Life in Shackles? The Quantitative Implications of Reforming the Educational Loan System

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Abstract

In this paper we conduct a quantitative analysis of a number of stylized educational loan systems. We develop a stochastic general equilibrium model of a closed economy with a competitive firm sector and a government that levies taxes and administers educational loans. Individuals are heterogeneous in their talent for education and ability to learn on the job and face uninsurable idiosyncratic labour productivity risk during their working career. We calibrate the model to the US mortgage loan system and subsequently consider two possible reforms. The first is a Graduate Labour Tax (GLT) system whereby grants to students are financed by means of a tax on the labour income of educated individuals. We find that in the long run the proportion of uneducated workers stays roughly constant but the average educational attainment of students increases. As there exists a considerable amount of transitional dynamics in the model the welfare effects of the reform differ by generation. Cohorts alive at the time of the shock are worse off while ex-ante welfare of future cohorts increases. The gains to the latter are large enough to – at least in principle – compensate the losers from the policy reform and generate an overall welfare gain. The second possible reform we study is a Comprehensive Labour Tax (CLT). It is very similar to the GLT except for the fact that the educational tax is levied on all workers, including those who are uneducated. In contrast to the GLT reform the proportion of uneducated workers drops substantially. Generations that become economically active soon after the policy reform are worse off and the aggregate ex-ante welfare effect is negative.

JEL-Code: E100, E240, D910, I220, J240.

Keywords: human capital, experience effects, educational loans, uninsured labour market risk, incomplete markets, overlapping generations.

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1 Introduction

> "...student loan systems [...] are often badly designed for an extended period of high unemployment. In contrast to the housing crash, the risk from student debt is not of a sudden explosion in losses but of a gradual financial suffocation. The pressure needs to be eased."
>
> *The Economist* (October 29th, 2011)

Obtaining a college degree typically requires a large investment of time and money. In order to facilitate access to higher education most governments have instituted an educational loan system of one kind or another. For example, in the United States there are four major federal sources of loans (subsidized and unsubsidized Stafford loans, the PLUS program, and the Perkins loans) as well as private sector loans (Avery and Turner, 2012). In contrast, in Australia higher education is financed with income-contingent loans (Chapman, 1997). In such a system an individual starts the repayments of study debt only after a certain income threshold is reached. Finally, whereas in the Netherlands basic grants to students are currently paid out of general tax revenue, there are plans to move to a so-called Social Borrowing System. This is essentially a system of mortgage loans similar to the US.

The existence of these educational loan systems ensures that access to tertiary education in most developed countries is relatively good, but depending on the system in place graduates from colleges and universities may enter the labour force with a substantial amount of study debt. The quintessential horror story is that of the National Consumer Law Centre's client who has a $300,000 debt resulting from a failed attempt to become an airline pilot (*The Economist*, October 29th, 2011). Although this is an extreme case, some commentators suggest that the educational loan system is producing generations of the educated "indebted ones". In their view the government is hanging a mill stone around the necks of those youngsters who have to borrow funds in order to finance their tertiary education. The theoretical literature on this topic has suggested that the burden could be alleviated by moving away from a pure loan system to one involving graduate taxes. Under such a system individuals do not have an explicit debt but instead an implicit obligation to contribute to educational financing in the form of an additional tax on their labour income.

In a masterful chapter in *Capitalism and Freedom*, Milton Friedman strongly favours a system of graduate taxes (1962, p. 105). The arguments with which he supports his position are worth repeating here. First, he argues that tertiary education is unlikely to feature significant external effects and that higher education is "...a form of investment in human capital precisely analogous to investment in machinery, buildings, or other forms of non-human capital" (p. 100). With a perfect capital market there would be no role for government interference. Second, he notes that the rate of return on human capital investment is much higher than the rate of return on investment in physical capital and concludes that in the laissez faire economy there is underinvestment in human capital resulting from capital market imperfections. The main reason is that, "[i]n a non-slave state, the individual embodying the investment cannot be bought and sold". Third, he argues that private mortgage loans would be unattractive to borrowers because of the large risk-of-default premium that private lenders would require.

What is needed is some kind of limited-liability equity financing scheme. For education it would be advantageous if it were possible "…to 'buy' a share in an individual's earning prospects; to advance him the funds needed to finance his training on condition that he agree to pay the lender a specified fraction of his future earnings. In this way, a lender would get back more than his initial investment from relatively successful individuals, which would compensate for the failure to recoup his original investment from the unsuccessful. There seem no legal obstacle to private contracts of this kind, even though they are economically equivalent to the purchase of a share in an individual's earning capacity and thus to *partial slavery*." (p. 103) [emphasis added]. Finally, he closes the case in favour of graduate taxes by noting that the government is able to institute such a system of equity investment in human beings at a much lower cost than the private sector could because it already possesses the power to tax individuals.

Despite Friedman's strong advocacy of graduate taxes, many different systems of educational financing have been adopted around the world as we pointed out above. Is this because Friedman's message was not understood by policy makers, or are these financing systems – though theoretically distinct from graduate taxes – in practice more or less equivalent? To answer this inherently quantitative question, we conduct a formal computational analysis of a number of stylized study loan systems in this paper. Taking the US mortgage loan system as our point of departure we investigate the microeconomic and macroeconomic effects of two reforms. The first reform consists of a change from – what we call – the Subsidized Mortgage Loan (SML) system to one whereby the government implements a graduate labour income tax in order to finance its educational system. Only college and university graduates are liable for the tax, i.e. uneducated workers are exempt by assumption. In the second reform we consider the change from the SML system to one by which the government uses general labour income tax revenue to finance the educational system. We label this the Comprehensive Labour Tax (CLT). Of course, the key difference between GLT and CLT is that in the latter case all workers – even the uneducated ones – must contribute to the educational loan system. For each reform scenario we compute both the transitional and long-run effects of the policy change and we consider both the effects on the economic allocation and on welfare by pre-reform and post-reform generations.

The innovative features of our model are in the modeling of households. Following the pioneering work by Bewley (1977), Aiyagari (1994) and Huggett (1993, 1997) we assume that agents experience uninsurable idiosyncratic labour market risk during part of their life cycle. In the spirit of Krebs (2003), Abbott *et al.* (2013), Huggett *et al.* (2011), Kindermann (2012), and Krueger and Ludwig (2013) we enrich this workhorse model of modern quantitative macroeconomics by including features of the human capital accumulation process. At the start of adult life, an individual must choose the optimal amount of education. In the education phase agents are not experiencing any stochastic shocks by assumption. Once schooling is completed, however, the graduate joins the labour force and enters the risky part of the life cycle. By working an individual accumulates human capital via a learning-by-doing mechanism. Despite the fact that the rental rates on physical and human capital are deterministic in the absence of aggregate risk, labour market earnings are stochastic as a result of idiosyncratic labour productivity shocks. We introduce a tractable way to capture the notion of unemployment, i.e. periods with zero labour income. It is possible that somebody with a sizeable study debt experiences a bad run in the labour market and has trouble meeting the repayments. The different study loan systems that we discuss in this paper will influence the kind of financial distress that someone experiences during the repayment period and beyond.

Our main results regarding the policy change from SML to GLT are as follows. First, in the long run the proportion of uneducated workers stays roughly constant but here are significant changes in the shares of the different educational groups. With all graduates paying the tax, educated workers in the new steady state consistently have a higher schooling level than they did under the SML system. Indeed, the fraction of people with associate degrees is reduced whilst the shares of the more highly educated groups increase.

Second, there are sizeable effects on the macro-economy. In the long run, the capital stock and effective employment increase by, respectively, 2.72% and 0.23%. Since capital becomes relatively abundant its return drops by 0.14 percentage points whilst the wage rate increases by 0.56%. Finally, steady-state consumption and output increase by, respectively, 0.30% and 0.79%. So from a macroeconomic perspective it is hard to maintain that SML and GLT constitute more or less equivalent systems of educational financing.

Third, there exists a considerable amount of transitional dynamics in the model and it takes roughly half a century before the economy is close to its new steady state. The slow but realistic transition speed results from the fact that there are two slow-moving stocks in the model, namely physical and human capital (cf. Mankiw *et al.*, 1992).

Fourth, because of the slow transition, the ex-ante welfare effects experienced by existing generations display a distinct age profile whilst that for future generations features a noticeable time profile. For adults economically active at the time of the shock ex-ante welfare invariably falls. For working-age generations this result follows readily from the fact that they are – in a sense – paying the same bill twice. They must continue to pay off any existing study debt but are also hit by a higher labour-income tax. Students are hurt also, but to a lesser extent the younger they are (and thus the lower is the incurred study debt). Middle-aged and old generations have paid off their study debts and are hurt mainly by the graduate tax. Interestingly, all future generations gain from the policy change. Furthermore, their gains are large enough to – at least in principle – compensate the losers from the policy reform. From an ex-ante welfare perspective, therefore, we reach the conclusion that Friedman was right after all and that GLT is a better system than SML.

Fifth, from an ex-post welfare perspective, our results imply that individuals of all educational abilities are better off in the new steady state. This result holds even for the least able (who are not confronted by the educational tax at all) and the most able (who face income-contingent rather than fixed loan repayments under the new system).

Our quantitative results show that the effects of a policy change from SML to CLT differ from the first scenario along a number of dimensions. First, under the CLT scenario every worker pays for the educational system and this has an important effect on the educational composition of the labour force in the new steady state. Indeed, the proportion of unskilled workers drops from 52.02% in the base case to 40.90% in the CLT scenario whereas it hardly changed in the reform from SML to GLT. By confronting all workers with the educational tax, even those who previously chose not to take any tertiary schooling now choose to obtain an associate degree. Since they cannot avoid paying the tax they decide to enter school and reap at least some of the benefits of the system in the form of "free" educational grants. Second, future generations that are economically active close to the time of the reform are worse off as a result of it (whereas they were better off under the first reform) and the aggregate ex-ante welfare effect of the policy change is negative. Third, from an ex-post perspective individuals with the lowest educational ability are worse off in the new steady state. Such agents fall victim to reverse redistribution in the sense that they partially pay the bills for people who are more educated and wealthier than themselves.

Our paper relates to an growing literature. There are many theoretical contributions dealing with the financing of higher education. Prominent examples include García-Peñaloza and Wälde (2000), Jacobs and van Wijnbergen (2007), Cigno and Luporini (2009), and Del-Rey and Racionero (2010). These papers are invariably highly stylized in their description of economic decision making and are thus unsuitable for the quantitative analysis of study loan systems. In recent years, however, a literature had emerged which uses the techniques of modern stochastic macroeconomics – in particular the incomplete markets model – to study education subsidies. Examples include Akyol and Athreya (2005), Ionescu (2009), Krueger and Ludwig (2013), and Abbott *et al.* (2013). Of these, the paper by Abbott *et al.* (2013) is most closely related to ours. Although both papers use a common quantitative methodology, their focus is quite distinct. For example, Abbott *et al.* (2013) include a detailed description of how individuals decide about education and what exactly their resources are during the schooling period, i.e. both the study loan system and the borrowing constraints are modeled in detail. In addition, they include in vivo transfers from parents to offspring and assume that there exists an intergenerational transmission of ability. In their computational implementation they restrict attention to steady-state comparisons. In contrast, we focus mainly on the design of repayment schemes for government loans, keeping resources of individuals at the beginning of life (and during the time of study) constant. By adopting a less detailed description of the schooling phase we are able to compute the transitional dynamic effects of policy reforms. In doing so we can demonstrate the rather uneven distribution of costs and benefits over the different generations. We thus show that the actual implementation of policy reforms that improve long-run welfare may meet with a lot of political opposition.

The remainder of the paper is structured as follows. In Section 2 we formulate our base model. Section 3 discusses the calibration and visualizes some key features of the base model. Sections 4 and 5 present the quantitative results from our two reform scenarios. Section 6 summarizes and concludes. Technical issues are discussed in a number of brief appendices.

2 Model

In this section we develop a stochastic general equilibrium model of a closed economy. Compared to the existing literature on study loan systems the main innovation of our approach concerns the way in which we model individuals. Following the pioneering work by Bewley

(1977), Aiyagari (1994), and Huggett (1993, 1997) we assume that agents experience uninsurable idiosyncratic labour market risk during part of their life cycle. At the start of adult life, an individual must choose the optimal amount of education. During this phase the student receives study loans from the government which allow him/her to pay tuition fees and to consume goods. Since individuals are heterogeneous in their innate talent for education, the optimal educational choice and thus the amount of study debt will be different for different people. In the schooling phase agents are not experiencing any stochastic shocks by assumption. Once schooling is completed, however, the graduate joins the labour force and enters the risky part of the life cycle. By working the individual accumulates human capital via a learning-by-doing (LBD) mechanism. The magnitude of an agent's LBD parameter is only revealed at the start of the work career. Despite the fact that the rental rate on human capital is deterministic, labour market earnings are stochastic as a result of idiosyncratic labour productivity shocks. It is thus possible that somebody with a sizeable study debt experiences a bad run in the labour market and has trouble meeting the repayments. The different study loans systems that we discuss in this paper will influence the kind of financial distress that someone feels during the repayment period and beyond.

2.1 Firms

Perfectly competitive firms combine physical capital and efficiency units of labour in order to produce homogeneous output, the price of which serves as the numeraire. We abstract from aggregate uncertainty and capital adjustment costs so the representative firm essentially makes a sequence of static decisions regarding output supply and factor demands.

The production function is of the Cobb-Douglas type:

$$Y_t = \Phi^0 K_t^{\phi} [Z_t N_t]^{1-\phi}, \quad 0 < \phi < 1, \quad \Phi^0 > 0, \tag{1}$$

where t is the time index, Y_t is output, K_t is the stock of physical capital, and N_t is the amount of effective labour employed in production. The index of labour-augmenting technological change, Z_t, grows at an exogenous rate $n^z > 0$. The firm's stock of physical capital evolves according to $K_{t+1} = (1 - \delta^k)K_t + I_t$, where I_t is gross investment and δ^k is the (constant) rate of depreciation. The real profit flow at time t is given by $Y_t - w_t N_t - (r_t + \delta^k)K_t$, where r_t is the interest rate and w_t is the rental rate on effective labour. The profit-maximizing mix of inputs gives rise to the following marginal productivity conditions:

$$r_t + \delta^k = \phi\Phi^0 \left[\frac{K_t}{Z_t N_t}\right]^{\phi-1}, \qquad \frac{w_t}{Z_t} = (1-\phi)\Phi^0 \left[\frac{K_t}{Z_t N_t}\right]^{\phi}. \tag{2}$$

With these factor demands, profit is zero because of the linear homogeneity of the technology.

2.2 Individuals

Each individual lives for $\bar{U} + 1$ years with certainty, such that age $u \in \{0, 1, \ldots, \bar{U}\}$. At the start of each period v a cohort of size $P_{0,v}$ is born. The size of the cohort of age u in year t

is given by $P_{u,t} = P_{0,t-u}$. The total population in a given year is equal to $P_t = \sum_{u=0}^{\bar{U}} P_{u,t}$. We assume that the population grows at a constant rate n^p.

2.2.1 Stochastic environment over the life cycle

The individual reaches majority at age $u = M$ and starts making economic decisions from that age onward. The sequence of events in a person's life is summarized in Figure 1.

At different moments in the life cycle nature draws two important learning characteristics. First, at the age of majority the innate talent for education, θ, is drawn from a distribution with support $[0, 1]$ and cumulative distribution function F_θ. Educational talent affects the returns to education experienced by the individual. In particular, the stock of human capital at labour market entry given talent for education θ and years of education E is given by $\Gamma(\theta, E)$:

$$\Gamma(\theta, E) = 1 + \xi_1 \theta E - \xi_2 [1 - \theta] E^2, \quad \xi_1 > 0, \quad \xi_2 > 0. \tag{3}$$

The amount of start-up human capital is assumed to be deterministic, i.e. we abstract from input risk. Note furthermore that someone who chooses no education at all ($E = 0$) enters the labour force with one unit of human capital.

Second, upon completion of the optimally chosen schooling period, nature reveals the agent's ability to learn on the job which we denote by γ. For reasons of tractability we assume that the learning-by-doing (LBD) parameter γ can take on only two values, $\gamma \in \{\gamma_l, \gamma_h\}$, with $0 < \gamma_l < \gamma_h < 1$. Furthermore, we postulate that γ is correlated with θ and features a cumulative distribution function $F_{\gamma|\theta}$.

Third, there is a draw for the idiosyncratic labour productivity shock η in every period. For computational reasons we assume that the process for η takes the form of a four-state stationary Markov chain with the following features. First, $\eta \in \{0, \eta_l, 1, \eta_h\}$ with $0 < \eta_l < 1 < \eta_h$. Second, we assume that the transition probabilities depend on the individual's schooling level and write the conditional (cumulative) distribution function for η^+ as $F_{\eta^+|\eta,E}$, where η^+ is next period's productivity level.[1]

The structural features of the Markov scheme are illustrated in Figure 2. In the numerical implementation of the model we incorporate some real world features into this simple Markov scheme which we find relevant to workers who may have a sizeable explicit (or implicit) study debt resulting from their educational period. Whereas the financial obligations of graduates are crystal clear, the employment period is inherently risky and so is their ability to pay back the loan. First, we capture the notion of (temporary) unemployment by setting the lowest realization for η equal to zero. Second, we choose the typical elements of the Markov transition matrix in such a way that it captures some key characteristics of wage income data of employed individuals in the US (such as persistence and variability – see below). Third, we assume that the first draw of η upon labour market entry is equal to unity, which we call average productivity. Fourth, we impose a lot of additional structure on the Markov process in that (a) any productive worker can become unemployed, (b) barring moves to unemployment

[1]We use the notation of Cai and Judd (2010) by writing the productivity levels in the current and the immediately following period as, respectively, η and η^+.

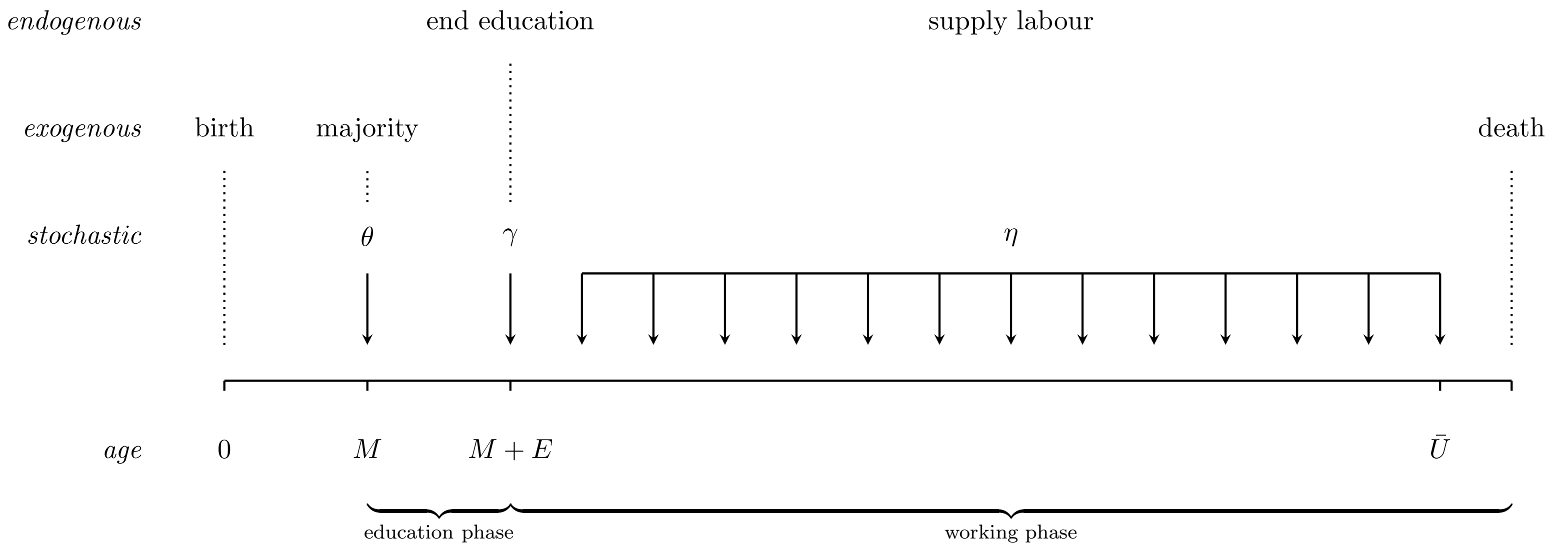


Figure 1: The individual's life cycle

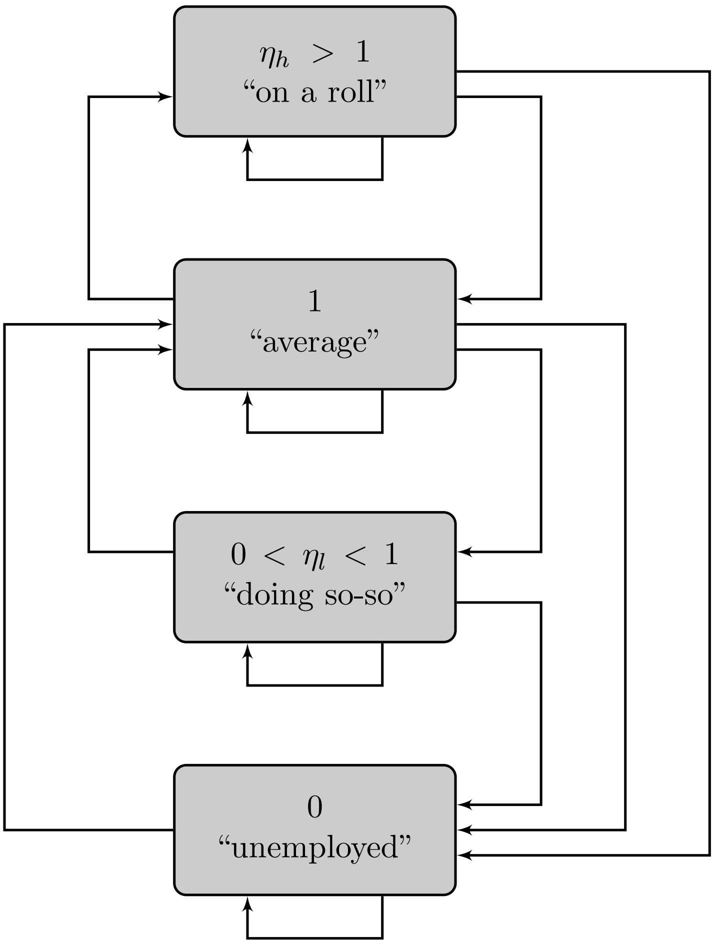


Figure 2: Markov process for labour productivity η

a productive worker can only move up or down by a single state level, and (c) a previously "unemployed" individual moves to the average productivity level or stays unemployed.

Individuals are assumed to be fully aware of the stochastic environment they live in and to formulate optimal life-cycle plans which maximize their utility subject to the constraints they face. It is most convenient to describe an individual's optimization problem backwards, i.e. starting with the employment phase and ending with the education phase.

2.2.2 Optimal decisions of a worker

Consider a worker who is of age u, has enjoyed E years of education, and features a LBD parameter γ. At the start of year t this individual owns stocks of financial assets a and human capital h and has a labour productivity level η. The individual chooses current consumption c and labour supply l as well as next-periods financial assets a^+ and human capital h^+ in order to maximize remaining lifetime utility. The optimization problem is characterized by the Bellman equation:

$$V_{u,t}(E,\gamma,a,h,\eta) = \max_{c,l,a^+,h^+} \left\{ \left[c^{\varepsilon}(1-l)^{1-\varepsilon}\right]^{1-1/\sigma} + \beta\left[\mathbb{E}_{\eta^+|\eta,E}\left[V_{u+1,t+1}(E,\gamma,a^+,h^+,\eta^+)^{1-\zeta}\right]\right]^{\frac{1-1/\sigma}{1-\zeta}} \right\}^{\frac{1}{1-1/\sigma}} \tag{4}$$

in combination with the laws of motion of the state variables and the constraints on the choice variables:

$$a^+ = [1+(1-\tau_t^r)r_t]a + (1-\tau_t^w)w_t\,\eta\,h\,l - (1+\tau_t^c)c + \nu_{u,t}\mathbb{1}_{\{\eta=0\}} - \Upsilon_{u,t}(E, w_t\,\eta\,h\,l) \tag{5a}$$

$$h^+ = (1-\delta_u^h)[1+\gamma l^{\alpha}]h \tag{5b}$$

$$0 \le l \le 1, \qquad c \ge 0, \qquad a^+ \ge 0, \tag{5c}$$

where β is the time discount factor, τ_t^r, τ_t^w, and τ_t^c are tax rates on, respectively, interest income, wage earnings, and consumption, $\nu_{u,t}$ is the unemployment benefit, $\mathbb{1}_{\{\eta=0\}}$ is an indicator function which equals unity if $\eta = 0$ and zero otherwise, and $\Upsilon_{u,t}(E,W)$ is the payment to the study loan system during period t for someone of age u with education E and gross wage income W.[2]

Several things are worth noting. First, the preference structure satisfies the King-Plosser-Rebelo conditions (see King *et al.*, 2002) so that – in the presence of ongoing labour productivity growth – a stationary decision problem is obtained by scaling the individual's consumption and financial assets (as well as wages, unemployment benefits, and repayments) by an index of productivity. See Appendix B for details.

Second, preferences are of the recursive form suggested by Epstein and Zin (1991) which allows us to disentangle the agent's attitudes towards risk and intertemporal consumption

[2]We include a highly stylized unemployment benefit system in which the benefit is the same for everyone and thus does not depend on the last-earned wage.

smoothing. Using the terminology of Backus *et al.* (2004, p. 341), the time-aggregator and certainty-equivalent functions are both of the CES type. In this formulation, σ parameterizes the *inter*temporal substitution elasticity ($\sigma > 0$) whilst ζ captures the degree of relative risk aversion ($\zeta \geq 1$). The instantaneous felicity function is a Cobb-Douglas aggregate of consumption and leisure (with $0 < \varepsilon < 1$). This implies a unitary *intra*temporal substitution elasticity between consumption and leisure.

Third, the expectation $\mathbb{E}_{\eta^+|\eta,E}$ is with respect to η^+ and conditional on information about η and E, i.e. it is computed using the cumulative conditional distribution function $F_{\eta^+|\eta,E}$. Fourth, since γ is revealed at the start of the working phase and E is predetermined both are constant. Finally, the value function depends on t because factor prices do. In addition it depends on the individual's age u because this determines the remaining length of life.

Expression (5a) states that the change in financial assets is equal to after tax income net of spending on consumption and payments to the study loan system. Expression (5b) shows that the accumulation of human capital during the working phase depends on two distinct mechanisms. The *learning-by-doing effect* (LBD) is captured by the term γl^α. Conditional on the agent's LBD coefficient γ, more experience is gained the more the individual works (though at a diminishing rate as $0 < \alpha < 1$). The *economic ageing effect* is captured by the term $1 - \delta_u^h$ and results from the fact that the depreciation rate on human capital is taken to be increasing in age (as in Heijdra and Reijnders, 2012). In particular for $M \leq u \leq \bar{U}$ we postulate:

$$\delta_u^h = 1 - \delta_0 \left(\frac{\bar{U} - u}{\bar{U} - M} \right)^{\delta_1}, \quad 0 \leq \delta_1 \leq 1, \quad 0 < \delta_0 \leq 1. \tag{6}$$

Finally, the expressions in (5c) show that labour supply, consumption, and financial assets must be non-negative. We thus impose the restriction – conventional in the macroeconomic literature on idiosyncratic risk – that individuals are unable to borrow for other purposes than financing their education. An often stated rationale for this borrowing constraint is that there is a positive probability of receiving zero wage income in one or more periods (Low, 2005, p. 951) and human capital is inalienable (Friedman, 1962, p. 102).

The solution to the worker's decision problem gives a set of policy functions for the working phase which we write as follows:

$$\mathbf{c}_{u,t}(E, \gamma, a, h, \eta), \quad \mathbf{l}_{u,t}(E, \gamma, a, h, \eta), \quad \mathbf{a}^+_{u,t}(E, \gamma, a, h, \eta), \quad \mathbf{h}^+_{u,t}(E, \gamma, a, h, \eta). \tag{7}$$

2.2.3 Optimal decisions of a student

Individuals enter adulthood without any financial assets and with an endowment of one unit of human capital. Formal education takes place when the agent enters adulthood at age M and requires a fixed time input of e^0 each period. Since the time endowment equals unity and working and studying are assumed to be mutually exclusive activities it follows that leisure during the educational phase is given by $1 - e^0$. In the absence of labour income students finance their living expenses with government-provided study loans. The student loan inflow q_t^0 and the tuition fee f_t^0 are exogenously determined and increase over time at the rate of economic growth, n^z. In addition we assume that consumption during the educational phase is also fixed, i.e. c_t^0 is the remainder of the study loan after paying the tuition fee:[3]

$$c_t^0 = \frac{q_t^0 - f_t^0}{1 + \tau_t^c} \tag{8}$$

The education decision constitutes a discrete choice in the sense that there are only four possible levels on offer, i.e. we postulate that $E \in \{0, 2, 4, 6\}$, where $E = 0$ stands for no (tertiary) education, $E = 2$ is an associate degree, $E = 4$ is a bachelor's degree, and $E = 6$ is a master's degree. The stock of human capital at labour market entry for a person with educational talent θ and years of education E is given by $\Gamma(\theta, E)$ in (3). Note that the functional form of $\Gamma(\theta, E)$ implies that individuals with a higher ability level experience weaker diminishing returns to education. Furthermore, for the most talented individuals ($\theta = 1$) the relation between startup human capital and education level is linear:

$$\frac{\partial \Gamma(\theta, E)}{\partial E} = \xi_1 \theta - 2\xi_2 [1 - \theta] E, \qquad \frac{\partial^2 \Gamma(\theta, E)}{\partial E^2} = -2\xi_2 [1 - \theta] \leq 0. \tag{9}$$

These properties of $\Gamma(\theta, E)$ ensure that the optimal education choice is increasing in ability. In the absence of the diminishing-returns effect (with $\xi_2 = 0$) this may not be true because high-ability individuals also have a higher opportunity cost of time.

We postulate that it is possible to choose the years of education only when not yet working. Consider a student with educational ability θ who is u years old at time t. We write this person's expected remaining life-time utility as follows:

$$\begin{aligned} S_{u,t}(\theta) = \max_{E \geq u - M} \Bigg[& \sum_{s=t}^{t-u+M+E-1} \beta^{s-t} \left[\left(c_s^0\right)^{\varepsilon} \left(1 - e^0\right)^{1-\varepsilon} \right]^{1-1/\sigma} \\ & + \beta^{M+E-u} \left[\mathbb{E}_{\gamma|\theta} \left[V_{M+E, t-u+M+E} \left(E, \gamma, 0, \Gamma(\theta, E), 1\right)^{1-\zeta} \right] \right]^{\frac{1-1/\sigma}{1-\zeta}} \Bigg]^{\frac{1}{1-1/\sigma}} \end{aligned} \tag{10}$$

Several things are worth noting. First, during the remaining period in school the student consumes fixed amounts of goods and leisure which gives rise to the first term on the right-hand side of (10). Second, the expectation $\mathbb{E}_{\gamma|\theta}[\cdot]$ is computed using the conditional cumulative distribution function $F_{\gamma|\theta}$. Since there is no uncertainty during the education years it is just a constant. Third, at labour market entry, γ is revealed, financial assets are zero, marketable human capital is given by $\Gamma(\theta, E)$, and the agent's startup productivity is equal to $\eta = 1$. This explains the arguments entering the value function at the age of school leaving. Finding the optimal years of education gives a policy function for the education phase:

$$\mathbf{E}_{u,t}(\theta) \tag{11}$$

[3]Implicitly we assume that students would like to – but cannot – borrow more than q_t^0, i.e. students face a binding borrowing constraint.

2.3 Educational loan systems

It remains to specify the details of the system of educational loans that is in place at any time. In particular we must formulate the functional form for $\Upsilon_{u,t}(E, W)$ that features in the worker's budget constraint. We consider three prototypical systems which differ in the way in which (explicit or implicit) study debt is redeemed by workers. The base case is the Subsidized Mortgage Loan (SML) system in which individuals pay off their own study loan debt during their working career. The second case we consider is the system of Graduate Labor Tax (GLT) financing in which all *educated* workers are confronted with a "graduate tax" on their wage income, the revenue of which is used to provide study loans to current students. Finally, the third case is called the Comprehensive Labour Tax (CLT) financing system in which *all* workers face the educational tax, even those who chose not to enjoy any schooling at all.

2.3.1 SML: Subsidized mortgage loans

In this system the payment $\Upsilon_{u,t}(E, W)$ does not depend on wage income W. Implicitly there is a level of study debt which at every time t depends on age u and years of education E only. We denote the debt level of someone of age u at time t by $\Omega_{u,t}(E)$. Debt evolves over the life cycle in the following fashion. First, everyone starts adulthood without debt, $\Omega_{M,t}(E) = 0$. Second, during the education phase study debt increases as a result of interest payments on existing debt and exogenous loan inflows:

$$\Omega_{u+1,t+1}(E) = [1 + (1 - \tau_t^r) r_t] \Omega_{u,t}(E) + q_t^0, \quad \text{for } M < u + 1 \leq M + E.$$

Note that interest payments on study debt are tax deductible (from asset income). However, since students do not earn any asset or wage income they effectively receive money from the government, i.e. they are allowed to borrow at a subsidized rate of interest.

Third, during the working phase debt decreases because loan repayments exceed interest payments from then on:

$$\Omega_{u+1,t+1}(E) = [1 + (1 - \tau_t^r) r_t] \Omega_{u,t}(E) - \Upsilon_{u,t}(E, W) \quad \text{for } M + E < u + 1 \leq \bar{U}.$$

There are only redemption payments from age $\underline{u}(E)$ up to and including age $\overline{u}(E)$, the *redemption period*. If $\underline{u}(E) > E$ then there is a *grace period*. The size of the redemption payment $\Upsilon_{u,t}(E, W)$ is determined in such a way that – in the absence of unanticipated changes to the interest rate – the loan will be paid off at age $\overline{u}(E) + 1$ if the payment remains constant during the remainder of the redemption period.

Under the SML system every individual settles his or her own account. Default does not happen because (a) there is a (small) social security system in place which covers zero-income periods ($\nu_{u,t}$ in a worker's budget constraint) and (b) rational individuals accumulate precautionary savings in order to avoid getting confronted with very low consumption levels in the future.

2.3.2 GLT: Graduate labour tax

Under the GLT system the study loan effectively disappears, i.e. $\Omega_{u,t}(E) = 0$. Of course, even though the agent does not have an explicit study debt there exists an *implicit* obligation in the sense that the government imposes a tax on all educated workers. The redemption period is the entire working phase and the payment is:

$$\Upsilon_{u,t}(E, W) = \tau_t^e \mathbb{1}_{\{E>0\}} W,$$

where τ_t^e is the graduate tax rate, $\mathbb{1}_{\{E>0\}}$ is an indicator function which equals unity provided E is positive and is zero otherwise, and $W \equiv w_t \, \eta \, h \, l$ is wage income. Note that in contrast to the SML system, under the GLT system an individual can avoid making payments by not working ($l = 0$). Furthermore, under the GLT system a lucky worker (with a high realization of η) contributes more per effective work hour than an unlucky worker does.

2.3.3 CLT: Comprehensive labour tax

Under the CLT system the payment $\Upsilon_{u,t}(E, W)$ does not depend on education E directly since everybody has to pay the tax. However, since educated individuals tend to have more human capital and are less likely to be unemployed they will have higher gross wages. Just as for the GLT system, under the CLT system there is no explicit study debt, $\Omega_{u,t}(E) = 0$, and – since all workers face the educational tax – implicit redemption payments during the working phase are described by:

$$\Upsilon_{u,t}(E, W) = \tau_t^e W.$$

2.4 Aggregation

Consider a cohort that reaches age M at the start of year t_0. Every individual in this cohort has an index $i \in \{1, 2, \ldots, P_{M,t_0}\}$, and the initial endowments for each person are given by:

$$a^i_{M,t_0} = 0, \quad h^i_{M,t_0} = 1, \quad d^i_{M,t_0} = 0.$$

For every person i in this cohort we can draw a talent for education θ^i. Then we can follow this person over time. The talent for education determines the years of education:

$$E^i = \mathbf{E}_{M,t_0}(\theta^i),$$

where $\mathbf{E}_{M,t_0}(\theta^i)$ is the policy function (11) evaluated for $u = M$. Of course, if there are unexpected shocks (e.g., a change in the system of educational loans) agents who are still in school can re-optimize and choose a different schooling level than the one they planned before the shock when they were at the age of majority. During the education phase consumption, labour supply, assets, and study debt are exogenous and given by $c^i_{u,t} = c^0_t$, $l^i_{u,t} = a^i_{u+1,t+1} = 0$, and $d^i_{u+1,t+1} = \Omega_{u+1,t+1}(E^i)$, where the form of $\Omega_{u+1,t+1}(E^i)$ depends on the particular study loan system in place.

At labour market entry person i has a startup human capital stock equal to:

$$h^i_{M+E^i,t_0+E^i} = \Gamma(\theta^i, E^i).$$

We can then draw a learning ability parameter γ^i (correlated with θ^i) and a sequence of idiosyncratic productivity shocks $\{\eta^i_{u,t}\}$ (dependent on E^i). Using the policy functions (7) this will give us the profile of consumption $c^i_{u,t}$, labour supply $l^i_{u,t}$, financial assets $a^i_{u+1,t+1}$ and human capital $h^i_{u+1,t+1}$ during the working phase:

$$\begin{aligned} c^i_{u,t} &= \mathbf{c}_{u,t}(E^i, \gamma^i, a^i_{u,t}, h^i_{u,t}, \eta^i_{u,t}), \quad & l^i_{u,t} &= \mathbf{l}_{u,t}(E^i, \gamma^i, a^i_{u,t}, h^i_{u,t}, \eta^i_{u,t}) \\ a^i_{u+1,t+1} &= \mathbf{a}^+_{u,t}(E^i, \gamma^i, a^i_{u,t}, h^i_{u,t}, \eta^i_{u,t}), \quad & h^i_{u+1,t+1} &= \mathbf{h}^+_{u,t}(E^i, \gamma^i, a^i_{u,t}, h^i_{u,t}, \eta^i_{u,t}), \end{aligned}$$

whilst study debt is given by:

$$d^i_{u+1,t+1} = \Omega_{u+1,t+1}(E^i)$$

Once individual variables are determined cohort averages can be calculated as follows:

$$\bar{c}_{u,t} \equiv \frac{1}{P_{u,t}} \sum_{i=1}^{P_{u,t}} c^i_{u,t}, \quad \bar{l}_{u,t} \equiv \frac{1}{P_{u,t}} \sum_{i=1}^{P_{u,t}} \eta^i_{u,t} h^i_{u,t} l^i_{u,t}, \quad \bar{a}_{u,t} \equiv \frac{1}{P_{u,t}} \sum_{i=1}^{P_{u,t}} a^i_{u,t}, \quad \bar{d}_{u,t} \equiv \frac{1}{P_{u,t}} \sum_{i=1}^{P_{u,t}} d^i_{u,t},$$

where $\bar{c}_{u,t}$ is average consumption, $\bar{l}_{u,t}$ is average effective labour, $\bar{a}_{u,t}$ is average financial assets, and $\bar{d}_{u,t}$ is average study debt. Population totals are defined as follows:

$$C_t \equiv \sum_{u=M}^{\bar{U}} P_{u,t} \bar{c}_{u,t}, \quad L_t \equiv \sum_{u=M}^{\bar{U}} P_{u,t} \bar{l}_{u,t}, \quad A_t \equiv \sum_{u=M}^{\bar{U}} P_{u,t} \bar{a}_{u,t}, \quad D_t \equiv \sum_{u=M}^{\bar{U}} P_{u,t} \bar{d}_{u,t},$$

where C_t is total consumption, L_t is total effective labour supply, A_t is total financial asset holdings, and D_t is total study debt. Note that – since there is no aggregate uncertainty – cohort averages and population totals (and thus also factor prices) are deterministic quantities.

2.5 Government

Apart from administering the study loan system on a balanced budget basis, the government also collects taxes on consumption, labour income and capital income which it uses to finance (intrinsically useless) public consumption and to fund the system of unemployment benefits. In the interest of clarity we split the governmental accounts into a regular budget and a study loan system budget.

2.5.1 Regular budget

There is an exogenous level of government spending G^0_t. It increases in line with economic growth and population growth:

$$G^0_{t+1} = (1 + n^z)(1 + n^p) G^0_t.$$

Total tax revenue, T_t, is equal to:

$$T_t \equiv \tau_t^c C_t + \tau_t^w w_t L_t + \tau_t^r r_t \Big[A_t - D_t \Big],$$

where we note that the tax deductibility of interest payments on study debt shows up here so the study loan system is not completely separated from the regular budget. Total spending on unemployment benefits, B_t, amounts to:

$$B_t \equiv \sum_{u=M}^{\bar{U}} \sum_{i=1}^{P_{u,t}} \nu_{u,t} \mathbb{1}_{\{\eta_{u,t}^i = 0\}}.$$

We abstract from debt financing so that the balanced budget requirement reduces to:

$$T_t = G_t^0 + B_t.$$

2.5.2 Budget of the educational loan system

Under the SML system, study loans are redeemed by the students themselves and by their very design the repayment schemes already ensure that all debt is paid back. In contrast, if study loans are financed by taxes (the GLT or CLT system) then in every period t tax revenues should cover total borrowing by current students:

$$\sum_{u=M}^{\bar{U}} q_t^0 \sum_{i=1}^{P_{u,t}} \mathbb{1}_{\{E^i > u-M\}} = \sum_{u=M}^{\bar{U}} \sum_{i=1}^{P_{u,t}} \Upsilon_{u,t}\big(E^i, w_t \eta_{u,t}^i h_{u,t}^i l_{u,t}^i\big).$$

2.6 Market clearing

The macroeconomic equilibrium is attained provided the following market clearing conditions are satisfied. First, the goods market equilibrium condition is given by:

$$Y_t = C_t + I_t + G_t^0 + F_t,$$

where F_t is the total amount of tuition fees:

$$F_t = \sum_{u=M}^{\bar{U}} f_t^0 \sum_{i=1}^{P_{u,t}} \mathbb{1}_{\{E^i > u-M\}}.$$

Second, the capital market equilibrium condition states that the productive capital stock is equal to the net stock of assets owned by the household sector:

$$K_t = A_t - D_t.$$

Finally, the labour market equilibrium condition requires equality between demand and supply of effective labour units:

$$N_t = L_t.$$

In the steady state of the model A_t, B_t, C_t, D_t, F_t, G_t^0, I_t, K_t, T_t and Y_t grow at rate $(1+n^z)(1+n^p)-1$, L_t and N_t grow at rate n^p and the wage rate w_t grows at rate n^z. By scaling these variables appropriately they will be constant along the balanced growth path, see Appendix B for more details.

3 Calibration

In this section we present and motivate the calibration of our model. In addition we visualize its main steady-state properties.

3.1 Distributions

We need to specify the distribution of the various stochastic model elements that have been discussed in Section 2.2.1. First, we assume that the talent for education θ follows a truncated normal distribution on $[0,1]$. This combines the convenience of a closed and bounded support with the flexibility of a bell-shaped curve. The distribution is characterized by:

$$F_\theta(x) = \frac{\Phi\left(\frac{x-\mu_\theta}{\sigma_\theta}\right) - \Phi\left(\frac{\mu_\theta}{\sigma_\theta}\right)}{\Phi\left(\frac{1-\mu_\theta}{\sigma_\theta}\right) - \Phi\left(\frac{\mu_\theta}{\sigma_\theta}\right)} \quad \text{for } 0 \leq x \leq 1,$$

where Φ is the cumulative distribution function of the standard normal distribution and μ_θ and σ_θ are the location and scale parameter, respectively. The second stochastic element is the ability to learn on the job γ which can only take two values. We specify the probability of each outcome conditional on θ as:

$$\mathbb{P}(\gamma = \gamma_h|\theta) = 0.5 + \rho_{\gamma\theta}\left[F_\theta(\theta) - 0.5\right]; \quad \mathbb{P}(\gamma = \gamma_l|\theta) = 1 - \mathbb{P}(\gamma = \gamma_h|\theta).$$

If $\rho_{\gamma\theta} > 0$ then there is a positive correlation between γ and θ. By setting $\gamma_l = \mu_\gamma - \sigma_\gamma$ and $\gamma_h = \mu_\gamma + \sigma_\gamma$ we ensure that the unconditional mean and variance are given by $\mathbb{E}[\gamma] = \mu_\gamma$ and $\text{Var}(\gamma) = \sigma_\gamma^2$. Finally we have to specify the transition matrix for the Markov process that governs idiosyncratic labour productivity η. We assume that there is an education-specific probability to enter into 'unemployment' (i.e, $\eta = 0$) denoted by $\pi(E)$. There is a probability κ of returning to $\eta = 1$ in the next period and a probability $1-\kappa$ of remaining unemployed for an additional year. Conditional on being employed (i.e., $\eta > 0$) labour productivity should mimic a log-AR(1) process with autocorrelation ρ_η and a stochastic innovation term with variance σ_ϵ^2. We impose some additional restrictions on transitions between states, see Section 2.2.1. The resulting transition matrix is given by:

$$\Pi(E) = \begin{pmatrix} 1-\kappa & 0 & \kappa & 0 \\ \pi(E) & [1-\pi(E)]\rho_\eta & [1-\pi(E)](1-\rho_\eta) & 0 \\ \pi(E) & [1-\pi(E)]\frac{1-\rho_\eta}{4} & [1-\pi(E)]\frac{1+\rho_\eta}{2} & [1-\pi(E)]\frac{1-\rho_\eta}{4} \\ \pi(E) & 0 & [1-\pi(E)](1-\rho_\eta) & [1-\pi(E)]\rho_\eta \end{pmatrix}.$$

The corresponding states are $\{0, \eta_l, 1, \eta_h\}$ with:

$$\eta_l = e^{-\sqrt{3\sigma_\epsilon^2/(1-\rho_\eta)^2}}, \quad \eta_h = e^{\sqrt{3\sigma_\epsilon^2/(1-\rho_\eta)^2}}.$$

A given entry of $\Pi(E)$ represents the probability of moving from the state corresponding to the row to the one associated with the column. For example, the entry in row 2 and column 3 is $\mathbb{P}(\eta^+ = 1|\eta = \eta_l, E)$. The probability of the first state in the limiting distribution is $\pi(E)/[\kappa + \pi(E)]$ which captures the unemployment rate for a given education level.

3.2 Parameter values

We calibrate the model to fit some key features of the US economy using a two-step procedure. First we assign to a subset of the parameters values that are taken directly from the data or the literature, see Table 1. İmrohoroğlu and Kitao (2009) provide an overview of estimates for the intertemporal substitution elasticity σ and we choose a value within the range they report. The coefficient of relative risk aversion is set in accordance with Cecchetti *et al.* (2000), who find that it is reasonable to have a value between 1 and 5.

Data from the World Bank for 2012 gives a population growth rate of 0.74% for the US. The maximum age is set equal to life expectancy at birth for the same year, rounded to the nearest integer. To obtain an estimate of the long run economic growth rate we collect data on GDP per capita from the Federal Reserve Economic Data of the St. Louis Federal Reserve Bank (measured in 2011 US dollars) and regress its log on a time variable. The resulting coefficient is 0.02 or 2% per year.

The growth rate of wages in the model depends on an individual's ability to learn on the job and is therefore not the same for every person. We take an estimate for the autocorrelation of the labour productivity process ρ_η from Guvenen (2009), who allows for heterogeneity in income growth rates. In order to capture the fact that long-term unemployment (more than one year) is very uncommon we choose a value close to 1 for the recovery rate κ. We set the probability of entering unemployment $\pi(E)$ such that the unemployment rate by education group approximately matches the average over the years 2000 up to and including 2006 as calculated from the March Current Population Survey (CPS). It follows that education offers some insurance against being out of work as more educated individuals are less likely to become unemployed.

We include a simple system of unemployment protection. In the absence of such a social security scheme individuals would work 'too hard' and save 'too much' in the years immediately following graduation compared to the data. As they enter the labour market without any savings but do face the risk of unemployment they have an incentive to accumulate precautionary savings at a quick rate. In addition, if there is no redistribution towards the unemployed in the benchmark case then we are likely to overstate the welfare changes from reforming the educational loan system in such a way that it offers more insurance against low income periods. We assume that all individuals between ages 18 and 60 whose labour productivity in a given year equals zero receive a fixed benefit independent of their employment history. Data from the US Department of Labor indicate that the average replacement ratio (definition 1) in the United States is about 47%. However, since entitlements are typically capped at six months and unemployment lasts for one year in our model we have chosen to set the unemployment benefit equal to 25% of average income in the calibration.

Our modeling of the education phase is very stylized and therefore it is not straightforward to choose parameter values for the annual amount of study loan and the tuition fees. Our main goal is to have a realistic level of student debt. To that end we use the average loan take up of undergraduate and graduate students in 2012 from the National Center for Education Statistics. This gives an amount of $\$11,887$ or approximately 24% of average income in the United States in the same year (which is about $\$50,000$). We set the tuition fee at 40% of this amount to capture the fact that part of the loan cannot be directly consumed. In the United States most types of study loans have no or a very brief grace period, but repayments can be deferred for up to 3 years during periods of unemployment or economic hardship. Although the standard repayment plan for federal loans is 10 years it is possible to arrange an extension up to 30 years. We simplify these provisions somewhat in the model by including a grace period of 4 years for everyone and by setting the redemption period equal to 15 years.

In the second step we calibrate the remaining parameters (Table 2) so as to match certain targets (Table 3). Some of these are quite standard: a capital to output ratio of about 3, an average work week of 40 hours for those that work at least 5,[4] and a net return to capital of 4% per year. We impose that investment and government spending take up 19% and 17% of yearly output, respectively. In addition we normalize the (scaled) return to effective labour to unity. The target for consumption tax revenue relative to output is taken from the OECD tax database.

The remaining targets require some more elaborate discussion. To calculate the education distribution we use information on educational attainment for individuals age 25 and above from the Current Population Survey (CPS) of 2012. We exclude individuals without a high school diploma and group those with some college but no degree with the high school graduates ($E = 0$). An associate degree (whether occupational or academic) corresponds to $E = 2$ while a bachelor's degree is $E = 4$. For individuals with a master's degree or above we set $E = 6$. In the resulting distribution more than half of the population has no tertiary education at all, while most of those that attend college obtain a bachelor's degree.

From Krueger and Ludwig (2013) we take two productivity profiles: one for individuals with no college education and one for individuals with some. These are normalized by the average productivity level of a high school graduate at age 23. We include the productivity at ages 25, 35, 45 and 55 for each profile among our targets. This will help us identify the parameters that govern the accumulation of labour market experience over the life cycle. We make sure that the implied college wage premium, the average hourly wage of individuals with at least 4 years of college education relative to that of individuals who are less educated, is comparable to the one calculated by Heathcote *et al.* (2010) for 2005. Finally, we include two measures of wage uncertainty. The first is the variance of the log of annual labour earnings at age 50 as reported by Storesletten et al. (2004). The second one comes from Guvenen (2009) and captures the variability among individuals in the extent to which wages increase with one

[4] We assume that the unit time endowment of individuals corresponds to about 14 hours a day (excluding sleep and personal care) or 100 hours a week. This means that a 40-hour work week equals 40% of the time endowment.

Table 1: Parameters taken from data or literature

Parameter		Value	Source
Preferences			
Intertemporal substitution elasticity	σ	0.500	İmrohoroğlu and Kitao (2009)
Coefficient of relative risk aversion	ζ	4.000	Cecchetti *et al.* (2000)
Demography			
Age of majority	M	18.000	
Population growth rate	n^p	0.007	WB for 2012
Maximum age	$\bar{U}$	79.000	WB for 2012
Technology			
Economic growth rate	n^z	0.020	FRED for 1970-2006
Wage uncertainty			
Autocorrelation of log productivity	ρ_η	0.821	Guvenen (2009)
Probability exiting unemployment	κ	0.990	
Probability entering unemployment	$\pi(0)$	0.048	March CPS for 2000-2006
Probability entering unemployment	$\pi(2)$	0.035	March CPS for 2000-2006
Probability entering unemployment	$\pi(4)$	0.027	March CPS for 2000-2006
Probability entering unemployment	$\pi(6)$	0.019	March CPS for 2000-2006
Study loans			
Annual loan to average income		0.238	NCES for 2012
Tuition fee as fraction of loan		0.400	
Length of grace period		4.000	
Length of redemption period		15.000	
Government			
Replacement rate unemployment		0.250	

Sources: CPS is the Current Population Survey. FRED is the Federal Reserve Economics Data of the St. Louis Federal Reserve Bank. NCES is the National Center for Education Statistics. WB is the World Bank.

more year of labour market experience. This corresponds to the variance of γl^{α} in our model.

The calibration then proceeds as follows. Parameters that are closely related to a specific target (those representing firm technology and government tax rates) or to which the model solutions are particularly sensitive (those affecting individual preferences) are updated in each iteration. These parameters and the corresponding moments are listed above the dashed line in Table 2 and 3. The parameters below the dashed line are determined using the Method of Simulated Moments (MSM). Let x^0 denote the vector of 15 targeted empirical moments and p the vector of 12 parameter values. For each choice of p we can solve the model and calculate the counterparts of the empirical moments from the simulated data. These are denoted by $x(p)$. Under the null hypothesis that the model has been correctly specified the following moment condition holds for the true parameter vector p^*:

$$\mathbb{E}\left[x(p^*) - x^0\right] = 0.$$

The MSM estimator $\hat{p}$ is then given by:

$$\hat{p} = \text{argmin } \left[x(p) - x^0\right]' W^0 \left[x(p) - x^0\right].$$

where W^0 is a weighting matrix. For W^0 we use the matrix with on the diagonal the inverse of the square of the successive elements in x^0 and zeros elsewhere. This means that effectively we minimize the sum of squared relative deviations of the simulated moments from their targets.

The resulting parameter values are reported in Table 2. We will briefly discuss some of them. As the ratio of government spending and investment to output are fixed and tuition fees are small, it follows that consumption will always constitute around 64% of income. As a consequence, setting the consumption tax at 7% will bring the resulting revenue close to the desired target of 4.35%. Note that we have imposed that the tax rate on wages and interest received and paid should be equal. The resulting uniform income tax rate is around 15%. The parameter values for μ_γ, σ_γ and α imply that for young individuals the return to experience given a 40-hour work week ranges between 4% and 6% depending on the ability to learn on the job. For older individuals these figures decline because of the ageing effect in human capital depreciation. The return to one year of education for the marginal student (the one who is indifferent between no education at all and 2 years of college) is around 8.5% based on our estimates of ξ_1 and ξ_2.

The model does a good job in matching the targeted moments, as can be seen from Table 3. In particular, the model is able to replicate the bimodal distribution of education levels. In Figure 3 we visualize some of the main features of the calibration. Panel (a) depicts the distribution of educational talent θ. It is single-peaked (by design) and features a lot of mass on the left-hand side and a thin tail at the right-hand side. This graph also shows the cutoff points defining the regions for which the different education choices are optimal. For example, individuals whose θ is such that $0 \leq \theta < \hat{\theta}_2 = 0.292$ find it optimal not to enjoy any tertiary education. The other cutoff points are at $\hat{\theta}_4 = 0.384$ and $\hat{\theta}_6 = 0.607$. As was asserted above, the optimal education choice is increasing in educational ability. Figure 3(b) depicts the calibrated

Table 2: Calibrated parameters

Parameter		Value
Preferences		
Time discount factor	β	0.983
Consumption share in felicity	ε	0.304
Technology		
Capital share in production	ϕ	0.227
Technology level	Φ^0	0.952
Capital depreciation rate	δ^k	0.036
Government		
Consumption tax rate	τ_t^c	0.070
Income tax rate	$\tau_t^w = \tau_t^r$	0.150
Education		
Location parameter talent for education	μ_θ	0.032
Scale parameter talent for education	σ_θ	0.402
Linear term in return to education	ξ_1	0.253
Quadratic term in return to education	ξ_2	0.001
Leisure cost of studying	e^0	0.290
Learning ability		
Strength of experience effect	α	0.638
Location parameter learning ability	μ_γ	0.093
Scale parameter learning ability	σ_γ	0.019
Relation talent for education and learning ability	$\rho_{\gamma\theta}$	0.800
Human capital depreciation		
Level parameter human capital depreciation	δ_0	0.981
Curvature parameter human capital depreciation	δ_1	0.053
Wage uncertainty		
Standard deviation of innovation term	σ_ϵ	0.205

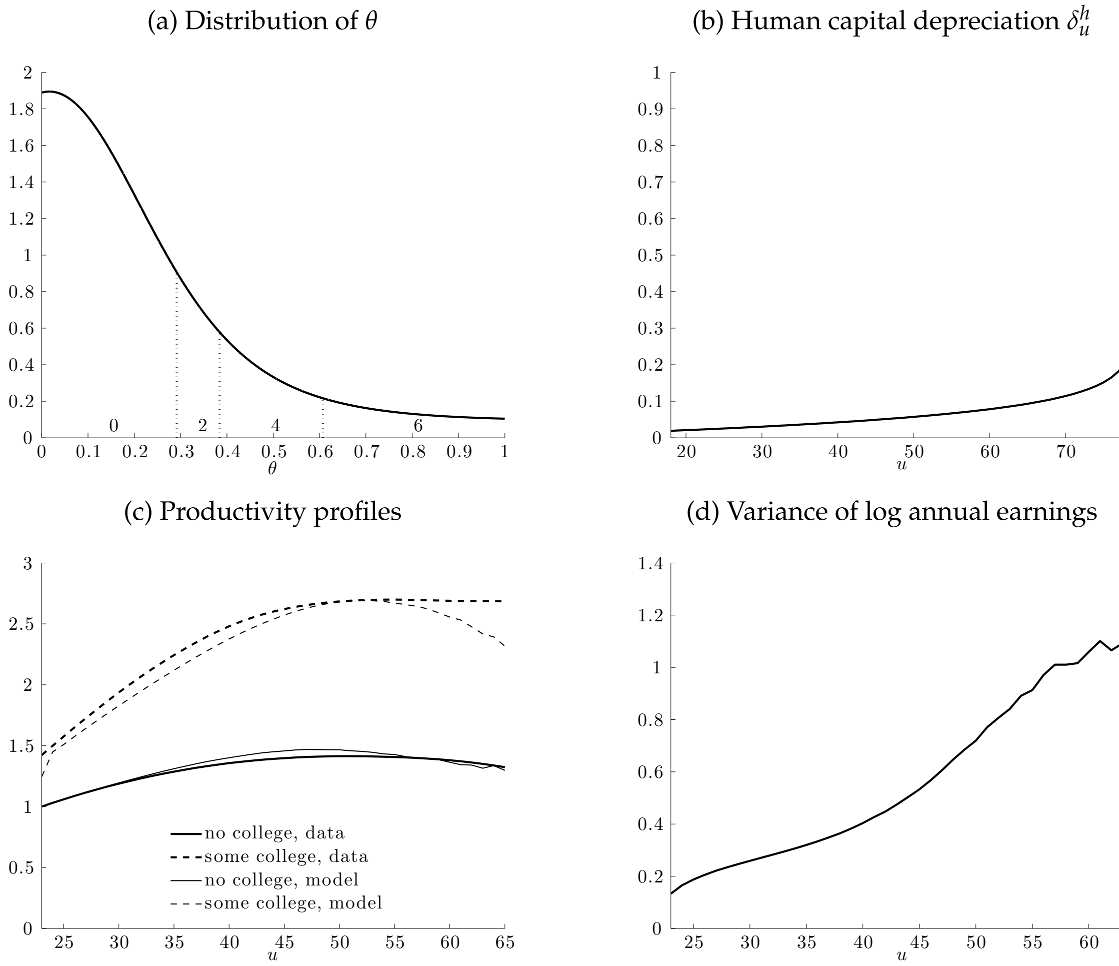
Table 3: Model fit on moments targeted by the calibration

	Model	Target	Source
Factor inputs and prices			
Capital to output	2.983	3.000	
Average hours worked by employed	40.101	40.000	
Net return to capital	0.040	0.040	
Return to effective labour	1.000	1.000	
Investment to output	0.190	0.190	
Government			
Consumption tax revenue to output	4.454	4.350	OECD for 2012
Government spending to output	0.170	0.170	
Education			
Fraction with 0 years	52.020	53.200	March CPS for 2012
Fraction with 2 years	13.120	11.130	March CPS for 2012
Fraction with 4 years	21.810	22.890	March CPS for 2012
Fraction with 6 years	13.050	12.790	March CPS for 2012
Cohort productivity profiles			
Productivity no college age 25	1.059	1.060	Krueger and Ludwig (2013)
Productivity no college age 35	1.311	1.287	Krueger and Ludwig (2013)
Productivity no college age 45	1.457	1.398	Krueger and Ludwig (2013)
Productivity no college age 55	1.427	1.407	Krueger and Ludwig (2013)
Productivity college age 25	1.509	1.576	Krueger and Ludwig (2013)
Productivity college age 35	2.119	2.243	Krueger and Ludwig (2013)
Productivity college age 45	2.572	2.622	Krueger and Ludwig (2013)
Productivity college age 55	2.672	2.700	Krueger and Ludwig (2013)
College wage premium (in %)	77.583	80.000	Heathcote *et al.* (2010)
Wage uncertainty			
Variance in income growth ($\times 10^3$)	0.357	0.380	Guvenen (2009)
Variance of log earnings at age 50	0.720	0.700	Storesletten *et al.* (2004)

Sources: BLS is the Bureau of Labor Statistics of the United States Department of Labor. CPS is the Current Population Survey. OECD is the Organisation for Economic Co-operation and Development.

function for the human capital depreciation function. Just as in Heijdra and Reijnders (2012), depreciation is quite low and virtually constant for a large part of life but shoots up very rapidly later on. Figure 3(c) plots the model-generated and actual productivity age-profiles for two groups of people, namely those without a college education ($E = 0$, solid lines) and those with some college ($E > 0$, dashed lines). As the graph shows, the data are matched quite well for the first group and up to age 55 for the second group. Finally, 3(d) depicts the variance of log annual labour earnings by age. Consistent with the data this variance increases smoothly up to about age 57. For higher ages the pattern becomes more irregular as more and more individuals stop supplying hours to the labour market. The convexity of this profile is consistent with the empirical findings of Guvenen (2009).

Figure 3: Calibration outcomes



3.3 Visualization of the base model

In Figure 4 we visualize some of the main life-cycle features of the calibrated base model. In each case the solid lines depict the cohort averages over the life cycle. In panel (a) we observe that average consumption is generally increasing in age. The jagged pattern at the start of life results from the fact that (a) four different education levels and school-leaving ages are distinguished in the model and (b) consumption is not a choice variable during the schooling period. The figure also shows average consumption for different education levels. Not surprisingly, when all agents have started to work (i.e., for ages $u \geq M + 6$) the level of consumption at a given age is increasing in the education level. Of course, there may be highly educated individuals (say featuring $E = 6$) who have encountered a lot of bad shocks and enjoy a lower consumption level than a lucky individual of the same age who only completed a bachelor's degree ($E = 4$). But the group averages are monotonic in the education level.

In Figure 4(b) the life-cycle labour supply profiles for the cohort average and by educational groups are plotted. For ages $u \geq M + 6$ two key features stand out. First, for each educational group average labour supply is roughly constant until middle age sets in. Second, holding age constant, the group-average labour supply is increasing in the education level.

Figure 4(c) shows that the age profiles of financial assets in the population and by educational group are bell shaped. This is not surprising in view of the fact that all individuals start life without financial assets and – in the absence of a bequest motive – plan to expire with zero assets as well. Since we abstract from mortality risk *all* agents run out of financial assets at the end of their final year of life.

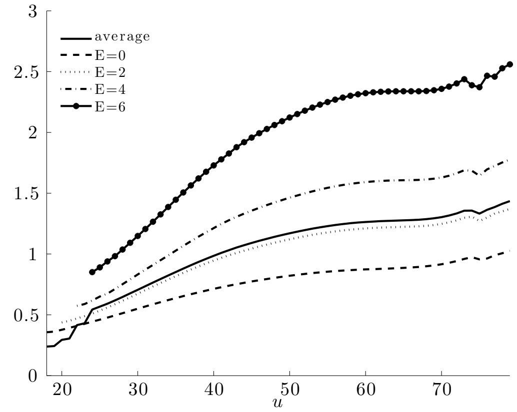
Finally, in Figure 4(d) we plot the age profiles for wage income. Obviously, for $u \geq M + 6$ wage income is increasing in the education level. Wage inequality is quite substantial during middle age. Furthermore, as most people have retired at age 70 wage income is close to zero for all educational groups.

In addition to cohort averages our model also provides quantitative evidence on age-dependent measures of dispersion for the different variables. In Figure 5 we present two commonly used measures of economic inequality, namely the variance of log consumption for the entire cohort (in panel (a)) and the variance in log hours for those who work at least five hours per week (in panel (b)). As is documented by Heathcote *et al.* (2010, pp. 34-35), household consumption inequality rises until about age 50 and flattens out thereafter. The increasing part is clearly evident in our model too but the flattening out occurs late in life and is very mild – see the solid line in panel (a). In the data the age profile for the variance of log hours is U-shaped, i.e. variance is high for young workers (due to high unemployment risk) and for older workers (due to early retirement). In contrast, in our model the profile is J-shaped because the unemployment rate is assumed to be age-independent – see the solid line in panel (b).

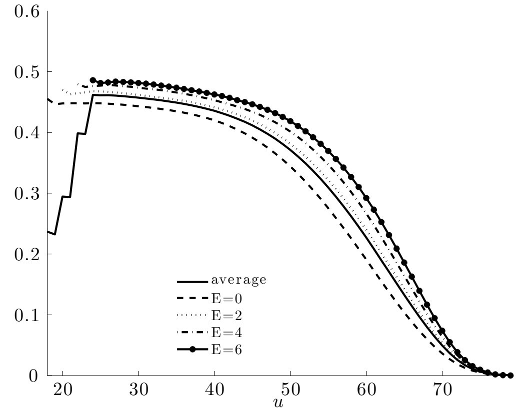
Figure 5 also provides information regarding the composition of the age-dependent variances. Suppose we group individuals by their education level. Panel (a) shows that within-group inequality (dashed line) is about twice as high as between-group inequality (dotted line) for consumption. Interestingly, as is shown in panel (b), the variance of log hours consists almost entirely of within-group inequality. Between-group inequality is tiny. This result follows from the structure of preferences which satisfy the growth-consistency conditions formulated by King *et al.* (2002) thereby ensuring that substitution and income effects on labour supply cancel out.

Figure 4: Age profiles of cohort averages

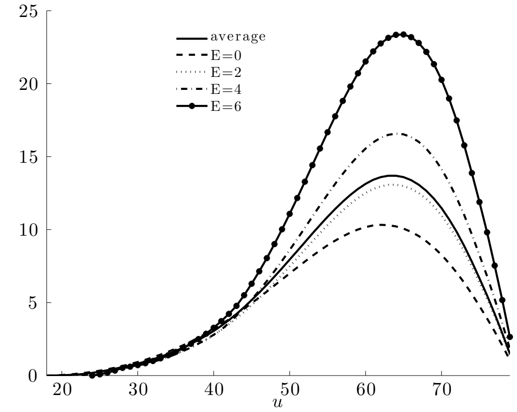
(a) Consumption



(b) Labour supply



(c) Financial assets



(d) Wage income

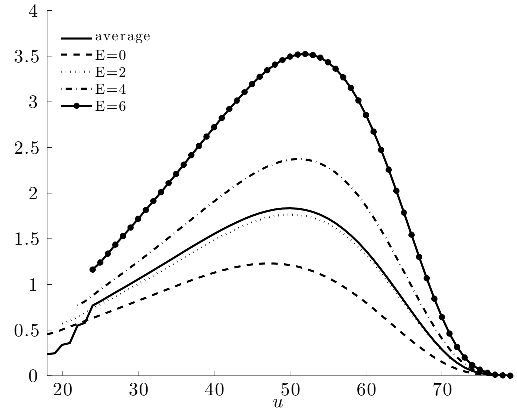
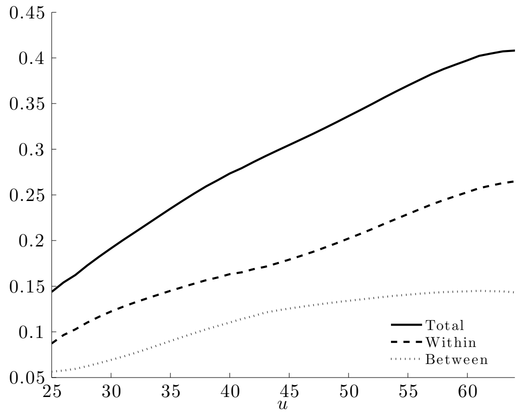
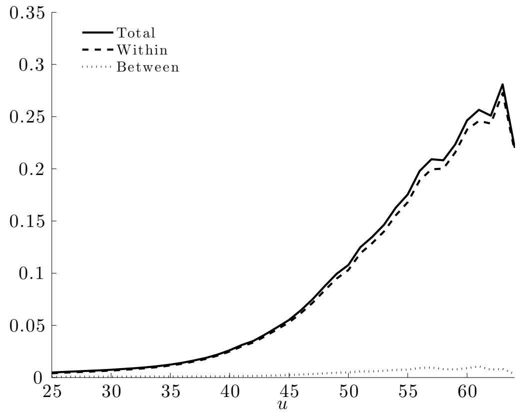


Figure 5: Measures of inequality

(a) Variance of log consumption



(b) Variance of log hours



4 Policy reform 1: From SML to GLT

In this section and the next we study possible policy reforms. Here we consider a change from the SML to the GLT system whilst in section 5 the reform from SML to CLT is discussed. The key difference between the two reforms is that in the latter case all workers – even the uneducated ones – must contribute to the educational loan system. For each case we compute both the transitional and long-run effects of the policy change and we consider both the effects on the economic allocation and on average welfare by pre-reform and post-reform generations. The reforms are initiated in an economy that is in its steady-state equilibrium.

4.1 Educational choices and transitional dynamics

The policy reform is implemented at time $t = 0$ and takes people unawares. Mortgage loans are not available anymore and existing study debt is paid off as was regulated under the old SML system. Existing (and future) students no longer incur an explicit debt but instead they will have to pay the graduate tax after completing their education and finding a job. From time $t = 0$ onward all educated workers are faced with an additional labour income tax τ_t^e which is levied over and above the regular tax τ_t^w. Uneducated workers, however, are exempt from the graduate tax.

Table 4: Steady-state education distribution and critical θ values

	(a) SML	(b) GLT	(c) CLT
0 years	52.02%	52.55%	40.90%
$\hat{\theta}_2$	0.29	0.29	0.22
2 years	13.12%	0.67%	12.84%
$\hat{\theta}_4$	0.38	0.30	0.30
4 years	21.81%	23.10%	23.60%
$\hat{\theta}_6$	0.61	0.48	0.49
6 years	13.05%	23.68%	22.66%

The long-run expansion of the educational sector can be gleaned from Table 4 in which we provide figures on the changing educational composition of the population. In the column labeled SML we report the calibrated distribution, e.g. 52.02% of the adult population is unskilled in the base model. The column labeled GLT records the steady-state result for the GLT scenario. Interestingly, though the proportion of uneducated workers stays roughly constant (a small effect at the extensive margin of zero or some college), there are significant changes in the shares of the different educational groups (a large effect at the intensive margin). With all graduates paying the tax, educated workers in the new steady state consistently have a higher schooling level than they did under the SML system. Indeed, the share of people with associate degrees ($E = 2$) is reduced whilst the shares of the more highly educated groups

($E = 4$ and $E = 6$) both increase.

The macroeconomic effects of the policy reform are illustrated in Figure 6. At the time of the shock, output, consumption, and effective employment drop by, respectively, 0.77%, 1.45%, and 1.00%. In the immediately following period these variables reach their maximum reductions at, respectively, 1.27%, 1.96%, and 1.65%. In the long run, the capital stock increases by 2.72% whilst effective employment rises by 0.23%. Not surprisingly, capital becomes relatively abundant so that its return drops by 0.14 percentage points and the wage rate increases by 0.56%. Finally, consumption rises by 0.30% whilst output increases by 0.79% in the long run. For the sake of convenience, Table 5 reports features of the initial steady state (in column (a)) as well as the long-run effects on the key macroeconomic variables (in column (b)).

Table 5: Long-run macroeconomic effects

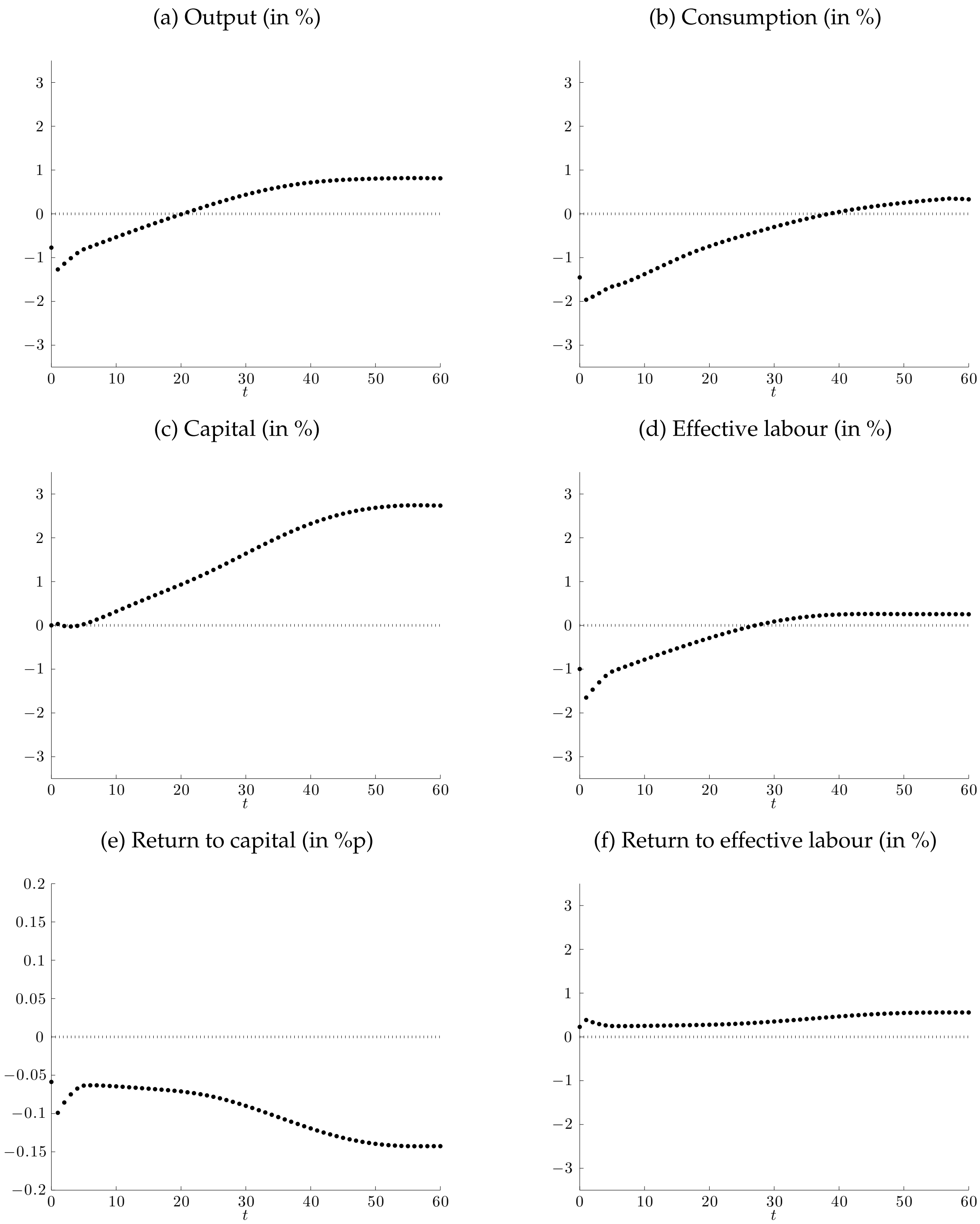
	(a) SML	(b) GLT	(c) CLT
Output	41.33	+ 0.79%	+ 1.03%
Consumption	26.30	+ 0.30%	+ 0.53%
Investment	7.85	+ 2.72%	+ 3.00%
Tuition fees	0.15	+22.30%	+33.39%
Government spending	26.30		
Effective labour	31.94	+ 0.23%	+ 0.46%
Capital	123.29	+ 2.72%	+ 3.00%
Rental rate of effective labour	1.00	+ 0.56%	+ 0.57%
Interest rate (in %p)	4.00	− 0.14%p	− 0.15%p
Income tax rate (in %p)	14.79	− 0.14%p	− 0.21%p
Educational labour tax rate (in %p)	0.00	+ 2.37%p	+ 1.56%p
Annuity payment	0.00	+ 0.08	− 0.29

The fact that there exists a considerable amount of transitional dynamics in the model is not surprising in view of the fact that there are two slow-moving stocks, namely physical and human capital. Macroeconomic convergence is more or less achieved after fifty years. This, of course, prompts the question concerning the welfare effects of the policy reform. Who are the winners and loser of the change in policy given that its effects are time variable?

4.2 Ex-ante welfare effects

In order to get a sense of the magnitude of welfare changes along the transition path we adopt the approach suggested by Fehr and Kindermann (forthcoming). We take factor prices and tax rates in each year as given. For every generation we calculate the transfer they should receive in order to make them, from an *ex-ante perspective*, equally well off under the new policy regime as in the initial steady state. The level of ex-ante welfare is calculated one second before individuals reach the age of majority so that they still face uncertainty about their educational talent θ. Since everyone is identical 'behind the veil of ignorance' this implies that there is only

Figure 6: Transitional changes in the reform from SML to GLT



one transfer needed for every cohort. Importantly, we do not want to provide the transfer at a moment in life when individuals are likely to be borrowing constrained (during the education phase or shortly thereafter). Therefore we impose that an individual cannot receive a transfer before age 27.

For future generations we let $\Lambda_t^j(\theta, I)$ denote the life-time utility of an individual who reaches the age of majority at time t under regime $j \in \{SML, GLT, CLT\}$. This person has learning ability θ and receives a transfer upon reaching age 27, the present value of which equals I at age M. Then we can calculate expected ex-ante welfare as:

$$\Psi_t^j(I) = \mathbb{E}_\theta \left[\Lambda_t^j(\theta, I)^{1-\zeta} \right]^{\frac{1}{1-\zeta}},$$

where we recall that ζ is the degree of relative risk aversion of individuals. The compensating transfer $I_{M,t}^j$ that this cohort should receive is the one that equalizes ex-ante welfare under subsidized mortgage loans to that under the new policy regime j:

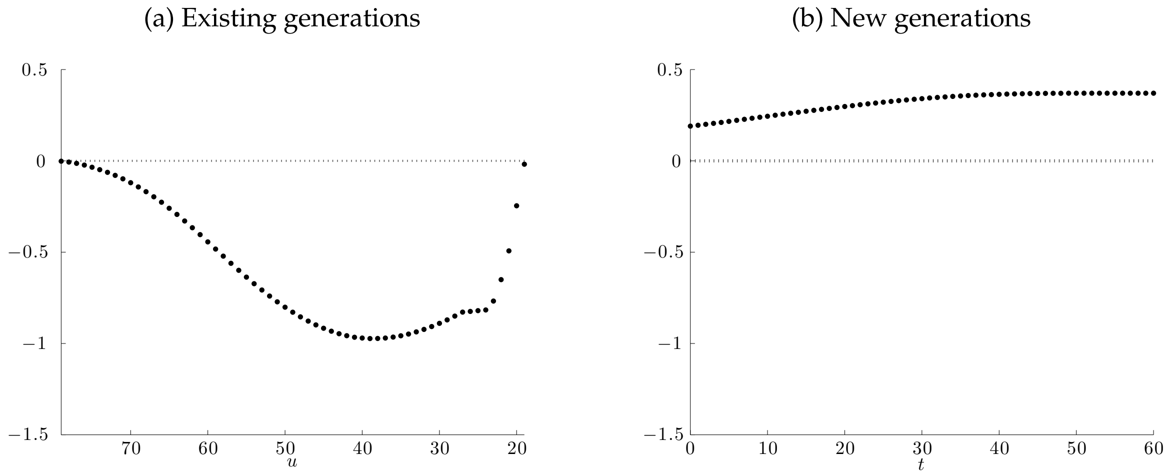
$$\Psi_t^{SML}(0) = \Psi_t^j(I_{M,t}).$$

If $I_{M,t}^j < 0$ (a payment instead of a gift) then the cohort is better off after the policy change. In Figure 7(b) we plot the negative of the transfer (corrected for economic growth) as a percentage of pre-reform aggregate consumption so that a positive number corresponds to a welfare gain. On average all future generations are better off as a result of the policy change.

For existing generations we make a similar calculation. Consider the cohort that is of age u at the time of the policy reform $t = 0$. All decisions that have been made in the past are predetermined and cannot be changed. We calculate the transfer that, when paid out either immediately or at age 27 (whichever comes sooner) makes the individuals in this cohort indifferent between policy regimes in terms of ex-ante welfare (again calculated from the perspective of a second before the age of majority). The present value of this transfer at age u is denoted by $I_{u,0}^j$. In Figure 7(a) we show the negative of the transfer relative to pre-reform aggregate consumption for each existing generation. Interestingly, all existing generations are worse off as a result of the policy change. For educated working-age generations this result follows readily from the fact that they are – in a sense – paying the same bill twice. They must continue to pay off any existing study debt but are also hit by a higher labour-income tax. Students are hurt also, but to a lesser extent the younger they are (and thus the lower is the incurred study debt). Middle-aged and old generations have paid off their study debts and are hurt mainly by the graduate tax.

The results indicate that some generations gain from a policy reform while others are worse off (if uncompensated). To get an aggregate measure of the change in welfare we calculate the present value of the negative of all the transfers using the (constant) interest rate in the initial steady state r for discounting. This will ensure that the weight given to each generation is the same and does not depend on the factor price changes generated by the reform. The

Figure 7: Compensating transfers from SML to GLT



resulting expression is:

$$PV^j = -\left[\sum_{u=M+1}^{\bar{U}} P_{u,0} I^j_{u,0} + \sum_{t=0}^{\infty} \frac{P_{M,t} I^j_{M,t}}{(1+r)^t}\right].$$

In order to facilitate interpretation we convert this present value into an annuity stream. That is, we determine a yearly payment AP^j that is indexed by population growth and economic progress and has the same present value:

$$PV^j = AP^j \sum_{t=0}^{\infty} \left(\frac{(1+n^z)(1+n^p)}{1+r}\right)^t$$

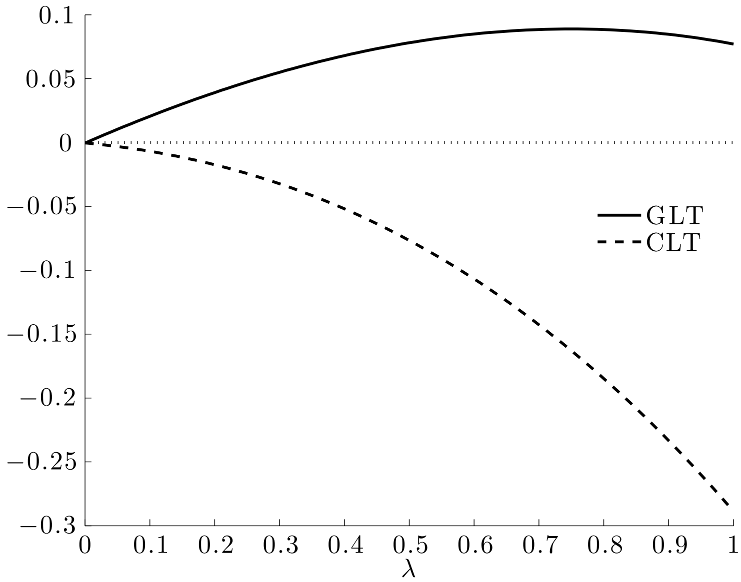
As before we express this compensating change in resources as a percentage of aggregate consumption in the initial steady state. We say that a policy reform leads to an aggregate welfare gain if the compensating annuity payment is positive. As we report in the final row of Table 5 for the graduate labour tax system (column (b)) the welfare gain is equal to 0.077%. Though relatively small, it implies that everybody can be made better off (in an ex-ante sense) if the reform from SML to GLT takes place and generations are appropriately compensated.

Interestingly, as is shown in Figure 8 (solid line), the government could improve aggregate welfare even more under the graduate labour tax system by only partially taking over the student loans. Let $\lambda \in [0,1]$ be the fraction of each student's loans that is paid for out of government tax revenue (either via a graduate labour tax (this section) or a comprehensive labour tax (the next section)) so that $1-\lambda$ has to be financed by subsidized mortgage loans. Under GLT the optimal value of λ is 0.750. A hybrid reform thus outperforms the pure GLT system.

4.3 Ex-post welfare effects by educational type

We are also interested in which individuals gain and lose by the policy reform within a cohort. To that end we want to compare ex-post steady-state welfare between policy regimes,

Figure 8: Aggregate welfare change as a function of λ



conditional on the realization of the talent for education parameter θ. Instead of calculating compensating transfers by θ-type we use an alternative (in this case, simpler) welfare metric, similar to that discussed in Auerbach and Kotlikoff (1987). Suppose that we multiply individual consumption in every year of life by a common factor $1+\omega$, then due to the linear homogeneity of the utility function we would find that the value function of a student becomes $(1+\omega)^{\varepsilon} S_{M,t}(\theta)$. We want to find the value of ω such that an individual of age M in period t is equally well off in the initial steady state equilibrium with subsidized mortgage loans as in the steady state under the policy reform j:

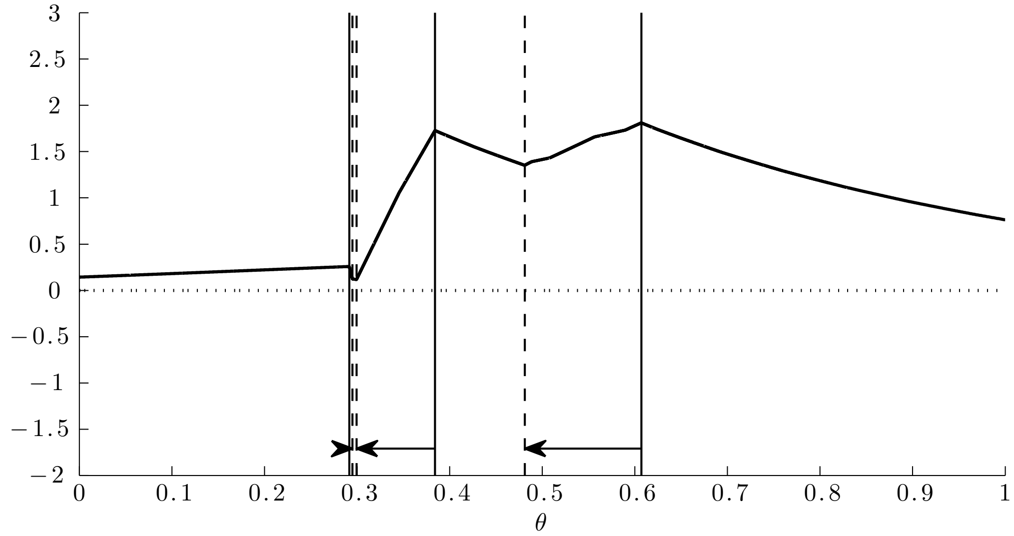
$$(1+\omega)^{\varepsilon} S_{M,t}^{SML}(\theta) = S_{M,t}^{j}(\theta).$$

It follows that:

$$\omega = \left(\frac{S_{M,t}^{j}(\theta)}{S_{M,t}^{SML}(\theta)} \right)^{1/\varepsilon} - 1$$

In Figure 9 we plot ω as a function of θ. The thin solid lines indicate the cutoff values $\hat{\theta}_2$, $\hat{\theta}_4$, and $\hat{\theta}_6$ under the SML system whilst the thin dashed lines plot these values for the GLT system. Several things are worth noting. First, $\hat{\theta}_4$ and $\hat{\theta}_6$ decline substantially whilst $\hat{\theta}_2$ rises marginally. Second, there are kinks at the old and new thresholds, reflecting the fact that the educational choice is a discrete one. Individuals located to each side of a kink differ by two years in schooling attainment in either the initial or new steady state. Third, even though the educational tax is not levied on them, the uneducated are better off under GLT and their welfare gain is increasing in θ. Fourth, the gain to the types that initially chose the maximum amount of education ($E = 6$) is decreasing in θ. Intuitively this is because the incurred study

Figure 9: Change in steady-state welfare from SML to GLT



debt was independent of θ under SML but graduate tax payments are increasing in θ under GLT.

4.4 Decomposition by key mechanisms

In order to highlight the key mechanisms that are operative in the model we decompose the long-run macroeconomic and welfare effects of the policy reform into several parts. To do so we initially shut down some adjustment channels and then open them one by one.

The starting point is the steady state equilibrium featuring subsidized mortgage loans. All long-run changes reported in Table 6 are with respect to this benchmark. We fix the interest rate and wage at their initial level by assuming we have a small open economy instead of a closed one. Any differences between output and domestic absorption are attributed to net exports and the discrepancy between domestic asset holdings and the capital stock determines net foreign asset holdings. In addition we keep the distribution of education levels constant so that each θ type makes the same schooling decision as in the benchmark. Finally we let individuals perceive the educational labour tax as being lump-sum, while in fact it is proportional to their gross labour income. Under these assumptions, the change in allocations and welfare as reported in column (a) can be attributed to a *redistribution effect*. There is an aggregate welfare gain equal to 0.14% of the initial level of aggregate consumption (the compensating annuity payment as explained above). By making contributions to the educational loan system proportional to labour income they automatically fall in periods of low productivity, in contrast to fixed mortgage loan payments. From an ex-ante perspective risk-averse individuals are better off with this kind of risk sharing.

Output, the capital stock, and effective labour all fall by 0.61% whilst consumption declines by 0.46%. The educational labour tax that balances the budget of the study loan system is 1.95% of labour income. Interestingly, despite the fact that the tax bases of the various taxes change, the regular labour income tax stays virtually unchanged.

In column (b) we keep the assumption of a small open economy and a constant education distribution but assume that individuals are aware that the tax they pay to finance the educational loan system is not lump sum but a percentage of their labour income. As a consequence the tax not only has an income effect but also a substitution effect which distorts the labour supply decision. Output, the capital stock, and effective labour all fall by 1.51% and consumption is reduced by 1.67%. The educational labour tax now equals 1.98% of labour income. Even though the increase in the tax wedge on labour resulting from this source appears to be quite small, the disincentive effect is so strong that the regular labour income tax must also be increased by 0.22%p. Aggregate welfare decreases by 0.04% of initial aggregate steady-state consumption and the *work incentive effect* is thus equal to -0.18% (the difference between -0.04 and 0.14).

In column (c) we allow individuals to optimally adjust their education decision which strongly dampens the effects on output, the capital stock, effective labour, and consumption. Indeed, the effects on these variables are very similar in columns (a) and (c). Aggregate welfare increases by 0.05% of initial aggregate steady-state consumption and the *educational incentive effect* is thus equal to 0.09% (the difference between 0.05 and -0.04).

In the final step we reinstate the assumption of a closed economy so that factor prices adjust to changes in domestic demand and supply. The numbers reported in column (d) correspond to those in Table 5 and Table 4 about the steady-state effects of the policy reform on macroeconomic quantities and the education distribution. Aggregate welfare increases by 0.08% of initial aggregate steady-state consumption and the *general equilibrium effect* is thus equal to 0.03% (the difference between 0.08 and 0.05).

We conclude that, in terms of long-run aggregate welfare changes, the work incentive effect is the strongest and negative, followed by a positive redistribution effect and educational incentive effect. The general equilibrium effect is quite small.

5 Policy reform 2: From SML to CLT

The CLT scenario is identical in all respects to the GLT case except for the fact that the educational labour tax is now paid by all workers, even those who have chosen not to pursue any tertiary education ($E = 0$). This case is not of purely theoretical interest only. For example, in the Netherlands all students receive basic grants that are financed in this way. In this section we briefly discuss the key effects of this policy change.

The policy reform from SML to CLT gives rise to transitional and long-run micro- and macroeconomic effects. Interestingly, the long-run educational composition of the labour force differs quite a lot for the two reform scenarios. Indeed, as Table 4 shows, the steady-state proportion of unskilled workers drops from 52.02% to 40.90% in the CLT scenario whereas it hardly changed in the GLT case at all. By confronting all workers with the educational tax even those who previously chose not to take any tertiary schooling now choose to obtain an associate degree, i.e. effects at the extensive margin are substantial. Intuitively, since they cannot avoid paying the educational tax anyway they decide to enter school and reap at

Table 6: Decomposition from SML to GLT

	(a)	(b)	(c)	(d)
Small open economy	yes	yes	yes	no
Fixed education	yes	yes	no	no
Individual lump-sum taxes	yes	no	no	no
Long-run changes in quantities (in %):				
Output	−0.61	−1.51	−0.43	0.79
Consumption	−0.46	−1.67	−0.36	0.30
Effective labour	−0.61	−1.51	−0.43	0.23
Capital	−0.61	−1.51	−0.43	2.72
Net financial assets	5.03	2.98	4.95	2.72
Long-run changes in factor prices:				
Wage (in %)	0.00	0.00	0.00	0.56
Interest rate (in %p)	0.00	0.00	0.00	−0.14
Long-run changes in tax rates (in %p):				
Income tax rate	0.00	0.22	−0.06	−0.14
Educational labour tax rate	1.95	1.98	2.41	2.37
Long-run changes in education (in %p):				
0 years	0.00	0.00	0.62	0.53
2 years	0.00	0.00	−12.74	−12.45
4 years	0.00	0.00	1.11	1.29
6 years	0.00	0.00	11.01	10.63
Aggregate welfare change:				
Annuity payment	0.14	−0.04	0.05	0.08

least some of the benefits of the system in the form of "free" educational grants. Indeed, the education-enhancing effect is quite pervasive, i.e. comparing columns (a) and (c) in Table 4 we observe that the cutoff values $\hat{\theta}_2$, $\hat{\theta}_4$ and $\hat{\theta}_6$ are all reduced substantially.

The long-run macroeconomic effects of the policy change are reported in column (c) of Table 5 whilst the transitional effects are illustrated in Figure 10. At the time of the shock, output, consumption, and effective employment drop by, respectively, 0.90%, 1.64%, and 1.16%. One period later these variables reach their maximum reductions at, respectively, 1.53%, 2.28%, and 1.99%. In the long run, the capital stock and effective employment increases by, respectively, 3.00% and 0.46%, the return on capital falls by 0.15 percentage points and the wage rate increases by 0.57%. Finally, consumption and output increase by, respectively, 0.53% and 1.03% in the long run. Comparing Figures 6 and 10 we note that the macroeconomic effects of the two policy initiatives are very similar.

In Figure 11 we plot the ex-ante welfare effects for existing generations (in panel (a)) and for future generations (in panel (b)). All existing generations, except the very oldest, are worse off. Whereas all new generations benefited from the move to the GLT scenario, in the CLT case the future generations reaching the age of majority close to the time of the shock are worse off. Cohorts that arrive later do gain but their welfare increase is much smaller than under the GLT reform. As is evident from column (c) in Table 5, aggregate welfare falls by 0.29% of initial total consumption under the CLT scenario. Moreover, as is shown in Figure 8 (dashed line), even under a hybrid version of CLT and SML it is never possible to generate a positive welfare change at the aggregate level, i.e. there is an aggregate welfare loss for all values of λ.

The ex-post welfare picture is plotted in Figure 12. The pattern that emerges from that figure is as follows. The lowest-ability types lose out as a result of the policy reform. For those who continue to choose zero schooling this result follows readily from the fact that they are forced to pay the educational tax. For those who switch from zero to two years of schooling the welfare effect is increasing in innate learning ability θ. Intuitively, this is because there is a positive correlation between θ and the ability to learn on the job γ. For all other types the welfare effect is positive.

In Table 7 we present a decomposition of the macroeconomic and welfare effects into a redistribution effect (column (a)), a work incentive effect (column (b)), an educational incentive effect (column (c)), and a general equilibrium effect (column (d)). For the sake of convenience, we report the quantitative realizations for these effect for GLT and CLT in Table 8. Several things are worth noting.

First, whereas the redistribution effect is positive under GLT, it is negative (and relatively large) under the CLT system. There is not only redistribution from individuals with a high productivity draw to those who are less fortunate but also from uneducated individuals to educated ones. Second, the work incentive effect is almost identical for the two cases. Third, the educational incentive effect is positive under both scenarios but much smaller for the GLT case. Finally, the general equilibrium effect is identical for the two scenarios.

Figure 10: Transitional changes in the reform from SML to CLT

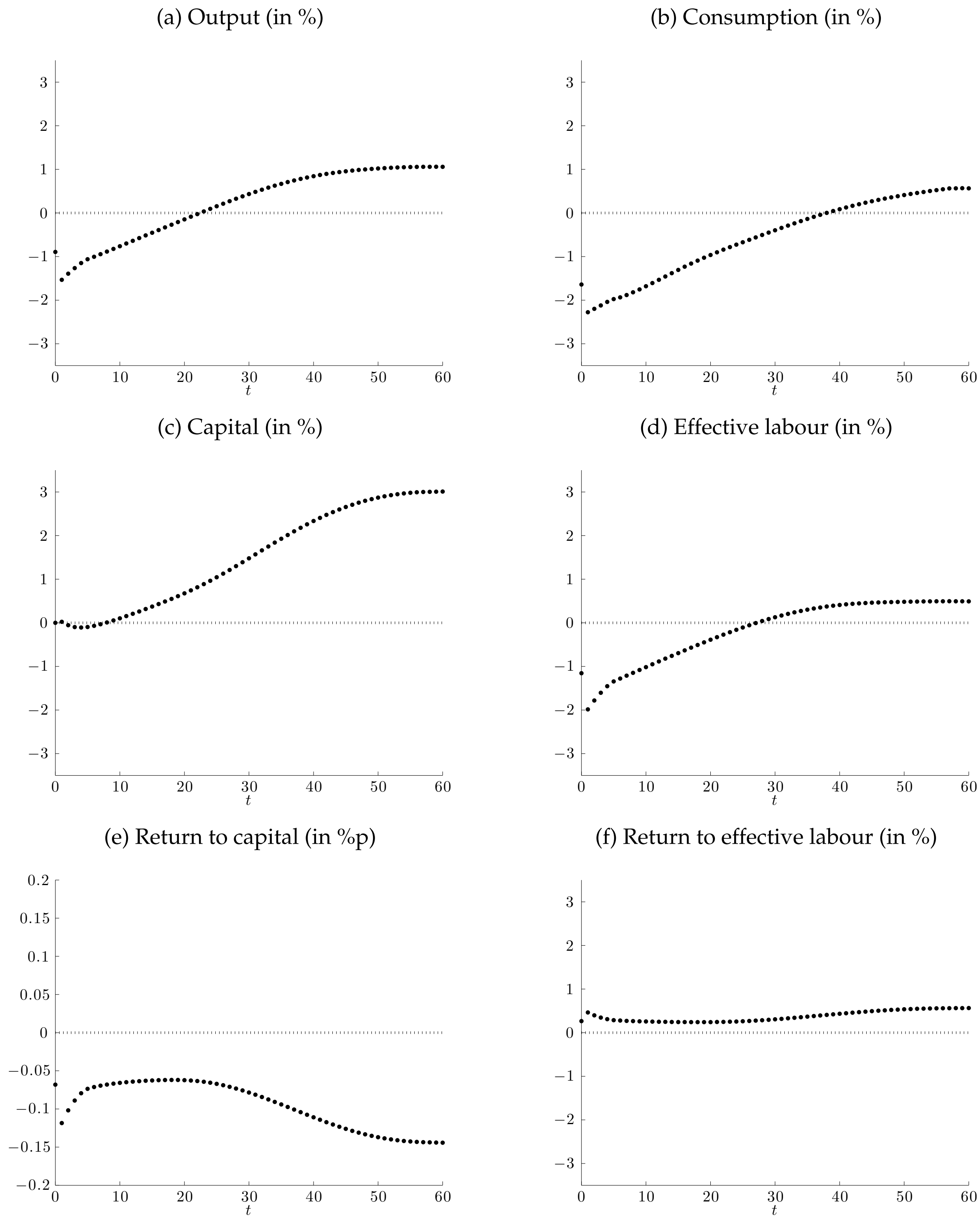


Figure 11: Compensating transfers from SML to CLT

(a) Existing generations | (b) New generations

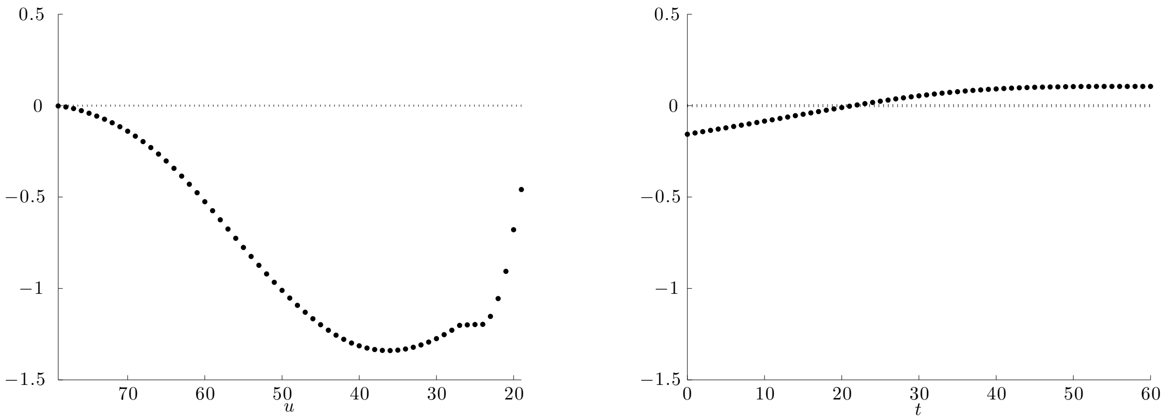


Figure 12: Change in steady-state welfare from SML to CLT

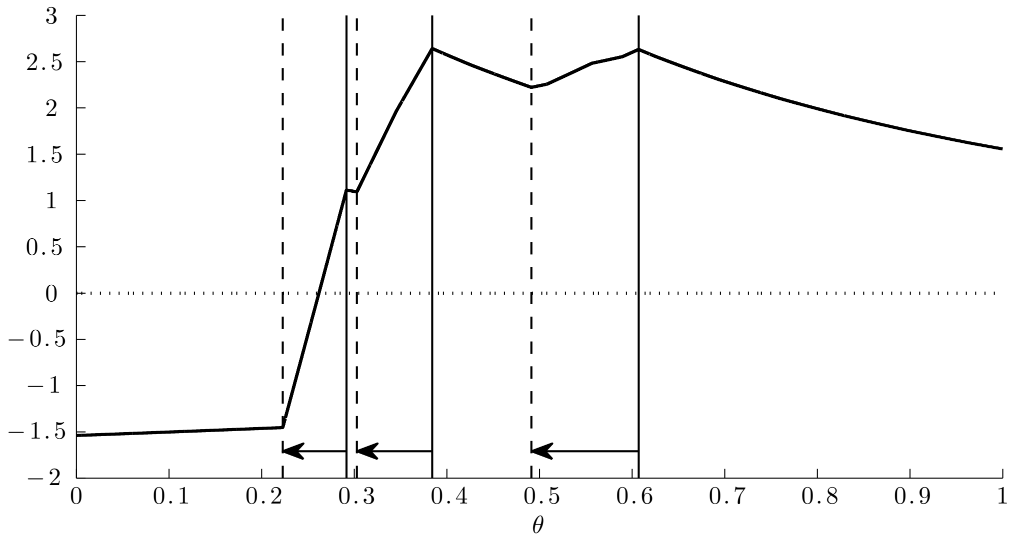


Table 7: Decomposition from SML to CLT

	(a)	(b)	(c)	(d)
Small open economy	yes	yes	yes	no
Fixed education	yes	yes	no	no
Individual lump-sum taxes	yes	no	no	no
Long-run changes in quantities (in %):				
Output	−0.62	−1.52	−0.21	1.03
Consumption	−0.47	−1.68	−0.14	0.53
Effective labour	−0.62	−1.52	−0.21	0.46
Capital	−0.62	−1.52	−0.21	3.00
Net financial assets	5.04	2.86	5.28	3.00
Long-run changes in factor prices:				
Wages (in %)	0.00	0.00	0.00	0.57
Interest rate (in %p)	0.00	0.00	0.00	−0.15
Long-run changes in tax rates (in %p):				
Income tax rate	0.00	0.22	−0.06	−0.21
Educational labour tax rate	1.19	1.20	1.59	1.56
Long-run changes in education (in %p:				
0 years	0.00	0.00	−11.24	−11.12
2 years	0.00	0.00	−0.35	−0.28
4 years	0.00	0.00	1.53	1.79
6 years	0.00	0.00	10.06	9.61
Aggregate welfare change:				
Annuity payment	−0.17	−0.36	−0.32	−0.29

Table 8: Comparing the mechanisms: GLT versus CLT

	(a) GLT	(b) CLT
redistribution effect	0.14	−0.17
work incentive effect	−0.18	−0.19
educational incentive effect	0.09	0.04
general equilibrium effect	0.03	0.03

6 Conclusions

In this paper we conduct a quantitative analysis of a number of stylized educational loan systems. We develop a stochastic general equilibrium model of a closed economy with a competitive firm sector and a government that levies taxes and administers educational loans. Individuals are heterogeneous in their talent for education and ability to learn on the job and face uninsurable idiosyncratic labour productivity risk during their working career.

We calibrate the model to the US mortgage loan system and subsequently consider two possible reforms. The first is a Graduate Labour Tax (GLT) system whereby grants to students are financed by means of a tax on the labour income of educated individuals. We find that in the long run the proportion of uneducated workers stays roughly constant but the average educational attainment of students increases. As there exists a considerable amount of transitional dynamics in the model the welfare effects of the reform differ by generation. Cohorts alive at the time of the shock are worse off while ex-ante welfare of future cohorts increases. The gains to the latter are large enough to – at least in principle – compensate the losers from the policy reform and generate an overall welfare gain.

The second possible reform we study is a Comprehensive Labour Tax (CLT). It is very similar to the GLT except for the fact that the educational tax is levied on all workers, including those who are uneducated. In contrast to the GLT reform the proportion of uneducated workers drops substantially. Generations that become economically active soon after the policy reform are worse off and the aggregate ex-ante welfare effect is negative.

Overall we conclude that Friedman was right and it might be advisable for policy makers in developed countries to consider introducing a graduate tax system to finance educational loans. However, as our analysis of the transitional dynamics shows, appropriate compensation of individuals who have already accumulated study debt is crucial in order to prevent them from paying the same bill twice. As always the devil is in the details.

A Aggregation

We assume that there is no aggregate uncertainty so that for a sufficiently large population each cohort average takes on a deterministic value. Hence, if we are only interested in these kind of *aggregate* statistics, then there is no need to trace the *individual* life-cycle choices.

To calculate the cohort averages we have to determine the distribution of individuals over the state space of the model, which is the set of possible values for each state variable. The relevant state variables are the talent for education $\theta \in \mathcal{Z} = [0,1]$, education $E \in \mathcal{E} = \{0,2,4,6\}$, learning ability $\gamma \in \mathcal{G} = \{\gamma_l, \gamma_h\}$, financial assets $a \in \mathcal{A} = [0,\infty)$, human capital $h \in \mathcal{H} = [0,\infty)$ and labour productivity $\eta \in \mathcal{X} = \{0, \eta_l, 1, \eta_h\}$. The distribution of individuals will be mixed for two reasons. First, some state variables take on a finite number of values while for others there is an uncountable set of possible values. Second, even the state variables with an uncountable domain can have 'mass points' in their marginal distributions. For example, all individuals are born without financial assets so that even though a can take on any non-negative value all mass is concentrated at $a = 0$. Because the distribution is mixed, it is not possible to characterize it by the probability mass at each point of its domain only (as for a discrete distribution). Instead we specify the cumulative distribution function.

Let $\chi_{u,t}$ denote the proportion of individuals of age u that are in the working phase in period t. Given that we know the policy function for the optimal choice of education and the distribution of the talent for education in a given cohort we can deduce:

$$\chi_{u+1,t+1} = \chi_{u,t} + \int_{\mathcal{Z}} \mathbb{1}_{\{\mathbf{E}_{u+1,t+1}(\theta)=u+1-M\}} \, \mathrm{d}F_\theta(\theta),$$

with initial condition $\chi_{M-1,t} = 0$. Let $\Psi_{u,t}(E,\gamma,a,h,\eta)$ denote the cumulative distribution function of workers over the product space $\mathcal{E} \times \mathcal{G} \times \mathcal{A} \times \mathcal{H} \times \mathcal{X}$ for a given age u and time t. As for any probability distribution the total mass is equal to unity:

$$\int_{\mathcal{E}\times\mathcal{G}\times\mathcal{A}\times\mathcal{H}\times\mathcal{X}} \mathrm{d}\Psi_{u,t}(E,\gamma,a,h,\eta) = 1.$$

Every individual who starts working immediately upon entering adulthood ($E = 0$) has no financial assets ($a = 0$), one unit of human capital ($h = 1$) and an average level of productivity ($\eta = 1$). The initial distribution of workers is therefore characterized by:

$$\Psi_{M,t}(E,\gamma,a,h,\eta) = \frac{1}{\chi_{M,t}} \int_{\mathcal{Z}} \mathbb{1}_{\{E\geq 0\}} \times \mathbb{1}_{\{a\geq 0\}} \mathbb{1}_{\{h\geq 1\}} \mathbb{1}_{\{\eta\geq 1\}} \mathbb{1}_{\{\mathbf{E}_{M,t}(\theta)=0\}} F_{\gamma|\theta}(\gamma|\theta) \, \mathrm{d}F_\theta(\theta),$$

where $\chi_{M,t}$ is used as a normalizing constant to ensure that the total mass is indeed equal to unity. The evolution of the distribution over time is given by:

$$\begin{aligned}\Psi_{u+1,t+1}(E,\gamma,a^+,h^+,\eta^+) = \frac{1}{\chi_{u+1,t+1}} \Big\{ \\ \chi_{u,t} \int_{\mathcal{A}\times\mathcal{H}\times\mathcal{X}} \mathbb{1}_{\{\mathbf{a}^+_{u,t}(E,\gamma,a,h,\eta)\leq a^+\}} \mathbb{1}_{\{\mathbf{h}^+_{u,t}(E,\gamma,a,h,\eta)\leq h^+\}} F_{\eta^+|\eta,E}(\eta^+|\eta,E) \, \mathrm{d}\Psi_{u,t}(E,\gamma,a,h,\eta) \\ + \int_{\mathcal{Z}} \mathbb{1}_{\{E\geq u+1-M\}} \mathbb{1}_{\{a^+\geq 0\}} \mathbb{1}_{\{h^+\geq \Gamma(\theta,u+1-M)\}} \mathbb{1}_{\{\eta^+\geq 1\}} \mathbb{1}_{\{\mathbf{E}_{u+1,t+1}(\theta)=u+1-M\}} F_{\gamma|\theta}(\gamma|\theta) \, \mathrm{d}F_\theta(\theta) \Big\}.\end{aligned}$$

The first part captures the mass of individuals that are working in period t. Their education level E and learning ability γ are given and constant over time. Conditional on a specific combination of state variables in the current period the optimal choice of next period's financial assets and human capital are described by the policy functions $\mathbf{a}^+_{u,t}$ and $\mathbf{h}^+_{u,t}$, respectively. The Markov process for labour productivity determines the probability of each possible draw of η^+. The last line captures the entry of current students into the labour market in a similar way as for the initial distribution.

For large cohorts ($P_{u,t} \to \infty$) we find the cohort averages by integrating over the distribution of households just derived. For example:

$$
\begin{aligned}
\bar{c}_{u,t} &= [1-\chi_{u,t}]c^0_t + \chi_{u,t} \int_{\mathcal{E}\times\mathcal{G}\times\mathcal{A}\times\mathcal{H}\times\mathcal{X}} \mathbf{c}_{u,t}(E,\gamma,a,h,\eta)\,\mathrm{d}\Psi_{u,t}(E,\gamma,a,h,\eta),\\
\bar{a}_{u,t} &= \chi_{u,t} \int_{\mathcal{E}\times\mathcal{G}\times\mathcal{A}\times\mathcal{H}\times\mathcal{X}} a\,\mathrm{d}\Psi_{u,t}(E,\gamma,a,h,\eta),\\
\bar{l}_{u,t} &= \chi_{u,t} \int_{\mathcal{E}\times\mathcal{G}\times\mathcal{A}\times\mathcal{H}\times\mathcal{X}} \eta\, h\, \mathbf{l}_{u,t}(E,\gamma,a,h,\eta)\,\mathrm{d}\Psi_{u,t}(E,\gamma,a,h,\eta).
\end{aligned}
$$

In order to actually calculate these values on a computer it is necessary to 'discretize' the state space, see the discussion in Appendix C.

B Scaling

The steady state or balanced growth path of the model has the property that all variables grow at a constant rate. If we know the steady-state growth rate of each variable then we can scale them appropriately such that resulting scaled values are time invariant along the balanced growth path.

B.1 Macroeconomic level

At the macroeconomic level of the economy we have factor prices, policy variables and aggregate quantities. For each we state the growth rate in the steady state and, if different from zero, define the corresponding scaled variable which is distinguished by a tilde.

(1) The interest rate r_t and the tax rates τ^c_t, τ^r_t, τ^w_t and τ^e_t are constant.

(2) The level of consumption during the education phase c^0_t, the tuition fee f^0_t, the annual study loan q^0_t, the wage rate w_t and the unemployment benefit $\nu_{u,t}$ grow at rate n^z.

$$
\tilde{c}^0_t \equiv \frac{c^0_t}{Z_t}, \quad \tilde{f}^0_t \equiv \frac{f^0_t}{Z_t}, \quad \tilde{q}^0_t \equiv \frac{q^0_t}{Z_t}, \quad \tilde{w}_t \equiv \frac{w_t}{Z_t}, \quad \tilde{\nu}_{u,t} \equiv \frac{\nu_{u,t}}{Z_t}.
$$

(3) Total effective labour supply L_t and effective labour demand N_t grow at rate n^p.

$$
\tilde{N}_t \equiv \frac{N_t}{P_{M,t}}, \quad \tilde{L}_t \equiv \frac{L_t}{P_{M,t}}.
$$

(4) Total asset holdings A_t, total unemployment benefits B_t, total consumption C_t, total study debt D_t, total tuition fees F_t, government spending G^0_t, gross investment I_t, the

capital stock K_t, total tax receipts T_t and output Y_t grow at rate $(1+n^z)(1+n^p)-1$.

$$\tilde{A}_t \equiv \frac{A_t}{Z_t P_{M,t}}, \quad \tilde{B}_t \equiv \frac{B_t}{Z_t P_{M,t}}, \quad \tilde{C}_t \equiv \frac{C_t}{Z_t P_{M,t}}, \quad \tilde{D}_t \equiv \frac{D_t}{Z_t P_{M,t}}, \quad \tilde{F}_t \equiv \frac{F_t}{Z_t P_{M,t}},$$
$$\tilde{G}^0_t \equiv \frac{G^0_t}{Z_t P_{M,t}}, \quad \tilde{I}_t \equiv \frac{I_t}{Z_t P_{M,t}}, \quad \tilde{K}_t \equiv \frac{K_t}{Z_t P_{M,t}}, \quad \tilde{T}_t \equiv \frac{T_t}{Z_t P_{M,t}}, \quad \tilde{Y}_t \equiv \frac{Y_t}{Z_t P_{M,t}}.$$

B.2 Microeconomic level

At the microeconomic level we can also apply scaling in order to turn the decision problem of an individual into a stationary one. This means that in the steady state the optimal choices only depend on a individual's age and not on the moment in time at which they are made. This only works if the preference structure satisfies some conditions, see King *et al.* (2002). The problem of a worker can be written as:

$$\hat{V}_{u,t}(E,\gamma,a,h,\eta) = \max_{c,l,a^+,h^+} \Bigg\{ \left[c^{\varepsilon}(1-l)^{1-\varepsilon}\right]^{1-1/\sigma} + \beta\left[\mathbb{E}_{\eta^+|\eta,E}\left[\hat{V}_{u+1,t+1}(E,\gamma,a^+,h^+,\eta^+)^{1-\zeta}\right]\right]^{\frac{1-1/\sigma}{1-\zeta}} \Bigg\}^{\frac{1}{1-1/\sigma}},$$

subject to:

$$\begin{aligned} a^+ &= [1+(1-\tau^r_t)r_t]a + (1-\tau^w_t)\hat{w}_{u,t}\,\eta\, h\, l + \hat{v}_{u,t}\mathbb{1}_{\{\eta=0\}} - \hat{\Upsilon}_{u,t}(E,\hat{w}_{u,t}\,\eta\,h\,l) - (1+\tau^c_t)c, \\ h^+ &= (1-\delta^h_u)[1+\gamma l^{\alpha}]h, \\ &0 \le l \le 1, \quad c \ge 0, \quad a^+ \ge 0, \end{aligned}$$

The growing factor prices and policy variables that appear in the constraints have been scaled by Z_{v+M}, which is the productivity level in the economy at the moment a person born at time v reaches the age of majority M:

$$\begin{aligned} \hat{w}_{u,t} &\equiv \frac{w_t}{Z_{t+M-u}} = \tilde{w}_t(1+n^z)^{u-M}, \\ \hat{v}_{u,t} &\equiv \frac{v_{u,t}}{Z_{t+M-u}} = \tilde{v}_{u,t}(1+n^z)^{u-M}, \\ \hat{\Upsilon}_{u,t}(E,W) &\equiv \frac{\Upsilon_{u,t}(E,WZ_{t+M-u})}{Z_{t+M-u}}. \end{aligned}$$

The solution to this problem gives a new set of policy functions indicated by a hat. The relationship with the unscaled policy functions as used in the main text is as follows:

$$\hat{\mathbf{c}}_{u,t}(E,\gamma,a,h,\eta) \equiv \frac{\mathbf{c}_{u,t}(E,\gamma,aZ_{t+M-u},h,\eta)}{Z_{t+M-u}}, \quad \hat{\mathbf{l}}_{u,t}(E,\gamma,a,h,\eta) \equiv \mathbf{l}_{u,t}(E,\gamma,aZ_{t+M-u},h,\eta),$$
$$\hat{\mathbf{a}}^+_{u,t}(E,\gamma,a,h,\eta) \equiv \frac{\mathbf{a}^+_{u,t}(E,\gamma,aZ_{t+M-u},h,\eta)}{Z_{t+M-u}}, \quad \hat{\mathbf{h}}^+_{u,t}(E,\gamma,a,h,\eta) \equiv \mathbf{h}^+_{u,t}(E,\gamma,aZ_{t+M-u},h,\eta).$$

Note that consumption and future financial assets are scaled because they grow over time, while labour supply and future human capital were already stationary in the original problem.

In order to determine how the new value function relates to the original one we start in the last period of life and write:

$$
\begin{aligned}
\hat{V}_{\bar{U},t}(E,\gamma,a,h,\eta) &= \hat{\mathbf{c}}_{\bar{U},t}(E,\gamma,a,h,\eta)^{\varepsilon}\Big[1-\hat{\mathbf{l}}_{\bar{U},t}(E,\gamma,a,h,\eta)\Big]^{1-\varepsilon} \\
&= \left[\frac{\mathbf{c}_{\bar{U},t}(E,\gamma,aZ_{t+M-\bar{U}},h,\eta)}{Z_{t+M-\bar{U}}}\right]^{\varepsilon}\Big[1-\mathbf{l}_{\bar{U},t}(E,\gamma,aZ_{t+M-\bar{U}},h,\eta)\Big]^{1-\varepsilon} \\
&= \frac{V_{\bar{U},t}(E,\gamma,aZ_{t+M-\bar{U}},h,\eta)}{Z^{\varepsilon}_{t+M-\bar{U}}}.
\end{aligned}
$$

Moving back in time using the recursive formulation of utility we find that this relationship holds in every period.

Similarly we can also scale the problem of a student:

$$
\begin{aligned}
\hat{S}_{u,t}(\theta) = \max_{E\geq u-M}\Bigg[&\sum_{s=t}^{t-u+M+E-1}\beta^{s-t}\Big[\big(\hat{c}^{0}_{u+s-t,s}\big)^{\varepsilon}\big(1-e\big)^{1-\varepsilon}\Big]^{1-1/\sigma} \\
&+\beta^{M+E-u}\Big[\mathbb{E}_{\gamma|\theta}\Big[\hat{V}_{M+E,t-u+M+E}\big(E,\gamma,0,\Gamma(\theta,E),1\big)^{1-\zeta}\Big]\Big]^{\frac{1-1/\sigma}{1-\zeta}}\Bigg]^{\frac{1}{1-1/\sigma}},
\end{aligned}
$$

where:

$$
\hat{c}^{0}_{u,t} \equiv \frac{c^{0}_{t}}{Z_{t+M-u}} = \tilde{c}^{0}_{t}(1+n^{z})^{u-M}.
$$

This gives a policy function $\hat{\mathbf{E}}_{u,t}(\theta) \equiv \mathbf{E}_{u,t}(\theta)$ and value function $\hat{S}_{u,t}(\theta) \equiv S_{u,t}(\theta)/Z^{\varepsilon}_{t+M-u}$.

Stationary choices on the individual level automatically lead to stationary cohort averages:

$$
\hat{\bar{c}}_{u,t} \equiv \frac{\bar{c}_{u,t}}{Z_{t+M-u}}, \quad \hat{\bar{l}}_{u,t} \equiv \bar{l}_{u,t}, \quad \hat{\bar{a}}_{u,t} \equiv \frac{\bar{a}_{u,t}}{Z_{t+M-u}}, \quad \hat{\bar{d}}_{u,t} \equiv \frac{\bar{d}_{u,t}}{Z_{t+M-u}}.
$$

We can then directly compute the scaled aggregate quantities at the macroeconomic level. For example:

$$
\tilde{C}_{t} = \sum_{u=M}^{\bar{U}} \frac{\hat{\bar{c}}_{u,t}}{(1+n^{z})^{u-M}(1+n^{p})^{u-M}}.
$$

C Program

C.1 Program structure

The structure of the computer program used to calculate how the economy moves from an initial steady state to a new one following a policy reform is visualized by means of a flow chart in Figure C.1. We start at the top with a guess for the time path of the tax rates and the factor inputs. The marginal productivity conditions of the firms then imply what the interest rate and return to effective labour should be. Given these prices we can solve for the policy functions and the value function of individuals of any given age in each time period. We can then aggregate across individuals to compute the average consumption, labour supply, financial assets and study debt by cohort. More details about these computations are given in the next section. By summing over all cohorts alive at a given moment in time we obtain the macro aggregates.

The solution has been found if the goods market is in equilibrium in every period. If not, then we update the guesses. The educational tax is set in such a way that tax receipts exactly cover student loans while one of the other tax rates is used to balance the regular government budget. The factor supplies are partially updated using the Gauss-Seidel rule:

$$\tilde{K}_t^{\text{new}} = \varphi \tilde{K}_t^{\text{old}} + (1-\varphi)\left[\tilde{A}_t - \tilde{D}_t\right],$$
$$\tilde{N}_t^{\text{new}} = \varphi \tilde{N}_t^{\text{old}} + (1-\varphi)\tilde{L}_t,$$

where $0 < \varphi < 1$ is a dampening factor. Greater dampening makes the solution algorithm slower but also more stable. Note that if the program converges then the capital market and the labour market clear so that by Walras' Law the goods market should also be in equilibrium.

C.2 Individual choices and aggregation

In this section we provide more detail about the methods used to compute the optimal life-cycle choices of individuals and the cohort averages. This corresponds to the two steps framed by dashed lines in the flow chart of Figure C.1.

C.2.1 Set up

We create a grid $\mathcal{U}$ for adult ages and a grid $\mathcal{T}$ for time periods:

$$u \in \mathcal{U} = \{M, M+1, \ldots, \bar{U}\},$$
$$t \in \mathcal{T} = \{0, 1, \ldots, \bar{T}\}.$$

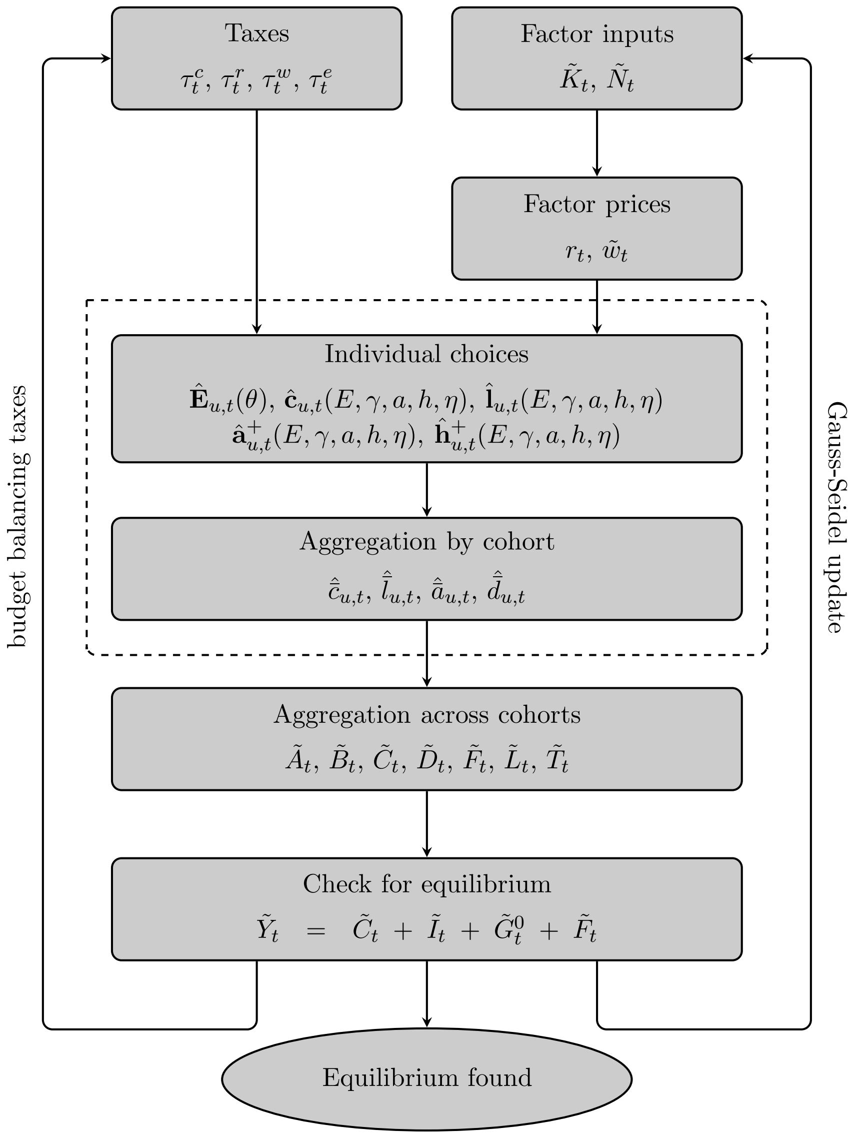
In addition we set up a grid for the discrete variables in the model:

$$\gamma \in \mathcal{G} = \{\gamma_l, \gamma_h\},$$
$$E \in \mathcal{E} = \{0, 2, 4, 6\},$$
$$\eta \in \mathcal{X} = \{0, \eta_l, 1, \eta_h\}.$$

There are two state variables which can theoretically take on a continuum of values, namely financial assets a and human capital h. As the computer cannot handle this unboundedness we have to 'discretize' the set of possible values by setting up a grid with a finite number of elements:

$$a \in \mathcal{A} = \left\{\mathcal{A}^{(j_a)} : j_a = 1, 2, \ldots, n_a\right\},$$
$$h \in \mathcal{H} = \left\{\mathcal{H}^{(j_h)} : j_h = 1, 2, \ldots, n_h\right\}.$$

Figure C.1: Program structure



To improve the accuracy of the computations we let the grid for human capital depend on age. The points on the asset grid are not evenly spaced, instead they are more closely concentrated at low levels.

Finally we introduce a variable m which is uniformly distributed with support $[0,1]$. There is a direct relation between this m and the talent for education θ in the model. Write $\theta = \Theta(m)$ with:

$$\Theta(m) = \Phi^{-1}\left(\Phi\left(\frac{\mu_\theta}{\sigma_\theta}\right) + m\left[\Phi\left(\frac{1-\mu_\theta}{\sigma_\theta}\right) - \Phi\left(\frac{\mu_\theta}{\sigma_\theta}\right)\right]\right),$$

where Φ is the cumulative distribution function of the standard normal distribution. This construction ensures that θ has a truncated normal distribution on $[0,1]$. In order to discretize the variable m we create an equidistant grid on $[0,1]$ with n_m elements:

$$m \in \mathcal{M} = \left\{\mathcal{M}^{(j_m)} : j_m = 1, 2, \ldots, n_m\right\} = \left\{0, \frac{1}{n_m - 1}, \frac{2}{n_m - 1}, \ldots, 1\right\},$$

and assign the same probability mass to each point on the grid:

$$\mathbb{P}(m = z) = \begin{cases} \dfrac{1}{n_m} & \text{if } z \in \mathcal{M} \\ 0 & \text{otherwise} \end{cases}$$

C.2.2 Value function and policy functions in the working phase

We use backward induction to compute the value function and the policy functions of an individual in the working phase. Consider a cohort that reaches the age of majority M at some time t_0. We start at the maximum age $u = \bar{U}$ with corresponding time period $t = t_0 + \bar{U} - M$.[5] At the end of this year the individual will die. Therefore we know that it is optimal to deplete the stock of financial assets ($a^+ = 0$) and that there is no human capital left at the end of the period irrespective of the labour supply decision ($h^+ = 0$, see the depreciation schedule). For a given state vector (E, γ, a, h, η) the individual's problem can be reduced to:

$$\hat{V}_{\bar{U},t}(E, \gamma, a, h, \eta) = \max_{l} \ c^{\varepsilon}(1 - l)^{1-\varepsilon},$$

where:

$$c = \frac{\left[1 + (1 - \tau_t^r) r_t\right] a + (1 - \tau_t^w)\hat{w}_{\bar{U},t}\eta\, h\, l + \hat{v}_{\bar{U},t}\mathbb{1}_{\{\eta=0\}} - \hat{\Upsilon}_{\bar{U},t}(E, \hat{w}_{\bar{U},t}\eta\, h\, l)}{1 + \tau_t^c}.$$

We already know that $\hat{\mathbf{a}}^+(E, \gamma, a, h, \eta) = 0$ and $\hat{\mathbf{h}}^+(E, \gamma, a, h, \eta) = 0$. We use Powell's algorithm to find the l that minimizes the negative of the objective function with an added penalty if one of the control variables is outside its domain. This gives us $\hat{\mathbf{l}}_{\bar{U},t}(E, \gamma, a, h, \eta)$, $\hat{\mathbf{c}}_{\bar{U},t}(E, \gamma, a, h, \eta)$ and the corresponding maximum level of utility $\hat{V}_{\bar{U},t}(E, \gamma, a, h, \eta)$. We repeat the procedure for every state vector in the product space $\mathcal{E} \times \mathcal{G} \times \mathcal{A} \times \mathcal{H} \times \mathcal{X}$.

[5] If $t > \bar{T}$ (the maximum entry in the time grid) then we set $t = \bar{T}$ as we assume that the economy has converged to a (new) steady state in the final period.

The next step is to go one period back to age $u = \bar{U} - 1$ and year $t = t_0 + \bar{U} - M - 1$. For every possible state vector we have to solve the problem:

$$\hat{V}_{u,t}(E, \gamma, a, h, \eta) = \max_{l,a^+} \left[\left[c^\varepsilon (1-l)^{1-\varepsilon} \right]^{1-1/\sigma} + \beta \left[\mathbb{E}_{\eta^+|\eta,E} \left[\hat{V}_{u+1,t+1}(E, \gamma, a^+, h^+, \eta^+)^{1-\zeta} \right] \right]^{\frac{1-1/\sigma}{1-\zeta}} \right]^{\frac{1}{1-1/\sigma}},$$

where:

$$c = \frac{\left[1 + (1-\tau_t^r) r_t\right] a + (1-\tau_t^w) \hat{w}_{u,t} \eta\, h\, l + \hat{v}_{u,t} \mathbb{1}_{\{\eta=0\}} - \hat{\Upsilon}_{u,t}(E, w_{u,t} \eta\, h\, l) - a^+}{1+\tau_t^c},$$
$$h^+ = (1-\delta_u^h)\left[1 + \gamma l^\alpha\right] h.$$

Note that in the previous step we have calculated next period's value function (as $u + 1 = \bar{U}$). Given the realization of the productivity shock in the current period we can calculate for every combination $(E, \gamma, a^+, h^+) \in \mathcal{E} \times \mathcal{G} \times \mathcal{A} \times \mathcal{H}$ the expectation as follows:

$$\mathbb{E}_{\eta^+|\eta,E} \left[\hat{V}_{u+1,t+1}(E, \gamma, a^+, h^+, \eta^+)^{1-\zeta} \right] = \sum_{x \in \mathcal{X}} \mathbb{P}(\eta^+ = x | \eta, E) \hat{V}_{u+1,t+1}(E, \gamma, a^+, h^+, x)^{1-\zeta}.$$

However, we also want to allow for the possibility that the optimal choice of next period's financial assets or human capital is not on the grid. In case $a^+ \notin \mathcal{A}$ or $h^+ \notin \mathcal{H}$ we have to use a linear interpolation method. That is, we determine the index j_a such that $\mathcal{A}^{(j_a)} \leq a^+ \leq \mathcal{A}^{(j_a+1)}$ and the index j_h such that $\mathcal{H}^{(j_h)} \leq h^+ \leq \mathcal{H}^{(j_h+1)}$. Define:

$$\phi_a \equiv \frac{a^+ - \mathcal{A}^{(j_a)}}{\mathcal{A}^{(j_a+1)} - \mathcal{A}^{(j_a)}},$$
$$\phi_h \equiv \frac{h^+ - \mathcal{H}^{(j_h)}}{\mathcal{H}^{(j_h+1)} - \mathcal{H}^{(j_h)}}.$$

The expectation can then be approximated by:

$$\begin{aligned}
&\mathbb{E}_{\eta^+|\eta,E} \left[\hat{V}_{u+1,t+1}\left(E, \gamma, a^+, h^+, \eta^+\right)^{1-\zeta} \right] \\
&\quad \approx (1-\phi_a)(1-\phi_h) \mathbb{E}_{\eta^+|\eta,E} \left[\hat{V}_{u+1,t+1}\left(E, \gamma, \mathcal{A}^{(j_a)}, \mathcal{H}^{(j_h)}, \eta^+\right)^{1-\zeta} \right] \\
&\qquad + \phi_a (1-\phi_h) \mathbb{E}_{\eta^+|\eta,E} \left[\hat{V}_{u+1,t+1}\left(E, \gamma, \mathcal{A}^{(j_a+1)}, \mathcal{H}^{(j_h)}, \eta^+\right)^{1-\zeta} \right] \\
&\qquad + (1-\phi_a) \phi_h \mathbb{E}_{\eta^+|\eta,E} \left[\hat{V}_{u+1,t+1}\left(E, \gamma, \mathcal{A}^{(j_a)}, \mathcal{H}^{(j_h+1)}, \eta^+\right)^{1-\zeta} \right] \\
&\qquad + \phi_a \phi_h \mathbb{E}_{\eta^+|\eta,E} \left[\hat{V}_{u+1,t+1}\left(E, \gamma, \mathcal{A}^{(j_a+1)}, \mathcal{H}^{(j_h+1)}, \eta^+\right)^{1-\zeta} \right].
\end{aligned}$$

By solving the individual's problem for different state vectors we obtain the value function and the policy functions for this age and time period. We continue moving backwards in time until we reach age $u = M$ in period $t = t_0$.

C.2.3 Education choice

Given that we have determined the value functions for individuals in the working phase we can derive the optimal education decisions by means of a grid search. Suppose an individual considers entering the labour market at age $u \in \mathcal{U}$ in period $t \in \mathcal{T}$ so that $E = u - M$. At that moment he will get a draw from the ability to learn on the job γ. For every $m \in \mathcal{M}$ we have a corresponding $\theta = \Theta(m)$ and we can calculate:

$$\mathbb{E}_{\gamma|\theta}\Big[V_{u,t}(E,\gamma,0,\Gamma(\theta,E),1)^{1-\zeta}\Big] = \sum_{x \in \mathcal{G}} \mathbb{P}(\gamma = x|\theta)\hat{V}_{u,t}(E,x,0,\Gamma(\theta,E),1)^{1-\zeta}.$$

We find the optimal E for a given $m \in \mathcal{M}$ by searching over the grid of possible values $\mathcal{E}_u = \{E \in \mathcal{E} : E \geq u - M\}$:

$$\hat{S}_{u,t}(\theta) = \max_{E \in \mathcal{E}_u} \left[\sum_{s=t}^{t-u+M+E-1} \beta^{s-t} \left[\left(\hat{c}^0_{u+s-t,s}\right)^{\phi} \left(1 - e^0\right)^{1-\phi} \right]^{1-1/\sigma} \right.$$
$$\left. + \beta^{M+E-u} \left[\mathbb{E}_{\gamma|\theta} \left[\hat{V}_{M+E,t-u+M+E}\big(E,\gamma,0,\Gamma(\theta,E),1\big)^{1-\zeta} \right] \right]^{\frac{1-1/\sigma}{1-\zeta}} \right]^{\frac{1}{1-1/\sigma}}.$$

Then we create a dummy variable $s_{u,t}(m)$ that is equal to 1 if a person is in school at age u in period t and zero otherwise.

C.2.4 Distribution of individuals in the working phase

As a consequence of discretizing the domain of the continuous state variables the state space now has a finite number of grid points. This means that the distribution of workers over the state space is completely characterized by the 'mass' at every point $(E, \gamma, a, h, \eta) \in \mathcal{E} \times \mathcal{G} \times \mathcal{A} \times \mathcal{H} \times \mathcal{X}$ on the grid. Instead of specifying a cumulative distribution function (as in Appendix A) it is sufficient to derive the corresponding probability density function which is denoted by $\psi_{u,t}$.

First we can determine for every age u and time period t the fraction of individuals in the cohort that are in the working phase:

$$\chi_{u,t} = \sum_{z \in \mathcal{M}} \mathbb{1}_{\{s_{u,t}(z)=0\}} \mathbb{P}(m = z).$$

This will be used as a normalizing constant to ensure that the total mass adds up to unity.

To find the probability density function we use forward iteration. Consider a cohort that reaches the age of majority M at some time $t = t_0$. Since everyone starts with $a = 0$, $h = 1$ and $\eta = 1$ the initial distribution of individuals in the working phase is given by:

$$\psi_{M,t}(0,x,0,1,1) = \frac{1}{\chi_{u,t}} \sum_{z \in \mathcal{M}} \mathbb{1}_{\{s_{M,t}(z)=0\}} \mathbb{P}(\gamma = x|\theta = \Theta(z)) \mathbb{P}(m = z),$$

for $x \in \mathcal{G}$. The probability density function is zero everywhere else.

We move one period forward so that the cohort is of age $u = M + 1$ in the year $t = t_0 + 1$.

The aim is to determine $\psi_{u,t}$ for any $(E,\gamma,a,h,\eta) \in \mathcal{E} \times \mathcal{G} \times \mathcal{A} \times \mathcal{H} \times \mathcal{X}$. First of all, there will be a group of former students that enter the labour market. To the point $(E,x,0,\Gamma(\Theta(z),1),1)$ we add:

$$\frac{1}{\chi_{u,t}} \mathbb{1}_{\{s_{u-1,t-1}(z)=1\}} \mathbb{1}_{\{s_{u,t}(z)=0\}} \mathbb{P}(\gamma = x|\theta = \Theta(z))\mathbb{P}(m = z),$$

for $x \in \mathcal{G}$ and $z \in \mathcal{M}$. In addition there are those who were already in the working phase. Consider a point $(E,\gamma,a,h,\eta) \in \mathcal{E} \times \mathcal{G} \times \mathcal{A} \times \mathcal{H} \times \mathcal{X}$ on the grid with a certain mass $\psi_{u-1,t-1}(E,\gamma,a,h,\eta)$ in the previous period. We want to determine where on the grid this mass ends up in the current period. To that end we find the optimal choices of a^+ and h^+ using the policy functions:

$$a^+ = \hat{\mathbf{a}}^+_{u-1,t-1}(E,\gamma,a,h,\eta), \quad h^+ = \hat{\mathbf{h}}^+_{u-1,t-1}(E,\gamma,a,h,\eta).$$

If $a^+ \in \mathcal{A}$ and $h^+ \in \mathcal{H}$ then we can immediately allocate the mass onto the grid. If not, then we use a linear interpolation method interpolation to distribute it over neighbouring points, see Figure C.2. The weights ϕ_a and ϕ_h are determined as described above such that the *average* amount of financial assets and stock of human capital are still correct:

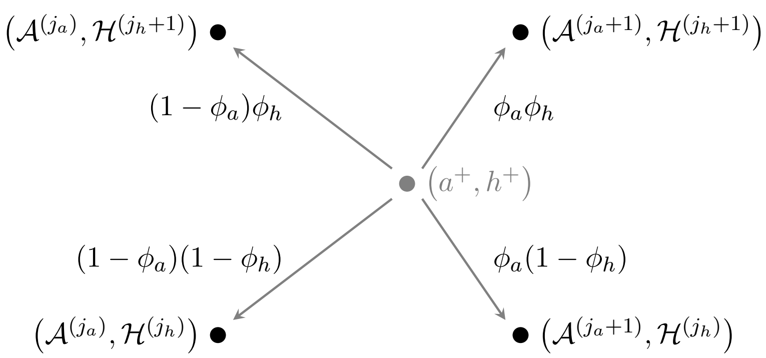
$$a^+ = (1-\phi_a)\mathcal{A}^{(j_a)} + \phi_a \mathcal{A}^{(j_a+1)},$$
$$h^+ = (1-\phi_h)\mathcal{H}^{(j_h)} + \phi_h \mathcal{H}^{(j_h+1)}.$$

For example, to the point $(E,\gamma,\mathcal{A}^{(j_a)},\mathcal{H}^{(j_h)},x)$ we add:

$$\frac{\chi_{u-1,t-1}}{\chi_{u,t}}(1-\phi_a)(1-\phi_h)\mathbb{P}(\eta^+ = x|\eta,E)\psi_{u-1,t-1}(E,\gamma,a,h,\eta),$$

for $x \in \mathcal{X}$.

Figure C.2: Distributing mass over grid points



C.2.5 Cohort averages

Knowing the policy functions and the distribution of workers over the state space is sufficient to calculate the cohort averages. For example:

$$\hat{\bar{c}}_{u,t} = (1-\chi_{u,t})\hat{c}^0_{u,t} + \chi_{u,t} \sum_{E\in\mathcal{E}} \sum_{\gamma\in\mathcal{G}} \sum_{a\in\mathcal{A}} \sum_{h\in\mathcal{H}} \sum_{\eta\in\mathcal{X}} \hat{\mathbf{c}}_{u,t}(E,\gamma,a,h,\eta)\, \psi_{u,t}(E,\gamma,a,h,\eta),$$

$$\hat{\bar{a}}_{u,t} = \chi_{u,t} \sum_{E\in\mathcal{E}} \sum_{\gamma\in\mathcal{G}} \sum_{a\in\mathcal{A}} \sum_{h\in\mathcal{H}} \sum_{\eta\in\mathcal{X}} a\, \psi_{u,t}(E,\gamma,a,h,\eta).$$

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