Market Integration, Wage Concentration, and the Cost and Volume of Traded Machines

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Abstract

We investigate the theoretical relationship between wage concentration and international market integration. Access to imported varieties lowers the cost of intermediate inputs ("machines") used to carry out production tasks, causing workers with different comparative abilities to be sorted across a narrower range of tasks and raising the concentration of earnings. The accompanying shift in input use further expands the range of traded varieties, which further lowers the cost of machines. Effects on the volume of intermediate goods trade and the number of varieties produced are mutually reinforcing, resulting in a multiplier effect of market integration on wage concentration.

There has been much debate on the causes of the observed increase in wage concentration over the last few decades.[1] In this paper we propose a simple mechanism that links wage concentration to the expansion of international trade in intermediate goods and to international market integration amongst identical countries.

We describe a model of trade and production choices by a large number of monopolistically competitive firms in a setting of symmetric product differentiation (Dixit and Stiglitz, 1977), where production involves a continuum of complementary tasks that can be carried out either by workers of varying skill types or by machines. Machines, in turn, are obtained through a technology that aggregates different product varieties. The productivity of machines is constant across tasks, whereas the productivity of workers varies with the task and with the skill type of a

[1] Starting in the early 1990s, most free-market economies (the USA and the UK particularly) saw wage inequality rapidly rising, with wage gaps progressively widening with the level of earnings: as the OECD (2011a) put it, 'earners in the top 10% have been leaving the middle earners behind more rapidly than the lowest earners have been drifting away from the middle.' Atkinson *et al.* (2011), provide a comprehensive discussion of historical patterns of income concentration at the top of the income distribution.

worker, with higher-productivity workers having a comparative advantage in tasks where machines are comparatively less productive.

In the model, an expansion in market size raises the number of varieties, and this in turn lowers the variety-adjusted cost of machines. The range of production tasks carried out by machines thus increases, resulting in a reassignment of workers across a narrower range of tasks. This amplifies the effects of comparative advantage gaps between workers of different skills and so systematically raises relative wage gaps in a proportionally increasing manner, resulting in unambiguously higher wage concentration (i.e. a ratio-dominant change in the wage distribution). In addition, since an increase in machine use raises gross output, the number of varieties increases more than proportionally with market size, further lowering the variety-adjusted cost of machines and further raising wage concentration.

We develop our arguments by focusing on a setting with identical countries and no international comparative advantage. In such a setting there is no scope for trade in tasks – neither explicitly through offshoring (as emphasised by the trade-in-tasks literature; Grossman and Rossi-Hansberg, 2006), nor implicitly through trade in intermediate or final goods that embody different production tasks. Effects of trade on the distribution of wages can thus only arise from the reallocation of workers across production tasks within countries, not because of a trade-induced intersectoral reallocation of factors associated with differential comparative factor endowments – the mechanism that has been traditionally emphasized in the literature on trade and income inequality (Leamer, 1998; Feenstra and Hanson, 1996).[2] Nevertheless, the mechanism we describe is akin to a chain of intersectoral factor reallocation effects occurring within economies across the production of different tasks, following a trade-induced displacement of factors from the production of some tasks. At a "macro level", this is observationally equivalent to technological complementarity between capital and skills (Krusell *et al.*, 2000), in the sense that higher mechanization will be associated with comparatively higher wages for higher-skill workers;[3] but this

[2]The relationship between trade, wage inequality and labour market sorting is documented, amongst others, by Haskel and Slaughter (2001), and by Davidson *et al.* (2010).

[3]Several accounts of increasing wage concentration and inequality have sought to rationalize these trends in terms of a capital-skill complementarity argument, which points to an increase in the de-

happens in our model because higher-skill workers have a comparative disadvantage in performing tasks carried out by machines rather than because capital (machines) and high skills are direct complements in production.

The role of input variety in the determination of productive efficiency, and hence of the gains from trade liberalization, was first highlighted by Romer (1994). This notion can be taken literally as directly implying production possibilities that reflect gains from input variety, or more metaphorically, as reflecting expanded possibilities for matching inputs with firms in a search framework.[4]

In methodological terms, the integration of a standard job assignment model with intermediate inputs with a conventional symmetric product differentiation framework is a very natural exercise, and one that is able to generate a rich set of predictions from a relatively parsimonious toolkit. While the trade-related implications of input variety for income growth have long been recognized (e.g. Acemoglu and Ventura, 2002), to the best of our knowledge the link with wage concentration has not been studied before.[5]

The mechanism we highlight is a direct implication of variety effects in the pres-

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mand for high-skilled labour arising from changes in non-labour input prices: a fall in the real user cost of capital – due either to a fall in the cost of capital goods (e.g. IT equipment; Berndt *et al.*, 1995), or to a fall in the cost of borrowing – raising the demand for those labour inputs (skilled labour) that are comparatively more complementary with capital inputs (Griliches, 1969). Krusell *et al.* (2000) provide both a theoretical framework that can rationalize this link and empirical evidence for it. However, as Katz and Autor (1999) point out, the time series of capital user costs is highly collinear with a time trend, and is sufficiently unreliable that year-on-year fluctuations may not give as strong evidence of causality as Krusell *et al.* (2000) argue. Acemoglu (2002) makes a similar point.

[4]The role of intermediate inputs trade on firm productivity is highlighted by Amiti and Konings (2007), and by Peng *et. al.* (2014). Compositional patterns of trade flows in terms of their value added and intermediate input components are documented by Johnson and Noguera (2012).

[5]Costinot and Vogel (2010) employ a job assignment model to examine effects of North-South trade liberalization on wage inequality in the North. Our analysis both has a different focus from theirs – the wage concentration effects of trade induced mechanization – and follows a different methodological approach – a model of trade in differentiated goods and inputs. Clearly, there is scope for integrating the mechanisms at work in these different models, which are in a sense complementary; but for simplicity we focus here on the intermediate inputs related channel only. We would argue that this is the most persuasive of the channels linking trade to increasing wage inequality, not least because of the scale of the long-term decline in observed real capital goods prices (Summers, 2014).

ence of trade in differentiated products when intermediate goods compete with workers of heterogeneous skills in the production of tasks. The prediction that, in such a setting, market integration will raise wage concentration can be shown to be quite robust. Endogenous skill investment responses can strengthen effects on wage concentration. If the ranking of wages by skill type does not reflect their comparative advantage in those tasks that can be more effectively performed by machines, progressive market integration will eventually realign the ranking of wages with comparative advantage to a point where further integration will raise wage concentration.

Our paper contributes to a large and expanding literature on the relationship between wage inequality and the organization of production (Autor, Levy and Murnane, 2003; Acemoglu and Autor, 2011; Sachs and Kotlikoff, 2012; Feng and Graetz, 2013; Brynjolfsson and McAfee, 2014; Hémous and Olsen, 2013), but, distinctively, links this line of analysis to trade-induced variety effects in a "vanilla" model of international trade under monopolistic competition, with intermediate goods trade generating a multiplier-like mechanism on productivity that is akin to the endogenous growth mechanism first proposed by Romer (1990).[6,7] Moreover, unlike outsourcing, the trade-related mechanism we describe here is able to account for the rise in inequality that has taken place both in countries that outsource tasks and in those that "export" outsourced tasks – like China (Han *et al.*, 2012). Admittedly, this trade channel is but one of many possible explanations for the observed distributional changes – alongside skill-biased technological change and capital-skill complementarity – although trade liberalization is possibly one of the most significant exogenous shifts to have occurred in the world economy in the last forty years (arguably more exogenous than even the IT revolution).

The rest of the paper is structured as follows. Section 1 describes the model and characterizes equilibria. Section 2 derives results about the effects of trade integra-

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[6]Analogously, Grossman and Rossi-Hansberg (2006) note that the effects of task outsourcing are akin to those of technological progress.

[7]In our model all producers have identical technologies. The wage effects we describe here are thus derived from a more parsimonious set of primitives than in models of trade-induced firm upgrading which directly assume a link between firm productivity types and the skill types of their workers (Yeaple, 2005).

tion on wage concentration. Section 3 extends the analysis to scenarios where the distribution of skill types arises from endogenous skill investment decisions. Section 4 discusses the relationship between the distribution of wage types and the distribution of skill types as specified in the model, and generalizes results to scenarios where wage concentration effects are not necessarily monotonic in market size. Section 5 concludes.

1 Tasks, workers, machines

In this section we describe a model of production and trade involving heterogenous, internationally immobile workers and tradable machines that can both be employed to carry out complementary production tasks. We characterize the equilibrium sorting of machines and worker types across production tasks,[8] and link it to market structure and market size. The implications of market integration for wage concentration are discussed in the next section.

Technologies and market structure

We focus on an economy with symmetrically differentiated varieties produced by monopolistically competitive firms under conditions of constant marginal costs, with a fixed cost, FC, per variety, expressed as a multiple of marginal cost.

All product varieties are produced through identical technologies all involving a continuum of production tasks indexed by τ, with $\tau \in [0, 1]$. Tasks are perfect complements – the production function is

$$y = \min_{\tau} \; k_{\tau}, \tag{1}$$

where k_{τ} is the input level of task τ. We choose to focus on a specification where tasks and (later on) skill types vary continuously, as this approach lends itself ideally to characterizing effects on wage concentration. Each task can be carried out either by workers or by machines, as we describe next.

[8]Job assignment models of the type we use here are surveyed by Sattinger (1993).

There is a continuum of worker skill types $[\underline{s}, \overline{s}] \equiv S$ distributed according to p.d.f. $f(s)$. The productivity of skill type s in carrying out task τ is $\pi(\tau, s)$. Productivity is increasing with respect to both τ and s, i.e. $\pi_\tau > 0$, $\pi_s > 0$. However, the proportionate gain in productivity as τ rises is increasing with respect to s, i.e. $\pi_{\tau s}/\pi > (\pi_\tau/\pi)(\pi_s/\pi)$, and so higher-$s$ types have a comparative advantage in carrying out higher-τ tasks. A specification that meets these requirements – and which we shall focus on later in order to derive comparative statics results – is $\pi(\tau, s) = \tau(1 + \tau s)$, for $s \in (0, 1)$; in this case, we have $\pi_\tau = 1 + 2\tau s > 0$, $\pi_s = \tau^2 > 0$, $\pi_{\tau s}/\pi - (\pi_\tau/\pi)(\pi_s/\pi) = 1/(1 + \tau s)^2$). The productivity of machines (intermediate inputs) in carrying out task τ is ϕ, the same for all τ.

Both final consumption and intermediate inputs (machines) consist of a Dixit-Stiglitz aggregate of available output varieties, with constant elasticity of substitution $\sigma > 1$, and unit cost $\left(\Sigma_{j=1}^{m} (p_j)^{1-\sigma}\right)^{1/(1-\sigma)}$, where m is the number of product varieties and p^j is the price of variety $j \in \{1, \ldots, m\}$. In a symmetric equilibrium the price of all varieties equals $1 + \mu$ times the marginal cost of output, where $\mu = 1/(\sigma - 1)$ is the monopolistically competitive markup rate (which is constant in a Dixit-Stiglitz "thick" monopolistic competition setting); so the unit cost of an aggregate of m varieties is

$$m^{1/(1-\sigma)}(1 + \mu) = (1 + \mu)/m^\mu \equiv z(m). \tag{2}$$

Equilibrium sorting of skill types across production tasks

Suppose that workers must perform the range of tasks $[\underline{\tau}, 1]$, taking $\underline{\tau}$ as exogenous for the time being.

Since firms behave as price takers in factor markets in a "thick" monopolistically competitive equilibrium, an equilibrium in the market for workers will satisfy constrained productive efficiency by general principles; and since costs are homogeneous across firms, this implies that, for any given number of firms, a competitive allocation of the available factors must be such that total output inclusive of fixed costs is maximal for the given technologies and skill endowments.

We next show that, in conjunction with our previous assumptions on $\pi(\tau, s)$, output maximization implies that an equilibrium allocation cannot be such that, given two skill types s_1 and $s_2 > s_1$, and two tasks τ_L and $\tau_H > \tau_L$, some labour of skill type s_1 is employed in carrying out a task of type τ_H and some labour of skill type s_2 is employed in carrying out a task of type τ_L: if this was the case, then reallocating a (differential) amount dx_1 of labour of type 1 from task τ_H to task τ_L would free an amount $dx_2 = \big(\pi(\tau_L, s_1)/\pi(\tau_L, s_2)\big)\, dx_1$ of labour of type 2 that could be devoted to task τ_H without affecting the volume of task τ_L; the resulting effect on the volume of task τ_H would then be equal to

$$\pi(\tau_H, s_2)\, dx_2 - \pi(\tau_H, s_1)\, dx_1 = \left(\frac{\pi(\tau_H, s_2)/\pi(\tau_H, s_1)}{\pi(\tau_L, s_2)/\pi(\tau_L, s_1)} - 1 \right) \pi(\tau_H, s_1)\, dx_1. \tag{3}$$

Given that $\pi_{\tau s}/\pi > (\pi_\tau/\pi)(\pi_s/\pi)$, the ratio within brackets in the above expression is greater than unity, implying that it would be possible to raise the volume of task τ_H without affecting the volume of task τ_L; and thus, by suitably re-allocating the surplus amount of skill type 2 labour (above the amount that is required to hold the volume of skill τ_H constant) amongst all tasks, it would be possible to raise the level of output. On the other hand, if no labour of skill type 2 is involved in carrying out task type τ_L, then such reallocation is not possible.

The above means that, for $\pi_{\tau s}/\pi > (\pi_\tau/\pi)(\pi_s/\pi)$, the equilibrium assignment of skill types to task types must be positively ordered; i.e. the assignment of skill types to task types can be represented by a mapping $s(\tau)$, where $s(\tau)$ denotes the minimum skill type involved in tasks above or equal to τ and where $s(\tau)$ is increasing in τ (i.e. $s'(\tau) > 0$).

Given this mapping, the cumulative volume of tasks produced for an interval of task types $[\tau, 1]$, $\tau \in [\underline{\tau}, 1]$, is defined by the integral

$$\int_\tau^1 s'(x) f\big(s(x)\big)\, \pi\big(x, s(x)\big)\, dx \equiv \Psi(\tau), \tag{4}$$

where we are making using of the fact that, given $f(s)$ (the p.d.f. of the distribution of skill types), the p.d.f. of the transformed variable $\tau(s) = s^{-1}(s)$ equals

$$g(\tau) = \left| \frac{1}{d\tau\big(\tau^{-1}(\tau)\big)/d\big(\tau^{-1}(\tau)\big)} \right| f\big(\tau^{-1}(\tau)\big) = s'(\tau)\, f\big(s(\tau)\big). \tag{5}$$

Since the task input requirement is constant across tasks and equal to y, we must have

$$\Psi(\tau) = (1 - \tau) y; \tag{6}$$

$$\Psi'(\tau) = -y. \tag{7}$$

As $\Psi'(\tau) = -s'(\tau)\, f\big(s(\tau)\big)\, \pi\big(\tau, s(\tau)\big)$, this requires

$$s'(\tau)\, f\big(s(\tau)\big)\, \pi\big(\tau, s(\tau)\big) = y. \tag{8}$$

The above first-order differential equation identifies an equilibrium assignment schedule, $s(\tau)$. The level of output, y, and the constant of integration, C, which features in the solution of the above differential equation, can be identified by the initial and terminal conditions

$$s(\underline{\tau}) = \underline{s}; \tag{9}$$

$$s(1) = \bar{s}. \tag{10}$$

Equilibrium wages and the price of tasks

Normalizing the marginal cost of output to unity, the unit cost of producing task τ with a worker of skill type s is $w(s)/\pi(\tau)$, where $w(s)$ is the wage rate of skill type s. In any equilibrium where workers of skill type $\check{s} = s(\tau)$, earning a wage $w(\check{s})$, are employed in the production of task τ, $\check{s}$ must then minimize $w(s)/\pi(\tau, s)$, i.e.

$$\frac{w'(\check{s})\, \pi(\tau, \check{s}) - w(\check{s})\, \pi_s(\tau, \check{s})}{\pi(\tau, \check{s})^2} = 0. \tag{11}$$

This implies

$$w'(\check{s}) = \frac{w(\check{s})\, \pi_s(\tau, \check{s})}{\pi(\tau, \check{s})} > 0; \tag{12}$$

i.e. the wage level must be increasing in s. Condition (12) is a first-order differential equation in s that identifies a schedule $w(s) = K\,\omega(s)$, where K is a multiplicative constant of integration. This can be pegged down by a condition that requires the total cost of output to be equal to the net-of-markup value of output (which equals the volume of output given that the net-of-markup price of output is normalized to unity):

$$\underline{\tau} y\big(z(m)/\phi\big) + K \int_{\underline{s}}^{\bar{s}} f(x)\, \omega(x)\, dx = y, \tag{13}$$

where $z(m)$ is the variety-adjusted cost of machines (as defined in (2)), and $z(m)/\phi$ is the unit cost of tasks performed by machines.

In equilibrium, the price of task τ, if carried out by workers, is

$$r(\tau) = \frac{w\big(s(\tau)\big)}{\pi\big(\tau, s(\tau)\big)}. \tag{14}$$

Differentiating (14) and combining it with (12), we obtain

$$r'(\tau) = -\frac{w\big(s(\tau)\big)\,\pi_\tau\big(\tau, s(\tau)\big)}{\pi\big(\tau, s(\tau)\big)^2} < 0; \tag{15}$$

i.e. the equilibrium price of tasks, if carried out by workers, is decreasing in τ.

The equilibrium level of mechanization

To summarize our results so far, under the curvature assumptions we have made about $\pi(\tau, s)$, we can conclude that: (i) $w(s)$ is increasing in s (i.e. higher s skill types earn a higher wage); (ii) $s(\tau)$ is increasing in τ (i.e. higher s skill types are allocated to higher-τ tasks); (iii) $r(\tau)$ is decreasing in τ (i.e. the higher τ the lower the price of task τ if carried out by workers).

We next characterize how the level of mechanization – measured by the minimum task level, $\underline{\tau}$, that is carried out by workers – is established in equilibrium. Let $\underline{\tau} \in (0, 1)$ be the level of τ for which

$$\frac{z(m)}{\phi} = r(\underline{\tau}) = \frac{w\big(s(\underline{\tau})\big)}{\pi\big(\underline{\tau}, s(\underline{\tau})\big)}, \tag{16}$$

assuming that parameters are such that such an interior solution exists. Provided that the difference between the left-hand size and the right-hand side of the above equality is increasing in τ, the value $\underline{\tau}$ obtained from (16) will identify a range of tasks in $[0, \underline{\tau})$ that will be carried out at minimum cost by machines and a range of tasks in $[\underline{\tau}, 1]$ that will be carried out at minimum cost by workers. We have already established that, for a given $\underline{\tau}$, the price of tasks $r(\tau)$, is decreasing in τ if carried out by workers. But this is by itself not sufficient, as $r(\tau)$ is also affected by changes in $\underline{\tau}$ through changes in $s(\tau)$ and $w(s)$. What must then be satisfied is a general equilibrium curvature condition – involving total rather than partial derivatives – that ensures that $\mathrm{d}\hat{w}(s)(\tau)/\mathrm{d}\underline{\tau}$ is decreasing in $\underline{\tau}$, i.e. that it crosses the level $z(m)/\phi$ from above, and that therefore reducing $\underline{\tau}$ below the point identified by (16) produces an increase, rather than a fall, in the difference $w(\underline{a})/\pi(\underline{\tau}, \underline{s}) - z(m)/\phi$. We shall characterize this condition in the next section, when we derive comparative statics results with respect to changes in market size.

Market integration and the equilibrium number of varieties

The total number of product varieties, m, available to consumers and producers in an equilibrium with q integrated identical economies equals qn,[9] where n is the number of varieties produced in each economy. In turn, the equilibrium level of varieties, $\hat{n}$, produced in each individual economy in a monopolistically competitive (zero profits) equilibrium is identified by equating the total monopolistic markups, $\mu\,\hat{y}$ with total fixed costs $\hat{n}\,FC$, i.e.

$$\mu\,\hat{y} = \hat{n}\,FC. \tag{17}$$

Overall equilibrium

For a given number of product varieties, m, an equilibrium for the model is identified by a value $\hat{\underline{\tau}}$, an equilibrium level of output $\hat{y}$, a mapping $\hat{s}(\tau, \hat{C})$, and a wage schedule $\hat{w}(s) = \hat{K}\hat{\omega}(s)$, such that:

(i) Workers are allocated to tasks according to an increasing mapping $\hat{s}(\tau, \hat{C})$ as

[9] In line with a long tradition in the trade literature (e.g. Krugman, 1979), we model market integration as an increase in the number of freely trading countries (implying autarky of those countries in relation to the excluded countries), rather than as a reduction in non-prohibitive tariffs or other trade costs. Albeit stylized, this modelling strategy generates clean analytical results and intuition, and is also a natural match for the assumed CES aggregation – which imposes no restrictions when modelling outcomes where (gross) variety prices are symmetric, but implies specific (and counterfactual) patterns of substitution as we move away from symmetry (Melitz and Ottaviano, 2008). Still, the key feature underlying the mechanism we present is a reduction in the price intermediate inputs from trade liberalization, and this feature would also be present in a specification that focuses on a reduction in non-prohibitive trade costs.

identified by (8) and (9) – re-stated below:

$$\hat{s}'(\tau)\, f\big(s(\tau)\big)\, \pi\big(\tau, \hat{s}(\tau)\big) = \hat{y}, \quad s \in [\underline{s}, \overline{s}]; \tag{18}$$

$$\hat{s}\big(\hat{\underline{\tau}}\big) = \underline{s}. \tag{19}$$

(ii) The wage schedule $\hat{w}(s) = \hat{K}\hat{\omega}(s)$ is such that the allocation of skill types is cost minimizing in the production of each task – condition (12) above, re-stated below in terms of the schedule $\hat{s}(\tau)$ – and such that the unit cost of production of output is unity, i.e.

$$\hat{\omega}'\big(\hat{s}(\tau)\big) = \hat{\omega}\big(\hat{s}(\tau)\big)\, \frac{\pi_s\big(\hat{\underline{\tau}}, \hat{s}(\tau)\big)}{\pi\big(\hat{\underline{\tau}}, \hat{s}(\tau)\big)}, \quad s \in [\underline{s}, \overline{s}]; \tag{20}$$

$$\hat{K} = \hat{y}\, \frac{1 - \hat{\underline{\tau}}\,\big(z(m)/\phi\big)}{\int_{\underline{s}}^{\overline{s}} f(x)\, \hat{\omega}(x)\, dx}. \tag{21}$$

(iii) The cost of having the marginal task $\hat{\tau}$ carried out by a machine is the same as the cost of having it carried out by a worker:

$$z(m) = \frac{\phi}{\pi\big(\hat{\underline{\tau}}, \hat{s}(\hat{\underline{\tau}}, \hat{C})\big)}\, \hat{K}\, \hat{\omega}\big(\hat{s}(\hat{\underline{\tau}}, \hat{C})\big). \tag{22}$$

(iv) There is full employment – which identifies the gross level of output, $\hat{y}$, through (10) – re-stated below:

$$\hat{s}(1) = \overline{s}. \tag{23}$$

In combination with (17), and replacing occurrences of $z(m)$ with the expression $(1+\mu)/m^{\mu} = (q\hat{n})^{1/(1-\sigma)}/(\sigma-1)$, the above conditions also identify an equilibrium value $\hat{n}$ for an exogenously given market size, q.

A parametric representation of workers' productivity in tasks

The analysis that follows will focus on the following specification of workers' productivity:

$$\pi(\tau, s) = \tau\,(1 + \tau s). \tag{24}$$

In this case we can derive (partially) explicit solutions, as detailed next. We cannot exactly solve for $s(\tau)$, but we can solve for $\tau(s) = s^{-1}(s)$ from the transformed condition

$$f(s)\,\pi\big(\tau(s), s\big) = y\tau'(s), \tag{25}$$

which, imposing the initial condition $\tau(\underline{s}) = \underline{\tau}$, gives

$$\tau(s) = \frac{\underline{\tau}\, y \exp\big(\big(F(s) - F(\underline{s})\big)/y\big)}{y - \underline{\tau}\int_{\underline{s}}^{s} \exp\big(\big(F(x) - F(\underline{s})\big)/y\big)\, x f(x)\, dx}, \tag{26}$$

where $F(s)$ denotes the cumulative of $f(s)$. In conjunction with the terminal condition

$$\tau(\bar{s}) = 1, \tag{27}$$

this implicitly identifies a combination of a schedule $\tau(s)$ and a corresponding level of output, y, for a given $\underline{\tau}$.

Condition (20) can be written as

$$\omega'(s) = \omega(s)\frac{\pi_s\big(\tau(s), s\big)}{\pi\big(\tau(s), s\big)}. \tag{28}$$

Together with the initial condition $z(m) = w(\underline{s})\,\phi \Big/ \Big(\tau(\underline{s})\big(1 + \tau(\underline{s})\underline{s}\big)\Big)$, (28) can be integrated to give

$$\omega(s) = \exp\left(\int_{\underline{s}}^{s} \frac{\tau(x)}{1 + \tau(x)x}\, dx\right); \tag{29}$$

$$w(s) = \frac{\tau(\underline{s})\big(1 + \tau(\underline{s})\underline{s}\big)\, z(m)}{\phi}\,\omega(s). \tag{30}$$

The critical value $\underline{\tau}$ can be derived from (13), which gives

$$\underline{\tau} = \frac{y - \int_{\underline{s}}^{\bar{s}} f(x)\, w(x) dx}{yz(m)/\phi}. \tag{31}$$

So, for an exogenously given market size, q, an equilibrium combination of a wage schedule $\hat{w}(s)$, inverse assignment rule $\hat{\tau}(s)$ (over the given domain $[\underline{s}, \bar{s}]$), gross output level $\hat{y}$, boundary task level $\hat{\underline{\tau}}$, and number of varieties per country, $\hat{n}$, is identified by (26), (27), (30), (31) and by the no-entry/no-exit condition (17).

2 Market integration and wage concentration

What we are principally interested in studying is the effect of a change in market size, q, on the wage schedule, $\hat{w}(s)$, and, in particular, whether an increase in q makes the wage distribution more concentrated. For this purpose, we will first discuss briefly how wage concentration can be measured in the model.

Ratio-dominant wage changes

Higher wage inequality does not in itself amount to higher wage concentration; for a systematic increase in wage concentration to occur, inequality must progressively increase moving up the wage distribution. To compare wage concentration levels across the two wage distributions, we can invoke the concept of ratio dominance. Adapting Preston's (2006) definition, we say that a discrete distribution f_1 of a variable, w, for J individuals indexed by $j = 1, \ldots, J$, ratio dominates distribution f_0 of the same variable if and only if

$$\ln w_1^{j+1} - \ln w_0^{j+1} > \ln w_1^j - \ln w_0^j, \qquad \forall j \in \{1, \ldots, J-1\}. \tag{32}$$

In other words, when individuals are ordered in terms of (increasing) wages, then the new distribution shows a greater proportional increase in the income of the $j+1$th individual compared to that of the jth individual, for any j.[10,11] This, in turn, implies that for any given pair of percentile levels the ratio of wage of the higher

[10]Ratio dominance is a stronger characterization of concentration than Lorenz dominance: it implies Lorenz dominance, and it also implies ratio dominance (and hence Lorenz dominance) for comparisons made over any given truncation of the original distribution, i.e. an increase in the Gini index for any percentile sub-range of the original distribution – a property that is not implied by Lorenz dominance.

[11]On the basis of publicly available OECD data on hourly pay deciles ratios (D5/D1, D9/D1, D9/D1) for various countries between 1997 and 2008, and years, one can conclude that there have been clear ratio-dominant increases in wage concentration for a number of countries (Australia, Czech Rep, Denmark, Finland, Germany, Korea and Norway). On the other hand, for other countries (Canada, Hungary, Ireland, Sweden, UK and USA), ratio dominance is only clearly seen at the top of the earnings distribution.

percentile/wage of the lower percentile will be greater in the new distribution than in the old one.

For a continuum distribution where the wage profile, $w(s)$ – defined over a domain, S, of individual types and increasing in s – is affected by a marginal change in an exogenous parameter, θ, the ratio dominance condition translates to the following condition

$$\frac{d}{ds}\left(\frac{\mathrm{d}\ln w(s)}{\mathrm{d}\theta}\right) > 0. \tag{33}$$

Ratio dominance effects of market integration

In order to study effects of market integration on wage concentration, we can focus on the normalized wage schedule

$$\hat{\omega}(s) = \exp\left(\int_{\underline{s}}^{s} \frac{\hat{\tau}(x)}{1+\hat{\tau}(x)x}\,dx\right), \tag{34}$$

as defined by (29). (This is normalized in the sense that $\hat{\omega}(\underline{s}) = 1$.)

An increase in q results in a ratio dominant distribution if the proportional change it generates in $\hat{\omega}(s)$ is increasing in s, i.e., if

$$\frac{d}{ds}\left(\frac{\mathrm{d}\ln\hat{\omega}(s)}{\mathrm{d}q}\right) > 0. \tag{35}$$

Expanding the left-hand side of the above condition and simplifying it, we can rewrite it as

$$\frac{\mathrm{d}\hat{\tau}(s)/\mathrm{d}q}{\left(1+\hat{\tau}(s)s\right)^{2}}. \tag{36}$$

The sign of this depends on the sign of $\mathrm{d}\hat{\tau}(s)/\mathrm{d}q$. In turn, because of the sorting condition derived earlier (as reflected by $\tau'(s) > 0$), any change that results in an increase in $\hat{\underline{\tau}} = \hat{\tau}(\underline{s})$ must bring about an increase in $\tau(s)$ for all $s \in [\underline{s}, \bar{s}]$. Thus, all that is left to prove is that $\mathrm{d}\hat{\underline{\tau}}/\mathrm{d}q > 0$.

We shall first derive results with reference to an exogenous, "first-step" change in the productivity-adjusted price of machines, $z(m)/\phi \equiv \zeta$, which would arise from an increase in q that raised the total number of traded varieties, $m = qn$, but left n otherwise unchanged:

Proposition 1 *For $\pi(\tau, s) = \tau(1 + \tau s)$, an exogenous reduction in the productivity-adjusted price of machines, ζ, produces a ratio-dominant change in the distribution of wages.*

PROOF: For a given ζ, an equilibrium in $\underline{\tau}$ and y is identified by the system of equations

$$\begin{cases} \underline{\tau}\, y\, \zeta + \underline{\tau}(1 + \underline{\tau}\underline{s})\, \zeta \displaystyle\int_{\underline{s}}^{\overline{s}} f(x)\omega(x)\, dx = y, \\ \tau(\overline{s}) = 1, \end{cases} \tag{37}$$

where the first condition is obtained from (13) after replacing K with the ratio that appears on right-hand side of (30). We can then totally differentiate the above system with respect to $\underline{\tau}$, y, and ζ, in order to sign the total derivative $\mathrm{d}\underline{\tau}/\mathrm{d}\zeta$. If this is negative, an exogenous decrease in ζ raises $\underline{\tau}$.

After expanding and simplifying, we can write

$$\frac{\mathrm{d}\underline{\tau}}{\mathrm{d}\zeta} = -\frac{\underline{\tau}}{\zeta \Phi}, \tag{38}$$

where

$$\Phi \equiv 1 + \underline{\tau}\underline{s}\,(2 - \underline{\tau}\zeta)\,/\,(1 + \underline{\tau}\underline{s}) - (1 - \underline{\tau}\zeta)y^2/\Xi, \tag{39}$$

and

$$\Xi \equiv y + \underline{\tau}(y - 1) \int_{\underline{s}}^{\overline{s}} \exp\big(F(z)/y\big) f(z) z\, dz + \underline{\tau} \int_{\underline{s}}^{\overline{s}} \exp\big(F(z)/y\big)\, f(z) F(z) z\, dz. \tag{40}$$

The expression $1 + \underline{\tau}\underline{s}\,(2 - \underline{\tau}\zeta)$ in (39) is positive, since $\underline{\tau}\zeta < 1$ from (13); the term involving the expression Ξ relates to a feedback effect of the resulting reduction in y on $\underline{\tau}$, and is negative: $1 - \underline{\tau}\zeta$ is positive, and so is Ξ – since $y > 1$. The denominator of (38) thus cannot be signed directly.

To sign the denominator of (38), we can focus on curvature conditions requiring that, as $\underline{\tau}$ increases, *with y and the equilibrium wage schedule adjusting accordingly*, the cost of carrying out the marginal non-mechanized task, $\underline{\tau}$, with labour inputs must be falling. Under an equilibrium assignment rule, the marginal task is always carried out by workers of skill level $\underline{s}$; but since the equilibrium wage schedule depends on $\underline{\tau}$, the wage level $w(\underline{s})$ changes with $\underline{\tau}$, which we can express by writing $\widetilde{w}(\underline{s}; \underline{\tau})$. Then the above requirement means that, as $\underline{\tau}$ increases, the schedule $\widetilde{w}(\underline{s}; \underline{\tau})\,/\,\big(\underline{\tau}(1 + \underline{\tau}\underline{s})\big)$ must cross ζ from above.[12] In formal terms, these

[12]Writing the equilibrium condition as a fixed-point condition, $\underline{\tau} = \Lambda(\underline{\tau})$, where $\Lambda(.)$ is an implicit function of τ, this condition amounts to the local fixed-point stability requirement $\Lambda'(\underline{\tau}) < 1$.

conditions can be derived by focusing on the system of equations

$$\begin{cases} \underline{\tau} y \zeta + K \int_{\underline{s}}^{\bar{s}} f(x)\omega(x)\, dx = y, \\ \tau(\bar{s}) = 1, \end{cases} \tag{41}$$

with unknowns K and y, and totally differentiating the system with respect to K, y, and $\underline{\tau}$ – treating the latter as an exogenous parameter – to derive an expression for the total derivative

$$\frac{\mathrm{d}}{\mathrm{d}\underline{\tau}}\left(\frac{K\omega(\underline{s})}{\underline{\tau}(1+\underline{\tau}\underline{s})}\right). \tag{42}$$

In order for condition (31) to identify a stable equilibrium, (42) must be negative. After simplification, (42) can be expressed as $-\zeta\Phi/\big(\underline{\tau}\,(1-\underline{\tau}\zeta)\big)$; stability then requires that Φ be positive, and thus $\mathrm{d}\underline{\tau}/\mathrm{d}\zeta$ negative. As long as the equilibrium is well behaved (in the sense of Samuelson, 1941), an exogenous reduction in the productivity-adjusted price of machines raises $\underline{\tau}$ and hence wage concentration (in the ratio dominance sense). □

Thus, a reduction in the productivity-adjusted price of machines – however it arises – reduces the range of task carried out by workers (it raises $\underline{\tau}$), and in doing so, generates a systematic increase in wage concentration. The effect we have derived in the above proposition, however, does not account for the negative relationship between y and ζ through n. This can be incorporated to obtain the following result:

Proposition 2 *For $\pi(\tau, s) = \tau(1+\tau s)$, and provided that a given increase in the number of varieties results in a less than proportional increase in output, an exogenous increase in market size produces a ratio-dominant change in the distribution of wages.*

PROOF: Signing the total derivative $\mathrm{d}\underline{\tau}/\mathrm{d}\zeta$ also allows us to establish that $\mathrm{d}y/\mathrm{d}\zeta < 0$: total differentiation of (37) with respect to $\underline{\tau}$, y, and ζ gives, after simplification,

$$\frac{\mathrm{d}y}{\mathrm{d}\zeta} = -\frac{y^3(1+\underline{\tau}\underline{s})}{\zeta\,\Xi\Phi}, \tag{43}$$

where Φ and Ξ are as defined in the proof of Proposition 1. This is negative if (38) is negative.

We can then obtain an expression for a total effect that also accounts for the relationship between y and n. Letting $\zeta = m^{-\mu}\,(1+\mu)/\phi$, we get $d\zeta/dm = -\mu m^{-(1+\mu)}(1+\mu)/\phi < 0$, and

$$\frac{\mathrm{d}y}{\mathrm{d}m} = \frac{\mathrm{d}y}{\mathrm{d}\zeta}\frac{d\zeta}{dm} > 0. \tag{44}$$

Totally differentiating (17) with respect to n and q, and using (17) to replace $\hat{n}\,F$ with $\mu\hat{y}$, we obtain

$$\frac{\mathrm{d}\hat{n}}{\mathrm{d}q} = -\frac{\mu\,(\mathrm{d}y/\mathrm{d}m)\,\hat{n}}{\mu(\mathrm{d}y/\mathrm{d}m)\,q - F} = -\frac{\hat{n}}{q}\,\frac{\mu\,\hat{y}\,\chi}{\mu\,\hat{y}\,\chi - \hat{n}\,F} = \frac{\hat{n}}{q}\,\frac{\chi}{1-\chi}, \tag{45}$$

where

$$\chi = \frac{m}{y}\,\frac{\mathrm{d}y}{\mathrm{d}m} = \frac{\zeta}{y}\,\frac{\mathrm{d}y}{\mathrm{d}\zeta}\,\frac{m}{\zeta}\,\frac{d\zeta}{dm} = -\mu\,\frac{\zeta}{y}\,\frac{\mathrm{d}y}{\mathrm{d}\zeta} > 0. \tag{46}$$

A necessary and sufficient condition for (45) to be positive is thus $\chi < 1$, i.e.

$$\left|\frac{\zeta}{y}\,\frac{\mathrm{d}y}{\mathrm{d}\zeta}\right| < 1/\mu = \sigma - 1. \tag{47}$$

In turn, the condition $\chi < 1$ coincides with the condition for local stability of an equilibrium in y and m as identified by (37) and (17).[13]

The total derivative $\mathrm{d}\hat{y}/\mathrm{d}q$ can then be obtained as

$$\frac{\mathrm{d}\hat{y}}{\mathrm{d}q} = \frac{\mathrm{d}y}{\mathrm{d}m}\left(\hat{n} + q\,\frac{\mathrm{d}\hat{n}}{\mathrm{d}q}\right) = \frac{\mathrm{d}y}{\mathrm{d}m}\,\hat{n}\,\frac{1}{1-\chi}. \tag{48}$$

Analogously, letting

$$\frac{\mathrm{d}\tau}{\mathrm{d}m} = \frac{\mathrm{d}\tau}{\mathrm{d}\zeta}\,\frac{d\zeta}{dm} > 0, \tag{49}$$

we obtain

$$\frac{\mathrm{d}\hat{\tau}}{\mathrm{d}q} = \frac{\mathrm{d}\tau}{\mathrm{d}m}\left(\hat{n} + q\,\frac{\mathrm{d}\hat{n}}{\mathrm{d}q}\right) = \frac{\mathrm{d}\tau}{\mathrm{d}m}\,\hat{n}\,\frac{1}{1-\chi}. \tag{50}$$

A necessary and sufficient condition for both (48) and (50) to be positive is then $\chi < 1$. □

The presence of the elasticity χ at the denominator of the last ratio in (48) and (50) reflects the mutually positive equilibrium interaction between n and y – a higher n lowers ζ and raises y; a higher y raises n. This can be thought of as a trade-related "multiplier effect" that is at work in the model: market integration raises the number of varieties; which lowers the variety-adjusted cost of machines; which raises the level of mechanization, the level of output, y, and the number of product

[13] Expressing the above two conditions in terms of implicit functions, respectively $y = \tilde{y}(m)$ and $m = \tilde{m}(y)$, each with derivatives $\tilde{y}'(m) \equiv \mathrm{d}y/\mathrm{d}m$ and $\tilde{m}'(y) \equiv \mathrm{d}m/\mathrm{d}y$, a fixed point for y is identified by $y = \tilde{y}(\tilde{m}(y))$. This fixed point is locally stable if $\tilde{y}'\tilde{m}' = \big(\mathrm{d}y/\mathrm{d}m\big)\big(\mathrm{d}m/\mathrm{d}y\big) < 1$. Noting that $\mathrm{d}m/\mathrm{d}y = q\mu/FC = m/y$, this amounts to the condition $\chi = \big(\mathrm{d}y/\mathrm{d}m\big)\big(m/y\big) < 1$.

varieties per economy; which raises the total number of varieties; which lowers the cost of machines; and so on.

It is thus unsurprising that additional conditions must be imposed for this process to be bounded. In principle, this feedback mechanism can result in an indefinite multiplication that allows a given mass of workers to be employed in a vanishingly small subset of tasks and to produce an arbitrarily large level of gross output. The elasticity condition described in the above proof is thus analogous to the boundedness constraints on production possibilities imposed in standard neoclassical general equilibrium analysis; here variety gains can effectively make production possibilities unbounded. In practice, there would be outside limiting factors that make the process bounded, i.e. such a space and natural resource constraints, which in the model would formally translate into technologies exhibiting decreasing rather than constant returns to scale – e.g., as with a production function of the form as $y = \big(\min_\tau k_\tau\big)^\gamma$, with $0 < \gamma < 1$, giving rise to fixed-factor rents $(1-\gamma)y$. A direct implication of an extended specification such as this is that the ratio of earning to total income inclusive of fixed factor rents should be rising in q.

The model we have presented is static, and so machines are modelled as being produced and simultaneously used as inputs – i.e. machines are one and the same as non-durable intermediate inputs. This static model can be thought of as a simplified long-run representation of a sequence of choices where machines are durable inputs that span more than one production period. The only change that fully accounting for the durable nature of machines would entail, with reference to a stationary-state representation of the economy, is that installing machines which last D periods would require an additional per-period servicing cost (capital cost) equal to $z(m)\kappa\,(D-1)/D$ for each machine employed, where κ is the unit servicing cost of capital, making their gross-of-capital price equal to $z(m)\big(1+\kappa(D-1)/D\big)$, and resulting in a total level of capital income in each country equal to $\underline{\tau} y\big(z(m)/\phi\big)\kappa(D-1)/D$. An implication of accounting for the capital cost of durable machines is then that, as with fixed-factor rents, the share of capital income in total income should be increasing with market size, q, and should therefore be positively related to wage concentration[14] – a pattern that is observationally equivalent to that which would be

[14]The decline in the labour's share of national income is well documented; see, for example,

implied by a direct technological complementarity between "high" skills and capital inputs. In addition, if capital supply is price-elastic – i.e. if κ is increasing with machine use – this would act as a further limiting factor on the expansion of mechanization to ensure boundedness.[15]

A calibrated example

To illustrate the model's predictions, and to gauge their potential quantitative contribution to observed changes in the concentration of earnings, we present numerical simulations result for a loosely calibrated numerical version of the model.

We select $\sigma = 5$, which at the lower end of the range of elasticity estimates reported by Anderson and van Wincoop (2004). In a Dixit-Stiglitz specification a given σ implies a constant μ and hence a certain ratio of fixed costs to total costs.

We follow Saez (2001), who, with reference to the US, assumes a Pareto distribution with p.d.f. $f_w(w) = \alpha\, \underline{w}^{\alpha} w^{-(1+\alpha)}$ with $\alpha = 2$, where $\underline{w}$ is a minimum wage level. Our model, however, is based on an unobservable distribution of skill types, s, which generates the distribution of wages as an equilibrium outcome; so we cannot directly specify parameters for the wage distribution. This mapping also depends on the assumed task productivity of intermediates, ϕ. Moreover, ϕ and the distribution of skills jointly determine the ratio of intermediate costs to total costs.

To parameterize the distribution of skill types and simultaneously select a value

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Karabarbounis and Neiman (2013), and Piketty (2014).

[15] As an example, suppose that $D = 2$ in an overlapping generations economy of workers each lived for two periods, with a stationary population and no bequests, where capital supply (savings) in each period comes from the young cohort and is identified by the condition $u'(c_1^t) = u'(c_2^t)(1+\kappa)/(1+\beta)$; with $u(.)$ denoting instantaneous utility, $\beta > 0$ the rate of time preference, and c_j^t, $j \in \{1, 2\}$ consumption at period $t + j - 1$ by the cohort born at t. With a stationary population and symmetric countries (implying zero international capital flows in equilibrium), and focusing on a stationary state (thus dropping the superscript t), the intertemporal budget constraint for a worker of type s in a representative cohort is $c_1(s) + c_2(s)/(1+\kappa) = w(s) + w(s)/(1+\kappa)$. Incorporating this into the optimal savings condition, we can re-write it as $u'(w(s) - v(s)) = u'\big(w(s) + (1+\kappa)v(s)\big)\,(1+\kappa)/(1+\beta)$, where $v(s)$ is savings by a worker of type s. Aggregate capital supply, $\int_{\underline{s}}^{\overline{s}} v(s)\, ds$ is then a function of κ, as well as of the other variables in the problem. In equilibrium, capital supply must equal capital demand $2\underline{\tau} y\big(z(m)/\phi\big)$, and this condition determines an equilibrium level of κ.

of ϕ, we proceed as follows. We arbitrarily set $\underline{s} = 1$ and assume a truncated Pareto distribution for s, whose p.d.f. has the form

$$f(s) = \frac{\xi (\bar{s}\, \underline{s})^{\xi}}{\left(\bar{s}^{\xi} - \underline{s}^{\xi}\right) s^{1+\xi}} = \frac{\xi}{\left(1 - (\bar{s})^{-\xi}\right) s^{1+\xi}}, \tag{51}$$

with $\xi > 0$. We then use the following information: (i) the average Gini coefficient for earnings of full-time workers in OECD countries in the mid-2000's was around 0.38 (OECD, 2011b); and (ii) the ratio of intermediate goods trade to total goods trade in developed countries in the late 2000's was between 0.5 and 0.6 (Miroudot *et al.*, 2009). On the basis of this information, and targeting a Pareto distribution for wages with $\alpha = 2$, we select $\bar{s} = 30$, $\xi = 2/3$ and $\phi = 3/5$. This choice yields a baseline ratio of intermediate use in total costs (which in our model this is the same as the ratio of intermediate goods trade to total trade[16]) approximately equal to 0.61, corresponding to a value of $\underline{\tau}$ approximately equal to 0.46; and a baseline wage distribution with a Gini coefficient of 0.374 – both values being in line with (i) and (ii). This wage distribution is not a Pareto distribution, but if we try to fit a Pareto distribution to it, the best-fit value of α for the fitted distribution is not far from 2 – roughly 1.8 to 2.0 depending on the best-fit method.

Once the numerical model is calibrated in this way for a baseline value of q equal to unity, we use it to simulate the effects of a reduction in market size from $q = 1$ to $q = 1/4$.

Figure 1 shows the baseline and counterfactual wage distributions. As mentioned above, the baseline distribution is approximately Pareto with $\alpha = 2$ and with a Gini coefficient of 0.375. The Gini coefficient in the counterfactual distribution equals approximately 0.31.

Figure 2 shows the equilibrium assignment of skill types – identified by their corresponding wage percentile in the wage distribution – to production tasks. In the

[16]Intermediate goods trade includes trade in intermediate goods that, strictly speaking, are not involved in the execution of production tasks as we have characterized it in our model (e.g., raw materials). However, with Leontief technologies, inframarginal tasks are formally indistinguishable from other intermediate inputs use that involve strict complementarity (e.g., using wood as an intermediate input in the production of wooden furniture), and there is no advantage to modelling or measuring such intermediate inputs separately from tasks.

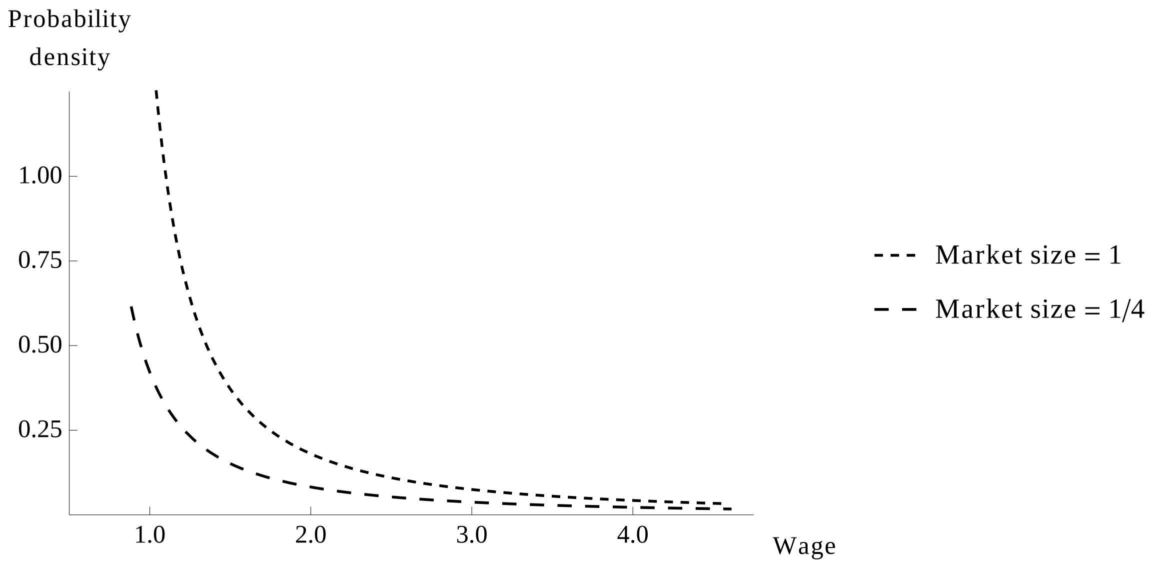

Figure 1: Wage distributions for $q = 1$ and $q = 1/4$

baseline equilibrium, tasks in the range $[0,\ 0.46)$ are carried out by machines and workers are assigned to the remaining tasks in increasing order of wage percentile. In the counterfactual equilibrium where market size is smaller, the range of tasks carried out by machines narrows to $[0,\ 0.19)$ and workers are spread through a wider range of tasks, still in increasing order of wage percentile.

Figure 3 shows the change in log normalized wages shares from the counterfactual to the baseline by wage percentile (the solid curve), where normalized wage wages by percentile are defined as the ratio of the wage share of a certain wage type to the mean wage share, i.e. the derivative of the Lorenz curve of the wage distribution, $L(t) \equiv \int_0^t F(y)\, dy \Big/ \int_0^1 F(y)\, dy$, with respect to the percentile level, t (where $F_w^1(t)$ is the inverse of the cumulative wage distribution). The change is systematically increasing from the lowest wage percentile to the highest, indicating a ratio-dominant change in the wage distribution.

The dashed curve in Figure 3 shows effects computed by shutting down the positive feedback channel of y on n, i.e. by holding n constant at its baseline level, and removing (17) from the equilibrium condition. In this case effects are approximately halved, and the Gini coefficient falls only to 0.34 rather than to 0.31. Without a better handle on the link between market integration in the model (as measured by q) and

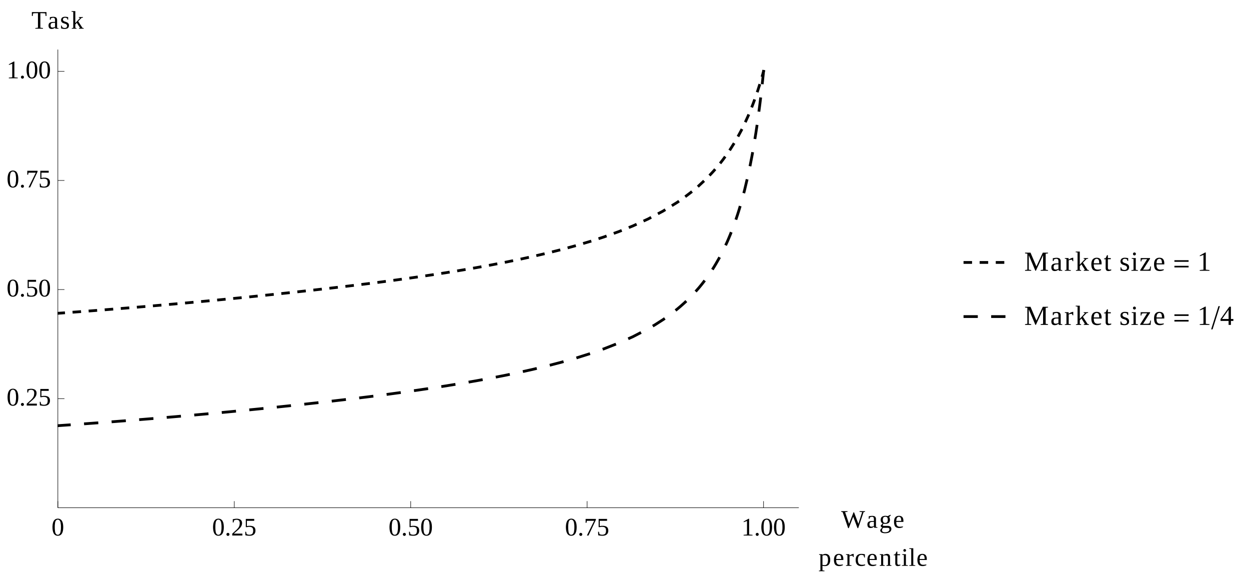

Figure 2: Assignment of skill types to task types for $q = 1$ and $q = 1/4$

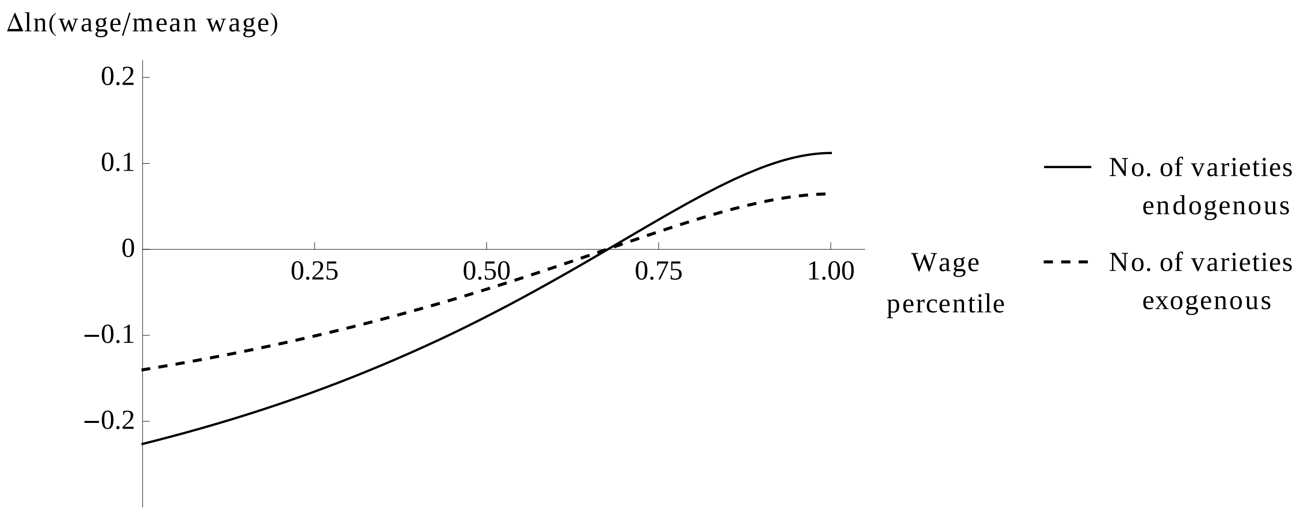

Figure 3: $\Delta \ln$ of normalized wage shares by wage percentile

the actual extent of market size (as resulting from changes in institutions, transportation costs, and population size), and without direct evidence on the responsiveness of $\underline{\tau}$ to market integration, the estimated size of the simulated responses in this calibrated example cannot be taken too seriously; but the exercise does illustrate the potential for market integration to significantly raise wage concentration through the mechanism we have highlighted. The *relative* size of the direct and indirect effects, on the other hand, is comparatively independent of the absolute size of the change – it derives from the model's basic structure and from parameter conditions about which we can be comparatively more confident (or, to be precise, comparatively less skeptical), such as σ and the shape of the wage distribution. In other words, irrespectively of how large the change in q is, or how responsive $\underline{\tau}$ is to changes in the variety-adjusted price of machines, under this model structure and parameterization we could expect the indirect channel to almost double the effects that flow through the direct channel.

3 Endogenous responses in skill investment

If the distribution of skill types is the result of endogenous investment choices by workers, then effects on wage concentration will also depend on how investment responses re-shape the distribution of skill types following a change in the wage schedule. In principle, first-order effects on wage concentration could be, at least in part, offset by changes in $f(s)$ that increase dispersion at the bottom end and decrease it at the top end.[17]

To look at this possibility, we can first focus on a setting where – unlike in the setting we have discussing – a change in the distribution of skill levels does not affect the wage schedule, and simply ask how changes in the distribution of skills affect wage concentration for a given exogenous change in the wage schedule. It is then easy to construct abstract examples where an exogenous change in the wage schedule that is neutral with respect to wage concentration, such as an equiproportional

[17]Our analysis here is related to the discussion in Acemoglu and Autor (2011) concerning the implications of endogenous skill supply for the effects of trade liberalization on wage inequality in a model of skill-biased technical change with only two skill levels.

increase in wages for all skill levels, may lead to lower wage concentration. Suppose, for example, that individuals have two choices: either stay at skill level s (different for different individuals) or upgrade to a common level $\bar{s} > s$; and suppose that, before the wage change, they face a cost of upgrading to skill $\bar{s}$ that is only marginally higher than $w(\bar{s}) - w(s)$. Then, before the wage increase, they would stay at s; but any equiproportional or increasingly proportional increase in wages would cause all worker types to choose to upgrade to $\bar{s}$, reducing wage concentration to zero.

What the preceding discussion neglects to account for is that, in the problem we are studying, a change in the distribution of skill types changes the equilibrium wage schedule – which can produce further effects on concentration. This is because, for any given skill type, s, an increase in the comparative supply of skills above s pushes the use of type-s inputs towards comparatively lower-τ tasks, where skill s is comparatively less productive, an effect that is larger the lower s is; individuals can offset this negative effect by acquiring higher skills, but individuals with different initial skill levels may be affected differently by the supply push from the top, and may also be differently able to respond to the change. Thus, for given partial-equilibrium responses in skill investment – which, by themselves, may produce positive or negative effects on wage concentration – the general-equilibrium wage effects that result from those partial-equilibrium responses may produce further effects on wage concentration.

To gauge the potential implications for wage concentration of both the partial- and the general-equilibrium effects that result from endogenous investment responses, we repeat the simulation experiment that we presented in the previous section, but now in a model variant where skill types are endogenous. We model skill investment choices as being optimally made by underlying (unobservable) individual learning types, λ, where λ is a learning productivity parameter that negatively affects the cost, $c(s, \lambda)$, of acquiring skill level s. This cost is assumed to be isoelastically increasing and convex in s and inversely related to λ – i.e. $c(s, \lambda) = (s - \underline{s})^{\eta}/(\eta\lambda)$, $\eta > 1$.[18] Assuming $\eta = 2$ (quadratic costs), the FOC for an optimal choice of s_0 by a given

[18] It can be shown that, under these cost conditions, skill investment responses are neutral in the following sense: if wages are linearly increasing in s, or, more generally, if they are isoelastically increasing in s, then endogenous changes in skill investment choices following an equiproportional

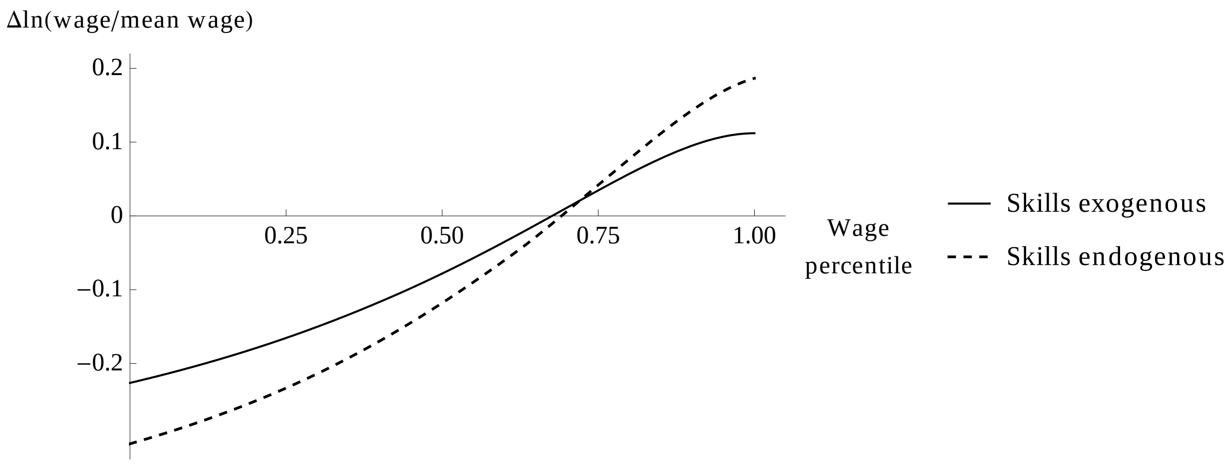

Figure 4: $\Delta \ln$ of normalized wage shares by wage percentile
– endogenous vs. exogenous skill supply

λ type under the wage profile $w_0(s) = w(s, \theta_0)$ then involves $c'(s) = w_0'(s)$, and so $\lambda = (s_0 - \underline{s})/w_0'(s_0)$. Analogously, an optimal choice of s_1 by the same learning type under the (changed) wage profile $w_1(s) = w_0(s, \theta_1)$ – changed as a result of a change in an exogenous parameter θ – implies $\lambda = (s_0 - \underline{s})/w_0'(s_0)$. Equating the right-hand sides of these two equalities and solving for s_0, we obtain

$$s_0 = \underline{s} + (s_1 - \underline{s})\,\frac{w_0'(s_0)}{w_1'(s_1)}. \tag{52}$$

This defines an implicit mapping $\tilde{s}_0(s_1)$ that determines an endogenous distribution of skills from a given initial distribution, through the relationship

$$F_1(s) = F_0\big(\tilde{s}_0(s)\big), \tag{53}$$

where $F_0(s)$ refers to the cumulative of the original distribution and $F_1(s)$ to the cumulative of the new, endogenous distribution. Other than replacing $F(s)$ with $F_1(s)$ as defined by (53), the model's equilibrium conditions are as before.

Figure 4 compares effects on wage shares by wage percentile with and without endogenous skill investment responses for a change from $q = 1/4$ to $q = 1$ – with the equilibrium for $q = 1$ being the same in both cases. This shows a markedly more

change in wages leave wage concentration unchanged.

pronounced ratio-dominant change in the distribution of earnings when skills are endogenous. When $q = 1/4$ and skills are endogenous, the Gini coefficient is 0.27 rather than the value of 0.31 obtained for a scenario where the distribution of skills is exogenous (implying a larger increase in concentration when expanding market size to $q = 1$), and the range of tasks carried out by machines falls to $[0,\ 0.16)$. As noted above, the effect on wage concentration in this case is the combined result of a first-order, ratio-dominant change resulting from an increase in τ (just as when skills are exogenous), of endogenous responses in skill choices, and of differential second-order changes in the skill-wage profile induced by skill supply changes. Thus, endogenous responses in skill supply can exacerbate effects of international market integration on wage concentration, rather than mitigating them.[19]

4 Trade, machines, and the ranking of skills

The trade-driven process of wage concentration we have described revolves around patterns of comparative advantage for workers of heterogeneous skill types in the execution of those tasks that can be carried out comparatively more effectively by machines; the fact that in the model workers that have a comparative disadvantage in lower-τ type tasks are higher wage workers (i.e. higher skill workers) only comes from an implicit choice of measurement units for labour inputs, and is not essential for the result. Indeed, which skill types are high-skill types (in the sense of receiving a higher wage) can be made fully endogenous in the model without any substantial modification to the arguments and conclusions. To see this, consider the following modified specification of $\pi(\tau, s)$:

$$\widetilde{\pi}(\tau, s) = \tau \, \frac{1 + \tau s^Z}{h(s^Z)}, \tag{54}$$

[19]When measured in terms of years of schooling, educational inequality has declined in most developed countries over the last three decades – if only because of legislated increases in the minimum school leaving age compulsory school attendance age. Depending on the shape of the mapping between years in schooling and (unobservable) levels of skill attainment, there need not be a conflict between decreasing concentration in one and increasing concentration in the other. Indeed, Lemieux (2006) suggests that there has been a dramatic increase in the dispersion of returns to post-secondary education.

where $s^Z \equiv s/(\bar{s} - \underline{s})$, and where $h(s^z)$ equals $\omega(s)$ (as defined in (34)) when this is evaluated under a linear assignment schedule $\tilde{\tau}(s^Z) = s^Z - \underline{s}^Z$. This re-specification of $\pi(\tau, s)$ modifies the productivity of skill types in a manner that is orthogonal to τ, and therefore amounts to a re-scaling of labour units by skill type – so that one unit of supply of skill s measured in the original unit equals an amount $h(s^Z)$ of supply of skill type s measured in the re-defined unit – in conjunction with a corresponding re-specification of the distribution of skill types in terms of the re-scaled units. The re-scaled model specification and its equilibrium properties are thus fully equivalent to the specification that we have used for our earlier derivations, although workers' wages would be different if each worker is now equated with the re-scaled unit rather than with the original unit. Consider then such a re-specification of $\pi(\tau, s)$ in a scenario where the re-scaled density distribution of the distribution of skill types is uniform, and suppose that machines are initially unavailable or prohibitively costly, implying $\underline{\tau} = 0$. By construction, then, the equilibrium assignment rule is linear and results in a uniform equilibrium wage schedule $\hat{w}(s) = 1$: for $\tilde{\tau}(s^Z) = s^Z - \underline{s}^Z$, productivity $\tilde{\pi}(\tau, s)$ is constant and equal to unity across skill types; and with a uniform $f(s)$, this assignment rule makes the supply of tasks uniform across tasks. Thus, in the absence of machines, no skill type would be intrinsically "higher" than any other. Starting from such an equilibrium, however, if machines are made available at a sufficiently low cost, the wage profile would become positively sloped in s, and so higher-s workers would become higher-skilled workers – in other words, the availability of machines is what makes high-s workers high-skill workers.

The above discussion highlights an unavoidable limitation of any theory attempting to derive fully general predictions on wage concentration in the presence of workers' heterogeneity: any measurement of wage concentration, by its very nature, hinges on a ranking of skill types by wage level – a ranking that is based not only on qualitative differences across skill types but also on productivity differentials by skill type. For example, the ratio dominance we have adopted to measure effects on concentration, as reflected in (33), relies on $w(s)$ being increasing in s. If s is positively related to a comparative disadvantage in tasks that can be carried out comparatively efficiently by machines, but we re-scale units of labour supply so that $w(s)$ is no longer increasing in s, while leaving the equilibrium exactly the same in terms of the re-scaled units, we still get to the conclusion that market integration produces an increasing proportional change $\mathrm{d} \ln w(s)/\mathrm{d}q$ by skill type s (condition (35)); but this result – the exact same result obtained in a fully equivalent, re-scaled model – now signifies a *reduction* rather than an increase in wage concentration. Thus, although the mechanism we have described only depends on the comparative advantage of different worker types and of machines in performing different tasks, its implications for wage concentration do rely on individuals who supply higher-s type skills initially receiving a higher wage – when this wage is measured as the product of the price and quantity of the skill type they individually supply. This is a pattern for which there is some empirical support (Katz and Margo, 2013); but, on the face of it, not one that can be supported by theoretical reasoning alone.

Nevertheless, it is possible to obtain a theoretical prediction on wage concentration that makes allowances for scaling and thus applies more generally, i.e. under weaker conditions than those implied by our previous analysis. Suppose that, for a given distribution of skill types and an initial level of q, the scaling of units is such that, to begin with, having a higher-s skill does not necessarily translate into receiving a higher wage. Then, a higher q will produce a proportionally increasing effect on $\omega(s)$, but not necessarily a ratio-dominant change in the wage distribution. However, if we keep increasing q indefinitely, pushing $\underline{\tau}$ arbitrarily close to unity, and as long as the initial, "inverted" wage gaps are not too large, when q increases beyond a certain level, wage levels will become increasing in s, and so, beyond that level of integration, wage concentration will be unambiguously increasing in q; i.e. even if the effects of market integration on wage concentration are initially negative or uneven in sign across the wage distribution, they may become uniformly positive as integration deepens:

Proposition 3 *For $q = q_0$, there exists a lower bound $\underline{R}_{q_0}(s_1, s_2)$ for each pair of skills levels s_1 and s_2 such that $s_2 > s_1$, such that if the ratio of the initial wages of skill types s_2 and s_1 lies above $\underline{R}_{q_0}(s_1, s_2)$ for all s_1, s_2, an increase in q asymptotically raises wage concentration.*

PROOF: For q approaching infinity, $z(m)$ approaches zero, $\underline{\tau}$ approaches unity, the equilibrium assignment schedule approaches $\hat{\tau}_{q\to\infty}(s) = 1$, and the normalized wage schedule approaches $\hat{\omega}_{q\to\infty}(s) = \exp\left(\int_{\underline{s}}^{s} (1/(1+x))\,\mathrm{d}x\right) = (1+s)/(1+\underline{s})$. Suppose now that we re-scale labour units so that the individual productivity of a worker of type s is $\tau(1+\tau s)/\varphi(s)$.

For any two skill levels, s_1 and s_2, such that $s_2 > s_1$, if it is the case that

$$\varphi(s_2)/\varphi(s_1) < (1 + s_2)/(1 + s_1), \tag{55}$$

then, as q approaches infinity, the ratio of re-scaled wages $\dfrac{\hat{\omega}_{q\to\infty}(s_2)/\varphi(s_2)}{\hat{\omega}_{q\to\infty}(s_1)/\varphi(s_1)}$ will lie above unity. Provided that (55) is satisfied for all pairwise combinations of skill levels, then for q sufficiently large, an increase in q will induce a ratio-dominant change in the distribution of skills.

Now let $\hat{\tau}_{q_0}(s)$ be the equilibrium assignment schedule for $q = q_0$ (as identified by (26)-(27)); let $\hat{\omega}_{q_0}(s)$ be the corresponding equilibrium normalized wage schedule (as identified by (34) with $\tau(t) = \hat{\tau}_{q_0}(s)$); and let $w^S_{q_0}(s_2)/w^S_{q_0}(s_1) \equiv R^w_{q_0}(s_2, s_1)$ be the ratio of the actual (scaled) wages for skill types s_2 and s_1 at $q = q_0$. The scaling ratio $\varphi(s_2)/\varphi(s_1)$ that must be implied in moving from the ratio $\hat{\omega}_{q_0}(s_2)/\hat{\omega}_{q_0}(s_1) \equiv R^{\hat{\omega}}_{q_0}(s_2, s_1)$ to the ratio $R^w_{q_0}(s_2, s_1)$ is then equal to $\varphi(s_2)/\varphi(s_1) = R^{\hat{\omega}}_{q_0}(s_2, s_1)/R^w_{q_0}(s_2, s_1)$. Combining this with (55), we can conclude that a sufficient condition at $q = q_0$ for a ratio dominance effect to occur asymptotically (for q sufficiently large) is

$$R^w_{q_0}(s_2, s_1) > \frac{R^{\hat{\omega}}_{q_0}(s_2, s_1)}{(1 + s_2)/(1 + s_1)}, \quad s_1, s_2 \in S. \tag{56}$$

Note that, since $(1 + s_2)/(1 + s_1) = R^{\hat{\omega}}_{q\to\infty}(s_2, s_1)$, and since $R^{\hat{\omega}}_{q\to\infty}(s_2, s_1) > R^{\hat{\omega}}_{q_0}(s_2, s_1)$ by the ratio dominance result previously obtained (Proposition 2), the right-hand side of (56) is less than unity – i.e. a ratio dominance effect can apply asymptotically even if the initial scaled wages are decreasing rather than increasing in s. □

5 Concluding remarks

We have shown how extending a job assignment model by allowing intermediate inputs to compete with workers in the execution of production tasks, and combining it with a conventional, monopolistically competitive model of international trade, naturally gives rise to the prediction of a positive relationship between market integration and wage concentration. Effects on wage concentration are compounded by the positive interaction between the level of output and the number of varieties produced in an economy, and can be further strengthened by endogenous skill investment responses.

The model we have presented delivers a rich set of predictions that allow us to derive differential effects of market integration on heterogeneous workers from an otherwise symmetric framework. While this is theoretically appealing, it means neglecting other dimensions of heterogeneity that are both theoretically important and practically relevant for empirical applications, such as firm heterogeneity (Melitz, 2003). In particular, this omission implies that our analysis cannot address the well-documented link between intra-sectoral and intra-occupational wage dispersion and export conduct at the firm level (Helpman *et al.*, 2012).

Our analysis also abstracts from a number of additional dimensions that are likely to be relevant to the relationship between market integration and wage concentration. Besides comparative advantage, we have also deliberately abstracted from directed technical progress. We have briefly mentioned the role of non-reproducible factors other than labour, but we have not examined the possibility that these factors may combine with other inputs in a non-separable way, introducing a further mechanism that would make the job assignment problem vary with the scale of market integration and production. Finally, our stylized representation of market integration abstracts from the role of trade costs, and specifically from the role that produced machines may play in transportation. These and other extensions are left for future research.

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