentions (on p. 48) the "clarification of the conditions under which social housing is attributed" as one of the main challenges facing housing policies in France within the next 10 years.

[11]This corresponds to a slight over-representation of social housing in the sample compared to the overall French situation.

[12]The income measure we use is total household income in 2006 (including labor income, capital income and social transfers) per unit of consumption –i.e., using the OECD equivalence scale to correct for the household size.

[13]We know the income and residency status of households in 2006, but not when first entering social housing. Selecting in the database the households who have moved recently to social housing results in a subsample that is too small to be exploited.

Even though p is first increasing in y, we obtain that the expression between square brackets in (2) is monotone decreasing in y and becomes nil at the 57.7 percentile of the income distribution. Our model then predicts that 57.7% of the french population should support a positive size of the public housing system, given the way housing is allocated as a function of income. Moreover, since our calibrated function p in Figure 2 is concave over y for all income levels who most-prefer a positive value of t, we obtain that the median income voter is decisive when voting over the size of the system (for given α).[14] Finally, observe that the shape of p departs sufficiently from the assumptions made in Lemma 1 that γ is actually very slightly lower than μ, with $F(\gamma) = 0.6$ while $F(\mu) = 0.61$. Indeed, the average income voter should be against the public housing scheme as implemented in 2006 in France.

4   Conclusion

The message of this short paper is that minority targeting can be supported by a majority of voters when the probability of receiving the benefit is decreasing and concave in income, however small the minority targeted. Departures from these assumptions are small enough in the case of social housing in France in 2006 that we indeed find majority support for this program benefitting roughly 20% of the population. Finally, making the probability that the median income voter receives the transfer sufficiently responsive to the fraction of the population targeted induces her to favor a smaller system when targeting is increased, a necessary condition for the paradox of redistribution to occur.

Our model can be extended in several directions. First, it can be adapted to the case where the probability of receiving the benefit is increasing with income (for instance, in the case of public provision of "elitist" goods, such as opera or higher education). Second, it can be enriched to investigate the effects of income inequality on the majoritarian support for targeted programs. Finally, voters could also vote over the degree of targeting of the system, as in De Donder and Hindriks (1998) or Cardak et al. (2013). These extensions belong to our research agenda.

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[14]Comparing the majority voting equilibrium with the actual size of the social housing system would require, among other challenging steps, estimating the utility function of the voters as well as the size of the social housing system. This would by far exceed our objective in this note.

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Figure 1: Proportion of social housing by income decile
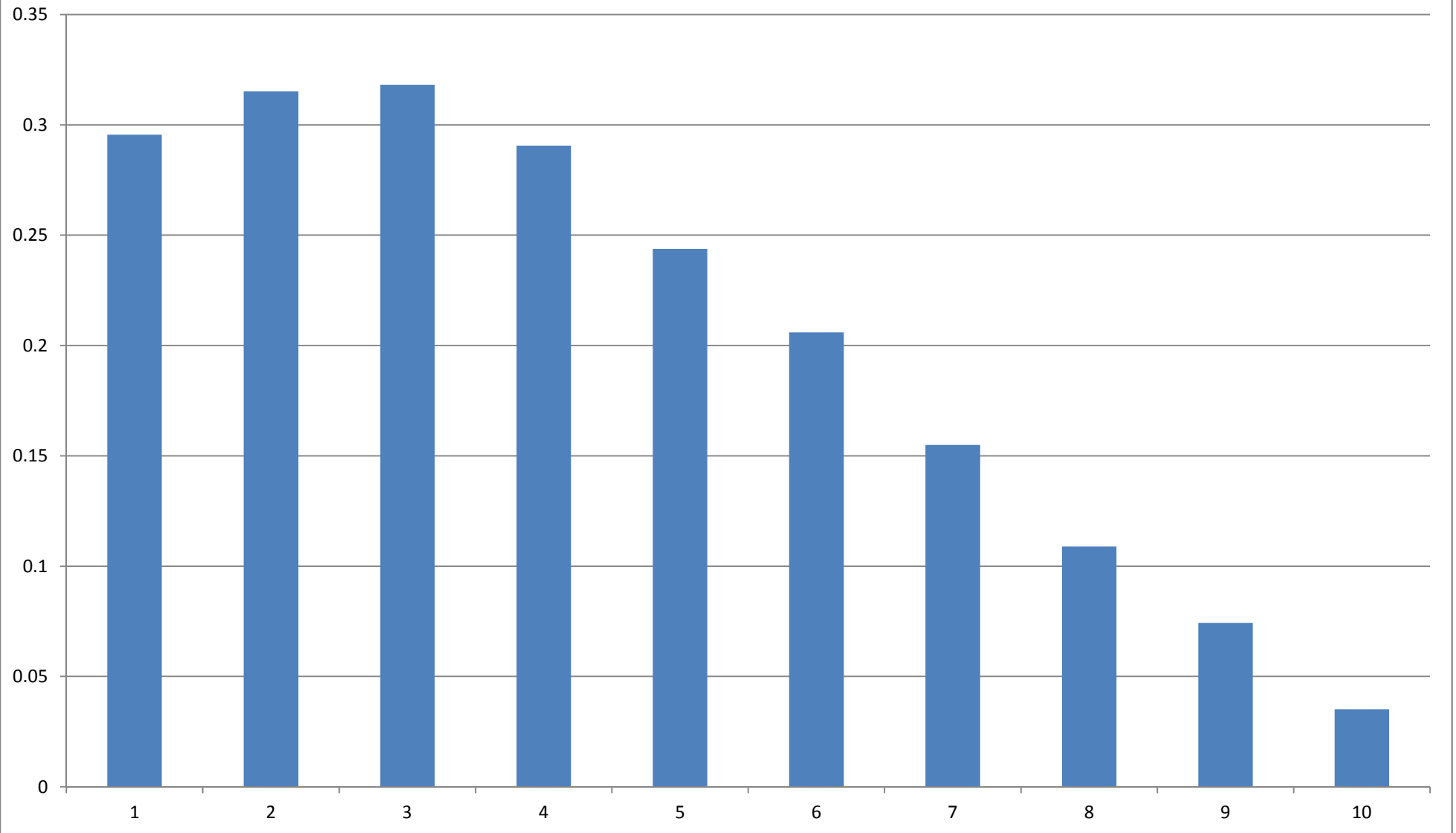
Figure 2: Proportion of social housing by household annual income (in €)